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Dominance concepts for discrete Fehr-Schmidt preferences with a focus on income inequality*

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Abstract

The evidence for other-regarding preferences is extensive. How should an individual with other-regarding preferences compare two distinct distributions of income? We show that the classical concepts of first and second order stochastic dominance are inadequate to answer this question. We develop the relevant stochastic dominance concepts for the case of the popular other-regarding preferences in Fehr and Schmidt (1999) that we call FS preferences; we consider the linear and non–linear forms of FS preferences. These new dominance concepts, that we call first and second order FS dominance provide sufficient conditions for ranking income distributions. We show that our concepts can be extended to uncertainty and are applicable to some other models of other-regarding preferences. Our use of a discrete framework is empirically realistic and avoids measure theoretic issues arising under the continuous case.

Keywords: Other-regarding preferences; first order FS dominance; second order FS dominance; weak FS dominance.

JEL Classification: D03 (Behavioral Economics: Underlying Principles); D63 (Equity, Justice, Inequality and other normative criteria of measurement); D64 (Altruism).

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1. Introduction

Consider the following motivation for stochastic dominance that arises in welfare economics.[1] There is a *self-regarding individual* with non-decreasing utility function of income, $u(y)$, who follows expected utility. Let the set of possible income levels for that individual be $\mathbf{Y} = \{y_1 < y_2 < ... < y_n\}$. The individual must choose between two income distributions P and Q over the set $\mathbf{Y}$ from behind a *veil of ignorance*, i.e., before the realized income level is known. We call this the *ex-ante perspective*. It is well known that if u is non-decreasing then the individual will prefer P to Q if, and only if, P first order stochastically dominates Q. Furthermore, when, in addition, u is concave and P and Q have the same mean, then the individual will prefer P over Q if, and only if, P second order stochastically dominates Q.[2]

Classical first and second order stochastic dominance have been used in both the positive sense (as a description of actual behaviour of decision makers) and in the normative sense (as describing what decision makers *ought* to do). In this paper we are interested in the positive sense.

Extensive empirical evidence shows that individuals have *other-regarding preferences*, i.e., they care, not just for their own consumption bundle, but for the consumption bundles of others.[3] Let us now interpret the set $\mathbf{Y} = \{y_1 < y_2 < ... < y_n\}$ as the set of n possible realized income classes of N individuals. Unlike the perspective in classical welfare economics, throughout this paper we take an *ex-post perspective*. In this perspective, individual j knows his/her income level, y_j. Given a distribution, P over $\mathbf{Y}$, of incomes of all individuals, let the utility function of individual j, when the preferences are other-regarding, be given by $U(y_j, P)$. This gives rise to a powerful framework of analysis that has superior predictive power relative to one based on self-regarding preferences alone.[4]

Several models of other-regarding preferences have been proposed; for surveys, see Camerer (2003), Fehr and Schmidt (2006) and Dhami (2016). One of the most salient models is that of Fehr and Schmidt, 1999; which we refer to as the FS model. The FS model

[1]We are not arguing that this is the only possible motivation for stochastic dominance but it is an important one and is the most relevant for our paper.

[2]See, Hanoch and Levy (1969), Rothschild and Stiglitz (1970, 1971), Shorrocks, 1983, Kolm (1969) and Atkinson (1970). See, also the book length treatment in Lambert (2001). The reader can also consult any standard textbook in microeconomics, such as Mas-Colell et al. (1995).

[3]See, for instance, Camerer (2003), Fehr and Fischbacher (2002), Fehr and Schmidt (2006), Kolm and Ythier (2006), Gintis (2009), and Dhami (2016).

[4]Other-regarding preferences can explain the evidence from a wide range of experiments such as the dictator game, the ultimatum game, the gift exchange game and the public-good game. The scope of the phenomena explained by such preferences spans much of economics. For instance, such preferences can also explain social comparisons at the workplace, the design of optimal incentive schemes under moral hazard, the structure of labour contracts, some surprising effects of incentives and provides a reason for the existence of firms (Dhami, 2016, Part 2). By contrast, self-regarding preferences sit uneasily with these experimental results.

explores a subset of other-regarding preferences known as *inequity averse* preferences. Under FS preferences, individuals care about their own payoffs (as in models of self-regarding preferences) and they also derive disutility from comparing their income with those who are (i) richer (*envy*). and (ii) poorer (*altruism*). In some situations, such as a race or a tournament among workers in an organization, such as yardstick competition, a worker may derive positive utility (*pride*) in coming out ahead. We also consider the role of pride.

Redistributive taxes offer individuals a choice among alternative income distributions such as P, Q over $\mathbf{Y}$; the relevant evidence supports the role of other-regarding preferences in this context. In experiments where people are asked to choose among alternative income distributions, the FS model often fits the data reasonably well (Ackert et al., 2007; Bolton and Ockenfels, 2006; Tyran and Sausgruber, 2006).[5]

With the growing acceptance of other-regarding preferences in economics, the challenge is to develop the theoretical analogues of the tools that play a critical role under self-regarding preferences. In particular, we are interested in answering the following question under an ex-post perspective (so we do not appeal to 'behind the veil of ignorance' arguments). Consider two cumulative distributions, P, Q, over the set of incomes $\mathbf{Y}$. We then ask the following novel question: Under what conditions on P and Q would a decision maker with FS preferences, who knows his own income level y_j, prefer P to Q? We now give a few motivating examples where this question may be important.

Example 1 *(Political economy): Rational choice theory considers a framework in which there are two political parties that care only about winning elections. Under well-known conditions, it can be shown that the optimal policy of each party caters to the preferences of the median voter (Downs, 1957). However, there might well be other motivations for political parties to cater to a particular group of swing voters or to those who have a particular ideological persuasion (Gilens and Page, 2014). Suppose that, for whatever reason, a political party is interested in winning the support of a particular pivotal income group that has FS preferences. Suppose also that the current policy debate centres around the choice of two societal income distributions, P and Q (or possibly any finite number of distributions). The campaign strategy of the political party is likely to depend on which of the alternative income distributions is preferred by the pivotal voter.*

Example 2 *(Location decisions): An individual with FS preferences is considering a move*

[5] From their experimental results on voting over alternative income distributions, Tyran and Sausgruber (2006) conclude that the FS model predicts much better than a model with self-regarding preferences. For the three income classes in their experiments, the FS model provides, in their words, "strikingly accurate predictions for individual voting in all three income classes." In the context of redistributive taxation, Ackert et al. (2007) find that the estimated coefficients of altruism and envy in the FS model are statistically significant and of the correct sign.

to one of two neighborhoods/clubs that have respective distribution of incomes given by P and Q. The neighborhoods are identical in all other respects. Which neighborhood/club should the individual choose?

Example 3 *(Inequality in public discourse): The aphorism 'rich get richer poor poorer' is typically invoked to describe situations of increasing inequality and to express ethical disapproval of the observed income distributions.*[6] *Suppose that there are two societal income distributions, P and Q. Under Q, relative to P, the rich are richer and the poor poorer. Under purely self-regarding preferences, if an individual's income is identical under P and Q, then he/she should be indifferent among the two distributions. Hence, the common use of the aphorism above is a puzzle. A natural question to ask is: Does an individual with FS preferences prefer P to Q, which might help us to account for the popularity of this aphorism?*

The plan of our paper is as follows. In Section 2, we describe the model and give some preliminary intermediate. We also give the results on classical first and second order stochastic dominance concepts.

In Section 3, we give the general form of FS preferences and some intermediate results. We show, by example, why an individual with FS preferences may be unable to rank income distributions under the classical notions of first and second order stochastic dominance.

In Section 4, we consider two main dominance concepts that are appropriate for the general version of FS preferences: *first and second order FS dominance* respectively. These are the analogues of classical first and second order stochastic dominance. Under FS preferences, individuals dislike income differences between themselves and those who are (1) poorer (altruism), and (2) those who are richer (envy). Thus, individuals with FS preferences feel better-off if (1) for incomes below their own income, the poor become richer, and (2) for incomes above their own income, the rich become poorer. In other words, they prefer distributions that are classically first order dominant below their incomes and dominated above their incomes; this gives rise to *first order FS dominance*.

There could be cases where we are unable to rank distributions by first order FS dominance. In this case, if the own utility function is non-decreasing and concave, then we may be able to rank distributions by *second order FS dominance*. This requires that, relative to one's income, the preferred distribution be classically second order *dominant* for poorer individuals and classically second order dominated for richer individuals.

For pedagogical clarity we structure Section 4 as follows. The main ideas are introduced through a simple example in subsection 4.1. Subsections 4.2 and 4.3 give formal treatments of first and second order FS dominance, respectively. Subsection 4.4 shows that FS dominance is a generalization of classical stochastic dominance.

[6] To judge our use of the word 'typically', we invite the reader to Google the aphorism.

Section 5 considers the appropriate dominance condition for the linear version of FS preferences, *weak FS dominance*, which is implied by first and second order FS dominance.

In Section 6, we derive the dominance results when *pride* (see discussion above) replaces *altruism*. Pride ensures that, relative to their own income, individuals prefer the poor to be poorer and envy ensures that they prefer the rich to be poorer as well. Hence, such individuals prefer *stochastically dominated* distributions of income in the classical sense. Thus, classical first and second order stochastic dominance suffice when pride replaces altruism in FS preferences. However, the direction of preference is the opposite of the case with self-regarding preferences.

In Section 7, we explore the applicability of our results to other models of inequity averse preferences that include the ERC model of Bolton and Ockenfels (2000) and the Charness and Rabin model (2002). We are well aware of other models of social preferences, particularly those that involve the role of intentions and type-based reciprocity, which is missing from models of inequity aversion. However, a rigorous consideration of intentions requires the use of psychological game theory (Dhami, 2016, Section 13.5) and an examination of inequality issues within this framework must await future research.

Section 8 concludes.

The proofs are in the Appendix (Section 9).

2. The Model

The purpose of this section is to set up our model and give some preliminary results that will be used later in the paper. We then give the classical first and second order stochastic dominance concepts and the main results concerning these.

2.1. Notation and some preliminaries

Consider a society or a group of individuals that considers each other a part of its reference group, with N members. Income, y, of any member belongs to one (and only one) of the income levels in the set of possible incomes $\mathbf{Y} = \{y_1 < y_2 < ... < y_n\}$.[7] We use the discrete case because (1) it is empirically realistic, and (2) avoids measure theoretic issues associated with the continuous case.

Let $p_i \geq 0$ be the proportion of individuals with income y_i, $i = 1, 2, ..., n$, $\sum_{i=1}^{i=n} p_i = 1$. The *cumulative probability distribution* is given by $P_0 = 0$, $P_j = \sum_{i=1}^{i=j} p_i$. Let Π be the set of all such distributions over $\mathbf{Y}$. The *cumulative of the cumulative distribution* is given by

[7] As explained in the introduction we take the more realistic ex-post perspective that does not rely on 'choosing behind a veil of ignorance' arguments.

$\widetilde{P}_0 = 0$, $\widetilde{P}_j = \sum_{i=1}^{i=j} P_i$, $j = 1, 2, ..., n$; let $\widetilde{\Pi}$ be the set of all such distributions.[8]

A total of $p_i N$ individuals have income y_i each, and $p_i N y_i$ is their total income. The proportion of individuals with incomes less than or equal to y_j is P_j; the total number of such individuals is $P_j N$. The total income accruing to the poorest $P_j N$ individuals is $N \sum_{i=1}^{i=j} p_i y_i$. The average, or mean, of $y_1, y_2, ..., y_n$ under the distribution $P \in \Pi$ is $\mu_P = \sum_{i=1}^{i=n} p_i y_i$.

We shall make use of the Kronecker-δ: $\delta_{ij} = 1$ if $i = j$ but $\delta_{ij} = 0$ if $i \neq j$.

For some results it will be useful to restrict the income levels to be equally spaced.

