

The Political Economy of Higher Education Admission Standards and Participation Gap



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Abstract

We build a political economy model in order to shed light on the empirically observed simultaneous increase in university size and participation gap. Parents differ in income and in the ability of their unique child. They vote over the minimum ability level required to attend public universities, which are tuition-free and financed by proportional income taxation. Parents can invest in private tutoring to help their child pass the admission test. A university participation gap emerges endogenously with richer parents investing more in tutoring. A unique majority voting equilibrium exists, which can be either classical or “ends-against-the-middle” (in which case parents of both low- and high-ability children favor a smaller university). Four factors increase the university size (larger skill premium enjoyed by university graduates, smaller tutoring costs, smaller university cost per student, larger minimum ability of students), but only the former two also increase the participation gap. A more unequal parental income distribution also increases the participation gap, but barely affects the university size.

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1 Introduction

The second half of the XXth Century has witnessed a large expansion of higher education, with the US leading the way with the G.I. Bill (1944), and other developed countries gradually following suit. By 1970, the global enrollment rate in universities was about 10 per cent, and it reached 20 per cent by the end of the century (Schofer and Meyer, 2005). In recent decades, the process has extended to most middle-income countries, including the BRIC (Brazil, Russia, India and China), and to a significant number of low-income ones. Currently, about one third of the world’s college age population participate in higher education (Marginson, 2016).

While greater equality of opportunity has often been one motivation to increase university size (see for instance the 1963 Robbins Report of the Committee on Higher Education in the UK), this massive expansion has often not been accompanied by a reduction in inequality of access. For instance, in the UK, studies have consistently found that better-off youths disproportionately benefited from the expansion (even though university education was tuition-free until 1997), so that participation gaps according to parental income actually grew instead of shrinking (Blanden and Machin, 2004; Galindo-Rueda et al., 2004; Machin and Vignoles, 2004; Blanden et al. 2005). This persistence (or even aggravation) of educational inequality across generations despite the expansion of higher education has also been documented in many countries where universities are basically tuition-free,¹ as well as in countries with high tuition fees like the US or Australia (e.g. Cameron and Heckman, 2001; Acemoglu and Pischke, 2001; Cardak and Ryan, 2009), and in the BRIC countries (Carnoy et al., 2012, 2013).

¹Vona (2012) investigates the educational outcomes of four cohorts born between 1940 and 1980, across twelve European countries and finds that the expansion of higher education “brought about an increase in background-related inequality”. Di Paolo (2012) studies the evolution of social disparities in access to higher education in Italy and Spain between the 1940 and 1980 cohorts, and obtains that the expansion disproportionately benefited better-off youths. These results are consistent with those obtained by Rahona-López (2009) for Spain, and by Checchi and his co-authors (Bratti et al. 2012, Checchi et al. 2013) for Italy.

In this paper, we build a simple and tractable political economy model with the objective of shedding light on the stylized fact that the expansion of higher education has not been accompanied by a decrease in the participation gap. The key ingredients of this model are as follows. Parents differ in income w and in the ability of their unique child, θ . Children can either attend a vocational program and become low-skilled, or attend university and become high-skilled. Their future wage is the product of their ability and of the reference wage of their skill level. The skill premium –difference in reference wage across the two skill levels– depends on the relative supply of each type of labor, and is thus a function of the ability threshold democratically chosen.² University is financed with an income tax on the whole (parents) population.

Parents first vote over the minimum ability requirement θ_u to access university. They then choose whether and how much to invest in costly tutoring for their child. Tutoring allows children to perform better in the university admission test, but does not increase productivity permanently.³ Children then take the test, and join universities in the case they pass the test. The resulting skill mix determines equilibrium wages.

Tutoring is an important ingredient in explaining access to higher education for at least three reasons: (i) it is frequent throughout both developed and developing countries;⁴ (ii)

²This assumption accords well with basic intuition and with state-of-the-art models of the labor market (e.g. Acemoglu, 2003; Acemoglu and Autor, 2011; Carneiro and Lee, 2011).

³This assumption is in line with the empirical literature. Although high quality empirical evidence strongly suggests the existence of positive *short-run* effects of tutoring in academic achievement (e.g. Lavy and Schlosser (2005), Banerjee et al (2007), Jacob and Lefgren (2004)), the latter two studies also find that this effect quickly fades away. We have not found empirical evidence of long term labor market effects. Our assumption is in line with Bray (2011)’s statement that, in the EU, “much tutoring is of low pedagogic value. It teaches to the test and is dominated by past examination papers, tips on likely questions, and strategies for answering questions within the time constraints”. It is also in accordance with the stated purpose of much tutoring. For instance, the web page of a major private tutoring firm in France (www.academia.fr) mentions prominently that students resorting to the firm increase their test scores by 3.2 points on average. Finally, we note that according to Kirby (2016), well over half the students (aged 11-16) taking an Ipsos MORI poll in the UK said they received private tuition “to help me do well in a specific test or exam”.

⁴Aurini et al. (2013) document “the global intensification of supplementary education”. For instance,

it is especially prevalent in upper secondary years to prepare for university entrance exams (Bray, 2013); (iii) its expansion is paralleling that of higher education.⁵ Observe also that we have made access to university easy to children of low income parents by assuming that there is no correlation between parental income w and child’s ability θ , by assuming that university is tuition-free and accessible based on ability,⁶ and by assuming away opportunity costs of going to university in terms of foregone labor market income.

Solving the model, we first establish that richer parents – who face a lower utility cost of tutoring expenses– are willing to pay more in order to raise their children’s signal of ability, as measured during the test, to θ_u .⁷ This result generates a participation gap, since for any given θ_u the fraction of students attending university increases with parental income w .⁸

We then prove existence of a unique majority voting equilibrium that can be of two

in the UK in 2000, 30% of year 13 students had some tutoring prior to A-levels exams that determine access to university (Ireson and Rushforth (2005)). In Canada, 8% to 20% of students aged 16 had some tutoring in 1999 (CME(2000)), and the number of formal tutoring businesses in major Canadian cities has grown between 200% and 500%, according to Aurini and Davies (2004). In Hong Kong, 70% of secondary year 6-7 students received tutoring in 1996-1998 (Bray and Kwok, 2003). In the US, in 1990-1992, 14%-21% of high school students took special courses at high school to prepare for the SAT or ACT, 8%-14% took group private tutoring (commercial coaching classes) and 6-8% took one-to-one private tutoring (Briggs, 2001).

⁵A detailed description of this phenomenon can be found in Bray (1999, 2009). Dang and Rogers (2008) reviews the empirical evidence.

⁶See De Donder and Martinez-Mora (2015) for a long list of countries where access to universities is organized this way.

⁷This result is in accordance with stylized facts. In the UK, Kirby (2016) shows that the proportion of state-school students who have ever received private tutoring is 30% for richer families but only 15% for poorer families. Referring to the EU, Bray (2011) argues that “If left to market forces, tutoring maintains and exacerbates inequalities. Families with higher income can afford both greater quantities and better qualities of tutoring”.

⁸We would observe a similar result if the tutoring technology were to increase permanently children’s ability. In that sense, the strong assumption that tutoring has no long term effect on ability, which allows to simplify the model, is not crucial to our results.

types.⁹ In a *classical* equilibrium, the half population who most prefer a higher-than-equilibrium value of θ_u is composed of parents of high ability children who favor a smaller university (i) to boost the high-skilled wage of their child, by restricting the supply of future high-skilled workers, and (ii) to decrease the tax cost of university. The other half of the polity wants a larger university, either (i) to enrol their child at university or (ii) to boost the vocational wage of their child by restricting the supply of future low-skilled workers. In an *end-against-the-middle* equilibrium (à la Epple and Romano (1996)), the half of the population preferring a smaller-than-equilibrium university size is made of two groups: the same group as in the classical equilibrium above, plus parents whose children have very low ability. This latter group wants a higher-than-equilibrium value of θ_u in order to decrease the tax cost of university, and pays little attention to the impact on the unskilled wage of a smaller university because of the low ability of their child. Another difference between the two types of equilibria is that, while a strict majority of students attend university in a classical equilibrium, this is not necessarily the case with an ends-against-the-middle equilibrium.

We next identify which factors may explain the empirically observed increase in university size and participation gap. Although we prove some analytical results concerning the university size and type of equilibrium, assessing how the participation gap is affected requires resorting to numerical examples. We obtain that raising exogenously the skill premium for any skill mix increases both the university size and participation gap: the larger skill premium increases the returns to both higher education and private tutoring, benefiting especially richer households who have a lower marginal utility cost of investment. We therefore identify a mechanism by which the rise in the skill premium observed since the late 1970s (e.g. Goos and Manning (2007) for the UK; OECD, 2011) could have harmed equality of opportunity while increasing university size. A reduction in the cost of tutoring also generates a simultaneous increase in university size and participation gap.

⁹The voting problem we consider has an inherent theoretical interest, since individual preferences are neither single-peaked nor single-crossing (à la Gans and Smart 1996) in θ_u , because of the switch from vocational schooling to university when a threshold value of θ_u is attained.

We also study the impact of two other factors which correspond to empirical observations: a decrease in the unit cost of university (spending per student in the UK has been halved in the last two decades of the past century according to Greenaway and Haynes, 2003) and a larger minimum ability of children (as a consequence of the expansion of secondary education and increase in the minimum school leaving age). Both factors increase university size but decrease the participation gap. Finally, a rise in household income inequality (such as the one documented by Goos and Manning (2007) since the 1970s) increases the participation gap, but it barely affects university size.

Our paper belongs to a relatively small but growing literature studying access to higher education and its financing. A large strand of that literature compares the impact of fees and of various subsidization policies. Fernández and Rogerson (1995) study voting over the size of a tax-financed subsidy and obtain that the political equilibrium subsidy level is not large enough to allow poor students to access higher education. Garcia-Peñalosa and Wälde (2000) compare the efficiency and equity effects of a traditional tax-subsidy scheme, a graduate tax and loans, and obtain that the latter two fare better than the former. Del Rey and Racionero (2012, 2014) analyze the political support for, and the efficiency and equity properties of, income-contingent loans. Borck and Wimbersky (2014) study numerically majority voting over a traditional subsidy scheme, a pure loan scheme, income contingent loans and graduate taxes. Surprisingly, they find that the poor favor the subsidy scheme, even though they pay part of its tax cost.¹⁰

Two papers study admission tests either together with, or instead of, (subsidized) tuition fees. Gary-Bobo and Trannoy (2008) study the socially optimal examination-

¹⁰Other important contributions in the area (e.g. Epple et al. 2006 and 2016; De Fraja and Valbonesi, 2012; Haupt, 2012; or Fu, 2014) are less closely related to this paper. Ichino et al (2011) develop a dynamic political economy model to study the political determinants of the intergenerational elasticity of income. They model education policy in a reduced form, as a parameter of the dynastic production function. A more progressive education policy reduces inequality by increasing the income of less talented individuals while decreasing that of more talented ones. Education policy thus distorts incentives to privately invest in the children's human capital, so that a lower correlation between father's and son's income may imply more inefficiency.

cum-fees policy. They assume that students observe only a private, noisy signal of their ability, and that universities can condition admission decisions on the results of noisy tests. Tests are part of the optimal policy provided that their results are not public knowledge. The paper most closely related to ours is De Fraja (2001). As in our model, parents differ in income and in the ability of their child and face a binary educational choice but, unlike here, universities charge fees to students, and the future income of children is random and determined only by their own education decision (assuming away general equilibrium labor market effects). Hence, the participation gap occurs because better-off parents are more willing to take the financial risk of enrolling their child at university. Our paper then extends the work of De Fraja (2001) in four directions: (i) we model general equilibrium labor market effects, so that the decision to attend university by an additional agent exerts an externality on others by lowering the skill premium; (ii) we study majority voting over the admission test level in the presence of (full) subsidy of fees; (iii) we allow for parental investments in tutoring; and (iv) we apply our framework to explain the stylized fact outlined above.

