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Abstract

The evidence shows that in many important economic domains, many people are either predisposed to engage in 'socially responsible actions' and/or required by regulations to do so. Examples include pollution abatement activity, behavior in a commons, and contributions to charity. We propose a general framework of analysis for modelling such actions and the role of public policy in encouraging these actions in an equilibrium setting. Multiple equilibria are endemic in these situations. We show that it is possible to conduct interesting and meaningful analysis in the presence of multiple equilibria. We examine the role of optimal public policy such as subsidies, taxes and direct government grants in engineering moves from less to more desirable equilibria. We highlight a new role for leadership contributions in facilitating moves between multiple equilibria. We also conduct a welfare analysis of the optimal mix between private and public actions.

Keywords: Social responsibility; multiple equilibria; optimal mix of public and private social responsibility; subsidies and direct grants; environmental economics, problem of the commons, charitable giving.

JEL Classification: D6, H2, H4.

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1 Introduction

Much evidence shows that many individuals choose to incur *socially responsible expenditures* (SRE). These expenditures benefit others even at a personal cost, primarily independent of strategic or reputation building considerations in repeated play. Evidence indicates that SRE arises when it benefits a small number of other players (Small and Loewenstein, 2003; Small et al. 2007; Bartling et al., 2015) or a large number of other players (Bartling et al., 2015). Economics has paid inadequate attention to the social responsibility embedded in human actions, preferring instead to model humans 'exclusively' as *sociopaths* (Gintis, 2009). Sociopaths have completely self-regarding preferences, they are amoral and will happily tell *selfish black lies*, i.e., lies that increase one's personal material utility but harm others (Erat and Gneezy, 2012). That all individuals behave in such a manner is rejected by the empirical evidence(Fehr and Schmidt, 2006; Gintis, 2009; Dhami, 2016).[1]

Even in anonymous market settings, human morality is not eroded (Falk and Szech, 2013). Experimental evidence supports a positive correlation between competition, market institutions, and trust/reciprocity (Henrich et al., 2001). In voting over redistribution, players often choose a smaller but more equally distributed cake relative to a larger more unequally distributed one (Tyran and Sausgruber, 2006; Ackert et al., 2007). Voting in large elections, where one is unlikely to be pivotal, is considered by many to stem from a sense of social responsibility (Heckelman and Miller, 2015; Gintis, 2016). Experimental evidence on social preferences demonstrates that pro-sociality in the form of SRE and the adherence to cooperative norms is widespread across many diverse domains (Fehr and Schmidt, 2006; Dhami, 2016). Humans exhibit *conditional reciprocity* (Gintis, 2009; Bowles and Gintis, 2011), so they have an inherent propensity to undertake SRE provided others do so.[2] Furthermore, human virtues are prized in society, morals play a central role in the belief systems of humans, most people will prefer not to tell selfish black lies, at least not for low stakes, and there is a particular preference for exhibiting human virtues in the form of SRE towards ingroup members (Dhami, 2016; Gintis, 2016).

The weight of the evidence, briefly alluded to above, must force a rethink of our theoretical frameworks to allow for at least some socially responsible economic agents. In this paper, we offer a theoretical rethink of an important class of problems in economics that can be nested within the following general framework; we give important examples that fit this framework, below.

Suppose that players are distributed continuously over the interval $[0, 1]$. The player

[1] Note that we allow the case where some, or many, individuals have exclusively self-regarding preferences. Indeed, our examples below allow for this.

[2] This does not require either an infinitely repeated game, nor any asymmetric information about the types of others, hence, such a propensity is quite distinct from the one that may be justified by appeal to repeated game results in classical game theory; see Part 2 in Dhami (2016) and Gintis (2009).

at location $x \in [0,1]$ is called player x and has exogenous income $m(x)$ that is taxed at the rate $t \in [0,1]$. The player chooses between private consumption, $c(x)$, and making some *private* SRE, $g(x)$, that is subsidized at the rate $s \in [0,1)$. The government uses its tax revenues to finance subsidies and to give a direct public grant, D, to fund the socially responsible task; this may be termed as *public* SRE. The aggregate of private and public SRE gives the total SRE, G, on some *socially responsible task.* Furthermore, consistent with much evidence reviewed above, at least some players derive utility from their private contributions $g(x)$ and from the aggregate SRE, G.

Each player is small enough that his/her effect on the aggregate G is not significant, hence, players do not choose their SRE in a strategic manner.[3] Rather, as in a competitive equilibrium, in optimally choosing $c(x)$ and $g(x)$, player x takes as given (1) the policy parameters s, t, D, and (2) the beliefs about the aggregate G. In a *competitive rational expectations equilibrium*, the beliefs about G come out to be true.

We examine the nature and implications of human morality and human virtues that lead to multiple equilibria in our general framework, as described above. We also consider the optimal welfare maximizing policy response to multiple equilibria. However, first we demonstrate that a very large set of economic problems fall within our framework. Consider the following three examples (we shall solve the first and third examples below).

Example 1 *(Pro-environmental expenditures by consumers and firms): The private players can be consumers or firms, and the socially responsible task is a greener environment. So $g(x)$ is a pro-environmental socially responsible expenditure (SRE) by player x; let us term this generically as pollution abatement (e.g., more expensive greener technology choices by firms, or the choice of more fuel efficient cars by consumers that are typically more expensive). The objective function of player x is $u(c, g, G, x)$. Player x has initial resources given by $m(x)$ and a part $g(x) \geq 0$ is expended on the private SRE, which is subsidized by the government at the rate s. Once player x decides on the pollution abatement expenditure, $g(x)$, the remaining monetary resources, $c(x)$, can be used to consume (if it is a consumer) or produce (if it is a firm). The government may also make direct grants, D, towards pollution abatement.*[4]
The function $u(c, g, G, x)$ is non-decreasing in all arguments, $u_c \geq 0$, $u_g \geq 0$ and $u_G \geq 0$.

[3] It should not be construed that we are anti game theory in any sense. Indeed, we make ample use of game theory in much of our research. However, the use of strategic interaction should not be a default assumption in situations where there is a large number of small players who individually have a negligible effect on the outcome. We give examples below.

[4] Information from Eurostat Statistics, 2015 shows that for 28 European countries (EU-28) in 2013: Specialised producers such as waste collectors (private and public) accounted for most environmental protection expenditure (EUR 145 billion), accounting for 51.1 % of the total level of expenditure. The remaining environmental protection expenditure was accounted for by the public sector (EUR 87.2 billion) and by private industry (EUR 51.6 billion).

u_g may be positive because a 'core intrinsic value' for player x could be reducing the carbon footprint. u_G may be positive for a firm because the firm feels that there will be fewer or less stringent environmental regulations on the industry if aggregate pollution is lower. u_G may be positive for a consumer because of the intrinsic value of G or because the cause appears more salient or more important, the larger is G, as often occurs in models of herding. For instance, a buyer of an electric car, may not experience feel a warm glow from his own purchase but might also be particularly pleased if larger number of people in society choose electric cars. The cross partial derivative between g, G is denoted by u_{gG}. If, for instance, $u_{gG} > 0$, then there are complementarities between g, G.

Example 2 *(Problem of the Commons) Suppose that the private players are farmers, the socially responsible task is maintenance of a commons, which we take to be the 'natural beauty and biodiversity in the countryside'. The private SRE, $g(x)$, of the farmers is private investment towards the maintenance and enhancement of the commons.*[5] *The government gives a tax-financed subsidy on this private SRE at the rate s. In the UK, this takes the form of monetary payments and accreditation schemes that signal to others that the farm has a commitment to sustainable farming. The government may also give direct grants, D, to promote and aid wildlife and farmland diversity. The sum of the private SRE and public SRE, G, goes towards the preservation of the commons. The utility function of farmer $x \in [0, 1]$ is given by $u(c, g, G, x)$. The restriction $u_g \geq 0$ reflects pride in own-contribution to the commons. The restriction $u_G \geq 0$ may reflect the own-enjoyment of the commons for the farmer and his family. These considerations, in conjunction with others that define appropriate property rights, may be critical to the maintainence of commons (Ostrom, 1990).*

Example 3 *(Charitable donations) Charitable donations is an important economic activity.*[6] *The following stylized facts can be culled from the literature. (1) Small dispersed individual private donors are the largest contributors. (2) Government direct grants to charities are significant in terms of magnitude. (3) Contributions to charity are typically tax deductible, i.e., they are subsidized. For instance, the rate of charitable deductions is 50% in the US and 17-29% for Canada. (4) Direct grants in the form of seed money or leadership contributions (that precede private giving) made by governments, foundations, the national lottery (as in the UK) or exceptionally rich individuals are particularly effi-*

[5] Farmers in the UK are encouraged to devote 2% of their land to aid wildlife and promote farmland biodiversity such as bees, wildflowers, spiders, beetles, voles, and birds such as barn owls. Their private investment, $g(x)$, includes planting hedges, and maintaining ditches, ponds or woodlands in their farmland.

[6] In 2003 for the USA, 89% of all households gave to charity with the average annual gift being \$1620, for a total of \$100 billion. The World Giving Report, 2015 finds that 4.6 billion people worldwide have donated their time, money or helped a stranger in a the last 5 years.

cacious in eliciting further contributions.[7]

These stylized facts suggest the following plausible formulation (see Figure 1 below for a diagrammatic representation). Consumers are distributed continuously over the interval $[0,1]$*. The utility function of consumer* x *is* $u(c,g,G,x)$*, where* g *is the charitable contribution of the consumer that is subsidized at the rate* s*. Order the location of consumers so that consumers* $x \in [0,p]$ *have positive income* $(m(x) > 0)$ *that is taxed at the rate* t *and consumers* $x \in (p,1]$ *have zero income* $(m(x) = 0)$ *so they pay no taxes. Among consumers with positive incomes,* $x \in [0,p]$*, consumers* $x \in [0,k]$ *care for the poor in the sense that* $(u_g > 0)$ *and the rest, consumers* $x \in (k,p]$*, do not care for the poor* $(u_g = 0)$*. The case* $u_g > 0$ *reflects the warm glow or prestige from own contribution (Cornes and Sandler, 1984; Andreoni, 1990).*[8]

The charitable contributions of the consumers $x \in [0,k]$ *and the direct government grants,* D*, financed by taxation, combine to produce the aggregate charitable contributions,* G*. The charity is a passive player that merely redistributes* G *equally among the poor (i.e., among consumers* $x \in (p,1]$*). In this case it is likely that* $u_{gG} > 0$ *because a bigger charity may be a signal that it is reputable, viable, and well-regarded by others, hence, one derives a greater marginal utility from own-contributions.*

Multiple equilibria are typically ruled out in problems that lie within the framework that we have described above.[9] In contrast, multiple equilibria arise naturally in our framework. Suppose that for some players $u_{gG} > 0$, as in Example 1 and 3 above. Thus, for any player x, the marginal utility of an extra unit of SRE, $g(x)$, is increasing in aggregate SRE, G. It follows that if the expectation is that G is high, then players are also induced to make high SRE, $g(x)$, which in turn leads to high G, validating the initial beliefs. Similarly, low initial expectation of G induce low private SRE, $g(x)$, ratifying the initial low beliefs. This gives rise to *multiple competitive equilibria*. In some equilibria, aggregate SRE is *high* (H), while in other equilibria, aggregate SRE is *low* (L). Multiple equilibria may provide one explanation of heterogeneity in SRE across societies.

[7] See Andreoni (2006a) for the empirical evidence.Consider briefly the evidence for stylized facts 1,2 and 4: (1) For US data, for 2002, individuals accounted for 76.3% of the total charitable contributions. Other givers are: Foundations (11.2%), bequests (7.5%), corporations (5.1%)). (2) For non-US data, governments are typically the single most important contributors to charities. On average, in the developed countries, charities receive close to half their total budget directly as grants from the government, while the average for developing countries is about 21.6%. (4) See, for instance, Karlan and List (2007), Potters et al. (2007), and Rondeau and List (2008).

[8] Warm glow implies that there is extra utility from one's own contribution, which mitigates the free rider problem arising from purely altruistic considerations. It also obviously implies that government grants to charities do not completely crowd out private donations because the two are imperfect substitutes from the point of view of givers.

[9] Andreoni (1998) considered multiple equilibria in charitable giving that arise from non-convexities in production; this approach is unrelated to our paper.

In particular, the low equilibrium, L, may have serious welfare implications in the case of Examples 1–3. Furthermore, in an environmental context, a country might face externally imposed carbon footprint targets and the low equilibrium might simply not be adequate to hit these targets. This may make it inevitable that the government considers a move from equilibrium L to equilibrium H, even if such a move cannot be justified on the criterion of maximizing a weighted utility of the players in the economy.

A particularly important feature of the low equilibrium, that we highlight, is that it could be associated with *perverse comparative statics*, i.e., an increase in subsidies may reduce private SRE, $g(x)$. In contrast, when subsidies enhance private SRE, we get *normal comparative statics*. Thus, in the case of perverse comparative statics, a proposed move from equilibrium L to equilibrium H will not occur by giving greater incentives (subsidies in this case) for private SRE. Classical economic incentives do not work in this case.

We conduct a classical welfare analysis in which a benevolent government maximizes a weighted utilitarian objective. This allows us to determine the optimal policy at any given equilibrium. However, as noted above, this might not be adequate and for a variety of reasons, society may wish to operate at equilibrium H rather than L. In this case, we need to determine how public policy may allow us to engineer moves from one equilibrium to another. We propose a novel solution to this problem that is based on tax financed temporary direct government grants, D. Once the economy arrives at the desired equilibrium, then one may engage in classical welfare analysis to determine the optimal mix of private and public contributions to SRE.

The main contributions in this paper are the following.

(1) Using temporary direct grants, D, a policy maker can *engineer a move* from the low equilibrium, L, to the high equilibrium, H. This is particularly desirable when comparative statics at the low equilibrium are *perverse* and those at the high equilibrium are *normal*. Unlike the thrust of many other approaches which try to rule out multiple equilibria, by restrictive assumptions in the model, we squarely face up to the possibility of multiple equilibria and the optimal public policy response to it. This allows us to highlight the powerful role of *leadership contributions* in facilitating moves between multiple equilibria, a factor that has not been recognized in this literature.

(2) When comparative statics at the low equilibrium are *normal* and those at the high equilibrium are *perverse*, the government can do better than simply encouraging subsidy-induced giving at the low equilibrium. Indeed, once the government successfully engineers a move to the high equilibrium using temporary direct grants, the perverse comparative statics at the high equilibrium ensure that a reduction in subsides will induce even greater private giving.

(3) By carrying out a welfare analysis, we give conditions that specify the *optimal mix* of public and private SRE.

It is worth closing on two important observations.

1. In this paper we concentrate exclusively on equilibrium analysis in a static model. The reader may wonder about the stability properties of the equilibria. This requires the specification of an adjustment process. However, there are no dynamic constraints in our model or in any static competitive equilibrium model. Therefore, the only natural dynamics for our model is the very simple one where economic agents jump immediately to whatever equilibrium is in line with their expectations. In this, we follow the classical and standard approach in economics.

