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Temptation in Markets with no Commitment: Give-aways, Scare-aways and Reversals

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Temptation in Markets with no Commitment: Give-aways, Scare-aways and Reversals[1]

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ABSTRACT. I study a two period model where the buyer suffers from self-control problems and his level of temptation is private information. I derive the optimal behaviour of a seller that offers her product to a buyer. In period 1, the latter decides whether or not to "enter the store" based on the prices posted by the seller. In period 2 he decides how much of the product to buy, if any. Differently from the existing literature, I assume that the seller cannot commit to the prices posted in period 1. I show how, under this framework, the presence of tempted consumers and asymmetric information can explain the existence of free vouchers offered by the seller to the consumer in exchange for entering the store. In contrast with classical contract theory, I show that the relatively untempted consumer (the "low type") can be better off when information about his type is private than when the seller is fully informed. Moreover, the presence of self-control may induce the seller to exclude the relatively strongly tempted consumer (the "high type") from the market.

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1. Introduction

"£100 free to get you started", "€5 free bet, no deposit required", "110 free spins when you join", "Get your first box of snacks free when you join", these are only a few of the several advertisements that we can find everyday on- and offline.[3] Sellers that advertise such offers often operate in the online gambling and sports betting industry and in the subscription boxes services (which I describe below). By registering to the seller's website, or subscribing to the delivery service, the buyer obtains a *free voucher*. This may be in the form of free credit to spend on the goods offered by the seller, or simply free products. Importantly, the benefits

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[3]The first advertisement is from sporting-index.com; the second from tipbet.com, the third from primeslot.com, the fourth from Graze. Further examples: netbet.com (betting), red32.com,, touch-lucky.com (gambling), Birchbox (subscription boxes).

of such vouchers are often independent from the future purchase decisions of the buyer. Vouchers come with "no strings attached". In other words, the buyer can obtain the free products, or spend the free credit, even if he decides not to purchase any further products from the seller.

In this paper, I investigate the reasons behind, and implications of, the use of free vouchers by a seller. Why do sellers offer such vouchers? Why is it optimal to offer these vouchers to all consumers in the population even when some of them may not purchase anything further? I address these questions by presenting a model with heterogeneously tempted consumers who suffer from self-control problems. Following the existing literature on temptation models with asymmetric information, I examine a two period game where the purchase takes place in period 2. Differently from other contributions (see Esteban and Miyagawa, 2005, 2006; Esteban, Miyagawa, and Schum, 2007, among others), however, I assume a seller that cannot commit in period 1 to the prices and quantities she sets in period 2.

Consider the following example. A seller offers her goods in a store to a buyer, who in period 1 is "outside the store". The latter, before "entering the store", has a clear idea of the quantity of the good he wants to buy. He knows, however, that, once inside, he will face temptation, that is, he will be willing to buy more (*upward* temptation), or less (*downward* temptation), than he was willing to buy when he was still outside the store. Since, the seller cannot commit to the prices she sets once the buyer is in the store, the latter knows that if he enters, the seller will take advantage of his temptation. Ultimately, if temptation is strong enough, the self-control cost of entering the store may be too high, and the buyer may decide to walk past the store instead of entering to make his purchases. In other words, the buyer's temptation creates a lock-in effect of the Diamond Paradox type (Diamond, 1971). While I discuss the link between this paper, the Diamond Paradox literature and the issue of commitment in section 2, notice that the simplest way to attract the buyer back into the store, is to compensate him for his self-control cost (the role-equivalent of the search cost in a Diamond model). In a world with no commitment and private information, however, compensating a type of buyer for his self-control cost, means compensating *all* types, regardless of whether they need a further incentive to enter the store or not.

Vouchers like the ones described above can be easily modelled: the seller in period 1 commits to offer the buyer a free quantity $\epsilon > 0$ at zero cost once he enters the store. In other words, she ensures the buyer that, once inside, he will always have the outside option of obtaining ϵ at no cost, i.e. buy nothing further and leave the store. In section 3.1 and Appendix A.1, however, I discuss how, in the framework considered, this is equivalent to assuming that the seller offers a monetary transfer to the buyer if the latter enters the store, i.e. an entry "bonus" (or negative entry fee). This approach simplifies the analysis and comparative statics, and it allows me to provide deeper insights on the model's mechanics.

The paper highlights three main points. First, free vouchers have the role of lessening the buyer's temptation problems, compensating them for their self-control efforts. Under private information, a seller may find it optimal to offer these vouchers to *all* buyers, even if some of them, once inside, make no further purchase. Second, the seller may find it optimal to "scare-away" the most tempted buyers, in order to efficiently extract all surplus from the less tempted ones. Finally, the presence of self-control problems decreases the period 1 willingness to pay of consumers who suffer from high temptation (i.e. the ones that value the good the most). This generates a "role reversal" between "high" (the one with a high willingness to pay) and "low" (the one with a low willingness to pay) types. In classical screening problems (Spence, 1977), low type consumers always obtain zero utility and are sometimes excluded from the market, while high types are better off when information is asymmetric. I show how, when self-control problems affect the buyers, not only this may not be the case, but also welfare results of high and low types may be completely reversed.

The paper is based on two key assumptions: the seller cannot commit in period 1 to the prices she sets in period 2, and buyers suffer from self-control problems.

Assuming a fully committed seller in the markets I consider would be a relatively strong assumption. Gambling sites do not commit to the costs of their slot machines or the offers they make in store. In fact, often they do not advertise these prices/services in details until one registers to their website. Betting sites have no ability to commit to all prices, since odds are constantly changing. Subscription boxes sellers also do not fully commit to the prices they offer to their subscribers. Consider the example of Graze, the leading snack subscription boxes service, that in recent years has also taken over the US market.[4] By signing up to Graze's service, the buyer commits to paying a given amount each week, in exchange for a box of snacks delivered weekly to his address. The first box is free of charge and it can be consumed without commitment to purchase any other boxes. While prices are fixed at the start of the service, several offers are made over time, with special discounts, new box offers, new snacks and add-ons. Hence, the seller offers prices and goods she did not commit to ex-ante.[5] The model can generally be applied to any kind of sellers who offer a free voucher

[4] *"Snack maker Graze.com smashes US targets"*, Rebecca Callander, The Telegraph, March 2014.

[5] Other companies also do not commit entirely to their price/quantity scheme. Laithwaites.co.uk, a leading wine subscription service, for example, changes the composition (and price) of the box with every delivery.

to new customers that agree to purchase goods from her over time. The "app" market is another example.[6]

While the link between temptation, self-control, obsessive impulses and gambling is well known and analysed in the psychology literature (see Nower and Blaszczynsky, 2004, and the references therein), it is worth exploring how temptation enters other markets. The timing is the key aspect. Over time, consumers receive their boxes of wine, food, cosmetics and so on. While some of these products may not have a direct connection with temptation (toiletries for example), it is reasonable to assume that consumers change in their preferences over time. They test the product, grow a habit, and are constantly invited to purchase more by the seller. The latter now has all their details and payment information. Hence, if the buyer wants to consume more, new products are only "one click away".

The paper is organised as follows. In section 2, I discuss the relevant related literature. In section 3, I present the main features of the model. In section 4 I discuss optimal contracts and the equilibrium of the game. Section 5 concludes the paper. Attached are two Appendices. In Appendix A, I discuss my assumptions further and present some extensions and comparative statics. Appendix B, contains all the proofs.

2. Related Literature

This paper contributes to the literature on temptation models. In particular, I model self-control preferences à la Gul and Pesendorfer (2001) which I describe in the following Section.

The economics literature has studied temptation and self-control in several different frameworks, some of which use a less general multi-self model approach. Kumru and Thanopoulos (2008) use self-control preferences to study social security systems. Galperti (2015) uses a multi-self model to explain the trade-off between commitment and flexibility in contracts offered by a seller to a consumer with dynamically consistent or inconsistent preferences; he shows how the low type (the consistent one) enjoys an information rent. In Foschi (2015), I study how consumers with self-control problems may provide a justification for the existence of loyalty schemes in the retailing industry. Christensen and Nafziger (2016) study the optimal packaging of "sin" goods in the presence of consumers that suffer from temptation.

The closest papers of this literature to the present one are Esteban and Miyagawa (2005, 2006) and Esteban, Miyagawa, and Schum (2007). The first studies

[6] Often developers offer their app for free together with some free credit that can be spent to "upgrade the app", via add-ons. Over time, however, they often modify the app and the menu of add-ons.

optimal contracting of a seller who offers a good to a tempted consumer with private information on his own level of temptation (his type). It shows how the seller can replicate first best by offering two separate menus and “decorating” the one designed for the less tempted consumer. They, however, implicitly assume that the seller can perfectly commit to specific menus of offers, and that she is unable to change them once the consumer is “in the store”. By doing so, they allow the seller to set different menus for different customers. Once this assumption is dropped, the seller sets a single menu of offers designed according to the ex-post utility. I show how, if this is the case, the result of Esteban and Miyagawa (2005) does not hold any longer. I also borrow their decoration result to study what I call in section 4 “scare-away menus”. The nature of these menus and the decorated ones of Esteban and Miyagawa (2005) is similar. There, however, they are a tool used by an uninformed seller to achieve first-best screening. In my model, instead, they are used to scare away strongly tempted buyers. Esteban and Miyagawa (2006) and Esteban, Miyagawa, and Schum (2007) extend the model to, respectively, a market with perfect competition and a continuum of types.

Finally, this paper contributes also to the literature on solutions to the Diamond Paradox (Diamond, 1971). When consumers have to pay a search cost in order to acquire information on the price of a good, firms can create a hold-up problem. Once a consumer is in a store, his willingness to pay raises by the search cost he has to bear if he were to look for the same good in another store. Firms exploit this hold-up problem and raise the price. Diamond (1971) shows that this creates an upward thrust on the equilibrium price that, eventually, reaches the one of joint profit maximisation. Consumers, however, anticipate the firms’ behaviour and decide not to “search” for the good in the first place. This leads to complete market break-down (and the paradox). I show how, in temptation models without commitment, the hold-up problem is endogenous and defined by the level of temptation that afflicts the consumer.

One of the many solutions to this paradox studied in the literature is for the seller to commit to a particular price format (Wernerfelt, 1994; Anderson and Renault, 2006).[7] If commitment is impossible, the easiest way for the seller to attract the consumer in the store is to compensate him for the search cost. In this paper I follow a similar logic. The consumer suffers from temptation ex-post.

[7] Varian (1980) assumes the presence of temporal dispersion of information on prices; Burdett and Judd (1983) introduce “noisy” search — which means that consumers may learn two, or more, prices every time they search; Stahl II (1989) assumes the presence of some fully informed consumers; Anderson and Renault (1999) establish the relationship between preferences for product differentiation and searching cost; Anderson and Renault (2006) introduce advertisement as a form of partial commitment that the seller can use to disclose an optimal amount of information; Rhodes (2015) builds on Anderson and Renault (2006) but considers the case of multi product retailing where the seller creates a “low price image” of himself by advertising low price products only.

When he enters the store, the seller may exploit his temptation and his higher willingness to pay. In period 1, outside the store, the consumer anticipates this behaviour and does not enter the store. Hence, the seller, being unable to commit, compensates the consumer for his negative ex-ante utility by means of a free voucher.

3. The Model

A seller (she) offers a good to a tempted consumer (he) in her store. She posts a menu M of offers $x = (t, q) \in \mathbb{R}^2_+$ where t is the transfer the consumer has to make in order to acquire quantity (or quality) q of the good.

There are two periods; in period 1, the *ex-ante stage*, the consumer is "outside of the store" and has to decide whether to "enter the store" or not. As in standard mechanism design problems, I normalise the consumer's payoff from the outside option to zero. Hence, if the consumer does not enter, both he and the seller obtain zero payoff.

If the consumer enters in period 1 the game continues in the *ex-post stage* (period 2). In this stage the seller sets a menu M of offers and the consumer chooses which $x \in M$ to buy. Following the existing literature discussed in Section 2, I assume that the seller cannot prevent the consumer from leaving the store having bought nothing. That is, offer *0* $= (0, 0)$ is always in the menu. Once the consumer has chosen an offer, payoffs are realised and the game ends.

