Ultrasound in Med. & Biol. Vol. 16, No. 1, pp. 81–91, 1990
Printed in the U.S.A.

0301-5629/90 $3.00 + .00
Pergamon Press plc

●*Original Contribution*

SELECTION OF THE ORDER OF AUTOREGRESSIVE MODELS FOR SPECTRAL ANALYSIS OF DOPPLER ULTRASOUND SIGNALS

FERNANDO S. SCHLINDWEIN
Federal University of Rio de Janeiro, Brazil

and

DAVID H. EVANS
Department of Medical Physics & Clinical Engineering, Leicester
Royal Infirmary, Leicester LE1 5WW, England[†]

(*Received* 21 *April* 1989; *in final form* 18 *July* 1989)

Abstract—Autoregressive modelling includes a model identification procedure, that is, it is necessary to choose the order of the autoregressive (AR) process that best describes the given finite record (frame) of the signal. Four previously suggested procedures to choose the "best order" of AR processes have been tested: The "first zero crossing" of the autocorrelation function (FZC), the "final prediction error" (FPE), "Akaike's information criterion" (AIC), and the "criterion autoregressive transfer-function" (CAT). It was found that: (i) For more than 98% of the 1280 frames of Doppler signals analyzed the order selected by the various criteria was ten or less. (ii) For the same records of Doppler signals, FPE, AIC and CAT behave in a very similar manner, but the FZC criterion underestimates the order in relation to the others. (iii) For true AR processes, the order selected is frequently different from the true AR order when frames of 64 samples are used. When more samples are used FPE, AIC and CAT tend to select the correct order. (iv) The effect on the spectral estimate of using too high a model order is usually insignificant, while using too low an order can change the estimate more dramatically, that is, overestimating the model order is better than underestimating it.

Key Words: **Ultrasound, Ultrasonic Doppler shift, Doppler power spectrum, Autoregressive spectrum analysis, Autoregressive model order.**

1. INTRODUCTION

Doppler ultrasound is widely used as a noninvasive method for the assessment of arterial blood flow. Under ideal conditions the Doppler power spectrum has a shape similar to the histogram of the blood velocities within the sample volume and thus spectral analysis of the Doppler signal produces information concerning the velocity distribution in the artery (Evans et al. 1989).

The estimation of the power spectral density of the Doppler signal is normally performed by applying a Fast Fourier Transform (FFT) directly to the sampled signal. This technique of transforming the data directly is referred to as the periodogram approach. The periodogram produces fairly good results for the typical analysis regime used (128 frequency components every 10 ms). For short data lengths, however the FFT-periodogram technique suffers from poor statistical stability and poor spectral resolution (Kay and Marple 1981), and autoregressive (AR) modelling has been shown to give better results (Kitney and Giddens 1986; Kaluzynski 1987; Vaitkus et al. 1988). The AR method has been implemented in real-time by Schlindwein and Evans (1989).

Autoregressive modelling includes a procedure for model identification, that is, it is necessary to choose the order of the autoregressive process that best describes the finite duration of signal available (frame). This paper is concerned with the behaviour of four techniques which have been suggested for identifying the "best order" of AR processes: The "first zero crossing" of the autocorrelation function (FZC), the "final prediction error" (FPE), "Akaike's information criterion" (AIC), and the "criterion autoregressive transfer-function" (CAT). The study is comprised of four parts. Firstly, the recommended "best orders" for describing 256 frames of 64 samples of five different kinds of Doppler signal using the four criteria were evaluated. Secondly, sets of a thousand

---

[†] Address for correspondence.

realizations of true AR processes were created (each realization consisting of 64 samples), using autoregressive coefficients corresponding to selected frames of a real signal, and the statistical performance of the different procedures for identifying the true AR order were tested. Thirdly, the effect on the behaviour of the techniques of using larger frames was studied by creating sets of 128 and 256 samples, with the same known coefficients, and testing the statistical performance of the techniques to identify the order. Finally, the effect of choosing an order different from the recommended one was assessed by modelling the selected frames with higher and lower orders and calculating the corresponding estimates of the power spectra.

2. THE AUTOREGRESSIVE POWER SPECTRAL DENSITY ESTIMATE

Spectral estimation techniques can be seen as attempts to fit measured data to an assumed model. The autoregressive model assumes that the current value of a process, x_n, can be described by a finite linear aggregate of the previous values of the process and the current value of a white noise driving signal, n_n. An autoregressive process of zero mean and order "p" is defined as (Box and Jenkins 1976)

$$x_n = n_n - a_1x_{n-1} - a_2x_{n-2} - a_3x_{n-3} - \cdots - a_px_{n-p}. \quad (1)$$

The autoregressive model contains $p + 1$ parameters which are estimated from the data: the p coefficients and the variance of the white noise. The estimation of these parameters for the AR model results in linear equations (Yule-Walker equations), which are computationally easy to implement (Kay and Marple 1981). An alternative (and faster) recursive method for calculating the Yule-Walker estimates of the autoregressive parameters is provided by the Levinson-Durbin algorithm (Kay and Marple 1981).

Once the parameters are computed, the AR power spectral density estimate of the data is given by

$$S(f) = \sigma^2T/|\sum_{n=0}^{p} a(n)\exp(-j2\pi fnT)|^2 \quad (2)$$

where σ^2 is the variance of the white noise driving function, T is the sampling period, and $a(0) = 1$. Notice that eqn (2) refers to a continuous function, $S(f)$, computed from a finite discrete series of parameters, $a(n)$, T, and σ^2.

2.1. *Maximum entropy method (MEM)*

Maximum entropy spectral estimation is based upon an extrapolation of the known values of the autocorrelation function (or estimate) using the autoregressive model as the basis for the extrapolation. Supposing that $p + 1$ values of the autocorrelation function, $R_{xx}(0)$ to $R_{xx}(p)$, are known, the maximum entropy extrapolation of the autocorrelation is

$$r_{xx}(n) = R_{xx}(n) \quad \text{for} \quad |n| \le p$$

$$r_{xx}(n) = -\sum_{k=1}^{p} a(k)r_{xx}(n-k) \quad \text{for} \quad |n| > p \quad (3)$$

and the MEM power spectrum density estimate is given by

$$S(k) = T\sum_{n=-M}^{M-1} r_{xx}(n)\exp(-j2\pi knT). \quad (4)$$

Since the true autocorrelation function (ACF) of the process is not known, the biased estimate of the ACF given by

$$\hat{R}_{xx}(k) = \frac{1}{N}\sum_{n=0}^{N-k-1} x(n)x(n+k), \quad (5)$$

is used since it has less variance than the unbiased estimate for finite data sets (Box and Jenkins 1976).

