Safety and Reliability of Distributed Embedded Systems

Technical Report ESL 04-02

Simulation of Motorway Traffic Flows

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**Project summary**

This technical report is one of a series (listed in full below). Together these reports describe a complete hardware-in-the-loop (HIL) simulation that reproduces the behaviour of a passenger car travelling down a motorway. In the simulation, the speed and position of the car are determined by an adaptive cruise control system implemented using one or more embedded microcontrollers.

The test bed is intended to be used to assess and compare different software architectures for use in distributed embedded systems, particularly those for which high reliability is a key design consideration.

**Full list of reports in this series**

**Available now:**

ESL04/01 “Simulation of Vehicle Longitudinal Dynamics”

ESL04/02 “Simulation of Motorway Traffic Flows”

ESL04/03 “Development of a Hardware-in-the-Loop Test Facility for Automotive ACC Implementations”

**Forthcoming:**

ESL04/04 “Control Technologies For Automotive Drive-By-Wire Applications”

ESL04/05 “10-Node Distributed ACC System: Co-Operative Implementation”

ESL04/06 “10-Node Distributed ACC System: Pre-Emptive Implementation”

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## Contents

1. Introduction ............................................................................................................ 1  
   1.1 The UK Motorway Network ........................................................................... 1  
   1.2 Simulation Principle ...................................................................................... 2  
       1.2.1 The Box-Muller Transformation .............................................................. 3  
   1.3 Dynamic Representation Of Road Users ...................................................... 3  
2. Microscopic Driver Modelling ................................................................. 6  
   2.1 Longitudinal Control Model ........................................................................... 6  
       2.1.1 Free Driving Model ............................................................................... 6  
       2.1.2 Car Following Model ............................................................................ 7  
       2.1.3 Model Switching .................................................................................... 8  
   2.2 Lane Changing Model .................................................................................. 8  
       2.2.1 Driver Side Lane Changes ................................................................. 10  
       2.2.2 Passenger Side Lane Changes ............................................................ 11  
       2.2.3 Lane Change Trajectory ................................................................. 14  
3. Simulation Optimisation And Macroscopic Results .......................... 16  
   3.1 Empirical Data .............................................................................................. 16  
   3.2 Optimisation Procedure ............................................................................. 17  
   3.3 Macroscopic Simulation Results .................................................................... 18  
4. Conclusion ........................................................................................................... 20  
5. References / Bibliography ............................................................................. 21
1. Introduction

This report describes the development of a computer simulation of traffic flow on a typical section of motorway in the UK. First a brief description of the motorway network itself is given, along with a discussion of UK driving conventions. Following this, the principle of microscopic simulation and the representation of road users are discussed. Driver models for both car following and lane changing are then developed. The microscopic parameters in the driver models are then optimised to fit the simulation results with existing empirical data. The simulation is to be used with the host vehicle dynamic model, details of which may be found in the related Technical Report ESL04/01. Details regarding the integration of these simulations, and their interface with the embedded system, may be found in Technical Report ESL04/03.

1.1 The UK Motorway Network

The first motorway to be opened in the UK was a two-lane, 8.25-mile long section of road called the M6 in 1958. Since this date, there has been a steady and consistent rise in both the number of motorways, the total length of motorways and the amount of traffic flowing upon them. Currently there are approximately 100 motorways totalling over 3500 Km, with the amount of lanes varying between two up to eight in some areas. This study is concerned with the simulation of a straight, three-lane section of road with no junctions and a maximum gradient of 7% (4 degrees) which is currently the design maximum allowed by legislation. Each lane has a typical width of 4m, as shown in the figure below:

![Figure 1: Simulated Motorway Section](image)

