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Strictness of Environmental Policy and Investment in Abatement

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Abstract

In this paper we model an oligopoly where firms invest in abatement technologies and emissions are taxed by the government. We show that a stricter environmental policy does not necessarily lead to an increase in firms' R&D investment into cleaner production methods. In fact, the emission-to-output ratio may be a U-shaped function of the environmental damage parameter. This result holds both when the government can commit and in the social optimum. When the government cannot commit, this relationship is ambiguous except in markets with few firms. Our results further suggest that if the emission-to-output ratio is decreasing throughout, output is a U-shaped function of the environmental damage.

Keywords: Environmental innovation, environmental taxation, commitment, oligopoly.

JEL classification: L12, Q55, Q58

1 Introduction

Environmental policy gives polluting firms an incentive to do R&D and invest in cleaner ways of producing, to reduce their compliance costs. There is a large literature on the effect of environmental policy on innovation (see e.g. Requate (2005) for an overview) starting from Kneese and Schultze (1978). One question that has received relatively little attention is: When the environmental policy becomes stricter and stricter, will firms invest more and more in environmental R&D? Our immediate intution might suggest that this should be the case, and indeed it is in models where a firm's marginal abatement costs only depend on its emissions (Downing and White (1986), Milliman and Prince (1989), Jung et al. (1996) and Requate and Unold (2003).) However, taking the output market into account, another element comes into play. Firms responds in two ways to an increase in the tax rate on emissions: by investing more in environmental R&D and by reducing output. The latter response, in turn, reduces firms' incentives to conduct environmental R&D. In other words, the profitability of investing in a reduction of the emissions-to-output ratio depends on the eventual level of output. If the firm will produce very little because of a very strict environmental policy, it also has little incentive to invest in environmental R&D. This suggests that while output is decreasing in the strictness of environmental policy, the emissions-to-output ratio might be U-shaped. However, the effects might conceivably also be reversed: When an ever stricter environmental policy prompts a firm to invest more and more in environmental R&D, production might eventually become so clean that it starts to increase again.

The effects of a stricter environmental policy on emission intensity and output, and the interactions between these variables, are therefore far from clear. However, this subject has hardly been researched so far.

Ulph (1997) is the most important contribution related to the topic of our paper. The author sets up a free-entry Cournot oligopoly model with very general functional forms, treating the environmental tax rate t as an exogenous variable. In stage one, firms decide whether to enter the market. Then in stage two of the "simultaneous entry" game, the firms in the market simultaneously spend R&D money F on reducing their emission-to-output ratio ε and set their output levels q.[1] In the "sequential entry" game, the firms first set F (in stage two) and then q (in stage three). Ulph (1997) finds that in both games, an increase in t reduces q if and only if:[2]

$$\varepsilon(F)\varepsilon''(F) - [\varepsilon(F)]^2 > 0 \tag{1}$$

The effect of t on R&D spending is ambiguous, however when t is low, an increase in t raises R&D spending in both models.

Katsoulacos and Xepapadeas (1996) set up a Cournot oligopoly with technology spillovers, where the government taxes emissions and subsidizes R&D. They assume, as we will do, a linear demand function and a quadratic R&D cost function. Their finding that the effect of t on q is ambiguous is in accordance with Ulph (1997), because the sign of the LHS of (1) is ambiguous for a quadratic R&D cost function. Katsoulacos and Xepapadeas (1996) further find that R&D spending is increasing in the tax rate.

A separate research strand has been dealing with the issue of R&D investment into cleaner technologies in the context of cross-border competition. In an international Cournot duopoly, Ulph (1994) finds that the effect of an increase in the domestic tax rate on domestic R&D is ambiguous, both when foreign output is given and when it is endogenous (but foreign R&D is given). An increase in the domestic tax rate reduces domestic full marginal cost (marginal production cost plus implicit tax $t\varepsilon$ on output) and increases foreign production if and only if (1) holds. Ulph and Ulph (1996) show that the same result holds if firms can choose how much to spend on reducing marginal production costs as well as on emission intensity. Simpson and Bradford (1993) also set up an international Cournot duopoly, with investment that simultaneously reduces marginal production costs and emission intensity. In spite of this complication, it seems that their results are still driven by the $\varepsilon(F)$ function, since with an exponential $\varepsilon(F)$ function, the LHS of (1) is zero and the domestic tax rate has no effect on domestic full production costs or foreign output, as Ulph (1994) has shown. However, with a logarithmic $\varepsilon(F)$ function, Simpson and Bradford (1993) show that the effect is ambiguous.

Finally, similar results have been found in different but related settings. In an interna-

[1] To assist with comparisons, we will use the notation of our own model when discussing other papers.
[2] Ulph (1997, p. 47) mistakenly writes $\varepsilon''(F) < 0$. This should be $\varepsilon''(F) > 0$.

tional Cournot duopoly where the two firms buy an environmental input from an eco-firm, Feess and Muehlheusser (2002) show that in general, the effect of the domestic tax rate on a firm's demand for the environmental input is ambiguous. In a model of perfect competition where firms can switch to a clean technology at a certain cost, Bréchet and Meunier (2011) show that the number of clean firms is inverse U-shaped in the emission tax rate.

In this paper we aim at disentangling the interplay of the two effects of the environmental policy mentioned above and shed some light on the circumstances that will make the emissions-to-output ratio be U-shaped or strictly decreasing in the strictness of the policy. To this aim, we build on and extend the framework in Ulph (1997). We depart from Ulph in that we endogenize the choice of the emissions tax rate. In particular, the government chooses this tax rate given the (perceived) environmental damage. We contemplace the cases where the government can and cannot commit to the emissions tax rate. The latter case is interesting because the government might be tempted to adjust its environmental policy once firms have chosen their emission intensity. Ulph (1997), along with the other papers discussed so far, assumes that the government will not give in to this temptation.[3] We also differ from Ulph (1997) in that we assume that the R&D cost function is quadratic. This assumption is widely used in the context of R&D investment as it implies the existence of diminishing returns to R&D investments. As a benchmark, we also solve the social optimum.

Our results show that the emission-to-output ratio may be a U-shaped function of the (perceived) environmental damage. This result holds both when the government is able to commit to the tax rate as well as when it is not. The same may also apply in the social optimum. Our results further suggest that if the emission-to-output ratio is decreasing throughout, output is a U-shaped function of the environmental damage. The U-shaped function of the emissions-to-output ratio will tend to arise in situations where the R&D costs and number of firms are relatively large and the size of the market is relatively small.

[3] There is a literature that compares the outcomes of games where the government can and cannot commit to its environmental policy, the latter is also called time consistency (Amacher and Malik, 2001, and Petrakis and Xepapadeas, 2001). However, the comparison of the two games is not the main focus of our paper.

