Theoretic analysis and experimental investigation of induced polarization effect of rocks/minerals

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Abstract: This paper considers the polarization phenomena and measurement techniques of rock materials in the view of system identification and parameter estimation. Induced Polarization method is used to measure the rock materials, which is a scheme for using electric fields in the earth to estimate physical structure. These fields are generated by the deliberate injection of electrical currents into the earth. The effective conductivity of rock materials for alternating current is not in general constant, but is variable and complex. The complex conductivity in the frequency domain gives rise to the overvoltage or induced polarization effect in the transient time domain. Usually traditional Cole-Cole model is used to describe the conductivity of rock materials in the frequency domain. But practical measuring data in the frequency domain proves that Cole-Cole model is not the best description of the characteristics of the polarization phenomena of rock materials. Also it is difficult to discriminate between a valid induced polarization (IP) response and electromagnetic (EM) coupling effects, which are caused by IP measurement and the complexity of the rock materials. Therefore we use a system identification method to find an adequate IP system model. Summarizing the analysis of the IP system model, it is concluded that the proposed IP system model fits the field data much better than the traditional Cole-Cole model. EM coupling effects can be removed using system identification method and the induced polarization effect in the transient time domain can be presented in Simulink via the IP system model.

Introduction

When current is applied to the ground, the measured voltage first rises rapidly and then approaches E_0 asymptotically. Similarly, when a steady current flowing in the ground is suddenly terminated, the voltage E_0 between two grounded electrodes drops abruptly to a small polarization voltage E_p, and then decreases asymptotically to zero (Fig .1).

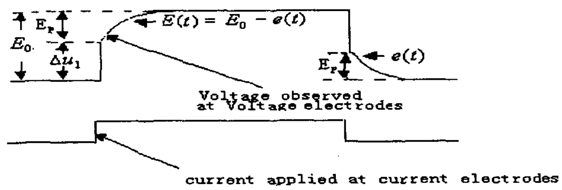
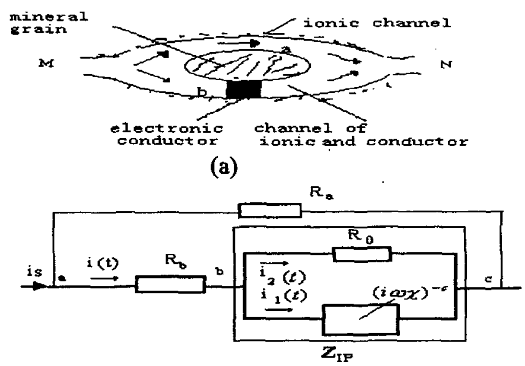


Figure 1: Ground response to a square wave signal
The ratio of E_p to E_0 is seldom more than a few percent

First we can discuss the IP phenomenon from the standpoint of the dynamic equivalent circuit. Let us investigate a small unit within a mineralized rock or a volume polarizable body (Fig 2a) following the method of [1-2]. In this unit, the host mineral acts as an insulator. Ionic conducting (fissure water) channels can either be blocked or not by electronic-conduction mineral grains. Only those channels that are not blocked (we will call them a-channels) have pure resistance R_a. Except for the resistance R_b of the ionic and electronic conductor, the blocked channels (b-channels) are connected by an equivalent impedance Z_{IP} of the surface polarization of the conducting grains.



(b)

Figure 2: (a) Polarizable element of mineralized rock and (b) its equivalent circuit

The frequency behaviour of the complex IP impedance of Fig 2a can be described by an equivalent network as Fig 2b.

From the parallel and series relationship of impedance, the frequency-domain impedance of the equivalent network can be obtained as

$$Z(j\omega) = Z(0)\{1 - m[1 - \frac{1}{1 + (j\omega\tau)^c}]\} \qquad (1)$$

where

$Z(j\omega)$: complex resistance;

$Z(0)$: resistance at zero frequency ($\omega = 0$);

m : limited polarizability (or chargeability);

τ : time constant,

c : a parameter associated with the frequency dependence.

Model (1) is called the Cole-Cole model because it is first proposed by K.S. Cole and R.H. Cole in [3]. The Cole-Cole representation has gained popularity as the model to determine time constants for the separation of responses due to clays, graphite and metallic-cluster (sulfide) minerals, but practical measuring data in frequency domain proves that Cole-Cole model is not the best description of the characteristics of the polarization phenomena of rock materials. Also it is difficult to discriminate between a valid induced polarization (IP) response and electromagnetic (EM) coupling effects, which are caused by IP measurement and the complexity of the rock materials.

If we consider the current as input, the overvoltage as output and the ground as an IP geophysical system, the theory of system can be used to analysis the IP phenomenon.

