

Odor to Sensor Space Transformations in Biological and Artificial Noses

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ABSTRACT

A geometric interpretation of population coding is described which gives an intuitive understanding of how individual sensor/receptor “tunings” define the overall performance of a sensory system based upon such a code. The approach is demonstrated by applying it to biologically realistic, and synthetic olfactory systems. The analytical measures developed as part of this treatment may be widely to optimize the detection performance of artificial “electronic nose” systems as well as for comparing different tuning scenarios in the biological olfactory pathway. The examples show clearly how a population coded sensory system can deliver both extreme selectivity to compounds of interest without comprising the range of stimuli that may be perceived.

Keywords - olfaction, artificial nose, population coding

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1. Introduction

Population coding describes one method that nature uses within sensory systems to encode information about a stimulus. A well known, and often cited, example is the cricket cercal system which is used for sensing wind direction. [1] Another example of population coding, but which is less understood, is the mammalian olfactory system. Even so, it is clear that olfaction relies upon vast amounts of sensory input from a large population of Olfactory Receptor Neuron (ORN) types that have overlapping specificities to different odor stimuli (between 300–1,000 in mammals). [2] The response from a sizeable proportion of the receptor population as a whole is required in order to make an accurate decision about the odor stimulus.

[Place Fig. 1.1 near here]

Artificial olfactory systems, or so-called “electronic nose” technology, also provides an interesting example of population coding. [3] Electronic nose systems rely upon the response of an array of broadly-tuned chemical sensors in order to generate a “fingerprint” of the odor stimulus, in a similar fashion to the first stages of the olfactory pathway in mammals. Only through the distributed, population-based, response of the array can the system classify stimuli according to a set of odor descriptors. The components of such an instrument are shown in Fig. 1.1, where a mix of volatile chemical compounds each of concentration, x_i , interacts with a chemical sensor array to generate a response vector, \mathbf{S} . The task for a pattern recognition engine is then to classify these array response vectors under one or more of the odor descriptors using a database of *a priori* sensor responses to class-labeled stimuli.

This paper considers the development of a simple analytical model of a sensor population. This model may be applied equally to biological and artificial sensory systems that are based upon population coding, and can be used to describe and visualize how detection performance is defined in these systems. The model may be used to define the performance of chemical sensor arrays and so provide the means to specify, and the foundation to optimize, electronic nose systems to a specific problem domain. The same model may also be of use in understanding how this “tuning” of biological receptors limits the

overall detection performance for a sensory system. How the model may be applied to both cases is demonstrated by exploring two important examples.

2. Linear Transformations

We begin by considering a linear model of the stimulus dependence for a *generalized sensor*. This term shall be used hereafter to refer to either a receptor neuron in the central nervous system or a man-made transducer in an artificial sensory system. The simplest description of the sensor response is given by,

$$S = a_0 + a_1x_1 + a_2x_2 + \dots + a_jx_j + \dots + a_nx_n, \quad (2.1)$$

where x_j gives the intensity of some stimulus j , and a_j defines the sensitivity of the sensor to the same stimulus. The response, S , may take the form of an analog electrical signal as is usual for many man-made chemical transducers, or perhaps a spike rate-code generated by a sensory neuron. For a generalized chemical sensor, x_j would be defined in terms of concentration to a single chemical compound, j . The term a_0 gives the sensor response when no stimulus is present, often referred to as the “baseline” for a transducer or the spontaneous firing rate for our rate-coded receptor neuron.

A generalized sensor population may be modeled as comprising m such sensors, each with potentially different sensitivity terms, a_{ij} . This linear model is convenient for analysis since we may apply linear algebra to represent the population as

$$\begin{pmatrix} S_1 \\ S_2 \\ \vdots \\ S_m \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} + \begin{pmatrix} a_{10} \\ a_{20} \\ \vdots \\ a_{m0} \end{pmatrix}, \quad (2.2a)$$

or simply

$$\mathbf{S} = \mathbf{A}\mathbf{X} + \mathbf{A}_0, \quad (2.2b)$$

where \mathbf{A} will be termed the *sensitivity matrix* and \mathbf{A}_0 the *residual baseline vector* for the population. Using this simplified view we may consider the population of sensors as carrying out a linear geometric transformation between stimulus space, \mathbf{X} , and sensor space, \mathbf{S} . For an olfactory system the stimulus space would comprise n distinct simple odor components (*i.e.* single chemical compounds) existing within a mixture. Within

this representation we can uniquely define any linear combination of stimuli and with it a specific population response. In terms of the nature of the transformation, the residual baseline vector, \mathbf{A}_0 , is of little interest since it has no effect on the population response to changing stimuli – it acts only as an offset term. Consequently, we will not consider \mathbf{A}_0 any further in this analysis. On the other hand, the sensitivity matrix, \mathbf{A} , uniquely determines the population response to changes in the stimulus and so is of fundamental importance to our analysis.