VanDenBos (1971) proved formally that for $M \rightarrow \infty$, eqn (4) is equivalent to eqn (2) for the one-dimensional case, that is, for sufficiently long extrapolations of the ACF, the MEM and the AR power spectral density estimates (PSDE) are the same.

In this study, the coefficients of the AR model were calculated using the Levinson-Durbin recursive algorithm with the biased estimate of the autocorrelation function (eqn 5), and the power spectrum, estimated according to the MEM approach (eqn 4 with $M = 128$).

3. FOUR CRITERIA TO ESTIMATE THE "BEST ORDER"

Four model identification procedures were implemented. The first zero crossing criterion (FZC), proposed by Kitney et al. (1986) chooses as the order for the AR model the point where the autocorrelation function crosses the time-lag axis. The FZC is intended to provide a "reliable means of obtaining the minimum model order consistent with adequate low frequency resolution" (Kitney et al. 1986). The other three criteria select as the order of the AR process the value which minimizes a function of the order, and differ only with respect to the kind of function used. The "final prediction error" criterion (Akaike 1969) was defined as

$$\mathrm{FPE}(k) = \frac{N+k+1}{N-k-1}\sigma(k)^2 \quad (6)$$

Table 1. The sampling frequencies and corresponding frequency ranges and frame lengths.

Sampling frequency	Frequency range	Frame length
40.96 kHz	20.48 kHz	1.563 ms
20.48 kHz	10.24 kHz	3.125 ms
10.24 kHz	5.12 kHz	6.25 ms
5.12 kHz	2.56 kHz	12.50 ms
2.56 kHz	1.28 kHz	25.00 ms

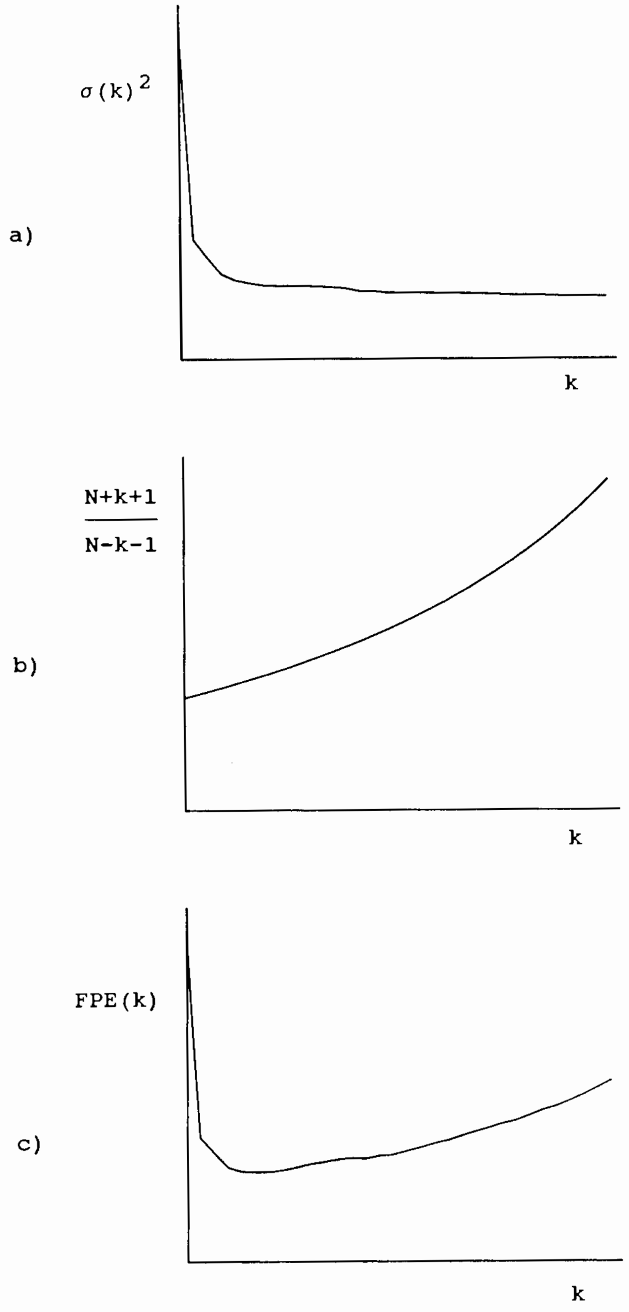

Fig. 1. (a) The variance of the residuals, $\sigma(k)^2$, as a function of the order, k, for an individual frame of 64 samples (k ranging from 1 to 32), (b) the weighting function for the FPE, representing the bias of the estimation of the AR coefficients, according to Akaike (1969), and (c) the corresponding "final prediction error" function.

where N is the number of samples in the frame, k is the trial model order and $\sigma(k)^2$, the variance corresponding to the order k. By scanning k successively from 1 to some upper limit L, the model order is given by the k which results in a minimum of FPE(k), $k = 1, 2, \ldots, L$. The function consists of two components: the variance (mean square error of the estimation) monotonically decreases with the order (Fig. 1a), while the first term, representing statistical deviation (bias) of the estimation of the AR coefficients from the true $a(n)$, increases with the order (Fig. 1b). As the product of these two components, the FPE function should have a minimum (Fig. 1c) corresponding to an autoregressive model of an order for which the bias is not very significant and at the same time would not produce a too big mean square prediction error (Akaike 1969).

In 1974 Akaike revised his definition of FPE and proposed an alternative "minimum information theoretical criterion" (Akaike 1974) defined as

$$\mathrm{AIC}'(k) = -2\ln(ML) + 2k, \tag{7}$$

where ML is the maximum likelihood estimate of the k AR parameters. This technique minimizes an estimate of a theoretical "information function" as the criterion of fitness of the model. In the same paper Akaike suggested that, for a stationary zero-mean Gaussian process, N times the variance could be used in place of the log-likelihood. This was interpreted by Kay and Marple (1981) as

$$\mathrm{AIC}(k) = \ln(\sigma(k)^2) + (2k+1)/N, \tag{8}$$

and it is this second form that has been used in this study. Finally, the "criterion autoregressive transfer-function" (Parzen 1975)

$$\mathrm{CAT}(k) = \left[\frac{1}{N}\sum_{j=1}^{k}\frac{N-j}{N\sigma(j)^2}\right] - \frac{N-k}{N\sigma(k)^2} \tag{9}$$

chooses the order of the AR process by minimizing the overall mean square error estimate of the infinite autoregressive transfer function of the filter which transforms the time series back to its innovations (white noise).

