Economic Growth and the Cultural Transmission of Attitudes towards Education

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Abstract

This analysis investigates path-dependencies in a growing economy where altruistic parents try to inculcate their children with a behavioural trait that is conducive to human capital formation. The initial stock of physical capital is critical because it shapes the population dynamics of behavioural traits which, in turn, impinge on the formation of physical and human capital. Despite the absence of a complementarity in the process of cultural instruction, the long-run equilibrium can also depend on the initial distribution of behavioural traits among the population, as long as the efficiency of the external elements of cultural transmission is sufficiently low for households with parents who did not adopt the human capital-promoting trait when they, themselves, were young.

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1 Introduction

The development and establishment of characteristics such as attitudes, values, and social norms are inherently dynamic, as they involve a process of cultural transmission (either intentional or through imitation) from parents to their offspring; from teachers to their students; from leaders and role models to their followers, and so on. Regardless of the actual process through which such characteristics become ingrained into the fabric of society, the existing empirical evidence leaves little doubt against the idea that they impinge on decisions that shape not only an individual's economic circumstances, but also the overall economic environment (Manz *et al.* 2006; Tabellini 2010; Algan and Cahuc 2010). However, there is also compelling evidence to support the view that characteristics such as attitudes and values are more or less symptomatic of the economic conditions that surround the people who carry them (Alesina and La Ferrara 2002; Bidner and Francois 2010; Butz and Yogeeswaran 2011). Moreover, there is an abundance of arguments and empirical studies that offer unequivocal credence to the view that the characteristics of intergenerational cultural transmission are particularly pertinent to the attitudes that affect the formation of human capital. Indeed, the empirical analysis of Patacchini and Zenou (2011) shows that all aspects of cultural transmission are relevant for educational attainment. Further evidence for the impact of parenting on educational outcomes is provided by Astone and McLanahan (1991) and Brown and Iyengar (2008), to name but a few, whereas other empirical studies suggest that such educational outcomes are also affected by role models (e.g., Nuygen 2008; Beaman *et al.* 2012; Macours and Vakis 2014). It should also be noted that Becker and Tomes (1986) cited 'motivation' as one of the channels through which parents can influence the children's human capital and, therefore, their future earnings.

The aforementioned evidence provides the backdrop to my analysis, given that the aim is to shed light on the interplay between the accumulation of (physical and human) capital and the process of intergenerational cultural transmission, along with the role of this interplay in creating an environment that is propitious to the emergence of multiple, path-dependent equilibria – circumstances where differences in current conditions can be amplified and ultimately generate differences in long-term economic performance. Such environments are not just theoretical curiosities. On the contrary, the plethora of

empirical analyses that find persistently large differences in the distribution of per capita GDP across countries (e.g., Durlauf and Johnson 1994; Quah 1997; Hansen 2000; Canova 2004) has always been viewed by researchers as a testament to the relevance and importance of economic models that generate multiple development regimes.[1] The framework I employ is an attempt to integrate a process of intergenerational cultural transmission *à la* Bisin and Verdier (2001) within a standard neoclassical growth model in the manner of Diamond (1965), i.e., a model where the process of physical capital accumulation is explicit and driven by optimal saving behaviour. The inclusion of physical capital formation is not a mere gimmick; it has a strong empirical grounding, given the wealth of evidence on the significant contribution of physical capital investment to the growth rates of developed and developing economies (e.g., De Long and Summers 1991; Bond *et al.* 2010). Contrary to some existing analyses, but similarly to Sáez-Martí and Sjögren (2008) and Patacchini and Zenou (2011), I assume that parental decisions towards the cultural instruction of their offspring are motivated from true, rather that paternalistic, altruism.[2] Particularly, parents care about their children's future prospects, thus they try to instil human capital-promoting attitudes and behavioural traits, irrespective of whether they, themselves, demonstrated similar attitudes when young. I also adopt a flexible parameterisation, according to which the positive externalities that permeate the transmission mechanism are less pronounced for individuals whose parents did not assume education-inducing attitudes when they were young. This is meant to capture the idea that, due to self-selection, the parents' social and geographic environment, as well as their interests and activities, make it less likely that their children will find suitable role models to motivate them into the adoption of human capital-promoting attitudes.[3] Finally, since my purpose is to examine novel sources of path-dependency, the parental effort function is designed to incorporate an external effect that rules out the type of cultural complementarities or substitutabilities that the existing literature has already identified as sources of multiple equilibria, or lack thereof (Bisin and Verdier 2008).

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[1] See Azariadis and Drazen (1990), Galor and Zeira (1993), and Ceroni (2001) among others.

[2] Paternalistic altruism applies when parents make a subjective evaluation on what contributes to their children's well-being, based on their own preferences.

[3] It should be noted that this idea is related to the notion of 'neighbourhood quality', for which Patacchimi and Zenou (2011) offer empirical support.

The results reveal that, when multiple equilibria exist, the initial stock of physical capital is always critical in determining both the economic and the cultural outcomes of the model's long-run equilibrium. Under different contexts, this possibility has been considered by previous analyses (e.g., Chakraborty 2004), but it has eluded the attention of previous research on the interplay between economic and cultural dynamics. In my model, the two-way causal effects that permeate the dynamics of capital accumulation and cultural transmission prove vital in generating this result. These two-way causal effects are also key in generating the possibility that the initial distribution of traits matters for both economic and cultural prospects in the long-run. Note, however, that this outcome materialises only when the efficiency of the external elements of cultural transmission is sufficiently low for households with parents who did not adopt the human capital-promoting trait when they, themselves, were young. This result is another novelty of the model, as it brings forth a new explanation on the conditions that perpetuate and amplify cultural differences in the long-run – an outcome with major implications for the economic prospects of countries with different cultural characteristics.

Before proceeding to my model's formal exposition, it is useful to consider its framework, its results, and its implications within the context of the existing literature. The analysis of intergenerational cultural transmission within economic frameworks gained prominence through the influential work of Bisin and Verdier (2001), who adopted ideas from Cavalli -Sforza and Feldman (1981), and Boyd and Richerson (1985), to analyse the population dynamics of preference traits in an optimising framework. They showed that multiple equilibria, where the initial distribution of traits matters for long-term outcomes, occur under a case of cultural complementarity, i.e., when parental efforts towards the cultural instruction of their children are increasing in the share of the population who carry the trait that parents seek to diffuse. On the contrary, under cultural substitutability – the scenario where parental efforts are decreasing in the share of the population who carry the same behavioural trait – they obtained a unique, interior long-run equilibrium, no matter what the initial distribution of traits is. Since then, other analyses have shown alternative environments under which the dynamics of cultural transmission may result in path-dependent outcomes (e.g., Hauk and Sáez-Martí 2002;

Sáez-Martí and Sjögren 2008).[4] Note, however, that none of these analyses have incorporated an explicit process of economic growth, or its interplay with the process of cultural transmission, as I do in my analysis. In fact, the number of studies that have done so is rather limited, despite the wealth of empirical evidence and intuitive arguments that corroborate this approach.[5] Klasing (2014) demonstrates how the cultural transmission of attitudes towards risk can impinge on the dynamics of occupational choice, and determine long-run growth through its effect on the number of individuals who opt for high-return, but riskier, entrepreneurial projects. In a similar vein, Doepke and Zilibotti (2014) show that the endogenous transmission of attitudes towards risk and patience can be a source of multiple, path-dependent growth rates. This outcome stems from the two-way causal effects between the fraction of entrepreneurs in the economy, and the magnitude of the mechanism that instils risk tolerance and patience in younger generations – attributes that are favourable to innovative, growth-promoting activities.[6] In Chakraborty *et al.* (2016), culturally-induced stagnation (a zero growth equilibrium where entrepreneurs do not adopt more advanced technologies) can be escaped by means of policies that may induce either a shift in aggregate productivity, or lower transferability of existing human capital across technologies. When this happens, long-run growth is not sensitive to cultural characteristics.

In this paper, I draw on themes from the aforementioned literature. Nevertheless, I modify and extend them with the purpose of generating novel mechanisms and results, hence contributing to our further understanding of the circumstances under which the interplay between cultural and economic dynamics is conducive to path-dependencies. To facilitate the exposition of these mechanisms and results, the remainder of the analysis is organised as follows: In Section 2, I discuss the characteristics of the model,

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[4] See also Bisin and Verdier (2008) and the references therein.

[5] Palivos (2001) and Kim and Lee (2015) have analysed growth models that incorporate social/cultural characteristics. Nevertheless, they do not consider an explicit mechanism for the intergenerational transmission of these characteristics.

[6] Doepke and Zilibotti (2014) consider parents who display true, rather than paternalistic, altruism. Another deviation of their cultural transmission mechanism is that they do not consider a process of oblique transmission. Using a similar cultural transmission set-up, applied to attitudes towards patience and leisure, Doepke and Zilibotti (2008) offer a mechanism that explains the socioeconomic transformation that followed the Industrial Revolution in Britain.

while in Section 3 I derive its equilibrium. Section 4 provides the characterisation of the economic/cultural dynamics, and Section 5 concludes.

2 The Economy

I consider an infinite horizon economy in which time is measured in discrete periods, indexed by $t = 0,1,2,\ldots$. This economy is populated by a sequence of overlapping generations of individuals who live for three periods. Henceforth, the first period of an individual's lifetime will be referred to as *youth*, whereas the two subsequent periods of maturity will be referred to as *middle age* (the second period) and *old age* (the third and final period of her lifespan). In terms of demographics, each adult gives birth to one young individual during middle age. Thus, the population mass of each age cohort is constant over time. Without loss of generality, each age cohort's population mass is also normalised to 1.