The remainder of the paper is organized as follows: after presenting the model in section 2, we solve it by backward induction, starting with the private tutoring decision in section 3. We then describe parents' preferences over the admission ability threshold in section 4. Existence of a majority voting equilibrium is studied in section 5. Section 6 provides a formal comparative statics analysis of the majority chosen university size and type of equilibrium. Section 7 uses numerical examples to shed light on the impact of various factors on the equilibrium university size and participation gap. Section 8 concludes.

2 The model

We model a static economy with a continuum of households of mass one. Parents differ in their (exogenous) income w and in the ability of their only child θ . We assume that income is distributed over $[\underline{w}, \bar{w}]$ according to the cdf $G(w)$ while ability is distributed over $[\underline{\theta}, \bar{\theta}]$

according to the cdf $F(\theta)$, so that income and ability are independently distributed.¹¹ Both distributions have full support. While the smallest conceivable ability may tend toward zero, the smallest ability level actually observed in the economy is $\underline{\theta}$. We denote by θ_{med} the median value of θ , and by Ew the average value of w . With a slight abuse of language, we denote by (w, θ) the type of the parent.

The (binary) skill level j of children is determined by education. Children who go to a vocational school ($j = V$) become low-skilled, while those who go to university ($j = u$) become high-skilled. A child's ability θ is known to her parent but not to the government, which must perform a test to elicit it. Access to university is rationed by the results of this admission test. We denote by θ_u the minimum level of the test mark required to be admitted to a university and to become high-skilled.

Parents can make a private tutoring investment in their child prior to the taking of the test, in order to boost her test mark. This investment is costly to the parents, and does not generate any lasting impact on the child's ability, beyond the improvement of her test mark for university entry. There is no uncertainty as to the result of the test. A student of ability θ who does not receive additional parental investment obtains a mark equal to her own ability. If a parent decides to invest privately, he will invest the minimum amount necessary for his child's mark to reach the threshold for university attendance. We denote with the function $p(\theta_u - \theta)$ the investment cost for the parent to bring his child's mark to the required level θ_u when her ability is $\theta \leq \theta_u$, and we assume that this cost is increasing and convex in the gap between requirement and ability: $p'(\cdot) > 0$, and $p''(\cdot) > 0$. We moreover assume that $\lim_{\theta \rightarrow \theta_u} p(\theta_u - \theta) = \lim_{\theta \rightarrow \theta_u} p'(\theta_u - \theta) = 0$, so that there is no fixed cost in the private tutoring technology. We denote by $H(\theta_u)$ (resp., $L(\theta_u)$) the fraction of the children population who accesses university (resp., who attends the vocational schools) when the threshold test level for university admittance is set at θ_u , with $H(\theta_u) + L(\theta_u) = 1$ by definition. We will compute these fractions $H(\theta_u)$ and $L(\theta_u)$ in section 3.

¹¹We make this simplifying assumption not to bias the model from the outset in favor of a university participation gap. All our analytical results hold when income and ability are correlated.

After completing school, children work and obtain a wage which is the product of their idiosyncratic ability, θ , and of the reference wage for their skill level, ω_i , $i \in \{L, H\}$.¹² High and low-skilled reference wages depend on the relative supply of each type of labor. As the supply of high-skilled labor $H(\theta_u)$ increases, the low-skilled reference wage ω_L goes up while the high-skilled wage ω_H falls. Thus, the skill premium $\omega_H - \omega_L$ is increasing (resp., decreasing) in the fraction of low-skilled labor supplied, $L(\theta_u)$ (resp., high-skilled, $H(\theta_u)$).¹³ As a shortcut, we denote the reference wages as functions of the test threshold: $\omega_H(\theta_u)$ and $\omega_L(\theta_u)$. Furthermore, to avoid unrewarding complication, we assume that the skill premium is always positive: $\omega_H(\theta_u) > \omega_L(\theta'_u)$, $\forall \{\theta_u, \theta'_u\} \in [\underline{\theta}, \bar{\theta}]$.²

Universities are costly while the cost of vocational education is normalized to zero. The (constant) cost per student of university education is c_u , and is financed through a proportional tax on income at rate t , paid by all parents.¹⁴ The government budget constraint is then

$$tEw = c_u H(\theta_u),$$

so that

$$t(\theta_u) = \frac{c_u H(\theta_u)}{Ew}. \quad (1)$$

¹²So, even though there are only two skill levels, the actual income of workers of a given skill level is continuously increasing in their ability. All results in this paper can be generalized to a setting with uncertainty (as to the probability of actually graduating or the future wage amount) as long as the expected wage of students increases with θ (for instance because of a lower dropout rate) and is larger when attending university rather than the vocational school, whatever θ .

¹³More precisely, we assume that ω_L (respectively, ω_H) is differentiable (and hence continuous) in $L(\theta_u)$ (respectively, $H(\theta_u)$) and bounded. This back box approach can be given micro-foundations by introducing for instance a CES production function with both types of labor as substitute inputs. One could then obtain the wage functions from usual profit-maximization conditions. Proceeding this way would lengthen the presentation of the model and complicate equations without bringing much new insight. Observe also that we assume that parents do not compete with their children in the job market.

¹⁴Since all children get some form of education in our model, adding a cost for vocational education would not change our results provided we interpret c_u as the difference between the per student university and vocational school costs. Also, the assumption of proportional taxation is made for simplicity only, with all our results continuing to hold provided that taxes paid increase with income.

Parents care both about current consumption and the wage of their child. A parent's utility is

$$U_u(\theta_u, w, \theta) = u(w(1 - t(\theta_u)) - p(\theta_u - \theta)) + \delta\theta\omega_H(\theta_u), \quad (2)$$

if their child attends university, where we assume that $p(\theta_u - \theta) = 0$ when $\theta \geq \theta_u$, since in that case the parent has no incentive to invest in the test preparation.¹⁵ A parent's utility is

$$U_V(\theta_u, w, \theta) = u(w(1 - t(\theta_u))) + \delta\theta\omega_L(\theta_u), \quad (3)$$

if their child attends vocational school. The parameter $\delta > 0$ measures the intensity of the altruism of parents towards their child, while the utility function u is continuous and twice differentiable with $u' > 0$ and $u'' < 0$. We assume for simplicity that parent's utility is linear in their child's wage, as more complex formulations would not bring any further insight.

The timing of the model is as follows. Parents first vote over the admission cut-off θ_u . They then choose individually whether to invest or not in private tutoring for their unique child to access university. Finally, they decide whether to enrol their child at university if she passes the admission test.

Solving the last stage is straightforward: since the skill premium is always positive and the investment cost $p(\theta_u - \theta)$ is a sunk cost at the final stage, all parents of children whose test marks are at least θ_u do enrol their child at university. We then solve the model backward, studying first which parents do invest in the preparation to the test (section 3), before turning to preferences over the threshold level θ_u (section 4) and to the aggregation of these preferences through majority voting (section 5).

¹⁵This formulation will prove easier to deal with than the alternative, which would define two different utility functions for parents of university educated children, depending on whether parents do invest in tutoring (because $\theta < \theta_u$) or not (since $\theta > \theta_u$).

3 Private tutoring decision

The tutoring investment stage takes place after the vote, once the cut-off of the admission test θ_u is known, and before the test. We therefore take the test cut-off ability θ_u as given in this section. Parents with children whose ability is above the threshold do not invest, since the investment is costly and generates no lasting effect beyond improving the test mark. We then focus on parents whose child's ability is below the threshold θ_u .

Proposition 1 (i) *For each income level w and test threshold θ_u , there exists a threshold ability, denoted by $\theta_m(\theta_u, w)$, with $\theta_m(\theta_u, w) < \theta_u$, such that parents with type (w, θ) such that $\theta \in [\theta_m(\theta_u, w), \theta_u[$ invest just enough for their child to qualify for university, while those with $\theta < \theta_m(\theta_u, w)$ do not invest and send their child to vocational school.*

(ii) *This threshold θ_m increases with θ_u and decreases with w .*

Proof. See Appendix A ■

Only parents whose child's ability is close enough to the required threshold invest in tutoring, while those whose child ability is too far below the threshold do not invest at all. This is intuitive, since tutoring costs increase in a convex way in the distance between child's ability θ and the test threshold θ_u . The fraction of children who become high-skilled is then given by

$$H(\theta_u) = \int_w^{\bar{w}} (1 - F(\theta_m(\theta_u, w))) dG(w),$$

with $L(\theta_u) = 1 - H(\theta_u)$.

As θ_u increases, the cost of investment goes up for every parent and so does the threshold θ_m . A richer parent has a lower marginal utility from consumption and is thus willing to pay more in order to raise his child to the test level required, so that θ_m decreases with w . This generates a higher education participation gap.

We next look at parents' preferences over the threshold level θ_u before aggregating these preferences through majority voting.

4 Individual preferences over θ_u

We proceed in two steps. We first look at individual preferences over θ_u as a function of the (for the moment, exogenous) type of education received by the child (academic or vocational). More precisely, we study separately the utility attained if the child becomes high-skilled (see (2)) or low-skilled (see (3)). In each case, we determine the individually most-preferred value of θ_u , and we perform some comparative statics analysis. We then study the two cases jointly, and we determine whose parents prefer to set θ_u so large (resp., small enough) that their child becomes low-skilled (resp., high-skilled). In all cases, when considering their preferred value of θ_u , parents correctly anticipate investment choices and the corresponding equilibrium allocation of students across educational tracks.

4.1 Preferences if the child attends vocational school

Assume first that the child attends vocational school and becomes low-skilled. The individually most-preferred value of θ_u maximizes $U_V(\theta_u, w, \theta)$ as given by (3), with the following FOC for an interior value of θ_u :

$$\delta\theta\omega'_L(\theta_u) = u'(w(1-t(\theta_u)))wt'(\theta_u). \quad (4)$$

This individually optimal size trades off the smaller low-skilled wage associated to a smaller university (the left-hand side of (4)) with the smaller tax bill (the right-hand side of (4)). We denote by $\theta_u^V(w, \theta)$ the value of θ_u satisfying (4) and we assume from now on that $\underline{\theta} < \theta_u^V(w, \theta) < \bar{\theta}$ for all (w, θ) and that the SOC

$$u''(\cdot)[wt'(\theta_u)]^2 - u'(\cdot)[wt''(\theta_u)] + \delta\theta\omega''_L(\theta_u) < 0$$

holds, which is the case for instance as soon as $t''(\theta_u) > 0$.