4. MATERIAL AND METHODS

The hardware of the system used for this investigation is comprised basically of a Research Machines

Table 2. The sources of the five signals used for the study.

Signal 1—Common femoral artery of an adult volunteer, 8 MHz CW Doppler unit, sampling frequency = 10.24 kHz.

Signal 2—Common carotid artery of an adult volunteer, 8 MHz CW Doppler unit, sampling frequency = 10.24 kHz.

Signal 3—Middle cerebral artery of a neonate, 5 MHz PW Doppler unit, sample length = 3 mm, sampling frequency = 10.24 kHz.

Signal 4—stenosed common carotid artery of a patient, 3 MHz PW Doppler unit, sample length = 8 mm, sampling frequency = 20.48 kHz.

Signal 5—stenosed ascending aorta of a patient, 1.9 MHz CW Doppler unit, sampling frequency = 10.24 kHz.

Nimbus PC2 (8 MHz 80186 based personal computer), a heterodyne unit built in our laboratory, a 16 bit 17 μs analog-to-digital converter (ADC), and an analog interface with amplifier and antialiasing filter (Schlindwein et al. 1988).

The input to the ADC is a single channel audio signal. The forward and reverse audio outputs of a bidirectional Doppler velocimeter are mixed and offset about a programmable heterodyne frequency of 500 Hz, 1 kHz, 2 kHz, 4 kHz or 8 kHz, and sampled with one of the programmable sampling rates given in Table 1. The sampling rate and the heterodyne frequency are chosen by the operator in such a way that the full range of frequency analysis is used without aliasing. The operator has real-time AR sonograms as a feedback (Schlindwein and Evans 1989) while choosing the combination of sampling frequency and heterodyne setting.

The Doppler audio signals were sampled and segments corresponding to up to 6.4 s were stored as

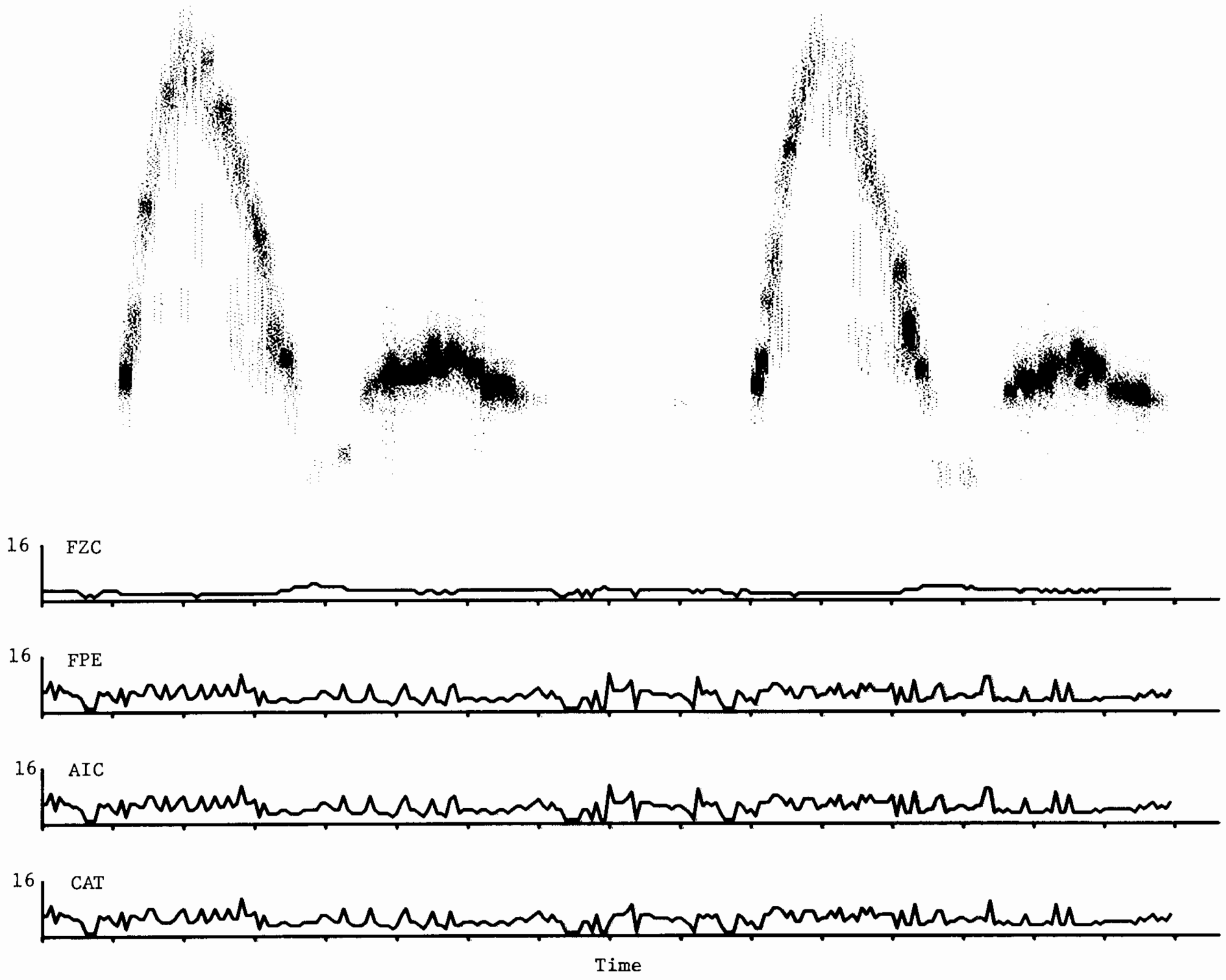

Fig. 2. AR sonogram from signal 1, together with the evolution of the "best orders" with time, according to the four criteria. Notice that FPE, AIC, and CAT are very similar but the orders have no obvious correlation with the sonogram. FZC presents, as expected, an inverse correlation with the mean frequency of the signal.

disk files. The waveforms analyzed included signals from normal common carotid and common femoral arteries of adult volunteers, the middle cerebral artery of a neonate, a stenosed common carotid artery, and an ascending aorta distal to an aortic stenosis, according to Table 2.