The overvoltage function, e(t), can be approximated very well by the following

$$e(t) \cong E_0 \sum_{n=1,2,3...} A_n \exp(-\alpha_n t)) \qquad (2)$$

where

$E_0 = E(t)$

$(t \to \infty)$

The response of the IP geophysical system to a step function is then

$$E(t) \cong E_0(1 - \sum_{n=1,2,3...} A_n \exp(-\alpha_n t)) \qquad (3)$$

The choice of the time factors depends on the form of the E(t) curves. Having selected suitable A_n and α_n factors, the frequency function can be written down immediately. The Laplace transform of .(2) gives

$$E(s) = E_0(\frac{1}{s} - \sum_{n=1,2,3...} \frac{A_n s}{s + \alpha_n}) \qquad (4)$$

then

$$G(s) = \frac{E(s)}{E_0(s)} = 1 - \sum_{n=1,2,3...} \frac{A_n s}{s + \alpha_n} \qquad (5)$$

where G(s) is the transfer function of the IP geophysical system.

The frequency-response characteristics of a system can be obtained directly from the sinusoidal transfer function, ie, the transfer function in which s is replaced by jw, where w is frequency.

Under the sinusoidal input, the output can be found by substituting jw for s in G(s) as follows

$$E(jw) = E_0(jw)(1 - \sum_{n=1,2,3...} \frac{A_n jw}{jw + \alpha_n}) \qquad (6)$$

For example, 20 per cent pyrite and per cent andesite plus 5% 0.01 N NaCl solution, the overvoltage-time curve is well approximated by the following.

$$E(t) = E_0(1 - 0.29\exp(-5.7t) - 0.11\exp(-40t) - 0.10\exp(-300t)) \qquad (7)$$

The corresponding frequency function is then

$$E(jw) = E_0(jw)(1 - 0.29\frac{jw}{5.7 + jw} - 0.11\frac{jw}{40 + jw} - 0.10\frac{jw}{300 + jw}) \qquad (8)$$

In practice, for time intervals between one and several tens of seconds, two exponential terms are sufficient to account for the IP phenomenon [4]. So a second-order IP system model is sufficient to analyse the IP phenomenon. In fact, The Cole-Cole model is essentially an infinite dimensional model, but the correlation analysis of the second-order IP system model and the Cole-Cole model shows Cole-Cole model can be approximated by a second-order IP system model very well.

Least Squares Approach

Assume $Z(j\omega)$ can be approximated by a finite dimensional model $Z_{fd}(j\omega)$.

$$Z(j\omega) \approx Z_{fd}(j\omega) = \frac{b_{n+1}(j\omega)^n + b_n(j\omega)^{n-1} + b_{n-1}(j\omega)^{n-2} + ... + b_1}{(j\omega)^n + a_n(j\omega)^{n-1} + ... + a_1} \qquad (9)$$

We use the data produced from the Cole-Cole model to identify the parameters in the finite model Z_{fd}.

Denote $z_i = Z(j\omega_i), \quad i = 1,2,...$ Clearing the denominator of (9), we have an over-determined equation

$$MX = B \qquad (10)$$

where

$$X = (a_1, a_2, ..., a_n, b_1, b_2, ..., b_{n+1})^T$$

Let N be the sampling data size. We may express M and B as the following

$$M = \begin{bmatrix} P_1^1 & P_1^2 \\ Q_1^1 & Q_1^2 \\ \vdots & \vdots \\ P_N^1 & P_N^2 \\ Q_N^1 & Q_N^2 \end{bmatrix} \qquad B = \begin{bmatrix} R_1 \\ I_1 \\ \vdots \\ R_N \\ I_N \end{bmatrix} \qquad (11)$$

where

$R_i = \mathrm{Re}(z_i(j \times w_i)^n);$

$I_i = \mathrm{Im}(z_i(j \times w_i)^n); \qquad i = 1,2,..., N$

$$P_i^1 = -\mathrm{Re}(z_i\underbrace{(1, j\omega_i, (j\omega_i)^2, ..., (jw_i)^n)}_{n+1});$$

$$P_i^2 = \mathrm{Re}(\underbrace{1, j\omega_i, (j\omega_i)^2, ..., (jw_i)^{n-1}}_{n}).$$

$$Q_i^1 = -\mathrm{Im}(z_i\underbrace{(1, j\omega_i, (j\omega_i)^2, ..., (jw_i)^n)}_{n+1});$$

$$Q_i^2 = \mathrm{Im}(\underbrace{1, j\omega_i, (j\omega_i)^2, ..., (jw_i)^{n-1}}_{n}).$$

Under the non-singularity assumption that M has full column rank, which can be easily satisfied and reported automatically in computation, the least square approximate solution of (10) will be

$$X = (M^T M)^{-1} M^T B \tag{12}$$

We use (12) as the coefficients of the finite dimensional approximation to the Cole-Cole model.

