Question 4

a) Derive a bound for the aperture error or jitter allowable in the sample-and-hold used as the input stage to an $N$-bit analogue-to-digital converter (ADC, in terms of the maximum sampling frequency. [8 MARKS]

b) Derive an expression for the root mean square of quantisation error for an $N$-bit ADC assuming an equiprobable error distribution within each quantisation step. [8 MARKS]

c) Use this to calculate the signal-to-noise ratio of a digitised triangular wave that spans the full input range of a 16-bit ADC. [4 MARKS]
Question 5

a) Compare and contrast the advantages and disadvantages for using reed relays compared to CMOS switches for switching analogue signals.

[4 MARKS]

A 16-channel multiplexer is required to switch inputs directly to a 16-bit resolution analogue-to-digital converter (ADC) up to a maximum signal frequency of 18 kHz. The multiplexer is to be implemented using CMOS switches which have a constant stray capacitance, $C_{DS} = 1 \, \text{pF}$, $R_{DS} = 75 \, \Omega$ in the ON state $R_{DS} = 400 \, \text{M}\Omega$ in the OFF state.

b) Calculate the worst-case crosstalk attenuation.

[8 MARKS]

c) Given a selection of CMOS switches each with different values for $C_{DS}$ in the OFF state, but identical values for $C_{DS}$ in the ON state, calculate the minimum value for $C_{DS(OFF)}$ such that the crosstalk within the multiplexer is less than 3 LSB of the input range of the ADC.

[8 MARKS]
Solution to Question 4

a) For the conversion of a sinusoidal signal

\[ x(t) = X_m \sin(2\pi ft) \]

the rate of change of the signal is given by

\[ \frac{dx(t)}{dt} = X_m 2\pi f \cos(2\pi ft) \]

and the maximum rate of change of a signal with frequency \( f_{\text{max}} \) is

\[ \left. \frac{dx(t)}{dt} \right|_{\text{max}} = X_m 2\pi f_{\text{max}} \]

To guarantee that the effect of the variation in amplitude of the signal over the time \( T_j \) is negligible, the following relation must stand

\[ \left. \frac{dx(t)}{dt} \right|_{\text{max}} T_j < \frac{\text{LSB}}{2} \]

and replacing \( \left. \frac{dx(t)}{dt} \right|_{\text{max}} \) with \( X_m 2\pi f_{\text{max}} \) we have

\[ X_m 2\pi f_{\text{max}} T_j < \frac{\text{LSB}}{2} \]

now replacing \( X_m \) with \( 2^{N-1} \) LSB results in

\[ 2^{N-1} 2\pi f_{\text{max}} T_j < \frac{1}{2} \]

or

\[ T_j < \frac{1}{2^{N+1} \pi f_{\text{MAX}}} \]

b) Let us first consider the root mean square value of the quantisation error:

\[ \text{rms}(\epsilon) = \sqrt{E[\epsilon^2]} = \frac{1}{N} \text{LSB} \]
The variance of the error is defined as

\[ \sigma^2_e = E[(\varepsilon - \mu_e)^2] = \int_{-\text{LSB}/2}^{\text{LSB}/2} (\varepsilon - \mu_e)^2 p(\varepsilon) \, d\varepsilon \]

and the probability distribution of the quantisation error is assumed to be uniform with limits -LSB/2 and LSB/2. That is, we are assuming that all errors within that range are equiprobable and the mean of the quantisation error is zero as shown in figure 7.

\[ \mu_e = 0 \quad \text{and} \quad p(\varepsilon) = \frac{1}{\text{LSB}} \]

\[ \sigma^2_e = \frac{1}{\text{LSB}} \int_{-\text{LSB}/2}^{\text{LSB}/2} \varepsilon^2 \, d\varepsilon = \frac{1}{\text{LSB}} \left[ \frac{\varepsilon^3}{3} \right]_{-\text{LSB}/2}^{\text{LSB}/2} = \frac{1}{3\cdot\text{LSB}} \frac{\text{LSB}^3}{4} = \frac{\text{LSB}^2}{12} \]

therefore the root mean square value of the quantisation error is

\[ \sigma_e = \frac{1}{\sqrt{12}} \text{LSB} \]
For triangular waveform of height $v$ the RMS value is given by

$$\sqrt{\frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f^2(t) dt} = \sqrt{\frac{1}{2} \left[ \int_{-\frac{T}{2}}^{0} (vt)^2 dt + \int_{0}^{\frac{T}{2}} (-vt)^2 dt \right]} = \sqrt{\frac{1}{2} \left[ \frac{v^2 t^3}{3} \bigg|_{-\frac{T}{2}}^{0} + \frac{v^2 t^3}{3} \bigg|_{0}^{\frac{T}{2}} \right]} = \frac{v}{\sqrt{3}}$$

So for the signal the rms value is

$$rms\ (signal) = \frac{2^N}{\sqrt{3}} = 37837\ LSB$$

Therefore the SNR is

$$SNR = 20 \log \left( \frac{37837}{1/\sqrt{12}} \right) = 102.4\ dB$$
Solution to Question 5.

a) **Advantages:** The ON resistance of a reed switch is of the order of 0.1Ω and the OFF resistance is the order of $10^{10}$Ω giving high isolation between channels and sensor when off. They are used for multiplexing low level signals such as thermocouples and strain gauges.

**Disadvantages:** Switching times are limited to a few hundred Hz and contacts can bounce giving rise to glitches on the output. Their working life is limited to $10^9$ operations and at 100Hz would only last a few months.

Worst case is at 18 kHz.

Switch reactance (when OFF)

\[
X_{DS_{OFF}} = \frac{1}{2\pi fC_{DS}} = \frac{1}{2\pi \times 18 \times 10^{-3} \times 1 \times 10^{-12}} = 8.84 \text{ MΩ}
\]

\[
\frac{1}{Z_{OFF}} = \frac{1}{X_{DS_{OFF}}} + \frac{1}{R_{DS(OFF)}}, \quad Z_{OFF} = 8.65 \text{ MΩ}
\]

Switch reactance (when ON)

\[
X_{DS_{ON}} = X_{DS_{OFF}}
\]

\[
\frac{1}{Z_{OFF}} = \frac{1}{X_{DS_{OFF}}} + \frac{1}{R_{DS(OFF)}}, \quad Z_{OFF} = 75 \Omega
\]

crosstalk :

\[
20 \log \left( \frac{Z_{OFF}}{Z_{ON}(N-1)} \right) = 77.7 \text{ dB}
\]

Less than 3 LSB of 16-bit ADC gives a required attenuation of

\[
20 \log(2^{14}) = 84.29 \text{ dB}
\]

So,

\[
20 \log \left( \frac{Z_{OFF}}{Z_{ON}(N-1)} \right) < 84.29 \text{ dB}
\]

Now,

\[
Z_{OFF} = Z_{ON}(N-1) \times 16384 = 75.15 \times 16384 = 1243 \text{ MΩ}
\]

\[
\therefore \frac{1}{X_{DS_{OFF}}} = \frac{1}{Z_{OFF}} - \frac{1}{R_{DS(OFF)}}, \quad X_{DS_{OFF}} = 19.3 \text{ MΩ}
\]

So,

\[
C_{DS_{OFF}} > \frac{1}{2\pi fX_{DS_{OFF}}} = 0.46 \text{ pF}
\]