Definition 1 : *We say that the income levels are equally spaced if for some positive real number, δ, $y_{i+1} - y_i = \delta > 0$ for $i = 1, 2, ..., n-1$.*

Equal spacing is without loss of generality because we can always introduce extra income levels, each with probability zero, to achieve equal spacing.[9]

Lemma 1 : *Suppose incomes are equally spaced (Definition 1).*
(a) Let $P \in \Pi$. Then $\mu_P = y_n - \delta \widetilde{P}_{n-1}$.
(b) Let $P, Q \in \Pi$. Then $\mu_P \lesseqqgtr \mu_Q$ if, and only if, $\widetilde{P}_{n-1} \gtreqqless \widetilde{Q}_{n-1}$.

Definition 2 : *Suppose that the utility function $u(y)$ is non-decreasing in income, y. The class of all such utility functions is denoted by $\mathbf{u}$. For $u \in \mathbf{u}$, let:*
$\Delta_i u = u(y_{i+1}) - u(y_i) \geq 0$, $i = 1, 2, ..., n-1$ *and*
$\Delta_i^2 u = \Delta_i u - \Delta_{i-1} u = [u(y_{i+1}) - u(y_i)] - [u(y_i) - u(y_{i-1})]$, $i = 2, 3, ..., n-1$.

2.2. Classical first and second order stochastic dominance

Consider the classical framework that takes an *ex-ante perspective*, in the sense that the individual operates behind a veil of ignorance. Here, each distribution $P, Q \in \Pi$ is taken to reflect the ex-ante uncertainty with which a set of possible incomes $\mathbf{Y} = \{y_1 < y_2 < ... < y_n\}$ is realized in the future for a single individual who (1) operates behind a veil of ignorance, and (2) follows expected utility theory. Then the classical concepts of first and second order stochastic dominance are defined as follows.

[8] $\widetilde{\Pi}$ plays a critical role in defining second order stochastic dominance in the classical analysis; and also in ours.

[9] For example, suppose $y_1 = 5$, $y_2 = 7$, $y_3 = 11$, which is not equally spaced, and $P_1 = \frac{1}{3}$, $P_2 = \frac{2}{3}$. Consider $y_1 = 5$, $y_2 = 7$, $y_3 = 9$, $y_4 = 11$, which are equally spaced, and $Q_1 = \frac{1}{3}$, $Q_2 = \frac{2}{3}$, $Q_3 = \frac{2}{3}$. Both P and Q describe the same reality.

Definition 3 *(Expected utility): Let $u \in \mathbf{u}$ and $P \in \Pi$. Then the expected utility is given by*[10]

$$EU(P) = \sum_{i=1}^{i=n} p_i u(y_i). \tag{2.1}$$

2.2.1. First order stochastic dominance

Definition 4 *: Let $P, Q \in \Pi$. Then P first order stochastically dominates Q ($P \succsim_1 Q$) if $P_i \leq Q_i$ for $i = 1, 2, ..., n-1$. If, in addition, the inequality is strict for some j then P strictly first order stochastically dominates Q ($P \succ_1 Q$).*

Proposition 1 *: Let $P, Q \in \Pi$. Then*
(a) $P \succsim_1 Q$ if, and only if, for all $u \in \mathbf{u}$, $EU(P) \geq EU(Q)$.
(b) $P \succ_1 Q$ if, and only if, for all $u \in \mathbf{u}$, $EU(P) \geq EU(Q)$ and $EU(P) > EU(Q)$ for some $u \in \mathbf{u}$.

2.2.2. Second order stochastic dominance

Definition 5 *: Let $P, Q \in \Pi$. Then P second order stochastically dominates Q ($P \succsim_2 Q$) if $\widetilde{P}_j \leq \widetilde{Q}_j$ for $j = 1, 2, ..., n-1$. If, in addition, one of these inequalities is strict then P strictly second order stochastically dominates Q ($P \succ_2 Q$).*

Proposition 2 *: Suppose incomes are equally spaced (Definition 1). Let $P, Q \in \Pi$ have equal means. Then*
(a) $P \succsim_2 Q$ if, and only if, for all concave $u \in \mathbf{u}$, $EU(P) \geq EU(Q)$.
(b) $P \succ_2 Q$ if, and only if, for all concave $u \in \mathbf{u}$, $EU(P) \geq EU(Q)$ and $EU(P) > EU(Q)$ for some $u \in \mathbf{u}$.

The final result in this section relies on the fact that second order stochastic dominance needs the extra condition of concavity of the utility function relative to first order stochastic dominance. Hence, first order stochastic dominance implies second order stochastic dominance, but the converse need not hold.

Corollary 1 *: Suppose incomes are equally spaced (Definition 1). Let $P, Q \in \Pi$ have the same mean. Then*
(a) $P \succsim_1 Q \Rightarrow P \succsim_2 Q$,
(b) $P \succ_1 Q \Rightarrow P \succ_2 Q$.

[10] Our notation EU for expected utility is not to be confused with the FS utility function U that we introduce later.

2.3. Looking forward

Propositions 3 and 4 of Section 4 give the analogues for Fehr-Schmidt preferences of the classical results in Propositions 1 and 2. Propositions 5 and 6 of Section 5 give specialized results for the linear version of FS preferences. Propositions 7 and 8 of Section 6 give the analogues for Fehr-Schmidt preferences when pride replaces altruism.

3. Fehr-Schmidt (FS) social preferences

In this section we begin by stating the general form of FS preferences. In subsection 3.1 we derive some intermediate results. In subsection 3.2 we show, by example, why an individual with FS preferences may be unable to rank income distributions under the classical notions of first and second order stochastic dominance.

In its most general discrete form, the Fehr-Schmidt (*FS*) *utility function* U of an individual with income $y_j \in \mathbf{Y}$ is defined below.[11]

Definition 6 *(General form of FS utility): Consider the income distribution* $P \in \Pi$ *and utility function* $u \in \mathbf{u}$*. The general form of the FS utility function for an individual with income* $y_j \in \mathbf{Y}$ *and distribution* P *over* $\mathbf{Y}$ *is given by*[12]

$$U(y_j, P) = u(y_j) - \beta \sum_{i=1}^{j-1} p_i \left[u(y_j) - u(y_i)\right] - \alpha \sum_{k=j+1}^{n} p_k \left[u(y_k) - u(y_j)\right], \quad (3.1)$$

$$\text{where, } \alpha > 0, 0 \leq \beta < 1. \quad (3.2)$$

For an individual with *self-regarding* preferences, $\alpha = \beta = 0$, so

$$U(y, P) = u(y). \quad (3.3)$$

From (3.1), an individual with FS-preferences derives utility from 'own payoff' just like an individual with self-regarding preferences (first term in (3.1)). But, in addition, the individual derives disutility from two sources: From payoffs relative to those where inequality is *advantageous* (second term in (3.1)) and from payoffs relative to those where inequality is *disadvantageous* (third term in (3.1)). We shall call the disutility arising from these two terms, respectively, as *advantageous inequity* and *disadvantageous inequity*; these terms capture respectively, *altruism* and *envy* in the context of FS-preferences. Notice that in FS-preferences, inequality is *self-centered*, i.e., the individual uses own payoff as a reference point with which everyone else is compared. In (3.1), β is bounded above by 1

[11] Neilson (2006) gives an axiomatization.

[12] We adopt a slightly different normalization than Fehr-Schmidt (1999) because we wish to interpret p_i, $i = 1, 2, ..., n$ as probabilities or proportions.

because $\beta > 1$ would imply that an individual would increase utility by merely destroying own income, which is counterfactual. The typical empirical finding is that $\beta < \alpha$; (Fehr and Schmidt, 1999; Eckel and Gintis, 2010).

3.1. Some preliminary implications of FS preferences for inequality

The next Lemma allows us to write FS utility in terms of utility levels rather than utility differences.

Lemma 2 *: The FS utility function for an individual with income y_j, (3.1), can be written in the following, equivalent, form:*

$$U(y_j, P) = \omega_j u(y_j) + \beta \sum_{i=1}^{j-1} p_i u(y_i) - \alpha \sum_{k=j+1}^{n} p_k u(y_k), \tag{3.4}$$

$$\text{where } \omega_j = 1 - \beta P_{j-1} + \alpha(1 - P_j) > 0,\ \alpha \geq 0, 0 \leq \beta < 1,\ u \in \mathbf{u}. \tag{3.5}$$

Suppose that u is strictly increasing, then it follows from (3.4) and (3.5) that an increase in own income y_j (keeping all other incomes fixed and the order of incomes fixed) increases $U(y_j, P)$, as in models of self-regarding preferences.

The next lemma gives useful intermediate results. Its motivation is that the machinery of first order dominance relies on using cumulative probabilities, while second order dominance relies on cumulative of the cumulative probabilities.

Lemma 3 *: Let $u \in \mathbf{u}, P, Q \in \Pi$. Then, the term $U(y_j, P) - U(y_j, Q)$ can be written in the following three equivalent ways, (a), (b) and (c) that employ, respectively, probabilities, cumulative probabilities, and cumulative of the cumulative probabilities:*

$$\text{(a) } \beta \sum_{i=1}^{j-1} (q_i - p_i)\left[u(y_j) - u(y_i)\right] + \alpha \sum_{k=j+1}^{n} (q_k - p_k)\left[u(y_k) - u(y_j)\right]. \tag{3.6}$$

$$\text{(b) } \beta \sum_{i=1}^{i=j-1} (Q_i - P_i)\Delta_i u + \alpha \sum_{k=j}^{k=n-1} (P_k - Q_k)\Delta_k u. \tag{3.7}$$

$$\text{(c) } \alpha\left(\widetilde{P}_{n-1} - \widetilde{Q}_{n-1}\right)\Delta_{n-1}u + \left(\widetilde{Q}_{j-1} - \widetilde{P}_{j-1}\right)\left(\alpha\Delta_j u + \beta\Delta_{j-1}u\right)$$
$$+ \beta \sum_{i=1}^{i=j-2}\left(\widetilde{P}_i - \widetilde{Q}_i\right)\Delta^2_{i+1}u + \alpha \sum_{k=j}^{k=n-2}\left(\widetilde{Q}_k - \widetilde{P}_k\right)\Delta^2_{k+1}u. \tag{3.8}$$

Remark 1 below considers implications of FS preferences that lead to novel and testable predictions of inequity-averse preferences that are in contrast to the predictions of self-regarding preferences.

Remark 1 *: Suppose that u is increasing, $\alpha > 0$ and $\beta > 0$.*
(a) From Lemma 2, we see that $U(y_j, P)$ is increasing in the utilities of the relatively poor $(i < j)$ and decreasing in the utilities of the relatively rich $(k > j)$. The former reduces advantageous inequity and the latter increases disadvantageous inequity.
(b) Suppose that an individual has the FS preferences given in Definition 6 and has income y_j. Begin with an income distribution $P \in \Pi$. Consider two income levels, y_s, y_t, such that $y_s < y_t < y_j$, and $p_s > 0, p_t > 0$. Suppose that we obtain the distribution Q from P by transferring a fraction δ, $0 < \delta < p_t$, of individuals from the income class y_t to y_s. By construction, $q_s - p_s = \delta > 0$, $q_t - p_t = -\delta$ and $q_i = p_i$ for $i \notin \{s, t\}$. Then, from Lemma 3a,

$$\begin{aligned} U(y_j, P) - U(y_j, Q) &= \beta(q_s - p_s)[u(y_j) - u(y_s)] + \beta(q_t - p_t)[u(y_j) - u(y_t)] \\ &= \beta\delta[u(y_j) - u(y_s)] - \beta\delta[u(y_j) - u(y_t)] \\ &= \beta\delta[u(y_t) - u(y_s)] > 0. \end{aligned}$$

Thus, individuals with FS preferences dislike 'rich to poor transfers' among people poorer than them.
(c) Suppose that an individual has the FS preferences given in Definition 6 and has income y_j. Start with an income distribution $P \in \Pi$. Now consider two income levels, y_l, y_m, such that $y_j < y_l < y_m$, and $p_m > 0$, $p_j > 0$. Now obtain a new distribution Q by transferring a fraction δ, $0 < \delta < p_m$, of individuals from the income class y_m to y_l. Then, from Lemma 3a,

$$U(y_j, Q) - U(y_j, P) = \alpha\delta[u(y_m) - u(y_l)] > 0.$$

Thus, individuals with FS preferences prefer 'rich to poor transfers' among people richer than them.