Driving convention in the UK states that motorists should drive on the left, and when driving on the motorway they should keep to the KLEWO (Keep Left Except When Overtaking) system. The left lane corresponds to lane 1 in figure 1, next to the hard shoulder, which is kept clear except for use by the emergency services and breakdowns. Should a motorist wish to overtake a slower driver, he is expected to return to the slow lane as soon as conditions permit. However, it is clear that the KLEWO system is abused by motorists at medium to high traffic flows, who effectively ‘hog’ faster lanes and undertake by passing on the left [McDonald et al. 1994]. Additionally, the speed limit on many sections of the motorway is restricted to 70 Mph for
passenger vehicles and 60 Mph for Heavy Goods Vehicles (HGV’s) and lorries. It is clear from survey data that many passenger vehicles drive in excess of this limit [DFT 2004]. The overall distribution of desired speeds for passenger vehicles possesses a slightly negatively skewed distribution with a mean centred on the speed limit. For HGV’s and lorries, many of which are fitted with speed restrictors, it has slightly more pronounced distribution centred at approximately 53 Mph. In both cases, since the amount of skew is relatively small, for simplicity the distribution of drivers’ desired speeds could be taken as normal.

It is also known that traffic flows and composition vary depending on the time of day, the day of week and even the month of year, and even vary depending on which section and direction of motorway is being considered [May 1990]. Each of these observations will have to be considered in the implementation of an effective simulation. The overall principle to used in this model of traffic flow will be described in the following section.

1.2 Simulation Principle

The simulation will involve a microscopic representation of road users. They will be represented over a two-kilometre section of road, at the centre of which will lay the host vehicle, which is to be controlled by the embedded system. The dynamics of each road user will be evaluated with respect to the host vehicle – that is, the difference in velocity between the host and each other vehicle will dictate their position in the simulation. In other words, the co-ordinates of the simulated road section can be thought of as a moving frame of reference which is attached to the host vehicle. There will be an initialisation procedure, where each simulated vehicle is assigned a desired speed and given a random position in the simulation plane. Vehicles occupying the same physical location will be assigned different lanes based on the higher of the desired speeds, and if all three lanes should be occupied at a given location a different random position shall be assigned until all vehicles are distributed in a correct manner. A short initialisation period will then follow, allowing each vehicle to obtain the correct following distance to its leader (if applicable) and the simulation proper shall start ‘on the fly’ at this point.

As the simulation evolves, faster vehicles will exit the frame of reference to the host vehicle’s front, while slower vehicles will exit to the rear. When this occurs, a new vehicle will be added, with a randomly generated speed. Again, if the desired speed is greater than that of the host vehicle the new vehicle will enter the rear of the simulation, and vice versa. The simulation principle is shown graphically in figure 2 below.

![Figure 2: Simulation Principle](image)

The methodology used to generate random samples with a normal distribution is described in the following section.
1.2.1 The Box-Muller Transformation

The output produced by a pseudo-random number generator, such as a computer’s random algorithm, possesses a uniform distribution. Given two normalised random numbers \(x\) and \(y\), both with a uniform distribution and value between 0 and 1, the normal form of the Box-Muller transformation may be used to produce two random numbers \(z_0\) and \(z_1\) both having a normal distribution centred about 0 with a standard deviation of 1 [Box & Muller 1958]. The transformation is given mathematically as follows:

\[
\begin{align*}
  z_0 &= \cos(2\pi x) \cdot \sqrt{-2\ln(y)} \\
  z_1 &= \sin(2\pi x) \cdot \sqrt{-2\ln(y)}
\end{align*}
\]

Equation 04/02/A

The more common polar form of the transformation is not utilised in this simulation because, although it requires fewer calls to maths functions and is statistically more efficient, approximately 21% of samples are discarded based on a simple testing criteria. This makes it unsuitable for the purposes of real time simulation because, however small the probability, it cannot be guaranteed that in a given time period two random samples will be generated that both satisfy the criteria. Now that the main simulation principle has been described, the methodology for dynamically representing the vehicles in the simulation will be outlined.

1.3 Dynamic Representation Of Road Users

In order to represent road users, with the exception of the host vehicle, a simplified dynamic model is employed. The car-following model that is described in section 2.1 generates the acceleration signal \(\alpha\), which acts as the main input to the model. Defining the state \(x_2\) as the vehicle velocity, and the state \(x_1\) as the vehicle position (within the simulation), if the host vehicle velocity is \(v_h\) the dynamics are represented as follows:

\[
\begin{align*}
  \dot{x}_1 &= v_h - x_2 \\
  \dot{x}_2 &= \min(\alpha, \alpha_{\text{max}})
\end{align*}
\]

