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Growth and Demographic Change: Do Environmental Factors Matter?

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Abstract

We incorporate health-damaging pollution into a three period overlapping generations model in which life expectancy, fertility and economic growth are all endogenous. We show that environmental factors can cause significant changes to the economy's demographics. In particular, the entrepreneurial choice of less polluting production processes, induced by environmental policy, can account for such demographic changes as higher longevity and lower fertility rates.

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1 Introduction

The question on whether demographic changes, and their corresponding implications for population growth, are inherently linked to environmental issues is by no means a new one. In the past, many analysts have argued that population growth contributes to the decay of the natural environment as it has been associated with such problems as deforestation; air and water pollution; global warming; increased waste etc (see Meadows *et al.* 1972; Ehrlich and Ehrlich 1990). Others have opted for a less pessimistic argument, based on empirical studies supporting the view that the quantitative effect of population growth on pollution, despite being statistically significant, is relatively small (see, for example, Preston 1996). What is common among these views is the underlying idea that the causality on the nexus between population growth and environment quality runs from the former to the latter.

In this paper, we develop an economic theory to illustrate *how and why environmental factors may actually cause changes in some important demographic aspects*. Our analysis is motivated by the striking demographic changes that occurred in industrialised economies during the second half of the last century – changes such as greater life expectancy; reduced mortality; and lower fertility rates.[1] Existing theories that have sought to explain the joint determination of economic growth, fertility, longevity and mortality have absconded from issues pertaining to environmental quality (e.g., Blackburn and Cipriani 2002; Lagerlöf 2003; Zhang and Zhang 2005; Cervelatti and Sunde 2007). Other theoretical analyses have incorporated environmental quality in models of growth and (endogenous) life expectancy but have neglected the issue of fertility choices (e.g., Pautrel 2009; Mariani *et al.* 2010; Varvarigos 2010; Jouvet *et al.* 2010). A recent strand of literature that examines the interactions between pollution and optimal fertility choices employ representative agent models where mortality and life expectancy are exogenous (Schou 2002; Jöst *et al.* 2006; Lehmijoki and Palokangas 2010). In contrast to this literature, and as the following description of our model's mechanisms reveals, the presence of endogenous lifetime is of crucial importance for our results.

To the best of our knowledge, our paper is the first to explicitly consider environmental issues, within a growth model where *both fertility and life expectancy* are endogenous, thus suggesting that some well-documented demographic changes, as well as changes to

[1] See Galor (2005) and the references therein for a comprehensive discussion on the issue.

economic outcomes such as economic growth, may be (partially) attributed to factors associated with environmental quality. We build a discrete-time overlapping generations model in which both fertility decisions and life expectancy are endogenous. With respect to the latter, we account for the negative repercussions of pollution for the population's health status. These repercussions are well-documented and quantitatively significant: for example, Pimentel *et al.* (1998) argue that the direct and indirect effects of environmental degradation can account for almost 40% deaths worldwide.

Our model shows that, in the presence of emission taxes, the process of economic growth will generate sufficient resources for entrepreneurs to opt for a less polluting production method.[2] When this happens, the reduction in emissions per unit of output causes an increase in longevity. Consequently, households will find optimal to increase their saving in order to carry more resources towards future consumption. In addition to a higher saving rate, the latter effect is also associated with a reduction in fertility. This is because households will try to smooth their consumption profile by providing more labour when young, with the purpose of counteracting the adverse effect of a higher saving rate on their current consumption. This can only be achieved by a reduction in the time/effort they devote towards child rearing; hence both the fertility rate and the growth rate of the population fall.

The structure of our analysis is as follows. Section 2 describes the economy's main characteristics In Section 3 we show how pollution impinges on endogenous life expectancy. Section 4 analyses the model's equilibrium while Section 5 derives the equilibrium growth rate. In Section 6 we describe the mechanism through which the emission rate falls endogenously in the process of economic development. Section 7 presents the meain results concerning the joint determination of pollution per unit of output, economic growth, fertility and longevity. In Section 8 we conclude.

[2] A study carried by the OECD (2007) supports the idea that environmentally related taxes encourage changes in production processes that are based on cleaner production techniques and environmental R&D. There is also support for another characteristic of our mechanism – that is, the fact that higher GDP growth is positively associated with the promotion of new technologies that are directed towards environmental improvements. See Komen *et al.* (1997), Requate and Unold (2003), and Requate (2005) among others.