The rest of this paper is organized as follows. In Section 2, we introduce our model. We derive the welfare optimum in Section 3. In Sections 4 and 5, we analyze the game where the government can and cannot commit, respectively. Section 6 concludes the paper.

2 The model

There are m firms producing a homogeneous good. Firm i, $i = 1, \cdots, m$, producing q_i faces the inverse demand function

$$P = a - Q \tag{2}$$

with P the product price, $Q \equiv \sum_{i=1}^{m} q_i$ and $a > 0$. Production is polluting. Firm i's total emissions E_i are a linear function of production q_i and are given by

$$E_i = \varepsilon_i q_i \tag{3}$$

where $\varepsilon_i \in [0, 1]$ is the emissions-to-output ratio, which depends on the abatement technology that the firm installs.[4] There is a fixed cost F of installing the technology, which is decreasing in the emissions-to-output ratio (i.e. increasing in the effectiveness of the technology):

$$F(\varepsilon_i) = \frac{\gamma}{2}(1 - \varepsilon_i)^2 \tag{4}$$

with $\gamma > 0$. The (perceived) environmental damage produced by pollution is given by:

$$D = \frac{\beta}{2} E^2 \tag{5}$$

where $E \equiv \sum_{i=1}^{m} E_i$ and the environmental damage parameter β measures the severity of the environmental problem or the strength of the policy maker's preference for the environment.

The policy maker imposes a tax t on emissions. We will assume throughout that all firms respond symmetrically to environmental regulation. We assume that β is high enough to guarantee a positive tax rate ($t > 0$).[5] Marginal costs of production are constant

[4] Alternatively we can interpret ε_i as a coefficient that identifies the type of manufacturing technology the firm uses where the alternative technologies differ in terms of their emissions per unit of output (see Asproudis and Gil-Molto, 2009).

[5] For small β, the policy maker will want to set $t < 0$, to induce the monopolist to produce more.

and normalized to zero. Thus firms do not incur other costs than the technology costs (F_i) and the tax payments (tE_i). Thus, using (2) to (4), firm i's profits can be written as

$$\pi_i = Pq_i - tE_i - F(\varepsilon_i) = (a - Q)q_i - t\varepsilon_i q_i - \frac{\gamma}{2}(1 - \varepsilon_i)^2 \tag{6}$$

The policy maker's objective function is the aggregation of consumer and producer surplus (CS and PS respectively) minus the environmental damage plus the revenues from taxing emissions (tE), ie

$$W = PS + CS - D + tE \tag{7}$$

where producer surplus is given by the sum of the firms' profits and consumer surplus is defined in the standard way. Substituting (2) to (6) into (7), welfare is given by:

$$W = \left(a - \frac{1}{2}Q\right)Q - \frac{1}{2}\gamma\sum_{i=1}^{m}(1-\varepsilon_i)^2 - \frac{1}{2}\beta\left[\sum_{i=1}^{m}\varepsilon_i q_i\right]^2 \tag{8}$$

We want to determine how ε_i responds to the strictness of environmental policy as measured by the environmental damage parameter β. In order to do this, we will study two cases:

1. Commitment c: The regulator moves first and sets the tax rate t in stage one. Each firm i chooses its technology level ε_i in stage two and its output level q_i in stage three.

2. No commitment n: The firms move first, each firm i choosing its technology level ε_i in stage one. The government responds by setting the tax rate t in stage two. Finally, each firm i sets its output level q_i in stage three.

We also solve the welfare optimum w (where both ε_i and q_i are chosen to maximize welfare). As usual we solve the different games by means of backwards induction to find their respective subgame perfect Nash equilibria.

3 Welfare optimum

In the welfare optimum w, the regulator chooses ε_i and q_i, $i = 1, \cdots, m$, to maximize welfare. As firms are symmetric, we focus on the symmetric equilibrium of the game; that is, where $q_i = q$ and $\varepsilon_i = \varepsilon$ for all $i = 1, \cdots, m$. In symmetry, welfare (8) can be written as $W = mw$ with:

$$w = \left(a - \frac{1}{2}mq\right)q - \frac{1}{2}\gamma(1-\varepsilon)^2 - \frac{1}{2}\beta m(\varepsilon q)^2 \tag{9}$$

The first order conditions are:

$$\frac{\partial w}{\partial q} = a - mq - \beta m \varepsilon^2 q = 0 \tag{10}$$

$$\frac{\partial w}{\partial e} = \gamma(1-\varepsilon) - \beta m \varepsilon q^2 = 0 \tag{11}$$

We cannot solve explicitly for the optimal levels of q and ε. However, we can analyze the effect of the environmental cost parameter β on the optimal output and emission intensity levels. Totally differentiating (10) and (11) with respect to β yields:

$$(1+\beta\varepsilon^2)\frac{dq}{d\beta} + 2\beta\varepsilon q\frac{d\varepsilon}{d\beta} + \varepsilon^2 q = 0 \tag{12}$$

$$\left(\gamma + \beta m q^2\right)\frac{d\varepsilon}{d\beta} + 2\beta\varepsilon^2 q\frac{dq}{d\beta} + m\varepsilon q^2 = 0 \tag{13}$$

Solving for $dq/d\beta$ and $d\varepsilon/d\beta$ and using (11):

$$\frac{dq_w}{d\beta} = \frac{\left(mq^2\beta - \gamma\right)q\varepsilon^2}{\gamma + \beta\gamma\varepsilon^2 + mq^2\beta - 3mq^2\beta^2\varepsilon^2} = \frac{[1-2\varepsilon]\,\gamma q\varepsilon}{\gamma + \beta\gamma\varepsilon^2 + mq^2\beta - 3mq^2\beta^2\varepsilon^2} \tag{14}$$

$$\frac{d\varepsilon_w}{d\beta} = \frac{\left(\beta\varepsilon^2 - 1\right)mq^2\varepsilon}{\gamma + \beta\gamma\varepsilon^2 + mq^2\beta - 3mq^2\beta^2\varepsilon^2} = \frac{[\beta\varepsilon^2 - 1]\,\gamma(1-\varepsilon)}{\beta\left(\gamma + \beta\gamma\varepsilon^2 + mq^2\beta - 3mq^2\beta^2\varepsilon^2\right)} \tag{15}$$

The denominator on the RHS of both expressions is positive, because this is a second order condition for welfare maximization (the Hessian must be negative definite):

$$\gamma + \beta\gamma\varepsilon^2 + mq^2\beta - 3mq^2\beta^2\varepsilon^2 = \frac{\gamma}{\varepsilon}\left(4\beta\varepsilon^3 - 3\beta\varepsilon^2 + 1\right) > 0 \tag{16}$$

The equality follows from (11). The signs of (14) and (15) thus depend on the respective terms in square brackets on the RHS.