Let us consider a person who is born in period t. When young, she is endowed with a unit of time and decides how much of it will be allocated to activities (e.g., educational attainment and studying) that will facilitate the formation of her human capital. Such activities are costly however, in the sense that they entail a direct loss of utility due to the effort devoted to them. This loss of utility need not be solely associated with physical strain; it also captures the mental and psychological strain of educational activities, e.g., stress, pressure, and the frustration emanating from some loss of leisure activities. The utility cost is not uniform across the whole population of young individuals though. Instead, there are two personality traits, indexed by $i = \{x, v\}$, that distinguish individuals according to the different utility costs they experience, for given levels of educational effort. Particularly, an individual who devotes $e_{i,t}$ units of time for human capital-promoting activities, faces a utility cost

$$\Psi_i(e_{i,t}) = \frac{\psi_i e_{i,t}}{1 - e_{i,t}}, \quad \psi_i > 0, \tag{1}$$

where

$$\psi_i = \begin{cases} \underline{\psi} & \text{if} \quad i = x \\ \overline{\psi} & \text{if} \quad i = v \end{cases}, \quad \overline{\psi} > \underline{\psi}. \tag{2}$$

In essence, we can think of a Type-v individual as one who, in comparison to a Type-x individual, attaches more weight to leisure activities when she is young.

Note that each young individual's trait is not exogenous. On the contrary, it will be determined endogenously through a process of intergenerational cultural transmission. This process will be formalised at a later point of the analysis. For now, it is instructive to specify the process through which educational effort is transformed into units of human capital. Following others (e.g., Bénabou 1996; Glomm 1997; Blankenau 2005) I will assume that educational activities by the young are complemented by public investment in education.[7] Particularly, every period the government imposes a tax on financial intermediaries, i.e., the firms that process the transformation of savings into units of physical capital. The tax takes the form of a fixed fraction of the physical capital stock that financial intermediaries produce. This capital is used by the government as an input in the education technology. Formally, for a young Type-i person, human capital will be determined according to

$$h_{i,t+1} = B e_{i,t}^{\beta} k_{h,t}, \quad B > 0, \ 0 < \beta \leq 1, \tag{3}$$

where $k_{h,t}$ is the amount of capital (per person) that the government levies from financial intermediaries in the form of taxation. Note that the presence of physical capital in the education technology follows Rebelo (1991), among others, and is meant to capture the contribution of educational facilities, equipment, labs etc. towards the formation of human capital.

During the first period of maturity, the middle-aged individual supplies her human capital (i.e., her efficient labour) to output-producing firms in exchange for the competitive wage rate w_{t+1}. She decides how much of her labour income to consume and how much to save in order to finance her consumption expenditures during old age – a period during which she does not have any other source of income because she does not possess the ability to work when old. Using $c_{i,t+1}$ to denote middle age consumption, and $d_{i,t+2}$ to denote old age consumption of a Type-i individual who was born in period t, her budget constraints are given by

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[7] According to World Bank data, public spending in education is a significant component of national accounts, as it absorbs roughly 4-4.5% of real GDP in low- and middle-income countries, and more than 5% of real GDP in high-income countries. See http://data.worldbank.org/.

$$c_{i,t+1} = w_{t+1}h_{i,t+1} - s_{i,t+1}, \tag{4}$$

and

$$d_{i,t+2} = (1 + r_{t+2})s_{i,t+1}, \tag{5}$$

respectively. Note that $s_{i,t+1}$ denotes the saving of a Type-i individual while r_{t+2} is the market interest rate.[8]

In addition to the choice regarding her intertemporal consumption profile, the middle-aged individual is also endowed with a unit of time and chooses how much of it will be dedicated to inculcate her offspring with attitudes that are conducive to the young individual's willingness to invest in human capital formation. This is done through a process of cultural instruction, involving socialisation and nurturing, whereby parents may seek to instil aspirations, habits, values and norms – in general, the characteristics that can affect their children's perceptions, hence inducing them to devote more effort towards educational activities. In other words, each parent's effort will be directed towards attempts to ingrain the x trait into her offspring, irrespective of whether she, herself, grew up as a Type-x or a Type-v individual when young. As we shall see later, the parent's incentive to engage in this type of direct (vertical) cultural transmission stems from the idea that a middle-aged person is altruistic, but not in a paternalistic manner. Each parent cares about her child's human capital and, therefore, her objective is to have her offspring instilled with the behavioural trait that induces more willingness to engage in educational activities.[9] In this respect, my framework deviates from the notion of paternalistic altruism that is employed by Bisin and Verdier (2001), and adopts an idea that is conceptually closer to the approach of Becker (1976).[10] Despite the fact that parents of both types try to encourage the same behavioural trait, the process of intergenerational transmission differs between households of different types. The sources of such differences are twofold and relate to the external aspects that

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[8] The lack of direct consumption expenditures by young individuals is an innocuous assumption. For example, their consumption could be embedded to their middle-aged parents' overall consumption expenditures.

[9] This is meant to capture the idea that parents care about the future prospects (e.g., income; social status) of their children.

[10] The adoption of this type of (impure) altruism in OLG models of economic growth is quite common in the literature. See Galor and Mountford (2008), and Aksan and Chakraborty (2014) among others.

permeate the cultural transmission process. The detailed characteristics of this process are discussed in the following section.

2.1 The Process of Intergenerational Cultural Transmission

The framework under which young individuals adopt either the v or the x traits draws on Bisin and Verdier (2001), albeit suitably adapted to accommodate the idea that, irrespective of their own type, parents care about the human capital of their offspring, therefore they strive to instil in their children the behavioural characteristics associated with the x trait. Let us consider a Type-i parent. By devoting $\gamma_{i,t+1} \in [0,1]$ units of time, she can be successful in having the x trait instilled into her offspring, with probability $z(\gamma_{i,t+1}) \in [0,1]$ where $z' > 0$. Doing so, however, entails a loss of utility (due to her effort towards socialisation and nurturing), captured by

$$\Phi_i(\gamma_{i,t+1}) = \frac{f_{i,t+1}\gamma_{i,t+1}}{1 - \gamma_{i,t+1}}. \tag{6}$$

The term $f_{i,t+1}$ includes both the innate and the external characteristics that determine how costly (in terms of utility) is a parent's effort in trying to improve the chances that her offspring adopts the x trait. This is a source of differentiation between Type-x and Type-v parents. To formalise this idea, henceforth I will be using $\eta_{t+1} \in [0,1]$ to denote the fraction of middle-aged parents who adopted the x trait when they were young. Given this, it is assumed that

$$f_{i,t+1} = [1 - \zeta_i(\eta_{t+1})]\varphi_i = \begin{cases} [1 - \zeta_x(\eta_{t+1})]\underline{\varphi} & \text{if} \quad i = x \\ [1 - \zeta_v(\eta_{t+1})]\overline{\varphi} & \text{if} \quad i = v \end{cases}, \tag{7}$$

where $\overline{\varphi} > \underline{\varphi} > 0$, $0 \leq \zeta_v(\eta_{t+1}) < \zeta_x(\eta_{t+1}) \leq 1$, and $\zeta_x' > \zeta_v' > 0$. The assumption $\overline{\varphi} > \underline{\varphi}$ captures the Type-x parent's innate ability in requiring less effort, compared to a Type-v parent, to achieve a given probability of success in transmitting the x trait to her offspring – an outcome attributed to her own experiences and abilities. The assumption $\zeta_x', \zeta_v' > 0$ captures an intragenerational externality, according to which a larger number of Type-x individuals results in an expanded set of experiences relating to the process that induced them to adopt the x trait when they, themselves, were young. These experiences are shared with other parents (e.g., through social interactions), thus

generating a positive learning externality that can facilitate all parents in their efforts to instil the behavioural characteristics associated with the x trait, in their own children. The assumptions $0 \leq \zeta_v(\eta_{t+1}) < \zeta_x(\eta_{t+1}) \leq 1$ and $\zeta_x' > \zeta_v' > 0$ capture the idea that the benefit from this externality is less pronounced for Type-v parents. As a means of justifying these assumptions, I appeal to the idea that, due to self-selection, the social interactions of Type-v parents with Type-x ones are limited, thus mitigating the extent to which they can benefit from the aforementioned sharing of experiences.

Now let us consider what happens in the event that, despite her efforts, the parent is not successful in inculcating her offspring with the desired personality trait directly. In this case, the young individual's type will be determined through a process of oblique transmission, whereby she will adopt the lifestyle choices of a role model who she picks out of the population of middle-aged individuals. Specifically, she will adopt one of the two traits with a probability that is increasing in the share of the middle-aged population who possess the same trait. Formally, $n_i(\eta_{t+1}) \in [0,1]$ ($n_i' > 0$) denotes the probability that a young individual who grows up with a Type-i parent, adopts the x trait through the process of oblique transmission. Once more, this external effect is a source of differentiation between Type-x and Type-v households, in the sense that $n_x(\eta_{t+1}) > n_v(\eta_{t+1})$ and $n_x' > n_v'$. In order to justify this assumption, I appeal once more to the idea that Type-x parents are more likely to interact with other middle-aged individuals of the same type. Hence, their offspring will have more opportunities, compared to young individuals who grow up in households with Type-v parents, for aspiring to role models who will induce them to adopt the x trait. Since the sources of the two external effects that permeate the process of cultural transmission are similar, henceforth I will set $\zeta_i(\eta_{t+1}) = n_i(\eta_{t+1})$ for $i = \{x, v\}$.

It should be noted that, apart from its conceptual relevance, the previous assumption serves another, more technical purpose. As we shall see during the model's solution, this assumption will render the optimal choices for $\gamma_{i,t+1}$ independent of the distribution of behavioural traits among the population (i.e., independent of η_{t+1}). Hence, it will allow me to show that multiple, path-dependent equilibria, for which the initial distribution of

traits is critical, can emerge despite the absence of cultural complementarities, i.e., circumstances where the equilibrium solution for $\gamma_{i,t+1}$ is increasing in η_{t+1}.

