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A new method to discriminate between a valid IP response and EM coupling effects

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ABSTRACT

The problem of discrimination between a valid induced polarization (IP) response and electromagnetic (EM) coupling effects is considered and an effective solution is provided. First, a finite dimensional approximation to the Cole-Cole model is investigated. Using the least-squares approach, the parameters of the approximate model are obtained. Next, based on the analysis of overvoltage, a finite dimensional structure of the IP model is produced. Using this overvoltage-based structure, a specific finite dimensional approximation of the Cole-Cole model is proposed. Summarizing the analysis of the finite dimensional IP model, it is concluded that the proposed IP model, which fits the field data much better than the traditional Cole-Cole model, is essentially an RC-circuit. From a circuit-analysis point of view, it is well known that an electromagnetic effect can be described by an RL-circuit. The simulation results on experimental data support this conception. According to this observation, a new method to discriminate between a valid IP response and EM coupling effects is proposed as follows: (i) use a special finite dimensional model for IP–EM systems; (ii) obtain the parameters for the model using a least-squares approach; (iii) separate RC-type terms and RL-type terms – the first models the IP behaviour, the latter represents the EM part. Simulation on experimental data shows that the method is very simple and effective.

1 INTRODUCTION

Commercial induced polarization (IP) surveys have been carried out for more than 80 years. Gubins (1997) stated: ‘Today research continues on the effects of hydrocarbons and other groundwater contaminants on the IP response. IP is used extensively in the search for precious metals by mapping areas hosting disseminated sulfides that may occur in conjunction with precious metals. Interest has been renewed in porphyry deposits in third-world countries and complex resistivity (CR) or spectral IP is being used in attempts to discern the source of IP response and to discriminate between a valid IP response and electromagnetic (EM) coupling effects’. Theoretical and experimental aspects of IP in both the frequency domain and the time domain show that the ground can be treated as an IP geophysical system when some part of the rock mass becomes electrically polarized. This means that IP–EM coupled systems may be described as a model with which simulations and other techniques in the area of control systems can be realized in computers (Bertin 1976; Loeb 1976; Luo and Zhang 1998).

The Cole-Cole representation has gained popularity as the model to determine time constants for the separation of responses due to clays, graphite and metallic-cluster (sulphide) minerals, so it is appropriate to consider the common characteristics of the Cole-Cole model and the IP system model. The Cole-Cole model can be expressed as

$$Z_{cc}(j\omega) = Z_{cc}(0)\left\{1 - m\left[1 - \frac{1}{1 + (j\omega\tau)^c}\right]\right\}, \tag{1}$$

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where $Z_{cc}(0)$ denotes zero-frequency impedance, m denotes limited polarizability (or chargeability), τ denotes the time constant of surface polarization, and c is a non-integer exponent, which characterizes the frequency dependence.

The Cole-Cole model is essentially an infinite dimensional model (Helbig 1994; Curtain and Zwart 1995). It is linear because we assume that the input $I(s)$, which is the Laplace transform of the current $I(t)$, and the output $V(s)$, which is the Laplace transform of the voltage $V(t)$, satisfy

$$V(s) = Z_{cc}(s)I(s). \tag{2}$$

Intuitively, one can be convinced by the following argument that the Cole-Cole model is an infinite dimensional model:

Assume it is a finite dimensional model. Then, from control theory, we know that the degree of the numerator of the transfer function of a physical system is less than or equal to the degree of the denominator. Thus, Z_{cc} can be expressed for a certain order n as

$$Z_{cc}(j\omega) = d + \frac{b_n(j\omega)^{n-1} + b_{n-1}(j\omega)^{n-2} + \ldots + b_1}{(j\omega)^n + a_n(j\omega)^{n-1} + \ldots + a_1} \tag{3}$$

for some real parameters d, a_i, b_i. But, as long as the exponent c in model (1) is a proper fraction, it is easy to see that (3) can never be exactly true for any choice of parameters.

When an IP–EM coupled system is considered, more than one Cole-Cole model is used to describe it. Usually, models of order 2–3 are used. An order-2 Cole-Cole model is given by (Luo and Zhang 1998)

$$Z_{cc}(j\omega) = Z(0)\left\{1 - m_1\left[1 - \frac{1}{1+(j\omega\tau_1)^{c_1}}\right] + m_2\left[1 - \frac{1}{1+(j\omega\tau_2)^{c_2}}\right]\right\}, \tag{4}$$

where the first part is the IP model and the last term, with respect to m_2, τ_2, c_2, is the EM model.

In order to realize computer-aided analysis and use the techniques developed in system and control theory and signal processing, a finite dimensional approximation to the Cole-Cole model is necessary. In the later investigation, the finite dimensional approximation is also used to reveal some properties of the IP model.

The main purpose of this paper is to propose a new method to discriminate between a valid IP response and the EM coupling effects. The paper is organized as follows. In the next section we use a standard least-squares approach to provide a finite dimensional approximation of the Cole-Cole model. Via this approximation some properties of the IP model are revealed. Section 3 uses the overvoltage description of the IP model (Luo and Zhang 1998) to find a specified finite dimensional model of the IP system. Then in Section 4, based on the specified model, the specified finite dimensional approximation of the Cole-Cole model is constructed and its properties are investigated again. Section 5 discusses a basic property of the IP model, i.e. that the IP model is basically an RC-circuit. In Section 6, based on the simulation of experimental data, the EM model is described by an equivalent RL-circuit. The main result concerning the discrimination between the IP response and the EM effects is discussed in Section 7. A new method is proposed for separating RC-type and RL-type terms. Section 8 is a concluding summary.

2 LEAST-SQUARES APPROACH

As mentioned above, the Cole-Cole model is widely used for IP surveys and a finite dimensional version is more convenient for system analysis and the consequent signal processing. We therefore introduce the method chosen to convert the Cole-Cole model into a finite dimensional IP model. In addition, such an approach reveals some properties of the IP model.

Assume $Z_{cc}(j\omega)$ can be approximated by a finite dimensional model. We consider two types of models:

Type 1:

$$Z_{cc}(j\omega) \approx Z^1_{fd}(j\omega) = \frac{b_n(j\omega)^{n-1} + b_{n-1}(j\omega)^{n-2} + \ldots + b_1}{(j\omega)^n + a_n(j\omega)^{n-1} + \ldots + a_1} \tag{5}$$

and

Type 2:

$$Z_{cc}(j\omega) \approx Z^2_{fd}(j\omega) = \frac{b_{n+1}(j\omega)^n + b_n(j\omega)^{n-1} + b_{n-1}(j\omega)^{n-2} + \ldots + b_1}{(j\omega)^n + a_n(j\omega)^{n-1} + \ldots + a_1}. \tag{6}$$

We use the data produced from the Cole-Cole model to identify the parameters in the finite model Z_{fd}. Let

$$z_i = Z_{cc}(j\omega_i), \quad i = 1, 2, \ldots.$$

Clearing the denominator of (1), we have an overdetermined equation

$$\mathbf{MX} = \mathbf{B}, \tag{7}$$

where, for Type 1,

$$\mathbf{X} = (a_1, a_2, \ldots, a_n, b_1, b_2, \ldots, b_n)^{\mathrm{T}}$$

and for Type 2,

$$\mathbf{X} = (a_1, a_2, \ldots, a_n, b_1, b_2, \ldots, b_{n+1})^{\mathrm{T}}.$$