4.2 Preferences if the child attends university

We now move to the case where the child becomes high-skilled, in which case the utility of the parent is denoted by (2). Recall that $p(\theta_u - \theta) = 0$ if $\theta \geq \theta_u$ while $p(\theta_u - \theta) > 0$

if $\theta < \theta_u$. The marginal utility with respect to θ_u is:

$$\frac{\partial U_u(\theta_u, w, \theta)}{\partial \theta_u} = -u'(w(1 - t(\theta_u) - p(\theta_u - \theta))) [wt'(\theta_u) + p'(\theta_u - \theta)] + \delta\theta\omega'_H(\theta_u). \quad (5)$$

Observe first that U_u is continuous and differentiable in θ_u even at $\theta_u = \theta$, since $\lim_{\theta \rightarrow \theta_u} p(\theta_u - \theta) = \lim_{\theta \rightarrow \theta_u} p'(\theta_u - \theta) = 0$. Also, equation (5) can be simplified to

$$\frac{\partial U_u(\theta_u, w, \theta)}{\partial \theta_u} = -u'(w(1 - t(\theta_u) - p(\theta_u - \theta)))wt'(\theta_u) + \delta\theta\omega'_H(\theta_u) > 0$$

when $\theta_u < \theta$: utility is increasing in θ_u as long as $\theta_u < \theta$, since a larger value of θ_u decreases the tax bill ($t'(\theta_u) < 0$) while it increases the reference wage ($\omega'_H(\theta_u) > 0$). We then have that the most-preferred value of θ_u , denoted by $\theta_u^u(w, \theta)$, is strictly larger than θ , and is such that (5) is equal to zero. In words, $\theta_u^u(w, \theta)$ balances the marginal benefits of a smaller university (derived from tax savings and from a larger high-skilled wage) with the marginal (utility) cost of raising the child's mark in the admission test.

We assume that the SOC holds:

$$u''(\cdot) [-wt'(\theta_u) - p'(\theta_u - \theta)]^2 + u'(\cdot) [-wt''(\theta_u) - p''(\theta_u - \theta)] + \delta\theta\omega''_H(\theta_u) < 0,$$

which is the case either if $t''(\theta_u) > 0$ or if p is sufficiently convex. Put together with the fact that U_u is increasing in θ_u for $\theta_u \leq \theta$ and that $\theta_u^u(w, \theta) > \theta$, we obtain that U_u is single-peaked in θ_u .

The following lemma performs the comparative statics analysis of θ_u^u and θ_u^V .

Lemma 1 (i) $\theta_u^V(w, \theta)$ decreases with θ and increases (resp., decreases) with w if the coefficient of relative risk aversion (CRRA) is smaller (resp., larger) than one. (ii) $\theta_u^u(w, \theta)$ increases with θ and with w .

Proof. See Appendix B ■

Parents with a brighter child put more weight on the reference wage (whether ω_L or ω_H) and are thus in favor of a larger (resp., smaller) university if their child becomes low-skilled (resp., high-skilled), as ω_L decreases (resp., ω_H increases) with θ_u . Richer

parents pay more taxes and are thus in favor of a smaller university, other things equal. This statement has to be qualified in the case where the child becomes low-skilled, since a larger income translates into a smaller marginal utility and thus a smaller utility cost of taxation. The first effect is then larger than the second when marginal utility does not decrease too fast –i.e., when the CRRA is smaller than one.

The following lemma will be useful in several proofs.

Lemma 2 *For each income level, there exists a unique value of θ , denoted by $\hat{\theta}(w)$, such that $\theta_u^V(w, \theta) > \theta_u^u(w, \theta)$ for all (w, θ) with $\theta < \hat{\theta}(w)$, and $\theta_u^V(w, \theta) < \theta_u^u(w, \theta)$ for all (w, θ) with $\theta > \hat{\theta}(w)$.*

Proof. See Appendix C ■

4.3 Preferences with endogenous educational choice

We now study the preferences over θ_u when the child’s educational track is endogenous. This means that a (w, θ) parent anticipates that his child will be low-skilled for any θ_u such that $\theta < \theta_m(\theta_u, w)$ and will attend university and become high-skilled for values of the cut-off satisfying $\theta \geq \theta_m(\theta_u, w)$. Her utility over θ_u is then given by

$$\begin{aligned} U(\theta_u, w, \theta) &= U_u(\theta_u, w, \theta) \text{ if } \theta \geq \theta_m(\theta_u, w), \\ &= U_V(\theta_u, w, \theta) \text{ if } \theta < \theta_m(\theta_u, w). \end{aligned}$$

Observe that U is continuous in θ_u provided $p(\theta_u - \theta)$ does not include a fixed cost. Preferences are single-peaked in θ_u for all (w, θ) parents with $\theta > \hat{\theta}(w)$ (see Figure 1) but may not be for (w, θ) parents with $\theta < \hat{\theta}(w)$ (see Figure 2).

Insert Figures 1 and 2 around here

The following proposition studies which parents most-prefer a university size compatible with their child becoming high-skilled.

Proposition 2 (i) For each income level w , there exists a unique value of θ , denoted by $\tilde{\theta}(w)$, such that all (w, θ) parents with $\theta < \tilde{\theta}(w)$ most-prefer putting their child in a vocational school with $\theta_u = \theta_u^V(w, \theta)$, while all (w, θ) parents with $\theta > \tilde{\theta}(w)$ most-prefer enrolling their child at university with $\theta_u = \theta_u^u(w, \theta)$. (ii) Moreover, we have that $\tilde{\theta}(w) < \hat{\theta}(w)$.

Proof. See Appendix D. ■

The parent of a higher ability child benefits relatively more from university, for two reasons: (i) the child benefits more from the skill premium and (ii) the investment to be made in order for the child to pass the university entry test is smaller and thus less costly to the parent. This explains why there exists a unique threshold value of θ for each income level w below (resp., above) which parents most-prefer a university size consistent with their child becoming low-skilled (resp., high-skilled).

We now move to the determination of the majority voting equilibrium threshold ability.

5 Majority voting equilibrium

We start by introducing a straightforward definition and an assumption.

Definition 1 Let θ_u^{MV} be the median most-preferred value of θ_u in the population.¹⁶

Assumption 1 $\max_w \left[\theta_u^V(w, \tilde{\theta}(w)) \right] \leq \theta_u^{MV}$.

Proposition 3 proves that θ_u^{MV} is the Condorcet winner when voting over θ_u and that the majority voting equilibrium can be of two types. Assumption 1 is essentially technical and guarantees the existence of a Condorcet winner in the second type of equilibrium.¹⁷

¹⁶ θ_u^{MV} exists since $\theta_u^u(w, \theta)$ and $\theta_u^V(w, \theta)$ are continuous and strictly monotone in w and θ for all (w, θ) and since $G(w)$ and $F(\theta)$ have full support.

¹⁷ We present in section 7 numerical examples where θ_u^{MV} is the majority voting equilibrium even though Assumption 1 is not satisfied. We refer the reader to Appendix E for a description of the equilibrium existence issues faced when Assumption 1 is not satisfied.

Proposition 3 (a) If $\max[\theta_u^V(\bar{w}, \underline{\theta}), \theta_u^V(\underline{w}, \underline{\theta})] < \theta_u^{MV}$, then θ_u^{MV} is the unique Condorcet winning value of θ_u , and we have a “classical” majority voting equilibrium, where, for any w , high- θ (resp., low- θ) agents prefer a larger-than- θ_u^{MV} (resp., smaller-than- θ_u^{MV}) value of θ_u .

(b) If $\max[\theta_u^V(\bar{w}, \underline{\theta}), \theta_u^V(\underline{w}, \underline{\theta})] > \theta_u^{MV}$ and if Assumption 1 is satisfied, then θ_u^{MV} is the unique Condorcet winning value of θ_u , and we have an “ends-against-the-middle” majority voting equilibrium where, for any w , both low- θ and high- θ agents prefer a larger-than- θ_u^{MV} value of θ_u , while agents with intermediate values of θ prefer a smaller-than- θ_u^{MV} value of θ_u .

Proof. See Appendix F ■

The type of majority voting equilibrium depends on the preferences of the richest (resp., poorest) parent of the lowest ability children when $CRRA < 1$ (resp. $CRRA > 1$). If such parents (who most-prefer not to enrol their children at university) prefer a relatively large university system, then the decisive voters (who most-prefer $\theta_u = \theta_u^{MV}$) all enrol their children in the university at equilibrium (see Figure 3).¹⁸ Parents with high ability children favor a smaller-than-equilibrium university size (to save on the tax cost of university, and to boost their children’s high-skilled wage) while parents with low ability children favor a larger-than-equilibrium university size (either to enrol their children in universities, or to boost their low-skilled wage).

Insert Figures 3 and 4 around here

If $CRRA < 1$ (resp., $CRRA > 1$) and the richest (resp., poorest) parents of the lowest ability children prefer a relatively small university system, the majority voting equilibrium is of the “ends-against-the-middle” type (see Figure 4). Decisive voters are then made of two groups of agents: parents with high-ability children who enrol in university at equilibrium (as above), but also parents of low-ability children who attend vocational schools.¹⁹

¹⁸ Figures 3 and 4 correspond to the case where $CRRA < 1$, and are easy to redraw when $CRRA \geq 1$.

¹⁹ See Figure 5 in the proof of Proposition 3 for a description of the set of voters in the (w, θ) space.

Parents with children of intermediate (resp., high) abilities prefer a university size larger (resp., smaller) than equilibrium for the same reasons as explained above. Parents with children of low abilities favor a smaller-than-equilibrium university size because they put little weight on variations of the reference unskilled wage, but care relatively more for the tax cost of the university.

Before turning to the comparative statics analysis of the majority voting equilibrium, we briefly study the equilibrium size of the university.

Proposition 4 *In a classical equilibrium, strictly more than one half of the children population attend university.*

Proof. See Appendix G ■

Observe first that $\theta_u^{MV} > \theta_{med}$ in both types of equilibria (because $\theta_u^u(w, \theta) > \theta$ and $\theta_u^V(w, \theta) > \theta$ for all parents), but that this by itself does not imply that less than one half of the children population attend university. Indeed, in a classical equilibrium, those who attend university at equilibrium are composed of the half of the population who prefer a larger-than- θ_u^{MV} value of θ_u , but also of agents whose parents would prefer a slightly lower-than- θ_u^{MV} value of θ_u (because they would like to economize on their tutoring costs). Hence Proposition 4. By contrast, in an ends-against-the-middle equilibrium, a fraction of the agents who prefer a larger-than- θ_u^{MV} value of θ_u do not enrol their child at university, so that the equilibrium size of the university may be lower than one half of the polity.

The next section performs some analytical comparative statics analysis of the university size and equilibrium type. Section 7 then studies numerically other factors and their impact on the participation gap as well on university size.

6 Analytical comparative statics analysis of the majority chosen university size

Various factors may affect both the type of equilibrium (classical or ends-against-the-middle) and the majority chosen size of the university, given the equilibrium type. The

next proposition addresses both issues.