2. The reader may ask why do we not introduce some noise into the knowledge of payoffs of the economic agents, and then use the machinery of *global games* to get a unique equilibrium (Carlsson and van Damme, 1993; Morris and Shin, 2003). Our response is as follows. First, in many of our examples with large number of small economic agents there is no strategic interaction in any fundamental sense. For instance, the Red Cross and Red Crescent has 17 million volunteers worldwide and the contributors to major charities within each country number in the millions. It beggars belief that these large number of individually small givers to charities (see Example 3 above) are engaged in strategic interaction with respect to each other. Similarly voting in 'large elections' on account of social responsibility where the electorate numbers in the millions is surely not a strategic game among the voters.[10]
   In contrast, the competitive equilibrium framework that we propose was developed precisely to deal with situations where there is a large number of economic agents, each individually too small to influence the aggregate outcome.
   Second, the information requirements in global games are relatively high, and the assumption of rational expectations in beliefs is empirically a strong one. This does not jive well with the evidence on bounded rationality of economic agents (Camerer, 2003, Dhami, 2016). In contrast, the information requirements in our competitive equilibrium setting are minimal.
   Third, we believe that public policy does often face the problem of multiple equilibria and artificially ruling out this possibility using a theoretical fix is neither intellectually satisfactory nor practical. Indeed, one novelty of our approach is that we propose a tractable way of dealing with such problems.

The rest of the paper is organized as follows. Section 2 formulates the theoretical model. Section 3 provides two formal examples of our framework: pro-environmental expenditures, and charitable giving (similar to Examples 1 and 3) whose solution will be derived in

[10] We do fully recognize that the contributions of very large individual givers to charity and the voting behavior in elections in small committees may be strategic. But we are not focussing on these cases.

subsequent sections. Section 4 uses standard techniques to show that a solution exists to the optimization problem of the private economic agents and derives some straightforward comparative static results. Section 5 describes the equilibrium conditions in the model and derives the conditions under which comparative statics can be normal or perverse.

Section 6 shows how multiple equilibria arise naturally in our framework, and particularly for the two examples that we give in Section 3. Section 7 shows how one might use direct government grants to engineer a move from one equilibrium to another in the presence of multiple equilibria. As preparation for the rest of the paper, Section 8 discusses a natural and standard condition in static models of competitive equilibria that we call the *stability of beliefs* condition. Once an equilibrium has been established for a sufficient length of time (e.g., driving on the left or the right side of the road) then expectations anchor on such an equilibrium.

In Section 9 we conduct a welfare analysis in the presence of multiple equilibria. Section 9.1 assumes a weighted utilitarian objective and conducts a welfare analysis. This analysis applies only at a chosen equilibrium– either the low equilibrium (L) or the high equilibrium (H). In Section 9.2, we show how society might wish to engineer a move from equilibrium L to equilibrium H and when the new equilibrium is established, choose policy optimally, as shown in Section 9.1. Finally, we revisit the examples from Section 6 and show how the question of engineering moves between equilibria and choosing optimal policy instruments may be addressed simultaneously. Section 10 concludes. All proofs are in the appendix.

2 The model

There are two main kinds of active players in the model.[11] A *public player*, which we refer to as the *government* and a continuum of *private players* located on the unit interval, $[0, 1]$. Each private player, possibly a firm or a consumer depending on the context, is uniquely identified by her location, $x \in [0, 1]$ and, hence, referred to as player x. Let $m(x) \geq 0$ be the exogenous income of player x. Aggregate income is

$$M = \int_{x=0}^{1} m(x)dx < \infty. \tag{1}$$

We assume that each private player can afford a consumption level greater than a subsistence level, $\underline{c}(x) \geq 0$; this gives a lower bound on consumption. For a firm, $\underline{c}$ could represent a minimum operating scale. Aggregate income is sufficient to cover subsistence

[11] In addition, technically we allow for the existence of a passive player who takes predetermined non-strategic actions in the game. An example is the Charity in Example 3 that simply collects all charitable contributions and redistributes them equally among the poor in a predetermined manner.

consumption (for a firm, it permits operating at the minimum operating scale)

$$\int_{x=0}^{1} \underline{c}(x)\, dx < M. \tag{2}$$

There is a single *socially responsible task* that is enhanced by private and public *socially responsible expenditures* (SRE) on the task (as in each of Examples 1–3).

2.1 Preferences of the private players

The payoff of player x is

$$u(c, g, G, x), \tag{3}$$

where c is private consumption and $g \geq 0$ is the expenditure on a costly socially responsible task. $G \geq 0$ is the aggregate level of the SRE, which is the sum of the private SRE and the public SRE (to be defined below). All functions of x, including $u, \underline{c}, m$ (and, later, τ and ω) are assumed integrable with respect to x. It follows that the solutions to our optimization problems are also integrable with respect to x.[12] $u(c, g, G, x)$ is a C^2 function of c, g, G for $g > 0$ and $c > \underline{c}(x)$.

Denote by u_i, $i = c, g, G$, the partial derivative of u with respect to the i^{th} argument and by u_{ij}, $j = c, g, G$, the cross partial derivatives. The behavioral assumptions are:

$$\begin{cases} u_c > 0, u_{cc} \leq 0, u_g \geq 0, u_{gg} \leq 0, u_G \geq 0 \text{ for all } x \in [0,1], \\ u_g > 0 \text{ for a set of consumers of positive measure.} \end{cases} \tag{4}$$

Consider the first row of (4). The first four inequalities require utility to be increasing and concave in consumption and in private SRE. The last inequality requires that everyone benefits from the socially responsible task, even if they do not wish to contribute towards it (see Examples 1–3). For some players, we allow $u_g = 0$, so they do not derive any satisfaction from private SRE. However, from the second row of (4), there is a positive measure of players for whom $u_g > 0$. Such players are either inherently socially responsible or wish to self-signal or signal to others their social responsibility credentials (see particularly Examples 1 and 3); we find it descriptively convenient to encapsulate these diverse factors into the statement that such players derive a *warm glow* from contributing to SRE.

2.2 Public policy and the government budget constraint

The government levies an income tax at the rate $t \in [0, 1]$, hence income tax revenues equal $t \int_{x=0}^{1} m dx = tM$. The tax revenues are earmarked for two purposes. (1) To subsidize the

[12] For the measure-theoretic foundations see, for example, Kharazishvili (2006).

private SRE at the rate $s \in [0,1)$, and[13] (2) to finance public SRE that takes the form of direct grants, D.[14] The government runs a balanced budget constraint, which is given by

$$tM = D + s\int_{x=0}^{1} gdx;\ s \in [0,1), t \in [0,1]. \tag{5}$$

The aggregate SRE, G, is the sum of the public and private SRE, and is given by

$$G = D + \int_{x=0}^{1} gdx. \tag{6}$$

From the binding government budget constraint (5), we can eliminate one of the three policy parameters, s, t, D. We shall, without loss of generality, choose to eliminate D. Then, we may speak of public policy as the pair s, t (with D determined residually from (5)). In some cases, depending on the context, the case $D < 0$ may be difficult to justify, in which case we must assume $D \geq 0$. For instance, in the case of charitable donations in Example 3, $D < 0$ means that the Charity gives some (or all) of its private donations to the government, which does not appear to have an empirical basis.

2.3 Optimization problem of player x

Player x, $x \in [0,1]$, maximizes (3), given the public policy, s, t, D, taking as given aggregate G, and subject to the private budget constraint that is given by

$$c(x) + (1-s)\,g(x) \leq (1-t)\,m(x) + \tau(x)\,G;\ \ \tau(x) \geq 0,\ \int_{x=0}^{1} \tau(x)\,dx \leq 1. \tag{7}$$

The LHS of (7) is total expenditure, which is the sum of private consumption, $c(x)$, whose price is normalized to unity, plus the net of subsidy private SRE, $g(x)$. Private SRE is costly because the opportunity cost of $1-s$ units of $g(x)$ is a unit of private consumption. The RHS of (7) is the post-tax income of player x plus a lump-sum transfer, $\tau(x)\,G$ (possibly zero if $\tau(x) = 0$), received by player x. G is total SRE, which is the sum of private plus public SRE, and $\tau(x)$ is the fraction of this total that is received by player x; we shall need this construction only for Example 3, in which consumer $x \in (p, 1]$ who does not have any income, receives a lumpsum transfer $\tau(x)\,G$ from the charity.

2.4 Technical Assumptions on preferences

The following technical assumptions on preferences guarantee existence of an interior solution.

[13] For the restriction $s < 1$, see Remark 1 below.

[14] As noted in Example 3, direct public grants to charities are significant. Likewise, in Example 1, direct government expenditure on cleaning the environment is significant.

1. To ensure strict concavity of the utility function, we assume for all $s \in [0, 1)$:

$$(1-s)^2 u_{cc} - 2(1-s) u_{cg} + u_{gg} < 0. \tag{8}$$

   If $u_{cg} > 0$, then, using (4), (8) holds. If $u_{cg} \leq 0$, then (8) puts a lower bound on u_{cg}.

2. To rule out the case of zero consumption, we assume that $u_c \to \infty$ as consumption tends to its lower bound from above:

$$u_c \uparrow \infty \text{ as } c \downarrow \underline{c}(x). \tag{9}$$

3. To rule out the uninteresting case where no consumer wishes to undertake private SRE, we assume

$$\begin{cases} u \text{ is extended to the boundary, } g = 0, \text{ as a } C^1 \text{ function,} \\ \text{or } u_g \uparrow \infty \text{ as } g \downarrow 0. \end{cases} \tag{10}$$

2.5 Objective function of the government

The benevolent government chooses s and t to maximize the weighted utilitarian social welfare function,

$$W(s,t) = \int_{x=0}^{1} \omega(x) v(s,t,x) \, dx, \tag{11}$$

where the individual-specific weights, $\omega(x)$, satisfy $\omega(x) > 0$ (so that each individual is socially important) and $\int_{x=0}^{1} \omega(x) \, dx = 1$. $v(s,t,x)$ is the indirect utility function of player $x \in [0,1]$, which is formally derived in Section 9. Notice that D is not included as an argument in v because, as explained above, we have used the government budget constraint (5) to eliminate D; thus, (5) is always satisfied in the analysis below.

2.6 Sequence of moves

The government moves first to credibly announce the policy parameters s, t, D in accordance with its optimization problem described in Section 2.5. The announcement is believed by all players. Then all private players simultaneously make their choices. Player x solves the optimization problem described in Section 2.3. The solution is given by backward induction.

3 Two examples of our framework

We now give two illustrative examples of our framework that elaborate on Examples 1 and 3 in the introduction; the solution is derived in Sections 6.1 and 6.2 below.

Example 4 *(Pro-environmental voluntary expenditures): A reduction in the quality of the environment has a significant effect on quality of life and, hence, by implication on the ability to enjoy private consumption. This has been well documented for developing countries in successive World Bank Development reports. However, a similar and a serious problem also exists for developed Western countries.*[15]

In this example, which formalizes Example 1 when players are consumers instead of firms, we consider the complementarities between private consumption and the overall quality of the environment. Let the utility function of player x be

$$u(c, g, G, x) = (1 - a(x)) \ln \left(c - \frac{b(x)}{G} \right) + a(x) \ln g, \tag{12}$$

where

$$0 < a(x) < 1,\ b(x) > 0,\ \frac{b(x)}{G} < (1-t)m(x). \tag{13}$$

Condition (13) ensures that player x has sufficient disposable income, $(1-t)m(x)$, to sustain a level of consumption, c, greater than $\frac{b(x)}{G}$ and also a positive level of SRE on pro-environmental actions, $g(x)$. It is straightforward to check that $u_c > 0$, $u_g > 0$, $u_G > 0$.

This example can be given the following interpretation. Aggregate private (voluntary) environmental protection expenditures, $\int_{x=0}^{1} g dx$, plus pro-environmental government expenditures, D, financed from income taxation, lead to an overall level of environmental quality, $G = D + \int_{x=0}^{1} g dx$ (higher values of G signify a cleaner environment). In turn, G is a complementary good for private consumption, c because $u_{cG} > 0$. Further, an increase in G, leads to a higher level of utility for a given level of consumption, as captured in $u_G > 0$.[16]

Individuals who make pro-environmental expenditures derive a warm glow from them ($u_g > 0$). For instance, consumers may prefer greener cars, low energy light bulbs, solar panels for electricity generation, recycling, reduction in waste, conservation of water, biodegradable products, organic foods, and locally sourced food with low travel miles. We are agnostic as to whether consumers do so for the intrinsic utility derived from such actions, or to signal to themselves or to others that they are socially responsible. Clearly many individuals may not have preferences for pro-environmental actions in which case $u_g = 0$. This can be easily accommodated in our example; indeed the next example has this feature.

[15] Consider the following quote for the European Environment Agency (2015, Chapter 5): "The degradation of the environment, through air pollution, noise, chemicals, poor quality water and loss of natural areas, combined with lifestyle changes, may be contributing to substantial increases in rates of obesity, diabetes, diseases of the cardiovascular and nervous systems and cancer. Reproductive and mental health problems are also on the rise. Asthma, allergies and some types of cancer related to environmental pressures are of particular concern for children. The World Health Organization (WHO) estimates the environmental burden of disease in the pan-European region at between 15 and 20% of total deaths, and 18 to 20% of disability-adjusted life years..."

[16] For instance, one may enjoy a walk in the neighbourhood or a drive in the countryside even more if the neighbourhood and the quality of the air are cleaner. A cleaner environment also keeps one and one's family healthier, allowing them to derive greater utility from material consumption.

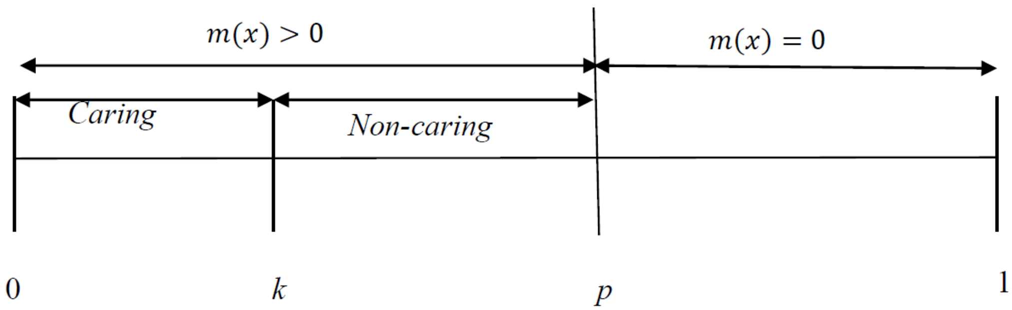

Figure 1: Types of consumers in the charitable donations example.

Example 5 *(Charitable contributions): We formalize here Example 3 in the introduction in which some players have no income and must rely on charitable contributions. The charitable funds come from donations, g, made by other 'caring' consumers with positive income and/or by tax-financed direct government redistributive payments, $D \geq 0$. Consider the indices for players k, p such that $0 < k \leq p < 1$, as shown in Figure 1. Player x has exogenously given income, $m(x)$, where $m(x) > 0$ for $0 \leq x \leq p$ and $m(x) = 0$ for $p < x \leq 1$. We assume $m(x)$ is continuous on $[0, p]$. The SRE of player x, $g(x) \geq 0$, has the interpretation of voluntary contributions to a single representative charity and D has the interpretation of direct public grants to the charity. The aggregate of all donations to charity (private and public) is given by G.*

The charity is a passive player. It has a predetermined and purely redistributive purpose, transferring an exogenously given fraction of G, $\tau(x) \geq 0$, to player x. However, the charity transfers a strictly positive amount only to consumers with no income, hence, the behavior of the charity is given by

$$\tau(x) \begin{cases} = 0 & \textit{for} \quad x \in [0, p] \\ > 0 & \textit{for} \quad x \in (p, 1] \end{cases} \quad \textit{and} \quad \int_{x=p}^{1} \tau(x)\, dx = 1. \tag{14}$$

Thus, the budget constraint of the players is given by

$$\begin{cases} c(x) + (1-s)\, g(x) \leq (1-t)\, m(x) & \textit{for} \quad x \in [0, p] \\ c(x) + (1-s)\, g(x) \leq \tau(x)\, G,\ x \in (p, 1] & \textit{for} \quad x \in (p, 1] \end{cases}. \tag{15}$$

Note that while the incomes of players $x \in [0, p]$ are taxed at the rate $t \in [0, 1]$, receipt of charitable income for consumers $x \in (p, 1]$ is not taxed.