Let N be the sampling data size. Then **M** and **B** can be expressed as follows:

$$\mathbf{M} = \begin{bmatrix} P_1^1 & P_1^2 \\ Q_1^1 & Q_1^2 \\ \vdots & \vdots \\ P_N^1 & P_N^2 \\ Q_N^1 & Q_N^2 \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} R_1 \\ I_1 \\ \vdots \\ R_N \\ I_N \end{bmatrix}, \tag{8}$$

where

$R_i = \mathrm{Re}(z_i(\mathrm{j} \times \omega_i)^n), \quad I_i = \mathrm{Im}(z_i(\mathrm{j} \times \omega_i)^n), \quad i = 1, 2, \ldots, N;$

for Type 1:

$$P_i^1 = -\mathrm{Re}(z_i(\underbrace{1, \mathrm{j}\omega_i, (\mathrm{j}\omega_i)^2, \ldots, (\mathrm{j}\omega_i)^{n-1}}_{n})),$$

$$P_i^2 = \mathrm{Re}(\underbrace{1, \mathrm{j}\omega_i, (\mathrm{j}\omega_i)^2, \ldots, (\mathrm{j}\omega_i)^{n-1}}_{n}),$$

$$Q_i^1 = -\mathrm{Im}(z_i(\underbrace{1, \mathrm{j}\omega_i, (\mathrm{j}\omega_i)^2, \ldots, (\mathrm{j}\omega_i)^{n-1}}_{n})),$$

$$Q_i^2 = \mathrm{Im}(\underbrace{1, \mathrm{j}\omega_i, (\mathrm{j}\omega_i)^2, \ldots, (\mathrm{j}\omega_i)^{n-1}}_{n});$$

for Type 2:

$$P_i^1 = -\mathrm{Re}(z_i(\underbrace{1, \mathrm{j}\omega_i, (\mathrm{j}\omega_i)^2, \ldots, (\mathrm{j}\omega_i)^{n}}_{n+1})),$$

$$P_i^2 = \mathrm{Re}(\underbrace{1, \mathrm{j}\omega_i, (\mathrm{j}\omega_i)^2, \ldots, (\mathrm{j}\omega_i)^{n-1}}_{n}),$$

$$Q_i^1 = -\mathrm{Im}(z_i(\underbrace{1, \mathrm{j}\omega_i, (\mathrm{j}\omega_i)^2, \ldots, (\mathrm{j}\omega_i)^{n}}_{n+1})),$$

$$Q_i^2 = \mathrm{Im}(\underbrace{1, \mathrm{j}\omega_i, (\mathrm{j}\omega_i)^2, \ldots, (\mathrm{j}\omega_i)^{n-1}}_{n}).$$

Under the non-singularity assumption that **M** has full column rank, which can be easily satisfied and reported automatically in computation, the least-squares approximate solution of (4) will be

$$\mathbf{X} = (\mathbf{M}^{\mathrm{T}}\mathbf{M})^{-1}\mathbf{M}^{\mathrm{T}}\mathbf{B}. \tag{9}$$

We use (9) as the coefficients of the finite dimensional approximation to the Cole-Cole model.

In the following example the data are taken from Luo and Zhang (1998).

Example 2.1. Let $Z_0 = 1.0$, $m = 0.81$, $\tau = 6.9$, $c = 0.85$ and choose a third-order model given by

$$\text{Type 1: } Z_{\mathrm{fd}}(\mathrm{j}\omega) = \frac{b_1 + b_2(\mathrm{j}\omega) + b_3(\mathrm{j}\omega)^2}{a_1 + a_2(\mathrm{j}\omega) + a_3(\mathrm{j}\omega)^2 + (\mathrm{j}\omega)^3}. \tag{10}$$

Let the frequency be $2^{-16} \le \omega \le 2^8$ and the sampling data size $N = 25$. The method proposed in (5)–(9) for Type 1 provides the following solution:

$$Z_{\mathrm{fd}}(\mathrm{j}\omega) = \frac{50.0925 + 167.8221(\mathrm{j}\omega) + 82.2318(\mathrm{j}\omega)^2}{50.9865 + 475.3094(\mathrm{j}\omega) + 420.5262(\mathrm{j}\omega)^2 + (\mathrm{j}\omega)^3}. \tag{11}$$

Note that the coefficient of the cubic term is very small compared with the coefficients of the lower-degree terms. When the degree of the model increases, this effect is more significant. For instance, when $n = 4$ we have $b_1 = 131.5$, $b_2 = 957.7$, $b_3 = 1077.2$, $b_4 = 234.5$, $a_1 = 133$, $a_2 = 1858.6$, $a_3 = 4292.1$ and $a_4 = 1212.1$. This fact appears for all simulations with different parameters. It suggests that the Type 1 model is not adequate because the highest-degree term seems to be unnecessary. Therefore, we use the second type of model and assume

$$\text{Type 2: } Z_{\mathrm{fd}}(\mathrm{j}\omega) = \frac{b_1 + b_2(\mathrm{j}\omega) + b_3(\mathrm{j}\omega)^2}{a_1 + a_2(\mathrm{j}\omega) + (\mathrm{j}\omega)^2}. \tag{12}$$

Using the same data, we get the following model:

$$Z_{\mathrm{fd}}(\mathrm{j}\omega) = \frac{5.5844 + 5.6736(\mathrm{j}\omega) + 0.1903(\mathrm{j}\omega)^2}{5.7483 + 28.011(\mathrm{j}\omega) + (\mathrm{j}\omega)^2}. \tag{13}$$

The simulation results are shown in Fig. 1. It can be seen that the IP model fits the field data much better than the traditional Cole-Cole model. Using the field data and the resulting Cole-Cole model of Jaggar and Fell (1988), Fig. 2 shows the results of comparing the 'goodness of fit' with the field data of both the Cole-Cole model and the new IP system model.

We propose the following method for correlation analysis. Let

$X_i = Z_{\mathrm{fd}}(\mathrm{j}\omega_i), \quad i = 1, 2, \ldots, N$

and

$Y_i = Z_{\mathrm{cc}}(\mathrm{j}\omega_i), \quad i = 1, 2, \ldots, N,$

then both

$\mathbf{X} = (X_1, X_2, \ldots, X_N)^{\mathrm{T}} \in \mathbf{R}^N$

and

$\mathbf{Y} = (Y_1, Y_2, \ldots, Y_N)^{\mathrm{T}} \in \mathbf{R}^N.$

We may then use the correlation of **X** and **Y** in $\mathbf{R}^N$ to test the 'goodness of fit' between the two models, i.e.