Proposition 5 (a) *A skilled wage less sensitive to supply (i.e., a lower absolute value of $\partial\omega_H(H)/\partial H$) increases the majority chosen university size in both types of equilibrium, and renders the ends-against-the-middle equilibrium more likely.*

(b) *An unskilled wage less sensitive to supply (i.e., a lower absolute value of $\partial\omega_L(L)/\partial L$) decreases the majority chosen university size in an ends-against-the middle equilibrium, does not affect the chosen size in a classical equilibrium, and renders the ends-against-the-middle equilibrium more likely.*

(c) *A lower level of altruism δ increases the majority chosen university size in a classical equilibrium, has an ambiguous impact on the chosen size in an ends-against-the middle equilibrium, and renders the ends-against-the-middle equilibrium more likely.*

(d) *A lower university cost per student c_u increases the majority chosen university size in both types of equilibrium, but has an ambiguous effect on the type of equilibrium.*

Proof. See Appendix H ■

The intuition for these results runs as follows. A skilled wage less sensitive to skilled labor supply decreases $\theta_u^u(w, \theta)$ for all individuals (by decreasing the incentives to restrict the university size in order to boost the skilled wage) but does not affect $\theta_u^V(w, \theta)$. The majority chosen university size then increases in both types of equilibrium. As for the type of equilibrium, recall that it depends on the comparison between $\theta_u^V(w, \underline{\theta})$ (for either $w = \bar{w}$ if $\text{CRRA} < 1$ or $w = \underline{w}$ if $\text{CRRA} > 1$) and $\theta_u^u(w, \theta)$ for some agents (w, θ) . Since the former is not affected, while the latter decreases for *all* (w, θ) , an ends-against-the-middle situation is made more likely.

An unskilled wage less sensitive to unskilled labor supply increases $\theta_u^V(w, \theta)$ for all individuals (by decreasing the incentives to enlarge the university size in order to boost the unskilled wage) but does not affect $\theta_u^u(w, \theta)$. The majority chosen university size then decreases in an ends-against-the-middle equilibrium, but not in a classical equilibrium. Since $\theta_u^V(w, \underline{\theta})$ increases, while $\theta_u^u(w, \theta)$ is not affected, an ends-against-the-middle situation is made more likely.

A lower level of altruism δ decreases $\theta_u^u(w, \theta)$ for all individuals (by decreasing the incentives to restrict the university size in order to boost the skilled wage) while it increases $\theta_u^V(w, \theta)$ for all individuals (by decreasing the incentives to enlarge the university size in order to boost the unskilled wage). This results in an increase in the majority-chosen university size in a classical equilibrium²⁰ but has an ambiguous impact on the chosen size in an ends-against-the-middle equilibrium. Since $\theta_u^V(w, \underline{\theta})$ increases while $\theta_u^u(w, \theta)$ decreases, an ends-against-the-middle situation is made more likely.

A lower university cost per student decreases both $\theta_u^u(w, \theta)$ and $\theta_u^V(w, \theta)$ for all individuals (by decreasing the tax cost of any university size), resulting in an increase in the majority-chosen university size in both equilibria. Since $\theta_u^V(w, \underline{\theta})$ and $\theta_u^u(w, \theta)$ move in the same direction, the impact of a lower c_u on the type of equilibrium is ambiguous.

7 Numerical comparative statics analysis of the majority chosen university size and participation gap

Going beyond the previous results to study the impact of other factors on the higher education participation gap as well as size (and equilibrium type) requires resorting to numerical simulations. Our base case in this section is built on functional forms and numerical assumptions detailed in Appendix I. These assumptions have not been chosen to fit any specific empirical observation, but because they generate an ends-against-the-middle majority voting equilibrium where Assumption 1 is satisfied (thereby showing that this assumption may indeed be satisfied). In this equilibrium, the ability threshold for access to university is set such that 55.9% of the population do attend university.²¹ The half of the electorate who want a smaller university is composed of 4.9% of parents

²⁰This counter-intuitive result is only valid locally: as δ decreases, $U_u(\theta_u^u(w, \theta), w, \theta)$ also decreases so that more agents prefer to send their child to vocational school and to restrict the university size.

²¹Although our objective is not to fit any specific empirical observation, we observe that this proportion is very close to the 55.4% reported for Scotland for the academic year 2013/2014 (Scottish Funding Council, 2016).

of low-skilled children (who would like to decrease the fiscal cost of university) and of 45.1% of parents of high-skilled children (who would like to increase the skill premium as well as decrease the fiscal cost of university). The participation gap is such that 54.1% of the children whose parent's income is in the bottom quintile (Q1 henceforth) attend university, while 57.5% of children in the top quintile (Q5) do attend university.²²

Raising the skill premium,²³ by increasing the value of ω_H for any value of θ_u , results in an increase of the university size (by 14%), as well as an increase in the ratio of participation between Q5 and Q1 (by 47%), compared to the base case. A higher skill premium makes university attendance more attractive, and induces especially richer parents to invest more in tutoring (because they have a lower marginal utility cost of tutoring expenses).

Decreasing the parental investment cost²⁴ both increases the university size (by 10%) and the participation gap (by 12%). Decreasing the tutoring cost induces more people to invest in preparation, resulting in a larger university size for any given value of θ_u . Agents react by increasing the majority voting equilibrium value of θ_u , but the first – direct – effect is larger than the second – indirect – one and the equilibrium university size increases compared to the base case. This bigger university goes hand in hand with a larger participation gap.

²²We could choose other functional forms which generate equilibria where the university participation rate is lower, or where the participation gap across income levels is larger. We have chosen this example because, while it is an ends-against-the-middle equilibrium satisfying Assumption 1, varying the parameters as listed below generates both ends-against-the-middle equilibria where Assumption 1 is not satisfied (proving that this assumption is not necessary for equilibrium existence, but only sufficient), and classical equilibria. When Assumption 1 is not satisfied, we have checked numerically that θ_u^{MV} is preferred by a majority of voters to any other value of θ_u .

²³In our simulations, we assume that ω_H is exogenous and we increase this value from 18.6 in the base case to 25. All analytical results presented above hold qualitatively when ω_H is exogenous, rather than a decreasing function of the university size. We have chosen to assume ω_H exogenous in this section for comparative statics purposes.

²⁴We move from $p(\theta - \theta_u) = 500x^2$ to $p(\theta - \theta_u) = 500x^3$, which reduces p for any $\theta - \theta_u$ since $\theta - \theta_u < 1$ for all θ, θ_u

At the polar extreme, we look at the impact of assuming away the possibility to invest in tutoring on the majority voting equilibrium. In the absence of a test preparation technology, we obtain an ends-against-the-middle equilibrium where only 45.8% of the children attend university. Without preparation technology, there is no participation gap in university attendance when income and ability are uncorrelated, so that the same fraction of children attend university irrespective of their parent's income.²⁵ Note that this result was not a foregone conclusion, as this move affects the individual preferences for university size. For instance, for any given θ_u , fewer individuals attend university when the investment technology is not available, which increases the skill premium and thus makes attending university more attractive.

We know from Proposition 5 that decreasing the cost of university c_u results in a larger university size. This is confirmed in our numerical exercise, where the university size increases by 8% when c_u is halved. Moreover, we obtain that the ratio of participation rates between Q5 and Q1 decreases by 18% compared to the base case. As explained in the previous section, a smaller value of c_u decreases the tax cost of university, leading to a larger equilibrium one. This smaller tax cost decreases the marginal utility cost of tutoring for all agents, but especially for the poorest ones (because of the concavity of utility), decreasing the participation gap.

Increasing the minimum value of θ (from $\underline{\theta} = 0.1$ to $\underline{\theta} = 0.2$, while keeping a uniform distribution over $[\underline{\theta}, 1]$) increases the equilibrium university size by 10%, while decreasing the participation gap by 5%. It is intuitive that removing the lowest-ability individuals, who in an ends-against-the-middle equilibrium favor a lower-than-equilibrium university size, results in a larger university size.²⁶

Performing a mean-preserving spread of the parental income distribution (by moving from a uniform distribution over $[5, 20]$ to one over $[2, 23]$) barely affects the university size

²⁵See De Donder and Martinez-Mora (2015) for analytical statements on equilibrium existence as well as a numerical analysis of the impact of introducing such a correlation.

²⁶Moreover, the equilibrium obtained with this larger value of $\underline{\theta}$ is a classical one, which exemplifies how one can move from one type of equilibrium to another as parameters get varied.

(which increases by 1%) but increases the university participation gap by 56%. Recall from Lemma 1 that the most-preferred university size is monotone in parental income. The parents added at both extremes of the income distribution then somehow counteract each other when voting over the university size. At the same time, the first income quantile is made poorer, and the fifth income quantile is made richer, which exacerbates the participation gap.

Finally, lowering the degree of altruism (from 0.5 to 0.3) decreases the equilibrium university size by 13%, and barely affects the participation gap, with the ratio of participation between Q5 and Q1 increasing by a mere 2% compared to the base case. Proposition 5 shows that the impact of a lower value of δ is ambiguous in an ends-against-the-middle equilibrium, while it unambiguously increases the equilibrium university size in a classical equilibrium. Our numerical example then shows that varying the degree of altruism can have impacts of opposite signs on the equilibrium university size according to the type of equilibrium. With the altruism modeled as an additive term, independent of income (see (2) and (3)), the incentives to invest in preparation are similarly affected across income levels when δ is varied, so that the participation gap is barely affected as δ is decreased.

To summarize, two modifications lead to an increase in both the equilibrium university size and the participation gap: a larger skill premium and a smaller tutoring cost. Two other modifications increase the equilibrium university size but decrease the participation gap: raising the minimum value of θ , and lowering the university cost. A more unequal parental income distribution (obtained through a mean-preserving spread) barely affects the university size but increases the participation gap. Finally, the degree of altruism impacts the university size (less altruism decreasing the university size), but does not affect much the participation gap.

8 Conclusion

We have built a political economy model in order to shed light on the empirically observed increase of university size and participation gap. In our model, the participation gap

emerges because tutoring investments (a widespread and growing phenomenon) increase with parental income. We obtain that an increase in the skill premium –as observed in many countries since the late 1970s– can replicate our stylized fact, by making university attendance more attractive and by inducing especially richer parents to invest more in tutoring (because they have a lower marginal utility cost of tutoring expenses). Two other phenomena, also empirically observed, increase the university size, but decrease the participation gap: a decrease in the per student university cost, and an increase in students’ minimum academic ability (the latter reflecting the expansion of participation in secondary education). Another empirically observed factor, greater inequality in parental income, increases the participation gap but barely affects the university size. Finally, we identify another potential factor, namely a change in the tutoring technology – making it cheaper – which increases both the university size and the participation gap.

We have voluntarily refrained from providing a welfare analysis of our model, as well as policy recommendations²⁷ because both would hinge crucially on whether tutoring only has short term effects (as in this paper) or allows to increase long term abilities of students. Empirical research is sorely needed to answer this question, although the preliminary evidence, on which this paper is based, is currently quite negative.