3.1. **Free Vouchers vs. Entry Fees.**

Differently from the existing literature on temptation models, here the seller cannot commit to a specific menu ex-ante and, therefore, sets M ex-post.

The main objective of this paper is to model markets where a seller attracts consumers into her store by offering them free goods (in the form of vouchers) in return. One way of modelling these types of markets is to assume that, ex-ante, the seller can commit to offering the consumer a quantity $\epsilon > 0$ that the latter can consume free of charge, once in the store. In other words, the seller commits ex-ante to adding offer $\boldsymbol{\epsilon} = (\epsilon, 0)$ to the menu she sets ex-post. I solve this case in Appendix A.1. In the paper, I simplify the analysis by assuming that, ex-ante, the seller sets an entry fee $F \in \mathbb{R}$. Transfer F takes place if and only if the consumer decides to enter the store. In other words, the seller offers a free *gift* (in the form of a monetary transfer) to a consumer that enters her store. In the Appendix, I show how this assumption (as opposed to vouchers) does not affect the equilibrium and the main results of the paper, while simplifying substantially the analysis.

There is a simple intuition behind this equivalence. Notice that a free voucher is in fact creating an ex-post outside option that grants positive utility to the consumer that enters the store. Given the single seller framework, assuming a free gift, in the form of a negative entry fee, transforms the ex-post utility obtained from quantity ϵ into "monetary" ex-ante utility. Further, since ϵ is restricted to positive values, $F \in \mathbb{R}$ provides a more general analysis.[8]

The consumer's preferences follow Gul and Pesendorfer (2001). He is affected by temptation in the second period, when choosing the offer from the menu, but he is able to anticipate this in period 1 when he is deciding whether or not to enter the store. Therefore, the latter decision depends crucially on the menu the seller sets in the ex-post stage. For instance, ex-ante, the consumer might be willing to choose offer *0* ex-post but knows that, once inside, he will fall victim to temptation and buy offer $x \neq$ *0* instead.

The consumer can be of two types: low (L), with probability β, or high (H). In the ex post stage he chooses an offer from menu M according to:

$$\max_{x \in M} [U(x) + V_i(x)] \qquad i = H, L. \tag{1}$$

Function U is called the *commitment (net) utility* while function V is called the *temptation (net) utility*. To understand the difference between these two functions, consider U as the base utility that the individual obtains from consuming the good, free of temptation. Function V, instead, measures the impulses of the individual in period 2. Ex-post, the individual considers both his commitment and his temptation and makes the choice. These utilities are assumed to be quasi-linear and to differ in the scaling of q:

$$U(x) = u(q) - t \tag{2}$$

$$V_i(x) = v_i(q) - t \tag{3}$$

[8]In this setting, however, one needs to assume some restrictions on this monetary transfer. A situation in which a seller is giving away money to everyone willing to enter his store, without any strings attached may seem implausible. As already argued, however, this is a simplifying assumption for a more realistic situation where the seller offers a free quantity of the good he sells, which is of interest only to a subset of people in the world.

Functions U, V_H and V_L all satisfy the single crossing property.[9] The temptation (gross) utility of the low type, v_L, values less, with respect to u, the quantity of each offer, while one of the high type, v_H, values it more. All functions u, v_L, v_H are increasing and concave in q. I assume, that they satisfy $u(0) = v_i(0) = 0$.[10] The direction of the temptation is therefore characterised by the slope of v_i relative to u. Following Esteban and Miyagawa (2005), I write $v_L \prec u$ to indicate that v_L is flatter than u and, therefore, I say that the low type is *downward* tempted. For high types instead, I say $v_H \succ u$ to indicate that v_H is steeper than u and, that the high type is *upward* tempted. Hence:

$$v_L \prec u \iff \left.\frac{\partial v_L}{\partial q}\right|_{q=q'} < \left.\frac{\partial u}{\partial q}\right|_{q=q'} \forall q' \iff \text{L is downward tempted}$$

$$v_H \succ u \iff \left.\frac{\partial v_H}{\partial q}\right|_{q=q'} > \left.\frac{\partial u}{\partial q}\right|_{q=q'} \forall q' \iff \text{H is upward tempted}$$

Clearly, $v_L \prec v_H$. Figure 1 illustrates this concept.

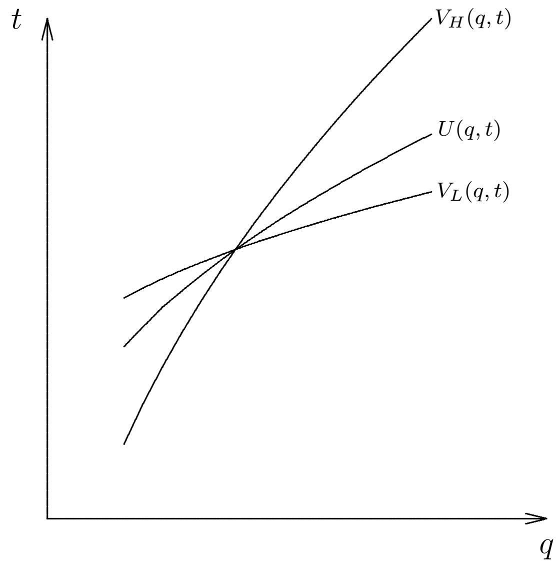

Figure 1. In the figure above, general concave temptation (net) utility indifference curves are drawn. The single crossing property implies that they cross only once in $\mathbb{R}^2_+$.

In the ex-ante stage, the consumer anticipates his ex-post decision and the menu

[9]Notice that, following the existing literature on temptation models, I assume the negative part of utility the consumer gets from paying tariff t is equal to the actual transfer t itself. This is true also for the temptation utility. This results in an ex-post utility $U(x)+V_i(x) = u(q)+v_i(q)-2t$ even if the transfer is made only once. This t cancels out in (4) below, if $\arg\max\,[U(x) + V_i(x)] = \arg\max V_i(x)$, since $W = u(q) + v_i(q) - 2t - (v_i(q) - t) = u(q) - t$. The peculiarity of this approach is that, if $\arg\max\,[U(x) + V_i(x)] \neq \arg\max V_i(x)$, the price of an offer which is not chosen enters the utility function as well.

[10]In Appendix A.4, I study an alternative case and show that results are qualitatively similar.

the seller sets ex-post. Hence, the preference representation of Gul and Pesendorfer (2001) implies an ex-ante utility of the type:

$$W_i(M,F) = \max_{x\in M}\,[U(x)+V_i(x)] - \max_{x\in M} V_i(x) - F \qquad i = H, L. \tag{4}$$

This is the utility the consumer obtains by accepting an entry fee F, anticipating that he will face menu M in the ex-post stage, i.e. the utility he gets by entering the store.[11] Notice that the first part is composed of the utility the consumer obtains in the ex-post stage minus the temptation utility that he is foregoing because he is exerting self-control effort. This is represented by the offer that would maximise his temptation utility, $\max_{x\in M} V_i(x)$.

To understand the intuition behind (4), consider the following example. A consumer on a diet is facing menu $M = \{s, h\}$ where s is a healthy salad and h is a (very tasty) hamburger. Suppose there is no entry fee. In this context, s is the offer that maximises his ex-post utility since, when choosing in the ex-post stage, he considers his commitment to his diet. Offer h, instead, is tempting the consumer, i.e. it maximises his temptation utility. The difference $V(h) - V(s)$ is known as the *self-control cost*. Notice that (4) then becomes $W(M) = U(s) - [V(h) - V(s)]$. Therefore, if the self-control cost of choosing the salad exceeds the commitment utility, the consumer will not accept the menu in the ex-ante stage. This is because he understands that in order to obtain utility $U(s) + V(s)$ in period 2 he also has to incur a self-control cost that makes his ex-ante utility negative.[12]

Given (4), type i consumer enters the store if and only if his ex-ante participation constraint $\underline{PC}_i$ is satisfied:

$$W_i(M,F) \geq 0. \tag{$\underline{PC}_i$}$$

The seller's ex-post profit is given by $\pi(x) = t - c(q)$, where function $c(q)$ is the total cost of production. It is assumed to be strictly increasing, convex in q and such that $c(0) = 0$. Following the existing literature, I assume that the seller faces no cost of adding an offer to a menu.

If there is perfect information about the buyer's type, in the ex-post stage the seller maximises her payoff given the consumer's ex-post participation constraint ($\overline{PC}_i$):

$$\max_{x_i} \pi(x_i) = \max_{x_i}\,[t_i - c(q_i)] \tag{5}$$

$$\text{s.t.} \quad [U(x_i) + V_i(x_i)] \geq 0 \tag{$\overline{PC}_i$}$$

[11] Alternatively, the entry fee F can also be included in U and V_i as an additional tariff, since its value does not depend on the menu.

[12] An alternative to this approach is a multi-self model (as in Strotz, 1955). However, as I show in Appendix A.6, a temptation model like the one considered here endogenises the change from a classical model with consumers's preferences $U(x) + V_i(x)$ to a multi-self model with self one's preferences $U(x)$ and self two's preferences $U(x) + V_i(x)$.

The solution to (5) s.t. ($\overline{PC}_i$) is the first-best offer, written x_i^*. A first best optimal menu set by the seller in the ex-post stage is, therefore, $M_i^* = \{\mathit{0}, x_i^*\}$.

In the ex-ante stage, the seller's profit are given by $\Pi(M, F) = \pi(x) + F$. Therefore, she sets the maximum possible F such that $\underline{PC}_i$ binds:

$$F^* = \{F \mid W_i(M_i^*, F) = 0\}. \tag{6}$$

Now consider the case of asymmetric information. In particular, suppose that the seller cannot observe the consumer's type, even when the latter is in the store. As mentioned above she knows that the probability the consumer is of low type is β (this constitutes her prior). Unless the seller is able to separate types ex-ante, in the ex-post stage, she faces a classical second-degree price discrimination problem (Spence, 1977):

$$\max_M \pi(M) = \max_M \left[\pi(x_L)\beta + \pi(x_H)(1-\beta)\right] \tag{7}$$

$$\text{s.t.} \quad U(x_H) + V_H(x_H) \geq 0 \tag{$\overline{PC}_H$}$$

$$U(x_L) + V_L(x_L) \geq 0 \tag{$\overline{PC}_L$}$$

$$U(x_L) + V_L(x_L) \geq U(x_H) + V_L(x_H) \tag{$\overline{IC}_L$}$$

$$U(x_H) + V_H(x_H) \geq U(x_L) + V_H(x_L), \tag{$\overline{IC}_H$}$$

where M is the menu of all offers set ex-post. Incentive compatibility constraints $\overline{IC}_L$ and $\overline{IC}_H$ are also introduced. They ensure that, ex-post, type i buys the offer designed for him and not the one set for type j. In the following section, I solve the model backwards, looking for the equilibrium.

4. Optimal Contracts and Fees

I now derive the optimal menu the seller sets ex-post and the optimal entry fee she charges ex-ante. Solving the game by backward induction, I first show what is best for the seller ex-post and then characterise the optimal entry fee.

The model has two natural benchmarks. The first benchmark is the case of full information, analysed below. As a second benchmark, consider a case where types are private information, but the consumer does not suffer from self-control problems. It is easy to see how this latter case simply replicates a classical screening problem (Spence, 1977). Since its results and intuitions are very well known, I relegate its analysis to Appendix A.2.

4.1. **Benchmark Case of Full Information.** As a benchmark I solve the full information problem finding the first best menu and offers. Suppose the seller is fully informed about the consumer's type both ex-post and ex-ante. The ex-post problem then becomes:

$$\max_{x_i} \left[t_i - c(q_i)\right] \tag{8}$$

$$\text{s.t. } u(q_i) + v_i(q_i) - 2t_i \geq 0 \tag{$\overline{PC}_i$}$$

$$\text{for } i = L, H.$$

Given the solution to (8) the seller sets the optimal entry fee according to (6). Since full information is assumed, she is able to set two different entry fees, one for each type.