[Place Fig. 2.1 near here]

We can visualize the action of the sensor population directly by considering a trivialized example of a 2-odor to 2-sensor transformation for a variety of sensitivity matrices, as shown in Fig. 2.1. Clearly the sensitivity matrix has a profound effect on the nature of the transformation. For perfectly orthogonal sensors, Fig. 2.1(a), no transformation occurs from the domain onto the range and so it preserves the area of the original odor space – in other words the transformation is isometric. However, as the orthogonality of the individual sensor sensitivities decreases, as shown in Fig. 2.1(b) and (c), the range collapses so as to restrict the possible population response to the point where the sensors are identical, Fig. 2.1(d), and all points within the domain are mapped onto a single line in the range. Clearly such an array would be unable to distinguish between the two stimulus qualities (in this case odor components), but would be able to provide an estimate of the combined stimulus intensity.

From these observations we can define an important performance parameter for a sensory population, the *hypervolume of accessible sensor space*, V_s , which in each example is equal to the area spanned by the range. Clearly in this example, V_s is in some sense related to the orthogonality of the two sensors, which is usually measured by a vector product or matrix determinant. Consequently, as odor stimuli cover a defined volume in the domain, which for the case of olfaction we term the *hypervolume of accessible odor space*, V_o , then in the 2-odor to 2-sensor example we have

$$V_s = V_o \text{abs}(|\mathbf{A}|), \quad (2.3)$$

where the absolute value must be taken since the determinant gives the “oriented volume”. One difficulty with (2.3) as a performance measure is that the determinant is only defined

for a square matrix, restricting its application to sensory systems where the number of separate stimulus components, n , is equal to the number of sensors, m , within the population. Consequently, we need to generalize the measure defined by (2.3) for a transformation of arbitrary dimensionality that may be carried out by a sensor population. Details are given elsewhere on the general result for the case where \mathbf{A} is non-square [4]

$$V_s = V_o \sum_{i=1}^n \text{abs}(M_i) \quad \text{for } n > m, \quad (2.4)$$

where M_i is a minor of order m that is obtained by taking the columns $(i, i+1, \dots, i+m-1)$ from \mathbf{A} . We can apply (2.4) to any population coded sensory system whose sensor elements behave linearly, whether biological or synthetic, as an important performance metric since the larger the transformed sensor space volume, V_s , the easier it will be to discriminate different stimuli in the presence of noise. So V_s provides an important measure of how easy it is to discriminate each stimulus component.

3. Noise Considerations

The performance of an orthogonal sensor population (one in which there is no correlation in response between any of the sensor elements, for example two distinct pheromone receptors) is simple to characterize. By simply measuring the detection limit of each sensor individually, the performance of the system is defined. The case for cross-sensitive sensors is less straightforward since the overall sensitivity of the population to an individual compound arises from the combined sensitivity of a number of sensors. Consequently, it is necessary to understand how these individual sensitivities contribute to the overall detection performance of the sensory system in the face of noise.

[Place Fig. 3.1 near here]

So far we have considered the sensor population to be noiseless, that is, there is a perfect correspondence between points within stimulus space and points within sensor space. However, any real-world sensor will generate a non-reproducible response to the stimulus. Instead of their being perfect correspondence between stimulus and sensor space, we must now view the noise process as mapping single points in the stimulus space onto a region (usually small) in sensor space where the likelihood of obtaining a particular measurement is determined by some probability density function (see Fig. 3.1). When

the noise process is introduced into the transformation, the magnitude of V_s becomes of great importance, since it restricts the total number of discriminable features for a given statistical significance. Where the noise in each sensor is considered to be Gaussian in nature and is independent of the stimulus, the confidence interval is defined as an m -dimensional hyperellipsoid, where the cross-sections along the principal axes are given by, $\delta S_i = \sigma_i^2$, the variance for each sensor i , as shown in Fig. 3.1. Since each ellipse corresponds to a single stimulus point in the domain, the number of ellipses that may be packed into the region V_s gives what we will term the *Number of Discriminable Odor Features*, N_o . This defines the theoretical limit of the number of features which can be discriminated by the array on the average, and is given by