In what follows, I will be using $\pi_{i,t+1}$ to denote the overall probability that a young individual, growing up with a Type-i parent, adopts the behavioural attributes of a Type-x person. Given that this probability must incorporate all the elements (direct and oblique) of cultural transmission, it follows that

$$\pi_{i,t+1} = z(\gamma_{i,t+1}) + [1 - z(\gamma_{i,t+1})]n_i(\eta_{t+1}). \tag{8}$$

By the law of large numbers, the share of the population who will grow up to maturity having adopted the x trait is given by

$$\eta_{t+2} = \pi_{x,t+1}\eta_{t+1} + \pi_{v,t+1}(1 - \eta_{t+1}). \tag{9}$$

2.2 Production Technology and Preferences

In the previous part of the analysis, I indicated that individuals enjoy utility from the consumption of the economy's homogeneous good, during the two periods of their maturity, as well as from the human capital of their offspring. They also face the utility costs associated with their efforts in forming their own human capital (when young) and nurturing their children towards the adoption of the x trait (when middle-aged). These characteristics can be summarised by the following lifetime utility function of a Type-i individual who is born in period t:

$$U_{i,t} = -\Psi_i(e_{i,t}) + \frac{\ln(c_{i,t+1}) + \pi_{i,t+1}h_{x,t+2}^\sigma + (1 - \pi_{i,t+1})h_{v,t+2}^\sigma - \Phi_i(\gamma_{i,t+1})}{1+\rho} + \frac{\ln(d_{i,t+2})}{(1+\rho)^2}, \tag{10}$$

where $\rho > 0$ is the rate of time preference and $\sigma > 0$ is the elasticity of a parent's utility with respect to her child's human capital.

Recall that individuals who are born in period t will be employed when middle-aged, by firms that produce and supply the economy's homogeneous good. These firms are perfectly competitive and their total mass is normalised to 1. They employ units of physical capital, purchased by financial intermediaries, and human capital, supplied by middle-aged individuals, in order to produce y_{t+1} units of output under a neoclassical technology[11]

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[11] I assume that physical capital depreciates completely during a period.

$$y_{t+1} = Ak_{y,t+1}^a (\Omega_{t+1} H_{t+1})^{1-a}, \quad A > 0, \ 0 < a < 1, \tag{11}$$

where $k_{y,t+1}$ is the amount of the economy's capital employed by private sector firms, H_{t+1} is the stock of human capital (efficient labour) used in production, and Ω_{t+1} is another factor affecting labour productivity (in addition to human capital). I am going to link this factor with workers' health status by considering a process whereby an individual's health stock is formed during youth – a health stock which, in turn, partially affects her labour productivity when middle-aged. Recent evidence has shown that the contribution of pollution on morbidity, and therefore its adverse effect on health, is among the significant factors in the determination of labour productivity (e.g., Hanna and Oliva 2015). This empirical fact will be incorporated in my model by assuming that Ω_{t+1} is an inverse function of the flow of pollution (denoted M_t) during each worker's youth. Furthermore, I follow Kijima *et al.* (2010) in assuming that the pollution flow is a negative externality, proportional to the use of physical capital in various economic activities (i.e., output production and education). Denoting the capital stock in period t by $k_t = k_{y,t} + k_{h,t}$, it follows that $M_t = mk_t$ where $m > 0$ a parameter that denotes emission intensity. Given the above, the adverse effect of pollution on labour productivity is specified through the following functional form:[12]

$$\Omega_{t+1} = \frac{1}{mk_t}. \tag{12}$$

3 Equilibrium

The maximisation problem of a Type-i individual will be solved by backward induction. I shall begin by considering the optimal choices for consumption and saving, as well the optimal choice regarding her child's cultural instruction, made during the individual's middle age. Subsequently, I will take account of these choices when

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[12] It should be noted that the presence of Ω_{t+1}, and its specification in Eq. (12), is nothing else other than a technical device to simplify the analysis of the model's equilibrium dynamics. Ultimately, this device allows me to avoid a situation where, due to the presence of the public input in the formation of human capital (see Eq. 3), the dynamics of capital accumulation would be characterised by a second-order difference equation. Nevertheless, absconding from this scenario will not affect the qualitative nature of the results because of the positive monotonicity of physical capital dynamics.

determining the optimal educational effort – a choice made when the same individual is young.

Given the above, the middle-aged person will choose $s_{i,t+1}$ and $\gamma_{i,t+1}$ in order to maximise her utility function in (10) subject to Eq. (4)-(8), while taking w_{t+1}, r_{t+2} and η_{t+1} as given. The first order conditions from this problem are given by

$$\frac{1}{w_{t+1}h_{i,t+1} - s^*_{i,t+1}} = \frac{1}{(1+\rho)s^*_{i,t+1}}, \tag{13}$$

and

$$z'(\gamma^*_{i,t+1})(h^\sigma_{x,t+2} - h^\sigma_{v,t+2}) = \frac{\varphi_i}{(1-\gamma^*_{i,t+1})^2}. \tag{14}$$

The condition in Eq. (13) is the familiar Euler equation: At the optimum, the individual should be indifferent about changes in the intertemporal consumption profile that are realised through changes in saving behaviour. The condition in Eq. (14) reveals that, at the optimum, the middle-aged parent's choice of effort in inducing her offspring to adopt the desired personality trait, is the one that equates the marginal benefits and marginal costs (in terms of utility) from the process of cultural instruction. Note that the benefit is proportional to the human capital increment resulting from the young individual's adoption of the x trait, whereas the cost depends on the innate characteristics that distinguish Type-x and Type-v parents, in terms of the effort they need to devote in order to achieve a given probability of success in inculcating their children with the behavioural attributes attached to a Type-x individual.[13]

The Euler equation in (13) can be solved to yield the optimal saving function

$$s^*_{i,t+1} = \frac{w_{t+1}h_{i,t+1}}{2+\rho}, \tag{15}$$

whereas mild conditions can be applied to the function $z(\gamma_{i,t+1})$ in order to ensure the uniqueness of the solution that can be derived from Eq. (14). Substituting (15) in (4) and (5), and the resulting demand functions, together with $\gamma^*_{i,t+1}$, in the lifetime utility function of Eq. (10), yields

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[13] As I argued previously, the assumption $\zeta_i(\eta_{t+1}) = n_i(\eta_{t+1})$ has eliminated cultural externalities from the determination of $\gamma^*_{i,t+1}$. Thus, it allows me to examine the possibility of path-dependencies, even in the absence of cultural complementarities (i.e., cases where $\gamma^*_{i,t+1}$ would be an increasing function of η_{t+1}).

$$U_{i,t} = -\Psi_i(e_{i,t}) + p\ln(h_{i,t+1}) + V_i^*, \tag{16}$$

where $p \equiv \dfrac{1}{1+\rho}\left(1 + \dfrac{1}{1+\rho}\right)$ is a composite term, and

$$V_i^* = p\ln\left(\frac{w_{t+1}}{2+\rho}\right) + \frac{\ln(1+\rho)}{1+\rho} + \frac{\pi_{i,t+1}^* h_{x,t+2}^\sigma + (1-\pi_{i,t+1}^*)h_{v,t+2}^\sigma - \Phi_i(\gamma_{i,t+1}^*)}{1+\rho} + \frac{\ln(1+r_{t+2})}{(1+\rho)^2}, \tag{17}$$

where $\pi_{i,t+1}^* = z(\gamma_{i,t+1}^*) + [1 - z(\gamma_{i,t+1}^*)]n_i(\eta_{t+1})$. Given these, the young individual chooses her optimal effort towards educational activities so as to maximise the utility function in (16) subject to (1) and (3), while taking $k_{h,t}$ as given. The first order condition associated with this problem is given by

$$\frac{p\beta}{e_{i,t}^*} = \frac{\psi_i}{(1-e_{i,t}^*)^2}. \tag{18}$$

At the optimum, the individual balances the marginal benefit and the marginal cost associated with her educational activities. The former is related to the increase of consumption intertemporally, due to higher disposable income during maturity – an effect that (i) is more pronounced if the education technology is more responsive to an individual's effort, and (ii) appropriately discounted to account for the individual's underlying impatience. The utility cost is determined by the innate characteristics that distinguish young individuals who have adopted either the x or the v trait.

The expression in (18) can be used to establish the existence of a unique, time-invariant equilibrium $e_i^* \in (0,1)$. To see this, define $\varepsilon(e_{i,t}) = \dfrac{p\beta}{e_{i,t}} - \dfrac{\psi_i}{(1-e_{i,t})^2}$ and check that

$\lim\limits_{e_{i,t}\to 0}\varepsilon(e_{i,t}) = +\infty$, $\lim\limits_{e_{i,t}\to 1}\varepsilon(e_{i,t}) = -\infty$ and $\varepsilon' < 0$. Furthermore, it is $\dfrac{\partial\varepsilon(\bullet)}{\partial\psi_i} < 0$ – a result that, combined with (2) and $\varepsilon' < 0$, allows us to infer that

$$e_i^* = \begin{cases} \overline{e} & \text{if } i = x \\ \underline{e} & \text{if } i = v \end{cases}, \text{ such that } \overline{e} > \underline{e}, \tag{19}$$

where $\overline{e}, \underline{e} \in (0,1)$ are composite parameter terms. Substituting (19) in Eq. (3), it follows that

$$h_{i,t+1} = \begin{cases} h_{x,t+1} = B\overline{e}^\beta k_{h,t} & \text{if } i = x \\ h_{v,t+1} = B\underline{e}^\beta k_{h,t} & \text{if } i = v \end{cases}, \tag{20}$$

and

$$h_{x,t+1} - h_{v,t+1} = Bk_{h,t}(\overline{e}^{\beta} - \underline{e}^{\beta}) > 0 , \qquad (21)$$

hence verifying the original conjecture of the analysis, as this is summarised in

Proposition 1. *Type-x individuals devote more effort towards their educational activities when young, thus their human capital when middle-aged is higher compared to the human capital of Type-v individuals.*

Proof. See the results in (19) and (21). ∎

As expected, the intuition rests with the idea that, due to the characteristics associated with the x trait, mainly that educational activities are less strenuous and, therefore, costly in terms of utility loss, young individuals of this type are willing to provide a greater amount of time, compared to Type-v individuals, in activities that facilitate the formation of human capital.