The signals were read from the disk files and analyzed off-line in frames of 64 samples in the microcomputer (using Pro Fortran 77). The spectral estimation produced 128 components according to eqn (4).

Initially all four criteria were applied to all the signals, to find out what sort of order is recommended by the various techniques to describe general Doppler signals, and also to verify whether the different criteria would or would not result in different orders for the same records (frames) of Doppler signals.

The recommended "best orders" were plotted together against time (frame number), in an attempt to identify any similar behaviour amongst the different criteria, and against the corresponding sonogram (Fig. 2) to highlight correlations between the frequency content and the order of the model. The histograms of Figs. 3–7 illustrate the relative occurrence of orders, from 1 to 16 (estimates of above 16 are plotted as 16) for the five signals used in this study.

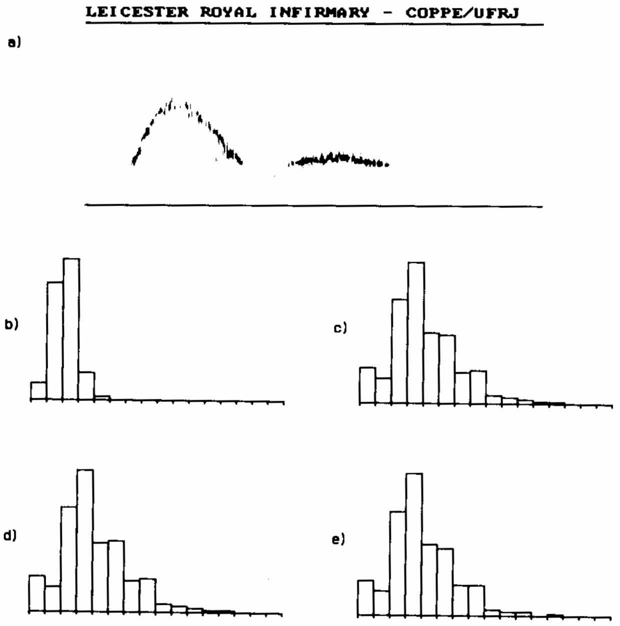

Fig. 3. (a) AR sonogram of signal 1—Doppler signal from the common femoral artery of a healthy volunteer, and normalized histograms of the "best orders" according to the four criteria: (b) First zero crossing, (c) final prediction error, (d) Akaike's information criterion, and (e) criterion autoregressive transfer function, for the 256 frames.

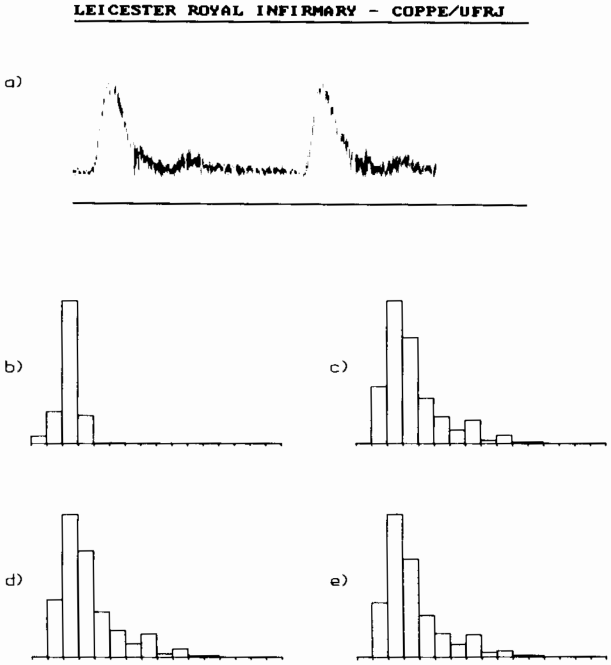

Fig. 4. (a) AR sonogram of signal 2—Doppler signal from the common carotid artery of a healthy volunteer. The histograms correspond to those of Fig. 3.

As can be seen in Fig. 2, apart from FZC, the other three criteria behave in a very similar fashion, predicting the same order for most of the frames. These results are in agreement with those of Jones (1974). None of the criteria predicted an order of bigger than 17 for any of the 1280 frames analyzed, and for more than 98% of the frames the order selected was 10 or less. There was no obvious correlation between the orders selected by FPE, AIC, or CAT and the spectral characteristics of the sonogram at that point. There is an evident correlation between the FZC criterion and the sonogram, as expected (the smaller the frequency, the bigger the FZC), but the FZC criterion produces underestimation of the "best" order when compared to the other three methods and also when used with simulated true AR processes (see section 5).

5. THE PERFORMANCE OF THE MODEL ORDER ESTIMATION TECHNIQUES WITH TRUE AR PROCESSES

If the Doppler signals *can* be described by an AR model of a certain order, a good technique to identify the model would be one that, for true autoregressive processes created with "real world" coefficients, correctly identifies the model order.

A simulation was set up to find out how well the different criteria work in these circumstances. For the

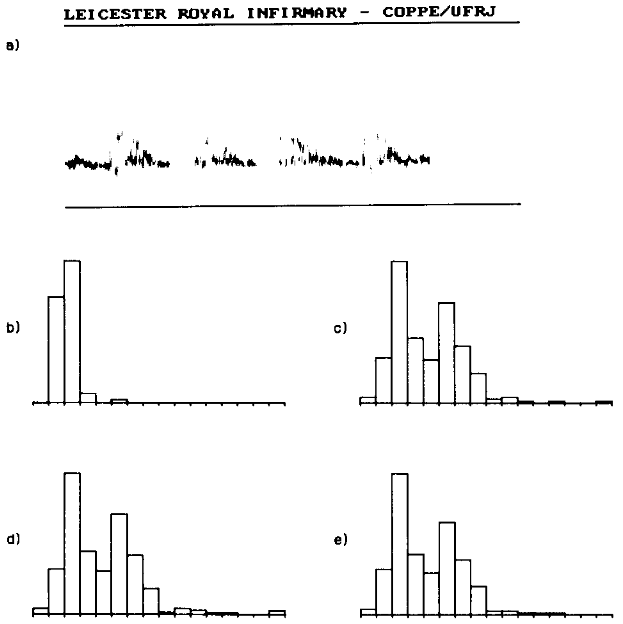

Fig. 5. (a) AR sonogram of signal 3—Doppler signal from the middle cerebral artery of a neonate. The histograms correspond to those of Fig. 3.