Proposition 1. *If the seller is capable of perfectly observing the type of the consumer, both types of consumer enter the store and obtain zero ex-ante and ex-post utility. The first best is characterised by an optimal offer $x_i = (q_i, t_i)$ and an ex-ante entry fee F_i for each type i where:*

$$t_i^* = \tfrac{1}{2}\left[u(q_i^*) + v_i(q_i^*)\right], \quad q_i^* = \left\{q \,\middle|\, \tfrac{1}{2}\left[u'(q) + v_i'(q)\right] = c'(q)\right\}, \tag{9}$$

$$F_H^* = \tfrac{1}{2}\left[u(q_H^*) - v_H(q_H^*)\right] < 0, \quad F_L^* = 0. \tag{10}$$

It is easy to see that, in equilibrium, both types of consumer get $U(x_i^*) + V_i(x_i^*) = 0$ and $W_i(\{\mathit{0}, x_i^*\}, F_i^*) = 0$. Notice that since the high type is upward tempted, in equilibrium, he buys more than he should, according to his ex-ante utility. Hence, in order to attract him into the store, the seller has to compensate him ex-ante with a negative entry fee (entry bonus).

As expected, both offers in the menu ensure that the marginal expected utility from consuming q_i^* is equal to the marginal cost of producing it.

4.2. **Asymmetric Information.** Assume now, instead, that the seller cannot observe the type of the consumer she faces. The model becomes a dynamic game of incomplete information.

At the start of the game, the seller is assumed to have a prior $\Pr[i = L] = \beta$, that she updates ex-post. I show, however, that, when focusing on pure strategies, updating is trivial.

Notice that the only tool the seller has to separate types ex-ante is the entry fee. Even if the seller were to set distinct entry fees, however, both types, being free to choose whichever they like, would enter choosing the lower of them, and without self-selecting themselves, making ex-ante separation impossible. This follows from $W_i(\cdot, F') > W_i(\cdot, F'')$ for all $F' < F''$, and all i. Hence, the only way to separate types ex-ante is by setting an entry fee such that only one type finds it optimal to enter the store, while the other stays out.[13]

Consider the situation where only the low type enters the store, while the high type stays out. In any equilibrium, the seller sets the first best offer given by (9) ex-post. Therefore, she can charge no positive entry fee, or the low type would not enter. Notice, however, that, since $U(x_L^*) + V_L(x_L^*) = 0$, then $U(x_L^*) + V_H(x_L^*) > 0$. In other words, the high type obtains a positive utility from the offer designed for the low type. Hence, facing menu $\{0, x_L^*\}$, he chooses x_L. Given this, the high type enters the store and buys the offer designed for the low type. He obtains an ex-ante utility given by $W_H(\{0, x_L^*\}, 0) > 0$. This would seem to imply that an equilibrium where the low type enters and the high type does not cannot happen. It is possible to show, however, that the seller can set up specific menus ex-post in order to tempt the high type and make his ex-ante utility negative. In this paper, I define these menus as follows.

Definition 1 (Scare-Away Menu)**.** *A scare-away menu contains an offer $z \in \mathbb{R}_+^2$ that tempts at least one type of consumer (i.e. it maximizes his $V_i(x)$), but is never chosen by any type in equilibrium. The presence of z makes the self-control cost of choosing from the scare-away menu so high that the consumer finds it optimal not to enter the store in the first place.*

An example of scare-away menus are the "high-stakes tables or machines" in casinos. By setting up a very tempting menu, the seller is screening out consumers who suffer from strong temptation and self-control problems. In this way, she can better exploit less tempted types and extract all of their surplus.

Scare-away menus, and similar tools, have already been observed in the literature. Esteban and Miyagawa (2005) define them as "decorated" menus. While the structure of these menus is the same of the scare-away menus defined here, however, their purpose is strongly different. In Esteban and Miyagawa (2005) they are a tool of price discrimination. They are used by the seller to regain first best when consumers' types are private information. This is impossible to reach in the present model because of the inability of the seller to commit ex-ante to the ex-post menu. The latter can, however, set a scare-away menu ex-post in order to exclude from the market the strongly tempted type and extract all surplus from the low type.

The tempting offers in the scare-away menus are also similar to the *imaginary offers* found in the literature on contracting with naïve agents (i.e. agents who are not aware of their true type and are not capable of estimating it "correctly";

[13] This depends on the assumption that the seller is not able to commit herself ex-ante to the menus she sets ex-post. If this assumption is dropped, the seller can "force" the consumer to buy from a specific menu when picking a given entry fee ex-ante.

among others see Eliaz and Spiegler, 2006, 2008; Foschi, 2016). These are offers added to a menu by the principal, but never chosen by any type of agent in equilibrium. Given that agents are naïve, however, ex-ante they may assign a positive probability to choosing an imaginary offer ex-post. Hence, these offers are used by the seller as screening tools, and as a way to extract more surplus from agents.

In the following, I show one way of creating a scare-away menu.

It is sufficient to show that there exists an offer z that tempts the high type only and makes his ex-ante utility W_H negative. This offer should not be chosen by either of the two types ex-post so as not to affect directly the seller's profits. Let $M' = \{0, x_L^*, z\}$ be the scare-away menu the seller sets ex-post, then z has to satisfy:

$$U(x_L^*) + V_L(x_L^*) \geq U(z) + V_L(z) \tag{11}$$

$$V_L(x_L^*) \geq V_L(z) \tag{12}$$

$$U(x_L^*) + V_H(x_L^*) \geq U(z) + V_H(z) \tag{13}$$

$$V_H(x_L^*) \leq V_H(z) \tag{14}$$

$$V_H(z) - V_H(x_L^*) \geq U(x_L^*). \tag{15}$$

The first two constraints ensure that z does not affect the low type's behaviour once he is inside the store. If they do not hold, the utility obtained by the low type is affected by the presence of z both ex-ante, if (12) fails and the presence of z increases the self-control cost, and ex-post, if (11) fails and he chooses z over x_L^*, moving away from equilibrium. The second two ensure that z tempts the high type when he chooses x_L^* from M' and the last one states that the self control cost of choosing x_L^* from M' is too high for the high type and, therefore, $W_H(M', 0) < 0$.

To see that such an offer exists consider the general case in Figure 2 (which is taken from Esteban and Miyagawa, 2005).

In the Figure, utility increases towards the bottom right of the graph. Offer z is above $U(x_L) + V_L(x_L) = 0$ and on the right of x_L. Therefore, z is also above $V_L(x_L)$ hence (11) and (12) hold. Also, x_L and z lie on the same $U + V_H$ indifference curve making (13) bind. It is easy to see that (14) holds. Finally, since $U(z) = 0$, $U(z) + V_H(z) = V_H(z)$. Hence, $U(x_L) + V_H(x_L) = V_H(z)$ and $V_H(z) - V_H(x_L) = U(x_L)$, which shows that (15) binds.

Setting menu M', the seller knows that when charging a zero entry fee only the low type enters the store, since $W_L(M', 0) = 0$ and $W_H(M', 0) < 0$.[14]

[14] Notice that, because of the lack of commitment from the seller to the menu she sets in period 2, the existence of this equilibrium candidate depends crucially on the assumption that adding an offer to the menu is costless. If this were not the case, then ex-post the seller would have no incentive to add an offer like the z described. Ex-ante, she would know that she would not set a scare-away menu ex-post and would not, therefore, be able to exclude the high type.

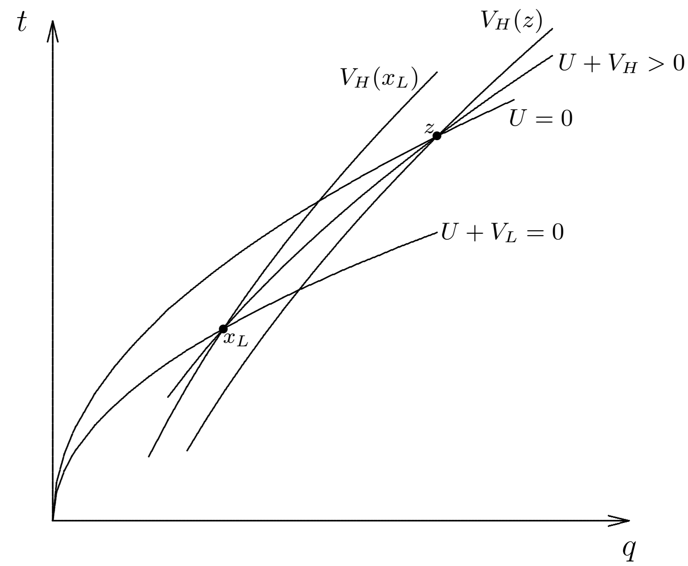

Figure 2. A "scare-away" menu to separate types ex-ante. With a menu $M' = \{0, x_L^*, z\}$ the low type is willing to enter the store while the high type stays out. The presence of the offer z makes the high type's ex-ante utility negative, while not affecting the low type's ex-ante and ex-post utility.

The following result highlights ex-ante profits and utilities for this case. I discuss later in this section the parameter space for this case to be an equilibrium.

Lemma 1 (Scare-Away Equilibrium)**.** *When the seller excludes the high type ex-ante, by setting no entry fee and menu M' ex-post, she earns ex-ante profits:*

$$\Pi^{EH} = \pi(x_L^*)\beta + 0 = \left[\tfrac{1}{2}\left[u(q_L^*) + v_L(q_L^*)\right] - c(q_L^*)\right]\beta \tag{16}$$

where EH stands for "exclude the high type". The low type enters the store and obtains zero ex-ante and ex-post utility. The high type stays out of the store.

Consider, now, the situation where only the high type enters the store, while the low type stays out. In any equilibrium, the seller is now certain to face a high type consumer. Hence, she sets the first best offer given by (9) ex-post. Therefore, she also has to charge a negative entry fee to induce the high type to enter the store, as in (10). Notice, however, that, since $U(x_H^*)+V_H(x_H^*) = 0$, then $U(x_H^*)+V_L(x_H^*) < 0$, and by the downward assumption of the low type, $\max\{V_L(x_H^*), 0\} = 0$. In other words, the low type obtains a negative utility from the offer designed for the high type. Hence, facing menu $\{0, x_H^*\}$, he chooses 0 and is also tempted by it. Given this, the low type enters the store, obtains the entry bonus, and buys nothing from the store. He obtains an ex-ante utility given by $W_L(\{0, x_H^*\}, F_H^*) > 0$. This implies that there can be no equilibrium where the high type enters and the low type does not. This is formally stated by the following Lemma.

Lemma 2 (No Scare-Aways for Low Types)**.** *There exist no scare-away menu that excludes the low type and sells the first best offer to the high type.*

The intuition behind the lemma is quite simple. Since the low type suffers from downward temptation, $v_H \succ u \succ v_L$, not only there is no way for the seller to tempt him without also tempting the high type, but it is also impossible to tempt him with an offer that he does not pick in equilibrium.

Given this, only one case remains: the one where consumer's types do not separate ex-ante and they both enter the store (i.e. pooling ex-ante).[15] Since self-selection does not take place ex-ante, the posterior beliefs of the seller are unchanged and she believes that the consumer is of low type with probability β. In this case, the problem she solves is a classical second-degree price discrimination as in (7). Hence, she has three options: (i) exclude low types from the market, offering ex-post only the first best offer for high types x_H^*, (ii) set a separating menu $M^S = \{\mathit{0}, x_L^S, x_H^S\}$ that induces types to self-select themselves ex-post, (iii) set a single offer x^P that both types are willing to buy — pooling.

In order to find the Perfect Bayesian Nash Equilibrium of the game, I first work out the optimal contracts in all three cases and the corresponding profits and ex-ante entry fees, then compare ex-post profits to obtain the equilibrium of the ex-post subgame. Finally, I compare ex-ante profits of the resulting equilibrium with the case of exclusion of the high type. The equilibrium derivation is described in Proposition 2.