The accumulation of physical capital takes place as follows. In every period, middle-aged individuals deposit their savings to financial intermediaries. These are perfectly competitive firms whose role is to collect all the saving deposits (denoted S_{t+1}) and use them as inputs in a technology that transforms units of output into units of physical capital, according to

$$k_{t+2} = k_{y,t+2} + k_{h,t+2} = S_{t+1} . \qquad (22)$$

Using Eq. (15), aggregate saving is

$$S_{t+1} = \eta_{t+1} s_{x,t+1}^* + (1 - \eta_{t+1}) s_{v,t+1}^* = \frac{w_{t+1}[\eta_{t+1} h_{x,t+1} + (1 - \eta_{t+1}) h_{v,t+1}]}{2 + \rho} . \qquad (23)$$

i.e., the sum of the saving deposits of both Type-x and Type-v individuals. Naturally, the equilibrium wage rate corresponds to the marginal product of effective labour in the production technology. That is,

$$w_{t+1} = (1 - a)Ak_{y,t+1}^a \Omega_{t+1}^{1-a} H_{t+1}^{-a} . \qquad (24)$$

Let us assume that, to provide the public input towards the formation of human capital, in every period t the government levies (in the form of taxation) a fraction $\tau \in (0,1)$ of the capital stock produced by financial intermediaries. It follows that

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$$k_{h,t} = \tau k_t \text{ and } k_{y,t} = (1 - \tau)k_t . \tag{25}$$

Taking account of the share of each group (Type-x and Type-v) in the total population of middle-aged individuals, the stock of human capital is $H_{t+1} = \eta_{t+1} h_{x,t+1} + (1 - \eta_{t+1}) h_{v,t+1}$. Substituting this, together with (12), (20), (23), (24) and (25), in Eq. (22) yields

$$k_{t+2} = g k_{t+1}^a [\eta_{t+1}(\overline{e}^\beta - \underline{e}^\beta) + \underline{e}^\beta]^{1-a} , \tag{26}$$

where $g \equiv \dfrac{(1-a)A(1-\tau)^a}{2+\rho} \left(\dfrac{B\tau}{m} \right)^{1-a}$ is a composite parameter term. The result in Eq. (26)

allows us to link the distribution of behavioural traits among the population, with the process of capital formation. This is done through

Proposition 2. *A higher population share of Type-x individuals is favourable to the accumulation of physical capital.*

Proof. Using Eq. (26), it is straightforward to establish that $\dfrac{\partial k_{t+2}}{\partial \eta_{t+1}} > 0$. ∎

In terms of intuition, a larger number of Type-x individuals means that the aggregate stock of human capital will be higher, simply because those are the people who were willing to dedicate more time towards their education when they were young. Consequently, aggregate productivity and, therefore, income will be higher as well. Since individuals save a fraction of their income during the first period of maturity, aggregate saving and, therefore, physical capital investment will be positively related to η_{t+1}.

Now, let us derive and analyse the equilibrium solution regarding a parent's effort to inculcate her offspring with the Type-x attributes. To do this, I follow others (e.g., Bisin and Verdier 2001; Hauk and Saez-Marti 2002) in employing the following functional form for $z(\gamma_{i,t+1})$:

$$z(\gamma_{i,t+1}) = \gamma_{i,t+1} . \tag{27}$$

Substituting (22), (25) and (27) in Eq. (14) yields

$$(B\tau k_{t+1})^\sigma (\overline{e}^{\beta\sigma} - \underline{e}^{\beta\sigma}) = \frac{\varphi_i}{(1 - \gamma_{i,t+1}^*)^2} . \tag{28}$$

Defining the composite term $\mu_i \equiv \sqrt{\dfrac{\varphi_i}{(B\tau)^\sigma (\overline{e}^{\beta\sigma} - \underline{e}^{\beta\sigma})}}$, and taking account of $\overline{\varphi} > \underline{\varphi}$, it

follows that

$$\mu_i = \begin{cases} \underline{\mu} & \text{if} \quad i = x \\ \overline{\mu} & \text{if} \quad i = v \end{cases} \text{, such that } \overline{\mu} > \underline{\mu}, \tag{29}$$

where $\overline{\mu}, \underline{\mu} > 0$ are composite parameter terms. Given these, the solution of Eq. (28)

yields

$$\gamma_{i,t+1}^* = \max\left\{0, 1 - \frac{\mu_i}{k_{t+1}^\omega}\right\} \equiv \gamma_i(k_{t+1}), \tag{30}$$

where $\omega \equiv \dfrac{\sigma}{2}$. The result in Eq. (30) allows us to derive implications on how economic

resources, captured by the stock of physical capital, impinge on parental decisions

regarding the process of cultural instruction. These implications are summarised in

Lemma 1. *There exist $\underline{k} > 0$ and $\overline{k} > 0$, such that $\underline{k} < \overline{k}$ and*

   i. $\gamma_v(k_{t+1}) = 0$, $\gamma_x(k_{t+1}) = 0$ *if $k_{t+1} \leq \underline{k}$;*

   ii. $\gamma_v(k_{t+1}) = 0$, $\gamma_x(k_{t+1}) = 1 - \dfrac{\underline{\mu}}{k_{t+1}^\omega} > 0$ *if $\underline{k} < k_{t+1} \leq \overline{k}$;*

   iii. $\gamma_v(k_{t+1}) = 1 - \dfrac{\overline{\mu}}{k_{t+1}^\omega} > 0$, $\gamma_x(k_{t+1}) = 1 - \dfrac{\underline{\mu}}{k_{t+1}^\omega} > 0$ *if $k_{t+1} > \overline{k}$.*

Furthermore, it is $\gamma_x(k_{t+1}) \geq \gamma_v(k_{t+1})$ and $\gamma_v', \gamma_x' \geq 0$.

Proof. Defining $\underline{k} \equiv \underline{\mu}^{1/\omega}$ and $\overline{k} \equiv \overline{\mu}^{1/\omega}$, all these results follow from the expressions in (29)

and (30). ∎

Irrespective of her type, the parent's optimal choice, which affects the probability of

her offspring adopting the x trait, is increasing in the economy's stock of physical

capital. This is because the capital stock determines the productivity of the human

capital technology, through the process of public investment in education. Parents who

care about their children's human capital, will be more willing to use the process of

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cultural instruction – thus inducing their children to exert more effort towards educational activities – because public spending in education improves the return to human capital investment.

An important result relates to the possibility of corner solutions in the determination of $\gamma_{i,t+1}^*$. Indeed, if the level of physical is not sufficient, the return to education is so low that the utility cost of engaging in the process of socialisation and nurturing, with the purpose of instilling the x trait in her child, is always greater than the expected benefit, which is measured in terms of the expected increase of the child's human capital. Naturally, this is a scenario where the parent will not devote any effort at all, whereas the young individual's type will be determined solely through the oblique transmission mechanism. Note that the threshold level of capital, below which corner solutions materialise, differs between Type-x and Type-v parents. Specifically, the stock of capital necessary to induce Type-v parents to exert any effort in inculcating their offspring with the x trait, is higher. This emanates from the innate characteristics that differentiate Type-x and Type-v parents. Due to $\underline{\varphi} < \overline{\varphi}$, the former group of parents require less effort to achieve the same probability of success in inducing their children to adopt their own attitudes and behaviour, compared to the latter group who try to induce attitudes and behavioural patterns to which they, themselves, did not actually abide as young individuals.

Recall that the external effects that permeate the process of cultural transmission are captured by a function $n_i(\eta_{t+1})$, such that $n_i' > 0$ and $n_x(\eta_{t+1}) > n_v(\eta_{t+1})$. As in other analyses (e.g., Bisin and Verdier 2001; Hauk and Saez-Marti 2002), I shall be using the specific functional form

$$n_x(\eta_{t+1}) = \eta_{t+1} , \tag{31}$$

to capture the external effects that apply to the cultural transmission process for a young individual who grows up with parents of the same type (i.e., Type-x). In order to capture the idea that (for the reasons detailed in Section 2) these external characteristics are less effective for a young individual who grows up with a Type-v parent, I shall employ

$$n_v(\eta_{t+1}) = \theta \eta_{t+1} , \tag{32}$$

where $\theta \in (0,1)$ is a parameter that quantifies the extent to which the external effects of the cultural transmission process differ between Type-x and Type-v households. The arguments that were used previously to justify the differences captured by θ indicate that a possible interpretation for this parameter is that lower values of θ capture a higher degree of social segregation between the two types of households. This may be the outcome of a self-selection mechanism that emanates from the differences in income and cultural characteristics between these two types.

Combining Eq. (8), (27) and (30)-(32) results in

$$\pi_{x,t+1} = \gamma_x(k_{t+1}) + [1 - \gamma_x(k_{t+1})]\eta_{t+1}, \tag{33}$$

and

$$\pi_{v,t+1} = \gamma_v(k_{t+1}) + [1 - \gamma_v(k_{t+1})]\theta\eta_{t+1}. \tag{34}$$

We can substitute (33) and (34) in Eq. (9) and manipulate algebraically in order to get

$$\eta_{t+2} = \eta_{t+1} + (1 - \eta_{t+1})\{\eta_{t+1}[\gamma_x(k_{t+1}) - (1 - \theta)] + (1 - \theta\eta_{t+1})\gamma_v(k_{t+1})\}. \tag{35}$$

Earlier, we established that a larger number of Type-x individuals facilitates the process of capital accumulation. As it turns out, the link between the stock of physical capital and the share of the population who adopt the x trait is two-way causal, given that the stock of physical capital may facilitate the process of cultural transmission. I formalise the latter idea through

Proposition 3. *As long as $k_{t+1} > \underline{k}$, a higher physical capital stock shifts the distribution of traits among the population, in favour of Type-x.*

Proof. It follows from Lemma 1 and $\dfrac{\partial \eta_{t+2}}{\partial \gamma_x(k_{t+1})}, \dfrac{\partial \eta_{t+2}}{\partial \gamma_v(k_{t+1})} > 0$. ∎

This is of course an intuitive result. As long as the effect of public education investment is sufficiently high, the increased return to the young individuals' human capital motivates (either some or all) parents to exert more effort towards the cultural instruction of their offspring. Consequently, a higher fraction of young individuals will adopt the x trait.