The following is an example. The data are from [3]

Example 1 Let $Z_0 = 1.0$, $m = 0.81$, $\tau = 6.9$, $c = 0.85$, and choose third order model.

Using the data from Cole-Cole model, we get the following model:

$$Z_{fd}(j\omega) = \frac{5.5844 + 5.6736\ (j\omega) + 0.1903\ (j\omega)^2}{5.7483 + 28.011\ (j\omega) + (j\omega)^2}$$

We propose the following method for correlation analysis:

Let

$$X_i = Z_{fd}(j\omega_i),\ i = 1,2,...,\ N.$$

and

$$Y_i = Z(j\omega_i), i = 1,2,..., N.$$

Then both

$$X = (X_1, X_2, ..., X_N)^T \in R^N$$

and

$$Y = (Y_1, Y_2, ..., Y_N)^T \in R^N$$

Then we may use the correlation of X and Y in R^N to test the 'goodness of fit' between the two models. That is

$$r = \frac{< X, Y >}{|X||Y|} \tag{13}$$

The correlation coefficient of Cole-Cole model and the second-order IP system model in example 2.1 is

r =0.9979.

Next, we consider the effect of the order of the model. The following example is one of many simulations with similar results.

Example 2 Let $Z_0 = 1.0$, $m = 0.70$, $\tau = 15.9$, $c = 0.45$, the frequency be $2^{-16} \le \omega \le 2^8$, the sampling data size $N = 25$, and using the general algorithm with (9)-(12).

(a) Let the order be 1. Then the parameters obtained are

$$b_1 = 0.2585, b_2 = 0.3627, a_1 = 0.2914$$

with correlation $r = 0.9889$

(b) Let the order be 2. Then the parameters obtained are

$b_1 = 0.2909$, $b_2 = 1.5949$, $b_3 = 0.3418$

$a_1 = 0.3193$, $a_2 = 3.5228$

with correlation $r = 0.9950$

(c) Let the order be 3. Then the parameters obtained are

$b_1 = 0.2830$, $b_2 = 3.9947$, $b_3 = 3.4100$, $b_4 = 0.3319$

$a_1 = 0.3053$, $a_2 = 7.4967$, $a_3 = 8.6556$

with correlation $r = 0.9973$

The second order approximation to Cole-Cole model is good enough, so we choose it as the IP system model

$$Z_{fd}(j\omega) = \frac{0.2909 + 1.5949\ (j\omega) + 0.3418\ (j\omega)^2}{0.3193 + 3.5228\ (j\omega) + (j\omega)^2}$$

Based on the examples, some of which are not shown here, we can conclude the following:

Remark

1. The Cole-Cole model is an infinite dimensional linear model. Since, as shown above, the terms of order greater than 2 on the finite dimensional approximations add only a small contribution to the correlation of the finite dimensional model and the full infinite dimensional model, we advocate the use of a simplified finite dimensional model of order 2 for analysis.

2. The proper model to approximate the Cole-Cole model should have the same order on both the numerator and the denominator polynomials.

In the next section, the arguments in the above remark will be reconfirmed.

It is noticeable that the IP model fits the field data much better than the traditional Cole-Cole model. Using the field data and the resulting Cole-Cole model provided by [5], we give the comparison results of the 'goodness of fit' to the field data of both the Cole-Cole model and the new IP system model in figure 3.

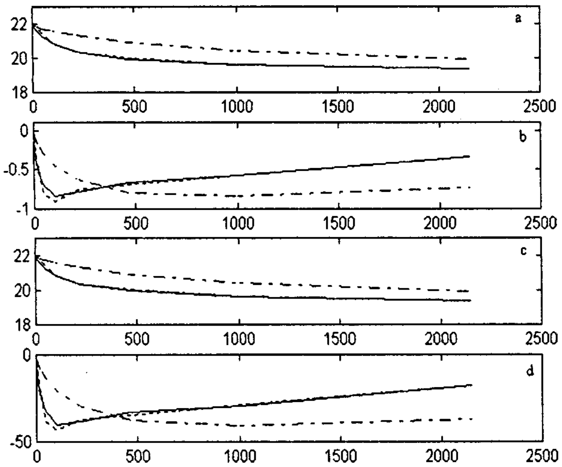


Figure 3: Comparison of the 'goodness of fit' between the traditional Cole-Cole model and IP system model (Full line: field data, Dot-and-dash line: Cole-Cole model, Dashed-line: IP system model) (a) Real part, (b) Imaginary part, (c) Amplitude, (d) Phase.