I start from case (i) that needs no computations, since the seller sets a single offer, and therefore a menu $\{\mathit{0}, x_H^*\}$ — recall, from above, that the low type chooses *0* from $\{\mathit{0}, x_H^*\}$. By doing so, the seller obtains ex-post profits:

$$\pi^{EL} = \pi(x_H^*)(1-\beta) = \left[\tfrac{1}{2}\left[u(q_H^*) + v_H(q_H^*)\right] - c(q_H^*)\right](1-\beta)\,. \tag{17}$$

Given the optimal menu, in order to attract high types to the store, the seller has to set an entry fee as in (10). Hence, both types will enter ex-ante. The high type consumer buys x_H^* in the ex-post stage while the low type simply walks out of the store, i.e. chooses *0*. Notice that the seller is paying an entry bonus ($F_H^* < 0$) to *both* types only to have the high type inside the store buying the first best offer. Hence, when, and if, this case is an equilibrium, the low type gets a positive ex-ante surplus and a zero ex-post utility while the high type obtains zero surplus both ex-ante and ex-post.

[15] The seller could, of course, also exclude both types ex-ante. In this case the game would end and her profits would be equal to zero.

Lemma 3. *When the seller excludes the low type ex-post, by setting entry fee F_H^* ex-ante and menu $\{0, x_H^*\}$ ex-post, she earns ex-ante profits:*

$$\Pi^{EL} = \pi^{EL} + F_H^* = u(q_H^*) - c(q_H^*) - \left[\frac{1}{2}\left[u(q_H^*) + v_H(q_H^*)\right] - c(q_H^*)\right]\beta \tag{18}$$

where EL stands for "exclude low type". The low type enters the store to buy nothing and obtains a strictly positive ex-ante utility. The high type enters the store to buy x_H^* and obtains zero ex-ante and ex-post utility.

Before moving to the cases of no exclusion of types, ex-post or ex-ante, it is important to stress the connection between this equilibrium candidate and the scare-away menu one. Generally, in classical price discrimination problems (e.g. Spence, 1977), the seller may find it optimal to exclude from the market the type of consumer with the lowest valuation of the good (i.e. the low type). Here, however, because of the self-control problems of the buyer, the role of types is inverted from one stage to the other. Ex-ante, the high type anticipates a stronger self-control problem which decreases his ex-ante willingness to pay. Ex-post, the low type is less tempted than the high type, and willing to pay less than the latter for the same quantity. Hence, ex-ante is the low type to be the *most valuable* consumer for the seller, while ex-post this role belongs to the high type. This is also a reason why the seller may find it optimal to exclude the high type ex-ante or the low type ex-post. Later in this section I argue that there exist a portion of the parameter space where this role "reversal" happens in equilibrium.

Returning to the ex-post analysis, case (ii), i.e. when the seller serves both types ex-post separating them, requires some computations. If the seller wants to separate types ex-post, she solves problem (7). In the next Lemma, I show how two of the four constraints of problem (7) can be ignored. This follows from the textbook solution of second degree price discrimination problems.

Lemma 4. *When types are private information and both types enter the store ex-ante, the sellers sets the optimal contracts according to (7), where the participation constraints of the low type and the incentive compatibility constraint of the high type are binding. Other constraints are slack.*

The optimal ex-post menu of case (ii) then solves:

$$\max_M \Pi(M) = \max_M \left[\pi(x_L)\beta + \pi(x_H)(1-\beta)\right] \tag{19}$$

$$U(x_L) + V_L(x_L) = 0 \tag{PC_L}$$

$$U(x_H) + V_H(x_H) = U(x_L) + V_H(x_L) \tag{IC_H}$$

which yields as a solution:

$$M^S = \{\mathit{0}, x_L^S, x_H^S\} \quad \text{where} \quad x_i^S = (q_i^S, t_i^S) \quad i = H, L \tag{20}$$

$$t_H^S = \tfrac{1}{2}\left[u(q_H^S) + v_H(q_H^S) - v_H(q_L^S) + v_L(q_L^S)\right] \tag{21}$$

$$q_H^S : \tfrac{1}{2}\left[u'(q) + v_H'(q)\right] = c'(q) \tag{22}$$

$$t_L^S = \tfrac{1}{2}\left[u(q_L^S) + v_L(q_L^S)\right] \tag{23}$$

$$q_L^S : \tfrac{1}{2\beta}\left[(u'(q) + v_H'(q))\beta - (v_H'(q) - v_L'(q))\right] = c'(q). \tag{24}$$

Lemma 5 below shows how (33)—(37) exhibit no distortion at the top and leave no surplus to the low type, as expected.

Lemma 5 (Efficiency of Ex-Post Separation)**.** *When the sellers wants to separate types ex-post serving both of them, she sets offers $x_i^S = (q_i^S, t_i^S)$, $i = H, L$, where:*

$$q_H^* = q_H^S > q_L^* \geq q_L^S$$

and q_i^* is the quantity of the first best offer designed for type i.

The quantity sold to the high type is unchanged from first best — efficiency at the top — while the quantity offered to the low type is lower — inefficiency at the bottom. On top of this, notice, from (34), that the tariff the high type pays is lower than the one paid in first best. This ensures the high type a positive (ex-post) surplus whilst the low type gets zero surplus. Hence, the second period separation outcome satisfies the classical properties of second degree price discrimination models.

Given M^S, ex-post profits from separation are:

$$\begin{aligned}\pi^S = &\left[\tfrac{1}{2}\left[u(q_L^S) + v_L(q_L^S)\right] - c(q_L^S)\right]\beta \\ &+ \left[\tfrac{1}{2}\left[u(q_H^S) + v_H(q_H^S) - v_H(q_L^S) + v_L(q_L^S)\right] - c(q_H^S)\right](1-\beta).\end{aligned} \tag{25}$$

Before moving to the ex-ante optimal entry fees, notice that menu M^S is only feasible if $q_L^S \geq 0$.

Lemma 6 (Feasibility of Ex-Post Separation)**.** *Ex-post optimal separation is feasible, i.e. $q_L^S \geq 0$, if and only if:*

$$\beta \geq \underline{\beta} \equiv \frac{v_H'(q_L^S) - v_L'(q_L^S)}{u'(q_L^S) + v_H'(q_L^S)} \tag{26}$$

Lemma 6 shows that separation becomes possible only when the probability that the consumer is indeed a low type is high enough. The intuition is quite simple: if the consumer is almost certainly a high type, the seller would like to decrease his positive surplus. Hence, in order for separation to be feasible, she has to sell a smaller quantity to the low type. If β is particularly low, the optimal q_L^S turns negative, making separation impossible.

Given this and the optimal menu of separation, it is easy to see that $W_L(M^S, 0) = 0$ and $W_H(M^S, 0) = U(x_H^S)$. Hence, the ex-ante utility of the high type is positive if:

$$U(x_H^S) \geq 0 \Longleftrightarrow v_H(q_L^S) - v_L(q_L^S) \geq v_H(q_H^S) - u(q_H^S), \tag{27}$$

which is not generally satisfied. When $v_H \longrightarrow u$ the RHS of (27) goes to 0 while the LHS remains positive. Hence as the temptation of the high type disappears, the LHS of the equation becomes relatively larger than the RHS. In other words, the quantity the high type buys ex-post gets closer to what is optimal according to his ex-ante preferences; hence the decreasing need to compensate him ex-ante.

Given this, when the seller separates types ex-post, ex-ante she sets an entry fee $F^S = min\left\{U(x_H^S), 0\right\}$. Recall that if $F \leq 0$ both types accept it when entering the store. Therefore, the ex-ante profits and utilities of ex-post separation are described in the following result:

Lemma 7. *When the seller separates types ex-post, by setting menu M^S and entry fee F^S, she earns ex-ante profits:*

$$\begin{aligned}\Pi^S &= \pi^S + F^S \\ &= (1-\beta)\left[\tfrac{1}{2}\left(u(q_H^S) + v_H(q_H^S) - v_H(q_L^S)\right) - c(q_H^S)\right] \\ &\quad + \tfrac{1}{2}v_L(q_L^S) + \left[\tfrac{1}{2}u(q_L^S) - c(q_L^S)\right]\beta + min\left\{U(x_H^S), 0\right\}.\end{aligned} \tag{28}$$

where S stands for "separation". Both types enter the store. The low type obtains ex-ante utility equal to F^S and zero ex-post utility. The high type obtains ex-ante utility equal to $\max\left\{U(x_H^S), 0\right\}$ and positive ex-post utility.

Finally, case (iii) is easy to derive. Since the seller is not interested in separating the two types,she drops IC_H and IC_L from problem (7) and sets the offer that solves:

$$\begin{aligned}&\max_{x^P} \pi(x^P) = t - c(q) \\ \text{s.t. } &U(x^P) + V_L(x^P) \geq 0 &&(PC_L)\\ &U(x^P) + V_H(x^P) \geq 0. &&(PC_H)\end{aligned}$$

Notice that PC_L is binding at x_L^* and that x_L^* maximises $\pi(x)$ subject to PC_L. Also, as in the general case — see Lemma 4—, if PC_L binds, PC_H is slack. Therefore, $x^P = x_L^*$ solves the pooling problem. Ex-post profits of pooling are given by:

$$\pi^P = \pi(x_L^*) = \tfrac{1}{2}\left[u(q_L^*) + v_L(q_L^*)\right] - c(q_L^*) \tag{29}$$

Lemma 8. *There exists no equilibrium where the seller sets a pooling menu ex-post.*

I prove this in the appendix using a similar argument to Rothschild and Stiglitz (1976). What Lemma 8 says is that there always exist a, non optimal, separation menu $\{0, x^P, \tilde{x}\}$ that makes consumers self-select and grants the seller higher ex-post profits than the pooling one. Given this result, when both types enter the store, in equilibrium, the seller either excludes the low type or she separates types selling offers x_L^S and x_H^S according to condition (26).

Alternatively, it can be shown with a graphical proof, that the ex-post profits from pooling are always dominated either by the ones of case (i) or the ones of case (ii). I show this in figure Figure 3 below.[16]

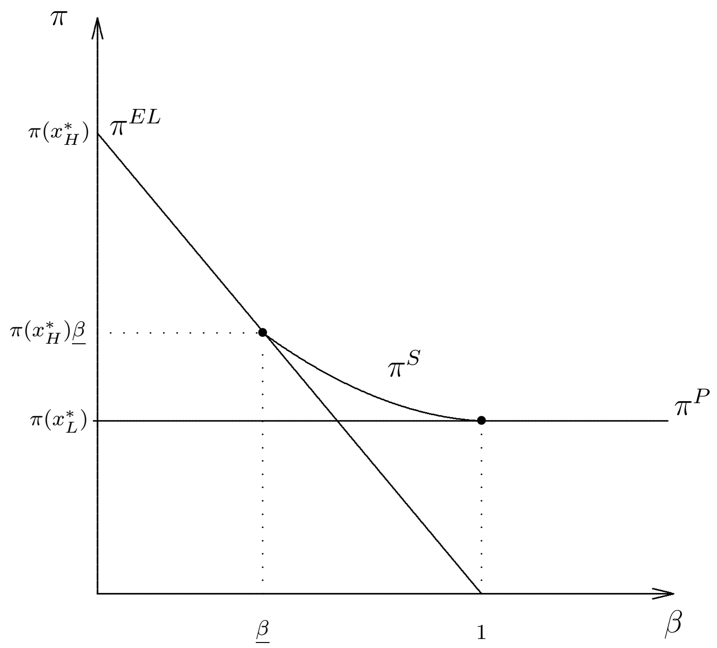

Figure 3. Cost and Gains of serving the low type. On the x-axis is the probability the consumer is a low type, β, while ex-post profits are measured on the y-axis. Following of Claims 1-4 in Appendix A.5, π^S takes the form shown. The curve is not plotted for values of $\beta < \underline{\beta}$ since separation is not feasible when (26) fails. The other profits are linear in β.

In the Figure, I plot the three different ex-post profits as a function of β. It is easy to see that as long as separation is possible, the ex-post profits it grants are the highest that the seller can obtain. When $\beta = \underline{\beta}$ the profits from separation equal those from the exclusion of the low type, and when $\beta = 1$ they equal those from pooling. This implies that, when condition (26) holds, ex-post the seller offers different positive quantities of the good to different types. When it fails, she excludes the low type from the market and only sells the first best offer to the high type.