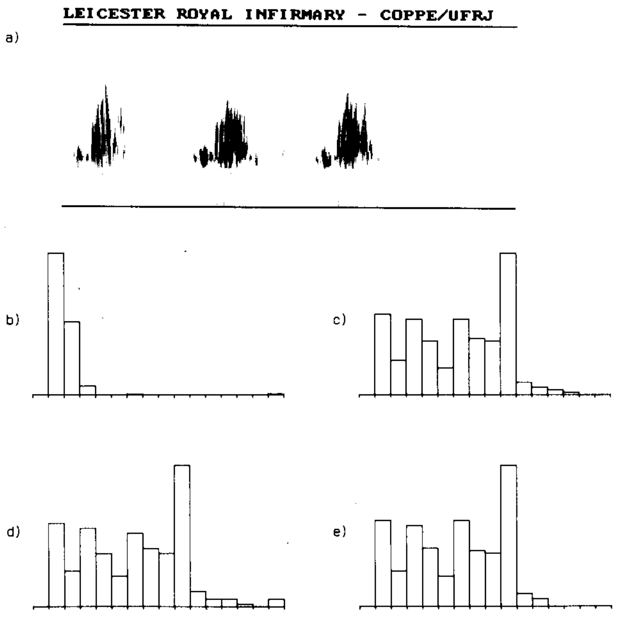

Fig. 7. (a) AR sonogram of signal 5—Doppler signal from a stenosed Aorta. The histograms correspond to those of Fig. 3.

simulation, six sets of 1000 realizations of true AR processes of known order were created and submitted to the four criteria.

Six particular frames from the signal 1 were chosen (three of them are shown in Figs. 8a–10a) and the coefficients corresponding to the order recommended by FPE (AIC and CAT agreed) for these frames were used as the AR coefficients to create the realizations of the AR processes used for the simulation (see Appendix and Table 3). Sets of 256 pseudonormal random numbers were generated and submitted to the AR filter defined by those coefficients. The last 64 values of the 256-valued output sequences of the AR filter were used as the realizations of the simulated AR process, in order to allow the filter to stabilize.

The "best orders" selected by the four criteria in 1000 realizations of each of the six processes were computed and the corresponding histograms of the occurrence of the orders for three of them are presented in Figs. 8–10. In all cases the histograms of FPE, AIC, and CAT present a local maximum corresponding to the true order, the three criteria behaving in a *very* similar manner. The FZC badly underestimated the correct order in all cases and we do not present the FZC histogram. Instead the true AR power spectral density function is shown in each of Figs. 8b–10b.

For the AR processes created with the coefficients of frames 43 and 45 (orders 6 and 5, respectively) the three criteria (FPE, AIC, and CAT) present a very strong (wrong) peak corresponding to order 2 in the histogram of "best orders" (Fig. 8).

For the other simulations (Figs. 9–10), the histograms representing the frequency of selection of the order of the model for FPE, AIC, and CAT presented

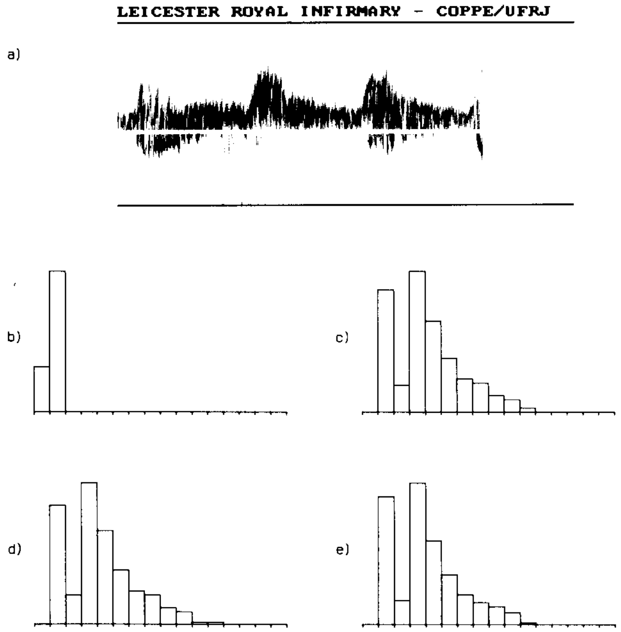

Fig. 6. (a) AR sonogram of signal 4—Doppler signal from a stenosed common carotid artery. The histograms correspond to those of Fig. 3.

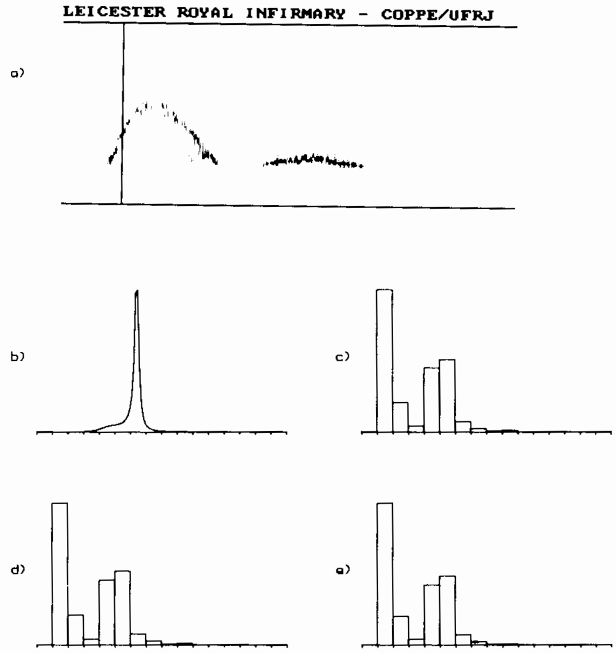

Fig. 8. According to FPE, AIC and CAT, frame 43, marked in (a), is best described by an AR model of order six. These six coefficients (Table 3) were used to create 1000 realizations of the AR process whose theoretical spectrum is given in (b). The normalized histograms of the orders found by FPE, AIC, and CAT are presented respectively in (c), (d), and (e).

a peak at the correct order, but for Fig. 9 the criteria more often produced an underestimation of the order than the correct model order.

6. EFFECTS OF USING LARGER FRAMES (128 AND 256 SAMPLES)

Observing the histograms of Figs. 8–10, it can be seen that none of the four methods can consistently estimate the correct order of short segments (frames of 64 samples) of AR processes. In order to verify if this behaviour is due to intrinsic lack of stability of the methods themselves or to statistical fluctuations of the simulated short segments of the "pseudo AR processes" used in the simulation, two other simulations were performed.