$$N_o \geq \frac{V_s}{\prod_{i=1}^m \delta S_i}. \quad (3.1)$$

The value obtained from Eq. (3.1) is an estimate since it assumes non-optimal packing of error volume in sensor space (regular x - y packing), but provides an approximate lower bound for N_o . By using the formulæ given for V_s in the non-orthogonal linear case, (2.4), and non-linear case, (4.2), it is possible to produce an estimate for N_o for any sensor population. This measure provides a theoretical limit to the number of separate features that can be discriminated by a sensor population within the stimulus space and under ideal operating conditions – that is, when the noise in the rest of the sensory system and the stimulus is negligible. Of course, when other noise sources within the sensory system dominate, the value obtained for N_o will not hold for the system as a whole, only for the sensor population. As such, this measure may be adopted for use in designing electronic nose sensor arrays, and also provides an estimate of the feature capacity for a hypothetical receptor population within a biological sensory system.

4. Non-linear Odor Space Transformations

Since only a small subset of chemical sensors can be considered to behave linearly up to some operating limit, it is necessary to extend the methods developed in Sections 2 & 3 so that they may be applied more generally.

[Place Table 4.1 near here]

The concentration dependences of the most popular chemical transducers to be used within practical electronic nose systems are shown in Table 4.1, alongside that for a ORN. For these, and other non-linear sensors, a locally linearized sensitivity matrix may be formed by using the total derivative, \mathcal{D}

$$\mathcal{D} = \left(\begin{array}{cccc} \frac{\partial S_1}{\partial x_1} & \frac{\partial S_1}{\partial x_2} & \cdots & \frac{\partial S_1}{\partial x_n} \\ \frac{\partial S_2}{\partial x_1} & \frac{\partial S_2}{\partial x_2} & \cdots & \frac{\partial S_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial S_m}{\partial x_1} & \frac{\partial S_m}{\partial x_2} & \cdots & \frac{\partial S_m}{\partial x_n} \end{array} \right) \bigg|_{x_1, x_2, \dots, x_n} \quad (4.1)$$

for some operating point (x_1, x_2, \dots, x_n) in odor space. This linearized sensitivity matrix may then be used in place of \mathbf{A} , defined by (2.2), so that the linear methods already described may also be applied in the general non-linear case. The determinant of the total derivative, termed the Jacobian, may then be used to approximate the localized hypervolume for the transformation at a particular operating point. This may then be used in a similar way to the determinant measures defined for the linear case, as a local approximation to transformed hypervolume. If the noise properties of the sensors within the population are independent of the stimulus, then the value for \mathcal{J} can be considered as a form of signal-to-noise ratio for the task of discriminating two infinitesimally close stimulus features.

Furthermore, the Jacobian may also be applied to calculate the total hypervolume of accessible sensor space in the non-linear case, since

$$V_s = \iiint \dots \int \mathcal{J} dx_1 dx_2 \dots dx_n. \quad (4.2)$$

Clearly for large sensor populations (4.2) will be difficult to solve analytically, but numerical methods may then be used to provide a quantitative result for V_s for use in (3.1).

[Place Fig. 4.2 near here][Place Fig. 4.1 near here]

Examples of calculations for the noiseless non-linear case are shown for ORNs in Fig. 4.1 and for conducting polymer sensors in Fig. 4.2, using the sensor models summarized in Table 4.1, (4.1), and (4.2). Both examples show the mapping onto 2-sensor space as being non-linear. However, the ORN model is capable of generating sensor space topologies with a high degree of non-linearity, principally due to this comprising two independent parameters, a slope or gain term, a_{ij} , which defines the sensitivity of the receptor and an offset parameter, b_{ij} , which defines the threshold for the receptor. By adjusting these parameters it is possible to generate sensor space topologies capable of marked expansion and contraction of different regions of the space, as can clearly be seen from Fig. 4.1. This results in extreme sensitivity to some regions of the stimulus space and virtually none in others. It would be straightforward to optimize the gain and offset parameters for a sensory system employing such receptors, in order to maximally separate salient odor stimuli whilst also suppressing interfering compounds. Nature may well have optimized these parameters within a receptor population in order to achieve maximum benefit to the animal. A similar approach can be taken to the optimization of artificial olfactory systems, although the number of parameters is more restricted and we may not be able to design a sensor with the exact parameter requirements.

5. Conclusions

Odor stimulus detection performance and range of stimulus appreciation are central to the capabilities of both biological and artificial olfactory systems. Understanding the emergent properties of complex sensory and processing neuron interaction at the cellular level of the biological olfactory bulb and cortex is still at an early stage. Even so, the use of a population coding strategy by nature clearly imparts properties of graceful degradation of detection performance in the face of noise and even damage. For artificial olfactory systems, on the other hand, analytical methods for optimizing a sensor array for a particular application will be central to the uptake of electronic nose technology in industry over the forthcoming decade.

