Environmental Investments in Mixed vs Private Oligopoly: What are the Implications of Privatization?

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Abstract

We compare economic and environmental outcomes under mixed and private oligopolies, in order to examine the effects of privatization when firms invest in abatement and emissions are taxed. We show that the number of competing firms in the market is an important factor in the determination of these effects. While privatization often involves a welfare trade-off, in the sense that higher (lower) output production implies higher (lower) pollution, there are also circumstances where it leads to both lower output and higher emissions simultaneously. Our results also indicate that privatization tends be associated with reductions in social welfare.

Keywords: Privatization; Pollution; Abatement; Mixed Oligopoly

JEL codes: L22; L32; Q52

1   Introduction

The issue of privatization has retained a prominent place in the agenda of policy makers for over three decades. Despite the fact that the views of governments and policy-inducing international organizations seem to very often favour programmes of extensive privatization, there are still many heated debates on the pros and cons of removing the direct engagement of the public sector from the procurement of goods and services such as utilities, transportation etc. Traditionally, these have focused on issues such as production efficiency; the transparency of managerial practices; the quantity and quality of the supplied goods and

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services, as well as their cost to the consumers and/or tax payers etc. The concerns over the widespread effects of global warming and other forms of environmental degradation have rekindled the interest on the merits and disadvantages of privatization. Particularly, a new point of discussion revolves around the following question: What are the implications for pollution, following a government's decision to relinquish its direct involvement in the production of goods and services?

The relevance of this question becomes obvious once we consider the fact that environmental damage is a by-product of economic activity. The amount of goods that an economy produces and consumes, as well as the environmental characteristics of the processes involved in the production of these goods, determine the impact of economic activity on environmental quality. As long as there are differences in the decisions and actions of private and public firms, originating from the markedly different objectives that these varying forms of ownership entail, then it is highly likely that privatization will have significant implications for the quality of the natural environment.

The presence of public firms is quite common in many sectors in both developed and developing countries. For example, public firms are major players in the energy sector all over the world. EDF, Enel or Vattenfall A.B. are some of the largest electricity generators and distributors in Europe and are controlled or owned by the French, Italian and Swedish governments respectively. The examples of publicly owned electricity generation and distribution companies are not exclusively limited to Europe. Many similar examples can be found in a variety of countries, such as South Korea (Korea Electric Power Corporation), Canada (Hydro-Quebec) or Brazil (Electrobras). Statoil and Petrobras are also major players in the oil industry and are controlled respectively by the Norwegian and the Brazilian governments. Another sector where the public presence is common is transportation. Matsumura and Sunada (2013) provide examples of public sector involvement in the airline markets, where many flag-carriers are still owned by the state and compete against private airlines.[1] Moreover, there is anecdotal evidence of public firms being major innovative players in sectors such as energy (Godø *et al.*, 2003). It is important to notice that the industries mentioned here are among the industries that raise more environmental concerns among policy makers and the wider public.

The objective of this paper is to compare the outcomes that transpire under mixed and private oligopolies, in an environment where pollution imposes a societal cost and firms decide on both their output and their investment in abatement. This comparison will allow us to offer some insights into the desirability of privatization when there are important environmental concerns due to the nature of the activity undertaken by firms. The acad-

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[1] Matsumura and Sunada (2013) cite the examples of Czech Airlines (CSA), Polish Airlines (LOT), and Portugal Airlines (TAP). Interestingly, the privatization of TAP is being negotiated at the time of writing this paper under the Portuguese bail-out plan agreed with the EU and the IMF.

emic literature on mixed oligopoly has so far focused on the advisability of privatization but has largely absconded from any environmental considerations, despite the fact that publicly owned or controlled firms are quite common in the real world, and particularly in highly polluting sectors such as energy generation or transport, as argued before. Some of the existing analyses have shown that privatizing a public firm improves social welfare under a number of different assumptions.[2] Other analyses have shown that if firms' outputs are subsidized, privatization is at best irrelevant from the point of view of welfare (White, 1996; Poyago-Theotoky, 2001 and Fjell and Heywood, 2004).[3] With respect to efficiency-enhancing innovation in the context of mixed oligopolies, it has been shown that privatization tends to reduce innovation and welfare (Tomaru, 2007, Heywood and Ye, 2009 and Gil-Moltó *et al.*, 2011). However, none of these papers contemplate investment to reduce emissions (that is, the social cost of production); rather, they focus on the (private) cost-reducing innovation.[4].

Some recent contributions have started introducing environmental aspects within the frame of analysis of mixed oligopoly. Using the framework by Beladi and Chao (2006), Saha (2012) shows that if an optimal pollution tax is employed, privatization of the public monopoly does not affect the resulting levels of output or pollution. However, in this simple framework, the decision of the firm is limited to output (there are no abatement investments). Bárcena-Ruiz and Garzón (2006) analyze the case of a quantity-competing mixed oligopoly with pollution where firms do choose their abatement levels. They show that when the government implements a pollution tax, privatizing the public firm results in lower pollution (and lower output). Using fairly general demand and cost functions, Pal and Saha (2014) show that in a mixed duopoly with pollution, the first best can be attained if a tax on output and a subsidy to abatement activities are jointly employed, while keeping one of the firm's under full ownership by the public sector.

An important point of departure of our paper from the existing literature on mixed oligopoly and environmental issues resides on our modelling of the abatement technology. Both Bárcena-Ruiz and Garzón (2006) and Pal and Saha (2014) consider the abatement effort not to have an effect on gross emissions. That is, they assume the abatement technologies to be "end-of-pipe" Examples of such technologies are the installation of filters or scrubbers, which allow firms to reduce emissions subsequently. In reality, abatement

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[2]Namely, that the public firm is less efficient than the private firm and the marginal cost of production is linear; the market is contestable; the number of private firms is large enough; and there are economies of scale or the public firm is incurring losses (De Fraja and Delbono 1989, 1990; Estrin and de Meza, 1995; Anderson *et al.*, 1997; Matsumura and Kanda, 2005).

[3]However, if there is an excessive burden of taxation, privatization does affect welfare (Matsumura and Tomaru, 2013).

[4]A separate literature strand, less related to our paper, has focused on the issues of product innovation with mixed results in terms of the desirability of privatization (see for example, Delbono and Denicolò, 1993; Poyago-Theotoky, 1998; Matsumura and Matsushima, 2004 and Ishibashi and Matsumura, 2006).

efforts will often affect gross emissions instead, in particular when firms employ "process integrated technologies". Examples of such technologies are switching to a cleaner fuel or cleaner production technique.[5] We will focus on this latter type of abatement technologies. Moreover, unlike Pal and Saha (2014), we consider the case where there is a public firm competing with n-private firms. In fact, the number of private firms in the market will be of significant importance for some of our results. We will assume, as standard, that the public firm is social-welfare maximizer[6] while private firms maximize profits.

Our results show that the tax on emissions has a positive effect on the investment in abatement and a negative effect on output for both public and the private firms. Moreover, a public firm in a mixed oligopoly will invest more in abatement but not necessarily produce more than its private competitors. Note that the result according to which public firms invest more in abatement activities corroborates with the existing empirical evidence (see Fowlie, 2010). After privatization, the newly privatized firm invests less in abatement (than it would do if it was kept under public ownership) and overall, emission intensities increase with the privatization. The effect on output is less clear-cut and depends on the number of firms: With a relatively small (large) number of private firms, privatizing the public firm leads to a decrease (increase) in aggregate output. All in all, emissions will increase with the privatization when the number of firms is relatively large. In such case, output and pollution increase with the privatization. For a relatively low number of firms, pollution will decrease after privatization but this will come at the cost of lower output. Interestingly, for intermediate values of firms, a privatization will result in both lower output and higher emissions. This result holds regardless of the emission tax being exogenously given or chosen to maximize welfare. Furthermore, our analysis suggests that the number of competing firms may also be critical in determining the overall impact on social welfare. However, the scenario that emerges for most of the cases is one where privatization has a negative effect on social welfare.

The remainder of the paper is structured as follows: In Section 2, we introduce our model. We solve the mixed and the private oligopolies in sections 3 and 4 respectively. Section 5 is devoted to the comparison across the two regimes, which allows us to derive some conclusion regarding the desirability of privatization. In Section 6, we provide an extension where the emission tax rate is endogenously chosen by the government to maximize welfare. Section 7 summarizes and concludes.

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[5] See Requate (2005) for more on this distinction.

[6] The assumption that the public firm maximizes welfare is standard in the literature on mixed oligopoly (see e.g., Anderson et al. (1997), De Fraja and Delbono (1989), Fjell and Heywood (2004), Matsushima and Matsumura (2003), Pal and White (1998), Poyago-Theotoky (2001) and White (1996, 2002)). Using unequal weights for the different components in the social welfare function would represent the case of partial-privatization, as in Matsumura (1998).