Equation 04/02/B

In order to make the simulation realistic, the acceleration signal must be saturated at a maximum level \(\alpha_{\text{max}}\) for both acceleration and braking. Since the maximum acceleration of each vehicle depends on its type, the road grade and weather conditions, these must be incorporated into the model. Previous research has shown that a simple speed-dependent maximum is sufficient to provide a realistic model [Yang 1997]. In this simulation, a linear dependence is assumed. For passenger vehicles, this relationship (on level ground) is taken as follows for acceleration:

[ESL04-02A.DOC - 11 October 2004]
\[ \alpha_{t_{\text{max}}} = 4 - 0.07 \ x_2 \quad m / s^2 \]

Equation 04/02/C

And as follows for braking:

\[ \alpha_{t_{\text{max}}} = 4.5 - 0.023 \ x_2 \quad m / s^2 \]

Equation 04/02/D

This relationship provides for realistic acceleration times and stopping distances at various velocities. It can be shown graphically as in figure 3:

![Car Acceleration/ Speed Relationship](image)

**Figure 3: Car Acceleration/ Speed Relationship**

Similarly, for HGV’s, the relationship (on level ground) is taken as follows for acceleration:

\[ \alpha_{t_{\text{max}}} = 2 - 0.05 \ x_2 \quad m / s^2 \]

Equation 04/02/E

And for braking:

\[ \alpha_{t_{\text{max}}} = 3 - 0.017 \ x_2 \quad m / s^2 \]

Equation 04/02/F

Again this can be shown graphically as in figure 4:
To incorporate the effect of road gradient, a simple formula may be used to modify the level ground acceleration $\alpha_{l\text{max}}$ as such [Yang 1997]:

$$\alpha_{g\text{max}} = \alpha_{l\text{max}} - \frac{Gg}{100} \text{ m/s}^2$$

*Equation 04/02/G*

Where $g$ is the acceleration of gravity and $G$ is the road gradient (%). Finally, to incorporate the effects of weather conditions, a co-efficient of friction $\mu$ is introduced as follows:

$$\alpha_{\text{max}} = \mu\alpha_{g\text{max}} \text{ m/s}^2$$

*Equation 04/02/H*

The table of figure 5 shows typical values for $\mu$ for a given set of weather conditions:

<table>
<thead>
<tr>
<th>Condition</th>
<th>Friction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry</td>
<td>1</td>
</tr>
<tr>
<td>Wet</td>
<td>0.8</td>
</tr>
<tr>
<td>Snow</td>
<td>0.3</td>
</tr>
<tr>
<td>Ice</td>
<td>0.1</td>
</tr>
</tbody>
</table>

*Figure 5: Friction Characteristics*
2. Microscopic Driver Modelling

Microscopic driver modelling deals with the creation of an ‘intelligent driver’ for each of the simulation vehicles. This intelligent driver, or ‘agent’, will control both the longitudinal and lateral vehicle motion, depending on the current road situation. Longitudinal motion is concerned with generating an acceleration signal for the vehicle, and lateral motion with deciding when to change lanes in response to certain stimuli. Although use of soft computing techniques, for example fuzzy logic [Wu et al. 2000] have been found extremely useful for modelling driver behaviours, the computational complexity and overheads in the use of these techniques make there use in a real-time, single processor PC based simulation impractical. The following section describes the longitudinal model.

2.1 Longitudinal Control Model

In this simulation three different types of longitudinal motion are to be modelled:

1. Free Driving – the agent endeavours to achieve and maintain the desired speed.

2. Following – the agent attempts to safely approach an oncoming vehicle in the same lane and maintain a safe distance to it.

3. Emergency Braking – the agent slows the vehicle severely in order to attempt to achieve the desired separation due to an unexpected event e.g. a cut in.

Two different models, with appropriate switching in between, can represent these three types of driving and will be considered in the following two sections. The emergency braking model is ingrained within the following model due to its similarity.

2.1.1 Free Driving Model

The free driving model used in this simulation is a basic proportional controller that acts on the error between the desired and actual velocities, as shown in figure 6.

Figure 6: Free Driving Controller

As the vehicle approaches its desired speed the error and hence the acceleration signal \( \alpha_{fd} \) will fall to zero, assuming the gain is positive. The selection of suitable gains for each driver is
based around a normal distribution to allow for different acceleration profiles when in free
driving mode, and is identical to the proportional following gain as used in the following model.
This is described in the following section.