We would like to conclude by explaining the price exacted by tractability on our model, simplifications which we would like to address in future research. First, we ignore the multi-tiered structure of university systems, with multiple tests determining whether a student graduates and on what terms, generating finely discriminating signals of student quality. Second, we ignore the signalling aspect of education, which is arguably the strongest argument for admissions requirements. Third, we do not address how admissions and subsequent testing requirements affect student effort, which presumably has an impact on their productivity as skilled workers. Fourth, another dimension absent from our analysis is immigration; this is a significant omission for its effects on the labor market,

²⁷For instance, the Sutton Trust recommends introducing a means-tested voucher scheme to enable lower-income families to provide supplementary education to their children and decrease the participation gap (Kirby, 2016).

and because of the growing internationalization of higher education. Finally, observe that we have assumed that the joint distribution of income and ability is exogenous. In reality, it is itself determined by the education system. This is a feedback effect which would be most interesting to study in a dynamic version of our model.

Appendix A: Proof of Proposition 1

(i) The first stage in the proof studies which parents invest in tutoring when faced with policy $(t, \omega_L, \omega_H, \theta_u)$ whose four components are scalars. In a second stage, we endogenize the values taken by $t, \omega_L,$ and $\omega_H,$ given θ_u and the investment decisions of the parents, and we show that there exists a fixed point of this mapping from (t, ω_L, ω_H) unto itself, for any given $\theta_u.$

Parents faced with the exogenous quadruplet $(t, \omega_L, \omega_H, \theta_u)$ invest in tutoring if their utility when paying for this investment is at least equal to their utility if they do not invest. This difference in utility is given by

$$\Delta(t, \omega_L, \omega_H, \theta_u; w, \theta) = \delta\theta(\omega_H - \omega_L) + u(c_p) - u(c_0),$$

with

$$c_0 = (1 - t)w$$

the consumption level in case of no investment and

$$c_p = (1 - t)w - p(\theta_u - \theta)$$

the consumption level with investment.

We claim that only parents (w, θ) such that $\theta \in [\theta_p(t, \omega_L, \omega_H, \theta_u; w), \theta_u]$ with $\theta_p(t, \omega_L, \omega_H, \theta_u; w) < \theta_u$ do invest. It is obvious that parents with $\theta \geq \theta_u$ do not invest since tutoring has no benefit for them (their child being accepted at university without tutoring). Focusing on $\theta < \theta_u,$ we obtain

$$\frac{\partial \Delta(t, \omega_L, \omega_H, \theta_u; w, \theta)}{\partial \theta} = \delta(\omega_H - \omega_L) + u'(c_p)p'(\theta_u - \theta) > 0.$$

Observe that $\Delta(t, \omega_L, \omega_H, \theta_u; w, \theta_u) > 0$ while $\Delta(t, \omega_L, \omega_H, \theta_u; w, \underline{\theta}) \leq 0.$ We then define by $\theta_p(t, \omega_L, \omega_H, \theta_u; w)$ the unique value of θ such that $\Delta(t, \omega_L, \omega_H, \theta_u; w, \theta) = 0$ if $\Delta(t, \omega_L, \omega_H, \theta_u; w, \underline{\theta}) < 0,$ and $\theta_p(t, \omega_L, \omega_H, \theta_u; w) = \underline{\theta}$ otherwise.

Up to now, the triplet (t, ω_L, ω_H) has been taken as exogenous. We now compute the value of this triplet given the tutoring choices of the parents. In other words, we construct

a mapping from (t, ω_L, ω_H) to (t, ω_L, ω_H) , using the investment choices of the parents, and we show that this mapping has a fixed point. This mapping is such that

$$\begin{aligned} t &= \frac{H(t, \omega_L, \omega_H, \theta_u)}{Ew} c_u, \\ \omega_L &= \omega_L(L(t, \omega_L, \omega_H, \theta_u)), \\ \omega_H &= \omega_H(H(t, \omega_L, \omega_H, \theta_u)), \end{aligned}$$

with

$$H(t, \omega_L, \omega_H, \theta_u) = \int_{\underline{w}}^{\bar{w}} (1 - F(\theta_p(t, \omega_L, \omega_H, \theta_u; w))) dG(w),$$

and $L(t, \omega_L, \omega_H, \theta_u) = 1 - H(t, \omega_L, \omega_H, \theta_u)$.

It is obvious from the definitions of t , $\theta_p(t, \omega_L, \omega_H, \theta_u; w)$, $\omega_L(L(\cdot))$ and $\omega_H(H(\cdot))$ that the mapping is a continuous self-map on a closed, bounded and convex subset of \Re^3 , so that, by Brouwer's fixed point theorem, this mapping has a fixed point. We then denote by $\theta_m(\theta_u, w)$ the threshold ability level of a parent of income w below which the parent does not invest in the test preparation and enrolls his child in the vocational school.

(ii) We assume a unique interior value of θ_m for the comparative static analysis.²⁸ We reformulate the problem (slightly abusing notation) as follows. The value of θ_m is given by the function

$$\Delta(t, \omega_L, \omega_H, \theta_u; w, \theta_m) = \delta\theta_m(\omega_H - \omega_L) + u(c_p) - u(c_0) = 0, \quad (6)$$

where t , ω_L and ω_H are given by the implicit functions

$$\begin{aligned} t &= \frac{H(\theta_m)}{Ew} c_u, \\ \omega_L &= \omega_L(1 - H(\theta_m)), \\ \omega_H &= \omega_H(H(\theta_m)), \end{aligned}$$

with

$$H(\theta_m) = \int_{\underline{w}}^{\bar{w}} (1 - F(\theta_m)) dG(w).$$

²⁸We have not found any numerical example where there exist multiple equilibria of the investment stage.

With this formulation, it is obvious that t , ω_L and ω_H depend directly on θ_m , but not on θ_u nor on w .

Applying the implicit function theorem on (6) while making use of the definitions of t , ω_L and ω_H above, we obtain that

$$\frac{\partial \theta_m}{\partial w} = -\frac{d\Delta/dw}{d\Delta/d\theta_m},$$

where

$$\frac{d\Delta}{dw} = (1-t)(u'(c_p) - u'(c_0)) > 0,$$

and where

$$\frac{d\Delta}{d\theta_m} = \frac{\partial \Delta}{\partial \theta} + \frac{\partial \Delta}{\partial t} \frac{dt}{d\theta_m} + \frac{\partial \Delta}{\partial \omega_L} \frac{d\omega_L}{d\theta_m} + \frac{\partial \Delta}{\partial \omega_H} \frac{d\omega_H}{d\theta_m} > 0,$$

since

$$\begin{aligned} \frac{\partial \Delta}{\partial \theta} &= \delta(\omega_H - \omega_L) + u'(c_p)p'(\theta_u - \theta_m) > 0, \\ \frac{\partial \Delta}{\partial t} &= w(u'(c_0) - u'(c_p)) < 0, \\ \frac{dt}{d\theta_m} &= \frac{c_u}{Ew} \frac{\partial H}{\partial \theta_m} = -f(\theta_m) \frac{c_u}{Ew} < 0, \\ \frac{\partial \Delta}{\partial \omega_H} &= \delta\theta_m > 0, \\ \frac{d\omega_H}{d\theta_m} &= -\frac{\partial \omega_H}{\partial H} f(\theta_m) > 0, \\ \frac{\partial \Delta}{\partial \omega_L} &= -\delta\theta_m < 0, \\ \frac{d\omega_L}{d\theta_m} &= -\frac{\partial \omega_L}{\partial H} f(\theta_m) < 0. \end{aligned}$$

We then have that

$$\frac{\partial \theta_m}{\partial w} < 0.$$

Likewise, we obtain that

$$\frac{\partial \theta_m}{\partial \theta_u} = -\frac{d\Delta/d\theta_u}{d\Delta/d\theta_m} > 0,$$

since

$$\frac{d\Delta}{d\theta_u} = -u'(c_p)p'(\theta_u - \theta_m) < 0.$$

Appendix B: Proof of Lemma 1

(i) Applying the implicit function theorem on the FOC (4) and using the SOC, we obtain that²⁹

$$\begin{aligned}\frac{\partial \theta_u^V(w, \theta)}{\partial \theta} &\stackrel{s}{=} \delta \omega'_L(\theta_u) < 0, \\ \frac{\partial \theta_u^V(w, \theta)}{\partial w} &\stackrel{s}{=} -t'(\theta_u) [u'(c_0) + c_0 u''(c_0)]\end{aligned}$$

with $c_0 = w(1 - t(\theta_u))$, which is positive if the CCRA $-u''(c_0)c_0/u'(c_0) \leq 1$, and negative otherwise.

(ii) Applying the implicit function theorem on the FOC (5) and using the SOC, we obtain that

$$\begin{aligned}\frac{\partial \theta_u^u(w, \theta)}{\partial \theta} &\stackrel{s}{=} \delta \omega'_H(\theta_u) + u'(c_p)p''(\theta_u - \theta) \\ &\quad - u''(c_p)p'(\theta_u - \theta) [wt'(\theta_u) + p'(\theta_u - \theta)]\end{aligned}$$

which is positive since $c_p = w(1 - t(\theta_u) - p(\theta_u - \theta))$, and all three terms are positive (with the last term between brackets positive when $\theta_u = \theta_u^u(w, \theta)$), and

$$\frac{\partial \theta_u^u(w, \theta)}{\partial w} \stackrel{s}{=} u''(c_p)(1 - t(\theta_u)) [wt'(\theta_u) + p'(\theta_u - \theta)] - u'(c_p)t'(\theta_u) > 0.$$

Appendix C: Proof of Lemma 2

Results from $\lim_{\theta \rightarrow 0} \theta_u^u(w, \theta) < \lim_{\theta \rightarrow 0} \theta_u^V(w, \theta) = \bar{\theta}$, $\theta_u^u(w, \theta) \geq \theta$ and $\partial \theta_u^V(w, \theta)/\partial \theta < 0$ while $\partial \theta_u^u(w, \theta)/\partial \theta > 0$.

Appendix D: Proof of Proposition 2

We denote by

$$U_u^*(w_i, \theta) = U_u(\theta_u^u(w, \theta), w, \theta)$$

the highest utility level a parent of type (w, θ) can attain by sending his child to university (i.e., when setting $\theta_u = \theta_u^*(w, \theta)$), and by

$$U_V^*(w_i, \theta) = U_V(\theta_u^V(w, \theta), w, \theta)$$

²⁹The notation $\stackrel{s}{=}$ means “has the same sign as”.

the highest utility level attained when his child attends vocational school (i.e., when setting $\theta_u = \theta_u^V(w, \theta)$).