$$r = \frac{<\mathbf{X}, \mathbf{Y}>}{|\mathbf{X}||\mathbf{Y}|}. \tag{14}$$

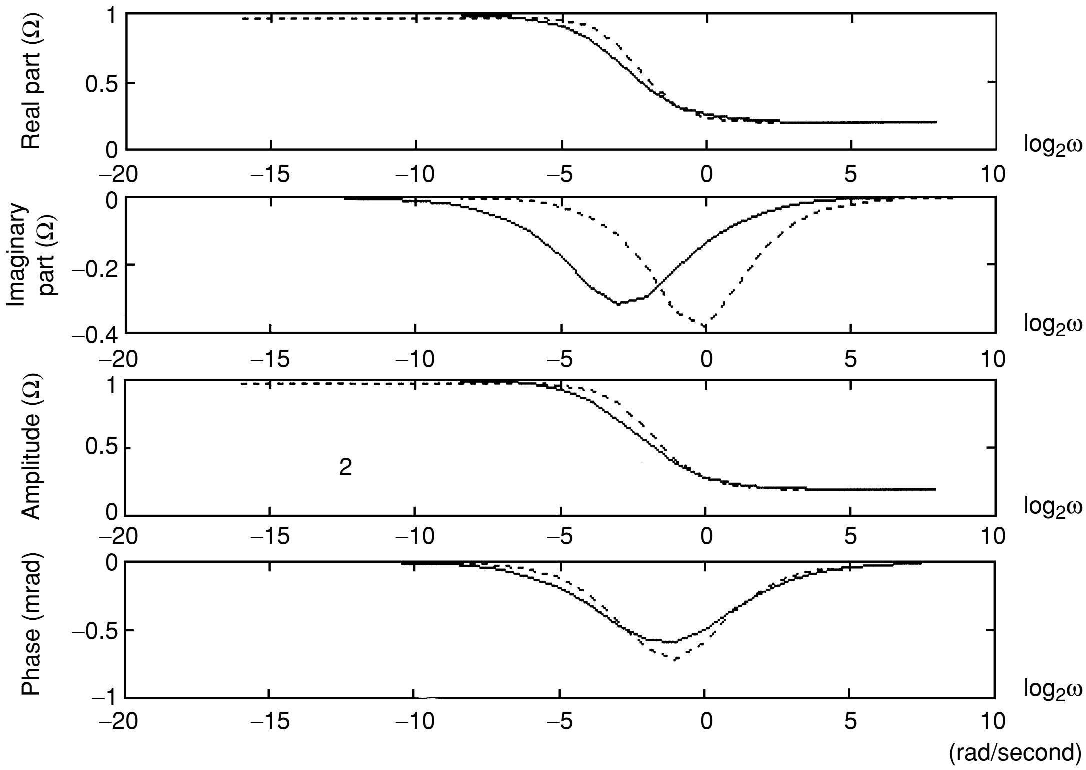

Figure 1 Second-order approximation to Cole-Cole model. Solid line: Cole-Cole model; dashed line: approximated model.

By comparing (13) with (1), using the two models and the data generated, the correlation coefficient is obtained as $r = 0.9979$.

The following weighted cost function may be used to improve the approximation:

$$\min \sum_{k=1}^{N} \lambda (P_k^1 a_k + P_k^2 b_k - B_{2k-1})^2 + (1-\lambda)(Q_k^1 a_k + Q_k^2 b_k - B_{2k})^2, \quad (15)$$

where $0 < \lambda < 1$. Let

$$\mathbf{M}_P = \begin{pmatrix} P_1^1 & P_1^2 \\ P_2^1 & P_2^2 \\ \vdots & \vdots \\ P_{N-1}^1 & P_{N-1}^2 \\ P_N^1 & P_N^2 \end{pmatrix}, \quad \mathbf{M}_Q = \begin{pmatrix} Q_1^1 & Q_1^2 \\ Q_2^1 & Q_2^2 \\ \vdots & \vdots \\ Q_{N-1}^1 & Q_{N-1}^2 \\ Q_N^1 & Q_N^2 \end{pmatrix},$$

and

$$\mathbf{B}_P = (R_1, R_2, \ldots, R_N),$$

$$\mathbf{B}_Q = (I_1, I_2, \ldots, I_N).$$

It is then a straightforward deduction to show that the least-squares solution of (15) is

$$\mathbf{X} = (\lambda \mathbf{M}_P^T \mathbf{M}_P + (1-\lambda)\mathbf{M}_Q^T \mathbf{M}_Q)^{-1} (\lambda \mathbf{M}_P^T \mathbf{B}_P + (1-\lambda)\mathbf{M}_Q^T \mathbf{B}_Q). \quad (16)$$

We use

$$E_R = \sqrt{\sum_{i=1}^{N} (\mathrm{Re}(X_i) - \mathrm{Re}(Y_i))^2}$$

and

$$E_I = \sqrt{\sum_{i=1}^{N} (\mathrm{Im}(X_i) - \mathrm{Im}(Y_i))^2}$$

to represent the approximation errors of the real part and the imaginary part, respectively.

Consider Example 2.1. Table 1 shows the results for different λs. These results show that the correlation increases with the increased weight of the real part and the errors in both the real part and the imaginary part decrease simultaneously. This result has been consistently observed via several simulations with different parameters.

Next, we consider the effect of the order of the model. The following example is one of many simulations with similar results.

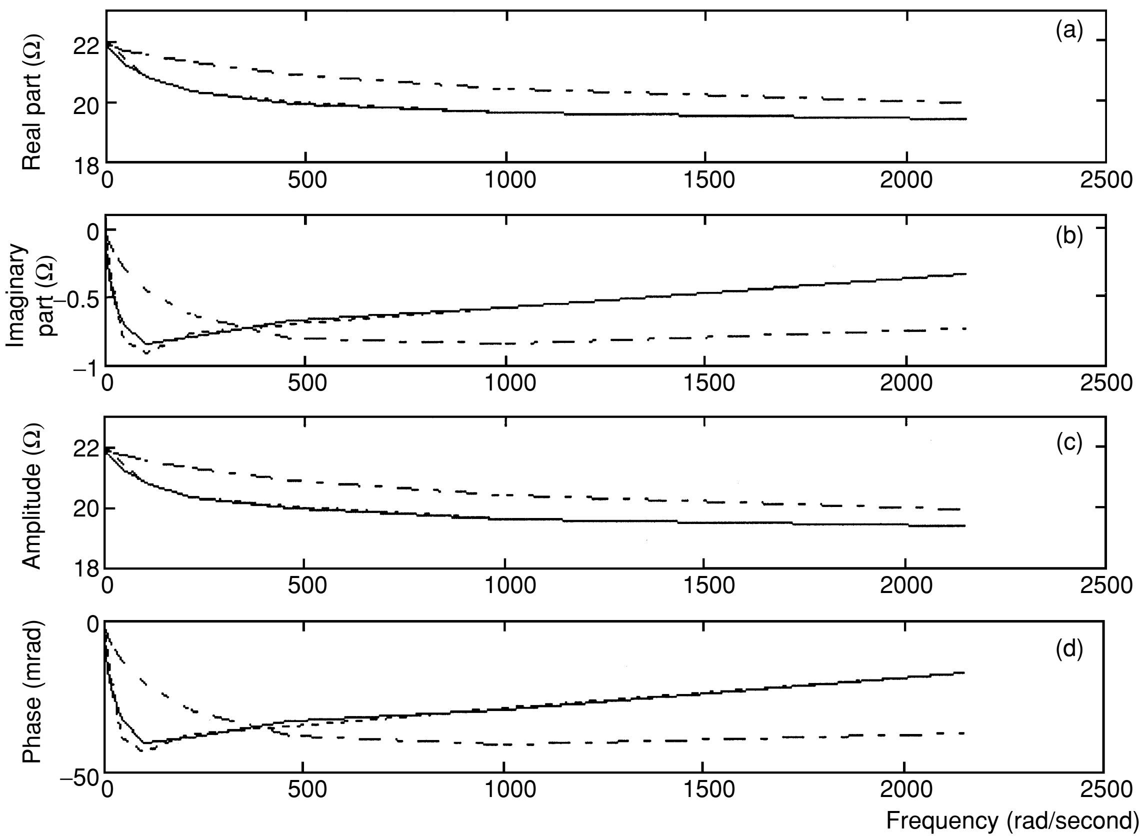

Figure 2 Comparison of the 'goodness of fit' between the traditional Cole-Cole model and the IP system model. Solid line: field data; dot-and-dash line: Cole-Cole model; dashed line: IP system model. (a) Real part; (b) imaginary part; (c) amplitude; (d) phase.