2.1.2 Car Following Model

The car following model to be used in this simulation has a slightly more complex nature,
and is based on a modified version of a proportional-derivative controller developed by Fang
[Fang et al. 2001]. As with most controllers of this nature, a constant time headway separation is
preferred to a constant distance headway separation. The concept of time headway is defined by
equation 5:

\[ t_h = \frac{v}{d_l} \]

Equation 04/02/1

Where \( v \) is the vehicle velocity and \( d_l \) is the distance to the lead vehicle. Thus as the velocity
of the lead increases, the separation distance increases for a given time headway, ensuring a
constant reaction time should the lead brake suddenly. The desired following time headway for
each driver is generated with a normal distribution, centred at 1.2 seconds, with a standard
deviation of 0.15 seconds to achieve realistic behaviour [Fang et al. 2001; May 1990]. A
schematic of the following controller is shown as figure 7 below:

![Car Following Controller Diagram]

**Figure 7: Car Following Controller**

It can be seen that a desired distance is generated based on the target’s velocity, as per
equation 5, and a proportional gain acts on the error between this distance and the actual distance
to the target. In addition, a velocity feedback gain \( K_v \) applies an additional acceleration signal
proportional to the relative velocity between the target vehicle and vehicle under consideration.
The controller effectively acts in two related dimensions, to equalise the velocities of both
vehicles and attain the required distance, by varying the acceleration \( \alpha_{cf} \) of the following vehicle.
To provide a realistic model, experiments on real data by Fang revealed a direct relationship
between the gains \( K_p \) and \( K_v \) [Fang et al. 2001]. A half-normal distribution of proportional gain
was found with mean value of 0.02, and standard deviation of 0.055. The relationship between \( K_v \)
and \( K_p \) was best described by a polynomial equation as given below:
\[
K_v = 0.1839 + 2.5719K_p - 5.5182K_p^2
\]

*Equation 04/02/J*

Giving an effective range for the velocity gain between 0.233 and 0.483. Although giving a range of realistic behaviours and being simple to implement, a problem was found in that small amounts of overshoot can occur in terms of the actual distance. This often occurs in practice, but at lower speeds and cut-in situations drivers increase the braking action to compensate. In this simulation, the compensatory behaviour is modelled by a simple non-linearity that magnifies the acceleration signal \(\alpha_{cf}\) when the actual distance is less than 70% of the desired distance, in direct proportion to the distance error:

\[
if (d_a < 0.7d_d) \alpha_{cf} = \alpha_{cf} \cdot 10 \frac{d_a}{d_d}
\]

*Equation 04/02/K*

Where \(d_a\) is the actual separation distance and \(d_d\) is the desired separation distance. This relationship ensures that any cut-in situations are responded with in an appropriate manner, i.e. large deceleration. The next section describes a simple yet effective method for switching between the two longitudinal control methods.

### 2.1.3 Model Switching

Examination of the operation of the free driving and car following models yields a simple switching method between the two, which is given in the following equation:

\[
\alpha = \min(\alpha_{cf}, \alpha_{fd})
\]

*Equation 04/02/L*

Where \(\alpha\) is the actual applied acceleration signal, \(\alpha_{cf}\) is the acceleration resulting from the car following/emergency braking model, and \(\alpha_{fd}\) is the acceleration signal resulting from the free driving model if they are both evaluated at each sample interval. The switching method basically decides the minimum of the two acceleration signals, ensuring that the car decelerates to the correct following distance should a lead vehicle be encountered. Additionally, should the lead vehicle start to accelerate, it will be followed at a safe distance but will not be ‘chased’ above the driver’s desired speed. Should a slow lead vehicle be encountered, the driver also has the option of overtaking if a faster lane is free. The following section describes the lane change model that has been developed for this simulation.