2 The Economy

We construct an overlapping generations model in which time takes form of discrete periods which are indexed by $t = 0,1,2,...$. In addition to a government, every period there are two groups of agents active in the economy. Henceforth, we shall be referring to these distinct groups as *households* and *entrepreneurs*.

At the beginning of each period, a unit mass of entrepreneurs comes into existence. Each of them lives for only one period and enjoys utility by consuming units of the economy's final good.[3] She is endowed with a technology that allows her to combine labour units from households, denoted L_{it}, and capital from financial intermediaries, denoted K_{it}, to produce a specific variety i of an intermediate product according to

$$y_{it} = BK_{it}^{\beta}(A_t L_{it})^{1-\beta}, \tag{1}$$

where $B > 0$ and $0 < \beta < 1$. The variable A_t indicates some type of labour-augmenting technological progress which, following Frankel (1962) and Romer (1986), we assume that is related to the average capital per worker ratio according to a learning-by-doing externality. That is

$$A_t = \Theta \int_0^1 \frac{K_{it}}{N_t}, \quad \Theta > 0. \tag{2}$$

where N_t is the total population of young households/workers.[4] The entrepreneur sells her product to perfectly competitive firms who combine all the available varieties of intermediate products to produce units of the economy's final consumption good according to

$$Y_t = \left(\int_0^1 y_{it}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}, \tag{3}$$

where $\sigma > 1$ is the elasticity of substitution between different varieties of intermediate inputs. We shall assume that the final good is the numéraire and that the price of each intermediate good is denoted ϱ_{it}.

As a result of her activity, each entrepreneur is responsible for the emission of $p_{it} > 0$ units of pollution per unit of intermediate good produced. Therefore, the total pollutants

[3] Thus, profit maximisation corresponds to utility maximisation.
[4] In what follows, the terms 'worker' and 'household' will be used interchangeably.

emitted by each entrepreneur are $p_{it}y_{it}$. We assume that the government follows an environmental policy characterised by a proportional emission tax $\tau > 0$ imposed to each entrepreneur. Given this, the net revenue available to each entrepreneur is $(1-\tau p_{it})y_{it}$. Naturally, we assume that $\tau p_{it} < 1$ is satisfied.

Denoting the marginal cost of production by m_t, we can write the entrepreneur's *variable* profits as

$$\pi_{it}^{variable} = [\varrho_{it}(1-\tau p_{it}) - m_t]y_{it}. \tag{4}$$

The reason why we have labelled the profits in (4) as variable is because entrepreneurs have the choice of reducing their emissions and, therefore, their tax obligation by incurring a fixed cost, denoted $\varepsilon > 0$, for a clean-up operation that decreases the emission rate of their technology. In particular, we assume that the entrepreneurial technology will either emit $p_{it} = \overline{p}$ pollutants per unit of production, if no such fixed cost is incurred, or $p_{it} = \underline{p}$ units of pollution per unit of production, if the entrepreneur decides to incur this cost. Naturally, we assume that $\overline{p} > \underline{p}$.[5] Thus, an entrepreneur's *total* profits are given by as

$$\pi_{it}^{\text{total}} = \begin{cases} \pi_{it}^{\text{variable}}, \text{ if } p_{it} = \overline{p} \\ \\ \pi_{it}^{\text{variable}} - \varepsilon, \text{ if } p_{it} = \underline{p} \end{cases}. \tag{5}$$

The economy is also inhabited by reproductive households who face a potential lifetime of three periods and belong to overlapping generations. The three periods of a household's lifetime are *childhood*, *young adulthood* and *old adulthood* and its members make their decisions only after they reach their adulthood. At the beginning of their young adulthood, they are endowed with a unit of time which they decide to allocate between labour and child rearing. For each unit of labour supplied to entrepreneurial firms households receive the competitive salary w_t while rearing each child caries a time/effort cost of $q > 0$. Denoting the total number of children raised by each household by n_t, the previous assumptions imply that household members will supply $1-qn_t$ units of labour.