Hence, (26) describes also the ex-post subgame equilibrium for the case of both types entering the store ex-ante.

[16]In Appendix A.5, I present Claims 1-4 to describe the shape of the profits depicted in the Figure.

The equilibrium of the game depends on whether the profit of setting a scare-away menu ex-post and a negative entry fee ex-ante, given by (16), is larger than the profit of letting both types in the store. Proposition 2 describes the equilibrium of the game.

Proposition 2. *In the equilibrium of the game with asymmetric information, the seller charges an ex-ante entry fee F and an ex-post menu of offers M, where:*

(i) *if optimal separation is not feasible, there exists a β_{EH}^{EL} such that if:*

$$\beta \leq \beta_{EH}^{EL} \tag{30}$$

$F = F_H^*$ and $M = \{0, x_H^*\}$, both types enter the store accepting the entry bonus ex-ante, the high type buys x_H^* ex-post while the low type chooses 0. If $\beta > \beta_{EH}^{EL}$, then $F = 0$ and $M = M' = \{0, x_L^*, z\}$, the seller charges a zero entry fee ex-ante, only the low type enters the store ex-ante and buys x_L^* ex-post.

(ii) *if optimal separation is feasible, there exists a β_{EH}^{S} such that if:*

$$\beta \leq \beta_{EH}^{S} \tag{31}$$

$F = F^S$ and $M = M^S = \left\{0, x_H^S, x_L^S\right\}$, both types enter the store accepting the entry bonus ex-ante, the high type consumer buys x_H^S ex-post while the low type buys x_L^S. If $\beta > \beta_{EH}^{S}$, then $F = 0$ and $M = M' = \{0, x_L^*, z\}$ as in (i).

The Proposition shows that an equilibrium exists for all values of $\beta \in [0, 1]$, and it is summarised by Figure 4 below.

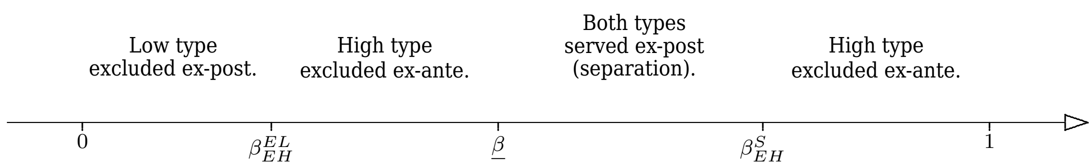

Figure 4. The equilibrium of the game for all values of β. The low type is excluded ex-post (but enters ex-ante) only when β is low. The high type is excluded ex-ante either because separation ex-post is not feasible or because β is too high.

Figure 4 shows only a *possible* and particularly interesting ordering of $\beta_{EH}^{EL}, \underline{\beta}$ and β_{EH}^{S}. I provide some comparative statics for this in Appendix A.3. From this particular ordering, two main points arise, highlighted in the following Corollaries.

Corollary 1. *There exists values for parameter β and temptation levels v_H and v_L such that the exclusion of the high type, via scare-away menus, is non-monotonic in β, the probability that the consumer is a low type.*

There exist a portion of the parameter space such that the ordering of $\beta_{EH}^{EL}, \underline{\beta}$ and β_{EH}^{S} is as in Figure 4. Given this, when the probability that the consumer is a low type is very low, i.e. $\beta \in \left[0, \beta_{EH}^{EL}\right]$, then the seller finds it optimal to exclude him from the market. However, she cannot do so ex-ante, but rather she is forced to let him in (paying him the entry bonus) in order to separate him from the high type once in the store.

As β starts to rise the seller finds it optimal to serve the low type as well, selling him a positive quantity. However, if she wants to continuing serving the high type, i.e. offering the menu of optimal ex-post screening M^S, she is force to leave the latter a positive utility and to offer him a non-positive entry fee F^S. If the probability of the buyer to be a high type, $(1-\beta)$, is high enough, $\beta \in \left[\beta_{EH}^{EL}, \underline{\beta}\right]$, this compensation becomes too costly and the seller is better off excluding the high type ex-ante and extracting all the surplus from the low type.

As $(1-\beta)$ decrease, $\beta \in \left[\underline{\beta}, \beta_{EH}^{S}\right]$, instead, the cost of leaving some utility to the high type decreases and the seller finds it optimal to induce both types in the store again.

Finally, for values of $\beta \in \left[\beta_{EH}^{S}, 1\right]$, the consumer is almost certainly a low type and therefore the seller finds it optimal to exclude the high type ex-ante again.

The second point arising from Figure 4 is that there exists a parameter space where the seller is optimally paying a positive amount to both types ex-ante but selling only to the high type ex-post. This implies two things. First, the presence of consumers who suffer from self-control problems generates an extra cost on the seller when she decided to exclude low types from the market. In other words, she has to "pay a fee" to low types consumers in order to sell at first best to high types. Second, it implies that under some conditions the low type obtains an information rent when information becomes asymmetric. This second point generates the following Corollary.

Corollary 2. *There exists values for parameter β and temptation levels v_H and v_L such that the low type consumer is better off when information is asymmetric than when types are common knowledge.*

This result holds under two circumstances: first, and more obviously, when both types are induced in the store but the low type is excluded ex-post; second when ex-post separation with a negative entry fee takes place. Notice that, here, the low type consumer obtains a zero ex-post surplus and a positive ex-ante surplus, precisely the opposite of what the high type obtains. This generates a discussion concerning the role that high and low types play in temptation models with self-control preferences.

In classical problems, where consumers do not suffer from self-control problems, the high type is usually considered to be the *best* type. He usually has a higher willingness-to-pay/ability, or induces the seller to face less risk. In temptation models with self-control preferences, this is only partially true. Ex-ante, in fact, the high type is no longer the best type in the market. His high willingness to pay for the good becomes a burden for him. Having a high valuation of the good (high temptation) now means he suffers from stronger self-control problems, and a lower ability to control his actions ex-post. Hence, while ex-post the roles are clear and the high type is the consumer with the highest valuation of the good, ex-ante these roles are reversed. The high type now becomes the type with the strongest self-control problem while the low type can control himself and bears a lower self-control cost. Hence, the role reversal in welfare results: the low type obtains positive rent ex-ante, while the high type obtains the ex-post.[17]

5. Conclusions

I construct a two period model where the buyer suffers from self-control problems (Gul and Pesendorfer, 2001) in order to study markets with free entry vouchers. Online casinos and betting websites, groceries subscription services and make-up companies, among others, often offer a free voucher to customers who sign up to their website or service. These vouchers take the form of free credit, to spend in the goods offered by the seller, or simply in free quantity of the goods and services on sale. Consumers first sign up, "enter the store", obtain the voucher, and make their purchase decision. This paper studies cases where the use of the voucher is completely independent from the purchase decision. In other words, I study vouchers "with no strings attached".[18] I highlight three main points.

First, in the markets I study, free vouchers are a way for the seller to lessen the buyer's self-control problems. Similarly to a Diamond Paradox (Diamond, 1971) model, the temptation that afflicts the buyer in period 2 creates a lock-in effect. The buyer's ex-ante willingness to pay is lower than the ex-post one since he anticipates the self-control cost he will bear once in the store. Because of the lack of commitment, the prices set in period 2 by the seller can turn out to be too high to the eye of a tempted consumer in period 1. Hence, the highly tempted buyer does not enter the store in the first place. However, by offering him a free voucher, the seller can compensate the buyer for his self-control costs ex-ante and incentives him to enter the store again.

[17] Or, indeed, none at all, as described above.

[18] For a non-exhaustive list of sellers that use this policy see the introduction.

The compensation needed to attract in the store some highly tempted buyers can turn out to be too much. In this cases, the seller would like to scare-away all the highly tempted buyers in order to extract all surplus from low tempted buyers. To do so she adds a tempting offer to the first best menu for the low type buyer (in the same fashion of Esteban and Miyagawa, 2005). This offer is neither chosen by the low type nor does it tempt him. It does, however, tempt the high type, to the point that he is driven away from the store.

Finally, I highlight the role reversal between high and low types from the ex-ante to the ex-post stage. In the model, the high type has the highest valuation for the good ex-post. His self-control problems, however, make his ex-ante willingness to pay the lowest in the market. The opposite is true for the low type. In other words, the *most valuable* consumer for the seller in the market is the low type ex-ante, and the high type ex-post.

The role reversal is also reflected in the equilibrium structure and in the buyer's utility. Under asymmetric information, and some conditions, the seller may (i) find it optimal to exclude the high type ex-ante or the low type ex-post; (ii) serve both types, leaving the low type with a strictly positive ex-ante utility and a zero ex-post utility, and the high type with a zero ex-ante utility and a strictly positive ex-post utility.

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Appendix A. Extensions and Comparative Statics

A.1. **Vouchers vs. Gifts.** In this section I show the equivalence between free gifts, as modelled in the paper (i.e. under the form of a monetary transfer from the seller to the buyer), and free vouchers. I do so by showing that the possible equilibria obtained when modelling free vouchers are qualitatively equivalent to the ones presented in the paper (with negligible minor differences).

First of all, notice that since the scare-away equilibrium does not feature any entry fee, it is independent from the choice of modelling vouchers or gifts.

The buyer's ex-post preferences are unchanged, while the ex-ante preference now take the form of the classical Gul and Pesendorfer (2001) self-control preferences:

$$W_i(M) = \max_{x \in M} [U(x) + V_i(x)] - \max_{x \in M} V_i(x) \qquad i = H, L.$$

The seller now does not offer a free transfer F to consumers that enter the store, but rather can commit ex-ante to offering the consumer a free quantity $\epsilon \in \mathbb{R}_+$ once he enters her store. Formally, she can commit ex-ante to adding offer $\boldsymbol{\epsilon} = (\epsilon, 0)$ to the menu she sets ex-post.

While $\boldsymbol{\epsilon}$ is set ex-ante, however, it affects the outside option of the buyer ex-post, and therefore affect the seller's ex-post problem. I now present the optimal menu set by the seller when she sets an $\boldsymbol{\epsilon}$ ex-ante. Given that the buyer is granted the free voucher independently of what quantity he ends up buying, every offer x_i is now given by $(q_i + \epsilon, t_i)$.

The scare-away menu case is unchanged, hence, I have to study only the case of both types entering the store ex-ante.[19]

A.1.1. *Separation Equilibrium.* I start from the case where the seller wants to separate types, while still selling them a positive quantity for a positive price. Hence, I present a modified version of problem (7).

$$\max_M \pi(M) = \max_M [(t_L - c(q_L + \epsilon))\,\beta + (t_H - c(q_H + \epsilon))\,(1 - \beta)] \tag{32}$$

$$\text{s.t.} \quad U(x_H) + V_H(x_H) \geq U(\boldsymbol{\epsilon}) + V_H(\boldsymbol{\epsilon}) \tag{$\overline{PC}_H$}$$

$$U(x_L) + V_L(x_L) \geq U(\boldsymbol{\epsilon}) + V_L(\boldsymbol{\epsilon}) \tag{$\overline{PC}_L$}$$

$$U(x_L) + V_L(x_L) \geq U(x_H) + V_L(x_H) \tag{$\overline{IC}_L$}$$

$$U(x_H) + V_H(x_H) \geq U(x_L) + V_H(x_L). \tag{$\overline{IC}_H$}$$

It is easy to show that even the presence of an outside option, the usual constraints selection holds. To see this, simply let ($\overline{PC}_L$) and ($\overline{IC}_H$)bind and rewrite the ($\overline{PC}_H$), ($\overline{PC}_L$) and ($\overline{IC}_H$) respectively in the following way:

$$\begin{aligned}
u(q_H + \epsilon) + v_H(q_H + \epsilon) - v_H(\epsilon) - 2t_H &\geq u(\epsilon)\\
u(q_L + \epsilon) + v_L(q_L + \epsilon) - v_L(\epsilon) - 2t_L &= u(\epsilon)\\
u(q_H + \epsilon) + v_H(q_H + \epsilon) - v_H(\epsilon) - 2t_H &= u(q_L + \epsilon) + v_H(q_L + \epsilon) - v_H(\epsilon) - 2t_L,
\end{aligned}$$

[19]I omit the derivation of the pooling case since Lemma 8 holds in this case too.

and notice that:

$$\begin{aligned}
&u(q_H+\epsilon)+v_H(q_H+\epsilon)-v_H(\epsilon)-2t_H \\
&\quad = u(q_L+\epsilon)+v_H(q_L+\epsilon)-v_H(\epsilon)-2t_L \\
&\quad > u(q_L+\epsilon)+v_L(q_L+\epsilon)-v_L(\epsilon)-2t_L \\
&\quad = u(\epsilon)
\end{aligned}$$

where the strict inequality comes from the fact that:

$$\begin{aligned}
&v_H(q_L+\epsilon)-v_H(\epsilon) > v_L(q_L+\epsilon)-v_L(\epsilon) \\
\Rightarrow\ & v_H(q_L+\epsilon)-v_L(q_L+\epsilon) > v_H(\epsilon)-v_L(\epsilon),
\end{aligned}$$

which is implied by $v_H \succ v_L$, as long as $q_L > 0$.[20] Hence (PC_H) is implied when (PC_L) and (IC_H) bind.