Table 1 Weighted least-squares approximations

λ	r	E_R	E_I
0.2	0.9977	0.2091	0.1587
0.4	0.9978	0.2032	0.1541
0.6	0.9980	0.1954	0.1485
0.8	0.9981	0.1869	0.1427

Example 2.2. Let $Z_0 = 1.0$, $m = 0.70$, $\tau = 15.9$, $c = 0.45$, the frequency be $2^{-16} \leq \omega \leq 2^8$, the sampling data size $N = 25$, and use the general algorithm with equations (6)–(9).
(a) For order 1, the parameters obtained are

$$b_1 = 0.2585, \quad b_2 = 0.3627, \quad a_1 = 0.2914,$$

with correlation $r = 0.9889$.
(b) For order 2, the parameters obtained are

$$b_1 = 0.2909, \quad b_2 = 1.5949, \quad b_3 = 0.3418,$$
$$a_1 = 0.3193, \quad a_2 = 3.5228,$$

with correlation $r = 0.9950$.
(c) For order 3, the parameters obtained are

$$b_1 = 0.2830, \quad b_2 = 3.9947, \quad b_3 = 3.4100, \quad b_3 = 0.3319,$$
$$a_1 = 0.3053, \quad a_2 = 7.4967, \quad a_3 = 8.6556,$$

with correlation $r = 0.9973$.
(d) For order 4, the parameters obtained are

$$b_1 = 0.1894, \quad b_2 = 5.8899, \quad b_3 = 13.5538, \quad b_4 = 5.1858,$$
$$b_5 = 0.3269,$$
$$a_1 = 0.2017, \quad a_2 = 9.7210, \quad a_3 = 30.1695, \quad a_4 = 13.8563,$$

with correlation $r = 0.9985$.

The second-order approximation to the Cole-Cole model is adequate, so we choose it as the IP system model,

$$Z_{\mathrm{fd}}(\mathrm{j}\omega) = \frac{0.2909 + 1.5949(\mathrm{j}\omega) + 0.3418(\mathrm{j}\omega)^2}{0.3193 + 3.5228(\mathrm{j}\omega) + (\mathrm{j}\omega)^2}. \tag{17}$$

Based on the examples given here and many other runs not shown here, we can conclude the following:

1 The Cole-Cole model is an infinite dimensional linear model. Since, as shown above, the terms of order greater than 2 on the finite dimensional approximations add only a small contribution to the correlation of the finite dimensional model and the full infinite dimensional model, we advocate the use of a simplified finite dimensional model of order 2 for analysis.

2 The proper model to approximate the Cole-Cole model should have the same order on both the numerator and the denominator polynomials.

In the next section, the arguments in the above conclusions will be reconfirmed.

3 OVERVOLTAGE APPROACH

In general, in the time domain, the connection between the shape of the complex conductivity curve and that of the overvoltage curve is not simple. The overvoltage, $e(t)$, however, can be approximated very well by the following (Luo and Zhang 1998):

$$e(t) \cong E_0\left(\sum_{n=1,2,3\ldots} A_n \exp(-\alpha_n t)\right), \tag{18}$$

where $E_0 = \lim_{t\to\infty} E(t)$. The relevant quantities are illustrated in Fig. 3 for a rectangular current pulse.

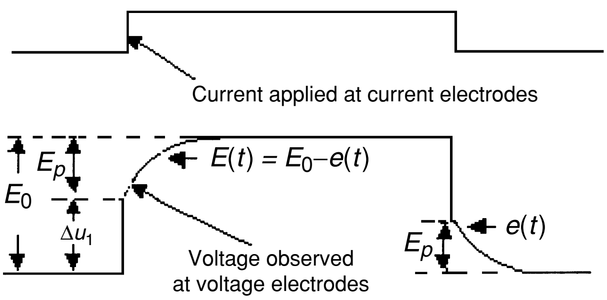

Figure 3 Ground response to a rectangular wave signal. The ratio E_p to E_0 is seldom more than a few per cent.

From (1), it is easy to deduce the response of the IP geophysical system to a step function. Then

$$E(t) \cong E_0\left(1 - \sum_{n=1,2,3\ldots} A_n \exp(-\alpha_n t)\right). \tag{19}$$

It is then also easy to convert the representation in the time domain to that in the frequency domain, described by a transfer function. To be precise, using the Laplace transform, the IP system in the frequency domain is described as follows:

$$G(s) = \frac{E(s)}{E_0(s)} = 1 - \sum_{n=1,2,3\ldots} \frac{A_n s}{s+\alpha_n}. \tag{20}$$

For 20% pyrite and 80% andesite plus 5% 0.01 N NaCl solution, the overvoltage–time curve is well approximated by the following:

$$E(t) = E_0(1 - 0.29\exp(-5.7t) - 0.11\exp(-40t) - 0.10\exp(-300t)). \tag{21}$$

The corresponding transfer function is then

$$G(s) = 1 - \frac{0.29s}{5.7+s} - \frac{0.11s}{40+s} - \frac{0.10s}{300+s}. \tag{22}$$

The IP system can be simulated using Simulink in Matlab. The responses of the IP system in the time and frequency domains can be obtained by driving the simulated IP system with a rectangular input and a pseudo-random input, respectively (van den Bosch and van der Klauw 1994).

Let us now consider a practical example. According to Bertin (1976), the IP model corresponding to a sample measurement of galena and bentonite is

$$G(s) = 1 - \frac{0.1s}{s+1/5.5} - \frac{0.023s}{s+1/62}. \tag{23}$$

Its tri-frequency pseudo-random input and response curves are shown in Fig. 4.

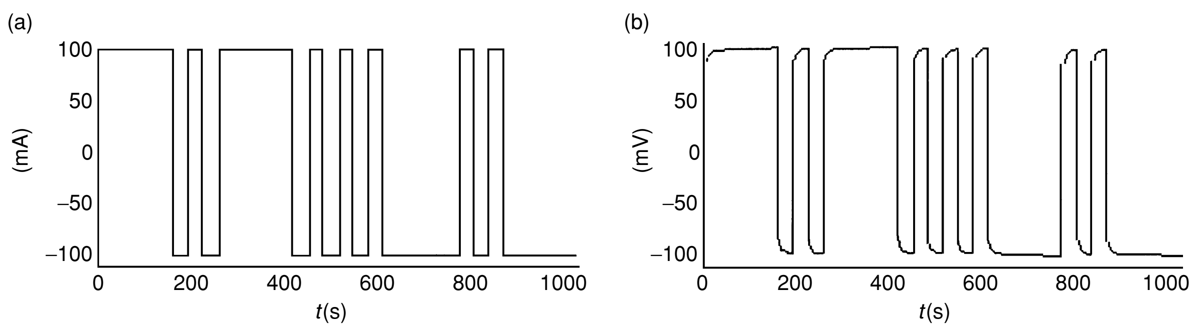

Figure 4 (a) Tri-frequency pseudo-random input; (b) tri-frequency pseudo-random response.