2   The model

Consider a market which consists of a unit-mass of identical consumers and an oligopoly producing and selling quantities of a homogeneous product. The price of this product is denoted by P. Each consumer is endowed with an exogenously given income of y and derives utility from consuming the good according to her utility function

$$U = \varphi + aQ - \frac{Q^2}{2}, \tag{1}$$

where Q is the consumption of the good, while φ denotes the quantity of all other goods and acts as the numéraire. Given the price of the good P, the budget constraint facing the consumer is given by

$$PQ + \varphi = y. \tag{2}$$

By rearranging and substituting (2) into (1), we get

$$U = y - PQ + aQ - \frac{Q^2}{2}. \tag{3}$$

The first order condition for maximization is therefore given by

$$\frac{\partial U}{\partial Q} = -P + a - Q = 0. \tag{4}$$

Solving the above equality, we obtain the demand function for the good at a price P

$$Q = a - P. \tag{5}$$

It is easy to verify that the above solution is indeed a maximum, since the second order condition for maximization is fulfilled ($\frac{\partial^2 U}{\partial Q^2} = -1 < 0$). Rearranging (5), we obtain the inverse demand function facing the oligopoly

$$P = a - Q,$$

where $a > 1$.[7]

The oligopoly comprises a public (welfare maximizing) firm and n private (profit maximizing) firms competing à la Cournot. We index the public firm with 0 and the private firms with i, where $i \in \{1, n\}$. Hence, total output is given by $Q = q_0 + \sum_{i=1}^{n} q_i$. Firms choose from a continuum of production technologies, indexed by $x \in \{0, 1\}$, which differ in their

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[7]We use the restriction $a > 1$ as this is one of the sufficient (but not necessary) conditions to guarantee that the equilibrium outputs, derived from the solution to the firms' problem, are non-negative.

environmental characteristics. In particular, a technology of index x is associated with an emission intensity (i.e., emissions per unit of output) given by

$$e_0 = (1 - x_0)^2 \; ; \; e_i = (1 - x_i)^2. \tag{6}$$

From (6), the reader can see that a choice of higher x corresponds to a cleaner production method (i.e., lower emission intensities).[8] In this respect, one can interpret x as an investment in an abatement technology. The quadratic functional form of e_0 and e_i is made to reflect the existence of diminishing returns to environmental investment. Given these, total emissions by each firm are given by

$$E_0 = e_0 q_0 \; ; \; E_i = e_i q_i. \tag{7}$$

Nevertheless, a cleaner production technology is associated with a higher cost of production. This assumption is made to reflect many real world observations. For example, according to U.S. Energy Information Administration, renewable energy power plants can be more expensive not only to build but also to operate (per unit of output) than natural gas or coal-fired plants.[9] In line with this assertion, the British Royal Society of Engineering estimates that the cost of electricity generated in, for example, wind farms can reach up to 7.2 pence per kWh whereas coal based plants operate at a cost of up to 3.2 pence per kWh.[10] The examples about cleaner production being less cost efficient are not restricted to the energy generation sector. The German Ship Owners Association calculates the costs of shipping to be substantially higher if using cleaner fuels (LNG) than heavy oil, which has important implications not only for this sector in particular, but also for any businesses shipping goods or transporting passengers.[11] Finally, according to the Food and Agriculture Organization of the United Nations, organic food is more expensive than conventional food, partly due to the higher labour costs involved in its production.[12] Given these observations, we assume that firms' production cost functions $C_0 = c(x_0, q_0)$ and $C_i = c(x_i, q_i)$ are such that

$$C_0 = x_0 q_0 + q_0^2 \; ; \; C_i = x_i q_i + q_i^2 \; . \tag{8}$$

Thus, it follows that firms face a trade-off between reducing their emissions and producing more efficiently (that is, with a lower marginal cost). The inclusion of the quadratic terms

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[8] That is, the abatement technology is integrated, since the abatement effort impacts on the emission intensities.

[9] See http://www.eia.gov/energy_in_brief/article/renewable_electricity.cfm

[10] See http://www.raeng.org.uk/news/publications/list/reports/Cost_Generation_Commentary.pdf.

[11] See http://www.spiegel.de/international/business/new-imo-regulations-push-shipping-industry-toward-cleaner-fuel-and-lng-a-916811.html

[12] See http://www.fao.org/organicag/oa-faq/oa-faq5/en/

in (8) is standard in the literature on mixed oligopoly to rule out the scenario where, in equilibrium, only the public firm stays in the market. This scenario is less interesting from the point of view of our paper, as we focus on the effect of competition between public and private firms on output and emissions levels.

We can rewrite C_0 and C_i as functions of the emission intensities and output. By inverting eq. (6), we get:

$$x_0 = 1 - \sqrt{e_0} \; ; \; x_i = 1 - \sqrt{e_i}. \tag{9}$$

Then, substituting the above into (8), we obtain:

$$C_0(e_0, q_0) = (1 - \sqrt{e_0})q_0 + q_0^2, \tag{10}$$

$$C_i(e_i, q_i) = (1 - \sqrt{e_i})q_i + q_i^2. \tag{11}$$

It is then straightforward to verify that the above functions comply with $C_e < 0$, $C_{ee} > 0$ and $C_{eq} < 0$, as in Requate (2005).

Given (8), a tax on emissions is required to induce (private) firms to invest in abatement.[13] As a consequence, we assume that emissions are taxed. Hence, firms' profits are given by

$$\pi_0 = Pq_0 - tE_0 - C_0, \tag{12}$$

$$\pi_i = Pq_i - tE_i - C_i,$$

where t is the tax rate on emissions. For most of our paper, we will assume that the tax rate is exogenously given. This assumption could reflect the scenario where the ownership of the public firm belongs to a local authority and emission tax is set at a national level. For example, in many countries electricity markets are organized at regional or local level and public utility companies are owned by the local authorities. Such is the case of Norway[14] or Canada[15] Or, we could think of a situation where the ownership of the firm is held at national level but the emission tax is set at federal level. For example, there is a substantial debate about the desirability of pan-European[16] or US-wide emission taxes.[17] Alternatively, the exogeneity of the emission tax could be related to budgetary constraints or pressure

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[13] Without taxation, there is no private benefit of investing to reduce emissions. However, even in the absence of emission taxation, the public firm would invest in abatement, since emissions negatively affect social welfare.

[14] See Von der Fehr and Vegard Hansen (2010).

[15] See https://www.neb-one.gc.ca/clf-nsi/rnrgynfmtn/prcng/lctrct/cndnmrkt-eng.html),

[16] See http://www.eubusiness.com/news-eu/aviation-transport.qxj, for the case of aviation.

[17] See http://www.cbo.gov/budget-options/2013/44857.

from lobbies not allowing the tax rate to be modified after privatization. In Section 6 we will undertake an extension where the emission tax is endogenously chosen to maximize welfare.[18]

The revenues from taxing emissions are employed by the government to mitigate the negative effects of pollution. We assume that (net) pollution Ω is given by

$$\Omega = (1 - dt)(E_0 + \sum_{i=1}^{n} E_i), \tag{13}$$

where the parameter $d \in [0, 1]$ measures how effective the cleaning-up technology at the government's disposal is. We impose that $dt \leq 1$ in order to to guarantee non-negativity of pollution.

All in all, the objective function of the public firm is given by the aggregation of producer surplus and consumer surplus minus pollution. As standard producer surplus (PS) is the aggregation of profits by all firms in this market

$$PS = \pi_0 + \sum_{i=1}^{n} \pi_i, \tag{14}$$

while consumer surplus (CS) is given by[19]

$$CS = \frac{Q^2}{2}. \tag{15}$$

Using (13)-(15), social welfare SW is given by

$$SW = \pi_0 + \sum_{i=1}^{n} \pi_i + \frac{Q^2}{2} - (1 - dt)(E_0 + \sum_{i=1}^{n} E_i). \tag{16}$$

We assume that firms simultaneously choose output and emission intensities. We will start by solving the mixed oligopoly in the next section (Section 3), while for comparison purposes, we will also solve the private oligopoly case in Section 4.

3 Mixed oligopoly

Recall that in this case we have one public firm and n private firms. The public firm chooses q_0 and x_0 to maximize social welfare while each private firm chooses q_i and x_i to maximize individual profits.

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[18]We can anticipate that our main results remain qualitatively the same.

[19]Note that substituting (5) into (3) yields $U = y + \frac{Q^2}{2}$. Since y is additive and exogenously given, it does not impact in our subsequent analysis. Therefore CS can therefore be summarized as $\frac{Q^2}{2}$.

The first order conditions for maximization for each of the private firms are given by

$$\frac{\partial \pi_i}{\partial q_i} \quad = \quad a - 4q_i - q_0 - \sum_{i' \neq i} q_{i'} - te_i - x_i = 0, \tag{17}$$

$$\frac{\partial \pi_i}{\partial x_i} \quad = \quad q_i[2t(1 - x_i) - 1] = 0, \tag{18}$$

where $\sum_{i' \neq i} q_{i'}$ indicates the aggregation of outputs by all other private firms.