4 Capital Accumulation and the Population Dynamics of Behavioural Traits

In the previous section, I showed that, as long as the capital stock is below a threshold – which, in turn, depends on the specific type of parents – there are corner solutions where $\gamma_i^* = 0$. I also established that the forces determining capital accumulation and the intergenerational transmission of behavioural traits are two-way causal. Now, let us examine how the combination of these characteristics determine the joint dynamics of k_t and η_t. The underlying sources of these dynamics can be summarised by combining Lemma 1 together with Eq. (26) and Eq. (35), expressed one period backward. That is

$$k_{t+1} = F(k_t, \eta_t) = g k_t^a [\eta_t(\overline{e}^\beta - \underline{e}^\beta) + \underline{e}^\beta]^{1-a}, \tag{36}$$

and

$$\eta_{t+1} = \Gamma(k_t, \eta_t) = \begin{cases} \eta_t - (1-\eta_t)\eta_t(1-\theta) & \text{if} \quad k_t \leq \underline{k} \\ \eta_t + (1-\eta_t)\eta_t[\gamma_x(k_t) - (1-\theta)] & \text{if} \quad \underline{k} < k_t \leq \overline{k} \\ \eta_t + (1-\eta_t)\{\eta_t[\gamma_x(k_t) - (1-\theta)] + (1-\theta\eta_t)\gamma_v(k_t)\} & \text{if} \quad k_t > \overline{k} \end{cases} \tag{37}$$

Now, consider the expression $\gamma_x(k_t) - (1-\theta)$. Taking account of (29) and (30), it is straightforward to establish that this expression is positive if and only if $k_t > \left(\dfrac{\underline{\mu}}{\theta}\right)^{1/\omega} \equiv \tilde{k}$.

Since $\theta \in (0,1)$, it is $\tilde{k} > \underline{k}$. However, the comparison between $\tilde{k}$ and $\overline{k}$ is not equally unambiguous. Specifically, whether $\tilde{k} > \overline{k}$ or $\tilde{k} < \overline{k}$ depends on whether $\theta < \tilde{\theta}$ or $\theta > \tilde{\theta}$ respectively, where $\tilde{\theta} \equiv \overline{\mu}^{-1}\underline{\mu}$. For now, I will focus on the case where the condition $\theta < \tilde{\theta}$ holds.

A look at the expression in (37) reveals that, as long as $k_t \leq \overline{k}$ holds, $\eta_{t+1} - \eta_t \leq 0 \; \forall t$. Hence, a steady state solution $\eta_{t+1} = \eta_t = \hat{\eta}$ for the fraction of the population who possess the x trait is $\hat{\eta}_1 = 0$ which, given (36), leads to the following steady state solution $k_{t+1} = k_t = \hat{k}$ for the stock of physical capital:

$$\hat{k}_1 = g^{\frac{1}{1-a}}\underline{e}^\beta. \tag{38}$$

Similarly, for $k_t \geq \tilde{k}$ the expression in (37) reveals that $\eta_{t+1} - \eta_t \geq 0 \; \forall t$. Therefore, the steady solution $\eta_{t+1} = \eta_t = \hat{\eta}$ is $\hat{\eta}_2 = 1$ which corresponds to the following steady state solution $k_{t+1} = k_t = \hat{k}$ for the physical capital stock:

$$\hat{k}_2 = g^{\frac{1}{1-a}} \overline{e}^{\beta} , \qquad (39)$$

where $\hat{k}_2 > \hat{k}_1$ by virtue of (19). Nevertheless, under the condition $\theta < \tilde{\theta}$ we have $\tilde{k} > \overline{k}$. Therefore, it is instructive to analyse the dynamics of η_t when $\overline{k} < k_t < \tilde{k}$. Combining (30) and (37), it can be established that there exist

$$\ddot{\eta}(k_t) = \frac{k_t^{\omega} - \overline{\mu}}{\underline{\mu} - \theta \overline{\mu}} , \qquad (40)$$

and $\breve{k} \equiv [(1-\theta)\overline{\mu} + \underline{\mu}]^{1/\omega}$, such that $\breve{k} \in (\overline{k}, \tilde{k})$ and

$$\eta_{t+1} - \eta_t \begin{cases} \leq 0 & \text{if} \quad k_t < \breve{k} \quad \text{and} \quad \eta_t \geq \ddot{\eta}(k_t) \\ > 0 & \text{if} \quad k_t < \breve{k} \quad \text{and} \quad \eta_t < \ddot{\eta}(k_t). \\ \geq 0 & \text{if} \quad k_t \geq \breve{k} \end{cases} \qquad (41)$$

Obviously, if there is a steady state solution $\hat{k}$ on the interval $\overline{k} < k_t < \breve{k}$, then this solution must satisfy $\ddot{\eta}(\hat{k}) = \hat{\eta}(\hat{k}) = \hat{\eta}$, where $\hat{\eta}(k_t)$ is derived from (36) after applying $k_{t+1} = k_t$ and solving for η_t. That is,

$$\hat{\eta}(k_t) = \frac{k_t - g^{1/(1-a)} \underline{e}^{\beta}}{g^{1/(1-a)} (\overline{e}^{\beta} - \underline{e}^{\beta})} . \qquad (42)$$

Now consider $\hat{k}_1 < \overline{k} \Leftrightarrow g^{\frac{1}{1-a}} \underline{e}^{\beta} < \overline{\mu}^{1/\omega}$ and check that $\ddot{\eta}(\overline{k}) = 0$, $\ddot{\eta}(\breve{k}) = 1$, $\hat{\eta}(\overline{k}) \in (0,1)$ and $\ddot{\eta}', \hat{\eta}' > 0$. It follows that, by virtue of the single crossing property, $\hat{\eta}(\breve{k}) < 1 \Leftrightarrow \breve{k} < \hat{k}_2 \Leftrightarrow [(1-\theta)\overline{\mu} + \underline{\mu}]^{1/\omega} < g^{\frac{1}{1-a}} \overline{e}^{\beta}$ is sufficient for the existence of steady state solutions $k_{t+1} = k_t = \hat{k}_3 \in (\overline{k}, \breve{k})$ and $\eta_{t+1} = \eta_t = \hat{\eta}_3 \in (0,1)$.

The preceding analysis can be utilised in order to provide a formal statement regarding the conditions under which multiple steady state equilibria exist. This is done by means of

Lemma 2. *Suppose that* $\theta < \tilde{\theta}$, $g^{\frac{1}{1-a}}\underline{e}^{\beta} < \overline{\mu}^{1/\omega}$ *and* $[(1-\theta)\overline{\mu}+\underline{\mu}]^{1/\omega} < g^{\frac{1}{1-a}}\overline{e}^{\beta}$ *hold. Then there exist three pairs of steady state equilibria* $\left(\hat{k}_1,\hat{\eta}_1\right)$, $\left(\hat{k}_2,\hat{\eta}_2\right)$ *and* $\left(\hat{k}_3,\hat{\eta}_3\right)$, *such that* $\hat{k}_1 < \hat{k}_3 < \hat{k}_2$ *and* $\hat{\eta}_1 < \hat{\eta}_3 < \hat{\eta}_2$.

Proof. It follows from the preceding analysis. ∎

As for the stability properties of the steady state equilibria, these are presented in

Lemma 3. *The equilibrium pairs* $\left(\hat{k}_1,\hat{\eta}_1\right)$ *and* $\left(\hat{k}_2,\hat{\eta}_2\right)$ *are stable, whereas the equilibrium pair* $\left(\hat{k}_3,\hat{\eta}_3\right)$ *is unstable.*

Proof. See the Appendix. ∎

In order to understand the forces that drive the economy's dynamics and its long-term prospects, given a pair of initial conditions (k_0,η_0), we shall make use of all the results derived so far, together with the phase diagrams in Figures 1 and 2. To begin with, consider a scenario with the same initial value η_0 but two different possible initial values for the stock of physical capital. At point *A*, the capital stock has the tendency to increase during the initial stages, but the share of the population who adopt the x trait gradually declines. This is because the capital stock is still not sufficient to induce high levels of efforts on cultural instruction by parents. Thus, the fraction of young individuals who adopt the behavioural characteristics that are conducive to educational activities declines over time – a process that is exacerbated by the fact that, as η_t falls, the external part of the cultural transmission process becomes weaker as well. Given the gradual fall in η_t, the decline in the aggregate level of human capital will become so pronounced that the process of physical capital accumulation will be reversed and the capital stock will, at some point, begin to decline as well. This is an additional impeding factor to the process of cultural transmission. In fact, at some point the capital stock will fall below the corresponding thresholds necessary to induce Type-v parents (initially)

and Type-x parents (subsequently) to devote any efforts in inculcating their offspring with the x trait. Eventually, what ensues is a vicious circle of mutually reinforcing declines in both k_t and η_t – a process that will lead the economy towards the low-income steady state $\left(\hat{k}_1, \hat{\eta}_1\right)$.