In fact, by performing a Laplace transform, (19) can be written as

$$Z(s) = \frac{E(s)}{I(s)} = Z(0)\left(1 - \sum_{n=1,2,3\ldots} \frac{A_n s}{s+\alpha_n}\right). \quad (24)$$

The limited polarizability can be calculated as (Luo and Zhang 1998)

$$m = \frac{Z(0) - Z(\infty)}{Z(0)}. \quad (25)$$

Let $s \to \infty$, then

$$Z(\infty) = Z(0)\left(1 - \sum_{n=1,2,3\ldots} A_n\right) = Z(0)(1-m), \quad (26)$$

giving

$$m = \sum_{n=1,2,3\ldots} A_n. \quad (27)$$

Substituting (27) in (24) yields

$$\begin{aligned} Z(s) &= Z(0)\left(1 - \sum_{i=1,2,3\ldots} A_n \left(\sum_{n=1,2,3\ldots} \frac{\frac{A_n}{\sum_{n=1,2,3\ldots} A_n} s}{s+\alpha_n}\right)\right) \\ &= Z(0)\left(1 - m\left(\sum_{n=1,2,3\ldots} \frac{\frac{A_n}{m} s}{s+\alpha_n}\right)\right) \\ &= Z(0)\left(1 - m\left(\frac{C_n s^n + C_{n-1}s^{n-1} + C_{n-2}s^{n-2} + \ldots + C_1 s}{s^n + B_{n-1}s^{n-1} + B_{n-2}s^{n-2} + \ldots + B_1 s + B_0}\right)\right), \end{aligned} \quad (28)$$

where

$$C_n = \frac{A_1}{\sum_{n=1,2,3\ldots} A_n} + \frac{A_2}{\sum_{n=1,2,3\ldots} A_n} + \ldots + \frac{A_n}{\sum_{n=1,2,3\ldots} A_n} = 1. \quad (29)$$

Some new notation is now necessary. Denote

$$C = s^n + C_{n-1}s^{n-1} + C_{n-2}s^{n-2} + \ldots + C_1 s, \quad (30)$$

$$B = s^n + B_{n-1}s^{n-1} + B_{n-2}s^{n-2} + \ldots + B_1 s + B_0, \quad (31)$$

$$\frac{C}{B} = 1 - \frac{1}{1+\frac{D}{E}} = \frac{D}{E+D}. \quad (32)$$

If we choose $D = C$, then $E = B - C$. Substituting D and E in (28), the impedance can be calculated as

$$Z(s) = Z(0)\left\{1 - m\left[1 - \frac{1}{1 + \frac{s^n + C_{n-1}s^{n-1} + C_{n-2}s^{n-2} + \ldots + C_1 s}{(B_{n-1} - C_{n-1})s^{n-1} + \ldots + (B_1 - C_1)s + B_0}}\right]\right\}. \quad (33)$$

By comparing (28) with (1), it can be seen that the apparent spectral parameters $\rho(0)$, m, τ_1, $\tau_2 \ldots \tau_n$ ($\rho = KZ$) (Luo and Zhang 1998), in which $\tau_n = \alpha_n$, $n = 1, 2, \ldots$, and the order of the model should be determined by inversion of the apparent spectrum. We can also use system identification techniques or Bode plots of control system methods to determine the parameters of the finite dimensional approximation.

In practice, for time intervals between one second and several tens of seconds, two exponential terms are sufficient to account for the IP phenomenon (Bertin 1976; Loeb 1976). This can also be seen from (22). Thus a second-order IP system model is sufficient to analyse the IP phenomenon.

The results and arguments in this section thus confirm the conclusions reached at the end of Section 2.

4 OVERVOLTAGE-BASED APPROXIMATION

We now consider the finite dimensional approximation for the Cole-Cole model again. We still use the least-squares approach, but with some predetermined parameters. In fact, we assume that some parameters such as $Z(0)$ and m are fixed, and then we start from the overvoltage-based model (equation (33)). Comparing (33) with (1) yields the following approximation:

$$(\mathrm{j}\omega\tau)^c \approx \frac{(\mathrm{j}\omega)^n + C_{n-1}(\mathrm{j}\omega)^{n-1} + \ldots + C_1(\mathrm{j}\omega)}{E_{n-1}(\mathrm{j}\omega)^{n-1} + \ldots + E_1(\mathrm{j}\omega) + E_0}, \quad (34)$$

where $E_i = B_i - C_i$, $i > 0$ and $E_0 = B_0$.

Using principal values, we have

$$(\mathrm{j})^c = \cos\left(c\frac{\pi}{2}\right) + \mathrm{j} \times \sin\left(c\frac{\pi}{2}\right).$$

In a similar procedure to that in Section 2, an overdetermined equation system is obtained as

$$\mathbf{MX} = \mathbf{B}, \quad (35)$$

where $\mathbf{X} = (E_0, E_1, \ldots, E_{n-1}, C_1, C_2, \ldots, C_{n-1})^{\mathrm{T}}$ are the parameters to be determined. $\mathbf{B}$ is given by

$$\mathbf{B} = \begin{cases} (-1)^{\frac{n-1}{2}}\tau^c(\omega_1^{n+c-1}, 0, \omega_2^{n+c-1}, 0, \ldots, \omega_N^{n+c-1}, 0,), & \text{for } n \text{ odd}, \\ (-1)^{\frac{n-2}{2}}\tau^c(\omega_1^{n+c-1}, 0, \omega_2^{n+c-1}, 0, \ldots, \omega_N^{n+c-1}, 0), & \text{for } n \text{ even}; \end{cases} \quad (36)$$

$\mathbf{M}$ can be expressed as

$$\mathbf{M} = \begin{pmatrix} P_1^1 & P_1^2 \\ Q_1^1 & Q_1^2 \\ \cdot & \cdot \\ P_N^1 & P_N^2 \\ Q_N^1 & Q_N^2 \end{pmatrix}, \quad (37)$$

where $P_k^1, Q_k^1 \in R^n, P_k^2, Q_k^2 \in R^{n-1}$ can be expressed as

$$P_k^1 = \tau^c\Big(\cos\Big(\frac{c\pi}{2}\Big)\omega_k^c, -\sin\Big(\frac{c\pi}{2}\Big)\omega_k^{c+1}, -\cos\Big(\frac{c\pi}{2}\Big)\omega_k^{c+2}, \sin\Big(\frac{c\pi}{2}\Big)\omega_k^{c+3}, \cos\Big(\frac{c\pi}{2}\Big)\omega_k^{c+4}, -\sin\Big(\frac{c\pi}{2}\Big)\omega_k^{c+5}, -\cos\Big(\frac{c\pi}{2}\Big)\omega_k^{c+6}, \sin\Big(\frac{c\pi}{2}\Big)\omega_k^{c+7}, \ldots\Big),$$

$$P_k^2 = \underbrace{(0, \omega_k^2, 0, \omega_k^4, 0, \omega_k^6, 0, \ldots)}_{n-1},$$

$$Q_k^1 = \tau^c\Big(\sin\Big(\frac{c\pi}{2}\Big)\omega_k^c, \cos\Big(\frac{c\pi}{2}\Big)\omega_k^{c+1}, -\sin\Big(\frac{c\pi}{2}\Big)\omega_k^{c+2}, -\cos\Big(\frac{c\pi}{2}\Big)\omega_k^{c+3}, \sin\Big(\frac{c\pi}{2}\Big)\omega_k^{c+4}, \cos\Big(\frac{c\pi}{2}\Big)\omega_k^{c+5}, -\sin\Big(\frac{c\pi}{2}\Big)\omega_k^{c+6}, -\cos\Big(\frac{c\pi}{2}\Big)\omega_k^{c+7}, \ldots\Big),$$

$$Q_k^2 = \underbrace{(-\omega_k, 0, \omega_k^3, 0, \omega_k^5, 0, \omega_k^7 \ldots)}_{n-1}.$$

Assume **M** has full column rank, then the least-squares approximate solution of (35) is

$$\mathbf{X} = (\mathbf{M}^{\mathrm{T}}\mathbf{M})^{-1}\mathbf{M}^{\mathrm{T}}\mathbf{B}, \tag{38}$$

which provides the coefficients of the overvoltage-based type finite dimensional approximation to the Cole-Cole model.