### 2.2 Lane Changing Model

Lane changing is a notoriously difficult and complex behavioural process to model. It is best considered in terms of a number of perception thresholds, which consider the risk involved in
accepting a gap in neighbouring lanes, compared to a benefit factor of some kind when performing the manoeuvre [McDonald et al. 1994]. Motorway lane changes may be described as either mandatory or discretionary [Ahmed et al. 1996]. This simulation is not concerned with mandatory lane changes, which generally involve a driver changing lanes in order to exit at a particular junction. A discretionary lane change is performed when the driver is not satisfied with the driving conditions in the current lane, and can be split into three different manoeuvres each with their own motivation:

1. Driver’s side change (overtaking) – this is performed when a driver perceives a certain speed benefit in moving to a faster lane.

2. Passenger side change (yield) – this is performed when the driver is currently occupying either the middle or fast lanes, and perceives a faster car approaching from the rear.

3. Passenger side change (KLEWO) – this manoeuvre is again performed when the driver is occupying either the middle or fast lanes, and moves over to the left after overtaking. The presence of a faster car approaching from the rear is not necessary.

The lane change model developed for this simulation consists of a driver side change model that implements rule 1, and a passenger side change model that follows rules 2 and 3. The models developed for this simulation consist of a simple ‘gap acceptance’ principle. The gap, in both cases, is defined as the time headway to nearest vehicle in the destination lane both to the front and rear of the considered vehicle. This is illustrated in figure 8:

![Figure 8: Lane Change Gap Principle](image)

Where the lead time headway and lag time headway are given by the following:

\[
T_{lead} = \frac{V}{D_{lead}}, \quad T_{lag} = \frac{V_{lag}}{D_{lag}}
\]

*Equation 04/02/M*

For naturalistic, unforced discretionary lane changes, it has been found that the minimum preferred lead gap is 1.93 seconds and the minimum preferred lag gap is 1.72 seconds [Olsen et al. 2002]. For all cases of lane changing, there is an absolute minimum clearance distance that must be exceeded for the movement to be initiated. These minimum lead and lag clearances may be modelled as follows [Gipps 1986]:

[ESL04-02A.DOC - 11 OCTOBER 2004]
Where the time $t_l$ is the time for the lane change to take place and will be discussed further in section 2.2.3. Additionally, it was also found that a minimum clearance separation from the lead vehicle in the same lane must be exceeded for a change to take place, which is given in equation P:

$$D_{front} = 2.2 \cdot (V - V_{front}) + 1$$

Equation 04/02/P

The following section describes the overtaking model that has been developed for this simulation.

### 2.2.1 Driver Side Lane Changes

The motivation for making a lane change on the drivers’ side is primarily due to speed inconvenience caused by a slower lead vehicle. As the inconvenience increases, the drivers’ willingness to accept a smaller gap also increases. In order to implement rule 1 from the previous section, the parameter $U_{overtake}$ was introduced as a measure of the urgency of the lane change. It is described mathematically below:

$$V_e = \frac{(V_d - V)}{\xi}$$

Equation 04/02/Q

$$U_{overtake} = KO_l \left(1 - e^{-5(1-V_e)}\right), \quad l = 1,2$$

Equation 04/02/R

The use of an exponential function in equation R is simply to generate a non-linear relationship between speed error and urgency. The urgency is saturated between the values 0 and 1. $V_d$ is the drivers desired velocity and the parameter $\xi$ is a sensitivity factor that may be adjusted to suit different driver types. The parameter $KO_l$ was introduced to provide a scaling gain dependent on the drivers current lane, as motivation for overtaking may be stronger if travelling in a slower lane. The urgency is related to the actual required gap for manoeuvre as follows:

$$t_{lead} = 3.86 \cdot U_{overtake}$$

Equation 04/02/S
\[ t_{\text{lag}} = 3.44 \cdot U_{\text{overtake}} \]

Equation 04/02/T

The values of 3.86 and 3.44 are equal to twice the preferred headways as described in the previous section, to emphasise that small speed inconveniences will require a large gap to be acted upon. However, the minimum distance criteria of equations R and V must still be satisfied even as the speed error increases to the value of \( \xi \) and the gap decreases to zero. The relationship used in this simulation is represented graphically as figure 9 below, after optimisation of the sensitivity \( \xi \) and gains \( K_0 \) (see section 4):

![Lane Gap Acceptance (Overtaking)](image)

