Question 4

a) Derive a bound for the aperture error or jitter allowable in the sample-and-hold used as the input stage to an N-bit analogue-to-digital converter (ADC, in terms of the maximum sampling frequency.

[8 MARKS]

b) Derive an expression for the root mean square of quanitisation error for an N-bit ADC assuming an equiprobable error distribution within each quantisation step.

[8 MARKS]

c) Use this to calculate the signal-to-noise ratio of a digitised triangular wave that spans the full input range of a 16-bit ADC.

[4 MARKS]

Question 5

a) Compare and contrast the advantages and disadvantages for using reed relays compared to CMOS switches for switching analogue signals.

[4 MARKS]

A 16-channel multiplexer is required to switch inputs directly to a 16-bit resolution analogue-to-digital converter (ADC) up to a maximum signal frequency of 18 kHz. The multiplexer is to be implemented using CMOS switches which have a constant stray capacitance, C_{DS} = 1 pF, R_{DS} = 75 Ω in the ON state R_{DS} = 400 MΩ in the OFF state.

b) Calculate the worst-case crosstalk attenuation.

[8 MARKS]

c) Given a selection of CMOS switches each with different values for C_{DS} in the OFF state, but identical values for C_{DS} in the ON state, calculate the minumum value for $C_{DS(OFF)}$ such that the crosstalk within the multiplexer is less than 3 LSB of the input range of the ADC.

[8 MARKS]

Solution to Question 4

a) For the conversion of a sinusoidal signal

$$x(t) = X_m \sin(2pft)$$

the rate of change of the signal is given by

$$\frac{dx(t)}{dt} = X_m 2pf \cos(2pft)$$

and the maximum rate of change of a signal with frequency f_{max} is

$$\left.\frac{dx(t)}{dt}\right|_{max} = X_m 2pf_{max}$$

To guarantee that the effect of the variation in amplitude of the signal over the time Tj is negligible, the following relation must stand

$$\left.\frac{dx(t)}{dt}\right|_{max} T_J < \frac{LSB}{2}$$

and replacing $\left.\frac{dx(t)}{dt}\right|_{max}$ with $X_m 2pf_{max}$ we have

$$X_m 2pf_{max} T_J < \frac{LSB}{2}$$

now replacing X_m with 2^{N-1} LSB results in

$$2^{N-1} 2pf_{max} T_J < \frac{1}{2}$$

or

$$T_J < \frac{1}{2^{N+1} pf_{MAX}}$$

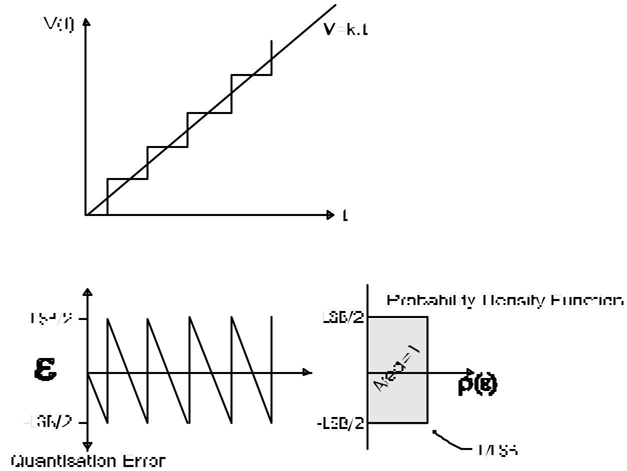
b) Let us first consider the root mean square value of the quantisation error:

$$\text{rms}(e) = \sqrt{s_e^2}$$

The variance of the error is defined as

$$s_e^2 = E\left[(e - m_e)^2\right] = \int_{-\frac{LSB}{2}}^{\frac{LSB}{2}} (e - m_e)^2 p(e) de$$

and the probability distribution of the quantisation error is assumed to be uniform with limits -LSB/2 and LSB/2. That is, we are assuming that all errors within that range are equiprobable and the mean of the quantisation error is zero as shown in figure 7