Understanding how the overall performance of a sensory system arises from the individual sensor elements as part of a population code has been the key question addressed in

this paper. By considering a simple linear model of sensor population behavior, an intuitive geometric theory of population coding has been described. This provides a framework for visualizing the action of a sensor population on the stimulus at the first stage of a sensory system. How this linear analysis may be more widely applied to man-made chemical sensors and ORNs has also been demonstrated. Examples of calculations for hypothetical ORN, and conducting polymer chemosensor populations demonstrate how, in the non-linear case, a wide range of detection performance may arise from a population code. By gaining a deeper insight into the functionality of population coded sensory systems using these and similar methods, we will in future be able to design better chemical sensing instrumentation whilst also generating more accurate models of information processing, in the olfactory pathway.

Acknowledgements

Figures 2.1, 4.1(a) and 4.2(a) were created using the MathematicaTM package for linear algebra written by Dr. J.R. Wicks (Department of Mathematics, North Park University, USA).

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Biosketch

Tim Pearce graduated from Warwick University with a BEng in Electronic Engineering in 1989 and subsequently accepted a position at Warwick as a Research Assistant developing electronic nose instrumentation for flavour monitoring supported by Neotronics (now EEV Ltd.) and Bass Breweries Ltd. The instrumentation developed under this project became the prototype for one of the first electronic nose instruments to be marketed internationally. He concurrently registered for a Doctorate by research in the same field, which led to the award of PhD in 1997. He has also held a Research Associate post at Warwick University funded by the. In 1997 he accepted a position as Visiting Research Assistant Professor at the Department of Neuroscience, Tufts University Medical School, Boston, USA. This work aimed at exploiting biological principles of olfactory information processing for practical instrumentation for unexploded ordinance detection. He currently holds a Lectureship in Bioengineering at Leicester University and is also a Royal Society funded Visiting Fellow at the Institute of Neuroinformatics at ETH, Zurich, conducting research on computational models of olfactory information processing and their application to practical chemical sensing instruments.

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Figure 3.1. Sensor space representation where sensor noise has been represented as ellipses superimposed on the region V_s . The addition of the noise components for each sensor $\delta S_i = \sigma_i^2$.

Figure 4.1. (a) Visualization of 2-odor to 2-sensor transformation using the sigmoid model for an Olfactory Receptor Neuron (ORN): $S_1 = \frac{1}{\exp(b_{11}-a_{11}x_1)} + \frac{1}{\exp(b_{12}-a_{12}x_2)}$, $S_2 = \frac{1}{\exp(b_{21}-a_{21}x_1)} + \frac{1}{\exp(b_{22}-a_{22}x_2)}$, and $a_{11} = 2$, $a_{12} = 5$, $a_{21} = 4$, $a_{22} = 2$, $b_{11} = 0.6$, $b_{12} = 0.4$, $b_{21} = 0.1$, and $b_{22} = 0.4$, (b) plot of the Jacobian for the same 2-sensor ORN population showing how the feature volume varies with the stimulus.

Figure 4.2. (a) Visualization of 2-odor to 2-sensor transformation using the non-linear Langmuir Isotherm model for conducting polymer devices: $S_1 = \frac{a_{11}x_1}{1+a_{11}x_1} + \frac{a_{12}x_2}{1+a_{12}x_2}$, $S_2 = \frac{a_{21}x_1}{1+a_{21}x_1} + \frac{a_{22}x_2}{1+a_{22}x_2}$, (b) plot of the Jacobian for the same 2-sensor conducting polymer device array showing how the feature volume varies with the stimulus.

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Table 4.1. Models of concentration dependence for a variety of chemical sensors and their behaviors. All models assume that chemicals act independently on the sensor.