The first order conditions for maximization for the public firm are

$$\frac{\partial SW}{\partial q_0} \quad = \quad \frac{\partial CS}{\partial q_0} + \frac{\partial PS}{\partial q_0} - \frac{\partial \Omega}{\partial q_0} = 0, \tag{19}$$

$$\frac{\partial SW}{\partial x_0} \quad = \quad \frac{\partial CS}{\partial x_0} + \frac{\partial PS}{\partial x_0} - \frac{\partial \Omega}{\partial x_0} = 0, \tag{20}$$

where $\frac{\partial CS}{\partial q_0}$, $\frac{\partial PS}{\partial q_0}$ and $\frac{\partial \Omega}{\partial q_0}$ are respectively given by

$$\frac{\partial CS}{\partial q_0} \quad = \quad q_0 + \sum_{i=1}^{n} q_i, \tag{21}$$

$$\frac{\partial PS}{\partial q_0} \quad = \quad a - 4q_0 - 2\sum_{i=1}^{n} q_i - te_0 - x_0, \tag{22}$$

$$\frac{\partial \Omega}{\partial q_0} \quad = \quad e_0(1 - dt). \tag{23}$$

Hence, the first order condition for maximization with respect to q_0 is given by

$$\frac{\partial SW}{\partial q_0} = -3q_0 + a - \sum_{i=1}^{n} q_i - te_0 - x_0 - e_0(1 - dt) = 0. \tag{24}$$

On the other hand, $\frac{\partial CS}{\partial x_0} = 0$, while $\frac{\partial PS}{\partial x_0}$ and $\frac{\partial \Omega}{\partial x_0}$ are respectively given by

$$\frac{\partial PS}{\partial x_0} \quad = \quad 2tq_0(1 - x_0) - q_0, \tag{25}$$

$$\frac{\partial \Omega}{\partial x_0} \quad = \quad -2(1 - dt)q_0(1 - x_0). \tag{26}$$

Hence, the first order condition for maximization of the public firm's objective function with respect to x_0 is given by

$$\frac{\partial SW}{\partial x_0} = 2q_0\{(1 - x_0)[1 + t(1 - d)] - 1\} = 0. \tag{27}$$

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Solving $\frac{\partial \pi_i}{\partial x_i} = 0$ and $\frac{\partial SW}{\partial x_0} = 0$ yields the following interior solutions in terms of the investment in abatement by each private firm (x_i^*) and the public firm (x_0):

$$x_i^* = 1 - \frac{1}{2t} \equiv x_{pr}^* \quad \forall i, \qquad (28)$$

$$x_0^* = 1 - \frac{1}{2[1 + t(1 - d)]}. \qquad (29)$$

Note that a larger tax rate will yield a higher investment in abatement for both types of firms. Note as well that $t \geq 1/2$ is required so that x_{pr}^* is non-negative. Thus, this condition is henceforth assumed to hold.

Now, substituting x_0 into $\frac{\partial SW}{\partial q_0}$ and solving for q_0, we find the public firm's (output) reaction function

$$q_0^R = \frac{1}{3}\left(a - \sum_{i=1}^{n} q_i - 1 + \frac{1}{4[1 + t(1 - d)]}\right). \qquad (30)$$

Likewise, substituting x_{pr}^* into $\frac{\partial \pi_i}{\partial q_i}$ and solving for q_i, we find each private firm's (output) reaction function

$$q_i^R = \frac{1}{4}\left(a - 1 - q_0 - \sum_{i' \neq i} q_{i'} + \frac{1}{4t}\right). \qquad (31)$$

Given that all the private firms are symmetric, in equilibrium, we will have that $q_i = q_{pr}$ $\forall i$. Hence, the above mentioned reaction functions reduce to

$$q_0 = \frac{1}{3}\left(a - 1 - nq_{pr} + \frac{1}{4[1 + t(1 - d)]}\right), \qquad (32)$$

$$q_{pr} = \frac{1}{n + 3}\left(a - 1 - q_0 + \frac{1}{4t}\right). \qquad (33)$$

Note that a larger tax rate has a direct negative effect on the output produced by both private firms and the public firm (although in this latter case modulated by d). Solving the above system yields the output solutions

$$q_0^* = \frac{1}{9 + 2n}\left\{3(a - 1) + \frac{1}{4t}\left[(3 + n)\frac{t}{1 + t(1 - d)} - n\right]\right\}, \qquad (34)$$

$$q_{pr}^* = \frac{1}{9 + 2n}\left\{2(a - 1) + \frac{1}{4t}\left[3 - \frac{t}{1 + t(1 - d)}\right]\right\}. \qquad (35)$$

All in all q_0^*, x_0^*, q_{pr}^*, x_{pr}^* constitute the (Nash) equilibrium solution of the mixed oligopoly case.[20] It is important to notice that $q_{pr}^* > 0$ because $\frac{t}{1 + t(1 - d)} < 1$ (since $0 < td \leq 1$). This implies that the private firms always produce. So that to guarantee $q_0^* \geq 0$, an additional

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[20] In the Appendix we show that the SOC hold as well.

constraint on n is needed. In particular we require that $n \leq \widehat{n}$, where

$$\widehat{n} = \frac{3t}{1 - td} \left\{ 1 + 4(a - 1) \left[1 + t(1 - d) \right] \right\}. \tag{36}$$

Note that $\widehat{n} > 1$, given that $t \geq 1/2$ and therefore $3t > 1$. Hence, the restriction on n that we require to guarantee the non-negativity of q_0^* can be summarized as follows.[21]

Condition 1 $n \, \epsilon \, [1, \widehat{n}]$.

Before we proceed to solve the case of the private oligopoly, a few points merit discussion here. First of all, the emission intensities in equilibrium are lower for the public firm than for the private firms, *i.e.* $e_0^* < e_{pr}^*$. The emission intensities can be calculated by substituting x_0^* and x_{pr}^* into e_0 and e_i, yielding

$$e_0^* = \frac{1}{4[1 + t(1 - d)]^2}, \tag{37}$$

$$e_{pr}^* = \frac{1}{4t^2}. \tag{38}$$

Given that $dt \leq 1$, it is easy to see that $e_0^* < e_{pr}^*$. The intuition behind this result is clear: The private firms choose their abatement levels to maximize individual profits. The public firm, however, chooses its abatement levels to maximize welfare, which includes not only the effect of its emissions on its profits but also on pollution. This constitutes our first result:

Lemma 1 *In the equilibrium of the mixed oligopoly, the equilibrium emission intensities of the public firm are lower than those of the private firms. That is, $e_0^* < e_{pr}^*$.*

The above result echoes the empirical findings in Fowlie (2010), which show that publicly owned plants are more likely to invest more in pollution control technologies than (unregulated) private firms. The comparison between the output levels of a private and the public firm does not produce equally unambiguous results. This can be seen from the comparison between q_0^* and q_{pr}^*:

$$q_0^* - q_{pr}^* = \frac{1}{9 + 2n} \left\{ (a - 1) + \frac{1}{4t} \left[\frac{t}{1 + t(1 - d)} (4 + n) - (3 + n) \right] \right\}. \tag{39}$$

Note that only if $(a - 1) > (<) \frac{1}{4t} \left[\frac{t}{1 + t(1 - d)} (4 + n) - (3 + n) \right]$, then $q_0^* > (<) q_{pr}^*$. The intuition is the following: In contrast with the private firms (which only care about their

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[21] In $n > \widehat{n}$, we find a corner solution where the public firm exits the market. In that situation, we would revert to a private oligopoly. If $n < 1$, we would be considering the case where there are no private firms in the market. As the focus of our paper is the interaction between mixed and private firms, these cases are less worth pursuing and therefore, we discard them by imposing the upper and lower bounds on n.

own profits), the welfare-maximizing public firm internalizes the effects of its decisions on both consumer (and producer) surplus as well as on environmental quality. Whereas the former effect offers the public firm incentives to increase production, the latter effect works on exactly the opposite manner (since higher production leads to an increase of pollution). Consequently the overall effect is ambiguous as it depends on the relative strengths of these effects.

Note that the public firm tends to produce more than the private firm for higher values of a and d and lower values of n. For higher values of a, there is more to gain (in terms of consumer surplus) from increasing output; hence the public firm tends to produce more so that to boost consumer surplus. Recall that d is the parameter that measures how effective the cleaning-up technology used by the government is (that is, how effectively tax revenues can be used to clean pollution). Hence, for higher values of d the public firm favours output, as emissions can be more effectively cleaned. Finally, the public firm tends to produce more than the private firms for lower values of n. With fewer private firms (lower n) overall output, and therefore emissions, tend to be lower. In that case, it is optimal for the the public firm to promote more production.