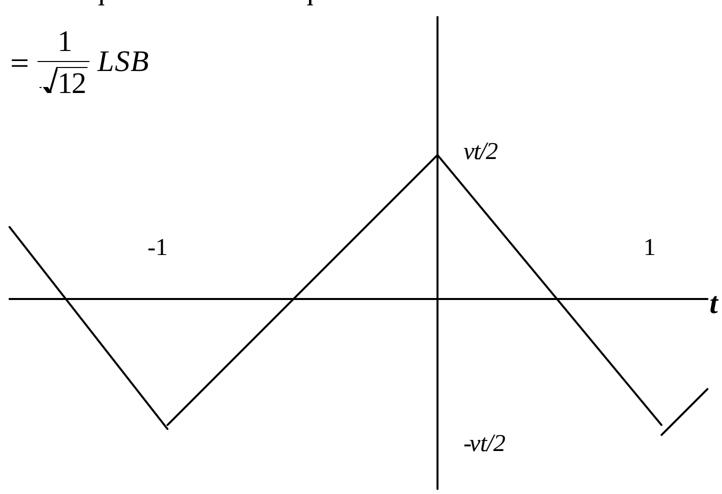
$m_e = 0$ and $\qquad p(e) = \dfrac{1}{LSB}$

$$s_e^2 = \frac{1}{LSB}\int_{-\frac{LSB}{2}}^{\frac{LSB}{2}} e^2 de = \frac{1}{LSB}\left[\frac{e^3}{3}\right]_{-\frac{LSB}{2}}^{\frac{LSB}{2}} = \frac{1}{3LSB}\frac{LSB^3}{4} = \frac{LSB^2}{12}$$

therefore the root mean square value of the quantisation error is

$$s_e = \frac{1}{\sqrt{12}} LSB$$

c)



For triangular waveform of height *v* the RMS value is given by

$$\sqrt{\frac{1}{T}\int f^2(t)dt} = \sqrt{\frac{1}{2}\left[\int_{-1}^{0}(vt)^2.dt + \int_{0}^{1}(-vt)^2.dt\right]} = \sqrt{\frac{1}{2}\left[\frac{v^2t^3}{3}\bigg|_{-1}^{0} + \frac{v^2t^3}{3}\bigg|_{0}^{1}\right]} = \frac{v}{\sqrt{3}}$$

So for the signal the rms value is

$$rms\,(signal) = \frac{2^N}{\sqrt{3}} = 37837\ LSB$$

Therefore the SNR is

$$SNR = 20\log\left[\frac{37837}{1/\sqrt{12}}\right] = 102.4\,\text{dB}$$

Solution to Question 5.

a) *Advantages:* The ON resistance of a reed switch is of the order of 0.1Ω and the OFF resistance is the order of 10^{10}Ω giving high isolation between channels and sensor when off. They are used for multiplexing low level signals such as thermocouples and strain gauges.

Disadvantages: Switching times are limited to a few hundred Hz and contacts can bounce giving rise to glitches on the output. Their working life is limited to 10^9 operations and at 100Hz would only last a few months.

Worst case is at 18 kHz.

Switch reactance (when OFF)

$$X_{DSOFF} = \frac{1}{2pfC_{DS}} = \frac{1}{2p\,18\,\mathrm{x}10^3.1\mathrm{x}10^{-12}} = 8.84\,\mathrm{M}\Omega$$

$$\frac{1}{Z_{OFF}} = \frac{1}{X_{DSOFF}} + \frac{1}{R_{DS(OFF)}}, \qquad Z_{OFF} = 8.65\,\mathrm{M}\Omega$$

Switch reactance (when ON)

$$X_{DSON} = X_{DSOFF}$$

$$\frac{1}{Z_{OFF}} = \frac{1}{X_{DSOFF}} + \frac{1}{R_{DS(OFF)}}, \qquad Z_{OFF} = 75\Omega$$

crosstalk :

$$20\log\left[\frac{Z_{OFF}}{Z_{ON}(N-1)}\right] = 77.7\,\mathrm{dB}$$

Less than 3 LSB of 16-bit ADC gives a required attenuation of

$$20\log(2^{14}) = 84.29\,\mathrm{dB}$$

So,

$$20\log\left[\frac{Z_{OFF}}{Z_{ON}(N-1)}\right] < 84.29\,\mathrm{dB}$$

Now,

$$Z_{OFF} = Z_{ON}(N-1).16384 = 75.15.16384 = 18.43\,\mathrm{M}\Omega$$

$$\therefore \frac{1}{X_{DSOFF}} = \frac{1}{Z_{OFF}} - \frac{1}{R_{DS(OFF)}}, \qquad X_{DSOFF} = 19.3\,\mathrm{M}\Omega$$

So,

$$C_{DSOFF} > \frac{1}{2pfX_{DSOFF}} = 0.46\,\mathrm{pF}$$