Mixed IP-EM Model

The electromagnetic (EM) coupling effect is a strong disturbance of the IP anomaly. It hinds the application and the investigation depth of the IP method, especially in the areas covered by a low resistivity stratum, or in other research such as looking for oil deposits, gas field, coal fields and underground water using weak IP anomaly.

An IP-EM mixed model can be expressed by

$$E_0(1-\sum_{k=1}^{n}\frac{A_k s}{s+a_k}+\sum_{k=1}^{m}\frac{B_k s}{s+b_k}),\quad A_k>0,\quad B_k>0 \tag{14}$$

The IP model can be expressed as

$$IP(s)=E_0(1-\sum_{k=1}^{n}\frac{A_k s}{s+a_k}),\quad A_k>0 \tag{15}$$

and the EM model as

$$EM(s)=E_0\sum_{k=1}^{m}\frac{B_k s}{s+b_k},\quad B_k>0 \tag{16}$$

It follows that the discrimination between a valid IP response and electromagnetic (EM) coupling effects can be achieved by the following steps:

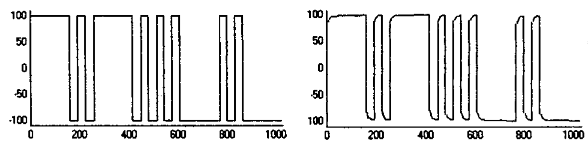
Determine the order and identify parameters of (9) by apparent spectral data using system identification techniques, as proposed in the above sections.

Convert (9) into (14), in which the negative Laplace transform terms ($-A_k<0$), present the IP effects. Collecting them to form the IP model as (15). The positive Laplace transform terms ($B_k>0$), which represent electromagnetic effects, form the EM model as (16).

Let's now consider a practical example. According to [4], the IP model corresponding to a sample measurement of galena and bentonite is

$$G(s)=1-\frac{0.1s}{s+\frac{1}{5.5}}-\frac{0.023\,s}{s+\frac{1}{62}} \tag{17}$$

Its tri-frequency pseudo-random input and response curves are shown in the following Figure 4.



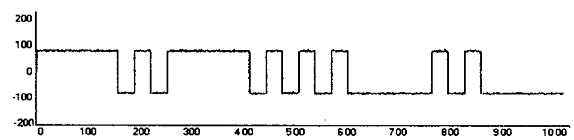
(a) (b)

Figure 4: (a) Tri-frequency pseudo-random input. (b) Tri-frequency pseudo-random response.

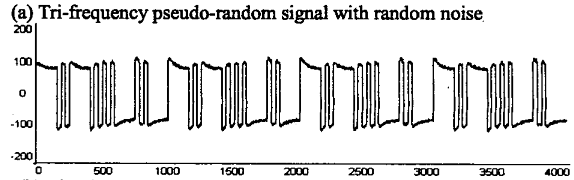
Choose an EM model as

$$EM(s)=\frac{0.3s}{s+0.02}, \tag{18}$$

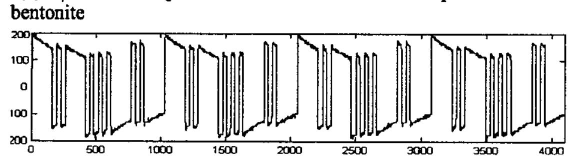
We add (18) to (17) and get the IP-EM mixed model. The simulation results of the IP-EM mixed model with a random noise is shown if Fig 5a and b. A set of field measuring data (Fig 5c) shows that EM exists in practical field measurement, which lays emphasis on the importance of discriminating between the valid IP response and electromagnetic (EM) coupling effects.



(a) Tri-frequency pseudo-random signal with random noise



(b) Simulation output of IP-EM mixed model. Sample: Galena and bentonite



(c) Field data: Pb-Zn

Figure 5: Comparison of simulation result and field measurement

Inversely, we can use the Least Squares Approach and the simulated data in Fig 5a and 5b to get (17) and (18).

Conclusions

A finite dimensional approximation to the Cole-Cole model was considered. Using the least square approach directly, the parameters for the approximated models were obtained. Certain properties of the IP model have been revealed via the finite dimensional approximation of the Cole-Cole model. These properties have been reconfirmed by the overvoltage-based approach to the IP model [3].

Using field measurement data, the EM model was discovered. The simulation results and the theoretical analysis confirmed the IP- EM model.

A new method has been proposed to discriminate between the valid IP response and electromagnetic (EM) coupling effects. Both the theoretical analysis and the simulation results show that the new method is simple and effective.

Acknowledgments

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