An increase in the cost of pollution leads to a decrease in the welfare-maximizing level of pollution:

$$\frac{dE_w}{d\beta} = m\varepsilon\frac{dq_w}{d\beta} + mq\frac{d\varepsilon_w}{d\beta} = \frac{-q\gamma\left(3\beta\varepsilon^3 - 2\beta\varepsilon^2 - \varepsilon + 1\right)}{\beta\left(\gamma + \beta\gamma\varepsilon^2 + mq^2\beta - 3mq^2\beta^2\varepsilon^2\right)} < 0 \tag{17}$$

The second equality follows from (14) and (15). The denominator on the RHS is positive by (16). The sign of the RHS then follows from:

$$3\beta\varepsilon^3 - 2\beta\varepsilon^2 - \varepsilon + 1 = (1-\varepsilon)\left(4\beta\varepsilon^3 - 3\beta\varepsilon^2 + 1\right) + \beta\varepsilon^2(1-2\varepsilon)^2 > 0$$

The first equality follows from (11). The inequality follows from (16).

We can now state:[6]

Proposition 1 *Define $\gamma^w \equiv a^2/m$. Then in the welfare optimum:*

1. *If $\gamma < \gamma^w$, then $d\varepsilon_w/d\beta < 0$ for all $\beta > 0$ and*

$$dq_w/d\beta \begin{matrix} < \\ = \\ > \end{matrix} 0 \qquad \textit{for } \beta \begin{matrix} < \\ = \\ > \end{matrix} \frac{\gamma}{2m\left(a+\sqrt{a^2-m\gamma}\right)^2}$$

2. *If $\gamma > \gamma^w$, then $dq_w/d\beta < 0$ for all $\beta > 0$ and*

$$d\varepsilon_w/d\beta \begin{matrix} < \\ = \\ > \end{matrix} 0 \qquad \textit{for } \beta \begin{matrix} < \\ = \\ > \end{matrix} \frac{2}{\sqrt{m\gamma}+\sqrt{m\gamma-a^2}}$$

3. *If $\gamma = \gamma^w$, then for $\beta < \frac{1}{4}$, both $dq_w/d\beta < 0$ and $d\varepsilon_w/d\beta < 0$. For $\beta = \frac{1}{4}$, $dq_w/d\beta = d\varepsilon_w/d\beta = 0$. For $\beta > \frac{1}{4}$, either $q_w = a/(2m)$ and $d\varepsilon_w/d\beta < 0$ or $\varepsilon_w = \frac{1}{2}$ and $dq_w/d\beta < 0$.*

Initially, as β starts to increase from zero, both output q_w and emission intensity ε_w are decreasing in β. However, there comes a point when either q_w or ε_w starts increasing in β. When $\gamma > a^2/m$, at this point output has decreased by so much that it is not worthwhile investing any further in reducing emissions per unit of output. Indeed, it becomes optimal to increase emission intensity. When $\gamma > a^2/m$ on the other hand, production has become so clean at the turning point that it becomes optimal to increase production again.

The significance of the comparison between a^2 and $m\gamma$ can be explained as follows. When a is high, demand is high, so that the regulator does not want to reduce output by too much and is anxious to increase it again if possible. When $m\gamma$ is high, the cost of reducing emission intensity per firm γ and for all firms m is high. Then the regulator

[6] All proofs are in the Appendix.

does not want to spend too much on reducing emission intensity and is happy to increase emission intensity again if possible.

All in all, the U-shape function of ε_w is more likely to appear for large γ and m relative to a. That is, when the market size is small relative to the R&D costs and the number of firms. In other words, when the profitability of investing on environmental R&D is lower (therefore making the option of reducing output a more efficient way to reduce emissions).

4 Regulator moves first (commitment)

In the commitment scenario c, the regulator sets the tax rate t in stage one. Subsequently, each firm i chooses abatement technology ε_i in the second stage and output q_i in the third and last stage. Thus the regulator can commit to a tax rate and is not going to adjust it after the firms have chosen their abatement technologies.

In the last stage, each firm i chooses q_i to maximise its profit (6), taking all $q_j, j = 1, \cdots, m, j \neq i$ as given. The FOC can be written as:

$$q_i = a - Q - t\varepsilon_i \tag{18}$$

Summing over i and solving for Q yields:[7]

$$\bar{Q} = \frac{ma - t\sum_{i=1}^{m}\varepsilon_i}{m+1} \tag{19}$$

Substituting (19) back into (18), we can solve for q_i:

$$\bar{q}_i = \frac{a - mt\varepsilon_i + t\sum_{j\neq i}\varepsilon_j}{m+1} \tag{20}$$

In stage two of the game, each firm i chooses ε_i to maximize its profit, taking all $\varepsilon_j, j = 1, \cdots, m, j \neq i$, as given and anticipating the effect of ε_i on q_i and Q as given by (20) and (19) respectively. Substituting (19), (20) and (4) into (6), firm i's profits can be written as:

$$\bar{\pi}_i = \bar{q}_i^2 - \frac{\gamma}{2}(1-\varepsilon_i)^2 \tag{21}$$

Maximizing with respect to ε_i yields, from (20):

$$\frac{\partial \pi_{i,c}}{\partial \varepsilon_i} = -\frac{2mtq_i}{m+1} + \gamma(1-\varepsilon_i) = 0 \tag{22}$$

[7] Expressions for Q, q_i and π_i that still contain ε_j are denoted with a bar.

The second order condition is, from (20):

$$\frac{\partial^2 \pi_{i,c}}{\partial \varepsilon_i^2} = \frac{2m^2t^2}{(m+1)^2} - \gamma < 0 \tag{23}$$

By simple comparative statics, it is easy to see that the second order condition is more likely to be satisfied the higher γ is and the lower m and t are.

In a symmetric solution, $\varepsilon_i = \varepsilon$ so that $q_i = q$ for all $i = 1, \cdots, m$. Then (20) becomes:

$$q_c|_{\varepsilon_i = \varepsilon} = \frac{a - t\varepsilon}{m+1} \tag{24}$$

Note that $q_c > 0$ only if $a - \varepsilon t > 0$. To guarantee that there is positive production for any ε, we impose that $a > t$. Also note that q_c is decreasing in t and in ε. That is, for a given ε, the higher the tax rate is, the less each firm produces. Likewise, for a given t, the higher ε is (the higher the emissions per unit are), the less firms produce. The reason for these results is that firms's marginal cost is determined by both ε and t. Profit maximisation implies that a higher marginal cost will lead to lower output.