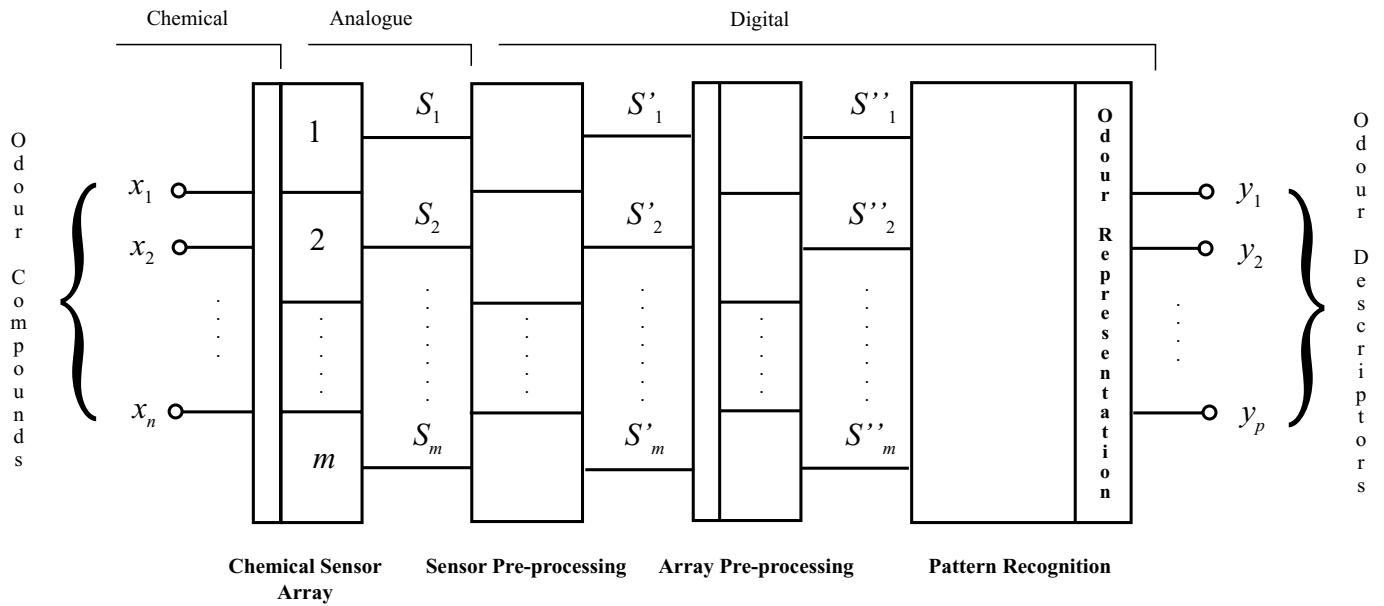


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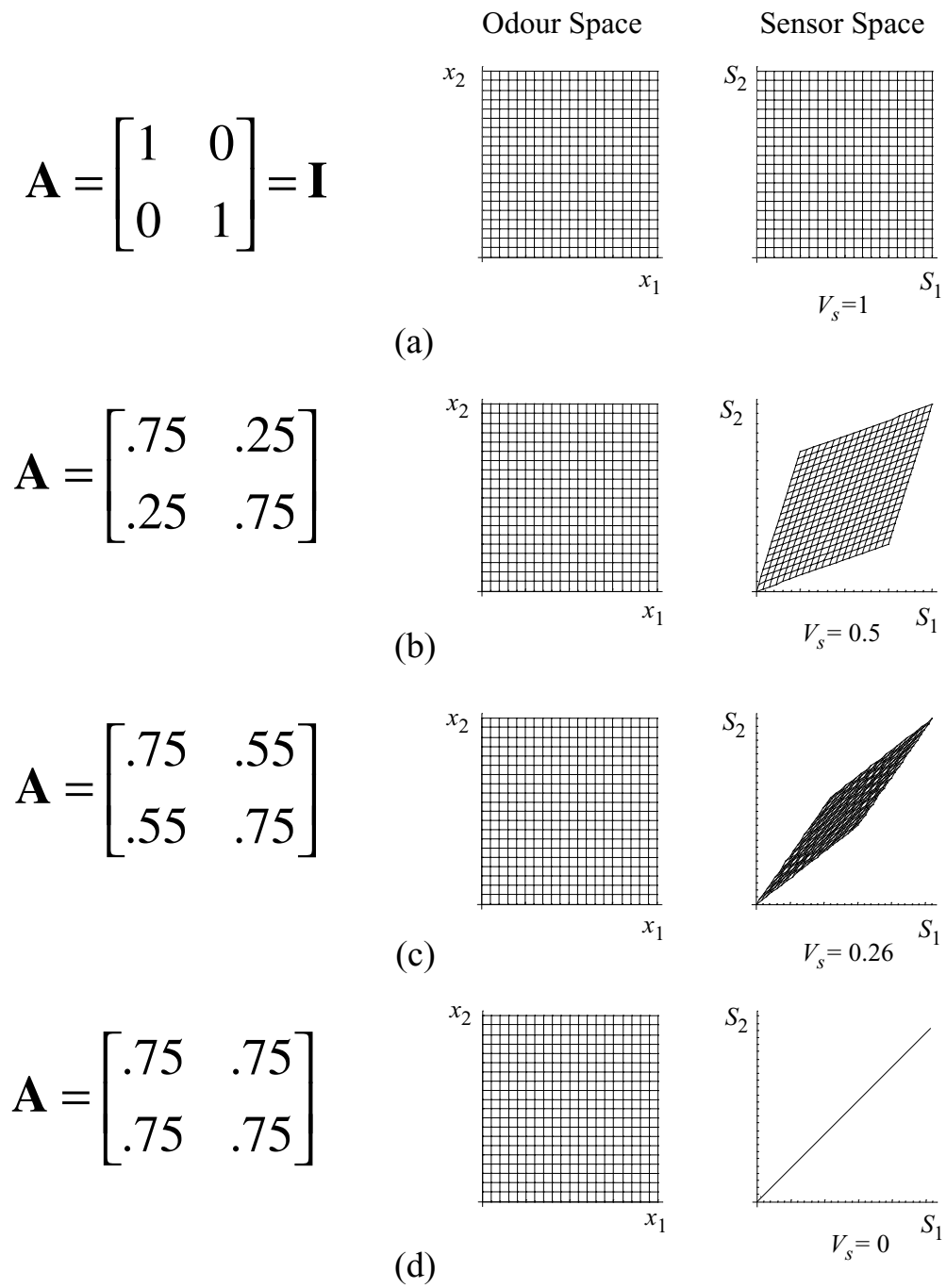