It is not difficult to see that the basic characteristic of the overvoltage-based model is that if the model is expressed in the form of (6), then $a_1 = b_1$. Based on this, an alternative algorithm may be proposed as follows.

Let $\mathbf{X} = (a_1, a_2, \ldots, a_n, b_2 \ldots, b_{n+1})^{\mathrm{T}}$. Let N be the sampling data size. We may express **M** and **B** as in (8), with **B** as described there. For **M,** we need

$$P_i^1 = -\mathrm{Re}(z_i - 1, z_i(\underbrace{\mathrm{j}\omega_i, (\mathrm{j}\omega_i)^2, \ldots, (\mathrm{j}\omega_i)^n}_{n})),$$

$$P_i^2 = \mathrm{Re}(\underbrace{\mathrm{j}\omega_i, (\mathrm{j}\omega_i)^2, \ldots, (\mathrm{j}\omega_i)^{n-1}}_{n-1}),$$

$$Q_i^1 = -\mathrm{Im}(z_i - 1, (\underbrace{\mathrm{j}\omega_i, (\mathrm{j}\omega_i)^2, \ldots, (\mathrm{j}\omega_i)^n}_{n})),$$

$$Q_i^2 = \mathrm{Im}(\underbrace{\mathrm{j}\omega_i, (\mathrm{j}\omega_i)^2, \ldots, (\mathrm{j}\omega_i)^{n-1}}_{n-1}).$$

Then **X** can be solved by using (9) again.

As an illustration, let us reconsider Example 2.1. Using the same data, we obtain the following model:

$$Z_{\mathrm{fd}}(\mathrm{j}\omega) = \frac{-0.0841 + 1.8522(\mathrm{j}\omega) + 0.1903(\mathrm{j}\omega)^2}{-0.0841 + 7.9458(\mathrm{j}\omega) + (\mathrm{j}\omega)^2}. \tag{39}$$

For this model we have $r = 0.9290$, $E_1 = 1.0771$, and $E_{\mathrm{R}} = 1.0815$. Compared with model (13), it is easy to see that (13) is a better approximation. However, it will be seen below that this model is particularly suitable for describing the IP–EM mixed model because the discrimination can be carried out easily.

5 SPECTRAL CHARACTER OF IP MODEL

As mentioned in Sections 2 and 3, for most practical purposes order 2 is a proper choice. In this section we concentrate on the second-order approximation of the Cole-Cole model.

Consider the overvoltage-based model (24). One important property of this model is that all the coefficients $A_n > 0$, $n = 1, 2, \ldots$. This property derives from the characteristics of RC-circuits and we refer to it as the basic property of the IP model.

Since we believe that a Cole-Cole model is a good description of the IP model, it should also possess this basic property to a certain extent, but since a Cole-Cole model is an infinite dimensional model, it is hard to characterize it. Physically, it can be explained by assuming that the Cole-Cole model is basically an RC-circuit equivalent model. To reconfirm the basic property of the IP model, we offer the following hypothesis.

Hypothesis 5.1. Any overvoltage-based least-squares finite dimensional approximation to the Cole-Cole model has the property that

$$A_n > 0, \quad n = 1, 2, \ldots. \tag{40}$$

At this stage this is only a hypothesis because it could not be proved rigorously. But as discussed above, since in practical usage the second-order model is a reasonable choice, we may show that the second-order approximation has this property.

The second-order overvoltage-based model is expressed as

$$Z(s) = Z(0)\left(1 - \frac{As}{s+a} - \frac{Bs}{s+b}\right). \tag{41}$$

The least-squares approach is similar to that shown previously. The only additional procedure is that, after the parameters have been determined in the general form (33), some algebraic calculations must be performed to convert them to the new parameters*:* a, b, A and B.

The following are the results of numerical testing.

Example 5.2. Let the frequency be $2^{-8} \leq f \leq 2^{10}$, the sampling data size $N = 25$, $c = 0.20, 0.24, 0.28, \ldots, 0.96$, $Z_0 = 1$ (Z_0 does not affect the signs of A and B, so we can simply set it to 1). The other parameters are (a) $m = 1.0$, $\tau = 50.0$; (b) $m = 0.82$, $\tau = 16.0$. Then the least-squares approximated system (41) has the parameters listed in Table 2. From Table 2

Table 2 Overvoltage-based second-order approximation to Cole-Cole model

c	a	b	A	B	a	b	A	B
0.20	130.8835	0.4311	0.0814	0.8095	134.7587	0.5556	0.0773	0.6335
0.24	121.1258	0.4230	0.0752	0.8506	125.5074	0.5803	0.0749	0.6670
0.28	11.5105	0.3928	0.0661	0.8841	116.1354	0.5730	0.0693	0.6957
0.32	102.2869	0.3505	0.0562	0.9107	106.9551	0.5430	0.0619	0.7198
0.36	93.5624	0.3040	0.0466	0.9316	98.1446	0.4991	0.0538	0.7397
0.40	85.3606	0.2583	0.0379	0.9478	89.7874	0.4485	0.0459	0.7559
0.44	77.6632	0.2164	0.0305	0.9603	81.9077	0.3965	0.0386	0.7690
0.48	70.4346	0.1795	0.0242	0.9698	74.4976	0.3464	0.0321	0.7796
0.52	63.6369	0.1480	0.0191	0.9770	67.5343	0.3002	0.0264	0.7880
0.56	57.2368	0.1216	0.0150	0.9825	60.9909	0.2589	0.0216	0.7947
0.60	51.2084	0.0999	0.0118	0.9867	54.8422	0.2227	0.0176	0.8000
0.64	45.5343	0.0822	0.0092	0.9899	49.0675	0.1914	0.0142	0.8042
0.68	40.2049	0.0678	0.0071	0.9923	43.6520	0.1648	0.0114	0.8076
0.72	35.2173	0.0562	0.0055	0.9941	38.5861	0.1423	0.0091	0.8102
0.76	30.5742	0.0469	0.0042	0.9956	33.8652	0.1235	0.0072	0.8124
0.80	26.2822	0.0395	0.0032	0.9967	29.4884	0.1078	0.0056	0.8142
0.84	22.3493	0.0336	0.0024	0.9976	25.4572	0.0948	0.0042	0.8157
0.88	18.7836	0.0289	0.0017	0.9983	21.7737	0.0840	0.0030	0.8169
0.92	15.5904	0.0252	0.0011	0.9989	18.4392	0.0753	0.0020	0.8180
0.96	12.7705	0.0223	0.0005	0.9995	15.4526	0.0682	0.0010	0.8190

and some other runs, we are confident that the hypothesis is correct. It supports the RC-circuit property of the IP model.