Solving for ε and q from (22) and (24) yields:

$$\varepsilon_c = \frac{\gamma(1+m)^2 - 2amt}{\gamma(1+m)^2 - 2mt^2} \tag{25}$$

$$q_c = \frac{(a-t)\gamma(1+m)}{\gamma(1+m)^2 - 2mt^2} \tag{26}$$

The denominator in (25) and (26) is positive by (23). This also implies that $\varepsilon_c > 0$, if $\gamma > \frac{2amt}{(m+1)^2}$. In other words, γ has to be large relative to a so that $\varepsilon > 0$. In addition, $\varepsilon_c < 1$ given that $a > t$.

For the firm's choice of ε as a function of t, we find:

Lemma 1 *Define $\gamma^c \equiv \frac{2ma^2}{(1+m)^2}$. Let the regulator set the environmental tax rate t in stage one. Then if $\gamma \leq \gamma^c$, ε_c is strictly decreasing in t. If $\gamma > \gamma^c$, ε_c is first decreasing and then increasing in t.*

Several interesting aspects can be highlighted from Lemma 1. First, the relevance of the size of the market (a), number of firms (m) and the technology costs (γ). If a is large enough and/or m low enough relative to γ ($\gamma < \gamma^c$), it is very profitable to invest in the abatement technology and therefore, increases in t can only lead to higher investments in abatement and consequently lower ε. However, if a is not so large or m not so low relative to γ ($\gamma > \gamma^c$), increases in the tax rate do not have such an unequivocal effect in the level of investment in abatement. This leads us to the discussion of the second relevant aspect in Lemma 1. As t increases, firms initially increase their investment in abatement (they lower ε) but after a critical value of t, they actually reduce their investment in abatement. The intuition is that a higher t generates incentives to invest in abatement (because a firm can save more on the tax payment), but also increases the marginal cost of production, making firms produce less (recall that q_c is decreasing in t for a given ε). The lower the output is, the less profitable it is to invest in abatement. If t is large enough, this second effect will outweigh the first effect, implying that higher t might actually lead to higher ε. Finally, note that as m increases $\gamma > \gamma^c$ is more likely to hold as the RHS of the inequality is decreasing in m. In other words, the higher m is, the more likely it is that the emissions to output ratio in equilibrium is U-shaped in the emissions tax rate.

Lemma 1 is in accordance with, but more specific than, Ulph (1997) who shows that with a quadratic R&D function, the effect of t on ε is ambiguous. In our model, depending on the parameter values, ε is either decreasing throughout or U-shaped as a function of t. Our result is in contrast with Katsoulacos and Xepapadeas (1996) who find that ε is decreasing throughout in t. However, Katsoulacos and Xepapadeas (1996) assume that the government is also subsidizing R&D investment, whereas in our model it is only taxing emissions.

We are now in a position to explore the total effect of a change in t on q_c. This effect can be decomposed into two separate effects:

$$\frac{dq_c}{dt} = \frac{\partial q_c}{\partial t} + \frac{\partial q_c}{\partial \varepsilon}\frac{d\varepsilon_c}{dt}$$

It is easy to see from (24) that the direct effect of t on output is negative ($\frac{\partial q_c}{\partial t} < 0$), as discussed above (a higher t implies a higher marginal cost and therefore a lower output in equilibrium). The same applies to $\frac{\partial q_c}{\partial \varepsilon} < 0$. On the other hand, $\frac{d\varepsilon_c}{dt}$ turns from negative to positive as t increases for $\gamma > \gamma^c$ as shown in Lemma 1. Thus, for $\gamma > \gamma^c$, the total effect of t on q_c could potentially be positive for low values of t. The next lemma states that this does not occur and and that, in fact, the output level is strictly decreasing in t for any t for $\gamma > \gamma^c$. From Lemma 1 we also know that ε_c is strictly decreasing in t for

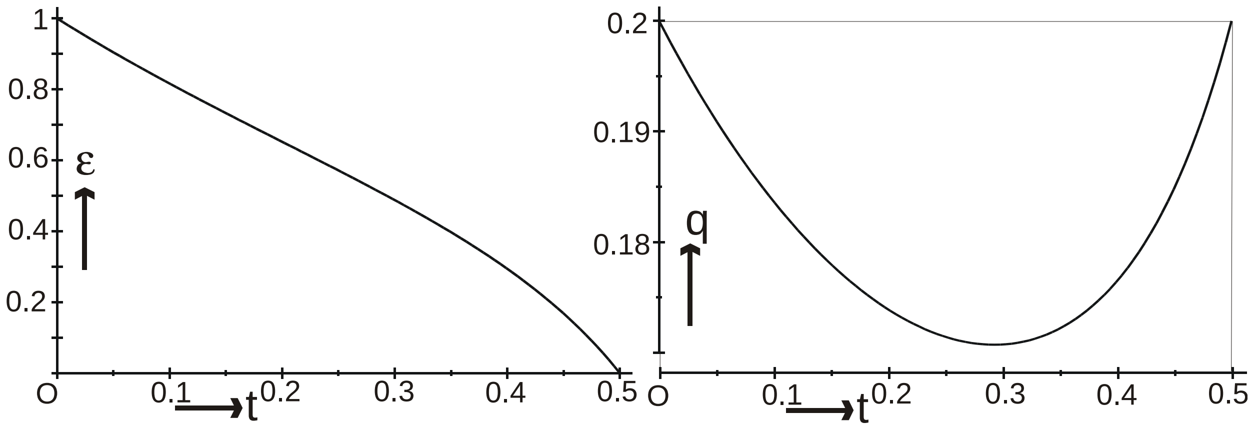

Figure 1: Emission intensity ε and output q as a function of the tax rate t for $\gamma < \frac{2ma^2}{(1+m)^2}$ ($a = 1$, $\gamma = 4/25$, $m = 4$).

$\gamma < \frac{2ma^2}{(1+m)^2}$ (that is, $\frac{\partial \varepsilon}{\partial t} < 0$). This could potentially lead to the total effect of t on output to be positive. Our next lemma also shows that this may actually happen for high enough values of t.

Lemma 2 *Let the regulator set the environmental tax rate in stage one. If $\gamma < \gamma^c$, then q_c^* is first decreasing and then increasing in t. If $\gamma \geqslant \gamma^c \equiv \frac{2ma^2}{(1+m)^2}$, then q_c^* is decreasing in t.*

To sum up, increases in the tax rate may lead to higher investments in abatement both if $\gamma \leq \gamma^c$ and if $\gamma > \gamma^c$. However, only in the first case (when a is high and m is low relative to γ) a higher tax rate could lead to a subsequent increase in output. This requires that the tax rate is sufficiently high (where the equilibrium emission-to-output ratio will be very low).

Lemma 2 is in accordance with, but more specific than, previous results in the literature. Katsoulacos and Xepapadeas (1996) and Ulph (1997) have shown that with a quadratic R&D cost function, the effect of t on q is ambiguous. In our model, depending on the parameters values, q is either decreasing throughout or U-shaped as a function of t. Combining Lemmas 1 and 2, we also show that q is decreasing in t when ε is U-shaped in t and vice versa.