Consumer $x \in [0, 1]$ has the utility function

$$u(c, g, G, x) = \begin{cases} \ln c + a(x)\, gG,\ a(x) > 0 & \textit{for} \quad x \in [0, k] \\ \ln c & \textit{for} \quad x \in (k, 1] \end{cases}. \tag{16}$$

We assume $a(x)$ is continuous on $[0, k]$. If $x \in [0, k]$, then consumer x cares about the plight of poor consumers $x \in (p, 1]$ with no income. However, all consumers $x \in (k, p]$ with strictly positive income are uncaring in the sense that they have purely self–regarding preferences.

4 Some intermediate results

This section collects some intermediate results that are used later in the paper and ease the burden of proof on subsequent propositions.

Lemma 1 *: There is a tax rate, t, a subsidy, s, a direct grant, D and transfers, $\tau(x)$, under which each consumer (firm) $x \in [0,1]$ can afford a level of consumption greater than the subsistence level, $\underline{c}(x)$ (can afford to operate at a scale greater than the minimum operating scale, $\underline{c}(x)$).*

Remark 1 *: If $s = 1$, then the budget constraint (7) of player x reduces to $c \leq (1-t)\,m(x) + \tau(x)\,G$. If $u_g > 0$ (see (4)) then g would become a free good so it follows that desired $g = \infty$. This is, clearly, not feasible, so, $s < 1$.*

For any player x, since $u_c > 0$, the budget constraint (7) holds with equality. Hence, we can use (7) it to eliminate c from the utility function (3). Letting $U(g, G, s, t, x)$ be the result, we have that for player x

$$U(g, G, s, t, x) = u\left((1-t)\,m(x) + \tau(x)\,G - (1-s)\,g, g, G, x\right). \tag{17}$$

From (8) and (17), it follows that U is strictly concave in own SPE,

$$U_{gg} < 0. \tag{18}$$

Hence, we can restate the optimization problem of player x, that is originally stated in Section 2.3, as follows. Given, s, t, G, player x chooses optimal g^* such that

$$g^* \in \arg\max U(g, G, s, t, x) \tag{19}$$

subject to the feasibility constraint

$$0 \leq g \leq \frac{1}{1-s}\left[(1-t)\,m(x) + \tau(x)\,G - \underline{c}(x)\right], \tag{20}$$

which follows from the budget constraint (7) and from the assumption that $g \geq 0$, $c \geq \underline{c}(x)$.

If subsistence consumption $\underline{c}(x) = (1-t)\,m(x) + \tau(x)\,G$ then the maximization problem has the unique solution, $g = 0$. But if $\underline{c}(x) > (1-t)\,m(x) + \tau(x)\,G$ then it has no solution. In this case we set $g = 0$ and $U = -\infty$. The more interesting case, $\underline{c}(x) < (1-t)\,m(x) + \tau(x)\,G$, is considered in Proposition 2, which outlines some standard properties of the solution that are based on the theorem of the maximum.

Proposition 2 *: Suppose* $\underline{c}(x) < (1-t)\,m(x) + \tau(x)\,G$*. Consider the maximization problem (19), subject to the constraint (20):*
(a) There is a unique solution, $g^*(G,s,t,x)$*,*
(b) g^* *is a continuous function of* G, s, t *and an integrable function of* x*,*
(c) $0 \leq g^*(G,s,t,x) < \frac{1}{1-s}\left[(1-t)\,m(x) + \tau(x)\,G - \underline{c}(x)\right]$*,*
(d) $0 \leq tM + (1-s)\int_{x=0}^{1} g^* dx$*,*
(e) If, in addition, the second row of (10) holds, then $g^*(G,s,t,x) > 0$*.*

In Lemma 3 below, we consider the comparative static effects of s, t, G on g^*, which follow directly from the implicit differentiation of the first order conditions. We shall interpret these conditions when they are used later.

Lemma 3 *: Suppose that* $g^*(G_0, s_0, t_0, x_0) > 0$*. Let* $c^*(G,s,t,x) = (1-t)\,m(x) + \tau(x)\,G - (1-s)\,g^*(G,s,t,x)$*. Then, at* G_0*,* s_0*,* t_0*,* x_0*,* $g^*(G_0,s_0,t_0,x_0)$*,* $c^*(G_0,s_0,t_0,x_0)$*:*
(a) $U_g = 0$*,*
(b) $(1-s)\,u_c = u_g$*,*
(c) $\frac{\partial g^*}{\partial G} = \frac{U_{gG}}{-U_{gg}} = \frac{u_{gG} + [\tau(x) - (1-s)]u_{cG} - (1-s)\tau(x)u_{cc}}{2(1-s)u_{cg} - (1-s)^2 u_{cc} - u_{gg}}$*,*
(d) $\frac{\partial g^*}{\partial s} = \frac{U_{gs}}{-U_{gg}} = \frac{u_c + [u_{cg} - (1-s)u_{cc}]g^*}{2(1-s)u_{cg} - (1-s)^2 u_{cc} - u_{gg}}$*,*
(e) $\frac{\partial g^*}{\partial t} = \frac{U_{gt}}{-U_{gg}} = \frac{m[(1-s)u_{cc} - u_{cg}]}{2(1-s)u_{cg} - u_{gg} - (1-s)^2 u_{cc}}$*.*

5 Equilibrium SRE and comparative statics

5.1 The aggregate desire for SRE

Player x determines her *desired* SRE, $g^*(G,s,t,x)$, by solving the optimization problem in Section 2.3, taking as given s, t, D and aggregate *actual* SRE, G (see (6)). The aggregate of all *desired* private and public SRE, $D + \int_{x=0}^{1} g^*(G,s,t,x)\,dx$, need not equal *actual* aggregate SRE, G. Therefore, we introduce a new function, F, which represents the *aggregate of all desires* (public and private) *to contribute to the* SRE,

$$F = D + \int_{x=0}^{1} g^*(G,s,t,x)\,dx. \tag{21}$$

The government budget constraint, (5), can be written as

$$D(s,t,G) = tM - s\int_{x=0}^{1} g^*(G,s,t,x)\,dx;\ \ s \in [0,1), t \in [0,1], \tag{22}$$

i.e., any tax revenues that are not spent by the government to subsidize private SRE can be used to make direct public contributions, D, to aggregate SRE. From (21) and (22) we can substitute out D to get

$$F(s,t,G) = tM + (1-s)\int_{x=0}^{1} g^*(G,s,t,x)\,dx. \tag{23}$$

The discussion above suggests the following definition.

Definition 1 *: The aggregate desire to contribute to SRE is defined by the mapping, $F(s,t,G)$, given by (23).*

Note two important features of the aggregate desire to contribute to SRE, F :

1. The government budget constraint is always satisfied because we have used (22) to derive (23).

2. It bears repeating that of the three policy instruments, s, t, D, we have eliminated D. Hence, in the subsequence analysis, we can without loss in generality, focus exclusively on the two instruments s, t; the instrument D is determined residually from the government budget constraint (22).

5.2 A digression on competitive market equilibrium

We digress here to highlight the analogy of our model with a static competitive market equilibrium that readers will be familiar with. Suppose that consumers are distributed over the interval $[0,1]$ and $\boldsymbol{\theta}$ is some vector of public policy (e.g., taxes, regulations, or specification of property rights). Consumer x is located at location $x \in [0,1]$. The supply curve of a single good is given by $q = S(p, \boldsymbol{\theta})$, where q is the quantity and p is the price. Consumer $x \in [0,1]$ takes the market price, p, as given and optimally chooses the individual demand, $D(p, \boldsymbol{\theta}, x)$, that maximizes utility. A price p^* is then a competitive equilibrium price if $S(p^*, \theta) = \int_{x=0}^{1} D(p^*, \boldsymbol{\theta}, x)dx$. In other words, if consumers make their demand decisions, taking as given p^*, then the sum of their demands equals supply at the price p^* that was initially taken as given. This is the sense in which there is a rational expectations in beliefs. We may then examine the comparative static effects of $\boldsymbol{\theta}$ on equilibrium p^*. Note well the following features of this analysis that will be relevant to our subsequent analysis.

(1) There is no notion of the dynamic path from one equilibrium to another, and no notion of the stability of equilibrium.[17] The focus is only on equilibrium analysis.

(2) There is no role for the belief formation process. This is achieved by assuming rational expectations of beliefs.

The analogue of p^* in our model is G^* (defined below) and the analogue of $D(p, \boldsymbol{\theta}, x)$ is $g^*(G, s, t, x)$.

[17] One may, of course, specify some ad-hoc dynamic processes and incorporate a stability analysis. However, since there are no costs of adjustment in our model, the only natural adjustment process is for players to jump straight to a new equilibrium.

5.3 Equilibrium in SRE

The relevant notion of equilibrium in our model is essentially the same as outlined in Section 5.2; in equilibrium, the actual and desired SRE are identical.

Definition 2 *(Equilibrium in SRE): Suppose that the public policy parameters are s, t and players take as given, s, t and the aggregate SRE, G.*[18] *Let $g^*(G, s, t, x)$ be the solution to the maximization problem (19), subject to the constraint (20). Then $G^* \in [0, M]$ is an equilibrium in SRE if it is a fixed point of the equation*

$$G = F(s, t, G), \tag{24}$$

where $F(s, t, G)$ is given by (23). We can rewrite the equilibrium condition (24) in two alternative but equivalent ways by using the definition of F in (21) or (23):

$$G = \begin{cases} tM + (1-s)\int_{x=0}^{1} g^*(G, s, t, x)\,dx \\ D(s, t, G^*) + \int_{x=0}^{1} g^*(G, s, t, x)\,dx \end{cases}. \tag{25}$$

Remark 2 *: In examining the comparative static effects of changes in policy, s, t, on G^*, there is no dynamic path (hence, no explicit stability analysis of equilibrium), and we assume rational expectations of beliefs without any explanation of how such beliefs may arise in actual practice. This is a standard assumption in equilibrium analysis.*

Next, we define a *regular equilibrium* which allows us to apply the implicit function theorem to compute the relevant comparative static effects of policy parameters, s, t, on the level of the equilibrium SRE, G^*.

Definition 3 *(Regular equilibrium): For given s, t, an equilibrium, G^*, is regular if $F_G(s, t, G^*) \neq 1$.*

5.4 Equilibrium analysis: Normal, neutral and perverse comparative statics

We now investigate how equilibrium aggregate SRE, G^*, responds to the policy instruments, s, t at a regular equilibrium (Definition 3). Using the implicit function theorem, we can then regard G^* as a C^1 function, $G^*(s, t)$, of s and t in that neighborhood. Let G_s^*, G_t^* be the partial derivatives of G^* with respect to the two policy instruments s and t respectively. Implicitly differentiating (24) directly gives the next proposition.

[18] Just so that there is no confusion, players also take government direct grants, D, as given. However, in our formulation we have eliminated D from the binding government budget constraint, which always holds in our analysis. Given s, t we can always recover D from the binding government budget constraint.

Proposition 4 *: Let G^* be a regular equilibrium. Then*
(a) $G_s^*(s,t) = \frac{F_s}{1-F_G}$, *(b)* $G_t^*(s,t) = \frac{F_t}{1-F_G}$, *(c)* $G_{tt}^*(s,t) = \frac{(F_{tt}+F_{tG}G_t^*)(1-F_G)+F_t(F_{tG}+F_{GG}G_t^*)}{(1-F_G)^2}$.

From Proposition 4(a),(b), if $F_G = 1$ (non-regular equilibrium) then we cannot compute the relevant partial derivatives.

We now define the critical concepts of *normal*, *neutral* and *perverse* comparative statics, which hinge on the response of the aggregate equilibrium, SRE, G^*, to changes in the subsidy, s.

Definition 4 *(Normal, neutral and perverse incentives): Comparative statics are normal if $G_s^* > 0$, neutral if $G_s^* = 0$ and perverse if $G_s^* < 0$.*

In the presence of multiple equilibria, we shall show that the comparative statics may be perverse at one equilibrium and normal at another. From Definition 4, comparative statics are perverse when an increase in subsidy, s, reduces total equilibrium SRE. In this case, one cannot use subsidies to private SRE in order to promote aggregate SRE, G^*. At the same time, temporary public SRE, D, financed through taxation, t, may be effective in boosting aggregate equilibrium SRE to a new, higher level where the comparative statics are normal. Once the economy is established at the new equilibrium, with normal comparative statics, then the temporary public SRE may be replaced by subsidies to private SRE. This is one of the critical insights of our analysis and will drive many of our policy proposals.

In order to understand why comparative statics may be perverse, consider the two partial derivatives F_G and F_s. Players make their desired private SRE decisions based on their expectations of G. Hence, from Definition 1, the response of the aggregate desired F to changes in expectations about G is given by

$$F_G(s,t,G) = \frac{\partial F(s,t,G)}{\partial G} = (1-s)\int_{x=0}^{1} \frac{\partial g^*(G,s,t,x)}{\partial G}dx. \tag{26}$$

The size of F_G (e.g., $F_G \gtrless 1$) depends on the aggregate response of individual SRE to the changes in beliefs about G. Not only does this depend on the underlying preferences, as (say) in models of exchange rate overshooting, completely rational individual behavior might lead to overshooting or overreacting (undershooting) relative to expectations and one may get the case $F_G > 1$ ($F_G < 1$). Since the shape of F is unrestricted, the reader may wish to draw a few diagrams to see how the alternative cases arise.

From Definition 1, we may also compute the partial derivative F_s :

$$F_s(s,t,G) = \frac{\partial F(s,t,G)}{\partial s} = -\int_{x=0}^{1} \frac{\partial g^*(G,s,t,x)}{\partial G}dx + (1-s)\int_{x=0}^{1} \frac{\partial g^*(G,s,t,x)}{\partial s}dx. \tag{27}$$

There are two opposing effects of an increase in subsidies on the aggregate SPE. First, a higher level of subsidies reduces the tax revenue available to the government to make public SRE (first term in (27)). Second, an increase in subsidies reduces the relative price of private SRE vis-a-vis the private consumption expenditure. If private SRE is not a Giffen good, then the second term in (27) is positive. However, it is not clear if the net effect (RHS of (27)) is positive or negative, hence, $F_s \gtrless 0$. A similar argument shows that $F_t \gtrless 0$.

Proposition 4 and Definition 4 immediately imply Corollary 5, below.

Corollary 5 *: Let G^* be a regular equilibrium.*
(a) Comparative statics are normal at G^* if (i) $F_s > 0$ and $F_G < 1$ or if (ii) $F_s < 0$ and $F_G > 1$.
(b) Comparative statics are neutral if $F_s = 0$ at G^*.
(c) Comparative statics are perverse at G^* if (i) $F_s > 0$ and $F_G > 1$ or if (ii) $F_s < 0$ and $F_G < 1$.