The dynamics are quite different though when we consider B as the starting point of the transition, despite the fact that the initial level η_0 is the same. The capital stock is high enough to induce relatively high levels of cultural instruction by both Type-x and Type-v parents. As a result, the gradual shift in the distribution of behavioural traits, in favour of Type-x, generates such an increase in human capital that, at some point, the stock of physical capital will gradually begin to increase due to the high level of saving. From that moment onward, there is a mutually reinforcing, virtuous circle of rising k_t and η_t, as the economy converges to the high-income steady state $\left(\hat{k}_2, \hat{\eta}_2\right)$.
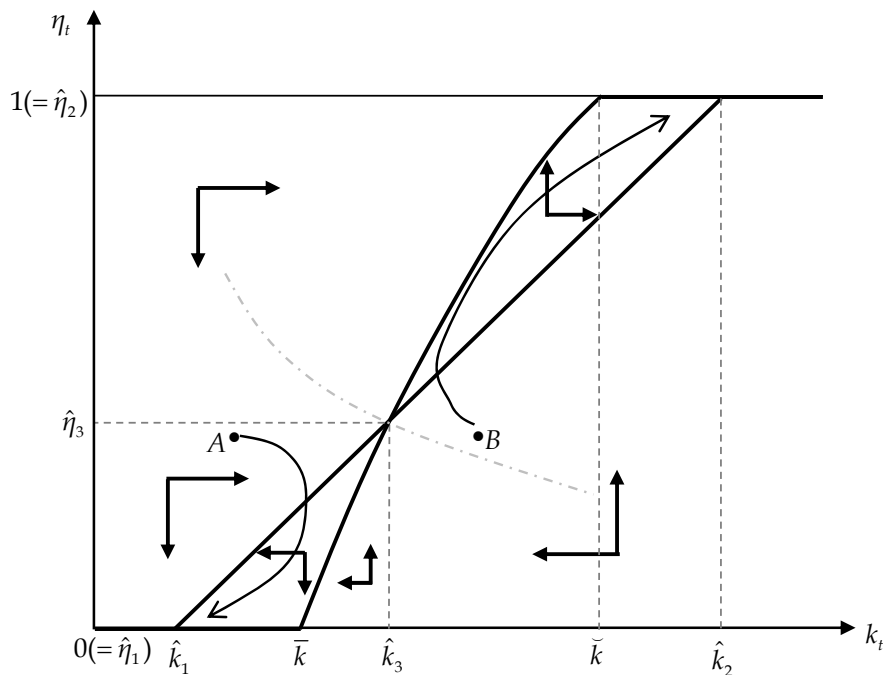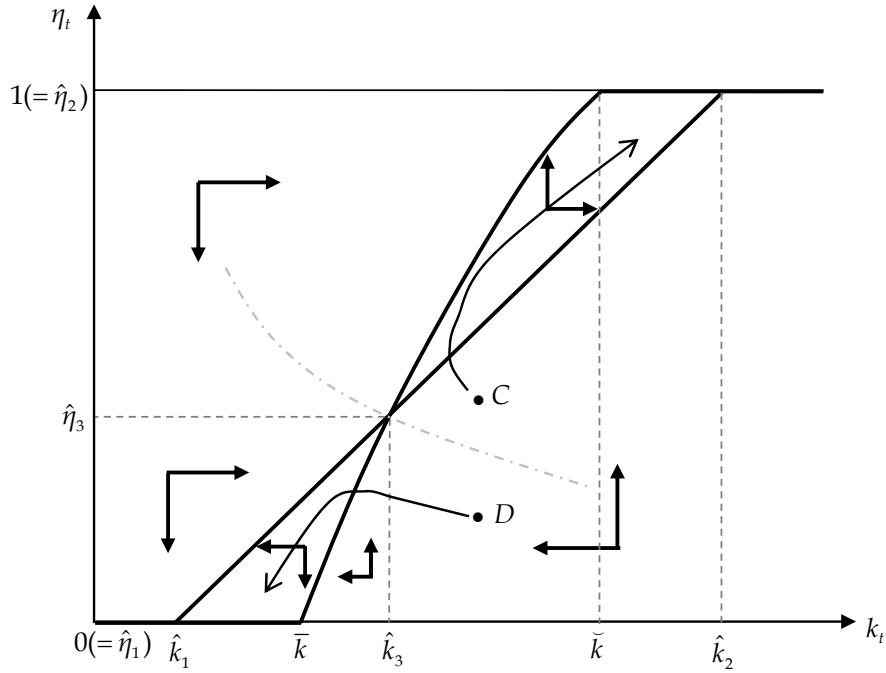

Figure 1. $\theta < \tilde{\theta}$

Figure 2. $\theta < \tilde{\theta}$

Now let us move to a scenario that entails the same initial value k_0 but two different initial conditions for the share of the population carrying the x trait. At point C, the stock of physical capital has the tendency to increase due to the high level of human capital that emanates from the relatively high proportion of people who dedicate more effort towards their educational activities when young. As the capital stock increases, it becomes more likely that, for a given η_t, the number of individuals who adopt the x trait will keep increasing over time. This process promotes the accumulation of physical capital even further which, in turn, supports a further increase in η_t due to the increased activities on cultural instruction by both Type-v and Type-x parents. This self-reinforcing process will eventually direct the economy towards the high-income steady state $\left(\hat{k}_2, \hat{\eta}_2 \right)$. On the contrary, the low initial value for η_t at point D generates mutually reinforcing forces that will gradually converge to the low-income steady state $\left(\hat{k}_1, \hat{\eta}_1 \right)$, despite the fact that the initial stock of physical capital is the same. This is because the low level of aggregate human capital, associated with the low fraction of individuals

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who possess the x trait, will generate a gradual decline in the stock of physical capital. Consequently, it will become more likely that η_t will keep declining over time, mainly owing to the initially reduced, and subsequently absent, cultural instruction activities by parents of both types, but also reinforced through the oblique transmission process.

A formal representation of all the implications we discussed above is provided in

Proposition 4. *Consider $\theta < \tilde{\theta}$. When multiple, path-dependent equilibria exist, both the initial stock of physical capital and the initial share of the population who adopt behavioural characteristics that are conducive to human capital formation, are critical in determining the economy's long-term prospects, i.e., whether it will converge to the low- or the high-income equilibrium.*

Proof. It follows from the preceding results and analysis. ∎

Next, I will analyse the dynamics and the long-run outcomes that transpire when $\theta > \tilde{\theta}$. Note that this is a situation where $\bar{k} > \tilde{k}$ holds. Hence, we can combine (37) together with $k_t > \tilde{k} \Rightarrow \gamma_x(k_t) > (1-\theta)$ to infer that

$$\eta_{t+1} - \eta_t = \begin{cases} \leq 0 & \text{if} \quad k_t < \tilde{k} \\ \geq 0 & \text{if} \quad k_t > \tilde{k} \end{cases}. \tag{43}$$

In this case, the possible outcomes associated with path-dependent equilibria can be summarised by means of

Lemma 4. *Suppose that $\theta > \tilde{\theta}$ and $g^{\frac{1}{1-a}} \underline{e}^\beta < \left(\dfrac{\mu}{\underline{\theta}} \right)^{1/\omega} < g^{\frac{1}{1-a}} \overline{e}^\beta$ hold. Then there exist two pairs of stable steady state equilibria $\left(\hat{k}_1, \hat{\eta}_1 \right)$ and $\left(\hat{k}_2, \hat{\eta}_2 \right)$ such that $\hat{k}_1 < \hat{k}_2$ and $\hat{\eta}_1 < \hat{\eta}_2$, separated by a threshold level of physical capital $\tilde{k} \in \left(\hat{k}_1, \hat{k}_2 \right)$.*

Proof. See the Appendix. ∎

Note that, similarly to the preceding analysis, the steady state solutions are $\hat{\eta}_1 = 0$, $\hat{k}_1 = g^{\frac{1}{1-a}} \underline{e}^{\beta}$, $\hat{\eta}_2 = 1$ and $\hat{k}_2 = g^{\frac{1}{1-a}} \overline{e}^{\beta}$. Given this, we can infer the characteristics that determine the transition towards the long-run equilibrium through

Proposition 5. *Consider $\theta > \tilde{\theta}$. When multiple, path-dependent equilibria exist, only the initial stock of physical capital is critical in determining the economy's long-term prospects, i.e., whether it will converge to the low- or the high-income equilibrium.*

Proof. It follows from Lemma 4. ∎

The dynamics under this scenario are illustrated on the phase diagram of Figure 3. What becomes evident here is that, given the result in (43), the only factor that determines whether the economy will converge to the low- or the high-income equilibrium is the initial stock of physical capital. At point *E*, the stock of physical capital is sufficiently high to induce an increase in η_t. As this happens, the resulting increase in aggregate human capital will promote the process of physical capital accumulation which, in turn, will motivate both Type-x and Type-v parents to engage more into the kind of activities that may instil the x trait in their children. At point *F*, however, the share of the population who adopt the x trait declines over time because, on the one hand, Type-v parents do not dedicate any effort to promote the adoption of the x trait by their children, whereas, on the other hand, the corresponding efforts by Type-x parents are too limited to ensure that a sufficiently large number of young individuals will adopt the behavioural patterns that are conducive to the formation of human capital. These effects are reinforced by the external factors of the cultural transmission process and generate a vicious circle where the gradual decline in η_t impedes the process of capital formation (due to the decrease in human capital) – an outcome that reinforces the net reduction of the population adopting the behavioural characteristics of the x trait.
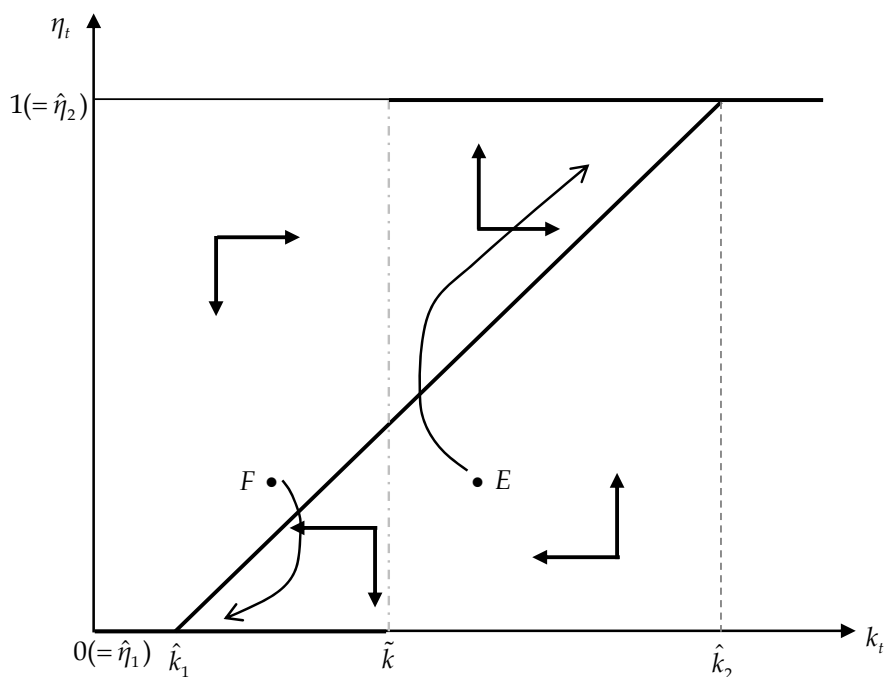
Figure 3. $\theta > \tilde{\theta}$