As mentioned above, the direct effect of a higher emission tax on the output level by the private firm is negative, as it raises the marginal cost of firms. For the public firm, the effect of an increase in the emission tax on its output decision is more ambiguous. In the absence of clean-up efforts by the government ($d = 0$), the emission tax t has only a negative effect on the output level by the public firm, due to the same reason as before. However, if $d > 0$, t also has positive effect on welfare through the government clean-up activities.

4 Private oligopoly

In this section we solve the case where there are only private firms in the market. So that to keep this case comparable to the mixed oligopoly, we assume that there are $n + 1$ private firms in the market (that is, it is as if the public firm has been privatized). In all other respects, the modelling of the private oligopoly is identical to the mixed oligopoly. Each firm chooses q_i and x_i to maximize individual profits. The first order conditions for profit maximization for each firm are given by

$$\frac{\partial \pi_i}{\partial q_i} = a - 4q_i - \sum_{i' \neq i} q_{i'} - te_i - x_i = 0, \tag{40}$$

$$\frac{\partial \pi_i}{\partial x_i} = q_i[2t(1 - x_i) - 1] = 0, \tag{41}$$

where $\sum_{i' \neq i} q_{i'}$ indicates the aggregation of outputs by all other private firms.

As before solving $\frac{\partial \pi_i}{\partial x_i} = 0$ yields the following interior solutions in terms of the investment in abatement by each private firm (x_{pr}^{**}):

$$x_{pr}^{**} = 1 - \frac{1}{2t}, \tag{42}$$

i.e, it is symmetric across firms and, once more, it implies that a higher tax will induce greater investments towards cleaner technologies.

Now, substituting x_{pr}^{**} into $\frac{\partial \pi_i}{\partial q_i}$ and solving for q_1, we find each firm's (output) reaction function

$$q_i^R = \frac{1}{4} \left(a - 1 - \sum_{i' \neq i} q_{i'} + \frac{1}{4t} \right). \tag{43}$$

Given that all the private firms are symmetric, in equilibrium, we will have that $q_i = q_{pr}$ $\forall i$. Hence, the above mentioned reaction functions reduce to

$$q_{pr}^{**} = \frac{1}{4 + n} \left(a - 1 + \frac{1}{4t} \right). \tag{44}$$

All in all q_{pr}^{**}, x_{pr}^{**} constitute the (Nash) equilibrium solution of the private oligopoly case.[22] Therefore, emission intensities in equilibrium are

$$e_{pr}^{**} = \frac{1}{4t^2}. \tag{45}$$

Note that the private firm invests the same under both types of oligopoly, while the newly privatized firm invests less in the abatement technology $(e_{pr}^{**} < e_0^*)$.

5 Comparisons

In this section we compare the equilibrium solutions in terms of output, emission rates and total emissions. This will allow us to make some statements regarding the desirability of privatization for outcomes relating to output and aggregate pollution. We start with the comparison of outputs. To save on notation, the subsequent analysis will be employing the composite terms $A \equiv a - 1$ and $\lambda \equiv \frac{t}{1 + t(1-d)}$.

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[22] As with the mixed oligopoly case, the SOCs for maximization are fulfilled. Details are provided in the Appendix.

5.1 Output

The effect of the privatization of the public firm on the private firm's individual equilibrium levels of output is given by:

$$\Delta q_{pr} = q_{pr}^{**} - q_{pr}^* = \frac{1}{(9+2n)(4+n)} \left\{ A - \frac{1}{4t} \left[3 + n - (4+n)\lambda \right] \right\}. \tag{46}$$

On the other hand, the effect of the privatization on the level of output of the privatized firm is given by:

$$\Delta q_0 = q_{pr}^{**} - q_0^* = -\frac{3+n}{(9+2n)(4+n)} \left\{ A - \frac{1}{4t} \left[3 + n - (4+n)\lambda \right] \right\}. \tag{47}$$

Given that $Q^* = q_0^* + n q_{pr}^*$ and $Q^{**} = (n+1)q_{pr}^{**}$, the change in the total level of output ($\Delta Q = Q^{**} - Q^*$) can be written as:

$$\begin{aligned} \Delta Q &= (n+1)q_{pr}^{**} - (q_0^* + n q_{pr}^*) = \\ &= n(q_{pr}^{**} - q_{pr}^*) + (q_{pr}^{**} - q_0^*) = \\ &= n\Delta q_{pr} + \Delta q_0, \end{aligned} \tag{48}$$

or, using (46) and (47)

$$\Delta Q = -\frac{3}{(9+2n)(4+n)} \left\{ A - \frac{1}{4t} \left[3 + n - \lambda(4+n) \right] \right\}. \tag{49}$$

Using the result of eq. (49), we can establish the following:

Proposition 1 *Total output decreases (increases) with the privatization of the public firm ($\Delta Q < (>)0$) if $n < (>)N$, where $N = \max\{1, \widetilde{n}\}$ and $\widetilde{n} \equiv \frac{t}{1-td}\{1 + 4A[1 + t(1 - d)]\} - 3$.*

Proof. Note that $\Delta Q \leq 0$ if $A > \frac{1}{4t}\left[(3 + n) - \lambda(4 + n)\right]$. Solving the above inequality for n, we get the critical value of n, $\widetilde{n}$, below (above) which, ΔQ is negative (positive). In particular, $\widetilde{n} = \frac{t}{1-td}\{1 + 4A[1 + t(1 - d)]\} - 3$, which by virtue of (36), can be written as $\widetilde{n} = \frac{\widehat{n}}{3} - 3 < \widehat{n}$. Given that we have to restrict our attention to cases where $n \geq 1$, then, as long as $\widetilde{n} \leq 1$, it will be that $\Delta Q > 0$, $\forall n$. Using (36) and $\lambda \equiv \frac{t}{1+t(1-d)}$, we can establish that $\widetilde{n} \leq 1$ corresponds to the condition $A \leq \frac{1-t(0.25+d)}{t[1+t(1-d)]}$. When $A > \frac{1-t(0.25+d)}{t[1+t(1-d)]}$, however, then $\widetilde{n} > 1$. Together with Condition 1, this implies that $\Delta Q < 0$ for $n \in [1, \widetilde{n})$ and $\Delta Q > 0$ for $n \in (\widetilde{n}, \widehat{n}]$, thus completing the proof. ■

As we can see, the number of competing firms is a critical factor determining the effect of privatization on aggregate production. The literature on mixed oligopoly has so far noted that aggregate output will decrease (see de Fraja and Delbono, 1989, 1990) or, at

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best, stay the same. The latter requires that output subsidies are provided and firms move simultaneously (see for example, White, 1996; Poyago-Theotoky, 2001; Kato and Tomaru, 2007). If the public firm becomes Stackelberg leader after privatization - and even if output subsidies are provided - then a privatization would be followed by a decrease in aggregate output (Fjell and Heywood, 2004). The same result applies if the government provides R&D subsidies, instead of output subsidies (Gil-Moltó *et al*, 2011). Our result is quite different, as it shows that in less competitive industries (lower n), privatizing the public firm will lead to a decrease in aggregate output while for more competitive markets (higher n), the opposite will apply. Note that comparing (46) and (47) with (49), we can state that for a high number of firms, privatizing the public firm will lead to lower output by each individual private firm ($\Delta q_{pr} < 0$) but also to higher output by the newly privatized firm ($\Delta q_0 > 0$), resulting in an overall increase in aggregate output ($\Delta Q > 0$) Similarly, when the number of private firms is small, the effects of privatizing the public firm take the opposite sign ($\Delta q_{pr} > 0$ and $\Delta q_0 < 0$), leading to a reduction in aggregate output $\Delta Q < 0$).

The intuition for this result can be explained by considering the objective function of the public firm and the effect of the privatization on its behavior. The public firm, in its effort to maximize welfare, has to trade-off the effect of its output decisions on surplus and emissions, while when it is privatized, it does not have environmental concerns anymore (it only maximizes profits). With a relatively high number of (private) firms, aggregate (private) output tends to be high. Therefore, the public firm in the mixed oligopoly finds optimal to produce less due to the effect of its output on emissions. Hence, when privatized, it raises its output, as it does no longer have any environmental concerns. Given that it increases output, and output choices are strategic substitutes, the private firms respond by decreasing their individual levels of output. With a relatively low number of firms, however, aggregate output tends to be small. In this situation, in the mixed oligopoly the public firm focuses on producing more output to boost surplus, as emissions are relatively low (due to the lower output). As a consequence, when it becomes privatized, it will decrease its own output, which will induce an increase in the output produced by the private firms. Overall, aggregate output will decrease.

5.2   Emissions

First of all, note that in the move from the mixed to the private market, average emission intensities increase. The reasons is that the privatized firm will invest less in abatement than if it were public ($e_{pr}^{**} > e_0^*$) while the private firms' choice will not be affected by the privatization ($e_{pr}^{**} = e_{pr}^*$). However, the effect on total emissions is less clear-cut, given that following a privatization the individual and aggregate levels of output may increase or decrease.