Figures 1 and 2 illustrate Lemmas 1 and 2. In Figure 1, $\gamma < \gamma^c$, so that emission intensity is monotonically decreasing in t, but output is U-shaped in t. For low levels of

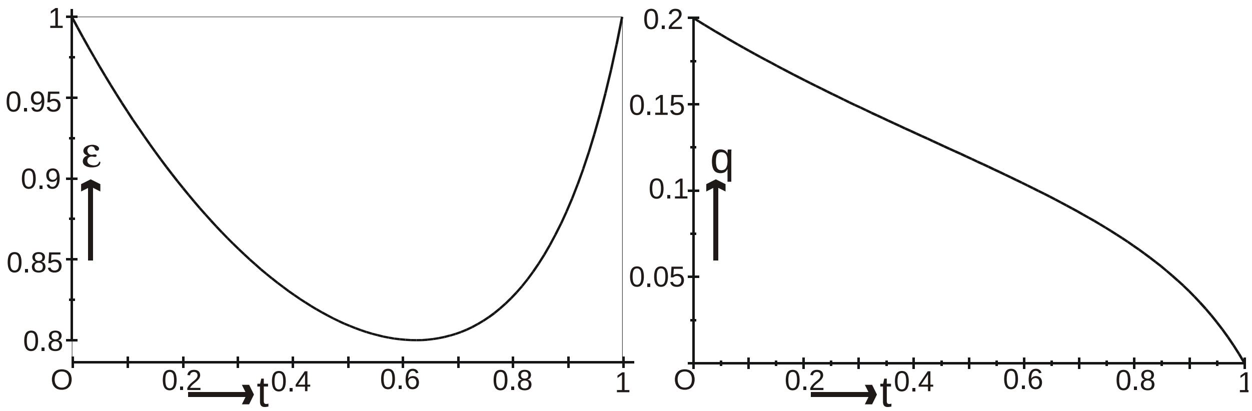

Figure 2: Emission intensity ε and output q as a function of the tax rate t for $\gamma > \frac{2ma^2}{(1+m)^2}$ ($a = 1$, $\gamma = 1/2$, $m = 4$).

t, it is profitable for the firms to reduce emission intensity and output as environmental policy becomes stricter. However, an increasingly strict environmental policy leads to lower and lower emission intensity, so that eventually the effective tax rate εt on output decreases and output rises again. For completely clean production ($\varepsilon = 0$), output is back at the level without environmental policy.

In Figure 2, $\gamma > \gamma^c$, so that output is monotonically decreasing in t, but emission intensity is U-shaped in t. For low levels of t, it is profitable for the firm to reduce emission intensity and output as environmental policy becomes stricter. However, an increasingly strict environmental policy leads to lower and lower output levels, which makes it less worthwhile to reduce the emission intensity. For $q = 0$, there is no point investing in emission reduction at all and ε is back to one.

In the first stage, the government chooses t to maximise its objective function. Finding an explicit solution is difficult. However, it is possible to characterise t_c as a function of β. The following lemma presents this characterisation:

Lemma 3 *Let the government set the environmental tax rate t in stage one. Then the tax rate t_c^* is strictly increasing in β.*

As β increases, the relative weight in the government's obejctive function of the damage made by the emissions increases. This leads the government to increase the tax per unit of emissions. To finish this section, we wish to characterize the equilibrium emissions-to-output ratio as a function of β. We present this characterization in the following proposition:

Proposition 2 *Let the government set the environmental tax rate t in stage one. Then if $\gamma < \gamma^c$, ε_c is strictly decreasing in β, whereas q_c is first decreasing and then increasing in β. If $\gamma > \gamma^c \equiv \frac{2ma^2}{(1+m)^2}$, ε_c is first decreasing and then increasing in β, whereas q_c is strictly decreasing in β.*

Interestingly, as β increases, the government raises the tax per unit of emissions with the objective of reducing the environmental damage. As reaction to this, firms will reduce their emissions levels via investments in abatement and via reduction of output. As firms reduce their output levels, they have a lower incentive to invest in abatement. When this second effect dominates, a higher β will lead to a higher emissions-to-output ratio in equilibrium. Therefore, even when the tax rate is chosen endogenously by the government, a stricter environmental technology can lead to more emissions per unit of output.

Comparing the critical levels γ^w for the welfare optimum and γ^c for the commitment case, from Propositions 1 and 2 respectively, we see that $\gamma^c < \gamma^w$ for $m \leq 2$ and $\gamma^c > \gamma^w$ for $m \geq 3$. This means that a U-shaped relation between strictness of environmental policy and emission intensity is more prevalent with commitment than in the welfare optimum for $m \leq 2$, but less prevalent with commitment for $m \geq 3$. When $m \leq 2$, there is very little competition between firms (none at all for $m = 1$), so that the regulator sets a low tax rate in order to increase output, but output will still be below the optimal level. With output and the emission tax rate being very low, firms have little incentive to invest in cleaner technology, and there is more likely to be a turning point from where emission intensity starts increasing with the regulator's environmental preference. When $m \geq 3$, output and the tax rate are relatively higher, so that it becomes more profitable to invest in cleaner technology, and emission intensity is more likely to be decreasing monotonically in the regulator's environmental preference.

5 Firms move first (no commitment)

In the no-commitment scenario n, each firm i chooses its abatement technology ε_i in stage one. The government then sets its environmental tax rate t in stage two. Finally, each firm i sets its output level q_i in the third stage. Thus the government cannot commit to a tax rate ahead of the firm's choice of technology.

The third stage, where each firm i sets q_i for given t and $\varepsilon_j, j = 1, \cdots, m$, is the same as in the commitment game. Thus, the equilibrium output level and profits are $\bar{q}_i$ in (20) and $\bar{\pi}_i$ in (21) respectively.