3.2. Inappropriateness of classical first and second order stochastic dominance for FS preferences

Consider an individual who is a member of a group of N individuals. Let the set of possible income levels of these individuals be $\mathbf{Y} = \{y_1 < y_2 < ... < y_n\}$. Consider two cumulative distributions over $\mathbf{Y}$: P and Q. An individual knows his/her own income, y_j, has FS preferences (Definition 6), and wishes to rank the distributions P and Q. Thus, our perspective is ex-post. The next example shows that the classical concepts of first and second order stochastic dominance (Definitions 4 and 5) are inappropriate for this purpose.

Example 4 *: Consider three income levels, $0, 25, 50$ and two distributions, P, Q defined in Table 3.1, where $\frac{1}{6} \leq \varepsilon \leq \frac{5}{6}$. Under the distribution Q, for instance, a sixth of the*

y	$y_1 = 0$	$y_2 = 25$	$y_3 = 50$
$p(y)$	0	1/3	2/3
$q(y)$	1/6	ε	(5/6) - ε
$P(y)$	0	1/3	1
$Q(y)$	1/6	(1/6) + ε	1
$\tilde{P}(y)$	0	1/3	4/3
$\tilde{Q}(y)$	1/6	(1/3) + ε	(4/3) + ε

Table 3.1: Hypothetical income distribution data to illustrate unsuitability of classical dominance concepts for Fehr-Schmidt preferences.

population has income $y_1 = 0$, a proportion ε has the income $y_2 = 25$ and a proportion $\frac{5}{6} - \varepsilon$ has the income $y_3 = 50$.

It can be seen that $P_j \leq Q_j$ for $j = 1, 2, 3$; hence, $P \succeq_1 Q$. In fact, $P \succ_1 Q$. Also, $\widetilde{P}_j \leq \widetilde{Q}_j$ for $j = 1, 2, 3$ and $\mu_P = \frac{125}{3} > \frac{125}{3} - 25\varepsilon = \mu_Q$; hence, $P \succeq_2 Q$ (see Definition 5). In fact, $P \succ_2 Q$. For the individual with income $y_2 = 25$, using Lemma 3a, we get

$$U(25, P) - U(25, Q) = \frac{\beta}{6}\left[u\left(25\right) - u\left(0\right)\right] + \alpha\left(\frac{1}{6} - \varepsilon\right)\left[u(50) - u(25)\right]. \tag{3.9}$$

It is simple to construct examples where an individual with FS preferences strictly prefers the stochastically dominated distribution Q to P. For example, let $\varepsilon = \frac{1}{3}$ $\alpha = 1$ and $\beta = 0$ and take u to be any strictly increasing own utility function. Then, from (3.9) we get $U\left(25, Q\right) - U\left(25, P\right) = \frac{u(50)-u(25)}{6} > 0$.
Consider now some implications of pride[13] ($\beta < 0$) rather than altruism ($\beta > 0$). Let u be any strictly increasing own utility function. Take $\alpha = 1$ and $\varepsilon = \frac{1}{6}$. Then $U\left(25, Q\right) - U\left(25, P\right) = -\frac{\beta}{6}\left[u\left(25\right) - u\left(0\right)\right]$. Clearly the choice between P, Q now hinges solely on the sign of β. An altruistic individual ($\beta > 0$) prefers P to Q but one with pride ($\beta < 0$) strictly prefers the strictly first and second order stochastically dominated distribution Q to P.

Example 4 shows the inadequacy of the first and second order stochastic dominance concepts in the presence of other-regarding preferences. Such decision makers may choose distributions that are stochastically dominated in the classical sense. Economists are often

[13] See Section 6, below for a more formal treatment.

y	$y_1 = 0$	$y_2 = 25$	$y_3 = 50$	$y_4 = 75$	$y_5 = 100$
$p(y)$	0	1/3	(1/3) + 2ε	(1/3) - 2ε	0
$q(y)$	ε	(1/3) - ε	1/3	(1/3) - ε	ε
$P(y)$	0	1/3	(2/3) + 2ε	1	1
$Q(y)$	ε	1/3	2/3	1 - ε	1
$\tilde{P}(y)$	0	1/3	1 + 2ε	2 + 2ε	3 + 2ε
$\tilde{Q}(y)$	ε	(1/3) + ε	1 + ε	2	3

Table 4.1: Hypothetical income distribution data to illustrate dominance concepts suitable for Fehr-Schmidt preferences.

reluctant to allow stochastically dominated options to be chosen unless such dominance is not obvious, which might be the case for complex distributions. Yet, there is nothing irrational about having other-regarding preferences. Hence, what is required is a more appropriate concept of stochastic dominance. This forms the subject matter of the rest of the paper.

4. Dominance concepts for FS preferences

In this section we consider two main dominance concepts that are appropriate for the general version of FS preferences. These are *first and second order FS dominance* that are, respectively, the analogues of classical first and second order stochastic dominance. The main ideas are introduced through a simple example in subsection 4.1, immediately below. Subsections 4.2 and 4.3 give formal treatments of first and second order FS dominance, respectively. Subsection 4.4 shows that FS dominance is a generalization of classical stochastic dominance.

4.1. Motivating example

We now consider an example that illustrates the main ideas behind several results in the paper.

Example 5 : *Consider the data given in Table 4.1, where $0 < \varepsilon < \frac{1}{6}$ and society has 5 different levels of income, $0, 25, 50, 75, 100$. There are two income distributions P and Q; the respective probability densities p and q show the proportions of individuals with each of the 5 income levels under each distribution. We are specifically interested in the*

individual with income $y_3 = 50$ who has FS preferences.
Income distribution Q is derived from the distribution P as follows. (1) Below income $y_3 = 50$, a fraction ε of richer individuals with income 25 are moved to a relatively poorer income level of 0. (2) Above income $y_3 = 50$, a fraction ε of poorer individuals with income 75 are moved to a relatively richer income level of 100. From Remark 1b, we would expect the individual with income y_3 to prefer P to Q. An examination of Table 4.1 reveals that the following restrictions apply

$$P_i \leq Q_i \text{ for } i < 3 \text{ and } P_k \geq Q_k \text{ for } k \geq 3. \tag{4.1}$$

$$\widetilde{P}_i \leq \widetilde{Q}_i \text{ for } i < 3 \text{ and } \widetilde{P}_k \geq \widetilde{Q}_k \text{ for } k \geq 3. \tag{4.2}$$

From (4.1), (4.2) neither distribution first or second order stochastically dominates the other (see Definitions 4 and 5). Thus, a-priori, the classical dominance framework cannot predict which distribution is preferred by an individual with self-regarding preferences who takes an ex-ante perspective (see Propositions 1 and 2).
However, the restrictions in (4.1), (4.2) are ideal to determine which of the two distributions is preferred by an individual with FS preferences and income $y_3 = 50$. One can make the following simple calculations using Lemma 3a for any $u \in \mathbf{u}$,

$$U(50, P) - U(50, Q) = \beta\varepsilon\left[u(25) - u(0)\right] + \varepsilon\alpha\left[u(75) - u(50) + u(100) - u(50)\right] > 0.$$

Thus, the individual with income y_3 prefers P to Q, which is consistent with Remark 1b. ■

The perspective in classical analysis is ex-ante, and invokes the veil of ignorance. However, our perspective is ex-post, hence, it is unavoidable but to consider the preferences over alternative distributions *for each level of income*. In particular, in Example 5 above, we consider the preferences of an individual with income $y_3 = 50$. A similar analysis can be carried out for individuals with income levels $0, 25, 75, 100$. If the FS preferences of individuals with different incomes levels over the alternative income distributions are different, then the choice of a societal income distribution, if such a choice exists, is an important question for future research.[14]

In the case of first and second order stochastic dominance (Definitions 4, 5) the dominant distribution, P, has a mean that is no lower than the dominated distribution, Q. This need not be the case under FS dominance. From the data given in Table 4.1:

$$\mu_Q - \mu_P = 50\varepsilon > 0.$$

Thus, the strictly preferred distribution, P, for an individual with FS preferences and income $y_3 = 50$ has a lower mean. The reason is that, due to $\alpha > 0$, a reduction in the

[14] For some progress in this direction, see Dhami and al-Nowaihi (2010a,b).

mean may be associated with a reduction in disadvantageous inequality for an individual without affecting that individual's income. From the perspective of an individual with income $y_3 = 50$ this could happen by moving some individuals with incomes y_5 to the income level y_4.[15] This is not just a theoretical curiosity. It is supported by the evidence where experimental subjects prefer a smaller cake which is is more equitably distributed.[16]

The conditions in (4.1), (4.2) are examples of the more general conditions that we give in the formal development of the theory, below. We call these conditions, respectively, *first order FS dominance* and *second order FS dominance*. We denote the binary relations corresponding to these dominance concepts, respectively, by $\succeq_{FS1_{y_j}}$ and $\succeq_{FS2_{y_j}}$; the subscript y_j reflects the ex-post perspective from the point of view of an individual who knows that his/her income is y_j. Thus, in Example 5 above, we have $P \succeq_{FS1_{y_j}} Q$ and $P \succeq_{FS2_{y_j}} Q$.

Notice from the data given in Table 4.1 that

$$\sum_{i=1}^{2} p_i \left(y_3 - y_i\right) \leq \sum_{i=1}^{2} q_i \left(y_3 - y_i\right), \tag{4.3}$$

$$\sum_{k=4}^{5} p_k \left(y_k - y_3\right) \leq \sum_{k=4}^{5} q_k \left(y_k - y_3\right). \tag{4.4}$$

Consider now the linear form of FS preferences given in Definition 6 with $u(y) = y$. Simple calculations will show that when (4.3), (4.4) hold, then an individual with income y_3 will prefer the distribution P to Q. Conditions (4.3), (4.4) are examples of what we shall call as *weak FS dominance* in Section 5, below. We shall denote the binary relation based on weak FS dominance by $\succeq_{WFS_{y_j}}$ and show formally that this is implied by first and second order FS dominance.

These examples enable us to anticipate the main results in the paper. We will, formally, show the following:

1. $P \succeq_{FS1_{y_j}} Q \Leftrightarrow U(y_j, P) \geq U(y_j, Q)$ for all non-decreasing own utility functions, u, and all $\alpha \geq 0$, $\beta \in [0, 1)$.

2. $P \succeq_{FS2_{y_j}} Q \Leftrightarrow U(y_j, P) \geq U(y_j, Q)$ for all non-decreasing concave own utility functions, u, and for all $\alpha \geq 0$, $\beta \in [0, 1)$, when incomes are equally spaced.

We shall also show that for the linear form of FS preferences ($u(y) = y$):

1. $P \succeq_{WFS_{y_j}} Q \Leftrightarrow U(y_j, P) \geq U(y_j, Q)$ for all $\alpha \geq 0$, $\beta \in [0, 1)$.

[15] Technically, this is because of the presence of the term $\alpha\left(\widetilde{P}_{n-1} - \widetilde{Q}_{n-1}\right)\left[u(y_n) - u(y_{n-1})\right]$ on the right hand side of (3.8) in Lemma 3.

[16] See, for instance, Ackert et al. (2007), Bolton and Ockenfels (2006) and Tyran and Sausgruber (2006).

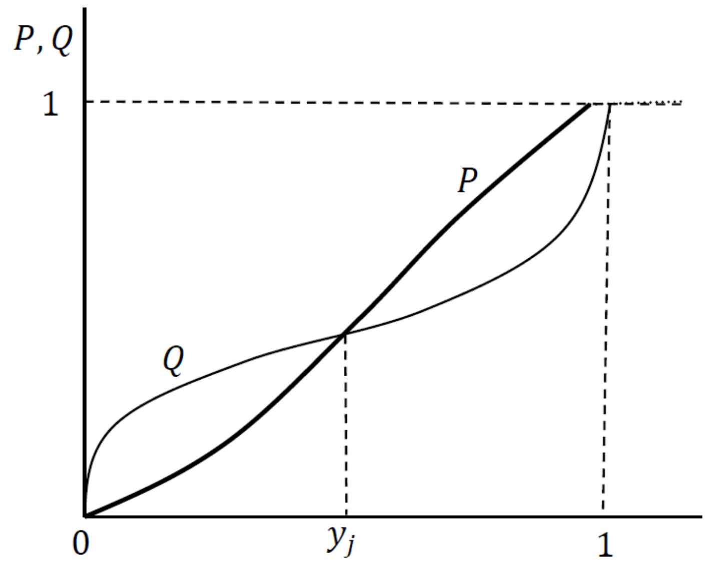

Figure 4.1: Distribution P *first order FS dominates* Q. The dominant distribution P is shown in bold.