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[5] We do not necessarily need to associate this scenario with a technology choice. We can equivalently interpret this choice as one where, by incurring the fixed cost, entrepreneurs can eliminate a fraction $\zeta \in (0,1)$ of their total 'end of pipe' emissions. In this case, $\underline{p} = (1-\zeta)\overline{p}$.

Each young household also receives a transfer, H_t^{young}, from the government – a transfer which is proportional to labour income according to $H_t^{young} = h_t^{young} w_t (1 - qn_t)$ $(h_t^{young} > 0)$. Households decide how much to consume and how much to save for retirement, given that, when old, nature does not bestow to them a labour endowment and, therefore, any alternative source of income from which they could finance their future consumption needs. With the purpose of introducing endogenous lifetime we follow Bhattacharya and Qiao (2007) by assuming that households face a limited lifetime once they enter their old adulthood. In particular, they will live for only a fraction $\psi_t \in [0,1)$ of their prospective maturity period. We also assume that retirement income (i.e., the income accrued from saving) is received by agents at the very beginning of their old age and that it is augmented by a proportional subsidy H_{t+1}^{old}. Denoting saving by s_t and the gross rate of interest on deposits by r_{t+1}, we have $H_{t+1}^{old} = h_t^{old} r_{t+1} s_t$ $(h_t^{old} > 0)$.[6] Consequently, a household's lifetime utility is given by[7]

$$U^t = \ln c_t^{t-1} + \gamma \ln n_t + \psi_t \ln c_{t+1}^{t-1}, \quad \gamma > 0, \tag{6}$$

where c_t^{t-1} denotes consumption during young adulthood and c_{t+1}^{t-1} denotes consumption during old adulthood. Notice that we follow the standard approach of assuming that households have preferences over the number of children they raise.[8]

Earlier, we indicated that the government imposes a tax $\tau \in (0, 1/p_t)$ on total emissions by each entrepreneur. With a unit mass of entrepreneurs, this action results in total revenues of $\tau \int_0^1 p_{it} y_{it} di$. The government uses its revenues to finance the income transfer to all young households, $H_t^{young} N_t = h_t^{young} w_t (1 - qn_t) N_t$, the subsidy to the retirement income of all old households, $H_t^{old} N_{t-1} = h_t^{old} r_t s_{t-1} N_{t-1}$, and government consumption which is denoted g_t. The government has to abide by a balanced budget rule. Hence,

$$\tau \int_0^1 p_{it} y_{it} di = h_t^{young} w_t (1 - qn_t) N_t + h_t^{old} r_t s_{t-1} N_{t-1} + g_t. \tag{7}$$

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[6] The same assumption behind the use of government subsidies is employed in Varvarigos (2011).

[7] In the utility function, a superscript indicates the period where the agent is born while the subscript indicates the period in which the actual activity takes place.

[8] See Galor and Weil (1996), Palivos (2001), Blackburn and Cipriani (2002), Azarnert (2004) and Liao (2011) among others.

As we noted earlier, the presence of endogenous longevity is crucial for the interactions between saving and fertility choices. In the section that follows, we describe how the emission of pollutants impinges on the population's life expectancy.

3 Longevity and Pollution

Following others (Chakraborty 2004; Bhattacharya and Qiao 2007; Varvarigos 2010) we assume that a household's lifetime is endogenous. Particularly, we assume that ψ_t is given by

$$\psi_t = \Psi(x_t), \tag{8}$$

where x_t is a variable that describes the health profile of the household.[9] The function in (8) satisfies $\Psi' > 0$, $\Psi'' < 0$, $\Psi(0) = 0$, $\Psi(\infty) = \lambda \in (0,1)$, $\Psi'(0) = \varphi > 0$ and $\Psi'(\infty) = 0$.

Existing empirical evidence shows that as economies develop and people become more educated, they are more prone to adopt a lifestyle that contributes to an improvement in their overall health status (e.g. Smith 1999). Another crucial factor that seems to have a profound effect on health is environmental quality. For instance, various by-products of economic activity, such as toxins, smoke, chemicals and litter, erode the quality of air as well as the quality of natural resources such as water, soil etc. Consequently, they result in significant adverse effects on the health status of people who are exposed to such environments. Various empirical studies appear to confirm this conjecture (e.g. Pimentel *et al.* 1998; Brunekreef and Holgate 2002; Donohoe 2004; Lacasaña *et al.*, 2005).