Firstly, the procedure described in section 5 was repeated, but this time 1024 pseudonormal values were used as the inputs to the AR filter in order to allow for better transient stabilization. The results showed the same kind of scattered histograms as Figs. 8–10. Transient effects of the application of the AR filter were ruled out as the cause of the problem.

Secondly, frames of 128 and 256 samples were generated using the coefficients corresponding to frame 60 of the signal 1, according to the same technique described in section 5, and the histograms corresponding to the orders selected for these larger frames of the *same* underlying AR process were generated. Figure 11 presents the histogram correspond-

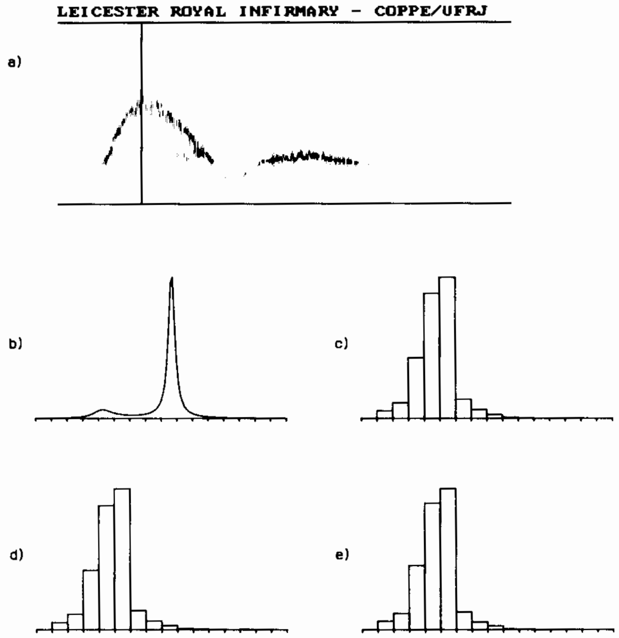

Fig. 9. Frame 60, marked in (a), is best described by an AR model of order six. The six coefficients were used to produce histograms as described in Fig. 8.

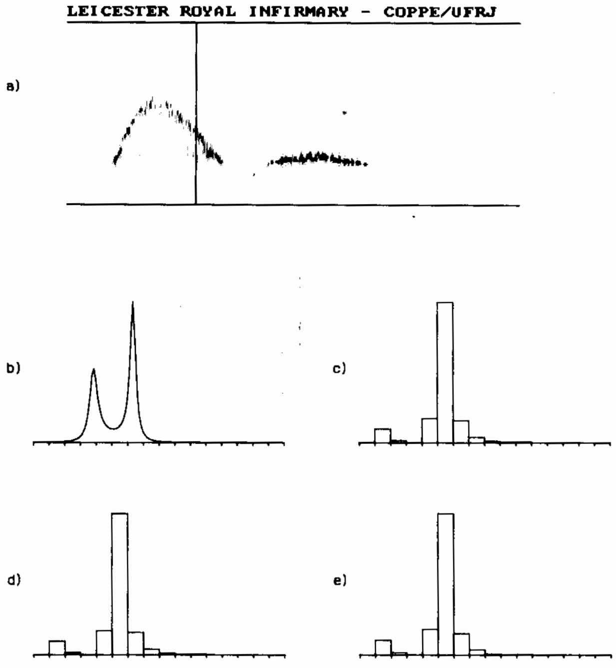

Fig. 10. Frame 92, marked in (a), is best described by an AR model of order six. The six coefficients were used to produce histograms as described in Fig. 8.

Table 3. The coefficients used in the simulations of AR processes.

Frame 43—order recommended = 6
a (1 to 6) = −0.7016, 1.0240, 0.2093, 0.0487, 0.1764, 0.2162

Frame 45—order recommended = 5
a (1 to 5) = −0.9380, 1.1127, −0.0454, −0.1594, 0.2665

Frame 60—order recommended = 6
a (1 to 6) = −0.2537, 1.0138, −0.2322, 0.4718, 0.2134, 0.1903

Frame 63—order recommended = 7
a (1 to 7) = −0.4650, 1.3020, −0.5012, 0.7128, 0.2514, 0.2216, 0.2696

Frame 86—order recommended = 5
a (1 to 5) = −0.9332, 1.2862, −0.2756, 0.1502, 0.2688

Frame 92—order recommended = 6
a (1 to 6) = −0.9424, 1.0181, 0.0733, 0.0904, 0.0676, 0.3532

ing to using 256 values/frame (compare with Fig. 9, where 64 values were used). From these, it could be seen that the larger the frame, the better the FPE, AIC, and CAT techniques behaved in estimating the correct order, six, for the underlying AR process. This seems to show that the three techniques are good in the sense that they converge to the correct order as the number of samples used to describe the process tends to infinity. This practical result seems to contradict the theoretical claim by Kashyap (1980), that the AIC is inconsistent for the estimation of the order of AR models.

Our tests show that for too short segments the determination of the correct order for the AR process is a problem and that this seems to have to do with the intrinsic instability of the data itself, and not to a failure of the techniques for selecting the order. Unfortunately the length of the data segment cannot be arbitrarily increased to permit accurate model order estimation because the maximum length of data that can be validly used for a single PSDE is related to the stationarity of the Doppler signal. Indeed a major advantage of the AR modelling over the traditional Fourier technique is that it allows the use of short data segments that fulfill the stationarity requirement.

7. EFFECTS OF UNDER/OVER ESTIMATING THE ORDER

Since none of the methods seem to consistently predict the correct order of true AR processes when short frames are used, under/overestimation of the order of the model will occur. Because of this, the effects of the wrong selection of the model order on the power spectral density estimation were investigated.