2. $P \succeq_{FS1_{y_j}} Q \Rightarrow P \succeq_{WFS_{y_j}} Q$.

3. When incomes are equally spaced, $P \succeq_{FS2_{y_j}} Q \Rightarrow P \succeq_{WFS_{y_j}} Q$.

4.2. First order Fehr-Schmidt dominance

In this section, we propose *first order FS dominance* (FS_{y_j}). It captures one sense in which an individual with income y_j and FS preferences prefers income distribution P to distribution Q. In Figure 4.1, distribution P *first order FS dominates* Q for an individual with income y_j.

Definition 7 *: Consider an individual with income $y_j \in \mathbf{Y}$. Let $P, Q \in \Pi$. Then P first order FS_{y_j} dominates Q ($P \succeq_{FS1_{y_j}} Q$) if*
(a) $P_i \leq Q_i$ for each $i = 1, 2, ..., j-1$, and
(b) $P_k \geq Q_k$ for each $k = j, j+1, ..., n-1$.
If, in addition, one of these inequalities is strict, then we say that P strictly first order FS_{y_j} dominates Q ($P \succ_{FS1_{y_j}} Q$).

First order FS dominance is illustrated in Figure 4.1. As noted earlier, our perspective is ex-post, unlike the classical ex-ante 'behind the veil of ignorance' approach. This requires us to assess income inequality from the point of an individual who knows that his/her income is y_j. The next proposition establishes the economic foundations of first order FS dominance.

Proposition 3 : *Consider an individual with income* $y_j \in \mathbf{Y}$*. Let* $P, Q \in \Pi$*. Then*
(a) (Dominance) P first order FS_{y_j} *dominates Q* ($P \succeq_{FS1_{y_j}} Q$) *if, and only if,* $U(y_j, P) \geq U(y_j, Q)$ *for all* $u \in \mathbf{u}$, $\alpha \geq 0$ *and* $\beta \in [0,1)$.
(b) (Strict dominance) Suppose that $U(y_j, P) \geq U(y_j, Q)$ *for all* $u \in \mathbf{u}$, $\alpha \geq 0$ *and* $\beta \in [0,1)$*. Also, suppose that* $U(y_j, P) > U(y_j, Q)$ *for some* $u \in \mathbf{u}$, $\alpha \geq 0$ *and* $\beta \in [0,1)$*. Then P strictly first order* FS_{y_j} *dominates Q* ($P \succ_{FS1_{y_j}} Q$).
Conversely, suppose that P strictly first order FS_{y_j} *dominates Q* ($P \succ_{FS1_{y_j}} Q$)*. Then* $U(y_j, P) \geq U(y_j, Q)$ *for all* $u \in \mathbf{u}$, $\alpha \geq 0$ *and* $\beta \in [0,1)$*; and* $U(y_j, P) > U(y_j, Q)$ *for some* $u \in \mathbf{u}$, $\alpha \geq 0$ *and* $\beta \in [0,1)$.

The intuition for Proposition 3 follows directly from Remark 1b. Inequity averse individuals prefer a income distribution that is more clustered around their own incomes. Spreading this distribution further away from their incomes (poor becoming poorer and the rich, richer) lowers their FS utility.

Corollary 2 : *Suppose that P first order* FS_{y_j} *dominates Q* ($P \succeq_{FS1_{y_j}} Q$)*. Let* $u \in \mathbf{u}$*. Suppose one of the following holds:*
(i) $\beta > 0$ *and for some* $i \in \{1, 2, ..., j-1\}$, $P_i < Q_i$ *and* $u(y_{i+1}) > u(y_i)$*, or*
(ii) $\alpha > 0$ *and for some* $k \in \{j, j+1, ..., n-1\}$, $P_k > Q_k$ *and* $u(y_{k+1}) > u(y_k)$,
then $U(y_j, P) > U(y_j, Q)$.

4.3. Second order FS dominance

When individuals have FS preferences, if neither P nor Q first order FS dominates the other for an individual with income y_j, then we need further restrictions on the utility function, u, to compare the distributions. A similar contrast exists between classical first and second order stochastic dominance when individuals have self-regarding preferences. As in the classical analysis, this requires us to introduce the cumulative of the cumulative distributions of P, Q, denoted respectively by $\widetilde{P}, \widetilde{Q}$ (see Section 2.1, above).

Definition 8 : *Let* $P, Q \in \Pi$, $y_j \in \mathbf{Y}$*. Then P second order* FS_{y_j} *dominates Q* ($P \succeq_{FS2_{y_j}} Q$) *if*
(a) $\widetilde{P}_i \leq \widetilde{Q}_i$ *for each* $i = 1, 2, ..., j-1$ *and*
(b) $\widetilde{P}_k \geq \widetilde{Q}_k$ *for each* $k = j, j+1, ..., n-1$.
If, in addition, one of the inequalities in (a) or (b) is strict, then we say that P strictly second order FS_{y_j} *dominates Q* ($P \succ_{FS2_{y_j}} Q$).

Remark 2 *From Definition 8 we get that* $P \succeq_{FS2_{y_j}} Q \Rightarrow \widetilde{P}_{n-1} \geq \widetilde{Q}_{n-1}$*. Using Lemma 1, we get that* $\mu_P \leq \mu_Q$*. Contrast this to the converse restriction,* $\mu_P \geq \mu_Q$*, when second order stochastic dominance holds under self-regarding preferences (the latter follows from Definition 5 and Lemma 1).*

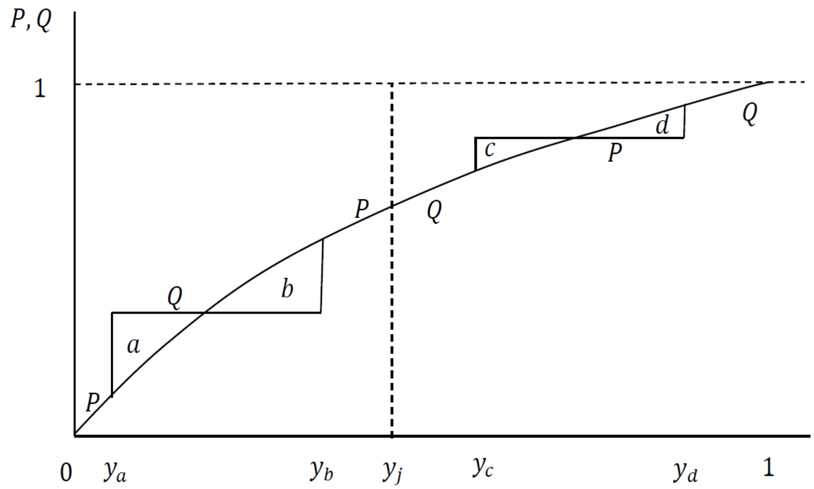

Figure 4.2: The distribution P second order stochastically dominates Q to the left of y_j and area a equals area b. Distribution Q second order stochastically dominates P to the right of y_j and area c equals area d.

Figure 4.2 illustrates the case $P \succeq_{FS2_{y_j}} Q$ for an *elementary increase in risk*, also known as a mean preserving increase in risk, which is constructed as follows. Pick two income levels to the left of $y_j : y_a < y_b$. We take the entire mass assigned to the interval $[y_a, y_b]$ by the distribution P and move it to the end-points of the interval to obtain the distribution Q such that the mean is preserved (area a equals area b). Analogously, pick two income levels to the right of $y_j : y_c < y_c$. Shift the entire mass assigned to the interval $[y_c, y_d]$ by the distribution Q and move it to the end-points of the interval to obtain the distribution P such that the mean is preserved (area c equals area d). In classical terminology, the distribution P second order stochastically dominates Q to the left of y_j, while the distribution Q second order stochastically dominates P to the right of y_j.

We now formalize the intuition given above in Section 4.1 that decision makers with FS preferences derive higher utility from a distribution P that second order FS dominates another distribution Q.

Proposition 4 *: Suppose that incomes are equally spaced (Definition 1). Let $y_j \in \mathbf{Y}$ and $P, Q \in \Pi$.*
(a) Suppose that P second order FS$_{y_j}$ dominate Q ($P \succeq_{FS2_{y_j}} Q$). Then $U(y_j, P) \geq U(y_j, Q)$ for all $\alpha \geq 0$, all $\beta \in [0, 1)$ and all concave $u \in \mathbf{u}$.
Conversely, suppose $U(y_j, P) \geq U(y_j, Q)$ for all $\alpha \geq 0$, all $\beta \in [0, 1)$ and all concave $u \in \mathbf{u}$. In addition, suppose $\widetilde{Q}_{j-1} = \widetilde{P}_{j-1}$. Then P second order FS$_{y_j}$ dominates Q ($P \succeq_{FS2_{y_j}} Q$).
(b) Suppose that P strictly second order FS$_{y_j}$ dominates Q ($P \succ_{FS2_{y_j}} Q$). Let $\alpha > 0$, $\beta > 0$ and let $u \in \mathbf{u}$ be strictly increasing and strictly concave. Then $U(y_j, P) > U(y_j, Q)$.
Conversely, suppose that $U(y_j, P) \geq U(y_j, Q)$ for all $\alpha \geq 0$, all $\beta \in [0, 1)$ and all concave

$u \in \mathbf{u}$. *Also, suppose that* $U(y_j, P) > U(y_j, Q)$ *for some* $\alpha \geq 0$, *some* $\beta \in [0, 1)$ *and some* $u \in \mathbf{u}$ *(not necessarily concave). In addition, assume that* $\widetilde{P}_{j-1} = \widetilde{Q}_{j-1}$. *Then* P *strictly second order* FS_{y_j} *dominates* Q $(P \succ_{FS2_{y_j}} Q)$.

Our final result in this section relies on the fact that second order FS_{y_j} dominance needs the extra condition of concavity of the utility function relative to first order FS_{y_j} dominance. Hence, first order FS_{y_j} dominance implies second order FS_{y_j} but the converse need not hold.

Corollary 3 : *Assume incomes are equally spaced (Definition 1). Assume* $\widetilde{P}_{j-1} = \widetilde{Q}_{j-1}$. *Then*
(a) $P \succeq_{FS1_{y_j}} Q \Rightarrow P \succeq_{FS2_{y_j}} Q$,
(b) $P \succ_{FS1_{y_j}} Q \Rightarrow P \succ_{FS2_{y_j}} Q$.

4.4. Relation between classical stochastic dominance and Fehr-Schmidt dominance.

Note that for $j = n$ part (b) in each of Definitions 7 and 8 is not applicable. Hence first and second order Fehr-Schmidt dominance reduce to classical first and second order stochastic dominance, respectively (Definitions 4 and 5), i.e., $\succeq_{FS1_{y_n}} = \succeq_1$, $\succ_{FS1_{y_n}} = \succ_1$, $\succeq_{FS2_{y_n}} = \succeq_2$ and $\succ_{FS2_{y_n}} = \succ_2$.

We can now see the role played by the extra condition $\widetilde{Q}_{j-1} = \widetilde{P}_{j-1}$ in Proposition 4. Consider the special case $j = n$. In this case, the condition $\widetilde{Q}_{j-1} = \widetilde{P}_{j-1}$ becomes $\widetilde{Q}_{n-1} = \widetilde{P}_{n-1}$. From Lemma 1, recall that $\widetilde{Q}_{n-1} = \widetilde{P}_{n-1}$ implies that the distributions P and Q have equal means; which is a condition needed in Proposition 2. We have seen, just above, that in the case $j = n$ second order Fehr-Schmidt dominance reduces to classical second order stochastic dominance. Thus, the condition $\widetilde{Q}_{j-1} = \widetilde{P}_{j-1}$ is the analogue for second order Fehr-Schmidt dominance of the condition of equal means for classical second order stochastic dominance.