We try to capture the aforementioned ideas by assuming that the variable x_t is related to average income per capita, $\overline{Y}_t$, and pollution, denoted μ_t, according to $x_t = X(\overline{Y}_t, \mu_t)$. In general, this function satisfies $X_{\overline{Y}_t} > 0$ and $X_{\mu_t} < 0$ but, for analytical purposes, we shall be focusing our attention to the specific functional form

$$x_t = \frac{\overline{Y}_t}{\mu_t}. \tag{9}$$

Other analyses that introduce the negative effect of pollution on longevity are those of Varvarigos (2010), Mariani *et al.* (2010) and Jouvet *et al.* (2010). Recall that, in our setting, pollution is a by-product of entrepreneurial activities in the production of intermediate

[9] Notice that the expected lifespan of a household is $2+\psi_t$. For this reason, we will be making use of such terms as 'life expectancy' and 'longevity' interchangeably.

goods. To maintain analytical tractability, without altering the strength of the mechanisms that permeate our subsequent results, we follow Stokey (1998), Jones and Manuelli (2001) and Hartman and Kwon (2005) and focus our attention the flow of pollution. Given our previous discussion, this is generated by

$$\mu_t = \int_0^1 p_{it} y_{it} di . \tag{10}$$

The preceding discussion completes the description of our theoretical framework. In the following section, we derive and characterise the equilibrium of our model.

4 Equilibrium

We shall begin the derivation of the model's equilibrium by solving the profit maximisation problem of an entrepreneur. As we indicated in Section 2, the entrepreneur's choice on the cleanliness of technology she will employ is discrete; hence it can be separated from her other choices. For this reason, we shall solve the problem using two distinct steps. In the first step, an entrepreneur will choose the amount of capital and labour she will employ, as well as the price of her product, for any technology described by p_{it}. In the second step, she will choose the technology she will implement by comparing her total after-tax profits in each case, taking account of the results from the first step of the optimisation procedure.

First of all, we can use (3) to find that profit maximisation by the (perfectly competitive) producers of final goods will lead to the demand function

$$y_{it} = \varrho_{it}^{-\sigma} Y_t . \tag{11}$$

Next, we substitute (11) in (4) and maximise with respect to ϱ_{it} to get

$$\varrho_{it} = \frac{\sigma}{(\sigma-1)(1-\tau p_t)} m_t . \tag{12}$$

The result in (12) is the standard condition according to which the price is set as a mark up over the marginal cost of production m_t.

Concerning the choice of capital and labour employed in production, cost minimisation leads to[10]

[10] The cost minimisation problem is $\min_{K_{it}, L_{it}} w_t L_{it} + R_t K_{it}$ subject to equation (1). It is solved by using the Lagrangean $\Lambda_t = w_t L_{it} + R_t K_{it} + m_t \left[y_{it} - BK_{it}^{\beta} (A_t L_{it})^{1-\beta} \right]$.

$$w_t = m_t(1-\beta)BK_{it}^{\beta}A_t^{1-\beta}L_{it}^{-\beta}, \tag{13}$$

and

$$R_t = m_t\beta BK_{it}^{\beta-1}(A_tL_{it})^{1-\beta}, \tag{14}$$

where R_t is the rental cost of capital while the marginal cost m_t is associated with the Lagrange multiplier of the cost minimisation problem. Furthermore, the fact that intermediate good producers operate under monopolistic competition implies that the equilibrium will be symmetric across entrepreneurs. That is, $\varrho_{it} = \varrho_t$, $K_{it} = K_t$, $L_{it} = L_t$, $p_{it} = p_t$ and $y_{it} = y_t$ for every i. For this reason, we drop the subscript i from the subsequent analysis.