Table 1 provides a quick reference to the possible comparative static results with respect to subsidies ($i = s$) and taxes ($i = t$); these results are obvious from Proposition 4a,b, and Definition 4. We assume that G^* is a regular equilibrium and all partial derivatives are evaluated at G^*.

	$F_i > 0$	$F_i < 0$
$F_G < 1$	$G_i^* > 0$ (normal)	$G_i^* < 0$ (perverse)
$F_G > 1$	$G_i^* < 0$ (perverse)	$G_i^* > 0$ (normal)

Table 1: Summary of the comparative static results with respect to subsidies and taxes, i = s,t

6 Multiple equilibria and the role of expectations

In making their private SRE decisions, private players take as given the aggregate SRE, G. For any particular expectation of G, Proposition 2 shows that the privately optimal SRE of any player x is unique. However, the solution to G obtained from (24) need not be unique and it depends on the shape of the aggregate desire to contribute to the SRE, F. Each possible solution to (24) captures different potential beliefs of the private players when they decide on their private SRE.

Different expectations about the level of G give rise to different rational expectations competitive equilibria. For instance, in the utility function in (16) of Example 5, $u_{gG} > 0$. Thus, if expectations of G are high, then the marginal utility of g is high, hence private players engage in higher private SRE, validating the original expectations that G is high. Similarly, if expectations of G are low then low private SRE validates low expectations. In other words, multiple equilibria may be endemic in such problems.

Recall that at an equilibrium, G^*, $G^* = F(s,t,G^*)$. The case $F(s,t,0) < 0$ is not feasible because $G \in [0,M]$. Thus, we always have one of the two cases. (1) If $F(s,t,0) = 0$ then, clearly, 0 is an equilibrium. (2) If $F(s,t,0) > 0$ then, the equilibrium, if it exists, is an interior point. Proposition 6 considers the problem of existence and uniqueness of equilibrium.

Definition 5 *(Isolated equilibrium): An equilibrium, G^*, is isolated if there is a neighborhood of G^* in which it is the only equilibrium.*

Proposition 6 *: (a) An equilibrium, $G^* \in [0,M]$, exists and satisfies $0 \leq G^* < M$.*
(b) If $F_G < 1$ for all $G \in [0,M]$ or if $F_G > 1$ for all $G \in [0,M]$, then an equilibrium, G^*, is unique.
(c) If $[F_G]_{G^*} \neq 1$, then G^* is an isolated equilibrium.

There is nothing in the equilibrium condition (24) that imposes the monotonicity required in Proposition 6(b). Indeed, the shape of the aggregate desire to give, F, is unrestricted (as is the shape of excess demands in competitive equilibrium analysis).

Consider, for instance, the case

$$F_s > 0, F_t > 0, F_G > 0, F_{GG} < 0. \tag{28}$$

Figure 2 sketches the following specific shape of F that frequently occurs in the case of Examples 4 and 5 (see Sections 6.1 and 6.2 below for the details).

$$\begin{cases} F(s,t,G) = 0 & for\ G \in [0, G^-(s,t)] \\ F(s,t,G) > 0, F_G > 0, F_{GG} < 0 & for\ G > G^-(s,t) \end{cases} \tag{29}$$

Let us plot F against G. From (24), the equilibrium must lie on the $F = G$ line. In Panel-A of Figure 2, we hold the tax rate fixed and increase the subsidy from s_1 to s_2; this is achieved by reducing direct government grants, D to loosen the government budget constraint. In Panel-B of Figure 2, we hold fixed the subsidy and increase the tax rate from t_1 to t_2. From (29), F is strictly increasing and strictly concave in G, for $G > G^-(s,t)$. For any policy pair s,t, in Figure 2 there are two equilibria, $G^- < G^+$.

Consider Panel-A which shows the case $s_1 < s_2$. From (28) $F_s > 0$, hence it follows that the graph of $F(s_2,t_1,G)$ is strictly above that of $F(s_1,t_1,G)$. There are four equilibria: a and b corresponding to the policy parameter values (s_1,t_1); and c and d corresponding to the policy parameter values (s_2,t_1). Furthermore, we see that $F_G > 1$ at points a and c but $F_G < 1$ at points b and d. In Panel-B we have $t_1 < t_2$ and since $F_t > 0$, $F(s_1,t_2,G)$ lies strictly above $F(s_1,t_1,G)$. Equilibria a and b correspond to (s_1,t_1), while e and f correspond to (s_1,t_2). At equilibrium points a and e, $F_G > 1$, while at e and f we have $F_G < 1$.

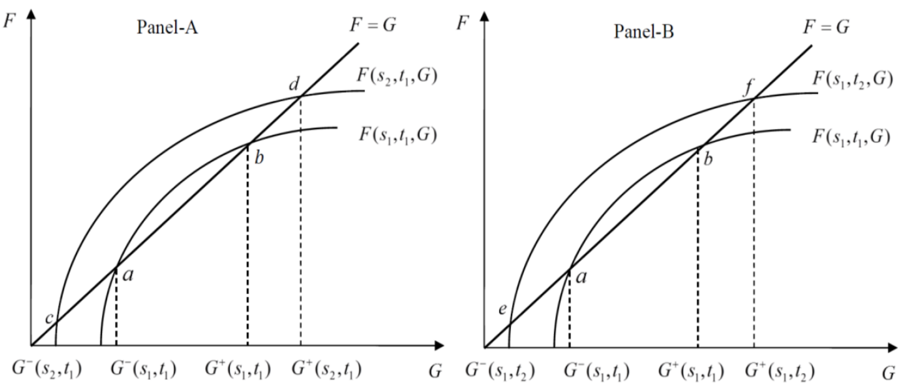

Figure 2: The case $s_1 < s_2$ with t_1 fixed (Panel-A). The case $t_1 < t_2$ with s_1 fixed (Panel-B).

Since we have $F_i > 0$, $i = s, t$, from the second column of Table 1 we know that comparative statics are *perverse* when $F_G > 1$ (points a and c in Panel-A and a and e in Panel-B) and *normal* when $F_G < 1$ (points b and d in Panel-A and e and f in Panel-B).

We now show how multiple equilibria arise naturally for the two concrete cases of Examples 4 and 5. We will return to these examples again in Sections 7 and 9, below to examine the role of optimal public policy, i.e., the optimal choice of s, t, D.

6.1 Multiple equilibria in pro-environmental actions (Example 4)

Consider the framework of Example 4 in Section 3. Define the constants B, C as:

$$B = \int_{x=0}^{1} a(x)\, m(x)\, dx + t \int_{x=0}^{1} [1 - a(x)]\, m(x)\, dx,\ C = \int_{x=0}^{1} a(x)\, b(x)\, dx. \tag{30}$$

The main results for this example are listed in Proposition 7, below.

Proposition 7 *: Assume $B^2 > 4C$.*
(a) Multiple equilibria: We have (at least) two loci of distinct real positive equilibria $0 < G^-(s,t) < G^+(s,t)$ (see Figure 3). These are given by

$$G^{\pm}(s,t) = \frac{1}{2}\left(B \pm \sqrt{B^2 - 4C}\right), \tag{31}$$

and, the levels of private pro-environmental expenditures of player $x \in [0,1]$ are

$$g^{\pm}(G^{\pm}(s,t), s, t, x) = \frac{a(x)}{1-s}\left[(1-t)\, m(x) - \frac{b(x)}{G^{\pm}(s,t)}\right],\ x \in [0,1].$$

(b) Increasing and concave desire for SRE, F: The aggregate desire for SRE, F, (i) responds positively to taxes, i.e., $F_t > 0$, (ii) is unresponsive to subsidies, i.e., $F_s = 0$,

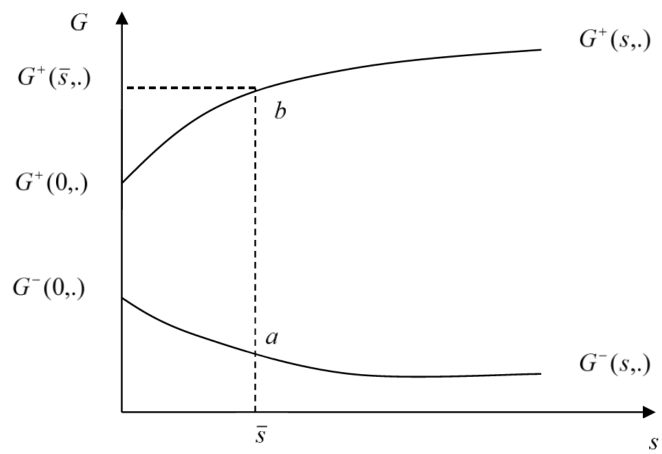

Figure 3: Two loci corresponding to $G^-(s,t)$ and $G^+(s,t)$ as we vary subsidies, s.

and, (iii) it is increasing and concave, i.e., $F_G > 0$, $F_{GG} < 0$ *(see Figure 2). Furthermore,*

$$G = \begin{cases} G^+ & \Rightarrow \quad F_G < 1 \\ G^- & \Rightarrow \quad F_G > 1 \end{cases} .$$

(c) Neutral comparative statics: Comparative statics are neutral, i.e., $G_s^{\pm} = 0$. *Thus, subsidies are ineffective in influencing SRE. Furthermore,* $G_t^- < 0$ *and* $G_t^+ > 0$.

From Proposition 7, we know that the economy has two equilibria, the low equilibrium $G^-(s,t)$ and the high equilibrium, $G^+(s,t)$. As we vary the policy parameters s, t, we trace out the loci of equilibria as shown in Figure 3. Each point on a locus gives a particular level of equilibrium SRE corresponding to a fixed set of policy parameters s, t. In Sections 7 and 9 below, we consider $G^-(s,t)$ as the status quo locus and $G^+(s,t)$ as the desired locus. We then discuss how the government can engineer a move from $G^-(s,t)$ to $G^+(s,t)$.

1. The locus of low equilibria is characterized by low private SRE contributions, causing low aggregate SRE, G^-. From (12), if aggregate SRE is low, then to achieve any specific utility level, high private consumption expenditure is needed. From the budget constraint, (7), we see that, as a consequence, less income can be contributed to the SRE, which is a complement in consumption, thus, perpetuating the low aggregate SRE.

2. The locus of high equilibria is characterized by high private SRE, causing high aggregate SRE, G^+. In turn, high aggregate SRE implies that relatively less private consumption expenditure is needed to reach any specific utility level. Hence, relatively more income is left over to contribute to SRE, perpetuating the high SRE.

6.2 Multiple equilibria in charitable contributions (Example 5)

Consider Example 5 and recall that in this case we have restricted $D \geq 0$. Depending on parameter values, this example can have multiple equilibria. We shall focus on two loci of equilibria in aggregate giving, $G^{-}(s,t) < G^{+}(s,t)$ (see Figure 3). It is convenient (but not necessary) to order consumers on $[0,1]$ in decreasing order of $a(x)\,m(x)$, i.e., $x \leq y \Leftrightarrow a(x)\,m(x) \geq a(y)\,m(y)$.

We restrict our analysis to a few tractable cases (see Proposition 8 and Corollary 9, below) that give rise to closed form solutions. The intuition is as follows. By assumption, players $x \in [0,k]$, have a caring disposition towards the poor players who have no incomes, $x \in (p,1]$. However, there is an opportunity cost of private contributions to charity in terms of the sacrifice in own-consumption. If players expect G to be high enough (made precise in Proposition 8 that requires $G > G_{\max}(s,t) = \frac{1-s}{(1-t)a(x)m(x)}$), then the marginal utility of their charitable giving is high enough for them to contribute. Otherwise they do not contribute. Players $x \in (k,p]$ are completely self-regarding so they contribute nothing although they have strictly positive incomes. Finally, players $(p,1]$ have no income, nor do they care about others, so they too do not contribute. Since some players $x \in [0,k]$ may choose not to contribute to charity, just like players in the interval $(k,1]$, it is best to group players into those who contribute and those who do not– we do so in the proof of Proposition 8 below.

Proposition 8 *: Let* $A = \int_{x=0}^{k} \frac{dx}{a(x)}$ *and* $G_{\max}(s,t) = \frac{1-s}{(1-t)a(k)m(k)}$.
(i) $[m + t(M-m)]^2 > 4(1-s)A$.
(ii) $m + t(M-m) - \sqrt{[m + t(M-m)]^2 - 4(1-s)A} > 2G_{\max}(s,t)$.
Then, for any $s \in [0,1), t \in [0,1)$ *that satisfy restrictions (i) and (ii) we have,*
(a) For $G > G_{\max}(s,t)$*, the aggregate desire to give is*

$$F(s,t,G) = tM + (1-t)\,m - \frac{1-s}{G}A. \tag{32}$$

(b) There are at least two equilibria, $G^{+}(s,t)$ *and* $G^{-}(s,t)$*, that are given by:*

$$\begin{aligned} G^{\pm}(s,t) &= \frac{1}{2}\left[m + t(M-m) \pm \sqrt{[m + t(M-m)]^2 - 4(1-s)A}\right], & (33)\\ G^{+}(s,t) &> G^{-}(s,t) > G_{\max}(s,t) > 0. & (34) \end{aligned}$$

There are two corresponding levels of charitable giving of player x*,* $g^{+}(s,t)$ *and* $g^{-}(s,t)$*, given by:*

$$g^{\pm}(G^{\pm}(s,t), s, t, x) = \begin{cases} \frac{1-t}{1-s}m(x) - \frac{1}{a(x)G^{\pm}(s,t)} & if \quad x \in [0,k] \\ 0 & if \quad x \in (k,1] \end{cases}. \tag{35}$$

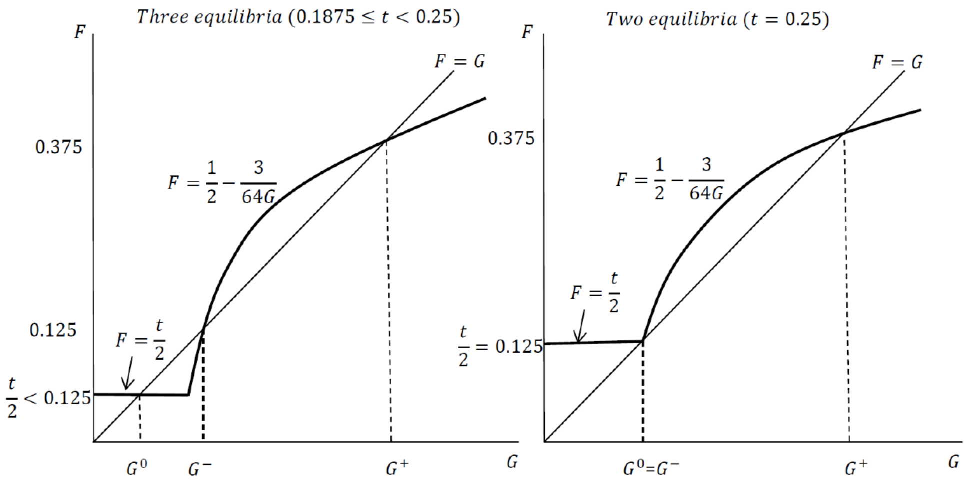

Figure 4: Three equilibria (left panel) and two equilibria (right panel) for the case shown in Corollary 9.