(i) We have

$$U_u^*(w, \theta) - U_V^*(w, \theta) = u(c_p(\theta_u^u(w, \theta))) - u(c_0(\theta_u^V(w, \theta))) + \delta \theta [\omega_H(\theta_u^u(w, \theta)) - \omega_L(\theta_u^V(w, \theta))],$$

where

$$\begin{aligned} c_p(\theta_u^u(w, \theta)) &= w(1 - t(\theta_u^u(w, \theta))) - p(\theta_u^u(w, \theta) - \theta), \\ c_0(\theta_u^V(w, \theta)) &= w(1 - t(\theta_u^V(w, \theta))). \end{aligned}$$

Using the envelope theorem, we obtain

$$\frac{\partial (U_u^*(w, \theta) - U_V^*(w, \theta))}{\partial \theta} = u'(c_p(\theta_u^u(w, \theta)))p'(\theta_u - \theta) + \delta [\omega_H(\theta_u^u(w, \theta)) - \omega_L(\theta_u^V(w, \theta))] > 0.$$

For any income level w , we have that $\lim_{\theta \rightarrow \bar{\theta}} U_u^*(w, \theta) > \lim_{\theta \rightarrow \bar{\theta}} U_V^*(w, \theta)$ since $\lim_{\theta \rightarrow \bar{\theta}} \theta_u^V(w, \theta) < \bar{\theta} = \lim_{\theta \rightarrow \bar{\theta}} \theta_u^u(w, \theta)$ so that $\lim_{\theta \rightarrow \bar{\theta}} c_p(\theta_u^u(w, \theta)) > \lim_{\theta \rightarrow \bar{\theta}} c_0(\theta_u^V(w, \theta))$. Hence the existence and unicity of $\tilde{\theta}(w)$: if $U_u^*(w, \underline{\theta}) < U_V^*(w, \underline{\theta})$ then the solution is interior; otherwise, i.e. if $U_u^*(w, \underline{\theta}) \geq U_V^*(w, \underline{\theta})$, the solution is $\tilde{\theta}(w) = \underline{\theta}$.

(ii) $U_u^*(w, \hat{\theta}(w)) > U_V^*(w, \hat{\theta}(w))$, together with $\partial (U_u^*(w, \theta) - U_V^*(w, \theta)) / \partial \theta > 0$, implies that $\tilde{\theta}(w) < \hat{\theta}(w)$.

Appendix E: Assumption 1

To convey the intuition for why Assumption 1 is needed to establish the existence of an end-against-the-middle equilibrium, assume that there is only one income level, w . When Assumption 1 is **not** satisfied, the individual $\tilde{\theta}$ is indifferent between $\theta_u = \theta_u^u(w, \tilde{\theta}) < \theta_u^{MV}$ and $\theta_u^V(w, \tilde{\theta}) > \theta_u^{MV} > \tilde{\theta}$. Unlike in the proof of Proposition 3 (b), θ_u^{MV} is not preferred to all $\theta < \theta_u^{MV}$ by individual $\tilde{\theta}$, since this individual attains a higher utility level with $\theta_u = \tilde{\theta} - \varepsilon$ with $\varepsilon > 0$ low enough. This opens up the possibility of a Condorcet cycle and of the inexistence of a Condorcet winning value of θ_u .

Appendix F: Proof of Proposition 3

To prove that our candidate θ_u^{MV} is indeed a Condorcet winner, we first define the set of decisive voters, that is, the set of voters whose global peak is exactly at θ_u^{MV} (i.e. at the median of the distribution of peaks). We define the global peak of (w, θ) voters as $\theta_u^*(w, \theta)$, with

$$\theta_u^*(w, \theta) = \begin{cases} \theta_u^V(w, \theta), & \forall (w, \theta) \text{ such that } \theta \leq \tilde{\theta}(w), \\ \theta_u^u(w, \theta), & \forall (w, \theta) \text{ such that } \theta > \tilde{\theta}(w). \end{cases}$$

We denote by $D(x)$ the set of agents (w, θ) who most-prefer $\theta_u = x$, so that $D(\theta_u^{MV}) \equiv \{(w, \theta) \text{ such that } \theta_u^*(w, \theta) = \theta_u^{MV}\}$. Lemma 3 characterizes this set.

Lemma 3 (a) *If $\max[\theta_u^V(\bar{w}, \underline{\theta}), \theta_u^V(\underline{w}, \underline{\theta})] \leq \theta_u^{MV}$, then the set of decisive voters $D(\theta_u^{MV})$ corresponds to the locus of $(w, \theta_d^u(w, \theta_u^{MV}))$ pairs with*

$$\theta_d^u(w, \theta_u^{MV}) \equiv \{\theta \text{ such that } \theta_u^u(w, \theta) = \theta_u^{MV}\}. \quad (7)$$

We have that $\theta_d^u(w, \theta_u^{MV})$ decreases with w , and that all (w, θ) voters with $\theta > \theta_d^u(w, \theta_u^{MV})$ (resp., $\theta < \theta_d^u(w, \theta_u^{MV})$) have $\theta_u^(w, \theta) > \theta_u^{MV}$ (resp. $\theta_u^*(w, \theta) < \theta_u^{MV}$).*

(b) *If $\max[\theta_u^V(\bar{w}, \underline{\theta}), \theta_u^V(\underline{w}, \underline{\theta})] > \theta_u^{MV}$ and Assumption 1 holds, then $D(\theta_u^{MV})$ equals the union of two loci:*

$$D(\theta_u^{MV}) = (w, \theta_d^u(w, \theta_u^{MV})) \cup (w, \theta_d^V(w, \theta_u^{MV})),$$

with $\theta_d^V(w, \theta_u^{MV}) \equiv \{\theta \text{ such that } \theta_u^V(w, \theta) = \theta_u^{MV}\}$. We have that $\theta_d^V(w, \theta_u^{MV})$ increases (resp., decreases) with w when $\text{CRRA} < 1$ (resp., $\text{CRRA} > 1$), and that all (w, θ) voters with $\theta < \theta_d^V(w, \theta_u^{MV})$ or $\theta > \theta_d^u(w, \theta_u^{MV})$ have $\theta_u^*(w, \theta) > \theta_u^{MV}$, while all other voters have $\theta_u^*(w, \theta) < \theta_u^{MV}$.

Proof of Lemma 3

(a) Given that $\partial \theta_u^V(w, \theta) / \partial \theta < 0$ (see Lemma 1), $\max[\theta_u^V(\bar{w}, \underline{\theta}), \theta_u^V(\underline{w}, \underline{\theta})] < \theta_u^{MV}$ implies that $\theta_u^V(w, \theta) < \theta_u^{MV}$ for all parents. Hence, only (w, θ) parents with $\theta > \tilde{\theta}(w)$ may be decisive, and so D is given by locus (7). Recall from Lemma 1 that $\partial \theta_u^u(w, \theta) / \partial \theta > 0$ and $\partial \theta_u^u(w, \theta) / \partial w > 0$, so that the implicit function theorem implies that $\theta_d^u(w, \theta_u^{MV})$ is

decreasing in w . The rest of part (a) follows from the fact that $\theta_u^V(w, \theta) < \theta_u^{MV}$ and that $\partial\theta_u^u(w, \theta)/\partial\theta > 0$ for all (w, θ) .

(b) Part (a) has established that $(w, \theta_d^u(w, \theta_u^{MV})) \in D(\theta_u^{MV})$ in case (b) as well. Application of the intermediate value theorem given that $\max[\theta_u^V(\bar{w}, \underline{\theta}), \theta_u^V(\underline{w}, \underline{\theta})] > \theta_u^{MV}$, that Assumption 1 holds and that Lemma 1 has established that $\partial\theta_u^V(w, \theta)/\partial\theta < 0$ means that $D(\theta_u^{MV})$ is also composed of another set of voters, namely those with $(w, \theta_d^V(w, \theta_u^{MV}))$.³⁰ Recall from Lemma 1 that $\partial\theta_u^V(w, \theta)/\partial\theta < 0$ and $\partial\theta_u^V(w, \theta)/\partial w > 0$ (resp. < 0) when $\text{CRRA} < 1$ (resp. $\text{CRRA} > 1$), so that the implicit function theorem implies that $\theta_d^V(w, \theta_u^{MV})$ increases (resp., decreases) with w when $\text{CRRA} < 1$ (resp., $\text{CRRA} > 1$). Finally, (i) agents with $\theta < \theta_d^V(w, \theta_u^{MV})$ prefer $\theta_u^*(w, \theta) = \theta_u^V(w, \theta)$, which is larger than θ_u^{MV} since $\partial\theta_u^V(w, \theta)/\partial\theta < 0$, (ii) agents with $\theta > \theta_d^u(w, \theta_u^{MV})$ prefer $\theta_u^*(w, \theta) = \theta_u^u(w, \theta)$, which is larger than θ_u^{MV} since $\partial\theta_u^u(w, \theta)/\partial\theta > 0$, while (iii) the other agents prefer either $\theta_u^*(w, \theta) = \theta_u^V(w, \theta)$ (if $\theta < \tilde{\theta}(w)$) which is smaller than θ_u^{MV} since $\partial\theta_u^V(w, \theta)/\partial\theta < 0$, or $\theta_u^*(w, \theta) = \theta_u^u(w, \theta)$ (if $\theta > \tilde{\theta}(w)$) which is smaller than θ_u^{MV} since $\partial\theta_u^u(w, \theta)/\partial\theta > 0$.

We now prove Proposition 3.

(a) Assume first that $\max[\theta_u^V(\underline{w}, \underline{\theta}), \theta_u^V(\bar{w}, \underline{\theta})] < \theta_u^{MV}$, so that we claim that

$$\theta_u^{MV} = \theta \text{ such that } \int_{\underline{w}}^{\bar{w}} (1 - F(\theta_d^u(w, \theta))) dG(w)$$

is preferred by a majority of parents to any other value of θ_u . In this case, Lemma 3 shows that the set of agents with $\theta_u^*(w, \theta) > \theta_u^{MV}$ are such that $\theta > \theta_d^u(w, \theta_u^{MV})$. They prefer θ_u^{MV} to any value of $\theta_u < \theta_u^{MV}$, since $U_u(\theta_u, w, \theta) > U_V(\theta_u, w, \theta)$ for all $\theta_u < \theta_u^{MV} \leq \theta_u^u(w, \theta)$ and since $U_u(\theta_u, w, \theta)$ increases with θ_u when $\theta_u \leq \theta_u^u(w, \theta)$. Since this group by definition represents one half of the polity, θ_u^{MV} cannot be beaten by any $\theta_u < \theta_u^{MV}$.

We now look at agents with $\theta_u^*(w, \theta) < \theta_u^{MV}$ —i.e., those with $\theta < \theta_d^u(w, \theta_u^{MV})$. They are all such that $\theta_u^V(w, \theta) < \theta_u^{MV}$ (since $\max[\theta_u^V(\underline{w}, \underline{\theta}), \theta_u^V(\bar{w}, \underline{\theta})] < \theta_u^{MV}$ and $\partial\theta_u^V(w, \theta)/\partial\theta < 0$)

³⁰ Assumption 1 together with $\max[\theta_u^V(\bar{w}, \underline{\theta}), \theta_u^V(\underline{w}, \underline{\theta})] > \theta_u^{MV}$ imply that $\theta_d^V(w, \theta_u^{MV})$ exists for at least some values of w , but may be not for all. All statements below must then be qualified as “when $\theta_d^V(w, \theta_u^{MV})$ exists for a given w ”.

and $\theta_u^u(w, \theta) < \theta_u^{MV}$ (since $\partial \theta_u^u(w, \theta) / \partial \theta > 0$ and $\partial \theta_u^u(w, \theta) / \partial w > 0$). Hence, their utility $U(\theta_u, w, \theta)$ decreases with θ_u for any $\theta_u \geq \theta_u^{MV}$, so that this half of the population prefers θ_u^{MV} to any larger value of θ_u , and θ_u^{MV} constitutes the unique Condorcet winner.