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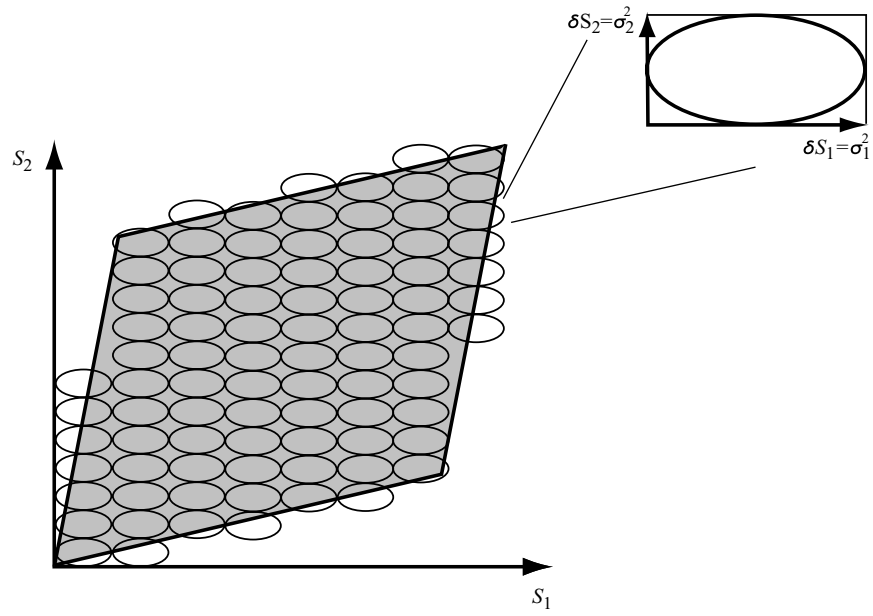
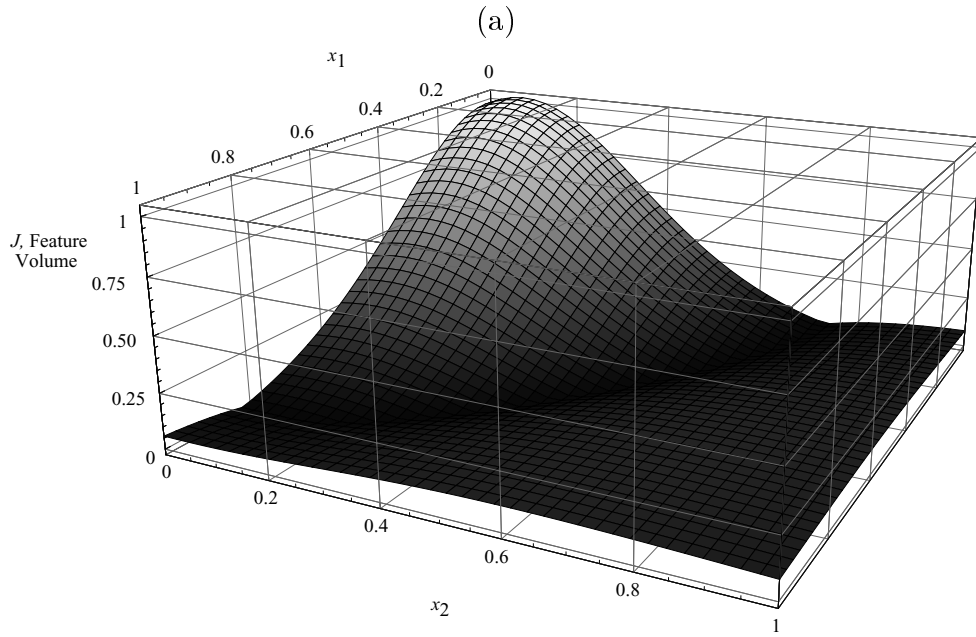
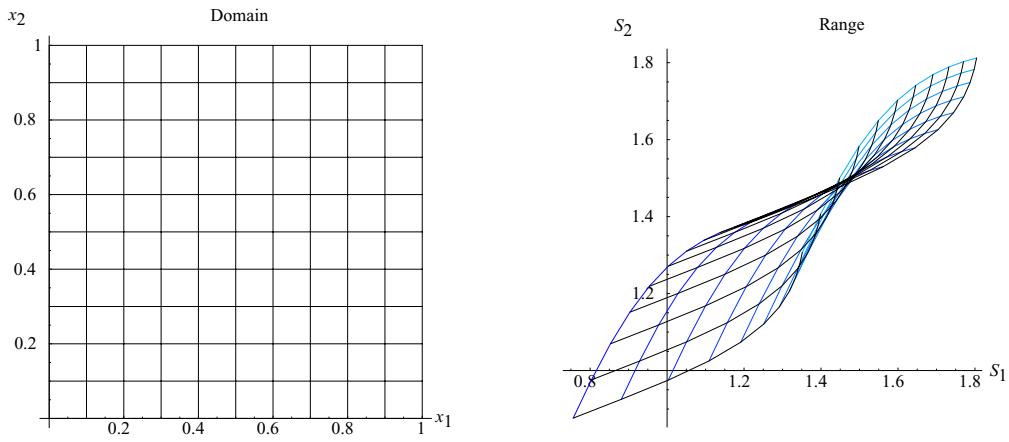
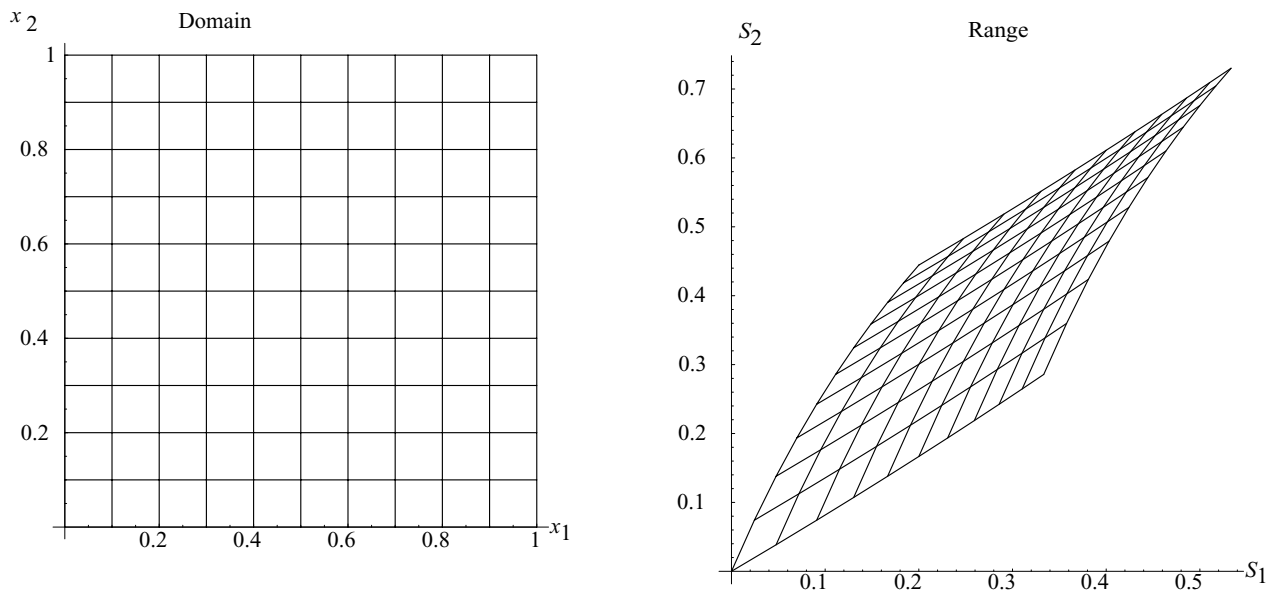