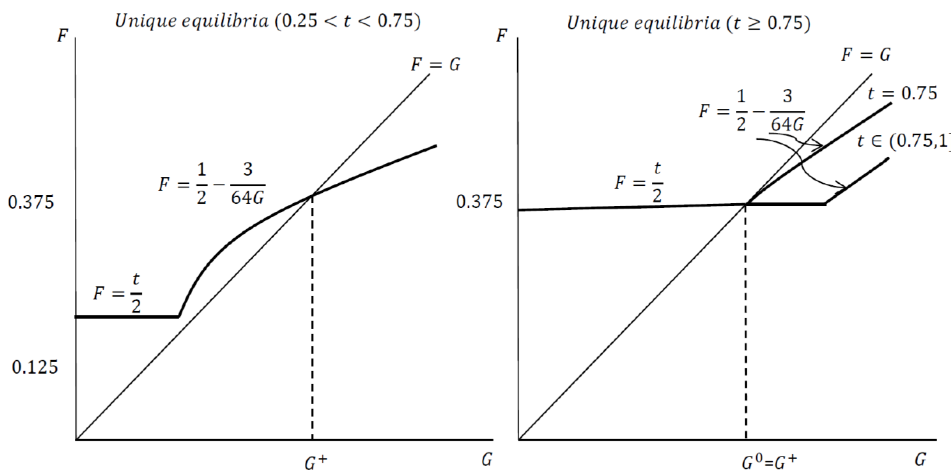

Figure 5: Two cases of unique equilibrium for the case shown in Corollary 9.

(c) Increasing and concave desire to contribute: For all G, the aggregate desire to give, F, (i) responds positively to subsides, i.e., $F_s > 0$ and, (ii) it is increasing and concave, i.e., $F_G > 0$, $F_{GG} < 0$ (Figures 2A,B). Furthermore,

$$G = \begin{cases} G^+ & \Rightarrow \quad F_G < 1 \\ G^- & \Rightarrow \quad F_G > 1 \end{cases} .$$

(d) Perverse and normal comparative statics: The comparative statics with respect to the subsidy are perverse at the locus of low equilibria and normal on the locus of high equilibria, i.e., $G_s^- < 0$, and $G_s^+ > 0$. For $m < M$ (and so $k < p$) the same holds for the comparative static result with respect to the tax rate, i.e., $G_t^- < 0$, $G_t^+ > 0$. For $k = p$, $G_t^{\pm} = 0$.

Corollary 9 below, considers the case where (1) all consumers are caring (in terms of Figure 1, $k = p$), and (2) the subsidy rate s is fixed but the tax rate t is varied. For illustrative values of the parameters, this allows us to give a rich illustration of the possible multiple equilibria in the model that are shown in Figures 4 and 5. We show that for $0.185 \leq t < 0.25$ we have three equilibria (see left panel of Figure 4); for $t = 0.25$ we have two equilibria (see right panel of Figure 4); and for $t > 0.25$ (see Figure 5) we have a unique equilibrium.

Since, we choose an arbitrary parametrization to illustrate these cases, the cut-off tax rates are illustrative rather than realistic. In each case, we highlight the intuition in Proposition 8. Namely, that if G is high enough, then the complementarity between private charitable giving and aggregate giving G in the utility function ensures that private contributions are high. The government uses the tax revenues collected to subsidize private charitable giving at a fixed exogenous rate (for endogenous policy parameters, see Section 9), and to give direct grants to charity. The full statement that covers all these cases is given in Corollary 9 below.

Corollary 9 : *Consider the special case $k = p = 0.5$ so all consumers with positive income are caring (see Figure 1), $m(x) = 1$ for $x \in [0, 0.5]$ and $m(x) = 0$ for $x \in (0.5, 1]$. Aggregate income is then $M = 0.5$. Let $a(x) = 8$ for $x \in [0, 0.5]$ and $a(x) = 0$ for $x \in (0.5, 1]$. Let the subsidy rate to private charitable giving be $s = 0.25$. Then the aggregate desire to give (Definition 1), is given by*

$$F(0.25, t, G) = \begin{cases} \frac{1}{2} - \frac{3}{64G} & if \quad G > \frac{3}{32(1-t)}, \\ \frac{t}{2} & if \quad 0 \leq G \leq \frac{3}{32(1-t)}. \end{cases}$$

Depending on the tax rate, t, we get one, two, or three, equilibria, as follows.
(a) *For $0.1875 \leq t < 0.25$ we have the following three equilibria. $G^0(0.25, t)$, $G^-(0.25, t)$ and $G^+(0.25, t)$ (see left panel of Figure 4) that we describe next in (ai)–(aiii).*[19]
(ai) Equilibrium $G^0(0.25, t) = 0.5t < 0.125$: In this equilibrium, private giving to charity is zero: $g^*(0.5t, 0.25, t, x) = 0$, $x \in [0, 1]$. Therefore, the only source of income for the poor with no income is a government grant, $D = 0.5t$, to the charity financed by taxation. The charity distributes this among the consumers with no income.
(aii) Equilibrium $G^-(0.25, t) = 0.125$: Here each consumer with positive income contributes a low amount to the charity, $g^*(0.125, 0.25, t, x) = \frac{1-4t}{3} > 0$, $x \in [0, 0.5]$. Those with no income give nothing: $g^*(0.125, 0.25, t, x) = 0$, $x \in (0.5, 1]$. The government makes the positive grant to charity, $D = \frac{16t-1}{24} > 0$. The charity uses this, together with the private donations, to give to those with no income. The government uses its tax revenue

[19] The restriction $0.1875 \leq t$ ensures that for all cases considered in part (a), we do not have the case $D < 0$, which has been ruled out by assumption. The cutoff $0.1875 \leq t$ depends on our hypothetically chosen values of k and p.

to subsidize giving and finance the grant to charity.

(aiii) $G^+(0.25, t) = 0.375$*: Here each consumer with positive income gives to charity the large amount* $g^*(0.375, 0.25, t, x) = \frac{3-4t}{3} > 0$*,* $x \in [0, 0.5]$*. Those with no income give nothing:* $g^*(0.375, 0.25, t, x) = 0$*,* $x \in (0.5, 1]$*. The government makes the positive grant to charity,* $D = \frac{16t-3}{24} > 0$*. The charity uses this, together with the private donations, to give to those with no income. The government uses its tax revenue to subsidize giving and finance the grant to charity. The grant is zero for* $t = 0.1875$*.*

(b) *For* $t = 0.25$ *we have the following two equilibria.* $G^0(0.25, 0.25) = G^-(0.25, 0.25)$ *and* $G^+(0.25, 0.25)$ *(see right panel of Figure 4). We characterize these equilibria below.*

(bi) Equilibrium $G^0(0.25, 0.25) = G^-(0.25, 0.25) = 0.125$*: In this equilibrium, private giving to charity is zero:* $g^*(0.125, 0.25, 0.25, x) = 0$*,* $x \in [0, 1]$*. The government makes the grant,* $D = 0.125$*, to the charity, financed by taxation. The charity distributes this among the consumers with no income.*

(bii) $G^+(0.25, 0.25) = 0.375$*: Here each consumer with positive income gives the relatively 'large' amount to charity,* $g^*(0.375, 0.25, 0.25, x) = \frac{2}{3} > 0$*,* $x \in [0, 0.5]$*. Those with no income give nothing:* $g^*(0.375, 0.25, 0.25, x) = 0$*,* $x \in (0.5, 1]$*. The government makes the grant to the charity,* $D = \frac{1}{24} > 0$*. The charity uses this, together with the private donations, to give to those with no income. The government uses its tax revenue to subsidize giving and finance the grant to charity.*

(c) *For* $0.25 < t \leq 1$ *we have a unique equilibrium (see Figure 5) which, depending on* t*, is given by the following three cases.*

(ci) For $0.25 < t < 0.75$*, the unique equilibrium is* $G^+(0.25, t) = 0.375$*. This case is shown in the left panel of Figure 5. Here each consumer with positive income gives to charity the amount* $g^*(0.375, 0.25, t, x) = \frac{3-4t}{3} > 0$*,* $x \in [0, 0.5]$*. The government uses its tax revenue,* $0.5t$*, to subsidize charitable giving at the rate* $s = 0.25$ *and give the charity the grant,* $D = \frac{16t-3}{24} > 0$*. The charity distributes this among the consumers with no income.*

(cii) For $t = 0.75$*, the unique equilibrium is* $G^0(0.25, 0.75) = G^+(0.25, 0.75) = 0.375$*. In this equilibrium, shown in the right panel of Figure 5 (see curve labelled* $t = 0.75$*), there is no private giving to charity:* $g^*(0.375, 0.25, 0.75, x) = 0$*,* $x \in [0, 1]$*. The government makes the grant,* $D = 0.5t$*, to the charity financed by taxation. The charity distributes this among the consumers with no income.*

(ciii) For $0.75 < t \leq 1$*, the unique equilibrium is* $G^0(0.25, t) = 0.5t > 0.375$*. In this equilibrium, shown in the right panel of Figure 5 (see curve labelled* $t \in (0.75, 1]$*), private giving to charity is zero:* $g^*(0.5t, 0.25, t, x) = 0$*,* $x \in [0, 1]$*. The government makes the grant,* $D = 0.5t$*, to the charity financed by taxation. The charity distributes this among the consumers with no income.*

7 Equilibrium analysis with multiple equilibria

Our theoretical analysis highlights the presence of multiple equilibria; see, for instance, Figures 2, 3 and 4. Suppose that we have two loci of equilibria in aggregate giving, $G^- < G^+$, as shown in Figure 3. Recall that the two loci plot the behavior of each equilibrium as the policy parameters s and t vary (Figure 3 shows only the case of variation in s).

In our analysis of multiple equilibria below, we assume that for any policy parameters s, t and any two distinct corresponding equilibria, $G^-(s,t) < G^+(s,t)$, the following holds.

$$\int_{x=0}^{1} g\left(G^-(s,t), s, t, x\right) dx \neq \int_{x=0}^{1} g\left(G^+(s,t), s, t, x\right) dx. \tag{36}$$

This requires making the entirely reasonable assumption that, ceteris-paribus, the aggregate private SRE changes when expectations of the equilibrium SRE change. Multiplying both sides of (36) by s and subtracting tM from both sides, we get

$$tM - s\int_{x=0}^{1} g\left(G^-(s,t), s, t, x\right) dx \neq tM - s\int_{x=0}^{1} g\left(G^+(s,t), s, t, x\right) dx. \tag{37}$$

From (22) and (37) we get that unless $s = t = 0$, we have

$$D\left(s, t, G^-(s,t)\right) \neq D\left(s, t, G^+(s,t)\right). \tag{38}$$

Suppose that the current public policy is given by s, t, at least one of s, t is non-zero, and the economy is at a point on the low equilibrium G^- locus. Suppose also that, *for whatever reason*, we wish to move the economy to a point on the high equilibrium locus G^+. How can public policy in the form of direct government grants, D, assist in this movement?[20] Using (25), at the equilibrium $G^-(s,t)$, the following condition must hold

$$G^-(s,t) = D\left(s, t, G^-(s,t)\right) + \int_{x=0}^{1} g^*\left(G^-(s,t), s, t, x\right) dx, \tag{39}$$

where $D\left(s, t, G^-(s,t)\right)$ is the direct government grant that ensures a balanced government budget at the policy parameters s, t, when the initial expectation of the private players is that $G = G^-(s,t)$. If, however, the initial expectations are that $G = G^+(s,t)$, then using (25), the necessary condition for $G^+(s,t)$ to be an equilibrium is that

$$G^+(s,t) = D\left(s, t, G^+(s,t)\right) + \int_{x=0}^{1} g^*\left(G^+(s,t), s, t, x\right) dx. \tag{40}$$

[20]Here we do not analyze the welfare maximizing public policy triple s, t, D; this task is carried out in Section 9. Our argument is rigorous and standard within the domain of static competitive equilibrium analysis; see Sections 5.2, 5.3 and Remark 2.

where $D\left(s,t,G^{-}(s,t)\right)$ is the direct government grant that balances the government budget at the policy vector (s,t) when the initial expectation are that $G=G^{+}(s,t)$.

Recall from Section 2.6, our assumption that for any policy parameters s,t, the government moves first to credibly choose D, which is believed by the private players who then choose their private SRE. In order to ensure that (40) holds, suppose that the government credibly commits up-front to direct public spending of

$$D\left(s,t,G^{+}(s,t)\right)=G^{+}(s,t)-\int_{x=0}^{1} g^{*}\left(G^{+}(s,t),s,t,x\right)dx. \tag{41}$$

This will rule out the equilibrium $G^{-}(s,t)$. To see this, suppose that the private players were to observe $D\left(s,t,G^{+}(s,t)\right)$ and yet hold expectations that $G=G^{-}(s,t)$. Then, for $G^{-}(s,t)$ to be sustained as an equilibrium, Definition 2 implies that (39) must hold. However, using (38), $D\left(s,t,G^{+}(s,t)\right)\neq D\left(s,t,G^{-}(s,t)\right)$. Hence, if private players expect $G=G^{-}(s,t)$ and the government commits to a contribution of $D\left(s,t,G^{+}(s,t)\right)$, then

$$G^{-}(s,t)\neq D\left(s,t,G^{+}(s,t)\right)+\int_{x=0}^{1} g^{*}\left(G^{-}(s,t),s,t,x\right)dx,$$

which rules out the equilibrium $G^{-}(s,t)$ (because (39) does not hold). From Proposition 6, we know that an equilibrium must exist. Hence, the only surviving equilibrium is $G^{+}(s,t)$ and the only reasonable expectations for the private sector to hold when they observe $D\left(s,t,G^{+}(s,t)\right)$ is that $G=G^{+}(s,t)$. This completes the proof of how public policy in the form of direct government grants, D, can move the economy from the G^{-} to the G^{+} locus.

Discussion: Our analysis highlights the powerful role of 'leadership contributions' in the presence of multiple equilibrium, which correspond in our model to direct grants by the government prior to the determination of private SRE. This is well supported by a large body of empirical evidence, particularly from leadership contributions in charitable giving, which applies to Example 5.[21] A recent field experiment from Bolivia shows the efficacy of leadership contributions made by local authorities for public goods contribution (Jack and Recalde, 2015). Our explanation, based on a multiple competitive equilibria, differs from other explanations. For instance, it differs from the strategic signaling explanation for leadership contributions that is based on information asymmetries (Vesterlund, 2003; Andreoni, 2006b) or differences in social status among the contributions of leaders and followers (Kumru and Vesterlund, 2010; Eckel et al., 2010). The explanation of political endorsements prior to elections may also be potentially explained in a reformulated model.

We wish to go further than just consider problems of engineering moves between equilibria. Does such a D exist? Once G^{+} is established, can we phase out D? Would this

[21] See, for instance, List and Lucking-Reiley, (2002), List and Rondeau (2003), Karlan and List (2007), Potters et al. (2007), Rondeau and List (2008), Huck and Rasul (2011) and Smith et al. (2015).

cause G^+ to decline or increase further? Could the economy revert back to G^-? It is questions like these that we address below. Table 1 suggests several possible cases that we could use to answer these questions. In what follows, we investigate only one case in detail: $F_s > 0, F_t > 0, F_G > 0, F_{GG} < 0$. The case $F_s = G_s^* = 0$ (neutral comparative statics) is considered in detail via Example 4 in subsection 6.1 and 9.2.1. All other cases can be dealt with in a similar fashion.

8 Stability of beliefs

Prior to conducting the welfare analysis in Section 9 below, we wish to highlight an implicit assumption in the static analysis of multiple equilibria in economics, which we call *stability of beliefs*. Indeed, no static equilibrium analysis in the presence of multiple equilibria can be conducted without making such an assumption or a similar one.