(b) Assume now that $\max[\theta_u^V(\underline{w}, \underline{\theta}), \theta_u^V(\bar{w}, \bar{\theta})] > \theta_u^{MV}$, so that we claim that

$$\theta_u^{MV} = \theta \text{ such that } \int_{\underline{w}}^{\bar{w}} F(\theta_d^V(w, \theta)) dG(w) + \int_{\underline{w}}^{\bar{w}} (1 - F(\theta_d^u(w, \theta))) dG(w) = 0.5$$

is preferred by a majority of parents to any other value of θ_u . Figure 5 illustrates the preferences over θ_u of the voters in the (w, θ) when $\text{CRRA} < 1$ (with $\text{CRRA} > 1$, the function $\theta_d^u(w, \theta_u^{MV})$ is decreasing in w).

Insert Figure 5 around here

By the same argument as in part (a) of this proof, voters with $\theta_u^*(w, \theta) < \theta_u^{MV}$ strictly prefer θ_u^{MV} to any other $\theta_u > \theta_u^{MV}$ and constitute by definition one half of the electorate. Thus, θ_u^{MV} cannot be defeated in pairwise contest by any alternative $\theta_u > \theta_u^{MV}$. The set of agents with $\theta_u^*(w, \theta) > \theta_u^{MV}$ is now given by all (w, θ) agents with $\theta < \theta_d^V(w, \theta_u^{MV})$ or $\theta > \theta_d^u(w, \theta_u^{MV})$. Again, part (a) applies to prove that (w, θ) agents with $\theta > \theta_d^u(w, \theta_u^{MV})$ strictly prefer θ_u^{MV} to any $\theta_u < \theta_u^{MV}$. We then have to prove that the remaining group, (w, θ) voters with $\theta < \theta_d^V(w, \theta_u^{MV})$ prefer θ_u^{MV} to any lower value of θ_u . For this group, we then have that $\theta < \theta_u^u(w, \theta) < \theta_u^{MV} \leq \theta_u^V(w, \theta)$. A necessary and sufficient condition for θ_u^{MV} to be preferred to any lower value of θ_u is that $U_u^*(w, \theta) = U_u(\theta_u^u(w, \theta), w, \theta) < U_V(\theta_u^{MV}, w, \theta)$ for all $\theta < \theta_d^V(w, \theta_u^{MV})$. It is easy to see (from the proof of Proposition 2) that $\partial (U_u^*(w, \theta) - U_V(\theta_u^{MV}, w, \theta)) / \partial \theta > 0$ so that, since $U_u^*(w, \theta_d^V(w, \theta_u^{MV})) < U_V(\theta_u^{MV}, w, \theta_d^V(w, \theta_u^{MV}))$, all agents with $\theta < \theta_d^V(w, \theta_u^{MV})$ strictly prefer θ_u^{MV} to any lower value of θ_u . We then have that θ_u^{MV} cannot be defeated at the majority voting and is the unique Condorcet winner.

Appendix G: Proof of Proposition 4

In a classical equilibrium, θ_u^{MV} is the most-preferred value of θ_u of parents whose type is (w, θ) with $\theta = \theta_d^u(w) < \theta_u^{MV}$, where the inequality stems from the fact that $\theta_u^u(w, \theta) > \theta \forall (\theta, w)$. These parents plan on paying the investment cost $p(\theta_u^{MV} - \theta)$ in order for their children to boost their test marks to the required level θ_u^{MV} –i.e., $\theta_m(\theta_u^{MV}, w) < \theta_d^u(w)$. It follows from this inequality that all parents with $\theta_m(\theta_u^{MV}, w) < \theta < \theta_d^u(w)$ also manage to enrol their children at university when the threshold is θ_u^{MV} .

Appendix H: Proof of Proposition 5

The value of $\theta_u^V(w, \theta)$ is determined by the following FOC (where we have made use of (1) and (4))

$$\delta\theta \frac{\partial \omega_L(H(\theta_u))}{\partial H} = u'(w(1 - t(H(\theta_u)))) \frac{w_i}{Ew} c_u. \quad (8)$$

The value of $\theta_u^u(w, \theta)$ in turn is determined by the FOC where equation (5) is set to zero, which we can rewrite as follows:

$$\left[-u'(c_p(\theta_u, \theta, w)) c_u \frac{w}{Ew} + \delta\theta \frac{\partial \omega_H(H(\theta_u))}{\partial H} \right] \frac{dH(\theta_u)}{d\theta_u} = u'(c_p(\theta_u, \theta, w)) p'(\theta_u - \theta), \quad (9)$$

where recall that $c_p(\theta_u, s, \theta, w) = w(1 - t(H(\theta_u))) - p(\theta_u - \theta)$.

In a classical equilibrium, showing that $\theta_u^u(w, \theta)$ decreases for all agents is sufficient to prove that θ_u^{MV} decreases (we need not bother with how $\tilde{\theta}(w)$ is affected since, under Assumption 1, variations in $\tilde{\theta}(w)$ only affect individuals with $\theta_u^u(w, \theta) < \theta_u^{MV}$.) Repeated applications of the implicit function theorem on (9) then proves the parts of statements (a) to (d) related to the impact on the majority chosen university size in a classical equilibrium.

In an ends-against-the-middle equilibrium, we have to compute the impact of variations on both $\theta_u^V(w, \theta)$ and $\theta_u^u(w, \theta)$, using each time the implicit function theorem on, respectively, (8) and (9). Hence the parts of results (a) to (d) pertaining to this type of equilibrium.

Finally, by Proposition 3, the type of equilibrium depends on the comparison between $\theta_u^V(w, \theta)$ (for either $w = \bar{w}$ (if $CRRA < 1$) or $w = \underline{w}$ if $CCRA > 1$) and $\theta_u^u(w, \theta)$ for some agents (w, θ) . We then look at all factors that increase $\theta_u^V(w, \theta)$, decrease $\theta_u^u(w, \theta)$ or both.

Application of the implicit function theorem on (8) yields result (b). Its application on (9) gives result (a). Application of the same theorem to both (8) and (9) yields (c). Application of the implicit function theorem on (8) and (9) shows that a lower ratio c_u/Ew simultaneously decreases $\theta_u^V(w, \underline{\theta})$ and $\theta_u^u(w, \theta)$, and so has an ambiguous impact on the type of equilibrium.

Appendix I: Functional forms for base case numerical example

We assume the following:

$$\begin{aligned} u(x) &= x^{1/2}, \\ p(\theta_u - \theta) &= 500(\theta_u - \theta)^2 \text{ if } \theta_u - \theta > 0 \text{ and } 0 \text{ otherwise,} \\ \omega_H &= 18.6, \\ \omega_L(H) &= 2 + 16.5H^{1/2}, \\ c_u &= 5, \\ \delta &= 0.5, \end{aligned}$$

together with θ uniformly distributed over $[\underline{\theta}, \bar{\theta}] = [0.1, 1]$ and w uniformly distributed over $[\underline{w}, \bar{w}] = [5, 20]$ (so that w and θ are independently distributed and not correlated).

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Figure 1. Single-peaked preferences