5. Dominance in the linear version of FS preferences

The preferences in Definition 6 do not restrict u to be linear. Hence, this might be termed as the general version. A special version of FS utility, predominant in applied work, is the *linear version*, in which $u(y) = y$ (Definition 9 immediately below). We then consider the appropriate dominance condition for the linear version of FS preferences, *weak FS dominance*, which is implied by each of first and second order FS dominance.

Definition 9 *(Linear version): The linear version of the FS utility function for an individual with income y_j is given by*

$$U(y_j, P) = y_j - \beta \sum_{i=1}^{j-1} p_i (y_j - y_i) - \alpha \sum_{k=j+1}^{n} p_k (y_k - y_j), \; \alpha \geq 0, 0 \leq \beta < 1. \tag{5.1}$$

We now propose a concept of dominance, *weak FS dominance*, that is suited to the linear form of FS preferences (Definition 9). A formal justification is given by Proposition 5, further below. We shall show that it is strictly weaker than both first order FS dominance and second order FS dominance.

Definition 10 *(Weak FS dominance): Let $y_j \in \mathbf{Y}$ and $P, Q \in \Pi$. Then P weakly FS_{y_j} dominates Q ($P \succeq_{WFS_{y_j}} Q$), if*

$$\sum_{i=1}^{j-1} p_i (y_j - y_i) \leq \sum_{i=1}^{j-1} q_i (y_j - y_i), \tag{5.2}$$

$$\sum_{k=j+1}^{n} p_k (y_k - y_j) \leq \sum_{k=j+1}^{n} q_k (y_k - y_j). \tag{5.3}$$

If, in addition, at least one of these inequalities is strict, then P strictly weakly FS_{y_j} dominates Q ($P \succ_{WFS_{y_j}} Q$).[17]

Proposition 5 *: Consider the linear version of FS utility (Definition 9). Let $y_j \in \mathbf{Y}$ and $P, Q \in \Pi$. Then:*
(a) $P \succeq_{WFS_{y_j}} Q$ if, and only if, $U(y_j, P) \geq U(y_j, Q)$ for all $\alpha \geq 0$ and all $\beta \in [0, 1)$.
(b) $P \succ_{WFS_{y_j}} Q$ if, and only if, $U(y_j, P) \geq U(y_j, Q)$ for all $\alpha \geq 0$ and all $\beta \in [0, 1)$ and $U(y_j, P) > U(y_j, Q)$ for some $\alpha \geq 0$ or some $\beta \in [0, 1)$.

We now show that weak FS dominance is implied by first order FS dominance and also by second order FS dominance.

Proposition 6 *: Let $y_j \in \mathbf{Y}$ and $P, Q \in \Pi$.*
(a) If P first order FS_{y_j} dominates Q, then P weakly FS_{y_j} dominates Q.
(b) If P strictly first order FS_{y_j} dominates Q, then P strictly weakly FS_{y_j} dominates Q.
(c) Suppose incomes are equally spaced (Definition 1). If P second order FS_{y_j} dominates Q, then P weakly FS_{y_j} dominates Q.

The converse of the three results in Proposition 6 does not hold because weak FS_{y_j} dominance applies only to the class of linear functions while first and second order FS_{y_j} dominance hold for all $u \in \mathbf{u}$.

[17] One may use the alternative terminology: "*P weakly FS_{y_j} dominates Q in the strict sense*".

Remark 3 : *Although Proposition 6(b) is a strict version of Proposition 6(a), there is no strict version of Proposition 6(c). The reason is that such a proposition would require the own utility function, u, to be strictly concave. But this is not possible for the linear version of FS preferences (Definition 9).*

6. Dominance concepts when pride replaces altruism

In FS preferences, Definition 6, $\beta > 0$ captures altruism or compassion towards individuals who are poorer. Hopkins (2008) differentiates between the sign of β in two different literatures. In behavioral economics, where models of social preferences have gained increasing acceptance, $\beta > 0$. But in happiness economics, $\beta < 0$ is allowed for. Hopkins (2008) refers to $\beta < 0$ as *pride*, *competitiveness* or *downward envy*.[18] Pride might also result from competitive situation such as salary compensation schemes that are determined in a tournament setting.

The FS utility function in (3.1) for the case $\beta \leq 0$, then becomes

$$U(y_j, P) = u(y_j) - \beta \sum_{i=1}^{j-1} p_i [u(y_j) - u(y_i)] - \alpha \sum_{k=j+1}^{n} p_k [u(y_k) - u(y_j)], \ \alpha \geq 0, \beta < 0. \tag{6.1}$$

Lemmas 2 and 3 continue to apply but with $\beta \leq 0$. We now briefly consider the consequences of $\beta \leq 0$ (see (6.1)) for our dominance concepts. Individuals with $\beta \leq 0$ prefer that below their own income the poor become poorer (pride) and above their own income the rich become poorer (envy). Hence, individuals with preferences in (6.1) prefer distributions that are stochastically *dominated* in the classical sense. Thus, while the classical dominance concepts such as first and second order stochastic dominance suffice in this special case, the preferred distribution is diametrically opposite to the one preferred by an expected utility maximizer with purely self-regarding preferences who chooses behind a veil of ignorance (see Section 2.2).

6.1. First order dominance

Proposition 7 : *Consider an individual with income $y_j \in \mathbf{Y}$. Let $P, Q \in \Pi$. Then*
(a) P first order dominates Q ($P \succeq_1 Q$) if, and only if, $U(y_j, P) \leq U(y_j, Q)$ for all $u \in \mathbf{u}$, $\alpha \geq 0$ and $\beta \leq 0$.
(b) Suppose that P strictly first order dominates Q ($P \succ_1 Q$). Then $U(y_j, P) \leq U(y_j, Q)$ for all $u \in \mathbf{u}$, $\alpha \geq 0$ and $\beta \leq 0$, and $U(y_j, P) < U(y_j, Q)$ for some $u \in \mathbf{u}$, $\alpha \geq 0$ and $\beta \leq 0$.
Conversely, suppose that $U(y_j, P) \leq U(y_j, Q)$ for all $u \in \mathbf{u}$, $\alpha \geq 0$ and $\beta \leq 0$. Also,

[18] Hopkins credits the term 'pride' to a 2005 working paper by Daniel Friedman.

suppose that $U(y_j, P) < U(y_j, Q)$ for some $u \in \mathbf{u}$, $\alpha \geq 0$ and $\beta \leq 0$. Then P strictly first order dominates Q ($P \succ_1 Q$).

The next corollary considers some implications of the cases $\beta < 0$ and $\alpha > 0$ for first order dominance.

Corollary 4 *Suppose that P first order dominates Q ($P \succeq_1 Q$). Let $u \in \mathbf{u}$. Suppose one of the following holds:*
(i) $\beta < 0$ and for some $i \in \{1, 2, ..., j-1\}$, $P_i < Q_i$ and $u(y_{i+1}) > u(y_i)$, or
(ii) $\alpha > 0$ and for some $k \in \{j, j+1, ..., n-1\}$, $P_k < Q_k$ and $u(y_{k+1}) > u(y_k)$,
then $U(y_j, P) < U(y_j, Q)$.

6.2. Second order dominance

Proposition 8 *: Suppose incomes are equally spaced (Definition 1). Let $y_j \in \mathbf{Y}$ and $P, Q \in \Pi$. In addition, suppose $\widetilde{Q}_{j-1} = \widetilde{P}_{j-1}$. Then*
(a) Let P second order dominate Q ($P \succeq_2 Q$), then $U(y_j, P) \leq U(y_j, Q)$ for all $\alpha \geq 0$, all $\beta \leq 0$ and all concave $u \in \mathbf{u}$.
Conversely, if $U(y_j, P) \leq U(y_j, Q)$ for all $\alpha \geq 0$, all $\beta \leq 0$ and all concave $u \in \mathbf{u}$, then P second order dominates Q ($P \succeq_2 Q$).
(b) Suppose that P strictly second order dominates Q ($P \succ_2 Q$). Let $\alpha > 0$, $\beta < 0$ and let $u \in \mathbf{u}$ be strictly increasing and strictly concave. Then $U(y_j, P) < U(y_j, Q)$.
Conversely, suppose that $U(y_j, P) \leq U(y_j, Q)$ for all $\alpha \geq 0$, all $\beta \leq 0$ and all concave $u \in \mathbf{u}$. Suppose that, in addition, $U(y_j, P) < U(y_j, Q)$ for some $\alpha \geq 0$, some $\beta \leq 0$ and some $u \in \mathbf{u}$ (not necessarily concave). Then P strictly second order FS_{y_j} dominates Q ($P \succ_2 Q$).

7. Other models of inequity-averse preferences and dominance concepts

In this section, we show that the FS dominance concepts are also applicable to some of the other leading models of inequity-averse preferences.

7.1. The ERC model

Bolton and Ockenfels (2000) propose a model of *equity*, *reciprocity* and *competition* (ERC), which is closely related to the model in Bolton (1991). Suppose that there are n income classes and each income class contains one player only. Denote by $y_i \geq 0$, the income of player $i = 1, 2, ...n$. The sum of the total incomes is $S = \sum_{i=1}^{n} y_i$, which we assume to be strictly positive. The relative income of player i is given by $r_i = r_i(y_i, S, n) = y_i/S$. The utility function of player i is given by

$$u^i = u^i\left(y_i, r_i\right). \tag{7.1}$$

The assumptions on the utility function are as follows: (A1) u^i is twice continuously differentiable in y_i, r_i. (A2) u^i is increasing and concave in the first argument, y_i ($u_1^i \geq 0$, $u_{11}^i \leq 0$). (A3) For any given income, u^i is maximized at $r_i = 1/n$ and is strictly decreasing and strictly concave in r_i around this point. Assumption A3 builds into the model the importance of an equal division of the social surplus. The critical trade-off is created by assumptions A2 and A3. From A2, the individual prefers to increase her monetary payoff. However, from A3, for any monetary payoff, the individual prefers equal division. Neither of ERC or FS nests the other, and they make different empirical predictions.[19] Our focus here is only on the income distribution implications of the ERC model.

In the ERC model, a comparison of one's payoff relative to the aggregate payoff of others captures inequity aversion. However, individuals using ERC preferences do not care about their interpersonal income differences with the incomes of others. In particular, there is no notion of altruism or envy. Suppose that a player has the mean payoff. Now consider a mean preserving spread of the payoff distribution. ERC predicts that the player's utility should be unaffected. The experimental results of Charness and Rabin (2002) do not support this prediction; utility falls with the mean preserving spread, which suggests that income differences are important. If an individual's income and the aggregate income stay the same, then an individual who has ERC preferences is indifferent between two distributions that, under FS preferences, can be ranked by first and/or second order FS dominance.

7.2. The Charness-Rabin model

Charness and Rabin (2002) propose a general model of social preferences. For some parameter values, their model nests FS preferences that allow for both altruism and pride. In this sense our results also apply to their model. Their experimental results also highlight a concern for other possible objectives of players such as efficiency and a Rawlsian concern for the worst-off individual in society. Inspired by the framework suggested by Charness and Rabin (2002), suppose that we generalize FS preferences for an individual with income y_j under the distribution P as follows.

$$W\left(y_j, P\right) = \delta V(y_1, y_2, ..., y_n) + (1-\delta)U\left(y_j, P\right),\ \ \delta \in [0,1], \tag{7.2}$$

[19] For a detailed account of the differences and similarities in the predictions made by the two models, see Fehr and Schmidt (2006) and Dhami (2016, Section 6.3).

where U is the FS utility function (Definition 6) and V is a function of all incomes. To reflect a Rawlsian concern, we may specify $V = \min\{y_1, y_2, ..., y_n\}$, while a concern for economic efficiency may give rise to $V = \sum_{i=1}^{i=n} y_i$. We have a fixed set of incomes, $y_1, y_2, ..., y_n$ in our model, so the minimum income and the sum of incomes is unchanged. If we allow the distribution P to change, then all of the results in Section 4 continue to hold.