In the second stage the government chooses t to maximize welfare, given the ε_i chosen by the firms in stage one. From (8), the regulator's first order condition in stage two is:

$$\frac{\partial W}{\partial t} = (a - Q)\frac{dQ}{dt} - \beta \left[\sum \varepsilon_i q_i\right] \left[\sum \varepsilon_i \frac{dq_i}{dt}\right] = 0$$

Using (19) and (20) and setting $\varepsilon_1 = \cdots = \varepsilon_m = \varepsilon$, we can solve for t:

$$t_n|_{\varepsilon_i=\varepsilon} = \frac{a\left(\beta m \varepsilon^2 - 1\right)}{m\varepsilon(1 + \beta\varepsilon^2)} \tag{27}$$

Note that $t_n > 0$ requires $\beta m \varepsilon^2 - 1 > 0$. In other words, β has to be sufficiently high for the government to tax the firms' emissions.[8]

In general, we can state:

Lemma 4 *If the firms set $\varepsilon_1 = \cdots = \varepsilon_m = \varepsilon$ in stage one, then in stage two*

$$\frac{dt}{d\varepsilon_i} = \frac{dt}{d\varepsilon_j} = \frac{1}{m}\frac{dt}{d\varepsilon} \qquad \forall i, j = m$$

Using straightforward comparative statics we can state the following:

Lemma 5 *Let each firm i set its emissions-to-output ratio ε_i in stage one. Then we find for the emission tax rate t_n^* in the symmetric equilibrium with $\varepsilon_1 = \cdots = \varepsilon_m = \varepsilon$:*

$$\begin{aligned} \frac{dt_n}{d\beta} &= \frac{a\varepsilon\,(m+1)}{m\left(\beta\varepsilon^2+1\right)^2} > 0, \qquad \frac{dt_n}{dm} = \frac{a}{m^2\varepsilon\left(\beta\varepsilon^2+1\right)} > 0 \\ \frac{dt_n}{d\varepsilon} &= \frac{a\left[(3+m)\beta\varepsilon^2 - m\beta^2\varepsilon^4 + 1\right]}{m\varepsilon^2\left(\beta\varepsilon^2+1\right)^2}, \qquad \frac{dt_n}{d\varepsilon_i} = \frac{a\left[(3+m)\beta\varepsilon^2 - m\beta^2\varepsilon^4 + 1\right]}{m^2\varepsilon^2\left(\beta\varepsilon^2+1\right)^2} \end{aligned} \tag{28}$$

[8] The SOC for a maximum is fulfilled for any $\beta > 0$ and $\varepsilon > 0$. Moreover, we require $\beta < (2+m)/m$ so that to guarantee that in equilibrium $W > 0$ for any $\varepsilon \in [0, 1]$. The proof of this is available from the authors upon request.

The lemma is intuitive. The higher the (perceived) environmental damage of emissions, the higher the tax on emissions is. The tax rate is increasing in the number of firms, because increasing competition between firms means they will be less successful in driving the product price up by limiting their output. Therefore the regulator is more worried about the pollution from production than about output being below the optimal level.

In the first stage of the game, each firm i chooses its emission intensity ε_i to maximise profits. The FOC is, from (21):

$$\frac{d\pi_{i,n}}{d\varepsilon_i} = 2q_{i,n}\left[\frac{\partial q_i}{\partial \varepsilon_i} + \frac{\partial q_i}{\partial t}\frac{dt}{d\varepsilon_i}\right] + \gamma(1-\varepsilon_i) = 0 \tag{29}$$

From (20) we find:

$$\frac{\partial q_{i,n}}{\partial t} = \frac{-m\varepsilon_i + \sum_j \varepsilon_j}{m+1} \qquad\qquad \frac{\partial q_{i,n}}{\partial \varepsilon_i} = \frac{-mt}{m+1}$$

Substituting this along with (20) and (28) into (29), we find:

$$\frac{-2aY}{m^3\varepsilon\left(\beta\varepsilon^2+1\right)^3(m+1)} + \gamma(1-\varepsilon) = 0 \tag{30}$$

with

$$Y \equiv m^3\varepsilon\left(1+\beta\varepsilon^2\right)^2 - am\beta^2\varepsilon^4 + (3+m)a\beta\varepsilon^2 + a > 0 \tag{31}$$

Given that finding the explicit solution to the above equation is very intricate, we resort to the implicit function theorem to characterise the relationship between β and ε_n. We can establish the following result:

Proposition 3 *Let each firm i set its emissions-to-output ratio ε_i in stage one. Then for $m \leq 3$ firms, ε_n is increasing throughout in β. For $m \geq 4$ firms, the relation between β and ε_n is ambiguous.*

In this case we can see again that the relationship between environmental damage and emissions per unit of output may non be monotonic, particularly for a relatively large number of firms. An increasingly strict environmental policy (higher β) tends to lead to a higher tax rate, increasing the incentives to invest in abatement technologies on the one hand but lowering output on the other. This latter effect will be stronger in markets with more firms (in particular, in markets with $m \geq 4$), where competition in output is stronger. The result of the interaction of these two effects will therefore be ambiguous in those markets.

Finally, we wish to comment on the relationship between the equilibrium output and β. After substituting t_n from (27) and $\varepsilon_i = \varepsilon_n$ into (20), we can write the equilibrium ouput as:

$$q_n = \frac{a}{m(1+\beta\varepsilon^2)} \tag{32}$$

It can be seen that β affects q_n both directly and indirectly, though its effect on ε_n. The direct effect of β is negative: Higher β leads to higher taxes and as a consequence of that, to lower output. However, higher taxes can also lead to more investment on abatement which tends to favour an increase of production. The interplay between these two effects determines whether the equilibrium level of output in increasing or decreasing in β. The next lemma formalizes this observation:

Lemma 6 *Let the firm set the emissions-to-output ratio ε in stage one. Then q_n is:*

i. Decreasing in β if $\frac{d\varepsilon_n}{d\beta} > 0$, or if $\frac{d\varepsilon_n}{d\beta} < 0$ and $\varepsilon_n > -2\beta\frac{d\varepsilon_n}{d\beta}$.

ii. Increasing in β if $\frac{d\varepsilon_n}{d\beta} < 0$ and $\varepsilon_n < -2\beta\frac{d\varepsilon_n}{d\beta}$.

6 Conclusion

Does an increasingly strict environmental policy spur on the polluting industry to invest more and more in finding cleaner ways to produce? The answer might seem obvious, but it is not once we take the output market into account. When stricter environmental policy leads to a reduction in output, investment in reducing the emissions-to-output ratio becomes less profitable.

We find that the emissions-to-output ratio can be a U-shaped function of the environmental damage parameter. This can happen in the welfare optimum, in the game where the regulator can commit to the emission tax rate before the firms decide on their environmental R&D, and in the game where the regulator cannot commit. In all these games, if the emissions-to-output ratio is decreasing throughout in environmental damage, it is output that is U-shaped in environmental damage. Thus, while initially both output and emission intensity are decreasing in environmental damage, eventually one of them will start to increase. Interestingly, the U-shaped function of the emissions-to-output ratio will tend to arise in situations where the R&D costs and number of firms are relatively large and the size of the market is relatively small. Such situations are associated with lower profitability of investing in environmental R&D.

Policy makers may wish to stimulate environmental R&D in order to reduce environmental compliance cost in the future or to strengthen their polluting or eco-industry in the global market. We show that a strict environmental policy, or a higher weight on environmental damage in the objective function, does not necessarily lead to more environmental R&D. Indeed, games between the government and industry aside, it may not even be optimal (in a static setting) for environmental R&D to keep increasing with the strictness of environmental policy.