Note that total emissions in the mixed oligopoly are given by:

$$\Omega^* = (1 - dt)(e_0^* q_0^* + n e_{pr}^* q_{pr}^*). \tag{50}$$

After substituting the relevant equilibrium solutions and simplifying, we obtain total emissions in mixed oligopoly:

$$\Omega^* = \frac{1 - dt}{(9 + 2n)4t^2} \left\{ A(2n + 3\lambda^2) + \frac{1}{4t}[(3 + n)\lambda^3 - \lambda^2 n + n(3 - \lambda)] \right\}. \tag{51}$$

In turn, total emissions in the private oligopoly are given by:

$$\Omega^{**} = (1 - dt)(n + 1)e_{pr}^{**} q_{pr}^{**}, \tag{52}$$

or:

$$\Omega^{**} = (1 - dt) \left(\frac{n + 1}{4 + n} \right) \frac{1}{4t^2} \left(A + \frac{1}{4t} \right). \tag{53}$$

Therefore, the effect of the privatization of the public firm on total emissions is given by $\Omega^{**} - \Omega^* = \Delta\Omega$ where

$$\Delta\Omega = (1 - dt) \times \left\{ \begin{array}{c} -\frac{A(2n + 3\lambda^2) + \frac{1}{4t}[(3 + n)\lambda^3 - \lambda^2 n + n(3 - \lambda)]}{(9 + 2n)4t^2} + \\ \left(\frac{n+1}{4+n} \right) \frac{1}{4t^2} \left(A + \frac{1}{4t} \right) \end{array} \right\} \tag{54}$$

which after some tedious, but straightforward algebra, can be written as

$$\Delta\Omega = -(1 - dt)\frac{[(9 + 2n)(4 + n)]^{-1}}{4t^2} F(n), \tag{55}$$

where

$$
\begin{aligned}
F(n) &= \frac{(1 - \lambda)^2(1 + \lambda)}{4t} n^2 - (1 - \lambda)\left[3A(1 + \lambda) - \frac{1 - 3\lambda - 7\lambda^2}{4t} \right] n + \\
&\quad + 3 \left\{ \left(A + \frac{1}{4t} \right) [1 - 4(1 - \lambda^2)] - \frac{\lambda^2(1 - \lambda)}{t} \right\}.
\end{aligned} \tag{56}
$$

As the reader can see, whether total emissions increase or decrease with a privatization depends on the sign of $F(n)$. Particularly, given (55) it will be $\Delta\Omega < 0 \ (> 0)$ as long as $F(n) > 0 \ (< 0)$. Note that the first and second derivatives of $F(n)$ with respect to n are:

$$F'(n) = \frac{2n(1-\lambda)^2(1+\lambda)}{4t} - (1-\lambda)\left[3A(1+\lambda) - \frac{1-3\lambda-7\lambda^2}{4t}\right], \qquad (57)$$

$$F''(n) = \frac{2(1-\lambda)^2(1+\lambda)}{4t}. \qquad (58)$$

Furthermore, it is $F''(n) > 0$; that is, $F(n)$ is a convex function of $n, \forall n$. Recall that $n \in [1, \widehat{n}]$. Using (36) and $\lambda \equiv \frac{t}{1+t(1-d)}$ in (56) we get

$$F(\widehat{n}) = \frac{-9(1-td) - 6t - 24A[1+t(1-d)]}{4t\left[1+t(1-d)\right]^3}, \qquad (59)$$

which is negative since $A > 0$ and $td \in [0,1]$. Thus, we want to check the sign of (56) at $n = 1$ (that is, at the minimum possible value of n) to check whether $F(n)$ is always negative or whether it changes sign from positive to negative as n increases. Given that we know that $F(n)$ is a convex function of $n, \forall n$, and that $F(\widehat{n}) < 0$, we know that $F(n)$ can change sign at most once, from positive to negative, within $n \in [1, \widehat{n}]$. If $F(1) > 0$, then we know that $F(n)$ changes sign once (from positive to negative) within $n \in [1, \widehat{n}]$. On the other hand, if $F(1) < 0$, then $F(n) < 0, \forall n$.

At $n = 1$, (56) becomes

$$F(1) = \frac{-7 - t\Phi - 12At\left[1+t(1-d)\right]\Psi}{4t\left[1+t(1-d)\right]^3}, \qquad (60)$$

where $\Phi = (1-dt)[26(1-dt) + 36t - 21d] - t^2(3+7d^3)$ and $\Psi = (1-dt)[4(1-dt) + 8t] - t^2$. Therefore, the sign of $F(1)$ depends on the sign of its numerator. Recalling that $t \geqslant 1/2$ by assumption, the following summarizes all the possible outcomes regarding the sign of $F(1)$.

Lemma 2 *There exist $\underline{t}$, $\overline{t}$, and A_0 such that:*
(i) If $t \in [1/2, \underline{t})$ then $F(1) < 0$;
(ii) If $t \in (\underline{t}, \overline{t})$ then $F(1) < 0 \ (> 0)$ if $A < A_0 \ (> A_0)$;
(iii) If $t > \overline{t}$ then $F(1) > 0$.

Proof. See the Appendix. ∎

Armed with this result and our previous analysis, we can derive the implications for the change in total emissions after privatization as follows:

Proposition 2 *After the privatization of the public firm, the change in pollution will be as follows:*
(i) Pollution will unambiguously increase if $t < \underline{t}$ or $t \in (\underline{t}, \overline{t})$ and $A < A_0$;
(ii) When $t \in (\underline{t}, \overline{t})$ and $A > A_0$ or when $t > \overline{t}$, pollution may increase or decrease depending

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on the number of firms competing in the market. Particularly, there exists $\overline{n}$, for which $F(\overline{n}) = 0$, such that $\overline{n} < \widetilde{n} < \widehat{n}$. Thus, in this case, pollution will decrease if $n \in (1, \overline{n})$ but it will increase if $n \in (\overline{n}, \widehat{n})$.

Proof. See the Appendix. ∎

Proposition 2 states that for relatively low values of t, emissions will necessarily increase with privatization. However, for relatively high values of t, privatizing the public firm will lead to an increase (decrease) in emissions as long as the number of firms is sufficiently high (low). In the next section we discuss the intuition of this result, as well as the joint implications of Propositions 1 and 2 regarding the advisability of privatization.

5.3 On the desirability of privatization

So far, we have established that the move from the mixed to a private oligopoly (i.e. following a privatization) has implications for both total production and aggregate emissions. The overall effects depend on structural characteristics such as parameters that determine the demand conditions, the strictness of environmental policies as well as the number of firms competing in the market. Naturally, the most desirable outcomes would entail cases where a reduction in emissions does not come at the cost of lower output, or equivalently when an increase in output does not necessarily entail the negative by-product of increased total emissions. Of course, cases where total output declines and aggregate emissions increase at the same time are the least desirable ones.

Note that scenarios such as the ones described above are possible given that total pollution is an outcome determined by two distinct factors - output and emission intensities. Combining the results from Propositions 1 and 2, we can determine the following possible outcomes. When parameter conditions are such that $\widetilde{n} < 1$, then it must be the case that $\overline{n} < 1$ as well. Therefore, after privatization, we have $\Delta Q > 0$ and $\Delta \Omega > 0$. If, however, parameter conditions satisfy $\widetilde{n} > 1$, then the final outcomes are determined as follows: For $t < \underline{t}$ or $t \in (\underline{t}, \overline{t})$ and $A < A_0$, then after privatization we have

$$\Delta \Omega > 0, \Delta Q < 0 \text{ for } n \in [1, \widetilde{n}), \tag{61}$$

$$\Delta \Omega > 0, \Delta Q > 0 \text{ for } n \in (\widetilde{n}, \widehat{n}]. \tag{62}$$

For $t \in (\underline{t}, \overline{t})$ and $A > A_0$ or $t > \overline{t}$, the outcomes that follow the privatization are

$$\Delta\Omega \quad < \quad 0, \ \Delta Q < 0 \quad \text{for } n \in [1, \overline{n}), \tag{63}$$

$$\Delta\Omega \quad > \quad 0, \ \Delta Q < 0 \quad \text{for } n \in (\overline{n}, \widetilde{n}), \tag{64}$$

$$\Delta\Omega \quad > \quad 0, \ \Delta Q > 0 \quad \text{for } n \in (\widetilde{n}, \widehat{n}]. \tag{65}$$

The previous summary allows us to derive some stark implications, which are summarized below:

Proposition 3 *Following the privatization, an increase in output will necessarily come at the cost of higher pollution. Similarly, a reduction in pollution will necessarily come at the expense of lower output. In fact, privatization may result in simultaneously lower output and higher pollution; an outcome where, by comparison, output is higher and pollution is lower can emerge only under a mixed oligopoly.*

The intuition behind these results is the following: As we have established earlier, the public firm internalizes the adverse effect of pollution on social welfare, thus it always invests more resources towards abatement compared to private firms. As a result, the privatization will unambiguously increase the average emission intensity in the economy. Consequently, when the move towards a purely private oligopoly results in an overall increase in production, pollution will be unambiguously higher. What is more, it may be possible that the reduction on abatement investment under private oligopoly is so strong and the average emission intensity so high, compared to the mixed oligopoly case, that privatization may lead to higher total emissions even under circumstances where total output declines (see the cases summarized by the expressions 61 and 64). On the contrary, it is only under the mixed oligopoly, where investments in environmental improvements by the public firm are stronger, that we have the possibility of the most desirable outcome, i.e. a situation where total pollution is still lower despite the fact that total output is higher. Naturally, such outcomes raise awareness to some circumstances under which privatization may lead to the least desirable outcomes, once we consider its combined impact on both output and the environment.