Using (3) and (11), these arguments imply that $\varrho_t = 1$. We can substitute this result in (12) to derive

$$m_t = \frac{\sigma-1}{\sigma}(1-\tau p_t). \tag{15}$$

Substituting (15) in (13) and (14) yields

$$w_t = (1-\tau p_t)\frac{\sigma-1}{\sigma}(1-\beta)BK_t^{\beta}A_t^{1-\beta}L_t^{-\beta}, \tag{16}$$

and

$$R_t = (1-\tau p_t)\frac{\sigma-1}{\sigma}\beta BK_t^{\beta-1}(A_tL_t)^{1-\beta}, \tag{17}$$

respectively. By virtue of (3) and (1), the symmetric equilibrium implies that

$$Y_t = y_t = BK_t^{\beta}(A_tL_t)^{1-\beta}, \tag{18}$$

while (4) and (15) imply that each entrepreneur's variable profits are equal to

$$\pi_t^{\text{variable}} = (1-\tau p_t)\frac{1}{\sigma}y_t. \tag{19}$$

We now turn our attention to the optimal decisions made by households. Given that $\varrho_t = 1$, the budget constraints faced by households during the two periods of their adulthood are $c_t^{t-1} = (1+h_t^{young})w_t(1-qn_t) - s_t$ and $c_{t+1}^{t-1} = (1+h_t^{old})r_{t+1}s_t$. Their objective is to choose c_t^{t-1}, n_t, s_t and c_{t+1}^{t-1} to maximise their lifetime utility in (6), taking ψ_t, w_t and r_{t+1} as given. It is straightforward to establish that the solutions to this problem are given by

$$s_t = \frac{\psi_t}{1+\psi_t}(1+h_t^{young})w_t(1-qn_t), \tag{20}$$

and

$$n_t = \frac{\gamma}{q(1+\gamma+\psi_t)}. \tag{21}$$

The intuition behind these results is straightforward. Equation (20) reveals that households will save a fraction of their total earnings (that is, labour income augmented by the government subsidy). Their propensity to save is increasing in the variable that determines their life expectancy. In particular, a higher ψ_t increases the marginal utility benefit of consuming when old; hence, it motivates agents to substitute future for current consumption. In equation (21), we can see that the fertility rate is inversely related to ψ_t because, as the utility from consuming when old increases, households will optimally want to carry more resources towards saving. Nevertheless, they will also try to smooth their consumption profile. They can do this by working more during their young adulthood in order to increase their available resources – an action which, nevertheless, leaves them with less time available to rear children.

Next, we can combine (8), (9) and (10) together with $y_t = \overline{Y}_t$ to get $\psi_t = \Psi(1/p_t)$, where $\Psi_{p_t} < 0$.[11] Substituting this in (21) yields

$$n_t = \frac{\gamma}{q[1+\gamma+\Psi(1/p_t)]} = n(p_t). \tag{22}$$

The result in equation (22) allows us to derive

Lemma 1. *The optimal fertility rate is positively related to the amount of emissions per unit on output. That is* $n'(p_t) > 0$.

Proof. It is $n'(p_t) = \frac{\partial n_t}{\partial \Psi(1/p_t)}\Psi_{p_t}$. Since $\frac{\partial n_t}{\partial \Psi(1/p_t)} < 0$ and $\Psi_{p_t} < 0$, we get $n'(p_t) > 0$. □

[11] The functional form in (9) allows us to eliminate the *direct* effect of $y_t = \overline{Y}_t$ on ψ_t due to the counterbalancing effects of economic development and pollution. This is actually a welcomed aspect because it permits us to focus on the demographic implications of different emission rates. In any case, later it will become clear that income still has a positive, albeit *indirect*, effect through the contribution of the growth process on the choice of a lower p_t.

In terms of intuition, a higher p_t reduces longevity because of the adverse health effect from the emission of harmful pollutants. As this reduces the relative importance attached to old age consumption, the equilibrium can only be restored by a reallocation of resources that favours the rearing of more children.

5 Capital Accumulation

The engine of output growth in our economy is the accumulation of physical capital. Furthermore, growth can be sustained in the long-run due to the presence of a learning-by-doing externality in the determination of labour productivity. Capital is accumulated by perfectly competitive financial intermediaries who accept deposits by young workers in exchange for the gross rate of return r_{t+1} per unit of deposited income. They subsequently transform these saving deposits into capital by accessing a technology that transforms one unit of time-t output into one unit of time-$t+1$ capital. The capital is supplied to intermediate good producers at a rental cost of R_{t+1} per unit.