6 MIXED IP–EM MODEL

In this section we consider the real geophysical model, which is a mixed IP–EM model (He *et al.* 1995a, b). First we observe the real data in Fig. 4(b), which are obtained via field measurements by using tri-frequency pseudo-random input. If the real geophysical model were a pure IP model, the output should be as shown in Fig. 4(b). We therefore believe that the real geophysical model is an IP model coupled with an EM model. The question now is how to describe the EM model. Initially, we add an RL-path to the model (23). The path is given by

$$EM(s) = \frac{As}{s+a}, \quad A > 0. \tag{42}$$

The reason for this will be explained below. The parameters, $A = 0.3$, $b = 0.02$, are chosen in such a way that the simulation result can be as close to the field measurement as possible.

The model is shown in Fig. 5. The system bounded by the dashed line is the proposed mixed IP–EM model (see Section 7 for a further explanation).

Figure 6 shows a comparison of the simulation result and the field measurement. It is obvious that the simulation result of the proposed IP–EM mixed model is very close to the real field data, but quite different from the model simulation of (23).

The simulation result is so convincing we believe that the EM model can be described by the model (42). This is explained as follows.

We know that when an electrical current through an inductor changes, an electromotive force will be induced in the conductor, which means there exists an electromagnetic effect when there are inductors in an electrical circuit. If a transfer function of an inductance element and a random noise are added to the simulation block of (17), then the simulation result is similar to the field measurement result, which indicates that the field data contain an electromagnetic effect (see Fig. 4). The transfer function of an inductance element may be written as

$$G_L(s) = \frac{V_L(s)}{I(s)} = \frac{Ls}{LGs+1} = \frac{As}{s+a}, \tag{43}$$

where

$$A = \frac{1}{G} > 0, \quad a = \frac{1}{LG}.$$

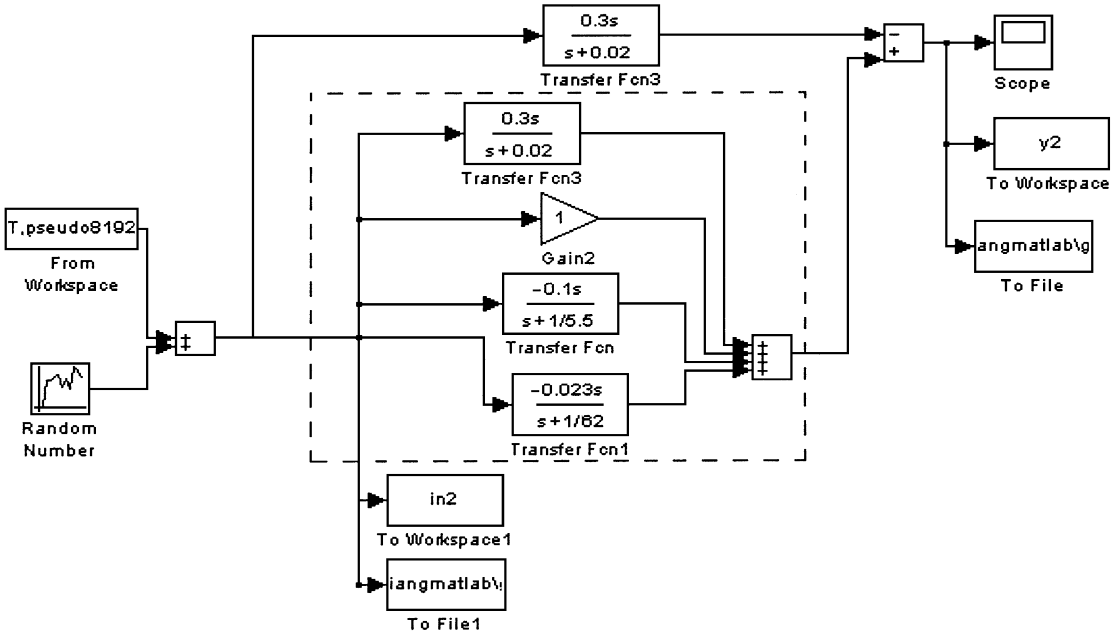

Figure 5 Simulation block of (23) with electromagnetic effect and added random noise.

The corresponding equivalent circuit of (43), which describes the electromagnetic effect, is introduced as shown in Fig. 7.

The circuit parameters $i_1(t)$, $i_2(t)$ and $v(t)$ are related through the first-order differential equation,

$$I(t) = Gv_L(t) + \frac{1}{L}\int v_L(t)\mathrm{d}(t). \tag{44}$$

When the current $I(t)$ is a step function, solving for $v_L(t)$, we have

$$v_L(t) = I\frac{1}{G}\mathrm{e}^{-t/\tau}, \tag{45}$$

where

$$\tau = LG. \tag{46}$$

7 DISCRIMINATION BETWEEN IP RESPONSE AND EM EFFECTS

In the light of the discussion above, we conclude that an IP–EM mixed model can be expressed by

$$E_0\left(1 - \sum_{k=1}^{n}\frac{A_k s}{s + a_k} + \sum_{k=1}^{m}\frac{B_k s}{s + b_k}\right), \quad A_k > 0, \quad B_k > 0. \tag{47}$$

The IP model and the EM model can then be discriminated as

$$IP(s) = E_0\left(1 - \sum_{k=1}^{n}\frac{A_k s}{s + a_k}\right), \quad A_k > 0, \tag{48}$$

and

$$EM(s) = E_0\sum_{k=1}^{m}\frac{B_k s}{s + b_k}, \quad B_k > 0. \tag{49}$$

It follows that the discrimination between a valid IP response and EM coupling effects can be achieved by the following steps:

1 Determine the order and identify parameters of (6) by apparent spectral data using system identification techniques, as proposed in the above sections.

2 Convert (6) into (47) in which the negative Laplace transform terms ($-A_k < 0$) represent the IP effects. Collect the terms to form the IP model as (48). The positive Laplace transform terms ($B_k > 0$), which represent the electromagnetic effects, form the EM model.

By subtracting an RL-path from the IP–EM mixed model, the IP model can be isolated. The method is shown in Fig. 4.

The following example is used to illustrate the processing of discrimination. The model and data are based on work by Pelton *et al.* (1978).

Example 7.1. Consider an order-2 Cole-Cole model, which is described as in (4). Let the parameters be as follows:

$$Z(0) = 1.0, \quad m_1 = 0.165, \quad \tau_1 = 33, \quad c_1 = 0.25,$$
$$m_2 = 0.309, \quad \tau_2 = 0.0013, \quad c_2 = 0.909.$$

When we use (6) to approximate (24) directly, the standard least-squares approach provides the following solution:

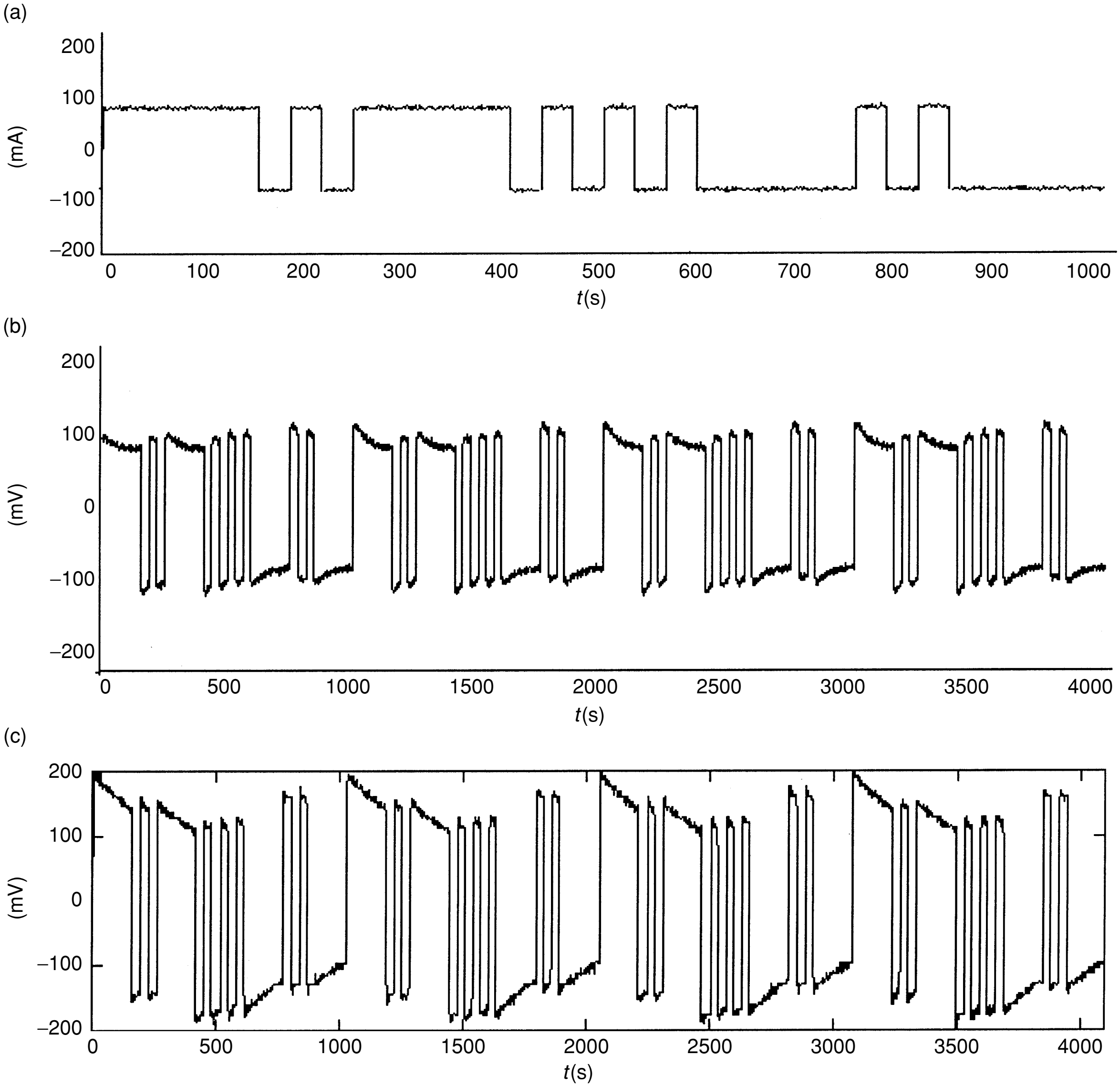

Figure 6 Comparison of simulation result and field measurement. (a) Tri-frequency pseudo-random signal with random noise. (b) Simulation output. Sample: galena and bentonite. (c) Field date: Pb–Zn.

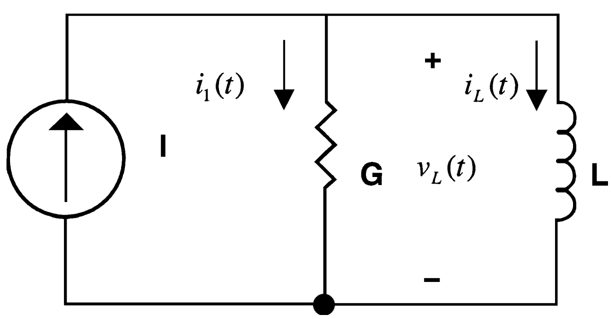

Figure 7 Equivalent circuit of electromagnetic effect.

$$
\begin{aligned}
Z(s) &= \frac{5.2023 + 75.6215s + 29.2708s^2 + 0.8285s^3}{5.2023 + 85.4107s + 33.84s^2 + s^3} \\
&= 1 - \frac{0.3602s}{s + 31.099} - \frac{0.1241s}{s + 0.0625} + \frac{0.3128s}{s + 2.6786}.
\end{aligned} \tag{50}
$$

The correlation is $r = 0.9993$, which shows the approximation is adequate.

According to the principle discussed above, it is easy to identify the IP and EM components of model (50) as

$$IP(s) = 1 - \frac{0.3602s}{s + 31.099} - \frac{0.1241s}{s + 0.0625}$$

and

$$EM(s) = \frac{0.3128s}{s + 2.6786}.$$

8 CONCLUSIONS

Two kinds of finite dimensional approximations to the Cole-Cole model were considered. Using the least-squares approach directly, the parameters for the approximated models were obtained. Certain properties of the IP model were revealed via the finite dimensional approximation of the Cole-Cole model. These properties were reconfirmed by the overvoltage-based approach to the IP model (Luo and Zhang 1998). In particular, it was shown that the IP model is essentially equivalent to an RC-circuit.

Using field measurement data, it was discovered that the EM model is basically equivalent to an RL-circuit. The simulation results and the theoretical analysis confirmed this claim.

Based on the above claims, a new method was proposed to discriminate the valid IP response and EM coupling effects. Both the theoretical analysis and the simulation results show that the new method is simple and effective.

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REFERENCES

Bertin J. (ed.) 1976. *Experimental and Theoretical Aspects of Induced Polarization: Presentation and Application of the IP Method Case Histories*, Vol. 1. Gebrüder Borntraeger, Berlin.

van den Bosch P.P.J. and van der Klauw A.C. 1994. *Modelling, Identification and Simulation of Dynamical Systems*. CRC Press, Inc., New York.

Curtain R.F. and Zwart H. 1995. *An Introduction to Infinite Dimensional Linear Systems Theory.* Springer-Verlag, Inc.

Gubins A.G. (ed.) 1997. Geophysics and geochemistry at the millennium. *Proceedings of Exploration 97. 4th Decennial International Conference on Mineral Exploration, Toronto*. GEO F/X Division of AG Information Systems.

He J., Wang S. and Tang J. 1995a. Chopping wave method for removing electromagnetic induction coupling. *Transactions of Nonferrous Metals Society of China* **5**, 1–10.

He J., Wang S. and Tang J. 1995b. Square wave coherent method for extracting IP effect. *Transactions of Nonferrous Metals Society of China* **5**, 1–6.

Helbig K. (ed.) 1994. *Modelling the Earth for Oil Exploration*. Pergamon Press, Inc.

Jaggar S.R. and Fell P.A. 1988. Forward and inverse Cole-Cole modelling in the analysis of frequency domain electrical impedance data. *Exploration Geophysics* **19**, 463–470.

Loeb J. (ed.) 1976. *Experimental and Theoretical Aspects of Induced Polarization: Macroscopic and Microscopic Theories*, Vol. 2. Gebrüder Borntraeger, Berlin.

Luo Y. and Zhang G. 1998. *Theory and Application of Spectral Induced Polarization*. Geophysical Monograph Series No. 8. Society of Exploration Geophysicists, Tulsa, OK.

Pelton W.H., Ward S.H., Hallof P.G., Sill W.R. and Nelson P.H. 1978. Mineral discrimination and removal of inductive coupling with multi-frequency IP. *Geophysics* **43**, 588–609.