Of course, an even more formal examination regarding the desirability of privatization would entail comparisons of the social welfare corresponding to the mixed and private oligopolies. This is a task we undertake in the following section where we also endogenize the tax rate on emissions. As our previous analysis and discussion suggests, apart from the cases where privatization leads to a combination of undesirable effects (i.e., when it can jointly account for higher emissions and lower output), in many cases the impact of privatization leads to outcomes with conflicting implications for social welfare. For example, some cases

with higher consumer surplus (due to higher output) are also associated with reductions in social welfare due to higher pollution. Analogously, some cases where improvements in social welfare result from lower pollution are also associated with losses to consumer surplus (resulting from lower output) - an outcome that has a negative effect on welfare. These conflicting effects, together with the complexity of the equilibrium solutions (even under our generally simple set-up) mean that it is not possible to provide any clear-cut implications analytically. It is for this reason that we undertake this task by means of numerical examples in the following section. In any case, we should also remember that, in this context, changes in social welfare do not have corresponding implications regarding Pareto optimality. The conflicting effects discussed above imply that some agents may be worse off (better off) even when social welfare improves (worsens). Still, however, this is an appropriate measure of well-being for the society as a whole, that is why we examine its implications in the next section.

6   Extension: Endogenously chosen tax rate

In the previous section, we have conducted the analysis of the mixed and the private oligopoly cases assuming that the tax rate is exogenously given. As explained before, this could reflect cases where the ownership of the public firm belongs to a local authority and emission tax is set at a national level or a situation where the ownership of the firm is held at national level but the emission tax is set at federal level. Alternatively, the exogeneity of the emission tax could also be related to budgetary constraints. In this section, we relax this assumption by solving the following game. In stage 1, the government sets the tax on emissions and commits to it. The tax is chosen to maximize social welfare which is defined as in (16). Then, in stage 2, firms choose their abatement investments and output. We solve the game for both the private and the mixed oligopoly. The solutions to the second stage are those in sections 3 and 4. Given the complexity of the problem, it is not possible to get a closed form solution for the emission tax in the mixed oligopoly. For this reason, we will undertake the analysis by means of numerical examples. As it will become clear from the numerical solutions, the main implications of the original set-up (see Proposition 3) survive even under this extension, where the emission tax is endogenously chosen.

In Tables 1 to 4 (see Appendix), the reader can find the equilibrium tax, output and emissions levels for different values of the parameters a and d. For example, consider Table 1, which provides the socially optimal tax rate and equilibrium levels of output and emissions for $a = 1.5$ and $d = 0$. In this case, for $n \leq 15$, privatization leads to lower emissions but this comes at the expense of lower output. For $n \geq 19$, total output is higher after privatization although this comes at the expense of higher pollution. For the intermediate

range of values of n ($15 < n < 19$), privatization does not only result in lower output but also on higher emissions, which implies that privatization leads to both lower private surplus and also more pollution. In Table 2, we have increased the value of d to 0.25, and we can see that the same qualitative results apply. In this case, for $n = 17$ and $n = 18$, a privatization yields lower output and higher emissions. For lower values of n, privatization may improve the environment, but at the cost of lower output, while the opposite applies for higher values of n. Tables 3 and 4, present two more examples with higher a, which again show the same qualitative results. All in all, what our numerical simulations show is that is that even with the socially optimal tax rate, the main implications of our analysis as presented in Proposition 3 still apply.

Finally, we have also computed the social welfare values before and after privatization. As discussed before, privatization may have conflicting effects on output (and therefore surplus) and total emissions, which implies that in principle the effect on social welfare is unclear. The reader can see that our simulations indicate that privatization will tend to negatively affect social welfare. Interestingly, for a relatively low number of firms, privatizing the public firm renders an overall negative effect on social welfare, because even though emissions are reduced, output is reduced as well. For an intermediate number of firms, privatization results in lower output but higher emissions, and again, in an overall negative effect on social welfare. For a relatively large number of firms, privatizing the public firm can lead to higher levels of both output and emissions, which generate opposite effects on social welfare. The simulations in our tables suggest that the interplay of these two effects results in lower levels of welfare, because the effect on emissions outweighs the positive effect derived from the increase in output. A priori, however, one can envisage that the comparison of these two effects could take the opposite sign (that is, the positive effect of privatizing the public firm through output may outweigh the negative effect through emissions); which would imply that a privatization is welfare enhancing. However, at least in our simulations, this could only happen for very large number of firms. For example, such an outcome would require about 80 and 46 private firms in the simulations in Tables 1 and 2 respectively. Therefore, the general message that can be extracted from our simulations is that a privatization in the context of our model will tend to damage social welfare.

7   Conclusions

The purpose of our paper was to contribute to the debate concerning the desirability of privatization by developing a theory that offers a positive analysis on the effect of privatization on both economic and environmental outcomes. In our model we consider an oligopoly composed of one (welfare-maximizing) public firm and n (profit-maximizing) private firms.

Firms face a proportional tax on emissions and decide on both output and the level of abatement investment. In order to evaluate the effect of privatization, we compare the results of this model with a setting where the public firm is privatized, in the sense that its objectives and behavior are identical to those of private firms.

Our model identified two distinct channels through which privatization impinges on the amount of emitted pollutants. Firstly, a public firm adopts a less polluting production process since it invests more resources towards abatement – an outcome that stems from the idea that a welfare maximizing public firm internalizes the societal cost of pollution. Secondly, for a given level of abatement, a public firm may produce either more or less compared to the amount of production that would undertake had it been privately owned. This is because of two opposing effects on the public firm's objective function: on the one hand, the internalization of the societal cost of emissions favours lower production; on the other hand, part of the public firm's objective is to maximize consumer surplus – an incentive that supports higher production. We identified the strength of competition, captured by the number of competing firms, as a critical factor in determining which of the two effects will dominate.

The combined effect of abatement investment and output decisions implies that, in principle, reductions in total emissions may be possible, even when the total production of the industry increases. This may happen as long as the reduction in emission intensity is particularly strong. Nevertheless, such an outcome never actually materializes following a privatization. Lower pollution entails the trade-off of lower output (and therefore lower consumer surplus). When production actually increases, following the privatization, pollution increases as well. More strikingly, there are circumstances where privatization may lead to the combined effect of lower output and increased pollution – an outcome related to the significant increase in average emission intensity following the public sector's surrendering of its direct involvement in the production of the industry's good. In terms of policy implications, our analysis indicates that, once we consider the issue of environmental quality, the merits of privatization may be less straightforward than has often been assumed. In fact, the combined effect of production and environmental technology decisions may result in a situation where privatization will result in a clear loss to the society. As private firms opt for more polluting production processes, the cost of higher emission taxes leads to lower production, but the increase in average emission intensity may be of such magnitude that total emissions could ultimately increase. These results emerge irrespective of whether the emission tax is either exogenously given or endogenously chosen (i.e., an emission tax set so as to maximize social welfare).

In terms of our methodological approach, we have opted for a framework that ensures the model's tractability and transparent characterization of all the equilibrium results. This

allowed us to retain a sharp focus on the interactions between (environmental) technology choice and production under the setting of both mixed and private oligopolies. Moreover, it allowed us to avoid blurring both their clarity of the results and their intuition by absconding from approaches that would have added significant technical complication while touching upon issues that go beyond the scope of our current analysis. Nevertheless, there are elements that would certainly enrich the current set-up and offer the possibility of additional implications. For example, one could consider the scenario whereby the public firm's objectives are permeated by issues of misgovernance and corruption in the public sector, meaning that its management may attach a reduced weight to social welfare. Another promising direction is to consider issues that offer additional incentives to private firms in undertaking investments in abatement – issues such as the presence of influential environmental activists or environmentally-aware consumers. Such extensions are indubitably important, thus they represent promising avenues for future research.