Evidently, the zero profit condition for financial intermediaries implies that[12]

$$r_{t+1} = R_{t+1}. \tag{23}$$

Furthermore, we have

$$K_{t+1} = N_t s_t, \tag{24}$$

which indicates that the collective savings by all young households (whose population is N_t) are the inputs in the investment process that leads to the formation of physical capital. Of course, the demographics of our economy imply that the population size of young households evolves according to

$$\frac{N_{t+1}}{N_t} = n_t. \tag{25}$$

Substituting (25) in (24) and using the notational standard $k_{t+j} = K_{t+j} / N_{t+j}$ ($j = 0,1,\ldots$), we can write (24) as

$$k_{t+1} = \frac{s_t}{n_t}. \tag{26}$$

[12] We assume full depreciation of capital.

Using (2), (26) and $L_t = N_t(1-qn_t)$ in (16) and (17) we get

$$w_t = (1-\tau p_t)\frac{\sigma-1}{\sigma}(1-\beta)B\Theta^{1-\beta}k_t(1-qn_t)^{-\beta}, \qquad (27)$$

and

$$\mathrm{R}_t = (1-\tau p_t)\frac{\sigma-1}{\sigma}\beta B\Theta^{1-\beta}(1-qn_t)^{1-\beta}, \qquad (28)$$

respectively.

Earlier, we indicated that the government imposes a proportional tax on emissions and uses the proceeds to finance a programme of transfers/subsidies to (young and old) households, as well as government consumption expenses. Now, we shall assume that this programme of transfers/subsidies is designed to eradicate the cost accrued to households, as a result of the taxation of pollutant emissions. We justify this assumption by appealing to the idea that workers/savers do not have any control on whether a cleaner production process will be applied or not. This choice rests with the entrepreneurs. For this reason, it may be proper to 'correct' any negative repercussions that accrue to them for choices over which they have no control whatsoever.

Given these arguments, we postulate that the programme of transfers/subsidies is designed so that

$$(1+h_t^{young})w_t(1-qn_t) = \frac{\sigma-1}{\sigma}(1-\beta)B\Theta^{1-\beta}k_t(1-qn_t)^{1-\beta}, \qquad (29)$$

and

$$(1+h_t^{old})r_t s_{t-1} = \frac{\sigma-1}{\sigma}\beta B\Theta^{1-\beta}(1-qn_t)^{1-\beta}s_{t-1}. \qquad (30)$$

Effectively, the scheme is designed in a manner that eliminates the term $(1-\tau p_t)$ from the returns to labour and capital (which is also the return to saving according to equation 23). Using equations (27)-(30), it is straightforward to establish that

$$h_t^{young} = h_t^{old} = \frac{\tau p_t}{1-\tau p_t}. \qquad (31)$$

Substituting (31) back to the government's budget constraint, we can eventually obtain government consumption as

$$g_t = \tau p_t \frac{1}{\sigma} y_t. \qquad (32)$$

We are now ready to obtain the economy's growth rate. First, we substitute (20), (22), (27) and (31) in (26). Subsequently, some straightforward algebra allows us to derive

$$\frac{k_{t+1}}{k_t} - 1 = \varkappa(p_t) = \frac{(\sigma-1)(1-\beta)B\Theta^{1-\beta}q}{\sigma\gamma}\Psi(1/p_t)\left[\frac{1+\gamma+\Psi(1/p_t)}{1+\Psi(1/p_t)}\right]^{\beta} - 1. \qquad (33)$$

As we can see, the growth rate of capital per worker is a function of the emission rate p_t. There are two ways through which the latter impinges on the economy's growth rate, both of them working through the emission rate's effect on life expectancy. On the one hand, the emission rate determines the marginal propensity to save – thus, the funds available for investment; on the other hand, it also affects fertility decisions and, correspondingly, the rate of population growth as well as the amount of labour that households offer. As it turns out, all these effects work on the same direction, thus leading to the result in

Lemma 2. *The growth rate of capital per worker is negatively related to the amount of emissions per unit of output. That is* $\varkappa'(p_t) < 0$.