**Figure 9: Overtaking Gap Acceptance**

### 2.2.2 Passenger Side Lane Changes

The motivation for making a lane change on the drivers’ side is primarily due to two factors, discussed in section 2.2. In order to implement rule 2, the parameter \( U_{\text{yield}} \) was introduced as a measure of the urgency of the lane change, i.e. the rear pressure from a faster driver. It is described mathematically below:

\[
V_i = \frac{(V_a - V_{fd})}{\xi}
\]

Equation04/02/U

\[
U_{\text{yield}} = K_{V_i} \left(1 - e^{-S(1-V_i)}\right), \quad l = 2,3
\]

[ESL04-02A.DOC - 11 OCTOBER 2004]
Where $V_d$ is the drivers desired velocity, while $V_{f/d}$ is the following drivers desired velocity, and the parameter $V_i$ is an indicator of the inconvenience factor of the following driver. Again, the use of an exponential function in equation V is simply to generate a non-linear relationship between speed inconvenience and urgency. The urgency is saturated between the values 0 and 1. The parameter $\zeta$ is a sensitivity factor that may be adjusted to suit different driver types. The parameter $K_{yl}$ provides a scaling gain dependent on the drivers’ current lane, as again the motivation for yielding may be stronger if travelling in a faster lane. The urgency is related to the actual required gap for manoeuvre as follows:

\[
\begin{align*}
t_{\text{lead}} &= 3.86 \cdot U_{\text{yield}} \\
t_{\text{lag}} &= 3.44 \cdot U_{\text{yield}}
\end{align*}
\]

Equation 04/02/W

Equation 04/02/X

The values of 3.86 and 3.44 are again equal to twice the preferred headways as described in section 2.2. Again, the minimum distance criteria of equations R and V must still be satisfied for a manoeuvre to occur, even when the inconvenience is large. The relationship used in this simulation is represented graphically as figure 10 below, after optimisation of the sensitivity $\zeta$ and gains $K_{yl}$ (see section 4):

![Lane Gap Acceptance (Yielding)](image)

Figure 10: Yielding Gap Acceptance
In addition to yielding as a result of pressure from a following driver, in order to implement rule 3 from section 2.2, another model was created and is described below. In this model, a stimulus urging the driver to return to the left lane of the motorway is implemented, in order to adhere to the KLEWO principle and implement rule 3 from section 2.2. However, since it has been found that adherence to the KLEWO principle decreases as overall traffic flow increases, this is again incorporated as a gap acceptance model where the lead gap increases in direct proportion to the traffic flow. This will ensure that a driver will be able to stay in the target lane for a minimum amount of time, otherwise the change will not be deemed worthwhile.

Although the traffic flow is actually a macroscopic simulation measure, an individual driver may easily predict traffic flow by the density and speed of vehicles in the visible horizon. However, in order to keep the simulation as simple as possible it is assumed that each driver has knowledge of the overall simulation traffic flow. Since this simulation is mainly concerned with traffic flows less than 6000 vehicles/hour, a KLEWO factor may be defined as follows:

$$KLEWO = \frac{F}{6000}$$

*Equation 04/02/Y*

Where $F$ denotes the actual traffic flow. This may then be used to generate a desired lead gap for adherence to this stimulus:

$$t_{lead} = \max\{KLEWO \cdot t_{k_{max}}, 1.93\}$$

*Equation 04/02/Z*

The required lag gap for KLEWO adherence is not dependent on flow and is a constant 1.71 seconds. The value of $t_{k_{max}}$ is effectively the gap size that must be present for the lane change to be worthwhile when the flow $F$ is equal to 6000. Figure 11 below shows the KLEWO gap acceptance relationship that is used for this simulation, after optimisation of the parameter $t_{k_{max}}$ (see section 4):

![Figure 11: KLEWO Gap Acceptance](Image)
Now that the motivation and acceptance models for a lane change have been described, it is necessary to describe the generation of a suitable motion profile for the lane change itself. This is discussed in the next section.