8 Appendix

8.1 Second Order Conditions

8.1.1 Mixed Oligopoly

- *For the Public Firm:*

$$\frac{\partial^2 SW}{\partial q_0^2} = -3 < 0 \tag{A.1}$$

$$\frac{\partial^2 SW}{\partial x_0^2} = -2q_0 \left[t + (1 - dt)\right] < 0 \tag{A.2}$$

and

$$\frac{\partial^2 SW}{\partial q_0 \partial x_0} = 2(1 - x_0) \left[t + (1 - dt)\right] - 1 \tag{A.3}$$

Note that in equilibrium, we know that

$$1 - x_0^* = \frac{1}{2 \left[t + (1 - dt)\right]} \tag{A.4}$$

and therefore

$$\frac{\partial^2 SW}{\partial q_0 \partial x_0} = \frac{2 \left[t + (1 - dt)\right]}{2 \left[t + (1 - dt)\right]} - 1 = 0 \tag{A.5}$$

Thus, given that $\frac{\partial^2 SW}{\partial q_0^2} < 0$, $\frac{\partial^2 SW}{\partial x_0^2} < 0$ and $\frac{\partial^2 SW}{\partial q_0 \partial x_0} = 0$, we know that the q_0^* and x_0^* maximize the public firms' objective function.

- *For the Private Firm:*

$$\frac{\partial^2 \pi_i}{\partial q_i^2} = -4 < 0 \tag{A.6}$$

$$\frac{\partial^2 \pi_i}{\partial x_i^2} = -2t q_i < 0 \tag{A.7}$$

and

$$\frac{\partial^2 \pi_i}{\partial q_i \partial x_i} = 2t(1 - x_i) - 1 \tag{A.8}$$

Note that in equilibrium, we know that

$$1 - x_i^* = \frac{1}{2t} \tag{A.9}$$

and therefore

$$\frac{\partial^2 \pi_i}{\partial q_i \partial x_i} = \frac{2t}{2t} - 1 = 0 \tag{A.10}$$

Thus, given that $\frac{\partial^2 \pi_i}{\partial q_i^2} < 0$, $\frac{\partial^2 \pi_i}{\partial x_i^2} < 0$ and $\frac{\partial^2 \pi_i}{\partial q_i \partial x_i} = 0$, we know that the q_i^* and x_i^* maximize the private firms' objective function.

8.1.2 Private Oligopoly

In this case:

$$\frac{\partial^2 \pi_i}{\partial q_i^2} = -4 < 0 \tag{A.11}$$

$$\frac{\partial^2 \pi_i}{\partial x_i^2} = -2t q_i < 0 \tag{A.12}$$

and

$$\frac{\partial^2 \pi_i}{\partial q_i \partial x_i} = 2t(1 - x_i) - 1 \tag{A.13}$$

Note that in equilibrium, we know that

$$1 - x_i^{**} = \frac{1}{2t} \tag{A.14}$$

and therefore

$$\frac{\partial^2 \pi_i}{\partial q_i \partial x_i} = \frac{2t}{2t} - 1 = 0 \tag{A.15}$$

Given that $\frac{\partial^2 \pi_i}{\partial q_i^2} < 0$, $\frac{\partial^2 \pi_i}{\partial x_i^2} < 0$ and $\frac{\partial^2 \pi_i}{\partial q_i \partial x_i} = 0$, we know that the q_i^{**} and x_i^{**} maximize the private firms' objective function.

8.2   Proof of Lemma 2

Firstly, consider the case where $d = 1$, a case where (given $dt < 1$), it is $t \in [0.5, 1]$. We have $\Psi = 4 - 5t^2 = \Psi(t)$ and $-7 - t\Phi = -7 - 5t - 5t^2 + 20t^3 = z(t)$ It is $\Psi(0.5) > 0$, $\Psi(1) < 0$ and $\Psi(\underline{t}) = 0 \Longrightarrow \underline{t} = \sqrt{4/5} \simeq 0.894$, meaning that $\Psi > 0$ (< 0) for $t \in [0.5, \underline{t})$ ($t \in (\underline{t}, 1]$). Furthermore, $z(0.5) < 0$, $z(1) > 0$ and $z'(t) = -5 - 10t + 60t^2 > 0 \nabla t \in [0.5, 1]$. Therefore there is a $\bar{t}$ such that $z(\bar{t}) = 0$, and $z < 0 (> 0)$ for $t \in (0.5, \bar{t})$. In fact, $\bar{t} \simeq 0.927$. Thus, we can conclude that, when $d = 1$, $F(1)$ is unambiguously negative (positive) when $0.5 \leq t < \underline{t}$ ($\underline{t} < t \leq 1$). For the intermediate values $t \in (\underline{t}, \bar{t})$, $F(1) < 0$ as long as $-7 - t\Phi - 12At\Psi < 0 \Longrightarrow -7 - t\Phi > -12At\Psi \Longrightarrow A < \frac{-7 - t\Phi}{-12At\Psi} = A_0$.

Next, let us consider the case where $d = 0$. now, given $dt < 1$, we can consider all values for which $t \in [1/2, \infty)$. for $d = 0$, we have $\Psi = 4 + 8t - t^2 = \Psi(t)$. It is $\Psi(0.5) > 0$, $\Psi(1) < 0$ and $\Psi(\underline{t}) = 0 \Longrightarrow \underline{t} = (2 + \sqrt{5}) \simeq 8.47$. Thus, $\Psi > 0$ (< 0) for $t \in [0.5, \underline{t})$ ($t \in (\underline{t}, 1]$). Furthermore, for $d = 0$, we have $-7 - t\Phi = -7 - 26t - 36t^2 + 36t^3 = z(t)$. Note that. $z(0.5) < 0$, $z(1) > 0$ and $z'(t) = -26 - 72t + 108t^2 > 0 \nabla t \in [0.5, 1]$. Therefore, there is a $\bar{t}$ such that $z(\bar{t}) = 0$, and $z < 0 (> 0)$ for $t \in (0.5, \bar{t})$. In fact, $\bar{t} \simeq 36.71$ in this case..Hence, for $d = 0$, we can conclude that $F(1)$ is unambiguously negative (positive) when $0.5 \leq t < \underline{t}$ ($\underline{t} < t \leq 1$). For the intermediate values $t \in (\underline{t}, \bar{t})$, $F(1) < 0$ as long as $-7 - t\Phi - 12At\Psi < 0 \Longrightarrow -7 - t\Phi > -12At\Psi \Longrightarrow A < \frac{-7 - t\Phi}{-12At\Psi} = A_0$.

The previous analysis has shown that qualitatively, the implications for the sign of $F(1)$ are identical irrespective of $d = 0$ and $d = 1$. Now denote the numerator of $F(1)$ as $k(d) = -7 - t\Phi - 12At[1 + t(1 - d)]\Psi$. It is $k'(d) = -t\frac{\partial \Phi}{\partial d} + \{12At[t\Psi - [1 + t(1 - d)]\frac{\partial \Psi}{\partial d}\}$. some straightforward can reveal that $\frac{\partial \Phi}{\partial d} = -t^2(26 - 26d) - t(26 - 21d) - 26t(1 - dt) - 21(dt)^2 - 21(1 - dt) < 0$ and $t\Psi - [1 + t(1 - d)]\frac{\partial \Psi}{\partial d} = 4t(1 - dt)[1 + t(1 - d)] + t(1 - dt)[4(1 - dt) + 8t] + [1 + t(1 - d)]4t(1 - dt) + (1 - dt)8t^2 + 7t^3 > 0$. Thus, $k'(d) > 0$. Therefore, given that the expression determining the sign of $F(1)$ is monotonic in d and that the qualitative characteristics of $F(1)$ are the same whether $d = 0$ or $d = 1$, we can establish that these qualitative characteristics will be identical for any $d \in [0, 1]$.

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8.3 Proof of Proposition 2

The first part of the proposition is straightforward because in these cases we have $F(n) < 0$, $\forall n$, therefore $\Delta E > 0$. for the second paper, we have cases for which $F(1) > 0$. together with $F(\widehat{n}) < 0$ and $F''(n) > 0$, which jointly imply that there is $\overline{n}$ such that $F(\overline{n}) = 0$ and $F(n) > 0 (< 0)$ if $n < \overline{n}$ $(n > \overline{n})$. Now consider the case where $d = 1$. One can show that in this scenario $F(\overline{n}) = 0$ leads to only one solution an the interval $(1, \widehat{n})$, equal to

$$\overline{n} = \frac{-1 + 4t + 12At + 4t^2 - 7t^3 - 12At^3 - \sqrt{\mu}}{2(1-t)^2(1+t)} \tag{A.16}$$

where $\mu = (1 - 4t - 12At - 4t^2 + 7t^3 + 12At^3)^2 - 4(1-t)^2(1+t)(-9 + 36At + 12t^3 + 48At^3$.