Suppose now that the specification of V is $V = \sum_{i=1}^{i=n} p_i y_i$. We know from Remark 2 that if $P \succeq_{FS2_{y_j}} Q$ then we get $\mu_P \leq \mu_Q$. In this case, in terms of the RHS of (7.2), the second term is higher under the dominant distribution, P, but the first term is higher under the dominated distribution, Q. The relative size of the weight δ (which is an empirical question) can then be used to see which of the two terms is relatively larger. In any case, without the developments in FS dominance that we have outlined in this paper, one cannot make progress in answering these questions.

7.3. Some other models of social preferences

Saito (2013) develops a model of FS-preferences but under uncertainty. It would be interesting to extend the dominance concepts of this paper to Saito's framework. There is a large number of other models of social preferences that incorporate additional factors such as *intentions-based reciprocity* and *type-based reciprocity*. However, these properly require the use of psychological game theory for formal modelling and lie outside the scope of our paper. Solving this problem will be a challenging theoretical task for future work.

8. Conclusions

Borrowing from welfare economics, classical first and second order stochastic dominance were developed under an ex-ante perspective in which individuals are self-regarding, use expected utility theory, and choose among income distributions behind a veil of ignorance. These concepts play a critical role in microeconomic models when self-regarding preferences are assumed. The usefulness of these concepts is evidenced by the fact that there is hardly any branch of economics where they are not used.

There is now enormous and growing evidence on other-regarding preferences in economics and the other social sciences. Thus, it becomes critical to develop the analogous dominance concepts for these preferences. This is the main value-added of our paper.

In this paper, we consider choices among alternative income distributions when individuals have *inequity-averse preferences*. Our main focus is on the inequity-averse model of Fehr and Schmidt (1999), that we called FS preferences. We use an ex-post perspective in which an individual with FS preferences knows his/her own income and wishes to rank two income distributions over their reference group.

We first show that the classical dominance concepts are not adequate in the case of other–regarding preferences. We then give the analogues of classical first and second order dominance concepts for FS preferences and call these *first and second order FS dominance*, respectively. We also introduce a dominance concept, *weak FS dominance*, that is suited to the linear version of FS preferences, which is popular in applied work. We show that our dominance concepts are also useful in other models of other-regarding preferences. In particular, we show that our concepts also extend to a joint consideration of uncertainty and other-regarding preferences.

We believe that our results can be extended in several directions. A potential extension is to preferences that incorporate type-based and intentions-based reciprocity. In this case, beliefs themselves are endogenous and one needs the specialized machinery of psychological game theory to address these problems. Thus, this is likely to pose a challenging problem for future research. Another possible extension is to a joint consideration of time and other-regarding preferences. However, in this case, we first need an appropriate preference representation before we can make further progress.

9. Appendix: Proofs

We now introduce the 'summation by parts' formula that is the analogue of the 'integration by parts' formula in the case of a continuous income distribution. As in the continuous case, this formula will play a critical role in our derivations.

Lemma 4 *(Summation by parts*[20]*): For any sequences* $\{F_i\}_{i=0}^{i=n}$, $\{G_i\}_{i=0}^{i=n}$,

$$\sum_{i=1}^{i=n} F_i \left(G_i - G_{i-1} \right) = F_n G_n - F_0 G_0 - \sum_{i=0}^{i=n-1} G_i \left(F_{i+1} - F_i \right). \tag{9.1}$$

Proof of Lemma 4: To check that (9.1) is correct, simply expand both sides. ■

[20] Summation by parts can, of course, take other, equivalent forms. But (9.1) is the most useful for us.

Proof of Lemma 1: Applying summation by parts (9.1) with $F = y$ and $G = P$, and using the facts that $P_0 = 0$ and $P_n = 1$, we get:[21]

$$\begin{aligned}
\mu_P &= \sum_{i=1}^{i=n} p_i y_i = \sum_{i=1}^{i=n} y_i \left(P_i - P_{i-1}\right) \text{ (since } p_i = P_i - P_{i-1}\text{)} \\
&= y_n P_n - y_0 P_0 - \sum_{i=0}^{i=n-1} P_i \left(y_{i+1} - y_i\right) \text{ (Using (9.1))} \\
&= y_n - \delta \sum_{i=0}^{i=n-1} P_i \text{ (since } \delta = y_{i+1} - y_i \text{ and } P_0 = 0\text{)} \\
&= y_n - \delta \widetilde{P}_{n-1}.
\end{aligned}$$

Similarly, $\mu_Q = y_n - \delta \widetilde{Q}_{n-1}$. Hence, $\mu_P \lesseqqgtr \mu_Q$ if, and only if, $\widetilde{P}_{n-1} \gtreqqless \widetilde{Q}_{n-1}$. ■

Lemma 5 : *Suppose income levels are equally spaced (Definition 1). Let $u \in \mathbf{u}$ be concave. Then $\Delta_i^2 u \leq 0$, $i = 2, 3, ..., n-1$. If u is strictly concave, then $\Delta_i^2 u < 0$, $i = 2, 3, ..., n-1$.*

Proof of Lemma 5: Immediate from the definitions of concavity and strict concavity. ■

Definition 11 : *Some useful families of functions are the following:*
(a) Step functions. Let $u_j(y) = 0$, if $y \leq y_j$ and $u_j(y) = 1$ for $y > y_j$, $j = 1, 2, ...n$.
(b) Two-piece linear functions. Let $u_j(y) = y$, if $y \leq y_j$ and $u_j(y) = y_j$ for $y > y_j$, $j = 1, 2, ..., n$.

Lemma 6 : *(a) For the step functions in Definition 11a, $\Delta_i u_j = u_j(y_{i+1}) - u_j(y_i) = \delta_{ij}$, $i = 1, 2, ..., n-1$, $j = 1, 2, ..., n$.*[22]
(b) Suppose income levels are equally spaced (Definition 1). Then, for the two-piece linear functions of Definition 11b, $\Delta_i^2 u_j = \Delta_i u_j - \Delta_{i-1} u_j = [u_j(y_{i+1}) - u_j(y_i)] - [u_j(y_i) - u_j(y_{i-1})] = -\delta\delta_{ij}$, $i = 2, 3, ..., n-1$, $j = 1, 2, ..., n$.[23]

Proof of Lemma 6: Immediate from Definitions 1, 2 and 11. ■

Lemma 7 :

$$EU(P) = u(y_n) - \sum_{i=1}^{i=n-1} P_i \Delta_i u \tag{9.2}$$

[21] Note that y_0 is not defined. However, since $P_0 = 0$, it follows that $y_0 P_0 = 0$, whatever real number y_0 is defined to be.
[22] Recall that we have already defined $\delta_{ij} = 1$ if $i = j$ but $\delta_{ij} = 0$ if $i \neq j$
[23] Recall that $\delta = y_{i+1} - y_i$ (Definition 1), while δ_{ij} is the Kronecker-δ.

Proof of Lemma 7: Substitute $P_i - P_{i-1}$ for p_i in (2.1), $i = 1, 2, ..., n$, then use summation by parts (9.1), recalling that $P_0 = 0$, $P_n = 1$ and $u(y_{i+1}) - u(y_i) = \Delta_i u$ (Definition 2), to get the required result. ■

Lemma 8 *: Let $u \in \mathbf{u}$ and $P, Q \in \Pi$. Then*
(a) $EU(P) \geq EU(Q)$ if, and only if, $\sum_{i=1}^{i=n-1} (Q_i - P_i) \Delta_i u \geq 0$.
(b) $EU(P) > EU(Q)$ if, and only if, $\sum_{i=1}^{i=n-1} (Q_i - P_i) \Delta_i u > 0$.

Proof of Lemma 8: Immediate from (9.2). ■

Lemma 9 *: Let $P, Q \in \Pi$. Then*
(a) $EU(P) \geq EU(Q)$ for all $u \in \mathbf{u}$ if, and only if, $P_i \leq Q_i$, $i = 1, 2, ..., n-1$.
(b) $EU(P) \geq EU(Q)$ for all $u \in \mathbf{u}$ and $EU(P) > EU(Q)$ for some $u \in \mathbf{u}$ if, and only if, $P_i \leq Q_i$, $i = 1, 2, ..., n-1$, with strict inequality for some i.

Proof of Lemma 9: (a) Suppose $P_i \leq Q_i$, $i = 1, 2, ..., n-1$. From Lemma 8a we get $EU(P) \geq EU(Q)$ for all $u \in \mathbf{u}$.
Conversely, suppose $EU(P) \geq EU(Q)$ for all $u \in \mathbf{u}$. In particular, for the step function u_j of Definition 11a, using Lemmas 6a and 8a, we get $\sum_{i=1}^{i=n-1} (Q_i - P_i) \delta_{ij} \geq 0$. Hence, $P_j \leq Q_j$, $j = 1, 2, ..., n-1$.

(b) Suppose $EU(P) \geq EU(Q)$ for all $u \in \mathbf{u}$ and $EU(P) > EU(Q)$ for some $u \in \mathbf{u}$. From part (a) we get $P_i \leq Q_i$, $i = 1, 2, ..., n-1$. Suppose $P_i = Q_i$, $i = 1, 2, ..., n-1$. From Lemma 8a we get $EU(P) = EU(Q)$ for all $u \in \mathbf{u}$, which is not the case. Hence, $P_i < Q_i$, for some $i = 1, 2, ..., n-1$.

Conversely, suppose $P_i \leq Q_i$, $i = 1, 2, ..., n-1$, with strict inequality for some i, say, $i = j$. From part (a) we get $EU(P) \geq EU(Q)$ for all $u \in \mathbf{u}$. For the step function u_j of Definition 11a, using Lemmas 6a and 7, we get $EU_j(P) - EU_j(Q) = \sum_{i=1}^{i=n-1} (Q_i - P_i) \delta_{ij} = Q_j - P_j > 0$. ■

Lemma 10 *:*

$$EU(P) = u(y_n) - \widetilde{P}_{n-1} \Delta_{n-1} u + \sum_{i=1}^{i=n-2} \widetilde{P}_i \Delta_{i+1}^2 u \tag{9.3}$$

Proof of Lemma 10: Substitute $\widetilde{P}_i - \widetilde{P}_{i-1}$ for P_i in (9.2), $i = 1, 2, ..., n-1$, then use summation by parts (9.1), recalling that $P_0 = 0$, $P_n = 1$ and $\Delta_{i+1} u - \Delta_i u = \Delta_{i+1}^2 u$ (Definition 2), to get the required result. ■

Lemma 11 : *Suppose incomes are equally spaced (Definition 1). Let* $u \in \mathbf{u}$*. Let* $P, Q \in \Pi$ *have equal means. Then*
(a) $EU(P) \geq EU(Q)$ *if, and only if,* $\sum_{i=1}^{i=n-2} \left(\widetilde{Q}_i - \widetilde{P}_i\right) \Delta_{i+1}^2 u \leq 0$.
(b) $EU(P) > EU(Q)$ *if, and only if,* $\sum_{i=1}^{i=n-2} \left(\widetilde{Q}_i - \widetilde{P}_i\right) \Delta_{i+1}^2 u < 0$.

Proof of Lemma 11: Since P and Q have the same mean, it follows from Lemma 1b that $\widetilde{P}_{n-1} = \widetilde{Q}_{n-1}$. The rest follows from Lemma (10). ■

Lemma 12 : *Suppose incomes are equally spaced (Definition 1). Let* $P, Q \in \Pi$ *have equal means. Then*
(a) $EU(P) \geq EU(Q)$ *for all concave* $u \in \mathbf{u}$ *if, and only if,* $\widetilde{P}_i \leq \widetilde{Q}_i$, $i = 1, 2, ..., n-2$.
(b) $EU(P) \geq EU(Q)$ *for all concave* $u \in \mathbf{u}$ *and* $EU(P) > EU(Q)$ *for some* $u \in \mathbf{u}$ *if, and only if,* $\widetilde{P}_i \leq \widetilde{Q}_i$, $i = 1, 2, ..., n-2$, *with strict inequality for some* i.