2.2.3 Lane Change Trajectory

This section describes the generation of a suitable lane change trajectory that a vehicle shall follow once a lane change has been initiated. The lane change to be implemented in this simulation has a sinusoidal trajectory - this was chosen mainly because of its simplicity. However, it still satisfies the criteria for an acceptable trajectory profile due to its continuous nature and accommodation of changing velocity. For a discussion on alternate trajectories that may be used in simulations of this nature, refer to [Sledge & Marshek. 1997]. If a lane change is initiated with vehicle velocity $v$ and initial position $y_s$ and $x_s$ at a time $t=t_0$, as shown in figure 12 below, the sinusoidal trajectory is defined as follows:

$$y_t = y_s + \frac{y_d}{2} + \sin\left(-\pi + 2\pi \frac{t-t_0}{t_1}\right) \cdot \frac{y_d}{2}$$

Equation 04/02/AA

Where $y_d$ is the desired $y$ position after the lane change and $t_1$ is the required time to complete the desired manoeuvre. In this simulation $y_d$ is equal to the width of a single lane, i.e. 4m.

![Figure 12: Lane Change Trajectory Coordinates](image)

A motorway study has shown that the mean change time $t_1$ over a large range of drivers is equal to 6.2 seconds, with a standard deviation of 2 seconds for an unforced discretionary change [Olsen et al. 2002]. As the perceived urgency of the manoeuvre increases, the change time decreases. In order to incorporate this into the simulation, the urgency, which was defined in equations R and V, is used to calculate the change time between the limits of 2 and 10 seconds. Thus, with urgency equal to 0.5, the change time and accepted gaps will both be close to the mean of the empirical data.

Simulation results for a typical change are given in figure 13. The change is initiated at a velocity $v = 5$ m/s, at an initial $x$ position of 40m and $y$ position of 2m. The change time $t_1$ is 3s in this case.
Figure 13: Lane Change Trajectory
3. Simulation Optimisation And Macroscopic Results

The longitudinal car following model described in section 2.1 has been shown to achieve realistic results in simulation [Fang et al. 2001]. However, due to the large number of sensitivity parameters in the lane change model, as discussed in section 2.2, a suitable (simple) calibration method is required to enable these parameters to be adjusted in order to produce realistic behaviour. To allow calibration to take place, a suitable quantifying method is needed that enables the quality of a certain set of parameters to be assessed. For the analysis of lane changing, the two most important measures are lane occupancy and lane change rate [McDonald et al. 1994]. These measures are normally given as a function of the total directional traffic flow rate. The traffic flow rate is defined as the number of vehicles to pass a given point over a given time period, and is related to density and velocity via the following equation [May 1990]:

\[ F = D \times V_a \]

Where \( D \) is the vehicle density (vehicles/km), and \( V_a \) is the average velocity of each vehicle (Km/Hr). In order to evaluate the quality of a given set of simulation parameters, a suitable empirical is data set is required. Key data from a suitable section of motorway is described in the following section.

3.1 Empirical Data

Of the many possible sections of motorway that could be used to obtain the statistical data that is necessary to optimise the simulation, it was decided to choose the M1, south of the M6 junction, as the basis for the data. The M1 runs 300 Km from London to Wetherby, past Leicester, and is one of the most frequently used motorways in the UK. In order to create an effective, realistic simulation, it is necessary to obtain data regarding traffic flows, traffic composition and average speeds for this section of motorway. According to latest figures published by the UK Department For Transport, the average traffic flow in 2002 was approx. equal to 4,167 vehicles/hour, with an absolute maximum flow of 6,750 recorded [DFT 2004]. The average percentage of HGV traffic was 12.9 [DFT 2003]. The speed limit for passenger vehicles, and hence the mean for generation of desired speeds, is 70 Mph with a standard deviation of 7.5 Mph. For HGV’s and lorries, the mean is taken to be 50 Mph with a standard distribution of 2.5 Mph. In both cases, it is assumed that no sample lies beyond 4 standard deviations.