Comparing with the value for $\widetilde{n}$ we determined before (see Proposition 1), we can show that the sign of the difference $\widetilde{n} - \overline{n}$ ultimately depends on he sign of the expression $4(1 + 4At)(1 - t^2)^2(3 - t + 8At)$, which is unambiguously positive for $t \in [1/2, 1]$ (recall $dt < 1$ must hold). Nevertheless, in the proof of Lemma 2, we showed that $k(d)$, the expression determining the sign of $F(1)$ is monotonic and increasing in d. This implies that $\forall d \in [0, 1]$, there is a unique $\overline{n} \in [1, \widehat{n}]$ such that $\overline{n} < \widetilde{n}$, thus completing the proof.

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TABLE 1: Extension. Equilibrium values for emission tax (t), output (Q) and total emissions (Ω) for $a=1.5$ and $d=0$

	Mixed oligopoly	Private oligopoly	After privatisation
$n<15$	...		$\Delta Q<0$, $\Delta \Omega<0$
$n=15$	$t=2.05242$	$t=2.08384$	$\Delta Q<0$ $\Delta \Omega<0$ $\Delta SW<0$
	$Q=0.523075$	$Q=0.522081$	
	$\Omega=0.030087$	$\Omega=0.030057$	
	$SW=0.14086$	$SW=0.14030$	
$n=16$	$t=2.06207$	$t=2.08871$	$\Delta Q<0$ $\Delta \Omega>0$ $\Delta SW<0$
	$Q=0.527428$	$Q=0.526734$	
	$\Omega=0.030139$	$\Omega=0.030184$	
	$SW=0.14170$	$SW=0.14118$	
$n=17$	$t=2.07081$	$t=2.09314$	$\Delta Q<0$ $\Delta \Omega>0$ $\Delta SW<0$
	$Q=0.53137$	$Q=0.53095$	
	$\Omega=0.03018$	$\Omega=0.03030$	
	$SW=0.14245$	$SW=0.14198$	
$n=18$	$t=2.07877$	$t=2.09719$	$\Delta Q<0$ $\Delta \Omega>0$ $\Delta SW<0$
	$Q=0.53496$	$Q=0.53477$	
	$\Omega=0.03022$	$\Omega=0.03039$	
	$SW=0.143121$	$SW=0.14269$	
$n=19$	$t=2.0861$	$t=2.1009$	$\Delta Q>0$ $\Delta \Omega>0$ $\Delta SW<0$
	$Q=0.53824$	$Q=0.53826$	
	$\Omega=0.030254$	$\Omega=0.030487$	
	$SW=0.143733$	$SW=0.14335$	
$n>19$	...		$\Delta Q>0$, $\Delta \Omega>0$

TABLE 2: Extension. Equilibrium values for emission tax (t), output (Q) and total emissions (Ω) for $a=1.5$ and $d=0.25$

	Mixed oligopoly	Private oligopoly	After privatisation
$n<16$	...		$\Delta Q<0$, $\Delta\Omega<0$
$n=16$	$t=1.62888$	$t=1.64411$	$\Delta Q<0$
	$Q=0.554852$	$Q=0.554249$	$\Delta\Omega<0$
	$\Omega=0.030246$	$\Omega=0.030191$	$\Delta SW<0$
	$SW=0.15997$	$SW=0.15954$	
$n=17$	$t=163416$	$t=1.64687$	$\Delta Q<0$
	$Q=0.559034$	$Q=0.558688$	$\Delta\Omega>0$
	$\Omega=0.030274$	$\Omega=0.0302654$	$\Delta SW<0$
	$SW=0.16084$	$SW=0.16045$	
$n=18$	$t=1.63895$	$t=1.64938$	$\Delta Q<0$
	$Q=0.562839$	$Q=0.562721$	$\Delta\Omega>0$
	$\Omega=0.0302975$	$\Omega=0.0303886$	$\Delta SW<0$
	$SW=0.16162$	$SW=0.16127$	
$n=19$	$t=1.64334$	$t=1.65169$	$\Delta Q>0$
	$Q=0.5663$	$Q=0.5664$	$\Delta\Omega>0$
	$\Omega=0.030317$	$\Omega=0.0304722$	$\Delta SW<0$
	$SW=0.16233$	$SW=0.16201$	
$n>19$	...		$\Delta Q>0$, $\Delta\Omega>0$

TABLE 3: Extension. Equilibrium values for emission tax (t), output (Q) and total emissions (Ω) for $a=1.25$ and $d=0.25$

	Mixed oligopoly	Private oligopoly	After privatisation
$n<10$	...		$\Delta Q<0$, $\Delta \Omega<0$
$n=10$	$t=1.65534$	$t=1.6776$	$\Delta Q<0$
	$Q=0.31397$	$Q=0.31357$	$\Delta \Omega<0$
	$\Omega=0.00162127$	$\Omega=0.0161696$	$\Delta SW<0$
	$SW=0.051177$	$SW=0.05085$	
$n=11$	$t=1.66592$	$t=1.68296$	$\Delta Q<0$
	$Q=0.318868$	$Q=0.318838$	$\Delta \Omega>0$
	$\Omega=0.016259$	$\Omega=0.0163017$	$\Delta SW<0$
	$SW=0.05175$	$SW=0.05127$	
$n=12$	$t=1.67516$	$t=1.68768$	$\Delta Q>0$
	$Q=0.323156$	$Q=0.323482$	$\Delta \Omega>0$
	$\Omega=0.0162955$	$\Omega=0.0164133$	$\Delta SW<0$
	$SW=0.05225$	0.05201	
$n>12$	...		$\Delta Q>0$, $\Delta \Omega>0$

TABLE 4: Extension. Equilibrium values for emission tax (t), output (Q) and total emissions (Ω) for $a=2$ and $d=0$

	Mixed oligopoly	Private oligopoly	After privatisation
$n<27$	...		$\varDelta Q<0$, $\varDelta\Omega<0$
$n=27$	$t=2.02426$	$t=2.04312$	$\varDelta Q<0$
	$Q=1.01456$	$Q=1.01375$	$\varDelta\Omega<0$
	$\Omega=0.06075$	$\Omega=0.06071$	$\varDelta SW<0$
	$SW=0.52721$	$SW=0.52654$	
$n=28$	$t=2.02797$	$t=2.04502$	$\varDelta Q<0$
	$Q=1.01771$	$Q=1.01704$	$\varDelta\Omega>0$
	$\Omega=0.06078$	$\Omega=0.0608$	$\varDelta SW<0$
	$SW=0.52837$	$SW=0.52772$	
$n=29$	$t=2.03146$	$t=2.04682$	$\varDelta Q<0$
	$Q=1.02067$	$Q=1.02013$	$\varDelta\Omega>0$
	$\Omega=0.06080$	$\Omega=0.06087$	$\varDelta SW<0$
	$SW=0.52945$	$SW=0.52883$	
$n=30$	$t=2.03475$	$t=2.04852$	$\varDelta Q<0$
	$Q=1.02346$	$Q=1.02304$	$\varDelta\Omega>0$
	$\Omega=0.06082$	$\Omega=0.06095$	$\varDelta SW<0$
	$SW=0.53046$	$SW=0.52988$	
$n=31$	$t=2.03785$	$t=2.05012$	$\varDelta Q<0$
	$Q=1.0261$	$Q=1.02578$	$\varDelta\Omega>0$
	$\Omega=0.0608$	$\Omega=0.0610$	$\varDelta SW<0$
	$SW=0.53140$	$SW=0.53086$	
$n=32$	$t=2.04079$	$t=2.05163$	$\varDelta Q<0$
	$Q=1.02859$	$Q=1.02837$	$\varDelta\Omega>0$
	$\Omega=0.06086$	$\Omega=0.06108$	$\varDelta SW<0$
	$SW=0.53230$	$SW=0.53178$	
$n=33$	$t=2.04357$	$t=2.05306$	$\varDelta Q<0$
	$Q=1.03094$	$Q=1.03082$	$\varDelta\Omega>0$
	$\Omega=0.06087$	$\Omega=0.06114$	$\varDelta SW<0$
	$SW=0.53315$	$SW=0.53266$	
$n=34$	$t=2.0462$	$t=2.05442$	$\varDelta Q<0$
	$Q=1.03317$	$Q=1.03313$	$\varDelta\Omega>0$
	$\Omega=0.06089$	$\Omega=0.06119$	$\varDelta SW<0$
	$SW=0.53394$	$SW=0.53348$	

$n=35$	$t=2.04871$	$t=2.05572$	$\Delta Q>0$
	$Q=1.03529$	$Q=1.03533$	$\Delta \Omega>0$
	$\Omega=0.06090$	$\Omega=0.06125$	$\Delta SW<0$
	$SW=0.5347$	$SW=0.53426$	
$n>35$	...		$\Delta Q>0,\ \Delta \Omega>0$