Proof of Lemma 12: (a) Suppose $\widetilde{P}_i \leq \widetilde{Q}_i$, $i = 1, 2, ..., n-2$. From Lemmas 5 and 11a we get $EU(P) \geq EU(Q)$ for all concave $u \in \mathbf{u}$.
Conversely, suppose $EU(P) \geq EU(Q)$ for all concave $u \in \mathbf{u}$. In particular, for the two-piece linear function u_j of Definition 11b, using Lemmas 6b and 11a, we get $\sum_{i=1}^{i=n-2} \left(\widetilde{Q}_i - \widetilde{P}_i\right)(-\delta\delta_{ij}) \leq 0$. Hence, $\widetilde{P}_j \leq \widetilde{Q}_j$, $j = 1, 2, ..., n-2$.

(b) Suppose $EU(P) \geq EU(Q)$ for all concave $u \in \mathbf{u}$ and $EU(P) > EU(Q)$ for some $u \in \mathbf{u}$. From part (a) we get $\widetilde{P}_i \leq \widetilde{Q}_i$, $i = 1, 2, ..., n-2$. Suppose $\widetilde{P}_i = \widetilde{Q}_i$, $i = 1, 2, ..., n-2$. From Lemma 11a we get $EU(P) = EU(Q)$ for all $u \in \mathbf{u}$, which is not the case. Hence, $\widetilde{P}_i < \widetilde{Q}_i$, for some $i = 1, 2, ..., n-1$.

Conversely, suppose $\widetilde{P}_i \leq \widetilde{Q}_i$, $i = 1, 2, ..., n-2$, with strict inequality for some i, say, $i = j$. From part (a) we get $EU(P) \geq EU(Q)$ for all concave $u \in \mathbf{u}$. For the two-piece linear function u_j of Definition 11b, using Lemmas 1b, 6b and 10, we get $EU_j(P) - EU_j(Q) = \sum_{i=1}^{i=n-2} \left(\widetilde{P}_i - \widetilde{Q}_i\right)(-\delta\delta_{ij}) = -\delta\left(\widetilde{P}_j - \widetilde{Q}_j\right) > 0$. ■

Proof of Proposition 1: Follows from Lemma 9. ■

Proof of Proposition 2: Follows from Lemma 12. ■

Proof of Corollary 1: Immediate from Propositions 1 and 2. ■

Proof of Lemma 2: Follows from Definition 6 by collecting terms corresponding to each $u(y_i)$, $i = 1, 2, ..., n$. ■

Proof of Lemma 3: We give an outline of the proof.

(a) (3.6) follows from (3.1).

(b) From (3.4) and (3.5) we get

$$U(y_j, P) = [1 - \beta P_{j-1} + \alpha(1 - P_j)] u(y_j) + \beta \sum_{i=1}^{j-1} p_i u(y_i) - \alpha \sum_{k=j+1}^{n} p_k u(y_k). \tag{9.4}$$

Replace p_i with $P_i - P_{i-1}$ and likewise for p_k. Apply summation by parts (9.1) to $\sum_{i=1}^{j-1}(P_i - P_{i-1})\,u(y_i)$ to get $P_{j-1}u(y_{j-1}) - \sum_{i=1}^{j-2} P_i\Delta_i u$ (recall that $P_0 = 0$). Likewise, $\sum_{k=j+1}^{n} p_k u(y_k) = u(y_n) - P_j u(y_j) - \sum_{k=j}^{n-1} P_k\Delta_k u$ (recall that $P_n = 1$). Substitute in (9.4) to get

$$U(y_j, P) = u(y_j) + \alpha u(y_j) - \alpha u(y_n) - \beta \sum_{i=1}^{i=j-1} P_i\Delta_i u + \alpha \sum_{k=j}^{k=n-1} P_k\Delta_k u$$

Do the same for $U(y_j, Q)$. Subtract, to get the required result.

(c) Replace P_i in (3.7) by $\widetilde{P}_i - \widetilde{P}_{i-1}$. Do likewise for P_k, Q_i and Q_k. Collect terms. Apply (9.1) to $\sum_{i=0}^{j-1}\left[\left(\widetilde{Q}_i - \widetilde{P}_i\right) - \left(\widetilde{Q}_{i-1} - \widetilde{P}_{i-1}\right)\right]\Delta_i u$ with $F_i = \Delta_i u$, $G_i = \widetilde{Q}_i - \widetilde{P}_i$, $G_{i-1} = \widetilde{Q}_{i-1} - \widetilde{P}_{i-1}$, and recalling that $\widetilde{P}_0 = \widetilde{Q}_0 = 0$. Repeat these steps for the term $\sum_{k=j}^{n-1}\left[\left(\widetilde{P}_k - \widetilde{Q}_k\right) - \left(\widetilde{P}_{k-1} - \widetilde{Q}_{k-1}\right)\right]\Delta_k u$. Substitute back, collect terms and simplify to get the required result. ■

Lemma 13 : *In (3.7) of Lemma 3b, set $\alpha = 1$, $\beta = 0.5$ and take u to be the step function u_h (Definition 11a). Then*

$$U(y_j, P) - U(y_j, Q) = \frac{1}{2}\sum_{i=1}^{i=j-1}(Q_i - P_i)\,\delta_{ih} + \sum_{k=j}^{k=n-1}(P_k - Q_k)\,\delta_{kh}.$$

Proof of Lemma 13: Follows from Lemma 6a. ■

Lemma 14 : *Let $P, Q \in \Pi$. Suppose $\widetilde{Q}_{j-1} = \widetilde{P}_{j-1}$. Then*

$$U(y_j, P) - U(y_j, Q) = \alpha\left(\widetilde{P}_{n-1} - \widetilde{Q}_{n-1}\right)\Delta_{n-1}u$$
$$+\beta\sum_{i=1}^{i=j-2}\left(\widetilde{P}_i - \widetilde{Q}_i\right)\Delta_{i+1}^2 u + \alpha\sum_{k=j}^{k=n-2}\left(\widetilde{Q}_k - \widetilde{P}_k\right)\Delta_{k+1}^2 u.$$

Proof of Lemma 14: Follows from Lemma 3c. ■

Lemma 15 : *Suppose $U(y_j, P) = U(y_j, Q)$ for all $\alpha \geq 0$, all $\beta \in [0,1)$ and all $u \in \mathbf{u}$. Then $P_i = Q_i$ for each $i = 1, 2, ..., n-1$.*

Proof of Lemma 15: From Lemma 3b, we get $\beta\sum_{i=1}^{i=j-1}(Q_i - P_i)\,\Delta_i u + \alpha\sum_{k=j}^{k=n-1}(P_k - Q_k)\,\Delta_k u = 0$. Set $\alpha = 1$, $\beta = 0.5$ and take u to be the step function u_h of Definition 11a. Lemma 6a then gives $0.5\sum_{i=1}^{i=j-1}(Q_i - P_i)\,\delta_{ih} + \sum_{k=j}^{k=n-1}(P_k - Q_k)\,\delta_{kh} = 0$. From this we get $P_h = Q_h$, $h = 1, 2, ..., n-1$. ■

Proof of Proposition 3: (a) Suppose $P \succeq_{FS1_{y_j}} Q$. Let $y_j \in [0,1]$, $u \in \mathbf{u}$, $\alpha \geq 0$ and $\beta \in [0,1)$. From Lemma 3b and Definition 7, it follows that $U(y_j, P) \geq U(y_j, Q)$.

Conversely, assume that $U(y_j, P) \geq U(y_j, Q)$ for all $u \in \mathbf{u}$, $\alpha \geq 0$ and $\beta \in [0, 1)$. From Lemma 13 we get $P_h \leq Q_h$ for $h = 1, 2, ..., j-1$ and $P_h \geq Q_h$ for $h = j, j+1, ..., n-1$.

(b) From part (a), it follows that $P \succeq_{FS1_y} Q$. Hence, $P_i \leq Q_i$ for each $i = 1, 2, ..., j-1$ and $P_k \geq Q_k$ for each $k = j, j+1, ..., n-1$. Hence, if $P \succ_{FS1_y} Q$ did not hold, then we would have $P_i = Q_i$ for each $i = 1, 2, ..., j-1$ and $P_k = Q_k$ for each $k = j, j+1, ..., n-1$. From Lemma 3b we would then get $U(y_j, P) = U(y_j, Q)$ for all $u \in \mathbf{u}$, $\alpha \geq 0$ and $\beta \in [0, 1)$, contrary to assumption. Hence, $P \succ_{FS1_y} Q$.

Conversely, suppose $P \succ_{FS1_y} Q$. From Definition 7, $P \succeq_{FS1_{y_j}} Q$. Hence, from part (a), $U(y_j, P) \geq U(y_j, Q)$ for all $u \in \mathbf{u}$, $\alpha \geq 0$ and $\beta \in [0, 1)$. Since $P \succ_{FS1_y} Q$ we must have $P_i < Q_i$ for some $i = 1, 2, ..., j-1$, or $P_k > Q_k$ for some $k = j, j+1, ..., n-1$ (Definition 7). In either case Lemma 13 gives $U(y_j, P) > U(y_j, Q)$. ■

Proof of Corollary 2: If either (i) or (ii) holds, then it follows from Lemma 3b that $U(y_j, P) > U(y_j, Q)$. ■

Proof of Proposition 4: (a) Let P second order FS$_{y_j}$ dominate Q ($P \succeq_{FS2_{y_j}} Q$). Then, from Definitions 1, 2, 8 and Lemmas 5, 3c we get that $U(y_j, P) \geq U(y_j, Q)$ for all $\alpha \geq 0$, all $\beta \in [0, 1)$ and all concave $u \in \mathbf{u}$.

Conversely, suppose $U(y_j, P) \geq U(y_j, Q)$ for all $\alpha \geq 0$, all $\beta \in [0, 1)$ and all concave $u \in \mathbf{u}$. In addition, suppose $\widetilde{Q}_{j-1} = \widetilde{P}_{j-1}$. We need to show that $\widetilde{P}_{n-1} \geq \widetilde{Q}_{n-1}$, $\widetilde{P}_h \leq \widetilde{Q}_h$ for each $h = 1, 2, ..., j-2$ and $\widetilde{P}_h \geq \widetilde{Q}_h$ for each $h = j, j+1, ..., n-2$. By assumption, $\widetilde{Q}_{j-1} = \widetilde{P}_{j-1}$. The other cases are considered below.

1. $\widetilde{P}_{n-1} \geq \widetilde{Q}_{n-1}$. *Proof*: Consider the utility function $u(y) = y$, which is clearly concave. Since incomes are equally spaced (Definition 1), we have $y_{i+1} - y_i = \delta > 0$ for $i = 1, 2, ..., n-1$. Hence, $\Delta_{n-1} u = \delta > 0$ and $\Delta^2_{i+1} u = \Delta^2_{k+1} u = 0$. Set $\alpha = 1$. Hence, Lemma 14 gives $U(y_j, P) - U(y_j, Q) = \left(\widetilde{P}_{n-1} - \widetilde{Q}_{n-1}\right)\delta$. Since $U(y_j, P) \geq U(y_j, Q)$, it follows that $\widetilde{P}_{n-1} \geq \widetilde{Q}_{n-1}$.