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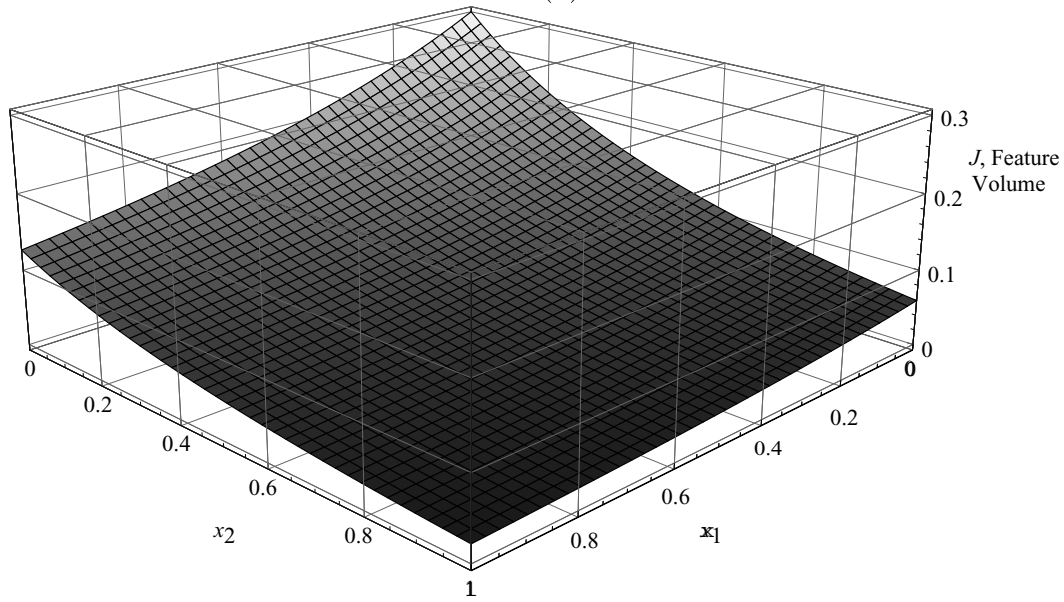