For a large selection of individual frames of real Doppler signals, the power spectrum was first estimated by the AR model using the order recommended (by FPE, AIC, and CAT). Then, the spectrum was re-estimated using a model with orders below and above the recommended one. The results indicated that choosing an order above the "correct" one does not affect the estimated spectrum as much as underestimating the order. In some cases, even doubling the order did not have significant effect on the resulting spectral estimate. The same procedure was carried out with individual frames of the simulated AR processes of section 5 with similar results. It seems than that although the estimation of the orders did not work well, when a model of order *above* the true order of the underlying process is used, the spectral density estimates are similar to the true power spectrum of the process. The costs associated with

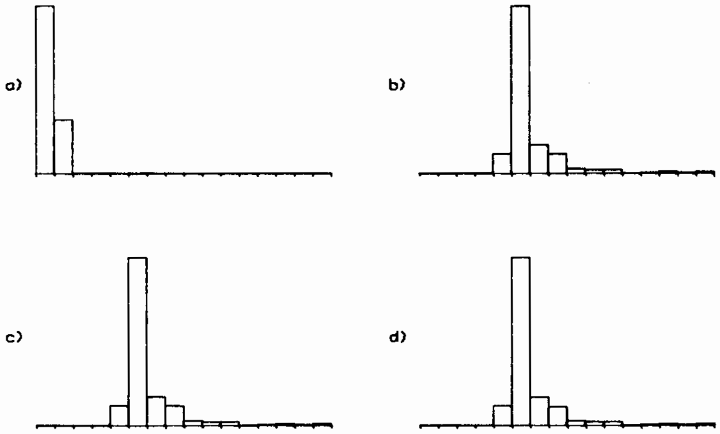

Fig. 11. The normalized histograms corresponding to the "best orders" found by (a) FZC, (b) FPE, (c) AIC, and (d) CAT for the simulation of 256 realizations of an AR process created with the six coefficients of frame 60 from signal 1 (marked on Fig. 9), using 256 samples/frame.

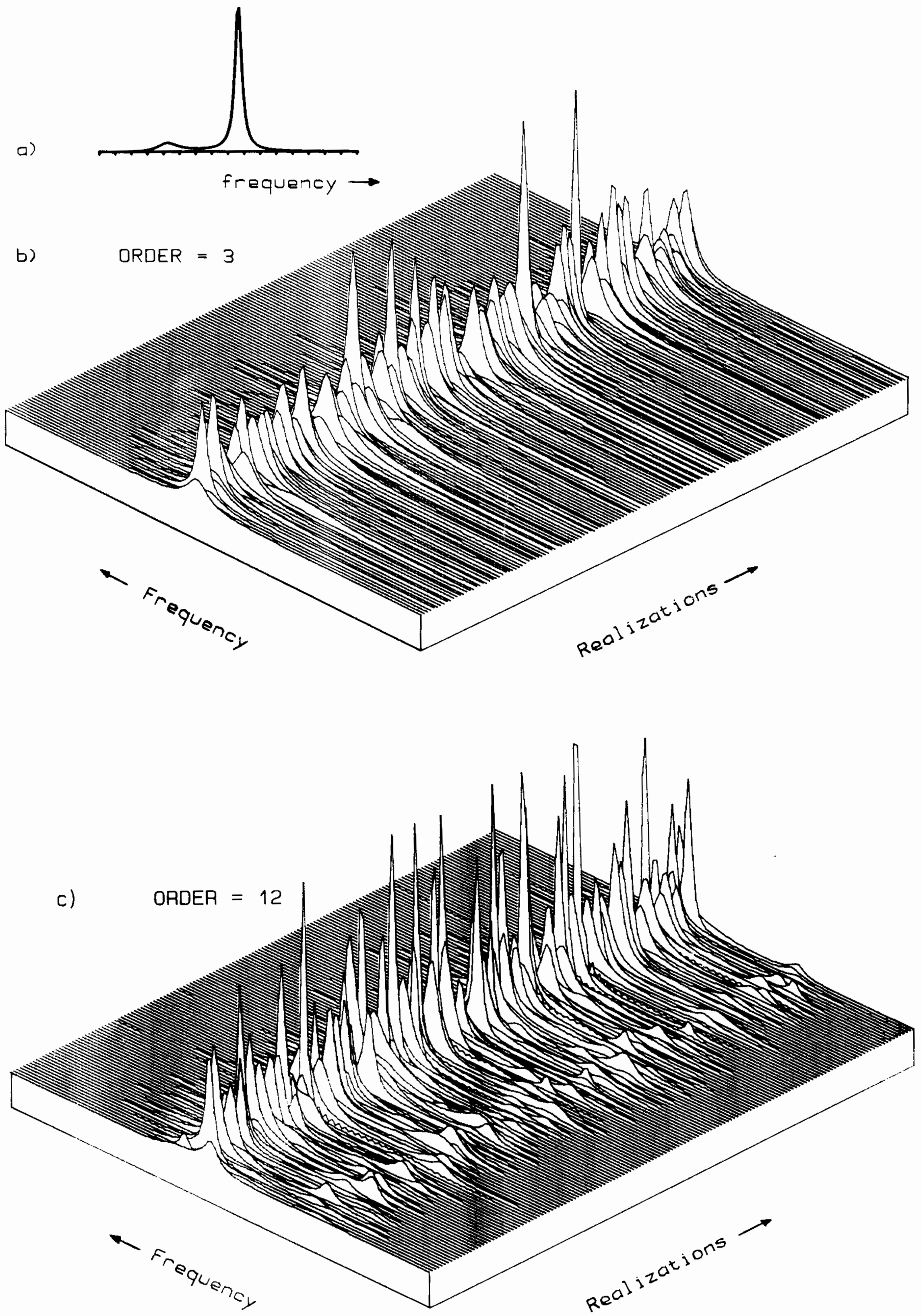

Fig. 12. (a) The theoretical AR power spectral density (eqn 2) corresponding to the six coefficients obtained from frame 60 of signal 1, and the sequence of spectral estimates obtained using a fixed order, $p = 3$ (b), and $p = 12$ (c), on 150 realizations of 64 samples of the process.

using a higher order for the model are that the computation time increases (Schlindwein and Evans 1989), and also, if an integer word Digital Signal Processor is used to estimate the AR coefficients recursively (for real-time applications), the errors associated with the computation of the coefficients become greater.

To illustrate the statistical variability of using short length records (64 samples) and the effect of choosing a fixed order smaller or bigger than the recommended one (the true AR order was $p = 6$ in this case), the maximum entropy AR spectral estimation has been applied to the first 150 realizations of the simulation corresponding to Fig. 9, but using a *fixed* order, $p = 3$ and again for $p = 12$. The theoretical true AR amplitude spectrum (calculated using eqn 2 with the true coefficients) is shown in Fig. 12(a) and the evolution of the spectrum estimates (using $p = 3$ and $p = 12$ and eqns 3 and 4) are shown in Figs. 12(b) and 12(c), respectively. From the figure it is seen that for $p = 3$ the main peak is present, but the smaller one is not. When using order $p = 12$ the general pattern of two peaks and their approximate frequencies are revealed, but there is still considerable degree of variation from one realization to the next.