The outcomes presented in Propositions 4 and 5 reveal the main implications of the model, in relation to the forces that permeate the formation of physical capital, and the formation of the personality traits underlying the attitudes and behavioural patterns that determine human capital. These can be summarised as follows:

Corollary. *When multiple, path-dependent equilibria exist, the initial stock of physical capital is always crucial in determining the economy's long-term prospects. The initial distribution of traits among the population can also be crucial, as long as the relative efficiency of the external aspects in the cultural transmission process for Type-v households is sufficiently low.*

To clarify the main implications, we should recall that, in this model, the characteristics of the cultural transmission process have rendered parental decisions independent of the existing distribution of traits among the population. In other words, a parent's optimal effort (irrespective of her type) is not subject to the cultural complementarity that is necessary for the existence of path-dependency in the analysis of Bisin and Verdier (2001). In my framework, however, differences in the initial

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distribution of traits, among economies that are otherwise identical in terms of initial economic resources and structural characteristics, can still be amplified in the long-run, despite the absence of such complementarities. This outcome occurs if the relative efficiency of the external elements in the cultural transmission process of Type-v households is sufficiently low. In terms of intuition, a relatively high initial value of η_t is central in counteracting the low value of θ, hence facilitating the dynamic process whereby the share of the population who adopt the x trait increases over time. Taking account of the underlying arguments behind the presence of the parameter θ, this is perhaps indicative of why and how social segregation can be central in perpetuating and eventually amplifying initial differences in the distribution of cultural characteristics. Put differently, if the degree of social segregation mitigates the benefits that could potentially accrue to Type-v households, through their interactions with Type-x ones, initial disparities in the distribution of traits among different economies, can be ingrained into their socioeconomic characteristics and, therefore, be established as permanent features that differentiate their long-term prospects.

Irrespective of the value of θ, the initial stock of physical capital is always central to the model's dynamics and long-run equilibrium. The interesting aspect is that the physical capital's role in generating outcomes that are sensitive to initial conditions, is attributed to its significance in shaping the population dynamics of behavioural traits. This is an outcome that has eluded the attention of the existing literature, despite the fact that it pinpoints to the important role of the economic growth process in influencing cultural differences across economies.

5 Conclusion

The aim of this paper was to offer a novel explanation behind the emergence of multiple, path-dependent equilibria in capital accumulation and, therefore, long-term economic performance. This was done by means of a full-fledged growth model where young generations are endogenously inculcated with attitudes that impinge on their willingness to devote more time and effort towards their educational activities and, therefore, the formation of their human capital – a process that affects and, at the same time, is affected by the accumulation of physical capital. The model adopted a process of

intergenerational cultural transmission with the following characteristics: (i) the vertical transmission mechanism involves the deliberate efforts of altruistic parents to inculcate their offspring with attitudes that are conducive to human capital-promoting activities; (ii) the marginal effort of cultural instruction is bounded away from zero, thus requiring a sufficient level of economic resources for inducing parental efforts towards their children's adoption of such attitudes; and (iii) the efficiency of the oblique transmission mechanism depends on whether parents adopted similar, human capital-promoting attitudes when they, themselves, were young.

Naturally, the analysis can be extended to derive implications on issues that – with the purpose of keeping the model tightly focused, and without blurring the mechanisms at work – were not examined in the current framework. One such extension could involve a dynamic externality whereby a young individual's human capital is positively affected by the human capital of her parent. This approach would be conducive to the study of income distribution dynamics and their interplay with the process of cultural transmission, thus it represents an interesting and rewarding avenue for future research.

Appendix

Proof of Lemma 3

Using the system in (36) and (37), let us write the Jacobian matrix of partial derivatives

$$\begin{pmatrix} F_{k_t}\left(\hat{k},\hat{\eta}\right) & F_{\eta_t}\left(\hat{k},\hat{\eta}\right) \\ \Gamma_{k_t}\left(\hat{k},\hat{\eta}\right) & \Gamma_{\eta_t}\left(\hat{k},\hat{\eta}\right) \end{pmatrix}',$$

where

$$F_{k_t}\left(\hat{k},\hat{\eta}\right)=a, \tag{A1}$$

$$F_{\eta_t}\left(\hat{k},\hat{\eta}\right)=(1-a)g^{\frac{1}{1-a}}(\overline{e}^{\beta}-\underline{e}^{\beta}), \tag{A2}$$

$$\Gamma_{k_t}\left(\hat{k},\hat{\eta}\right)=\begin{cases} 0 & \text{if} \quad k_t \leq \underline{k} \\ (1-\hat{\eta})\hat{\eta}\gamma_x' & \text{if} \quad \underline{k}<k_t\leq\overline{k} , \\ (1-\hat{\eta})[\hat{\eta}\gamma_x'+(1-\theta\hat{\eta})\gamma_v'] & \text{if} \quad k_t>\overline{k} \end{cases} \tag{A3}$$

$$\Gamma_{\eta_t}\left(\hat{k},\hat{\eta}\right)=\begin{cases}1-(1-\theta)(1-2\hat{\eta}) & \text{if} \quad k_t\le\underline{k}\\1+[\gamma_x(\hat{k})-(1-\theta)](1-2\hat{\eta}) & \text{if} \quad \underline{k}<k_t\le\overline{k}\\1+[\gamma_x(\hat{k})-(1-\theta)](1-2\hat{\eta})-\gamma_v(\hat{k})[1-\theta\hat{\eta}+\theta(1-\hat{\eta})] & \text{if} \quad k_t>\overline{k}\end{cases} \quad (A4)$$

For $k_t\le\overline{k}$, the steady state solutions are $\hat{\eta}_1=0$ and $\hat{k}_1=g^{\frac{1}{1-a}}\underline{e}^\beta$, therefore $\Gamma_{k_t}\left(\hat{k}_1,\hat{\eta}_1\right)=0$. Furthermore, note that $\Gamma_{\eta_t}\left(\hat{k}_1,\hat{\eta}_1\right)\in(0,1)\ \forall k_t\le\overline{k}$ because $\Gamma_{\eta_t}\left(\hat{k}_1,\hat{\eta}_1\right)=\theta$ if $k_t\le\underline{k}$, or $\Gamma_{\eta_t}\left(\hat{k}_1,\hat{\eta}_1\right)=\theta+\gamma_x(\hat{k}_1)$ if $\underline{k}<k_t\le\overline{k}$.[14] Given these, the trace and the determinant of the Jacobian matrix are $Tr=a+\Gamma_{\eta_t}(\bullet)$ and $Det=a\Gamma_{\eta_t}(\bullet)$ respectively. Since $Tr^2-4Det=[a-\Gamma_{\eta_t}(\bullet)]^2>0$ the eigenvalues are real and distinct numbers, given by

$$\lambda_{1,2}=\frac{Tr\pm\sqrt{Tr^2-4Det}}{2}. \quad (A5)$$

Substituting the existing results in (A5) yields $\lambda_1=a\in(0,1)$ and $\lambda_2=\Gamma_{\eta_t}(\bullet)\in(0,1)$, meaning that the steady state pair $\left(\hat{k}_1,\hat{\eta}_1\right)$ is a stable equilibrium.

Next, consider the case $k_t>\tilde{k}>\overline{k}$ where the steady state solutions are $\hat{\eta}_2=1$ and $\hat{k}_2=g^{\frac{1}{1-a}}\overline{e}^\beta$. Again we have $\Gamma_{k_t}\left(\hat{k}_2,\hat{\eta}_2\right)=0$, $Tr=a+\Gamma_{\eta_t}(\bullet)$ and $Det=a\Gamma_{\eta_t}(\bullet)$, where $\Gamma_{\eta_t}(\bullet)=1-\gamma_x(\hat{k}_2)+(1-\theta)(1-\gamma_v(\hat{k}_2))\in(0,1)$.[15] Once more, $Tr^2-4Det=[a-\Gamma_{\eta_t}(\bullet)]^2>0$ implies that the eigenvalues are real and distinct numbers, equal to $\lambda_1=a\in(0,1)$ and $\lambda_2=\Gamma_{\eta_t}(\bullet)\in(0,1)$. Thus, the steady state pair $\left(\hat{k}_2,\hat{\eta}_2\right)$ is a stable equilibrium.