In order to optimise the simulation, it is necessary to understand the basic relationships between traffic flow and lane occupancy. Lane occupancy is quoted as a figure for each lane, indicating the percentage of vehicles current occupying the given lane. Although the exact figures vary between countries, and also at different areas within the same countries, a basic flow/occupancy relationship exists that has the general basic shape as shown in figure 14 [May 1990; McDonald et al. 1994]:

[ESL04-02A.DOC - 11 OCTOBER 2004]
The points of interest in the above figure are the ‘lane inversion’ or crossover points – the points at which, as flow increases, lane occupancy becomes equal between two lanes. In the above figure, the first crossover point is between lanes 1 and 2, and occurs at approximately 710 vehicles/hour. Similarly, the 1/3 and 2/3 crossover points occur at approx. 1750 and 2620 veh/hr respectively. A 1994 video study of a section of the M1, south of the M6 junction found that the 1/3 and 2/3 crossover points occurred at flows of approximately 2700 and 3350 veh/hr [McDonald et al. 1994]. Sufficient data was not available regarding the 1/2 crossover point, however this occurred at a flow less than 2000 veh/hr. Extrapolating the existing data, it can be also estimated to occur at a point above 1500 veh/hr. At the 1/3 crossover point, the lane occupancy was approx 30% in both lanes, whilst at the 2/3 point it was 37% in each lane. During the study duration, the average percentage of HGV’s was greater than the 2003 data indicates (20%), yet it was found that the number of HGV’s did not affect the lane usage or change rate. The lane change rate was found to vary quite wildly, even for identical flow values, with an approximate mean located around 1100 events/Km/hr. This value remained quite steady over the whole flow regime, with fluctuations of up to 50% occurring at some flows. This provides enough information to optimise the simulation, which is discussed in the next section.

3.2 Optimisation Procedure

This section outlines the simple optimisation methodology that was utilised to achieve realistic simulation results. A simulation with a time-step of 1 second was allowed to run in a ‘super-loop’. Each test duration lasted for a benchmark 60 minutes, with an additional 10 minutes of initial data discarded to allow any transients to subside. Since changes to the random seed can cause variations in the simulation results [McDonald et al. 1994], each test was ran five times.
with different seeds and the averaged results taken. The overall flow was calculated by determining the time-averaged traffic flow using equation AC at each time instant. Lane occupancy was calculated at each time instant, and averaged over the test duration. The change rate was simply determined by summing the events, and halving the value at the end of each test (since the simulated motorway was 2Km in length).

The algorithm that was used simply made small changes to the value of each parameter, from an initial estimate, in order to reduce the squared error at the points of flow interest for the empirical data set outlined in the previous section. To ensure that the average flow of each test lied within the required flow window, the number of cars was either increased or decreased during the warm-up period. The target error was set at 5% of lane occupancy at approximate flows of 1000, 2000, 2700, 3350 and 4500 veh/h. The parameters that were determined to fulfil the error criterion are given below:

\[ \begin{align*}
\zeta &= 13.4 \\
\zeta &= 26.8 \\
K_{O_1} &= 0.35 \\
K_{O_2} &= 1 \\
K_{y_2} &= 1 \\
K_{y_3} &= 0.8 \\
t_{k_{\text{max}}} &= 20
\end{align*} \]

The following section presents simulation results of tests over the full range of flow using these parameters.

### 3.3 Macroscopic Simulation Results

Using the parameters that were determined in the previous section, and using a similar testing methodology, data were gathered regarding lane occupancy and change rate for approximate flow regimes from 500 to 4600 veh/hr. Figure 15 shows the lane occupancy. It can be seen that this figure has the basic relationship outlined in section 4.1, and the desired occupancy crossover points are similar to those found in the empirical data set. Figure 16 shows the lane change rate. It can be seen that flows greater than 2000, the approximate mean is 1250 events/hr/Km. This is approx 150 greater than the empirical data, but as mentioned in section 4.1 the empirical data set has large fluctuations in value for identical flows, and overall the simulation results fit into the data range.
Figure 15: Lane Occupancy vs. Traffic Flow

Figure 16: Lane Change Rate vs. Traffic Flow
4. Conclusion

This document has described the development of a simulation model for motorway traffic flows. The simulation principle was described, and suitable driver models were then developed. Empirical data for a suitable section of motorway was then described, and the simulation parameters then optimised based on key points of interest in the empirical data. The simulation results show a good correlation to this empirical data. The next report in this series, ESL 04/03, describes how this motorway simulation may be combined with the dynamic model that was developed in the first report, into a suitable test bed for embedded automotive designs.
5. References / Bibliography


[ESL04-02A.DOC - 11 October 2004]