Given the above, the seller then solves:

$$\max_M \pi(M) = \max_M \left[(t_L - c(q_L+\epsilon))\,\beta + (t_H - c(q_H+\epsilon))\,(1-\beta)\right]$$

$$U(x_L)+V_L(x_L) = U(\boldsymbol{\epsilon})+V_L(\boldsymbol{\epsilon}) \tag{$\overline{PC}_L$}$$

$$U(x_H)+V_H(x_H) = U(x_L)+V_H(x_L). \tag{$\overline{IC}_H$}$$

which yields as a solution:

$$M_\epsilon^S = \{\boldsymbol{\epsilon}, x_{L,\epsilon}^S, x_{H,\epsilon}^S\} \quad \text{where} \quad x_{i,\epsilon}^S = (q_{i,\epsilon}^S+\epsilon, t_{i,\epsilon}^S) \quad i=H,L \tag{33}$$

$$t_{H,\epsilon}^S = \tfrac{1}{2}\left[u(q_{H,\epsilon}^S+\epsilon)+v_H(q_{H,\epsilon}^S+\epsilon)-v_H(q_{L,\epsilon}^S+\epsilon)+v_L(q_{L,\epsilon}^S+\epsilon)-u(\epsilon)-v_L(\epsilon)\right] \tag{34}$$

$$q_{H,\epsilon}^S:\ \tfrac{1}{2}\left[u'(q+\epsilon)+v_H'(q+\epsilon)\right] = c'(q+\epsilon) \tag{35}$$

$$t_{L,\epsilon}^S = \tfrac{1}{2}\left[u(q_{L,\epsilon}^S+\epsilon)+v_L(q_{L,\epsilon}^S+\epsilon)-u(\epsilon)-v_L(\epsilon)\right] \tag{36}$$

$$q_{L,\epsilon}^S:\ \tfrac{1}{2\beta}\left[(u'(q+\epsilon)+v_H'(q+\epsilon))\beta-(v_H'(q+\epsilon)-v_L'(q+\epsilon))\right] = c'(q+\epsilon). \tag{37}$$

It is immediate to notice two things. First, $q_{i,\epsilon}^S+\epsilon = q_i^S$ for $i=L,H$. Which means that the optimal quantity sold by the seller is the same as in the case of entry fee/bonuses, and it is not affected by the outside option set ex-ante. Second, $t_{i,\epsilon}^S = t_i^S - u(\epsilon) - v_L(\epsilon)$ for $i=L,H$. This ensures that both types of buyer obtain at least the utility they would get from the outside option, i.e. only consuming the quantity granted by the free voucher at zero price. Notice that, while the low type gets ex-post utility exactly equal to $u(\epsilon)+v_L(\epsilon)$, the high type obtains a $u(\epsilon)+v_L(\epsilon)$ on top of the usual information rent, which he obtains in section 4.

[20]The proof that the (IC_L) is slack is the same of Lemma 4 and is therefore omitted.

Given the above, I now move to the derivation of the optimal voucher $\boldsymbol{\epsilon}$ set ex-ante by the seller. To do so, I study the ex-ante utility of both types when menu M^S_ϵ is set ex-post.

First, it is easy to show that the offer that, for any $\epsilon \leq q^S_{H,\epsilon}$ tempts the most the high type is $x^S_{H,\epsilon}$ while the offer that tempts the most the low type is $\boldsymbol{\epsilon}$. In fact:

$$\max\{V_H(x^S_{H,\epsilon}), V_H(\boldsymbol{\epsilon})\} = V_H(x^S_{H,\epsilon}) \qquad \text{if}$$
$$v_H(q^S_{H,\epsilon}+\epsilon) - u(q^S_{H,\epsilon}+\epsilon) + v_H(q^S_{L,\epsilon}+\epsilon) - v_L(q^S_{L,\epsilon}+\epsilon) \geq v_H(\epsilon) - u(\epsilon) + v_H(\epsilon) - v_L(\epsilon),$$

which always holds by $v_H \succ u \succ v_L$. Similarly

$$\max\{V_L(x^S_{L,\epsilon}), V_L(\boldsymbol{\epsilon})\} = V_L(\epsilon) \qquad \text{if}$$
$$v_L(q^S_{L,\epsilon}+\epsilon) - u(q^S_{L,\epsilon}+\epsilon) \leq v_L(\epsilon) - u(\epsilon),$$

which also holds for $v_H \succ u \succ v_L$. This implies that the low type obtains positive ex-ante utility $W_L(M^S_\epsilon) = u(\epsilon) \geq 0$ for any $\epsilon \geq 0$, while the high type obtains $W_H(M^S_\epsilon) = U(x^S_{H,\epsilon})$. Exactly as in (27), the sign of the high type ex-ante's utility is ambiguous. It is easy to see that $U(x^S_{H,\epsilon}) \geq 0$ if and only if:

$$u(\epsilon) + v_L(\epsilon) \geq v_H(q^S_{H,\epsilon}+\epsilon) - u(q^S_{H,\epsilon}+\epsilon) - v_H(q^S_{L,\epsilon}+\epsilon) - v_L(q^S_{L,\epsilon}+\epsilon). \tag{38}$$

The seller's profits are decreasing in ϵ. Hence, she finds it optimal to set the lowest possible ϵ in the market. That is, the lowest ϵ such that (38) holds with equality.[21]

Proposition 3. *if (38) fails, when the seller sets menu M^S_ϵ ex-post, she offers ex-ante a free voucher worth ϵ^S, where:*

$$\epsilon^S : u(\epsilon^S) + v_L(\epsilon^S) = v_H(q^S_{H,\epsilon}+\epsilon^S) - u(q^S_{H,\epsilon}+\epsilon^S) - v_H(q^S_{L,\epsilon}+\epsilon^S) - v_L(q^S_{L,\epsilon}+\epsilon^S),$$

and no voucher otherwise.

Proposition 3 shows that this equilibrium candidate is qualitatively equivalent to the equilibrium of ex-post separation with entry fees/bonuses. The two differ only in the ex-post utility of the low type. With entry fees, the low type always obtains zero ex-post utility. In the case of free vouchers, he obtains a strictly positive ex-post utility if $\epsilon^S > 0$.

[21] Of course, (38) may very well hold with the strict inequality for $\epsilon = 0$, in which case, the equilibrium features no voucher.

A.1.2. *Ex-post Exclusion of the Low Type.* First I derive the first best offer for the high type:

$$\max_{x_{H,\epsilon}} [t_{H,\epsilon} - c(q_{H,\epsilon} + \epsilon)] \tag{39}$$

$$\text{s.t. } u(q_H + \epsilon) + v_H(q_H + \epsilon) - 2t_H = u(\epsilon) + v_H(\epsilon) \tag{$\overline{PC}_H$}$$

$$\Rightarrow t^*_{H,\epsilon} = \tfrac{1}{2}[u(q^*_{H,\epsilon} + \epsilon) + v_H(q^*_{H,\epsilon} + \epsilon) - u(\epsilon) - v_H(\epsilon)], \tag{40}$$

$$\Rightarrow q^*_{H,\epsilon} : \tfrac{1}{2}[u'(q + \epsilon) + v'_H(q + \epsilon)] = c'(q + \epsilon). \tag{41}$$

Then I show that the low type chooses $\boldsymbol{\epsilon}$ from $\{x^*_{H,\epsilon}, \boldsymbol{\epsilon}, \mathit{0}\}$:

$$U(x^*_{H,\epsilon}) + V_L(x^*_{H,\epsilon}) < U(\epsilon) + V_L(\epsilon)$$

$$\Rightarrow v_L(q^*_{H,\epsilon} + \epsilon) - v_H(q^*_{H,\epsilon} + \epsilon) < v_L(\epsilon) - v_H(\epsilon),$$

which always holds. The ex-ante utility of the low type is given by $W_L(\{x^*_{H,\epsilon}, \boldsymbol{\epsilon}, \mathit{0}\}) = U(\boldsymbol{\epsilon}) \geq 0$, for all $\epsilon \geq 0$. The one of the high is instead given by $W_H(\{x^*_{H,\epsilon}, \boldsymbol{\epsilon}, \mathit{0}\}) = U(x^*_{H,\epsilon}) = v_H(\epsilon) + u(\epsilon) + u(q^*_{H,\epsilon} + \epsilon) - v_H(q^*_{H,\epsilon} + \epsilon) < 0$. Hence, the high type obtains a negative utility for $\epsilon = 0$.

In order to induce the high type to enter the store, the seller is therefore forced to give out a free voucher to all consumers.

Proposition 4. *When the seller sets menu $\{x^*_{H,\epsilon}, \boldsymbol{\epsilon}, 0\}$ ex-post, she offers ex-ante a free voucher worth ϵ^{EL}, where:*

$$\epsilon^{EL} : v_H(\epsilon^{EL}) + u(\epsilon^{EL}) + u(q^*_{H,\epsilon} + \epsilon^{EL}) - v_H(q^*_{H,\epsilon} + \epsilon^{EL}) = 0.$$

Similarly to Proposition 3, Proposition 4 shows that qualitative equivalence between vouchers and gifts for the equilibrium candidate where the low type is excluded ex-post. Here too, types obtain a positive ex-post utility.

While equilibria are qualitatively equivalent, the use of entry fees/bonuses simplifies a lot the derivation of all results, provides a sharper and more precise analytical result, and generalises the model allowing for $F > 0$.

A.2. **No Self-Control Benchmark.** In this Appendix I discuss the second benchmark of the model, the one where the buyer does not suffer from self-control problems and his type is private information. Both types' ex-ante utility is then identical to their ex-post utility and the only relevant actions are taken in period 2. Here, the seller sets the menu of offers and types decide whether to buy something or not. Formally, period 1 does not disappear, however, as I show in the derivations, the seller has no incentive to set any entry fee and types never strictly prefer to stay out of the store.

Given this, the ex-post problem of the seller is identical to the one she faces in the paper when both types enter ex-ante. Optimal contracts are also unchanged. In Equilibrium, the seller either excludes the low type or she sells to both types leaving a small surplus to the high type. This follows the same condition of the paper, (26). When it holds, in equilibrium, the seller serves both types and offers menu M^S. When it fails, she serves only the high type offering him the first best menu.

This implies that a model without self-control preference would fail to: explain the existence of scare-away menus, the one of free vouchers in a framework like the one described, and the role reversal between low and high types.

A.3. **Comparative Statics.** The main purpose of this paper is to show the effect of asymmetric information when the level of temptation of the consumer is private information and the seller cannot commit herself to the menus she sets ex-post. Proposition 2 characterises the equilibrium of this game. The qualitative features of the equilibrium depend on the probability the consumer is a low type, β, and on the temptation level of both types of consumer, i.e, the relative slope of v_L and v_H. Given the level of β, v'_H and v'_L, there exists a unique equilibrium in pure strategies. In this section, I focus on the effect of these variables on conditions (30) and (31), which, ultimately, identify the equilibrium of the game. Notice that both conditions are implicit in β since the RHS depends on it also.