(b)

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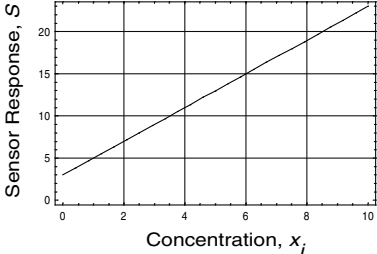
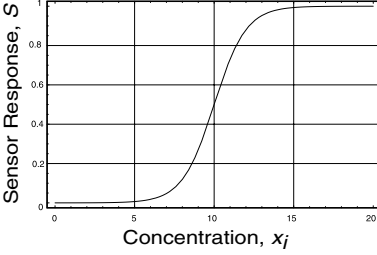
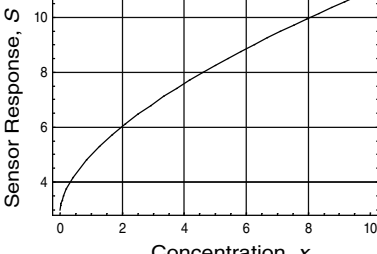
(a)



(b)

Figure 4.2: (a) Visualization of 2-odor to 2-sensor transformation using the non-linear Langmuir Isotherm model for conducting polymer device: $S_1 = \frac{a_{11}x_1}{1+a_{11}x_1} + \frac{a_{12}x_2}{1+a_{12}x_2}$, $S_2 = \frac{a_{21}x_1}{1+a_{21}x_1} + \frac{a_{22}x_2}{1+a_{22}x_2}$, (b) plot of the Jacobian for the same 2-sensor conducting polymer device array showing how the feature volume varies with the stimulus.

Table 4.1: Models of concentration dependence for a variety of chemical sensors and their behaviors. All models assume that no competition for sites within sensor takes place, and that chemicals act independently on the sensor.

Sensor	Model	Behavior
Electrochemical Fuel Cell, Fluorescent Indicators	Linear $S = \sum_i a_i x_i + a_0$	
Olfactory Receptor Neuron	Sigmoid $S = \sum_i \frac{1}{\exp(b_i - a_i x_i)} + a_0$	
Metal Oxide Semiconductor	Power $S = \sum_i a_i x_i^n + a_0$	
Conducting Polymer	Langmuir $S = \sum_i \frac{a_i x_i}{1 + a_i x_i} + a_0$	