Suppose that we have two loci of equilibria, $G^- < G^+$ (see Figure 3). Suppose that, for whatever reasons, G^+ is considered more desirable than G^-. Ideally, in considering a move from G^- to G^+, one would like to adopt an explicitly dynamic model in which beliefs endogenously evolve over time as individuals engage in learning. Yet there is no consensus in economics on which learning model to use.[22] In the absence of a satisfactory resolution of this problem, one needs to fall back on some assumptions about the stability of beliefs. This is an issue in all static economic models and, indeed in all equilibrium models with structural dynamics as well.

As an illustrative example, consider a coordination problem where people decide whether to drive on the left or the right of the road. People in the UK (respectively USA) drive on the left (respectively right), typically safe in their beliefs that all others will follow the rule. This example illustrates the idea that when a good equilibrium gets established, people come to expect that it will prevail. In other words, once established, beliefs may exhibit *inertia to change*, i.e., they may be *stable*. In the context of charitable contributions (see Example 5), say, once the Red Cross is established as a *large charity* (in the sense of attracting high private and public contributions) then people are likely to believe that it will be large next period. We call this the *stability of beliefs* assumption.

The stability of beliefs assumption is illustrated in Figure 3. Suppose that we have two loci of equilibria, $G^-(s,t)$ and $G^+(s,t)$ with t fixed and s variable, such that $G^-(s,t) < G^+(s,t)$ for any s,t. Suppose that the comparative statics along the G^- locus are perverse, i.e., $G_s^- < 0$, while those along G^+ are normal, i.e., $G_s^+ > 0$. This is reflected in the shapes of the two loci (see Definition 4); G^+ responds positively to an increase in subsidies while G^- responds negatively.

[22] Most traditional learning models do not perform too well when taken to the evidence although several behavioral models of learning do much better; see Camerer (2003) and Dhami (2016).

Suppose that we begin at point 'a' on the G^- locus, where $G_s^- < 0$. As we vary s we will trace out various points along the G^- locus. Now suppose that we can somehow engineer a jump to the G^+ locus, say, to point 'b' where $G_s^+ > 0$ (point b need not be directly above point a). Once the economy gets established on the G^+ locus (at point, b, say) for a sufficient length of time, then the *stability of beliefs* requires that as we adjust s, the economy moves along the G^+ locus.[23] Without this assumption, usually left implicit, it would be difficult to make any significant progress in any equilibrium model in economics (whether static or with structural dynamics) even with a unique equilibrium.

8.1 An Example

Suppose that the objective of the government is to use the policy parameters, s, t, D to move the economy from the G^- locus with perverse comparative statics to a locus of high equilibria with normal comparative statics, G^+. We will also note the limitations of this approach in the end and then conduct an explicit welfare analysis of the optimal policy choice in Section 9 below.

Consider the situation shown in Figure 2 for the case $F_s > 0, F_t > 0, F_G > 0, F_{GG} < 0$. In particular, the aggregate desire to give function, F is given in (29). Let us begin with the policy s_1, t_1 and the economy is initially at point a in Panel-A, with $G = G^-(s_1, t_1)$. Now the government wishes to shift the economy to equilibrium b in Panel-A, which corresponds to $G = G^+(s_1, t_1)$. Since $F_s > 0$ and $F_G > 1$ at point a, it follows, from Corollary 5ci (or see Table 1), that comparative statics are perverse ($G_s^- < 0$).

Suppose that the equilibrium $G = G^-(s_1, t_1)$ has had time to get established, so using the argument about the *stability of beliefs* and Figure 3, any change in the parameter s will move the economy along the G^- locus. Hence, an *increase* in s, from s_1 to s_2, will *reduce* aggregate giving, from $G^-(s_1, t_1)$ to $G^-(s_2, t_1)$, reflecting the case $G_s^- < 0$. A *decrease* in s would move the economy to another point on the G^- locus but will not enable the economy to move to the G^+ locus. Thus, subsidies are ineffective in moving the economy from a to the desired point b.

In Section 7, we showed that by keeping fixed the policy parameters at s_1, t_1, $s_1 \in (0, 1)$ if the government gives a direct grant $D(s_1, t, G^+(s_1, t_1))$ towards the SRE, then the only surviving equilibrium is $G^+(s_1, t_1)$. In this case, the economy moves from point a to b in Panel-A. We now use a slight variation of this method to demonstrate how the economy may be moved from a to b by employing instead an increase in taxes from t_1 to t_2; see Panel-B. In actual practice, very little is known about how players change beliefs in such

[23] In order to make this argument completely rigorous one would have to specify precisely what a "sufficient length of time" is. But this would require a dynamic model of learning and updating of beliefs that is subject to the caveats mentioned above. This observation applies to any situation of multiple equilibria in static models in economics.

situations. Since income tax changes are particularly salient, changing the policy regime from (s_1, t_1) to (s_1, t_2) may have greater salience in altering private beliefs about which equilibrium, G^- or G^+, is likely at the new policy parameters. We proceed as follows.

Starting from the policy parameters (s_1, t_1), the government (1) announces a new policy (s_1, t_2), and (2) credibly commits to the leadership contributions, $D(s_1, t_2, G^+(s_1, t_2))$. From (38), we know that $D(s_1, t_2, G^-(s_1, t_2)) \neq D(s_1, t_2, G^+(s_1, t_2))$. Thus, private agents cannot, in equilibrium, hold expectations that $G = G^-(s_1, t_2)$ because

$$G^-(s_1, t_2) \neq D\left(s_1, t_2, G^+(s_1, t_2)\right) + \int_{x=0}^{1} g^* \left(G^-(s_1, t_2), s_1, t_2, x\right) dx,$$

hence, the equilibrium condition required for G^- to be an equilibrium, (39), does not hold. Thus, the only surviving equilibrium is $G^+(s_1, t_2)$ because

$$G^+(s_1, t_2) = D\left(s_1, t_2, G^+(s_1, t_2)\right) + \int_{x=0}^{1} g^* \left(G^+(s_1, t_2), s_1, t_2, x\right) dx. \tag{42}$$

In Panel-B, the economy, thus, moves from the equilibrium point a to the equilibrium point f. Since $F_G < 1$ at point f, thus, the comparative statics at the equilibrium $G^+(s_1, t_2)$ are normal.

Once the equilibrium $G^+(s_1, t_2)$ with normal comparative statics is established for a sufficient period, the *stability of beliefs* ensures that as we adjust the policy parameters s, t, the economy moves along the G^+ locus. Thus, we can adjust s_1, t_2 back to the original policy parameters s_1, t_1, which allows us to achieve the objective of moving to the equilibrium $G^+(s_1, t_1)$ corresponding to point b in Panel-A.

The analysis in this section has not specified the optimization problem of the government. Hence, there is no analysis of the trade-offs in using the three policy instruments, s, t, D. For instance, it might be that players derive a warm glow from making private SRE, as in the case of charitable giving. However, government direct grants in the form of public SRE generate no such warm glow to private players. Suppose that the objective of the government is to maximize a weighted utilitarian function (11) to determine the welfare maximizing choice of the policy instruments, s, t, D. In this case, an extra dollar of private SRE, g, or public SRE, D, is no longer fungible from a welfare point of view, although both lead to the same increase in total SRE, G. This shows up the limitations of the analysis without an explicit welfare criterion. These shortcomings are addressed in Section 9 by performing an explicit welfare analysis.

9 Welfare analysis with multiple equilibria

We now ask what should be the optimal policy parameters s, t when the objective is to maximize a well defined social welfare function. We divide the discussion into two parts.

In Section 9.1, we consider the optimal public policy at a given equilibrium. So if $G^{-}(s,t)$ and $G^{+}(s,t)$ $(G^{-}(s,t) < G^{+}(s,t))$ are two equilibria corresponding to the policy s,t, then we consider the optimal policy separately at each equilibrium. However, welfare might be even higher when we allow policy to explicitly move from one equilibrium to another; this interesting question is taken up in the Section 9.2 below.

9.1 Welfare analysis without engineering moves between equilibria

In this section, we focus on welfare analysis at a given equilibrium even if we have multiple equilibria.

Substituting the optimal private SRE of player x, $g^{*} = g^{*}(G^{*}(s,t), s, t, x)$, and the equilibrium aggregate SRE, $G^{*} = G^{*}(s,t)$ in the individual utility function (17) gives player x's indirect utility function

$$v(s,t,x) = u((1-t)m(x) + \tau(x)G - (1-s)g^{*},\ g^{*},\ G^{*}, x). \tag{43}$$

Differentiating (43), using Lemma 3b, or using the envelope theorem, gives

$$v_s = g^{*}u_c + G_s^{*}(\tau u_c + u_G), \tag{44}$$

$$v_t = -mu_c + G_t^{*}(\tau u_c + u_G), \tag{45}$$

where the partial derivatives G_s^{*}, G_t^{*} are given by Proposition 4a,b, respectively.

Using (11), the government chooses s and t to maximize the weighted utilitarian social welfare function,

$$W(s,t) = \int_{x=0}^{1} \omega(x)v(s,t,x)\,dx. \tag{46}$$

Differentiating (46), using the chain rule, and (44), (45), we get

$$W_s = \int_{x=0}^{1} \omega g^{*}u_c dx + G_s^{*}\int_{x=0}^{1} \omega(\tau u_c + u_G)\,dx, \tag{47}$$

$$W_t = -\int_{x=0}^{1} \omega m u_c dx + G_t^{*}\int_{x=0}^{1} \omega(\tau u_c + u_G)\,dx, \tag{48}$$

where subscripts denote appropriate partial derivatives.

Propositions 10, 11 and 12, below, derive the optimal mix between private and public SRE in different cases. These propositions are used extensively below and are crucial in determining the optimal public policy at different equilibria.

Proposition 10, below, implies that in a social optimum where $G_t^{*} \leq 0$, no government intervention is needed, *warm glow* suffices to maximize social welfare.

Proposition 10 *: Let (s^*, t^*) be a social optimum where $G_t^* \leq 0$ and $g^* > 0$ for a subset of consumers of positive measure. Then $s^* = t^* = D^* = 0$. So all contributions to SRE are private contributions.*

Proposition 11, below, shows that if subsidies are effective then no direct government grant is needed. This is because when *private* SRE replace an identical amount of *public* SRE, D, welfare improves on account of the warm glow received by private actors.

Proposition 11 *: Let (s^*, t^*) be a social optimum where $G_s^* \geq 0$ and $g^* > 0$ for a subset of consumers of positive measure. Then $D^* = 0$ and s^* takes its maximum possible value in the feasible interval $[0, 1)$.*

The intuition behind the next result, Proposition 12, can be gleaned from the results summarized in Table 1. Both cases considered in Proposition 12 ensure that the comparative statics are normal, i.e., $G_s^* \geq 0$, hence, private SRE can be encouraged through the use of subsidies. Since private SRE leads to warm glow, and an improvement in welfare, it is optimal to generate all SRE through private SRE, rather than by direct government grants.

Proposition 12 *: Let G^* be a social optimum with $g^* > 0$ for a subset of consumers of positive measure. Suppose at G^* (i) $F_s \geq 0$ and $F_G < 1$, or (ii) $F_s \leq 0$ and $F_G > 1$. Then $D^* = 0$.*

9.2 Welfare analysis and engineering moves between equilibria

The results in Section 9.1 describe the optimal policy at a given equilibrium locus, $G^-(s, t)$ or $G^+(s, t)$. We can now use these results to revisit the question that we posed earlier. Suppose that comparative statics at G^- are perverse and we engineer a move to the G^+ locus. Once the economy gets established on the G^+ locus, where comparative statics are normal, i.e., $G_s^+ > 0$, what is the optimal policy? From Proposition 11 we know that, in this case, s attains its maximum value and, for the reasons given above, the government makes no direct contributions to charity, i.e., $D = 0$. We can say no more in an equilibrium analysis. The class of questions that deal with the mechanics of how the direct grant from the government is phased out and subsidies increased to attain their maximum values (e.g., gradually or suddenly), require a dynamic model that is outside the scope of our paper.

But why might we wish to engineer a move from G^- to G^+ in the first place? Societies do not always make choices by maximizing the weighted utilitarian objective functions of static utilities. Using cheaper but more polluting technologies may maximize the current utilitarian payoffs. However, they may also lead to a reduction in future societal

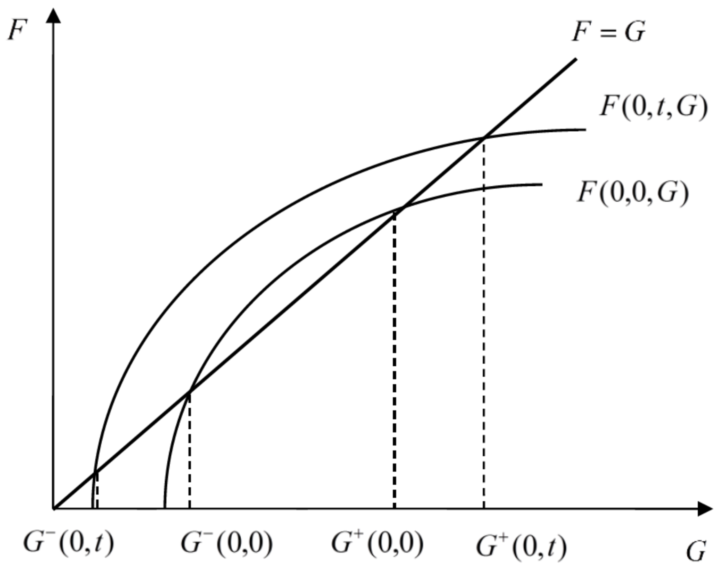

Figure 6: Engineering moves between multiple equilibria.

welfare. Thus, societies and international organizations often set targets to achieve better environmental quality for the future. This typically takes the form of making costly pro-environmental choices today that reduce current utilitarian welfare but improve future welfare. In this case, the analysis in Section 9.1 is incomplete. The most satisfactory alternative would be to model the trade-off between current costs and future benefits in a dynamic welfare framework in the presence of multiple equilibria. We leave this extremely ambitious task for future research but focus on an important aspect of this problem in this section.

Suppose that the economy is currently at a low equilibrium G^- corresponding to some policy parameters s, t, D. Suppose also that it is desirable to move the economy to the higher G^+ locus. This could arise due to the trade-off between the current costs and future benefits in some meta-welfare analysis or perhaps international agreements on climate change make such an action unavoidable. How can we then use the insights of Propositions 10, 11, 12, and the analysis in Section 9.1 to derive socially optimal policy? In order to be concrete, we use Examples 4 and 5 that were considered in sections 6.1 and 6.2 above to address this question.

9.2.1 Optimal policy intervention in Example 4

We now consider optimal public policy for Example 4. From Proposition 7, we know that the economy has two loci of equilibria and the situation is as described in Figure 6. Suppose that the economy is at the locus of low equilibria, G^-, and that it is socially desirable to move the economy to the locus of high equilibria, G^+. How can this transition be achieved? From Proposition 7c, we know that incentives in the form of a subsidy will not work because $G_s^* = 0$. From Proposition 7c, $G_t^- < 0$, so, from Proposition 10, it would appear that, at the low equilibrium G^-, the best feasible policy is no intervention: $s = t = 0$, leaving the economy at the low equilibrium $G^-(0,0)$. However, longer-term considerations may require a move to the G^+ locus and using public policy optimally. We show how this is done in two steps.