In future work we intend to generalize the market demand and R&D cost functions. It is especially interesting to look at cost functions where the sign of the LHS of (1) is ambiguous, so that the effect of environmental policy on environmental R&D is not immediately clear.

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A Appendix: Proofs

Proposition 1. From (10) to (17), it is clear that for low values of $\beta > 0$, both $dq/d\beta < 0, d\varepsilon/d\beta < 0$, but for higher values of β, either $dq/d\beta > 0$ or$d\varepsilon/d\beta > 0$, but not both.

Let us examine whether ε can reach a turning point in β ($d\varepsilon/d\beta = 0$), after which it will start increasing in β. When $\gamma < a^2/m$, then by (10) $dq/d\beta = 0$ for $\varepsilon = 1/2$ and:

$$\beta = \frac{\gamma}{mq^2} \tag{33}$$

Substituting both expressions into (10) yields:

$$a - mq - \frac{\gamma}{4q} = 0$$

Solving for q, we find:

$$q = \frac{a + \sqrt{a^2 - m\gamma}}{2m} \tag{34}$$

(The other solution is a local welfare minimum.) This confirms that there is only a solution for q if $\gamma < a^2/m$. Substituting (34) into (33), we find that $dq/d\beta = 0$ for

$$\beta = \frac{\gamma}{2m\left(a + \sqrt{a^2 - m\gamma}\right)^2}$$

This proves point 1 of the Proposition.

From (15), we see that $d\varepsilon/d\beta = 0$ when

$$\beta\varepsilon^2 = 1 \tag{35}$$

Substituting this into (10), we find $q = a^2/(4m^2)$. Substituting this and (35) into (11) yields:

$$\gamma(1 - \varepsilon) - \frac{a^2}{4m\varepsilon} = 0$$

Solving for ε, we find:

$$\varepsilon = \frac{1}{2} + \frac{1}{2}\sqrt{1 - \frac{a^2}{m\gamma}} \tag{36}$$

(The other solution is a local welfare minimum.) Thus there is only a solution for ε if $\gamma > a^2/m$. Substituting (36) into (35), we find that $d\varepsilon/d\beta = 0$ for:

$$\beta = \frac{2}{\sqrt{m\gamma} + \sqrt{m\gamma - a^2}}$$

This proves point 2 of the proposition.

When $\gamma = a^2/m$, both $dq/d\beta = 0$ and $d\varepsilon/d\beta = 0$ at the same point, namely where $\beta = \frac{1}{4}$, $\varepsilon = \frac{1}{2}$ and $q = \frac{a}{2m}$. For $\beta > \frac{1}{4}$, either ε remains at $\frac{1}{2}$ and $dq/d\beta < 0$ or q remains at $\frac{a}{2m}$ and $d\varepsilon/d\beta < 0$. This proves point 3 of the Proposition.

Lemma 1. From (25) we find:

$$\frac{d\varepsilon_c}{dt} = \frac{2m\left[\gamma(1+m)^2(2t-a) - 2mat^2\right]}{\left[\gamma(1+m)^2 - 2mt^2\right]^2} \tag{37}$$

It is easy to see that $\frac{d\varepsilon_c}{dt}$ is continuous in t in the relevant parameter space. As the denominator on the RHS of (37) is positive, the sign of $\frac{d\varepsilon_c}{dt}$ is the sign of the numerator. Solving $[\gamma(1+m)^2(2t-a) - 2mat^2] = 0$, we find the critical point (or points) at which $\frac{d\varepsilon_c}{dt}$ changes its sign. Two roots can be found, $\tau = \frac{\gamma(1+m)^2 - (1+m)\sqrt{\gamma(\gamma(1+m)^2 - 2a^2m)}}{2am}$ and $\upsilon = \frac{\gamma(1+m)^2 + (1+m)\sqrt{\gamma(\gamma(1+m)^2 - 2a^2m)}}{2am}$. Note that if $\gamma < \frac{2ma^2}{(1+m)^2}$, there are no real roots and therefore $\frac{d\varepsilon_c}{dt} < 0$ globally by continuity from $\frac{d\varepsilon_c}{dt} < 0$ at $t = 0$. Now, we focus on the case where there are real roots; that is where $\gamma > \frac{2ma^2}{(1+m)^2}$. When this holds, it is easy to see that $\upsilon > \tau$ and that $\upsilon > a$. Therefore, we can discard υ because we require $t < a$ so that output is positive. We therefore focus on what happens before and after τ. It is relatively easy to check that $\tau > 0$. Moreover, if $t = 0$, $\frac{d\varepsilon_c}{dt} < 0$. By continuity, given that $\tau > 0$, if $t < \tau$, ε_c is decreasing in t and if $t > \tau$, ε_c is increasing in t. Finally, we must consider the case where $\gamma = \frac{2ma^2}{(1+m)^2}$. In such a case $\tau = a$. Given that $t < a$ and that if $t = 0$, $\frac{d\varepsilon_c}{dt} < 0$, we know that if $\gamma = \frac{2ma^2}{(1+m)^2}$, ε_c is strictly decreasing in t. The lemma follows.

Lemma 2. From (26) we find:

$$\frac{dq_c}{dt} = \frac{\gamma(1+m)\left[2mt(2a-t) - \gamma(1+m)^2\right]}{(\gamma(1+m)^2 - 2mt^2)^2} \tag{38}$$

Notice that $\frac{dq_c}{dt}$ is continuous in the relevant parameter space and that $\frac{dq_c}{dt}\Big|_{t=0} < 0$. However, the sign of the derivative may change for higher values of t. Recall also that $t \in (0, a)$. The sign of $\frac{dq_c}{dt}$ is the sign of the term in square brackets on the RHS of (38). The roots for $\gamma(1+m)^2 - 4mat + 2mt^2 = 0$ are $t = \frac{2ma \pm \sqrt{2m(2ma^2 - \gamma(1+m)^2}}{2m}$. As $t < a$, the only relevant root is $t = \frac{2ma - \sqrt{2m(2ma^2 - \gamma(1+m)^2)}}{2m}$. We now study the behaviour of $\frac{dq_c}{dt}$ in the two cases highlighted in the lemma. It is easy to see that if $\gamma > \frac{2ma^2}{(1+m)^2}$, there are no real roots. By continuity, $\frac{dq_c}{dt} < 0$, for any t. Next, we turn our attention to the case where $\gamma < \frac{2ma^2}{(1+m)^2}$. Recall that $\frac{dq_c}{dt}\Big|_{t=0} < 0$ and it is straightforward to see that $\frac{dq_c}{dt}\Big|_{t=a} = \frac{\gamma(1+m)}{2ma^2-\gamma(1+m)^2} > 0$. Therefore if $\gamma < \frac{2ma^2}{(1+m)^2}$, $\frac{dq_c}{dt} < 0$ for $t < \frac{2ma\pm\sqrt{2m(2ma^2-\gamma(1+m)^2}}{2m}$ and $\frac{dq_c}{dt} > 0$ for $t > \frac{2ma\pm\sqrt{2m(2ma^2-\gamma(1+m)^2}}{2m}$. Finally, if $\gamma = \frac{ma^2}{2}$, the highlighted root is $t = a$. As $t < a$ and $\frac{dq_c}{dt}\Big|_{t=0} < 0$, by continuity we know that $\frac{dq_c}{dt} < 0$ if $\gamma = \frac{ma^2}{2}$. The lemma follows.