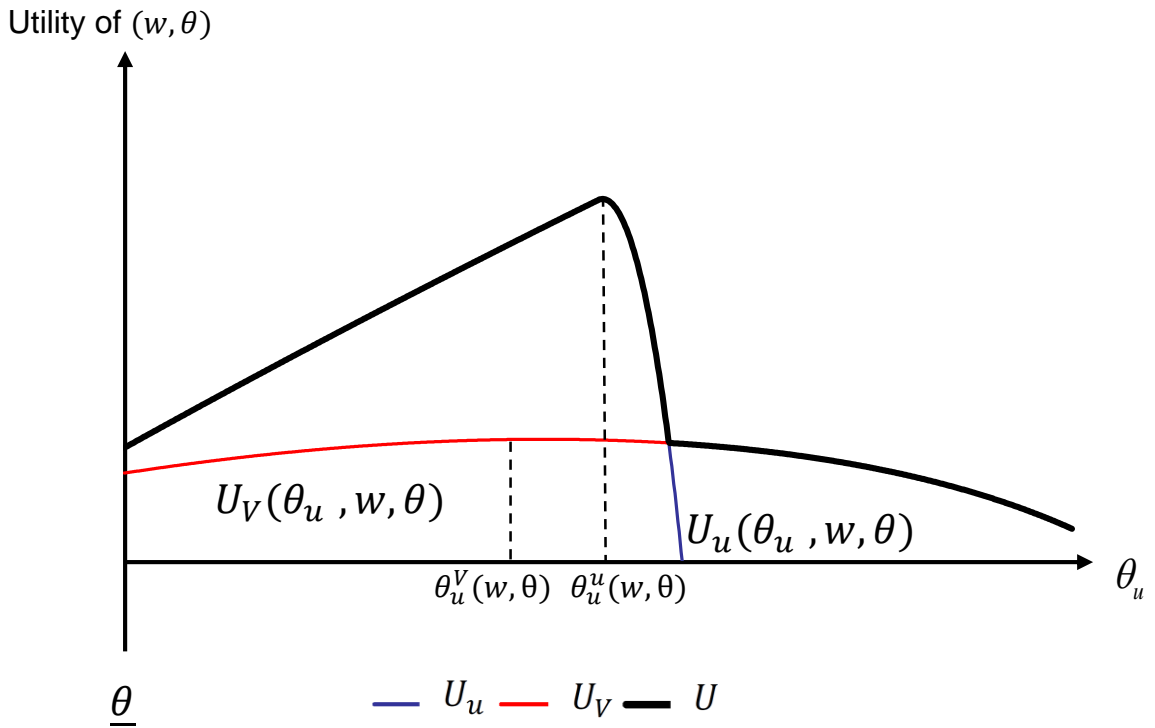


Figure 2. Non single-peaked preferences

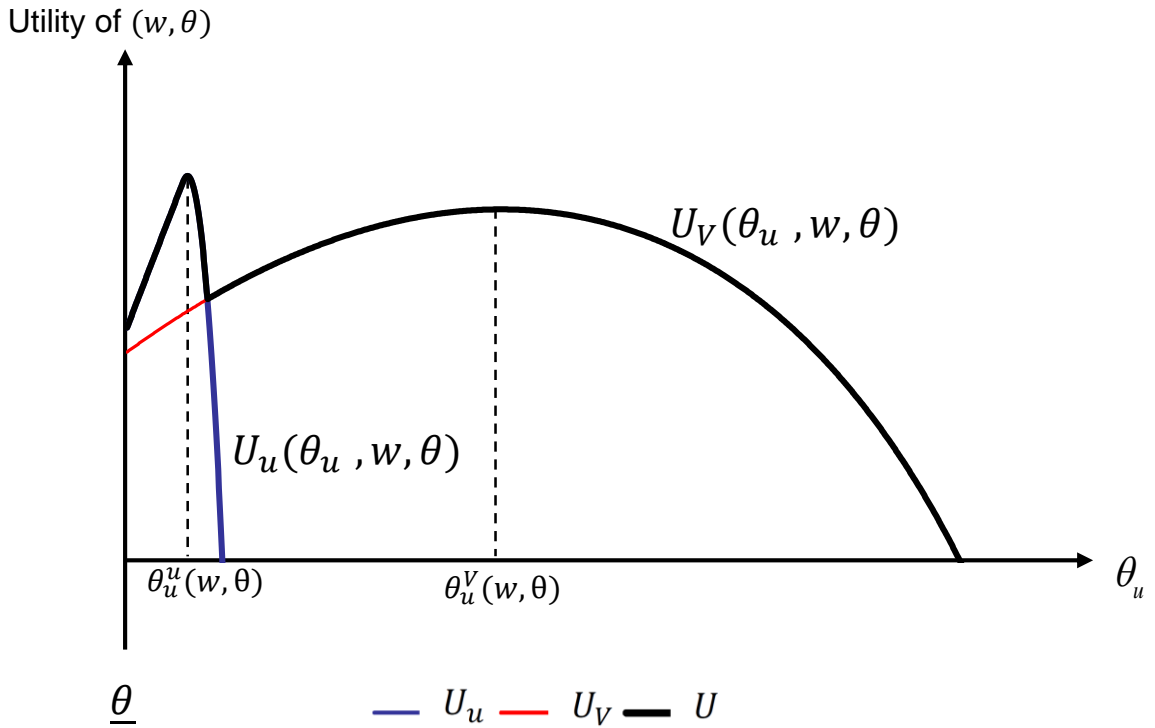


Figure 3. Classical Equilibrium

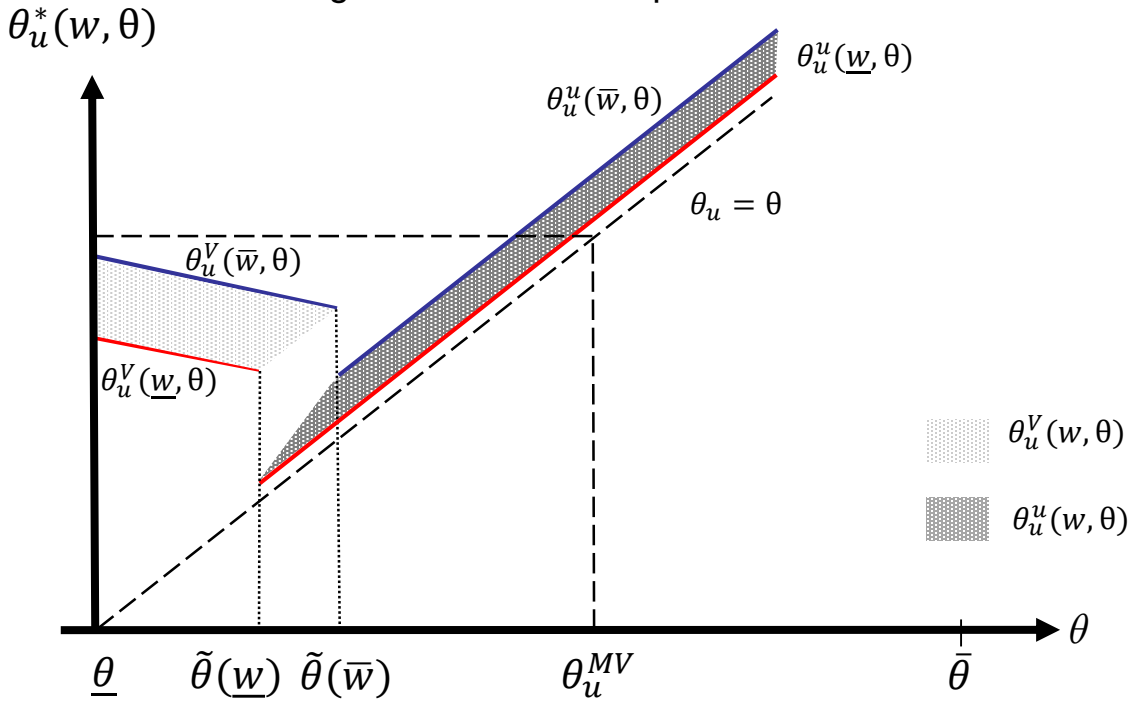


Figure 4. Ends-against-the-middle Equilibrium

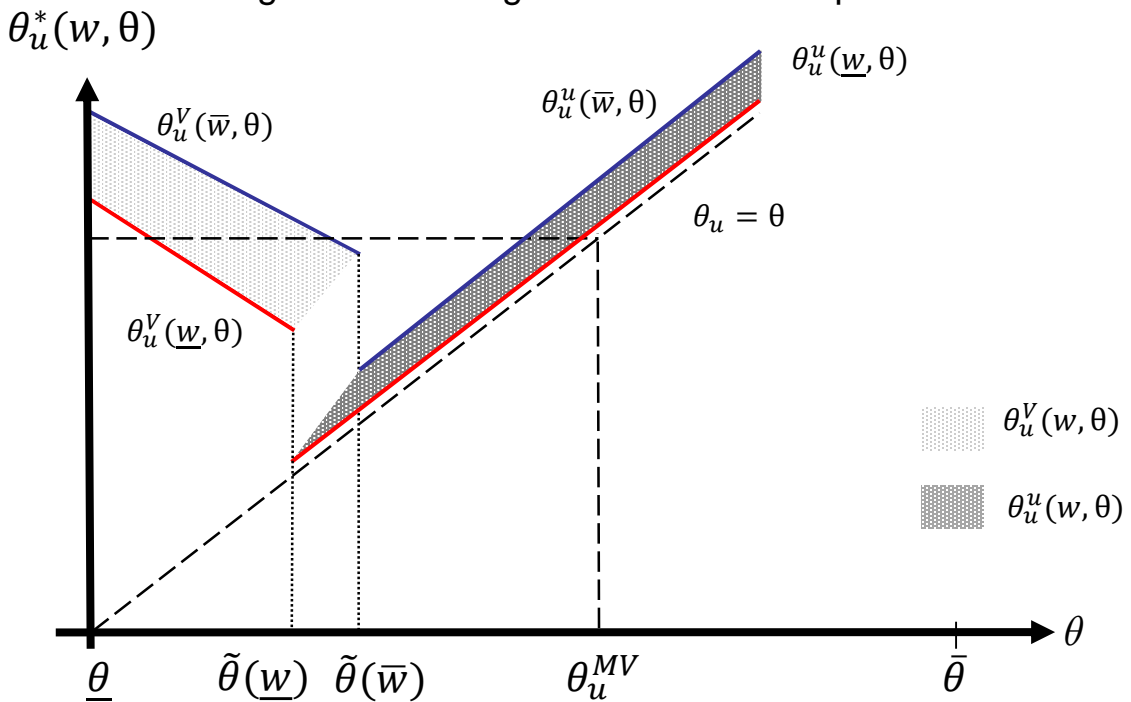
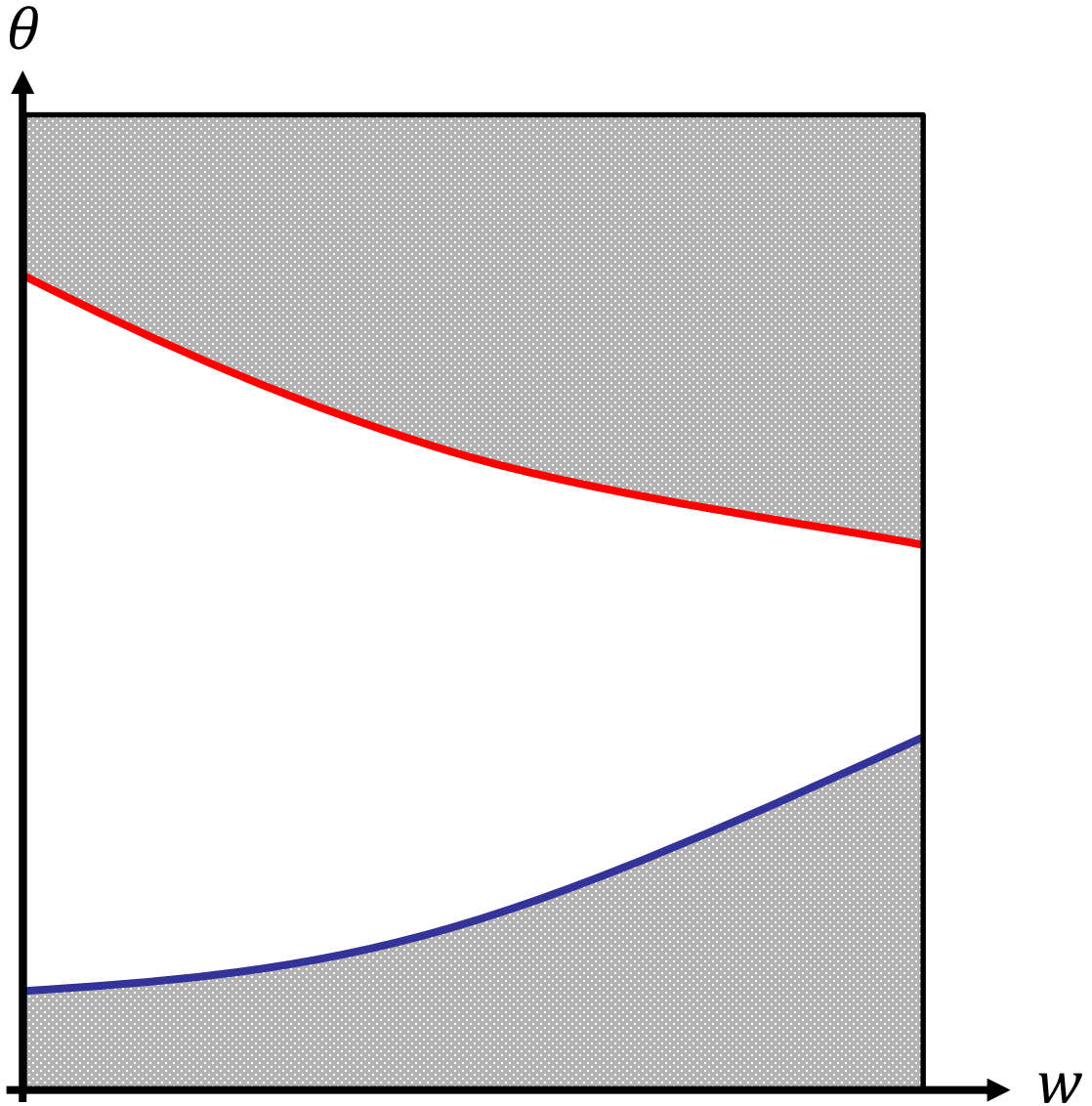






Figure 5. Decisive voters and coalitions –
Ends-against-the-middle Equilibrium



 Coalition of voters who prefer a smaller university

 Coalition of voters who prefer a larger university

Decisive voters:  $\theta_d^V(w, \theta_u^{MV})$  $\theta_d^u(w, \theta_u^{MV})$