Step-I: Use the method suggested in Section 7 to engineer a move from the equilibrium $G^-(0,0)$ to the equilibrium $G^+(0,t)$: The policy parameters are set as follows. A zero subsidy, $s = 0$, and, as in the example in Section 8.1, a direct grant in the form of leadership contributions, $D(0,t,G^+(0,t))$, from the government to the charity, financed by an income tax at the rate t :

$$D\left(0,t,G^+(0,t)\right) = G^+(0,t) - \int_{x=0}^{1} g^*\left(G^+(0,t),0,t,x\right)dx.$$

This gives rise to the $F(0,t,G)$ locus in Figure 6. As explained in Section 7 this rules out the equilibrium $G^-(0,t)$, leaving $G^+(0,t)$ as the only equilibrium.

Step-II: Use the welfare analysis in Section 9.1 to choose the optimal public policy at the new equilibrium: Once the economy establishes itself at the high equilibrium, $G^+(0,t)$, the stability of beliefs assumption (see Section 8 above) ensures that the economy is on the G^+ locus. The fiscal parameters, s,t can then be adjusted to their socially optimum values, as required in our analysis in Section 9.1. We now turn to this issue.

In the low equilibrium, G^-, we have seen above that the optimal policy solution is $s = t = 0$. From Proposition 7c, we know that at the high equilibrium, $G_t^+ > 0$, and so, the comparative static results are reversed from the low equilibrium. The intuition for $G_t^+ > 0$ is that each individual engages in low private SRE, relative to the social optimum. In this example, G is complementary with private consumption of other consumers. However, this externality on other consumers is ignored by any single consumer, thus, taxes here have the flavour of Pigouvian corrective taxes. The tax revenues are then used to subsidize private SRE to ensure socially optimum SRE.

The optimal tax rate, which can be found by setting W_t in (48) equal to zero, balances the loss in private consumption against the gain arising from the additional amount of G. Also, from (47), $W_s > 0$ (because $G_s^+ > 0$), hence, it is welfare improving to provide additional subsidies. Thus, all tax revenues are used to finance subsidies on private SRE. In the socially optimal solution at the high equilibrium, G^+, therefore, $s > 0$, $t > 0$ while $D = 0$ (see Proposition 11).

9.2.2 Optimal policy intervention in Example 5

We now consider the optimal policy intervention in Example 5. From Proposition 8, the equilibria are as shown in Figures 3, 4 and 5. Suppose that an economy is on the locus of low equilibria, G^-, and also suppose that it is socially desirable to move it to the locus of high equilibria, G^+. How can this be done?

From Proposition 8(d), on the locus of low equilibria, we have $G_s^- \leq 0$. Hence, it would appear that the best policy is no intervention, i.e., $s = t = 0$, leaving the economy at the low equilibrium, $G^-(0,0)$; see the $F(0,0,G)$ locus in Figure 6. However, an alternative policy is possible. Since the shape of the loci is as given in Figure 6, exactly the same two steps that are described in Section 9.2.1 will be employed.

Step-I: As in Step-I of Section 9.2.1, engineer a move from the equilibrium $G^-(0,0)$ to the equilibrium $G^+(0,t)$.

Step-II: Choose the optimal policy at the new equilibrium $G^+(0,t)$: Once the economy spends adequate time at the high equilibrium, $G^+(0,t)$, the stability of beliefs assumption (see Section 8) ensures that the economy is on the G^+ locus. The fiscal parameters, s, t can then be adjusted to their socially optimum values, following the analysis in Section 9.1. Having arrived at the G^+ locus, we consider two cases.

1. Let $k < p$, so that not all consumers with positive income are caring. From Proposition 8d, $G_s^+ > 0$, $G_t^+ > 0$, i.e., the comparative static effects are normal at the high equilibrium. Depending on the parameter values, the optimal tax rate may be positive, in which case it can be found by setting $W_t = 0$ in (48). From Proposition 8c, at the high equilibrium G^+, $F_s > 0$, and $F_G < 1$, so, Proposition 12 implies that $D = 0$. Thus, once the economy has moved from the G^- locus to the G^+ locus, the direct grant from the government is phased out. In the new, socially optimal equilibrium, all SRE are private (because only private contributions are associated with warm glow) and all income tax revenue is used to subsidize private donations to charity.

2. Let $k = p$, so that all consumers with positive income are caring. Hence, from Proposition 8d, $G_t^+ = 0$. It follows, from Proposition 10, that once the economy has established itself on the G^+ locus, $s = t = 0$. Thus, once the (one-off) direct government grant (financed by an income tax) has shifted the economy from the bad equilibrium, G^-, to the good equilibrium, G^+, no further government intervention is needed and the entire SRE arises from voluntary private social expenditures. At the new optimum, equilibrium is described by $G^+(0,0) = \int_{x=0}^{1} g^+(0,0,G^+(0,0),x)dx$.

10 Conclusions

We show that many diverse problems in economics such as the problem of the commons, environmental degradation, and charitable contributions in which there is a large number of individually small economic agents, have a common underlying structure. We study the properties of the set of rational expectations competitive equilibria in a general framework that encompasses these problems and potentially many others. Within this framework, multiple equilibria are endemic, which leads to an important role for public policy in moving the economy from less to more desirable equilibria. Furthermore, the comparative statics at the undesirable equilibrium may be perverse in the sense that positive incentives worsen matters. In contrast, the comparative statics at the desirable equilibrium may be normal, i.e., private activity responds positively to incentives.

We show that temporary direct government grants financed by taxation allow public policy to engineer moves from undesirable to desirable equilibria. This is particularly the case when comparative statics are perverse at a low equilibrium, so incentives such as subsidies cannot be relied upon to motivate private actions. This isolates a new role for leadership contributions in engineering moves between multiple equilibria. We also perform a welfare analysis and examine the optimality of public policy and alternative mixes of private and public expenditures on socially responsible activities. When comparative statics are normal, socially responsible activities should be entirely funded by private contributions, possibly subsidized through taxation. This may be welfare improving because private contributions to social expenditures generate warm glow, while direct government grants do not.

Throughout we focus on equilibrium analysis. The dynamics of time paths from one equilibrium to another involve fundamental questions about the precise learning mechanisms to be used. Although progress on learning mechanisms is being made, and in due course such mechanisms may enrich our model, currently such issues lie beyond the scope of this paper.

11 Appendix: Proofs.

Proof of Lemma 1: Consider $s = 0, t = 1, D = M$, $\tau(x) = \frac{1}{G}\left[\underline{c}(x) + M - \int_{x=0}^{1} \underline{c}(x)\,dx\right]$, then $\int_{x=0}^{1} \tau(x)\,dx = \frac{M}{G} = \frac{M}{D+\int_{x=0}^{1} g dx} = \frac{M}{M+\int_{x=0}^{1} g dx} \leq 1$ and, using (2), $\tau(x)\,G = \underline{c}(x) + \left(M - \int_{x=0}^{1} \underline{c}(x)\,dx\right) > \underline{c}(x)$, for each player $x \in [0,1]$. From (7), the budget constraint for player x becomes $c(x) + g(x) \leq \tau(x)\,G < \underline{c}(x)$ which, clearly, can be satisfied with $c > \underline{c}(x)$. ■

Proof of Proposition 2:

(a) U, as a function of g, is a continuous and strictly concave function defined on a compact interval; recall (17), (18). Hence, a maximum exists and is unique.

(b) $u(c, g, G, x)$ is a C^2 function of c, g, G for $g > 0$ and $c > \underline{c}(x)$, and u, τ, m, $\underline{c}$ are integrable in x. Hence, the theorem of the maximum immediately gives the result.

(c) From the restriction (9) we know that $c^*(x) > \underline{c}(x)$ and so
$g^* < \frac{1}{1-s}[(1-t)m(x) + \tau(x)G - \underline{c}(x)]$.

(d) Obvious.

(e) (10) implies that the marginal utility of the SRE is infinite as it approaches zero, hence, $g^* > 0$. ■

Proof of Lemma 3: If $g^* > 0$, then, in the light of Proposition 2c, g^* is an interior maximum and part (a) follows. Part (b) follows from (17). Appealing to the implicit function theorem, or differentiating the identity, $(1-s)u_c = u_g$, establishes parts (c), (d) and (e). ■

Proof of Proposition 6: Let $H(s, t, G) = G - F(s, t, G)$. Recall that at an equilibrium, G^*, $G^* = F(s, t, G^*)$.

(i) $F(s, t, 0) < 0$ is not feasible because $G \in [0, M]$.

(ii) If $F(s, t, 0) = 0$ then, clearly, 0 is an equilibrium.

(iii) Now, suppose that $F(s, t, 0) > 0$. Since $F(s, t, 0) > 0$, it follows that $H(s, t, 0) = -F(s, t, 0) < 0$. Given the budget constraint, the aggregate desire to give cannot exceed the total resources of the economy, $F(s, t, G) \leq M$. Further, assumption (9) implies that $c^*(x) > 0$ if $m(x) > 0$. Hence, $F(s, t, G) < M$ and in particular $F(s, t, M) < M$. So, $H(s, t, M) = M - F(s, t, M) \geq 0$. Since $H(s, t, G)$ is continuous, it follows that $H(s, t, G^*) = 0$, for some $G^* \in [0, M]$. Hence G^* is an equilibrium and $0 \leq G^* \leq M$.

(b) Since H is continuous, the set of equilibria $\{G : H(s, t, G) = 0\}$ is a closed subset of $[0, M]$ and, hence, is compact. By (a), it is not empty. Hence, it must contain minimum and maximum elements, $G^*_{\min}$ and $G^*_{\max}$, respectively, $G^*_{\min} \leq G^*_{\max}$, $H(s, t, G^*_{\min}) = 0$, $H(s, t, G^*_{\max}) = 0$. If $G^*_{\min} < G^*_{\max}$ then, since H_G is continuous, it follows that $H_G = 0$ for some $G \in (G^*_{\min}, G^*_{\max})$. Thus, $F_G = 1$ for some $G \in (G^*_{\min}, G^*_{\max})$, which cannot be the case because of the restriction $F_G < 1$ or $F_G > 1$ in Proposition 6(b). Hence, $G^*_{\min} = G^*_{\max}$, which implies that the equilibrium is unique.

(c) Suppose $[F_G]_{G^*} \neq 1$. Then $[H_G]_{G^*} \neq 0$. Hence, either $[H_G]_{G^*} < 0$ or $[H_G]_{G^*} > 0$. Since H_G is continuous, it follows that $H_G < 0$ (or $H_G > 0$) in some neighborhood of G^*. Thus, using an argument similar to that deployed in (b), G^* can be shown to be an isolated equilibrium. ■

Proposition 4: Let G^* be a regular equilibrium. By Definition 2, $G^* = F(s, t, G^*)$. Differentiating this implicitly, and rearranging, gives the required results. ■

Corollary 5: Immediate from Proposition 4 and Definition 4. ■

Proposition 7: Applying the first order condition of player x in Lemma 3b to the utility function (12), and using the budget constraint (7), which must hold with equality, gives the optimal choices of player x :

$$g^{*}(G,s,t,x)=\frac{a\left(x\right)}{1-s}\left[\left(1-t\right)m\left(x\right)-\frac{b\left(x\right)}{G}\right], \tag{49}$$

$$c^{*}(G,s,t,x)=\left[1-a\left(x\right)\right]\left[\left(1-t\right)m\left(x\right)-\frac{b\left(x\right)}{G}\right]+\frac{b\left(x\right)}{G}. \tag{50}$$

From (13) and (49), (50), we see that $g^{*}(G,s,t,x)>0$ and $c^{*}(G,s,t,x)>\frac{b(x)}{G}$. Furthermore, it is straightforward to verify that the second order conditions also hold. Hence, given s,t,G; $g^{*}(G,s,t,x)$, $c^{*}(G,s,t,x)$ maximize utility (12) subject to the budget constraint (7), and are unique.

Substituting from (49) into (23), the aggregate desire to give, $F(s,t,G)$ is:

$$F(s,t,G)=\int_{x=0}^{1}a\left(x\right)m\left(x\right)dx+t\int_{x=0}^{1}\left[1-a\left(x\right)\right]m\left(x\right)dx-\frac{1}{G}\int_{x=0}^{1}a\left(x\right)b\left(x\right)dx. \tag{51}$$

From (51) we get:

$$\begin{aligned}F_{s}&=0,\ F_{t}=\int_{x=0}^{1}\left[1-a\left(x\right)\right]m\left(x\right)dx>0, F_{G}=\frac{1}{G^{2}}\int_{x=0}^{1}a\left(x\right)b\left(x\right)dx>0,\\ F_{GG}&=-\frac{2}{G^{3}}\int_{x=0}^{1}a\left(x\right)b\left(x\right)dx<0.\end{aligned} \tag{52}$$

From (52) and Proposition 4a, we get

$$G_{s}^{*}=\frac{F_{s}}{1-F_{G}}=0. \tag{53}$$

To make further progress, we need to determine the equilibrium values of G. From (23), (51) and Definition 2, the equilibrium values of G are the solutions to the equation

$$G=\int_{x=0}^{1}a\left(x\right)m\left(x\right)dx+t\int_{x=0}^{1}\left[1-a\left(x\right)\right]m\left(x\right)dx-\frac{1}{G}\int_{x=0}^{1}a\left(x\right)b\left(x\right)dx. \tag{54}$$

Substituting (30) in (54) we get

$$G^{2}-BG+C=0, \tag{55}$$

with solutions

$$G^{\pm}\left(s,t\right)=\frac{1}{2}\left[B\pm\sqrt{B^{2}-4C}\right]. \tag{56}$$

If $B^{2}<4C$, then no equilibrium exists. If $B^{2}=4C$ then a unique equilibrium exists, given by $G=\frac{B}{2}=\sqrt{C}$. But then, from (30), (52), $F_{G}=1$. In this case, neither G_{s}^{*} nor G_{t}^{*} are defined, see Proposition 4a,b. Hence, the only interesting case is when $B^{2}>4C$.

In this case, (55) has two distinct real positive roots:

$$0 < G^{-}(s,t) < G^{+}(s,t)\,, \tag{57}$$

and, correspondingly,

$$g^{\pm}(G^{\pm}(s,t)\,,s,t,x) = \frac{a(x)}{1-s}\left[(1-t)\,m(x) - \frac{b(x)}{G^{\pm}(s,t)}\right],\ x \in [0,1]\,. \tag{58}$$

Using the fact that for real numbers $a > b > 0$: $\sqrt{a-b} > \sqrt{a} - \sqrt{b}$, as well as (52), (30) and (56)–(57), we get

$$G = G^{+} \Rightarrow F_G < 1,\ G = G^{-} \Rightarrow F_G > 1. \tag{59}$$

From (52), (59) and Proposition 4b, we get $G_t^{+} > 0$, $G_t^{-} < 0$. ■

Proof of Proposition 8: In order to simplify the calculations for this example, we first define some useful functions followed by a Lemma that these functions are well defined.