Lemma 3. From the implicit function theorem we know that

$$\frac{dt}{d\beta} = -\frac{\partial^2 W/\partial\beta\partial t}{\partial^2 W/\partial t^2}$$

The denominator on the RHS is negative, because this is the SOC for welfare maximization. It is straightforward to see from (8) that $\frac{\partial W}{\partial \beta} = -\frac{1}{2}\left(\sum \varepsilon_c q_c\right)^2$. Hence:

$$\partial^2 W/\partial\beta\partial t = -\left(\sum \varepsilon_c q_c\right)\sum\left(\frac{d\left[\varepsilon_c q_c\right]}{dt}\right) \tag{39}$$

Thus, the sign of $\partial^2 W/\partial\beta\partial t$ depends on the sign of $\sum\left(\frac{d[\varepsilon_c q_c]}{dt}\right)$ where from (25), (26), (37) and (38):

$$\frac{d\left[\varepsilon_c q_c\right]}{dt} = q_c\frac{d\varepsilon_c}{dt} + \varepsilon_c\frac{dq_c}{dt} = -\frac{\gamma(1+m)}{\left[\gamma(1+m)^2 - 2mt^2\right]^3}H$$

where $H = \gamma^2(1+m)^4 + 4am^2t^2(3a-2t) + 2\gamma m(1+m)^2(a^2 - 6at + 3t^2)$. By (23), the denominator on the RHS is positive. Recall that $t \in [0,a]$. Evaluating H at the maximum value of t, we have $H\ |_{t=a} = (2a^2m - \gamma(1+2m+m^2))^2 > 0$. And at the minimum value of t, we have $H\ |_{t=0} = 2a^2\gamma m(1+m)^2 + \gamma^2(1+m)^4 > 0$. Moreover, we know that $\frac{\partial H}{\partial t} = 12m(a-t)(2amt - \gamma\left[1+m\right]^2) < 0$ given (23) and $a > t$. Thus, we know that H is continuous and decreasing in t and given that H is positive at $t = a$ and at $t = 0$, we know that $H > 0$ for any feasible t. Thus, $\frac{d[\varepsilon_c q_c]}{dt} < 0$ and therefore from (39), we know that $\partial^2 W/\partial t\partial\beta > 0$. The rest of the lemma follows.

Proposition 2. By the chain rule, we know that $\frac{d\varepsilon_c}{d\beta} = \frac{d\varepsilon_c}{dt}\frac{dt}{d\beta}$ and $\frac{dq_c}{d\beta} = \frac{dq_c}{dt}\frac{dt}{d\beta}$. The sign of $d\varepsilon_c/d\beta$ and $dq_c/d\beta$ is given by Lemmas 1 and 2 respectively, whereas $dt/d\beta > 0$ by Lemma 3.

Lemma 4. In stage two, the regulator sets t according to $\partial W(\varepsilon_1, \cdots, \varepsilon_m, t)/\partial t = 0$. Then since $\varepsilon_i = \varepsilon_j = \varepsilon$:

$$\frac{dt}{d\varepsilon_i} = -\frac{\partial^2 W/\partial t\partial\varepsilon_i}{\partial^2 W/\partial t^2} = -\frac{\partial^2 W/\partial t\partial\varepsilon_j}{\partial^2 W/\partial t^2} = \frac{dt}{d\varepsilon_j} \qquad \forall i,j$$

Furthermore:

$$\frac{dt}{d\varepsilon} = \sum_{i=1}^{m} \left(-\frac{\partial^2 W/\partial t \partial \varepsilon_i}{\partial^2 W/\partial t^2} \right) = -m \frac{\partial^2 W/\partial t \partial \varepsilon_j}{\partial^2 W/\partial t^2} = m \frac{dt}{d\varepsilon_j} \qquad \forall j$$

Lemma 5. The first three results follow straightforwardly from (27), whereas $dt_n/d\varepsilon_i$ follows from (27) and Lemma 4.

Proposition 3. Totally differentiating firm i's first order condition $d\pi_i/d\varepsilon_i = 0$ with respect to β, we find:

$$\left(\frac{d^2\pi_i}{d\varepsilon_i^2} + \sum_{j \neq i} \frac{d^2\pi_i}{d\varepsilon_i d\varepsilon_j} \right) \frac{d\varepsilon}{d\beta} + \frac{d^2\pi_i}{d\varepsilon_i d\beta} = 0$$

Thus we have:

$$\frac{d\varepsilon_n}{d\beta} = -\frac{d^2\pi_i/d\varepsilon_i d\beta}{\left(\frac{d^2\pi_i}{d\varepsilon_i^2} + \sum_{j \neq i} \frac{d^2\pi_i}{d\varepsilon_i d\varepsilon_j} \right)}$$

The denominator on the RHS is negative by stability of the equilibrium (see Martin, 2001, p. 30). Thus, from (30), $d\varepsilon_n/d\beta$ has the sign of

$$\partial^2\pi_{i,n}/\partial\beta\partial\varepsilon = \frac{2a\varepsilon \left[Y + a \left(3\beta\varepsilon^2 - 1 \right) (m+1) \right]}{m^3 \left(\beta\varepsilon^2 + 1 \right)^4 (m+1)}$$

Since $Y > 0$ in (31) and $t_n > 0$ requires $\beta m \varepsilon^2 - 1 > 0$ by (27), the RHS is positive for $m \leq 3$, but could be negative for $m \geq 4$.

Lemma 6. Define $B_n \equiv \beta\varepsilon_n^2$. From (32), we can then write:

$$\frac{dq_n}{d\beta} = \frac{\partial q_n}{\partial B_n} \left[\frac{\partial B_n}{\partial \beta} + \frac{\partial B_n}{\partial \varepsilon_n} \frac{d\varepsilon_n}{dt} \right] = \frac{-a\varepsilon}{m(1+\beta\varepsilon_n^2)^2} \left[\varepsilon_n + 2\beta \frac{d\varepsilon_n}{dt} \right]$$

If the term in brackets is negative (positive), $\frac{dq_n}{d\beta} > (<) 0$.