8. SUMMARY

Autoregressive modelling requires, as part of the implementation procedure, the choice of the AR model order. Four previously described techniques to estimate the order of autoregressive processes were implemented, their general behaviour in estimating the order of Doppler signals was observed, and their performance in estimating the order of truly AR processes, evaluated. Three of the four techniques behave in a very similar manner. These are the "final prediction error," the "information criterion," and the "criterion autoregressive transfer-function." The "first zero crossing" technique appears to underestimate the order in the cases studied.

For the Doppler signals used in this study the order selected by FPE, AIC, and CAT ranged from 2 to 10 in 98% of the frames. Seventeen was the maximum value selected for the order of a particular frame of the set of signals used. The orders selected do not seem to correlate with the spectral characteristics of the Doppler signal and fluctuate erratically from frame to frame. In order to study the statistical behaviour of the orders selected according to the various techniques, further studies using true AR processes were carried out.

In six simulations in which 1000 realizations of 64 samples of true AR processes (with orders ranging from 5 to 7 and using coefficients found from real Doppler signals) none of the techniques performed very well in estimating the correct order (Figs. 8–10). Indeed, wrong order identification occurred more frequently than correct order identification. Wrong order selection seems to be due to the statistical variations of the process when very few samples of it are used rather than any intrinsic problem of the techniques—when larger frames are used the order selection gets better.

The effects of choosing the wrong order for the AR model on the power spectral estimates were investigated and our results indicate that if a sufficiently high order is selected, the spectral estimate is stable and corresponds to the underlying AR process. Therefore although it is difficult to correctly identify the model order for a short segment of Doppler signals, AR spectral estimation can still be used to derive a good spectral estimate of that segment.

9. CONCLUSION

It seems that none of the four techniques for AR model order estimation tested works well for short segments of Doppler signals. When larger frames are used, FPE, AIC, and CAT tend to select the correct AR model order. Overestimating the order of the AR model introduces less error in the spectral estimation of the signal than underestimating it. The practical implication of this is that, although the model order selection techniques could be implemented in real-time, a fixed order implementation seems to be a reasonable approach, and an order of 10 to 12 should produce good results for Doppler signals (if the sampling rate is correctly chosen), since this would correspond to overestimating the AR recommended order.

Acknowledgements—F.S.S. wishes to acknowledge the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) and the Federal University of Rio de Janeiro (UFRJ) for financial support. The authors gratefully acknowledge Aphrodite M'Cullough for providing the cardiac signals and Martin Warriner for the help with the graphics.

REFERENCES

Akaike, H. Fitting autoregressive models for prediction. Ann. Inst. Stat. Math. 21:243–247; 1969.

Akaike, H. A new look at the statistical model identification. IEEE Trans. Automatic Control AC-19:716–723; 1974.

Box, G. E. P.; Jenkins, G. M. Time series analysis forecasting and control. Oakland, CA: Holden-Day; 1976.

Evans, D. H.; McDicken, W. N.; Skidmore, R.; Woodcock, J. P. Doppler ultrasound—Physics, instrumentation, and clinical applications. Chichester: John Wiley & Sons Ltd; 1989.

Jones, R. J. Identification and autoregressive spectrum estimation. IEEE Trans. Automatic Control AC-19:894–897; 1974.

Kaluzynski, K. Analysis of application possibilities of autoregressive modelling to Doppler blood flow signal spectral analysis. Med. Biol. Eng. Comput. 25:373–376; 1987.

Kashyap, R. L. Inconsistency of the AIC rule for estimating the order of autoregressive models. IEEE Trans. Automatic Control AC-25:996–998; 1980.

Kay, S. M.; Marple Jr., S. L. Spectrum analysis—a modern perspective. Proc. IEEE 69:1380–1419; 1981.

Kitney, R. I.; Giddens, D. P. Linear estimation of blood flow waveforms measured by Doppler ultrasound. In: Salamon, R.; Blum, B.; Jorgensen, M., eds. MEDINFO 86. North Holland: Elsevier Science Publishers B.V.; 1986:672–677.

Kitney, R. I.; Talhami, H.; Giddens, D. P. The analysis of blood velocity measurements by autoregressive modeling. J. Theor. Biol. 120:419–442; 1986.

Parzen, E. Multiple time series: determining the order of approximating autoregressive schemes. Tech. Rep. No. 23. Buffalo: Statis. Sc. Div., State Univ. of New York; 1975.

Schlindwein, F. S.; Smith, M. J.; Evans, D. H. Spectral analysis of Doppler signals and computation of the normalized first moment in real-time using a Digital Signal Processor. Med. Biol. Eng. Comput. 26:228–232; 1988.

Schlindwein, F. S.; Evans, D. H. A real-time autoregressive spectrum analyzer for Doppler ultrasound signals. Ultrasound Med. Biol. 15:263–272; 1989.

Vaitkus, P. J.; Cobbold, R. S. C.; Johnston, K. W. A comparative study and assessment of Doppler ultrasound spectral estimation techniques. Part II methods and results. Ultrasound Med. Biol. 14:673–688; 1988.

VanDenBos, A. Alternative interpretation of maximum entropy spectral analysis. IEEE Trans. Inform. Theory IT-17:493–494; 1971.

APPENDIX

Description of the technique employed to implement the simulated realizations of the autoregressive processes used to evaluate the performance of the model order selection techniques:

1. An array of 256 "pseudonormal" numbers, $R(i)$ was generated, each of them created by adding 16 pseudorandom values uniformly distributed:

$$R(i) = \sum_{k=1}^{16} (\text{FORTRAN random numbers}) \quad i = 1 \text{ to } 256,$$

2. $R(i)$ was demeaned $\quad i = 1$ to 256,
3. $R(i)$ was submitted to the AR filter described by the p coefficients, $a(k)$, generating the autoregressive sequence:

$$A(i) = R(i) \quad i = 1 \text{ to } p,$$

$$A(i) = A(i) - \sum_{k=1}^{p} a(k)A(i-k) \quad i = p+1 \text{ to } 256,$$

4. The last 64 samples of $A(i)$ were demeaned and used as the realization of the AR process.

The sequence 1 to 4 was performed 1000 times to generate the 1000 realizations of the AR processes used in the simulations whose results are presented in Figs. 8 to 10. For the simulation corresponding to frames of 128 and 256 samples, 1024 "pseudonormal" numbers were generated and submitted to the AR filter. The last 128 (256) were used as the realizations of the AR processes mentioned in section 6. Only 256 such realizations were used in section 6.