Finally, we will focus on the steady state pair $\left(\hat{k}_3,\hat{\eta}_3\right)$ which exists on the interval $\overline{k}<k_t<\tilde{k}$. Combining (30), (40) and (A3), we get $\Gamma_{k_t}\left(\hat{k}_3,\hat{\eta}_3\right)=\frac{\omega(1-\hat{\eta}_3)}{\hat{k}_3}$ and

$\Gamma_{\eta_t}\left(\hat{k}_3,\hat{\eta}_3\right)=1-\frac{(\mu-\theta\overline{\mu})(1-\hat{\eta}_3)}{(\underline{\mu}-\theta\overline{\mu})\hat{\eta}_3+\overline{\mu}}=1-\frac{(\mu-\theta\overline{\mu})(1-\hat{\eta}_3)}{\hat{k}_3^\omega}\equiv1-\delta\in(0,1)$. Defining

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[14] Recall that $\gamma_x(\hat{k})<1-\theta$ when $k_t\le\overline{k}<\tilde{k}$.

[15] Recall that $\gamma_x(\hat{k})>1-\theta$ when $k_t>\tilde{k}$.

$$F_{\eta_t}(\bullet)\Gamma_{k_t}(\bullet) = (1-a)g^{\frac{1}{1-a}}(\overline{e}^\beta - \underline{e}^\beta)\frac{\omega(1-\hat{\eta}_3)}{\hat{k}_3} \equiv \xi > 0,$$

it follows that

$$Tr^2 - 4Det = [a-(1-\delta)]^2 + 4\xi > 0, \tag{A6}$$

meaning that the eigenvalues are real and distinct numbers. Given (A5) and (A6), let us focus on the following eigenvalue:

$$\lambda_2 = \frac{Tr + \sqrt{Tr^2 - 4Det}}{2} = \frac{a+1-\delta+\sqrt{[a-(1-\delta)]^2 + 4\xi}}{2}. \tag{A7}$$

The expression in (A7) is unambiguously positive, therefore it is sufficient to show that $\lambda_2 > 1$ as a means of proving that the steady state pair $\left(\hat{k}_3, \hat{\eta}_3\right)$ is unstable. Using (A7), the condition $\lambda_2 > 1$ requires

$$\xi > (1-a)\delta \Rightarrow$$

$$g^{\frac{1}{1-a}}(\overline{e}^\beta - \underline{e}^\beta)\frac{\omega}{\hat{k}_3} > \frac{\underline{\mu}-\theta\overline{\mu}}{\hat{k}_3^\omega} \Rightarrow$$

$$\frac{\omega\hat{k}_3^{\omega-1}}{\underline{\mu}-\theta\overline{\mu}} > \frac{1}{g^{1/(1-a)}(\overline{e}^\beta - \underline{e}^\beta)}. \tag{A8}$$

This is a condition that indeed holds, because the existence of a steady state on $\overline{k} < k_t < \tilde{k}$, requires that that the slope of Eq. (40) must be greater than the slope of Eq. (42), when evaluated at $\hat{k}_3$. ∎

Proof of Lemma 4

Using $\overline{k} > \tilde{k}$, the proof of Lemma 3 can be used as a guide to show the stability of steady state pairs $\left(\hat{k}_1, \hat{\eta}_1\right)$ and $\left(\hat{k}_2, \hat{\eta}_2\right)$. ∎

References

1. Aksan, A.M., and Chakraborty, S. 2014. "Mortality versus morbidity in the demographic transition," *European Economic Review*, 70, 470-492.

2. Alesina, A., and La Ferrara, E. 2002. "Who trusts others?" *Journal of Public Economics*, 85, 207–234.

3. Algan, Y., and Cahuc, P. 2010. "Inherited trust and growth," *American Economic Review*, 100, 2060-2092.

4. Astone, N.N., and McLanahan, S.S. 1991. "Family structure, parental practices and high school completion," *American Sociological Review*, 56, 309-320.

5. Azariadis, C., and Drazen, A. 1990. "Threshold externalities in economic development," *Quarterly Journal of Economics*, 105, 501-526.

6. Beaman, L. Duflo, E., Pande, R., and Topalova, P. 2012. "Female leadership raises aspirations and educational attainment for girls: A policy experiment in India," *Science*, 335, 582-586.

7. Becker, G.S. 1976. "Altruism, egoism, and genetic fitness: Economics and sociobiology," *Journal of Economic Literature*, 14, 817-826.

8. Becker, G.S., and Tomes, N. 1986. "Human capital and the rise and fall of families," *Journal of Labor Economics*, 4, S1-S39.

9. Bénabou, R. 1996. "Heterogeneity, stratification, and growth: Macroeconomic implications of community structure and school finance," *American Economic Review*, 86, 584-609.

10. Bidner, C., and Francois, P. 2010. "Cultivating trust: Norms, institutions and the implications of scale," *Economic Journal*, 121, 1097-1129.

11. Bisin, A., and Verdier, T. 2001. "The economics of cultural transmission and the dynamics of preferences," *Journal of Economic Theory*, 97, 298-319.

12. Bisin, A., and Verdier, T. 2008. "Cultural transmission," in Durlauf, S.N., and Blume, L.E. (eds): *The New Palgrave Dictionary of Economics, 2nd edition*, Palgrave Macmillan.

13. Blankenau, W. 2005. "Public schooling, college subsidies and growth," *Journal of Economic Dynamics and Control*, 29, 487-507.

14. Bond, S., Leblebicioğlu, A., and Schiantarelli, F. 2010. "Capital accumulation and growth: A new look at the empirical evidence," *Journal of Applied Econometrics*, 25, 1073-1099.

15. Boyd, R., and Richerson, P.J. 1985. *Culture and the Evolutionary Process*, University of Chicago Press, Chicago (IL).

16. Brown, L., and Iyengar, S. 2008. "Parenting styles: The impact on student achievement," *Marriage and Family Review*, 43, 14-38.

17. Butz, D.A., and Yogeeswaran, K. 2011. "A new threat in the air: Macroeconomic threat increases prejudice against Asian Americans," *Journal of Experimental Social Psychology*, 47, 22-27.

18. Canova, F. 2004. "Testing for convergence clubs in income per capita: A predictive density approach," *International Economic Review*, 45, 49-77.

19. Cavalli -Sforza, L.L., and Feldman, M.W. 1981. *Cultural Transmission and Evolution: A Quantitative Approach*, Princeton University Press, Princeton (NJ).

20. Ceroni, C.B. 2001 "Poverty traps and human capital accumulation," *Economica*, 68, 203-219.

21. Chakraborty, S., Thompson, J.C., and Yehoue E.B. 2016. "The culture of entrepreneurship," *Journal of Economic Theory*, 163, 288-317.

22. Chaktaborty, S. 2004. "Endogenous lifetime and economic growth," *Journal of Economic Theory*, 116, 119-137.

23. De Long, J.B., and Summers, L.H. 1991. "Equipment investment and economic growth," *Quarterly Journal of Economics*, 106, 445-502.

24. Diamond, P.A. 1965. "National debt in a neoclassical growth model," *American Economic Review*, 55, 1126-1150.

25. Doepke, M., and Zilibotti, F. 2008. "Occupational choice and the spirit of capitalism," *Quarterly Journal of Economics*, 123, 747-793.

26. Doepke, M., and Zilibotti, F. 2014. "Culture, entrepreneurship, and growth," Chapter 1 in Aghion, P., and Durlauf, S.N. (eds): *Handbook of Economic Growth, Volume 2*, North Holland.

27. Durlauf, S.N., and Johnson, P.A. 1995. "Multiple regimes and cross-country growth behaviour," *Journal of Applied Econometrics*, 10, 365-384.

28. Galor, O., and Mountford, A. 2008. "Trading population for productivity: Theory and evidence," *Review of Economic Studies*, 75, 1143-1179.

29. Galor, O., and Zeira, J. 1993. "Income distribution and macroeconomics," *Review of Economic Studies*, 60, 35-52.

30. Glomm, G. 1997. "Parental choice of human capital investment," *Journal of Development Economics*, 53, 99-114.

31. Hanna, R., and Oliva, P. 2015. "The effect of pollution on labor supply: Evidence from a natural experiment in Mexico City," *Journal of Public Economics*, 122, 68-79.

32. Hansen, B.E. 2000. "Sample splitting and threshold estimation," *Econometrica*, 68, 575-603.

33. Hauk, E., and Sáez-Martí, M. 2002. "On the cultural transmission of corruption," *Journal of Economic Theory*, 107, 311-335.

34. Kijima, M., Nishide, K., and Ohyama, A. 2010. "Economic models for the environmental Kuznets curve: A survey," *Journal of Economic Dynamics and Control*, 34, 1187-1201.

35. Kim, Y.J., and Lee, C.I. 2015. "Social values and economic dynamics," *Journal of Economic Dynamics and Control*, 53, 69-84.

36. Klasing, M.J. 2014. "Cultural change, risk taking behaviour and implications for economic development," *Journal of Development Economics*, 110, 158-169.

37. Macours, K., and Vakis, R. 2014. "Changing households' investment behaviour through social interactions with local leaders: Evidence from a randomised transfer programme," *Economic Journal*, 124, 607-633.

38. Manz, C. C., Sapienza, P., and Zingales, L. 2006. "Does culture affect economic outcomes?" *Journal of Economic Perspectives*, 20, 23-48.

39. Nguyen, T. 2008. "Information, role models and perceived returns to education: Experimental evidence from Madagascar," Unpublished manuscript.

40. Palivos, T. 2001. "Social norms, fertility and economic development," *Journal of Economic Dynamics and Control*, 25, 1919-1934.

41. Patacchini, E., and Zenou, Y. 2011. "Neighborhood effects and parental involvement in the intergenerational transmission of education," *Journal of Regional Science*, 51, 987-1013.

42. Quah, D.T. 1997. "Empirics for growth and distribution: Stratification, polarization, and convergence clubs," *Journal of Economic Growth*, 2, 27-59.

43. Rebelo, S. 1991. "Long-run policy analysis and long-run growth," *Journal of Political Economy*, 99, 500-521.

44. Sáez-Martí, M., and Sjögren, A. 2008. "Peers and culture," *Scandinavian Journal of Economics*, 110, 73-92.

45. Tabellini, G. 2010. "Culture and institutions: Economic development in the regions of Europe," *Journal of the European Economic Association*, 8, 677-716.