Proof. Using (33), it is straightforward to establish that

$$\varkappa'(p_t) = \frac{(\sigma-1)(1-\beta)B\Theta^{1-\beta}q\Psi_{p_t}}{\sigma\gamma}\left[\frac{1+\gamma+\Psi(1/p_t)}{1+\Psi(1/p_t)}\right]^{\beta}\left[1-\beta\frac{\Psi(1/p_t)}{1+\Psi(1/p_t)}\frac{\gamma}{1+\gamma+\Psi(1/p_t)}\right] < 0$$

because $\Psi_{p_t} < 0$. □

Earlier, we established that a higher p_t reduces longevity. This effect causes a reduction in the marginal propensity to save, thus reducing the amount of saving for a given amount of labour income. Furthermore, by leading to an increase in the fertility rate, the reduction in labour supply reduces disposable income available for saving. Finally, the higher rate of population growth implies a direct reduction in the amount of investment per household. All these effects result in a lower rate of growth. In what follows, and given the result in Lemma 2, we shall be assuming that parameter values are such that $\varkappa(\overline{p}) > 0$; that is, the growth rate of capital per worker is still positive even with the highest possible emission rate.

6 Endogenous Determination of the Emission rate

Recall that entrepreneurs will choose their emission per unit of production so as to maximise profits through the expression in (5), taking the supply of labour as given. Using (1), (2) and (19), we can rewrite this expression as

$$\pi_t^{\text{total}} = \begin{cases} (1-\tau\overline{p})\dfrac{1}{\sigma}B\Theta^{1-\beta}[1-qn_t]^{1-\beta}K_t, \text{ if } p_t = \overline{p} \\ \\ (1-\tau\underline{p})\dfrac{1}{\sigma}B\Theta^{1-\beta}[1-qn_t]^{1-\beta}K_t - \varepsilon, \text{ if } p_t = \underline{p} \end{cases} \tag{34}$$

Of course, (34) reveals that the emission rate will be endogenously determined from

$$p_t = \begin{cases} \overline{p}, \text{ if } K_t < \hat{K} \\ \\ \underline{p}, \text{ if } K_t \geq \hat{K} \end{cases}, \tag{35}$$

where

$$\hat{K} = \frac{\varepsilon\sigma}{B\Theta^{1-\beta}Z_t}, \tag{36}$$

and $Z_t = \tau(\overline{p}-\underline{p})(1-qn_t)^{1-\beta} > 0$. Intuitively, a choice of lower emissions per unit of production is beneficial in terms of variable profits because it reduces the fraction of revenues lost in the form of taxes. Nevertheless, given the fixed cost associated with a cleaner production process, this benefit will dominate only after the economy's resources (in terms of capital) exceed the endogenous threshold given by $\hat{K}$.

Let us assume that, given (22), the model's parameters allow $n(\underline{p})-1>0$. This can happen, for example, for a sufficiently low value for q. In this case, taking account of Lemma 1 and equation (25), we can see that the growth rate of the population is always positive. Recalling that $\kappa(\overline{p})>0$, it is true that the growth rate of the aggregate capital stock is positive as well, i.e., $\frac{K_{t+1}}{K_t} = \frac{k_{t+1}}{k_t}\frac{N_{t+1}}{N_t} > 1$; alternatively, $K_{t+1} > K_t$. Now, let us consider an economy for which $K_0 < \hat{K}$. Naturally, there must be some period $T \geq 1$ such that $K_{T-1} < \hat{K} < K_T$. Hence, the determination of the emission rate can be formally described through

Lemma 3. *There is a time period* $T \geq 1$ *such that*

$$p_t = \begin{cases} \overline{p}, \; for \;\; t = 0,...,T-1 \\ \\ \underline{p}, \; for \;\; t = T, T+1,... \end{cases}.$$

Proof. It follows directly from (35), $K_0 < \hat{K}$ and $K_{t+1} > K_t$. □

The result in Lemma 3 will have significant implications for issues pertaining to demographic changes in our economy. This is an issue to which we turn in the following section of our analysis.

7 Growth, Fertility, and Longevity

The result in Lemma 3 indicates that, at some point of its development process, the economy will experience a reduction in the pollutant emission rate. As we shall see, this outcome has significant implications for both demographic and economic outcomes. Concerning the former, one major result comes in the form of

Proposition 1. *The economy will undergo a demographic transition in the sense that it will experience an increase in life expectancy and a reduction in the rate of population growth. In particular, there is a time period* $T \geq 1$ *such that*

$$\Psi\left(\frac{1}{p_t}\right) = \begin{cases} \Psi\left(\frac{1}{\overline{p}}\right), \; for \;\; t = 0,...,T-1 \\ \\ \Psi\left(\frac{1}{\underline{p}}\right), \; for \;\; t = T, T+1,... \end{cases}, \quad \Psi\left(\frac{1}{\overline{p}}\right) < \Psi\left(\frac{1}{\underline{p}}\right),$$

and

$$n(p_t) = \begin{cases} n(\overline{p}), \; for \;\; t = 0,...,T-1 \\ \\ n(\underline{p}), \; for \;\; t = T, T+1,... \end{cases}, \quad n(\overline{p}) > n(\underline{p}).$$