2. $\widetilde{P}_h \leq \widetilde{Q}_h$ for each $h = 1, 2, ..., j-2$ and $\widetilde{P}_h \geq \widetilde{Q}_h$ for each $h = j, j+1, ..., n-2$. *Proof*: Set $\alpha = 1$ and $\beta = 0.5$ in Lemma 14. Take u to be the two-piece linear function u_{h+1} of Definition 11b. By Definitions 2 and 11b, $\Delta_{n-1} u_{n-1} = u_{n-1}(y_n) - u_{n-1}(y_{n-1}) = y_{n-1} - y_{n-1} = 0$. For $h \leq n-2$, Definitions 2 and 11b give $\Delta_{n-1} u_{h+1} = 0$. For $h = 1, 2, ..., j-2$, Lemmas 6b and 14 give $0 \leq U(y_j, P) - U(y_j, Q) = -0.5\delta\left(\widetilde{P}_h - \widetilde{Q}_h\right)$ and, hence, $\widetilde{P}_h \leq \widetilde{Q}_h$. For $h = j, j+1, ..., n-2$, Lemmas 6b and 14 give $0 \leq U(y_j, P) - U(y_j, Q) = -\delta\left(\widetilde{Q}_h - \widetilde{P}_h\right)$ and, hence, $\widetilde{Q}_h \leq \widetilde{P}_h$.
(b) Suppose that P strictly second order FS$_{y_j}$ dominates Q ($P \succ_{FS2_{y_j}} Q$). Let $\alpha > 0$, $\beta > 0$ and let $u \in \mathbf{u}$ be strictly increasing and strictly concave. It then follows from Definition 8 and Lemmas 5, 3c that $U(y_j, P) > U(y_j, Q)$.

Conversely, suppose that $U(y_j, P) \geq U(y_j, Q)$ for all $\alpha \geq 0$, all $\beta \in [0, 1)$ and all concave $u \in \mathbf{u}$. Also, suppose that $U(y_j, P) > U(y_j, Q)$ for some $\alpha \geq 0$, some $\beta \in [0, 1)$ and some $u \in \mathbf{u}$ (not necessarily concave). In addition, assume that $\widetilde{P}_{j-1} = \widetilde{Q}_{j-1}$. Then, from part (a), it follows that $P \succeq_{FS2_{y_j}} Q$. Suppose that $P \not\succ_{FS2_{y_j}} Q$. It then follows from Definition 8 that $\widetilde{P}_i = \widetilde{Q}_i$ for each $i = 1, 2, ..., n-1$. Hence, from Lemma 14 we would get $U(y_j, P) = U(y_j, Q)$ for all $\alpha \geq 0$, all $\beta \in [0, 1)$ and all $u \in \mathbf{u}$. This is not the case. Hence, $P \succ_{FS2_{y_j}} Q$. ■

Proof of Corollary 3: (a) follows from Propositions 3a and 4a.

(b) Since $P \succ_{FS1_{y_j}}$, it follows, from Definition 7, that $P \succeq_{FS1_{y_j}} Q$. Hence, from part (a), $P \succeq_{FS2_{y_j}} Q$. Suppose that $P \not\succ_{FS2_{y_j}} Q$. It then follows from Definition 8 that $\widetilde{P}_i = \widetilde{Q}_i$ for each $i = 1, 2, ..., n-1$. Hence, from Lemma 14 we would get $U(y_j, P) = U(y_j, Q)$ for all $\alpha \geq 0$, all $\beta \in [0, 1)$ and all $u \in \mathbf{u}$. Lemma 15 then gives $P_i = Q_i$ for $i = 1, 2, ..., n-1$. This is not the case since $P \succ_{FS1_{y_j}} Q$ (Definition 7). Hence, $P \succ_{FS2_{y_j}} Q$. ■

Proof of Proposition 5: Follows from Definitions 9 and 10. ■

Proof of Proposition 6: (a) Let $y_j \in \mathbf{Y}$ and $P, Q \in \Pi$. Suppose P first order FS_{y_j} dominates Q. From Proposition 3a it then follows that $U(y_j, P) \geq U(y_j, Q)$ for all $u \in \mathbf{u}$, $\alpha \geq 0$ and $\beta \in [0, 1)$. In particular, it follows that $U(y_j, P) \geq U(y_j, Q)$ for $u(y) = y$, all $\alpha \geq 0$ and all $\beta \in [0, 1)$. From Proposition 5 it then follows that $P \succeq_{WFS_{y_j}} Q$.

(b) Let $y_j \in \mathbf{Y}$ and $P, Q \in \Pi$. Suppose P strictly first order FS_{y_j} dominates Q. Hence, P first order FS_{y_j} dominates Q. From Part (a) it follows that P weakly FS_{y_j} dominates Q. From Proposition 5 it follows that for $u(y) = y$, $U(y_j, Q) \leq U(y_j, P)$ for all $\alpha \geq 0$ and all $\beta \in [0, 1)$. Since P strictly first order FS_{y_j} dominates Q, it also follows that (a) and (b) of Definition 7 hold, with one of them being strict. If (a) is strict, choose $\beta = \frac{1}{2}$. Take $u(y) = y$, then $u(y_i) = y_i < y_{i+1} = u(y_{i+1})$. Hence, (i) of Corollary 2 holds and, hence, $U(y_j, P) > U(y_j, Q)$. If (b) of Definition 7 is strict, choose $\alpha = 1$. Take $u(y) = y$, then $u(y_k) = y_k < y_{k+1} = u(y_{k+1})$. Hence, (ii) of Corollary 2 holds and, hence, again, $U(y_j, P) > U(y_j, Q)$. From Proposition 5 it then follows that P strictly weakly FS_{y_j} dominates Q.

(c) Let $P, Q \in \Pi$, $y_j \in \mathbf{Y}$. Suppose incomes are equally spaced (Definition 1). Let P second order FS_{y_j} dominate Q ($P \succeq_{FS2_{y_j}} Q$). From Proposition 4a it then follows that $U(y_j, P) \geq U(y_j, Q)$ for all $\alpha \geq 0$, all $\beta \in [0, 1)$ and all concave u. In particular, it follows that $U(y_j, P) \geq U(y_j, Q)$ for $u(y) = y$, all $\alpha \geq 0$ and all $\beta \in [0, 1)$. From Proposition 5 it then follows that $P \succeq_{WFS_{y_j}} Q$. ■

Lemma 16 : *In (3.7) of Lemma 3b, set $\alpha = 1$, $\beta = -1$ and take u to be the step function u_h (Definition 11a). Then*

$$U(y_j, P) - U(y_j, Q) = \sum_{i=1}^{i=n-1} (P_i - Q_i)\, \delta_{ih},$$

where $\delta_{ih} = 1$ if $i = h$ and $\delta_{ih} = 0$ if $i \neq h$.

Proof of Lemma 16: Follows from Lemmas 6a. ■

Proof of Proposition 7: (a) Suppose $P \succeq_1 Q$. Let $y_j \in [0,1]$, $u \in \mathbf{u}$, $\alpha \geq 0$ and $\beta \leq 0$. From Lemma 3b and Definition 4, it follows that $U(y_j, P) \leq U(y_j, Q)$.

Conversely, assume that $U(y_j, P) \leq U(y_j, Q)$ for all $u \in \mathbf{u}$, $\alpha \geq 0$ and $\beta \leq 0$. From Lemma 16 we get $P_h \leq Q_h$ for $h = 1, 2, ..., n-1$. Hence, $P \succeq_1 Q$ (Definition 4).

(b) Suppose that P strictly first order dominates Q ($P \succ_1 Q$). From part (a), it follows that $P \succeq_1 Q$. Hence, $P_i \leq Q_i$ for each $i = 1, 2, ..., n-1$ (Definition 4). Hence, if $P \succ_1 Q$ did not hold, then we would have $P_i = Q_i$ for each $i = 1, 2, ..., n-1$. From Lemma 3b we would then get $U(y_j, P) = U(y_j, Q)$ for all $u \in \mathbf{u}$, $\alpha \geq 0$ and $\beta \leq 0$, contrary to assumption. Hence, $P \succ_1 Q$.

Conversely, suppose $P \succ_1 Q$. From Definition 4, $P \succeq_1 Q$. Hence, from part (a), $U(y_j, P) \leq U(y_j, Q)$ for all $u \in \mathbf{u}$, $\alpha \geq 0$ and $\beta \leq 0$. Since $P \succ_1 Q$, we must have $P_i \leq Q_i$ for all $i = 1, 2, ..., n-1$, with strict inequality for at least one i (Definition 4). Then Lemma 16 gives $U(y_j, P) < U(y_j, Q)$. ■

Proof of Corollary 4: Since $P \succeq_1 Q$, it follows from Definition 4 that $P_h \leq Q_h$ for $h = 1, 2, ..., n-1$. If either (i) or (ii) holds, then it follows from Lemma 3b that $U(y_j, P) < U(y_j, Q)$. ■

Proof of Proposition 8: (a) Let P second order dominate Q ($P \succeq_2 Q$), then it follows from Definitions1, 2, 5 and Lemmas 5, 3c that $U(y_j, P) \leq U(y_j, Q)$ for all $\alpha \geq 0$, all $\beta \leq 0$ and all concave $u \in \mathbf{u}$.

Conversely, suppose that $U(y_j, P) \leq U(y_j, Q)$ for all $\alpha \geq 0$, all $\beta \leq 0$ and all concave $u \in \mathbf{u}$. By assumption, $\widetilde{Q}_{j-1} = \widetilde{P}_{j-1}$. The other cases are considered below.

1. $\widetilde{P}_{n-1} \leq \widetilde{Q}_{n-1}$. *Proof*: Consider the utility function $u(y) = y$, which is clearly concave. Since incomes are equally spaced (Definition 1), we have $y_{i+1} - y_i = \delta > 0$ for $i = 1, 2, ..., n-1$. Hence, $\Delta_{n-1} u = \delta > 0$ and $\Delta_{i+1}^2 u = \Delta_{k+1}^2 u = 0$. Set $\alpha = 1$. Hence, Lemma 14 gives $U(y_j, P) - U(y_j, Q) = \left(\widetilde{P}_{n-1} - \widetilde{Q}_{n-1}\right)\delta$. Since $U(y_j, P) \geq U(y_j, Q)$, it follows that $\widetilde{P}_{n-1} \geq \widetilde{Q}_{n-1}$.

2. $\widetilde{P}_h \leq \widetilde{Q}_h$ for each $h = 1, 2, ..., j-2$ and $\widetilde{P}_h \leq \widetilde{Q}_h$ for each $h = j, j+1, ..., n-2$. *Proof*: Set $\alpha = 1$ and $\beta = -1$ in Lemma 14. Take u to be the two-piece linear function u_{h+1} of Definition 11b. By Definitions 2 and 11b, $\Delta_{n-1} u_{n-1} = u_{n-1}(y_n) - u_{n-1}(y_{n-1}) = y_{n-1} - y_{n-1} = 0$. For $h \leq n-2$, Definitions 2 and 11b give $\Delta_{n-1} u_{h+1} = 0$. For $h = 1, 2, ..., j-2$, Lemmas 6b and 14 give $0 \geq U(y_j, P) - U(y_j, Q) = \delta\left(\widetilde{P}_h - \widetilde{Q}_h\right)$ and, hence, $\widetilde{P}_h \leq \widetilde{Q}_h$. For $h = j, j+1, ..., n-2$, Lemmas 6b and 14 give $0 \geq U(y_j, P) - U(y_j, Q) = -\delta\left(\widetilde{Q}_h - \widetilde{P}_h\right)$ and, hence, $\widetilde{Q}_h \geq \widetilde{P}_h$.

(b) Suppose that P strictly second order dominates Q ($P \succ_2 Q$). Let $\alpha > 0$, $\beta < 0$ and let $u \in \mathbf{u}$ be strictly increasing and strictly concave. From Definition 5 and Lemmas 5, 3c it follows that $U(y_j, P) < U(y_j, Q)$.

Conversely, suppose that $U(y_j, P) \leq U(y_j, Q)$ for all $\alpha \geq 0$, all $\beta \leq 0$ and all concave $u \in \mathbf{u}$. Suppose that, in addition, $U(y_j, P) < U(y_j, Q)$ for some $\alpha \geq 0$, some $\beta \leq 0$ and some $u \in \mathbf{u}$ (not necessarily concave). From part (a), it follows that $P \succeq_2 Q$. Suppose that $P \nsucc_2 Q$. It then follows from Definition 5 that $\widetilde{P}_i = \widetilde{Q}_i$ for each $i = 1, 2, ..., n-1$. Hence, from Lemma 14 we would get $U(y_j, P) = U(y_j, Q)$ for all $\alpha \geq 0$, all $\beta \leq 0$ and all $u \in \mathbf{u}$. This is not the case. Hence, $P \succ_2 Q$. ■

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