Definition 6 *: Define the functions $G_{\min}(s,t)$, $G_{\max}(s,t)$, $x_0(G,s,t)$, $m(G,s,t)$, $A(G,s,t)$ as follows:*
(a) Let $G_{\min}(s,t) = \frac{1-s}{(1-t)a(0)m(0)}$, $G_{\max}(s,t) = \frac{1-s}{(1-t)a(k)m(k)}$.
(i) For $G < G_{\min}(s,t)$, let $x_0(G,s,t) = 0$.
(ii) For $G > G_{\max}(s,t)$, let $x_0(G,s,t) = k$.
(iii) For $G \in [G_{\min}(s,t)\,, G_{\max}(s,t)]$, let $x_0(G,s,t) = \min\left\{x \in [0,k] : a(x)\,m(x) = \frac{1-s}{(1-t)G}\right\}$.
(b) $m(G,s,t) = \int_{x=0}^{x_0(G,s,t)} m(x)\,dx$, $m = \int_{x=0}^{k} m(x)\,dx$.
(c) $A(G,s,t) = \int_{x=0}^{x_0(G,s,t)} \frac{dx}{a(x)}$, $A = \int_{x=0}^{k} \frac{dx}{a(x)}$.

We first need to proved two Lemmas to generate useful intermediate results.

Lemma 13 *: (a) The function $x_0(G,s,t)$ (Definition 6) is well defined, $x_0(G,s,t) \in [0,k]$.*
(b) The functions $m(G,s,t)$, $A(G,s,t)$ (Definition 6) are well defined.
(c) $x > x_0(G,s,t) \Rightarrow a(x)\,m(x) \leq \frac{1-s}{(1-t)G}$.

Proof of Lemma 13: All we need to check is the soundness of Definition 6a(iii). The rest then follows. Since $a(x)\,m(x)$ is non-increasing in $x \in [0,1]$, it follows that $a(k)\,m(k) \leq a(0)\,m(0)$. Hence, $G_{\min}(s,t) \leq G_{\max}(s,t)$. Hence, $[G_{\min}(s,t)\,, G_{\max}(s,t)] \neq \emptyset$. Let $G \in [G_{\min}(s,t)\,, G_{\max}(s,t)]$. Let $X(G,s,t) = \left\{x \in [0,k] : a(x)\,m(x) = \frac{1-s}{(1-t)G}\right\}$. Since $G_{\min}(s,t) = \frac{1-s}{(1-t)a(0)m(0)}$, we get $0 \in X(G_{\min}(s,t)\,,s,t)$. Since $G_{\max}(s,t) = \frac{1-s}{(1-t)a(k)m(k)}$, we get $k \in X(G_{\max}(s,t)\,,s,t)$. Suppose $G_{\min}(s,t) < G < G_{\max}(s,t)$. Then $\frac{1-s}{(1-t)a(0)m(0)} < G < \frac{1-s}{(1-t)a(k)m(k)}$. Hence, $\frac{1}{a(0)m(0)} < \frac{(1-t)G}{1-s} < \frac{1}{a(k)m(k)}$. Therefore, $a(k)\,m(k) < \frac{1-s}{(1-t)G} <$

$a(0)m(0)$. Since $a(x)m(x)$ is continuous on $[0,k]$, it follows that $a(x)m(x) = \frac{1-s}{(1-t)G}$ for some $x \in (0,k)$. Hence, $X(G,s,t) \neq \emptyset$ for $G \in [G_{\min}(s,t), G_{\max}(s,t)]$. Since $a(x)m(x)$ is continuous on $[0,k]$, it follows that $X(G,s,t)$ is closed. Since $X(G,s,t) \subset [0,k]$, it is bounded. Hence, $X(G,s,t)$ is compact and non-empty. Hence, it contains a minimum element. This is $x_0(G,s,t)$. ■

Lemma 14 *: (a) The solution to the optimization problem of player $x \in [0,1]$ is:*

$$\text{For } x \in [0, x_0(G,s,t)] : \begin{cases} g^*(G,s,t,x) = \frac{(1-t)m(x)}{1-s} - \frac{1}{a(x)G} \geq 0 \\ c^*(G,s,t,x) = \frac{1-s}{a(x)G} > 0 \end{cases}, \tag{60}$$

$$\text{For } x \in (x_0(G,s,t), 1] : \begin{cases} g^*(G,s,t,x) = 0 \\ c^*(G,s,t,x) = (1-t)m(x) > 0 \end{cases} \tag{61}$$

(b) The aggregate desire to give (Definition 1), which is the sum of private and public contributions to the charity, is given by:

$$F(s,t,G) = tM + (1-t)m(G,s,t) - \frac{1-s}{G}A(G,s,t). \tag{62}$$

Proof of Lemma 14: (a) Let us carry out the following optimization exercises: (1) For players $x \in [0,k]$, maximize the first row of (16) subject to the budget constraint in the first row of (15). (2) For players $x \in (k,p]$, maximize the second row of (16) subject to the budget constraint in the first row of (15). (3) For players $x \in (p,1]$, maximize the second row of (16) subject to the budget constraint in the second row of (15). This gives the solution for optimal charitable contributions of each of these three types of players:

$$\text{For } x \in [0,k] : g^*(G,s,t,x) = \begin{cases} \frac{(1-t)m(x)}{(1-s)G}\left[G - \frac{1-s}{(1-t)a(x)m(x)}\right] > 0 & \text{for } G > \frac{1-s}{(1-t)a(x)m(x)} \\ 0 & \text{for } G \leq \frac{1-s}{(1-t)a(x)m(x)} \end{cases}$$

$$\text{For } x \in (k,1] : g^*(G,s,t,x) = 0$$

Using Definition 6a and Lemma 13a,c, we get

$$g^*(G,s,t,x) = \begin{cases} \frac{(1-t)m(x)}{1-s} - \frac{1}{a(x)G} \geq 0 & \text{for } x \leq x_0(G,s,t) \\ 0 & \text{for } x > x_0(G,s,t) \end{cases}.$$

The optimal values of consumption for each type of individual, $c^*(G,s,t,x)$, can then be recovered from the budget constraint in (15).

(b) Definitions 1 and 6b,c then give

$$F(s,t,G) = tM + (1-t)m(G,s,t) - \frac{1-s}{G}A(G,s,t).$$

Let $s \in [0,1), t \in [0,1)$ satisfy restriction (i) and (ii).

Let $G > G_{\max}(s,t)$. Then, by Definition 6 a(ii), $x_0(G,s,t) = k$. Hence, from Lemma 14 and Definition 6 (b, c), we get:

$$\begin{aligned} g^*(G,s,t,x) &= \frac{(1-t)\,m(x)}{1-s} - \frac{1}{a(x)\,G} \geq 0, \text{ for } x \in [0,k], \\ g^*(G,s,t,x) &= 0 \text{ for } x \in (k,1], \\ F(s,t,G) &= tM + (1-t)\,m - \frac{1-s}{G}A. \end{aligned}$$

From Definition 2 it then follows that an equilibrium, G^*, must satisfy

$$G^* = tM + (1-t)\,m - \frac{1-s}{G^*}A,$$

i.e.,

$$(G^*)^2 - [tM + (1-t)\,m]\,G^* + (1-s)\,A = 0,$$

which has the two roots

$$G^{\pm}(s,t) = \frac{1}{2}\left[m + t\,(M-m) \pm \sqrt{[m + t\,(M-m)]^2 - 4(1-s)A}\right],$$

which are real, by restriction (i), and satisfy $G^+(s,t) > G^-(s,t) > G_{\max}(s,t) > 0$, by restriction (ii). These establish (32), (33), (34) and (35).

From (32), we get

$$F_t = M - m \geq 0. \tag{63}$$

$$F_s = \frac{A}{G} > 0,\ F_G = (1-s)\,\frac{A}{G^2} > 0,\ F_{GG} = -2\,(1-s)\,\frac{A}{G^3} < 0. \tag{64}$$

The inequality in (63) is strict for $m < M$, i.e., $k < p$.

Using the fact that for real numbers $a > b > 0$: $\sqrt{a-b} > \sqrt{a} - \sqrt{b}$, as well as (33), (34) and (64), we get

$$G = G^+ \Rightarrow F_G < 1,\ G = G^- \Rightarrow F_G > 1. \tag{65}$$

From Proposition 4a,b, (63), (64), (65) we get

$$G_s^+ > 0,\ G_s^- < 0, \tag{66}$$

$$G_t^+ > 0,\ G_t^- < 0 \text{ (for } m < M\text{, i.e., } k < p), \tag{67}$$

$$G_t^{\pm} = 0 \text{ (for } m = M\text{, i.e., } k = p). \tag{68}$$

This completes the proof. ■

Proof of Corollary 9: We have $a(x)\,m(x) = 8$ for $x \in [0, 0.5]$ and $a(x)\,m(x) = 0$ for $x \in (0.5, 1]$, $k = 0.5$ and $M = 0.5$. Hence, from Definition 6a: $G_{\min}(0.25, t) =$

$G_{\max}(0.25,t)=\frac{3}{32(1-t)}$. Hence, $x_0(G,0.25,t)=0$ for $0\le G\le\frac{3}{32(1-t)}$ and $x_0(G,0.25,t)=0.5$ for $G>\frac{3}{32(1-t)}$. Hence, from Definition 6b: $m(G,0.25,t)=0$ for $0\le G\le\frac{3}{32(1-t)}$ and $m(G,0.25,t)=0.5$ for $G>\frac{3}{32(1-t)}$. Similarly, from Definition 6c: $A(G,0.25,t)=0$ for $0\le G\le\frac{3}{32(1-t)}$ and $A(G,0.25,t)=\frac{1}{16}$ for $G>\frac{3}{32(1-t)}$. Hence, from Lemma 14: $F(0.25,t,G)=\frac{t}{2}$ for $0\le G\le\frac{3}{32(1-t)}$ and $F(0.25,t,G)=\frac{1}{2}-\frac{3}{64G}$ for $G>\frac{3}{32(1-t)}$. The equilibria are given by $F(0.25,t,G)=G$, i.e., $G^0(0.25,t)=\frac{t}{2}$ for $0\le\frac{t}{2}\le\frac{3}{32(1-t)}$ and $G=\frac{1}{2}-\frac{3}{64G}$ for $G>\frac{3}{32(1-t)}$. These, in turn give $G^0(0.25,t)=\frac{t}{2}$ for $0\le t(1-t)\le\frac{3}{16}$ and $G^2-\frac{1}{2}G+\frac{3}{64}=0$ for $G>\frac{3}{32(1-t)}$. The latter gives $G^+(0.25,t)=\frac{3}{8}$ for $t<\frac{3}{4}$ and $G^-(0.25,t)=\frac{1}{8}$ for $t<\frac{1}{4}$. The corresponding values of individual giving are $g^*\left(\frac{t}{2},0.25,t,x\right)=0$ for $0\le t(1-t)\le\frac{3}{16}$, $g^*\left(\frac{3}{8},0.25,t,x\right)=1-\frac{4}{3}t$ for $t<\frac{3}{4}$ and $x\in[0,0.5]$, $g^*\left(\frac{1}{8},0.25,t,x\right)=\frac{1}{3}-\frac{4}{3}t$ for $t<\frac{1}{4}$ and $x\in[0,0.5]$ and $g^*(G,0.25,t,x)=0$ for $x\in(0.5,1]$. The direct government grant, D, can be found from $D(G,s,t)=tM-s\int_{x=0}^{x_0(G,s,t)}g^*(G,s,t,x)\,dx$. This gives $D\left(\frac{t}{2},0.25,t\right)=\frac{t}{2}$ for $t(1-t)\le\frac{3}{16}$, $D\left(\frac{3}{8},0.25,t\right)=\frac{2}{3}t-0.125$ for $t<\frac{3}{4}$ and $D\left(\frac{1}{8},0.25,t\right)=\frac{2}{3}t-\frac{1}{24}$ for $t<\frac{1}{4}$. ■

Proof of Proposition 10: Let (s^*,t^*) maximize social welfare (46). By assumption, $u_c>0$, $u_G\ge0$, $\omega(x)>0$, $\tau(x)\ge0$ and $m(x)\ge0$ with $m(x)>0$ on a subset of positive measure. If $G_t^*\le0$ then, from (48), it follows that $W_t<0$. Hence, necessarily, $t^*=0$. Since $D^*\ge0$, $s^*\ge0$ and $g^*>0$ on a subset of positive measure, it follows from the government budget constraint (5) that $s^*=D^*=0$. ■.

Proof of Proposition 11: We first prove two intermediate results in Lemmas 15 and 16:

Lemma 15 : $\int_{x=0}^{1}\left(\frac{\partial g^*}{\partial s}+\frac{\partial g^*}{\partial G}G_s^*\right)dx=\frac{1}{1-s}\left(G_s^*+\int_{x=0}^{1}g^*dx\right)$.

Proof of Lemma 15: From Definitions 1 and 2 we get $G^*(s,t)=tM+(1-s)\int_{x=0}^{1}g^*(G^*(s,t),s,t,x)\,dx$. Implicitly differentiate this identity and rearrange to get $\int_{x=0}^{1}\left(\frac{\partial g^*}{\partial s}+\frac{\partial g^*}{\partial G}G_s^*\right)dx=\frac{1}{1-s}\left(G_s^*+\int_{x=0}^{1}g^*dx\right)$. ■

Lemma 16 : $\frac{\partial s}{\partial D}=-\frac{1-s}{sG_s^*+\int_{x=0}^{1}g^*dx}$.

Proof of Lemma 16: From the government budget constraint, (22), evaluated at equilibrium, we get $D(s,t,G^*(s,t))=tM-s\int_{x=0}^{1}g^*(G^*(s,t),s,t,x)\,dx$. Rewrite this with D and t as the independent instruments, to get $D=tM-s(D,t)\int_{x=0}^{1}g^*\{s(D,t),t,G^*(s(D,t),t)\}\,dx$. Differentiate implicitly with respect to D and rearrange to get $\frac{\partial s}{\partial D}=-1/\int_{x=0}^{1}\left[g^*+s\left(\frac{\partial g^*}{\partial s}+\frac{\partial g^*}{\partial G}\frac{\partial G^*}{\partial s}\right)\right]dx$. Use Lemma 15 to get $\frac{\partial s}{\partial D}=-1/\left[\int_{x=0}^{1}g^*dx+\frac{s}{1-s}\left(G_s^*+\int_{x=0}^{1}g^*dx\right)\right]$. Simplify to get the required result. ■

Let (s^*,t^*) maximize social welfare (46). By assumption, $u_c>0$, $u_G\ge0$, $\omega(x)>0$, $\tau(x)\ge0$ and $g^*\ge0$ with $g^*>0$ on a subset of positive measure. Hence, if $G_s^*\ge0$ then (47) implies that $W_s>0$, so s^* takes its maximum possible value in the feasible interval $[0,1)$. Viewing D and t as the independent government instruments we get, $\frac{\partial W(s(D,t),t)}{\partial D} = W_s(s(D,t),t)\frac{\partial s(D,t)}{\partial D}$. From this and Lemma 16 we get, $\frac{\partial W}{\partial D} = -(1-s)W_s/\left(sG_s^* + \int_{x=0}^{1} g^* dx\right)$. Since $W_s > 0$, $G_s^* \geq 0$, $g^* \geq 0$ with $g^* > 0$ on a subset of positive measure, it follows that $\frac{\partial W}{\partial D} < 0$. Suppose $D^* > 0$. Since $\frac{\partial W}{\partial D} < 0$, it follows that W can be increased further by reducing D, which contradicts the assumption that (s^*, t^*) maximizes social welfare. Hence, $D^* = 0$. ■

Proof of Proposition 12: From Proposition 4a, $G_s^*(s,t) = \frac{F_s}{1-F_G}$. So when $F_s \geq 0$ and $F_G < 1$ at G^*, or if $F_s \leq 0$ and $F_G > 1$ at G^*, we get $G_s^*(s,t) \geq 0$. Proposition 11 then implies that $D^* = 0$. ■

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