Condition (30) compares the seller's ex-ante profits of excluding the high type ex-ante with those of excluding the low type ex-post. β must be lower than β^{EL}_{EH} in order for the seller to be willing to exclude low types ex-post. When β increases, the probability of paying the consumer to enter the store only to have him choose *0* ex-post increases. Therefore, excluding the low type ex-post becomes less appealing to the seller.

The effect of the temptation level is, unexpectedly, symmetric in (30). If v'_L rises, β^{EL}_{EH} decreases, hence the seller is less willing to exclude the low type ex-post. The intuition is straightforward: since the low type is now more tempted, the seller can exploit his temptation more and excluding him ex-post becomes less attractive.

To understand why a rise in the temptation level of the high type has the same effect, consider the following. As v'_H rises, q^*_H rises and β^{EL}_{EH} decreases. The condition becomes tighter and will, eventually, fail. There is a clear explanation for this. Notice that the profits compared here are *ex-ante* profits and, therefore, the entry fee plays an important role in the condition. The higher is the level of temptation of the high type, the higher is the entry bonus that the seller has to offer the consumer to attract him into the store. As condition (30) shows, if the temptation level is high, this effect is stronger than the incentive to attract the high type into the store and extract all his surplus. When v'_H is "too high", it becomes too costly to attract the high type and the seller finds it optimal to exclude him ex-ante.

Condition (31) compares the ex-ante profits of excluding high types ex-ante with the ones of separation. An examination of this conditions yields somewhat more ambiguous comparative statics. The reason for this is that changes in all the crucial variables of the model have "ex-ante effects" and "ex-post effects" which are in sharp opposition.

First, consider a rise in the parameter β. This has two opposite effects. On the one hand, excluding the high type becomes more attractive since the consumer is more likely to be of a low type. Moreover, ex-post separation becomes less attractive, since the surplus granted to the high type ex-post is higher when β is higher.[22] However, a third effect arises if $U(x_H^S) < 0$. Notice that in this case $F^S = U(x_H^S)$ and that $\left|U(x_H^S)\right|$ decreases in β. Therefore, the "ex-ante cost of separation", i.e. the entry bonus that the seller has to offer consumers, decreases in β, making separation more attractive. None of these effects dominates the other for all possible levels of temptations. Hence, a change in β can, eventually, make condition (31) hold or fail.

Consider, now, the case of a rise in v_H'. On the one hand, the seller is less willing to exclude the high type ex-ante in order to exploit his higher temptation ex-post. On the other, a higher temptation level implies a higher entry bonus ex-ante paid to all types that enter. Hence, a rise in v_H' has both positive and negative effects on the profits of ex-post separation. Similar is the intuition behind a rise in v_L'. On the one hand, the difference in temptation levels between types is lower and excluding the high type becomes more attractive. On the other, the cost of separation (intended as the surplus granted to the high type ex-post) decreases making separation more attractive. Hence, a rise in v_L' has positive effects on both the profits of excluding the high type ex-ante and those of separating ex-post.

A.4. **Generality of $v_H(0) = v_L(0) = u(0)$.** Given that the high type is more tempted than the low type, an interesting alternative assumption would be $v_H(0) > v_L(0) = u(0)$ — or, equivalently, $v_H(0) > u(0) > v_L(0)$; I study the former case for simplicity. In this case, function v_H lies strictly above v_L for every q. This has no qualitative implications on the result. Suppose $v_H(0) = \delta > 0$. In the first best, the optimal offer for the low type does not vary. The one for the high type changes in the following sense: the ex-post tariff and the ex-ante entry bonus decrease by $\delta/2$. Notice, this new assumption is equivalent to an ex-post outside option for the high type, since now offer *0* grants him an ex-post utility of δ. Qualitatively nothing changes: both types obtain ex-ante and ex-post utility equal to the one they would obtain from their outside option. Quantitatively, the ex-post surplus of the high type is now positive and equal to $v_H(0)$.

[22]To see this, notice that the ex post surplus is given by $t_H^* - t_H^S = \frac{1}{2}\left[v_H(q_L^S) - v_L(q_L^S)\right] > 0$, which is increasing in β.

If information is, instead, asymmetric then problem (7) becomes:

$$\max_{M} \pi(M) = \pi(x_L)\beta + \pi(x_H)(1-\beta)$$

$$\text{s.t.} \quad U(x_H) + V_H(x_H) \geq \delta \qquad (\overline{PC}_H)$$

$$U(x_L) + V_L(x_L) \geq 0 \qquad (\overline{PC}_L)$$

$$U(x_L) + V_L(x_L) \geq U(x_H) + V_L(x_H) \qquad (\overline{IC}_L)$$

$$U(x_H) + V_H(x_H) \geq U(x_L) + V_H(x_L). \qquad (\overline{IC}_H)$$

However, the maximisation can be solved in the exact same way as in Section 4. It remains to check whether the participation constraint of the high type holds with this new outside option. Plugging-in the solution from (33), I get: $U(x_H^S) + V_H(x_H^S) = v_H(q_L^S) - v_L(q_L^S)$. Since $v_H \succ v_L$ and $v_H(0) - v_L(0) = \delta$, then $U(x_H^S) + V_H(x_H^S) > \delta$.

A.5. **Figure 3 and Ex-post Equilibrium Analysis.** The next four Claims explain the shape of the profits in Figure 3.

I first show that $\pi^S(\beta)$ is decreasing and convex in β. Then I show what happens to x_H^S, x_L^S and $\pi^S(\beta)$ at the extreme values of $\beta \in [\underline{\beta}, 1]$. Claim 4 contains a technical requirement for the result.

Claim 1. *Profit $\pi^{EL}(\beta)$ is a linearly decreasing function of β while $\pi^S(\beta)$ is decreasing and convex in β.*

Proof. While the first is trivial, to see that the latter is true notice that:

$$\frac{\partial \pi^S}{\partial \beta} = \left\{\tfrac{1}{2}\left[u(q_L^S) + v_H(q_L^S)\right] - c(q_L^S)\right\} - \left\{\tfrac{1}{2}\left[u(q_H^*) + v_H(q_H^*)\right] - c(q_H^*)\right\} \tag{42}$$

where I used the fact that $\frac{1}{2}\left[v_L'(q_L^S) + u'(q_L^S)\beta - v_H'(q_L^S)(1-\beta)\right] - c'(q_L^S) = 0$, by definition of q_L^S. Hence:

$$\frac{\partial \pi^S}{\partial \beta} < 0 \quad \text{for all } \beta.$$

Moreover, since q_H^S is independent of β:

$$\frac{\partial^2 \pi^S}{\partial \beta^2} > 0 \quad \text{for all } \beta \tag{43}$$

since q_L^S is increasing in β. This proves the claim. □

Claim 2. *At $\beta = \underline{\beta}$, $x_H^S = x_H^*$, $x_L^S = 0$ and $\pi^S\left(\underline{\beta}\right) = \pi^{EL}\left(\underline{\beta}\right)$.*
At $\beta = 1$, $x_L^S = x_L^*$ and $\pi^S(1) = \pi^P$ (x_H^S is irrelevant).

Proof. Notice that $q_H^S = q_H^*$ and $t_L^S = t_L^*$ for all β. Moreover, at $\beta = \underline{\beta}$, $q_L^S = 0$ which makes $t_H^S = t_H^*$ and $t_L^S = 0$. Therefore the profit from the low type is zero and

$\pi^S\left(\underline{\beta}\right) = \pi(x_H^*)\left(1-\underline{\beta}\right) = \pi^{EL}\left(\underline{\beta}\right)$. This proves the first part.
At $\beta = 1$, instead, $q_L^S = q_L^*$ proving $x_L^S = x_L^*$. Moreover, since the probability of the consumer to be high type is zero, $\pi^S(1) = \pi(x_L^*) = \pi^P$. This concludes the proof of the claim. □

Claim 3. *When $\beta = \underline{\beta}$, separation yields higher profits than pooling.*

Proof. There is a simple way to prove this. Notice that, from claim 2, at $\beta = 1$, $\pi^S(1) = \pi^P$. Also, from claim 1, π^S is decreasing in β. Since $\underline{\beta} < 1$ it must then be that $\pi^S\left(\underline{\beta}\right) > \pi^P$. □

Claim 4. *At $\beta = \underline{\beta}$:*

$$\frac{\partial \pi^S}{\partial \beta}\left(\underline{\beta}\right) = \frac{\partial \pi^{EL}}{\partial \beta}\left(\underline{\beta}\right).$$

Hence $\pi^{EL}(\beta)$ is tangent to $\pi^S(\beta)$ in $\beta \in \left[\underline{\beta}, 1\right]$. The point of tangency is $\underline{\beta}$.

Proof. The proof is straightforward. π^{EL} is linear so its slope does not depend on β and it is equal to $-\pi(x_H^*)$. It follows from the fact that $q_L^S = 0$ at $\beta = \underline{\beta}$ that (42), evaluated at $\underline{\beta}$, is, also equal to $-\pi(x_H^*)$. To prove that the two functions intersect only at $\underline{\beta}$, where π^{EL} is tangent to π^S, simply notice that, from claim 1, the slope of π^S is strictly larger — less negative — than the slope of π^{EL} for every $\beta \in \left(\underline{\beta}, 1\right]$. □

These four Claims prove the representation of π^{EL}, π^S and π^P in Figure 3.

Because of Claims 1-4, π^{EL} is tangent to π^S in the interval $\left[\underline{\beta}, 1\right]$. Hence, π^{EL} lies below π^S for all $\beta \in \left[\underline{\beta}, 1\right]$.

This implies that, when $\beta \in \left[\underline{\beta}, 1\right]$, a further increase in the parameter value makes optimal separation more attractive. When $\beta \in \left[0, \underline{\beta}\right)$, of course, an increase in the parameter will, eventually, make separation possible.

Notice that this also implies that when pooling yields higher profits that excluding the low type, separation yields the highest possible profits. Therefore, $\pi^P < \max\{\pi^S, \pi^{EL}\}$. This is an alternate argument to Lemma 8 to show that pooling is never an equilibrium.

A.6. **Temptation in a Multi-self Model.** As mentioned in Section 3, the temptation aspect of decision-making can be also modelled as in a multi-self model (Strotz, 1955) with self one's preferences $U(x)$ and self two's preferences $U(x) + V_i(x)$. A temptation model like the one considered in this paper, however, endogenises the change from a classical model with consumers's preferences $U(x) + V_i(x)$ to a multi-self model. In this sense, the multi-self model is a special case of a model à la Gul and Pesendorfer (2001) (hereafter called *temptation model*) where consumers are always tempted by the offer they choose in period 2. Temptation models, instead, have the ability to account for the self-control cost the consumer bears. In other words, they account for the possibility of an offer non chosen ex-post to tempt the consumer and affect his choice ex-ante. This cannot happen in a multi-self model. Below I show how the equilibrium of a multi-self model like the one described is qualitatively different from the one derived in this paper. For the following analysis, notice that the ex-post problem of the seller does not change since period 2 preferences are still given by $U(x) + V_i(x)$.

Let information about types be private. First, suppose it is optimal for the seller to exclude the low type ex-post. The equilibrium does not change with respect to a temptation model. This is because the low type is always happy to enter when he chooses offer *0* ex-post. The high type, instead is tempted by x^*_H and, therefore, his period 1 utility is the same regardless of which model I consider. The entry fee is set to $F^*_H = U(x^*_H) < 0$.

Notice now, that an offer z that tempts the consumer, but is not chosen by him, ex-post (i.e. of the type described in Section 4.2) would have no effect on his behaviour in a multi-self model. Hence in equilibrium, in a multi-self model, the high type always enters ex-ante since there is no way for the seller to set up a scare-away menu.