Proof. It follows from Lemma 1, Lemma 3, and $\Psi_{p_t} < 0$. □

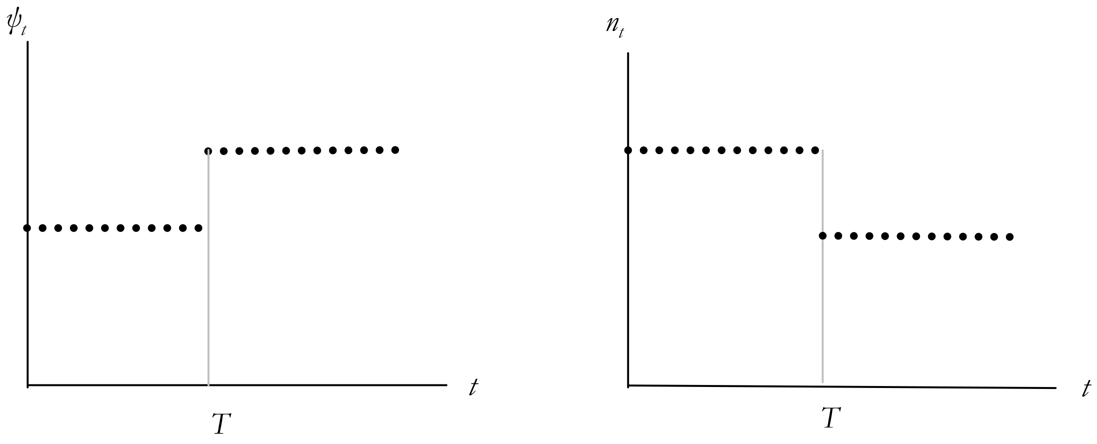

Figure 1. Demographic change

A similar distinct change can be observed in relation to the economy's growth rate. This becomes evident in

Proposition 2. *There is a time period* $T \geq 1$ *such that*

$$\varkappa(p_t) = \begin{cases} \varkappa(\overline{p}), \text{ for } t = 0,...,T-1 \\ \\ \varkappa(\underline{p}), \text{ for } t = T, T+1,... \end{cases}, \quad \varkappa(\overline{p}) < \varkappa(\underline{p}).$$

Proof. It follows from Lemma 2 and Lemma 3. □

The two previous propositions reveal that the economy will undergo a distinct change in both its economic (i.e., output growth) and demographic (i.e., fertility and longevity) outcomes. The novelty of our analysis rests on the idea that environmental factors – that is, the choice of less polluting production processes induced by environmental policy – are crucial in the joint determination of economic growth and various aspects of demographic change.

7 Conclusion

In this paper, we have sought to fill a gap in the literature by analysing a model which shows that the interactions between economic growth and environmental factors can account for historically observed changes in some important demographic characteristics. Specifically, we offer a novel mechanism according to which the endogenous change of the emission rate, which occurs in presence of environmental taxation, brings forth a joint change in both life expectancy and fertility.

Our model is constructed in a manner that allows analytical solutions. Thus it benefits from the clear-cut and detailed description of all the mechanisms involved whereas the absence of unnecessary complication allows us to avoid aspects that could blur the intuition. As always, the model can be enriched with elements that would allow us to study additional effects whose analysis do not comprise a part of this paper's objective. For example, our main purpose was to isolate and study the causal effects that run from environmental factors to aspects of demographic change. It will be worthwhile to examine a model where such effects are two-way causal. This can happen if we generalise the expressions describing pollution and life expectancy so as to account explicitly for the environmental strain caused by higher population growth. Furthermore, we could enrich the characteristics of population changes by allowing infant (in addition to adult) mortality. As stated earlier, these issues go beyond the purpose of our current study which seeks to focus on the causal effects of pollution on the economy's demography. Nevertheless, they are indubitably important; hence, they represent a potentially rewarding avenue for future research work.

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