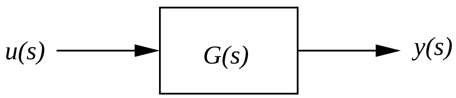
EG1110 SIGNALS AND SYSTEMS

Feedback and closed-loop systems

Basics

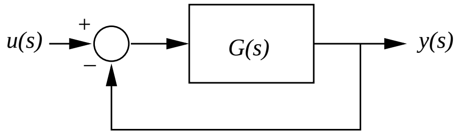
<u>Open-loop system</u>



$$y(s) = G(s)u(s)$$

- No connection between output $y(s)$ and input $u(s)$ except through $G(s)$ in "forward path".

<u>Closed-loop system</u>



$$\begin{aligned} y(s) &= G(s)[u(s) - y(s)] \\ &= \frac{G(s)}{1 + G(s)} u(s) \end{aligned}$$

- $y(s)$ and input $u(s)$ related through $G(s)$ in "forward path" and <u>feedback path</u>.

Effects of feedback

- Alters system's behaviour (normally to improve it)
- Affect's stability (important!)
- Present in many "applications": control systems, electronics, nature...
- Can be positve or negative
- Can have many different feedback loops

Feedback and block diagrams

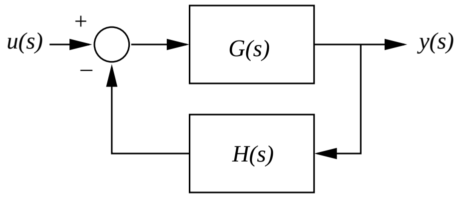
Recall useful rules for block diagrams

$$\begin{aligned} g(t) \star h(t) &= h(t) \star g(t) \\ g(t) \star [h(t) \star j(t)] &= [g(t) \star h(t)] \star j(t) \\ g(t) \star [h(t) + j(t)] &= g(t) \star h(t) + g(t) \star j(t) \end{aligned}$$

$$\begin{aligned} G(s)H(s) &= H(s)G(s) \\ G(s)[H(s)J(s)] &= [G(s)H(s)]J(s) \\ G(s)[H(s) + J(s)] &= G(s)H(s) + G(s)J(s) \end{aligned}$$

Using these rules many feedback problems can be simplified:

$$\begin{aligned} y(s) &= G(s)[u(s) - H(s)y(s)] \\ &= \frac{G(s)}{1 + H(s)G(s)} u(s) \end{aligned}$$



Stability and feedback

Overarching rule:
(there is no overarching rule)

***Open-loop** stability (instability) is neither necessary nor sufficient for **closed-loop** stability (instability)*

Meaning...

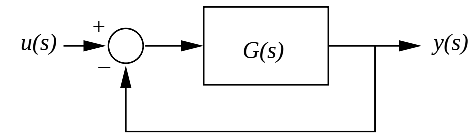
- If a feedback loop is "wrapped around" a stable system, the resulting *closed-loop* system may or may not be stable, depending on the properties of the open-loop system.

  i.e.

  – Open-loop stability *does not* imply closed-loop stability; and

  – Closed-loop stability *does not* imply open-loop stability.

Illustrative example



$$G(s) = \alpha \frac{1}{s+2}$$

Open-loop system In open-loop (assume feedback not connected) we have

$$\begin{aligned} y(s) &= G(s)u(s) \\ &= \alpha \frac{1}{s+2} u(s) \end{aligned}$$

OL system is stable - pole at $s = -2$, i.e. in left hald of s-plane.

Closed-loop system - case 1

$$\begin{aligned} y(s) &= \frac{G(s)}{1+G(s)} u(s) \\ &= \frac{\frac{\alpha}{s+2}}{1+\frac{\alpha}{s+2}} u(s) \\ &= \frac{\alpha}{s+2+\alpha} u(s) \end{aligned}$$

Thus pole at $s + 2 + \alpha = 0$.

If $\alpha > -2$, say $\alpha = -1$, then pole at

$$\begin{aligned} s &= -2 - \alpha = -2 - (-1) && (1) \\ &= -1 && (2) \end{aligned}$$

i.e system is *stable*

Closed-loop system - case 2

If $\alpha < -2$, say $\alpha = -6$, then pole at

$$\begin{aligned} s &= -2 - \alpha = -2 - (-6) && (3) \\ &= 4 && (4) \end{aligned}$$

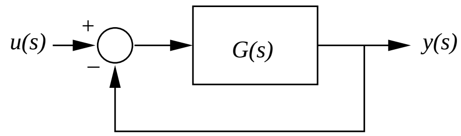
i.e system is *unstable*!

Thus systems with same open-loop poles may behave differently in closed-loop

More generally...

$$G(s) = \frac{n(s)}{d(s)} =: \left(\frac{\Pi_{j=1}^{m}(s + z_j)}{\Pi_{i=1}^{m}(s + p_j)} \right)$$

Hence in closed-loop



$$y(s) = \frac{G(s)}{1 + G(s)} u(s) = \frac{\frac{n(s)}{d(s)}}{1 + \frac{n(s)}{d(s)}} u(s) = \frac{n(s)}{n(s) + d(s)} u(s)$$

Thus

- Closed-loop zeros are open-loop zeros

  i.e. the roots of $n(s)$
- Closed-loop poles are *not* open-loop poles.

  i.e they are roots of $n(s) + d(s)$ *not* just $d(s)$
- Stability of closed-loop is not *trivially* related to stability of open-loop - relationship is system-specific.