Finally, the separation equilibrium strongly differs from the one described in the paper. The downward tempted consumer in a temptation model is *not* tempted by the offer he chooses ex-post, x^S_L. Hence, he behaves as a classical consumer not affected by self-control problems and decides whether or not to enter the store according to $U(x) + V_i(x)$. In the multi-self model presented in this appendix, instead, he evaluates entrance according to $U(x)$. Hence, his ex-ante utility from the separation menu is given by $W_L(M_S, 0) = U(x^S_L) \geq 0$. Recall from Section 4.2 that the sign of the ex-ante utility of the high type depends on condition (27). Let the condition hold, then the ex-ante utility of both types is positive and the seller can extract this surplus with a positive entry fee: $F^*_S = min\{U(x^S_L), U(x^S_H)\} \geq 0$.

To conclude, a multi-self model provides qualitatively different results from the ones derived in the paper. These results are, inevitably, less general, since a multi-self model would fail to account for the self-control cost of decision making and assume exogenously the difference in ex-ante and ex-post preferences. The temptation model, instead, endogenises this difference and provides more general results.

Appendix B. Proofs

B.1. **Proof of Proposition 1.** By backward induction, I start from the ex-post problem (8). Let the $(\overline{PC}_i)$ bind, then $t^*_i = \frac{1}{2}\left[u(q_i) + v_i(q_i)\right]$. Substitute this back into (8) and solve for q_i^*. Moving, then, to the ex-ante problem:

$$\max_{F_i} \Pi^S = \max_{F_i} \left[\pi(x_i^*) + F_i\right] \tag{44}$$

$$\text{s.t. } W_i(\{\mathit{0}, x_i^*\}, F_i) \geq 0 \tag{$\underline{PC}_i$}$$

$$\text{for } i = L, H.$$

Since F enters with a negative sign in the constraint, and with a positive sing in the profit, let the participation constraint bind to obtain F_H^* and F_L^*.

To see that $F_L^* = 0$, simply notice that $\max\{0, V_L(x_L)\} = 0$. Hence, $W_L(\{\mathit{0}, x_L^*\}, 0) = U(x_L) + V_L(x_L) = 0$ by the participation constraint. The seller cannot therefore increase F_L beyond 0 or the low type would not enter the store.

As for the entry fee for the high type, notice that $\max\{0, V_H(x_H)\} = V_H(x_H)$. Hence, $W_H(\{\mathit{0}, x_H^*\}, 0) = U(x_H) = \frac{1}{2}\left[u(q_H) - v_H(q_H)\right] < 0$. Hence, if the seller wants the high type to enter her store, she has to compensate him with a negative entry fee F_H^*.

B.2. **Proof of Lemma 2.** Recall that the low type suffers from downward temptation, $v_H \succ u \succ v_L$. Suppose there exists a scare-away menu $M'' = \{x_H^*, \mathit{0}, z''\}$, where $z'' = (q_z, t_z)$, such that when set together with entry fee F_H^*, the high type finds it optimal to enter and buy x_H^*, while the low type stays out of the store. From above, I know that between offers x_H^* and *0*, the low type both picks and is tempted by the latter. Hence, for M'' to be a scare-away menu I need at least:

$$U(\mathit{0}) + V_L(\mathit{0}) \geq U(z'') + V_L(z'') \tag{45}$$

$$V_L(\mathit{0}) \leq V_L(z''). \tag{46}$$

From (46), $v_L(q_z) \geq t_z$. By the downward temptation fo the low type then $u(q_z) \geq t_z$. This violates (45) and provides the desired contradiction.

B.3. **Proof of Lemma 4.** I will show that only PC_L and IC_H bind while PC_H and IC_L are redundant — I omit the upper bar on constraints, but all constraints are ex-post ones.

First of all, since $V_H \succ U \succ V_L$, PC_L implies that $U(x_L) + V_H(x_L) > 0$ which, along with IC_H, implies that PC_H is slack.

Constraint PC_L, instead, has to bind at the solution. Suppose this is not true then the seller can increase t_L and t_H of an amount $\epsilon > 0$ such that PC_L binds, not affecting the incentive compatibility constraints, raising her profits.

Similarly for IC_H: if it is slack, the seller can increase t_H by $\epsilon > 0$ such that IC_H binds, not affecting PC_L, relaxing IC_L, and raising profits.

Finally, consider IC_L. Suppose it is not redundant and suppose the solution to the reduced problem, subject only to PC_L and IC_H, is given by two different offers x'_H and x'_L. If IC_L is not redundant then the low type would be at least as happy to buy x'_H as to buy x'_L. Since IC_H binds, however, also the high type is as happy to buy x'_L as to buy x'_H. This means that the seller would be better off by simply offering the offer x'_i such that $\pi(x'_i) > \pi(x'_j)$. This contradicts two distinct offers, as x'_H to x'_L, to be the solution to profit maximisation. Since problem (19) in the paper yields two distinct solutions, IC_L can be considered slack and checked afterwards.

B.4. **Proof of Lemma 6.** Since c is increasing in q and such that $c(0) = 0$, in order for q_L^S to be positive, the slope $c'(q_L^S)$ has to be positive. This happens when the LHS in the definition of q_L^S from (37) is positive. Hence:

$$[(u'(q) + v'_H(q))] \geq \tfrac{1}{\beta}\,[(v'_H(q) - v'_L(q))] \Rightarrow\ \beta \geq \frac{v'_H(q_L^S) - v'_L(q_L^S)}{u'(q_L^S) + v'_H(q_L^S)} \equiv \underline{\beta}.$$

B.5. **Proof of Lemma 5.** $q_H^* = q_H^S$ is obvious. Recall that $c(q)$ is increasing and convex in q, all utility functions are increasing and concave in q and that q_H^S, q_L^*, q_L^S are described by the equations in (9), (33) and (37).

To see that $q_H^S > q_L^*$ suppose the contrary, $q_H^S \leq q_L^*$. Then $c'(q_H^S) \leq c'(q_L^*)$, $u'(q_H^S) \geq u'(q_L^*)$ and $v'_H(q_H^S) \geq v'_H(q_L^*)$. Also $v'_H(q_L^*) > v'_L(q_L^*)$ since $v_H \succ v_L$. This makes:

$$u'(q_H^S) + v'_H(q_H^S) > u'(q_L^*) + v'_L(q_L^*) \quad \text{and} \quad c'(q_H^S) \leq c'(q_L^*)$$

which contradicts (9) and (33). To see that $q_L^* = q_L^S$ is possible, notice that, for $\beta = 1$, (37) is identical to (9).

A similar proof for $q_L^* \geq q_L^S$ is possible. Suppose this is not true, i.e. $q_L^* < q_L^S$. Then $c'(q_L^S) > c'(q_L^*)$, $u'(q_L^S) < u'(q_L^*)$ and $v'_i(q_L^S) < v'_i(q_L^*)$ for $i = H, L$. I will show that his brings to a contradiction since when $q_L^* < q_L^S$, $c'(q_L^S) > c'(q_L^*)$ cannot happen. The latter is true if:

$$\tfrac{1}{2\beta}\left[(u'(q_L^S) + v'_H(q_L^S))\beta - (v'_H(q_L^S) - v'_L(q_L^S))\right] > \tfrac{1}{2}(u'(q_L^*) + v'_L(q_L^*))\beta$$

which can be rearranged as:

$$(u'(q_L^S) - u'(q_L^*))\beta + \left[v'_L(q_L^S) - v'_H(q_L^S)(1-\beta) - v'_L(q_L^*)\beta\right] > 0$$

which never holds. To see this, notice that the first element is negative, since $u'(q_L^S) < u'(q_L^*)$, and the second element is also negative, since $v'_L(q_L^S) < v'_L(q_L^*)$ and $v'_L(q_L^S) < v'_H(q_L^S)$ by assumption. hence, the desired contradiction.

B.6. **Proof of Lemma 8.** Define $\tilde{x} = (\tilde{q}, \tilde{t})$ where $\tilde{q} = q_L^* + \epsilon$ and $\tilde{t} = \frac{1}{2}\left[u(\tilde{q}) + v_L(\tilde{q}) + \delta\right]$. From Lemma 2, $q_H^* > q_L^*$, hence there exist a small $\epsilon > 0$ such that $q_H^* > \tilde{q} > q_L^*$.

Then it is easy to see that:

$$U(\tilde{x}) + V_L(\tilde{x}) = -\delta \tag{47}$$

$$U(\tilde{x}) + V_H(\tilde{x}) = v_H(\tilde{q}) - v_L(\tilde{q}) - \delta \tag{48}$$

$$\pi(\tilde{x}) = \tfrac{1}{2}\left[u(\tilde{q}) + v_L(\tilde{q})\right] - c(\tilde{q}) + \tfrac{1}{2}\delta. \tag{49}$$

To show that pooling is never an equilibrium, I prove that $\tilde{x}$ is not chosen by the low type in $\{0, x^P, \tilde{x}\}$, but it is chosen by the high type, and yields strictly higher profits for the seller—recall that $x^P = x_L^*$. Formally, there exist a $\delta > 0$ such that:

$$U(\tilde{x}) + V_L(\tilde{x}) \leq 0 \tag{50}$$

$$U(\tilde{x}) + V_H(\tilde{x}) \geq U(x^P) + V_H(x^P) \tag{51}$$

$$\pi(\tilde{x}) > \pi(x^P). \tag{52}$$

Equation (50) holds by (47) and the positivity of δ. Let (51) bind, then:

$$\delta = v_H(\tilde{q}) - v_L(\tilde{q}) - \left[v_H(q_L^*) - v_L(q_L^*)\right] > 0 \tag{53}$$

by definition of $\tilde{q}$. Substituting δ into (52) I get:

$$\left[\tfrac{1}{2}\left[u(\tilde{q}) + v_H(\tilde{q}) - v_H(q_L^*)\right] - c(\tilde{q})\right] - \left[\tfrac{1}{2}\left[u(q_L^*)\right] - c(q_L^*)\right] > 0. \tag{54}$$

To see why this is positive, notice first that $\frac{1}{2}u(\tilde{q}) > \frac{1}{2}u(q_L^*)$. Hence, if $\frac{v_H(\tilde{q})}{2} - c(\tilde{q}) > \frac{v_H(q_L^*)}{2} - c(q_L^*)$, (52) is satisfied. To see that this is true, notice that q_L^* maximizes $\frac{1}{2}[u(\cdot) + v_L(\cdot)] - c(\cdot)$ and that since $v_H(\cdot)$ is steeper than $\frac{u(\cdot)+v_L(\cdot)}{2}$, the maximum of $v_H(\cdot) - c(\cdot)$ is achieved at a higher quantity. Since $\tilde{q} > q_L^*$, $\frac{v_H(\tilde{q})}{2} - c(\tilde{q}) - \frac{v_H(q_L^*)}{2} - c(q_L^*) > 0$. This concludes the proof of the Lemma.

B.7. **Proof of Proposition 2.** The seller has two decisions to make. Ex-ante, she decides whether to exclude the high type or not. If she does not, then ex-post she decides whether to separate or exclude the low type. Solving the game backwards, I consider this latter choice first. This is described by (26). If (26) fails, then, in period 1, the seller compares the ex-ante profits of excluding the high type ex-ante with the ones of excluding the low type ex-post. Comparing (16) with (18) I obtain (30) where

$$\beta_{EH}^{EL} \equiv \frac{2\left[u(q_H^*) - c(q_H^*)\right]}{u(q_L^*) + v_L(q_L^*) - 2c(q_L^*) + u(q_H^*) + v_H(q_H^*) - 2c(q_H^*)}.$$

If (26) holds, instead, then he compares the ex-ante profits of excluding the high type in period 1 and the ones of serving both types ex-post. Comparing (16) with (28) I obtain (31) where

$$\beta_{EH}^{S} \equiv \frac{u(q_H^S) - 2c(q_H^S) + \left(v_H(q_H^S) - v_H(q_L^S) + v_L(q_L^S)\right)\mathbb{I}\left[U(x_H^S) > 0\right]}{2\pi(x_H^S) + u(q_L^*) + v_L(q_L^*) - 2c(q_L^*) - \left[u(q_L^S) + v_L(q_L^S) - 2c(q_L^S)\right]}.$$