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On the Joint Dynamics of Pollution and Capital Accumulation*

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Abstract

The current paper offers a new explanation on the emergence of threshold effects and multiple equilibria, for which the high (low) income equilibrium is associated with high (low) environmental quality. This new explanation rests on endogenous technological choice in the presence of environmental taxation – an idea whose foundations find strong support from existing empirical evidence. Thus, the interactions between environmental policy and technology choice, within a framework that accounts for the health effects of pollution, can explain some of the observed differences in income, life expectancy and environmental quality among countries.

JEL classification: O41; Q56

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1 Introduction

As the interest on the underlying characteristics and driving forces of economic growth rose considerably during the last two decades, it was inevitable that many researchers would turn their attention to the implications of sustained growth for the quality of the natural environment (e.g., Mourmouras, 1991; Gradus and Smulders, 1993; John and Pecchenino, 1994; Bovenberg and Smulders, 1996; Stokey, 1998; Agnani *et al.*, 2005; Jouvet *et al.*, 2005). The fact that various by-products of economic activity (e.g., gases, chemicals, toxins, smoke, radioactive substances, litter etc.) contaminate and erode the natural environment can hardly be disputed. Yet, contrary to what this fact may suggest, there is evidence showing that relatively high pollution is not always a feature associated with economies that produce more output. On the contrary, there are examples where developed countries perform better, in terms of environmental quality, when compared to some of their poorer counterparts.[1]

Understanding and explaining why richer economies may be more polluted is straightforward. The same is not true, however, for instances where the more polluted economies are relatively poorer. Perhaps this is the reason why different arguments have emerged in order to explain why per capita GDP is not always positively associated with environmental degradation. For many years, the dominant explanation has been the environmental Kuznets curve (e.g., Grossman and Krueger, 1995; Andreoni and Levinson, 2001; Millimet *et al.*, 2003; Aldy, 2005). According to this explanation, the relationship between some pollutants and per capita GDP is inverse-U-shaped, meaning that, once economies reach and exceed an income threshold, pollution declines as per capita GDP continues to grow. In a more recent contribution, Mariani *et al.* (2010) propose an explanation which relates to the idea of threshold effects and multiple, path-dependent equilibria. Using an overlapping generations model where life expectancy is endogenously determined by environmental quality, they find that multiple equilibria may emerge – in particular, the equilibrium with the higher stock of human capital is also associated with

[1] For example, while the United States (≈ 19 metric tons) and Canada (≈ 16.7 metric tons) have higher CO_2 emissions per capita than countries with lower per capita GDP such as Estonia (≈ 13.1 metric tons), Kazakhstan (≈ 12.6 metric tons), Oman (≈ 16.3 metric tons), the Czech Republic (≈ 11.3 metric tons) and Turkmenistan (≈ 9 metric tons), the majority of countries in the latter group appear to have higher CO_2 emissions per capita compared to richer nations such as France (≈ 6.2 metric tons), Sweden (≈ 5.6 metric tons), Switzerland (≈ 5.6 metric tons) and Germany (≈ 9.7 metric tons). The data, compiled by the United Nation's Department of Economic and Social Affairs, refers to the year 2006 and can be accessed electronically at http://mdgs.un.org/unsd/mdg/SeriesDetail.aspx?srid=751&crid.

better environmental conditions. This occurs because there are positive feedbacks in the determination of environmental and human capital dynamics: not only does environmental quality supports human capital accumulation and output growth, but, crucially, the quality of the environment improves monotonically as the level of income grows.

In this paper, my aim is to provide an alternative explanation on the possible emergence of multiple, path depended equilibria for which higher (lower) income is associated with better (worse) environmental quality and higher (lower) life expectancy. In contrast to the existing literature, the mechanism that I propose does not rely on an outcome whereby environmental quality improves monotonically with economic growth. Instead, I derive this result under a more plausible scenario in which, for a given emission-to-output ratio, output growth degrades the environment. What is crucial in my framework is that the growth process can complement environmental policy in inducing entrepreneurs to implement cleaner production methods. Hence, the use of environmentally friendlier technologies (induced by environmental taxation) is what leads to reduced pollution and higher life expectancy in relatively richer economies.[2]

The underlying motivation behind my analysis is as follows. Threshold effects in the joint determination of national income and environmental degradation require that the effects governing the environment-growth nexus are two-way causal. There is ample evidence to suggest that pollution may have a profoundly adverse impact to the overall health profile of the population and, therefore, their longevity prospects (e.g., Pimentel *et al*., 1998; Donohoe, 2003; Lacasaña *et al*., 2005). Thus, an idea similar to that of Mariani *et al.* (2010) – that is, the assumption according to which the stock of pollution has an adverse effect on life expectancy – is certainly an essential element of the interactions between economic growth and environmental quality. Nevertheless, the other side of the environment-growth relationship, i.e., the overall impact of income on environmental quality, needs to be reflected upon. In Mariani *et al.* (2010), the environmental impacts of polluting consumption and environmental maintenance spending are additively separable. Individuals who internalise the impact of their own actions on the environment, offer such a level of

[2] Another analysis that derives multiple equilibria in a model with pollution externalities and endogenous mortality is that of Varvarigos (2010). However, he does not take account of the entrepreneurial choice of the technology that determines the emission-to-output ratio, as I do in this paper. Furthermore, in his model the high income equilibrium is actually associated with the relatively higher (rather than the lower) flow of pollution. Thus, the implications are much different from what I seek to explain in this paper.

environmental maintenance that, collectively, it dominates the adverse environmental impact of (polluting) consumption. Since both types of expenditures (consumption and maintenance) are proportional to income, additive separability implies that the overall effect of higher income on the quality of the environment is always positive, even for given emission rates.

However, the empirical plausibility of an outcome whereby the benefit from activities targeted at environmental support is greater than the overall environmental cost of pollution, a cost that such activities are supposed to mitigate in the first place, is perhaps questionable. Actually, some existing analyses which employ a similar type of additive separability, identify this outcome as a shortcoming. They address it by imposing a non-negativity constraint that requires the environmental cost of emissions to dominate the benefit from abatement. Roussillon and Schweinzer (2010) justify this restriction on the basis that "requiring non-negative differences in the damage function…ensures that reductive efforts cannot substitute productive efforts" (Roussillon and Schweinzer, 2010; Footnote 5, p. 4). Economides and Philippopoulos (2008) use a similar restriction, arguing that the scenario for which environmental maintenance is stronger than the polluting effect of production is "too good to be true" (Economides and Philippopoulos, 2008; p.213). Evidently, these analyses avoid situations whereby, for given technological parameters, income growth represents a net benefit for the environment – after all, environmental degradation *is* largely a result of human activities.[3]

So what does the more realistic setting whereby environmental quality is a decreasing function of aggregate production imply for long-run outcomes? In the first part of my main analysis I show that, under any possible parameter configuration, the dynamics of pollution and capital accumulation converge to a unique stationary equilibrium. This is because the dynamics of capital accumulation and environmental quality do not complement each other: on the one hand, reduced pollution supports longevity and saving behaviour, thus promoting capital formation; on the other hand, however, capital accumulation adds to the stock of pollution, thus deteriorating the quality of the environment.

[3] In a recent paper, Palivos and Varvarigos (2010) use a similar criticism in order to justify the use of non-separable environmental impacts for pollution and abatement activities. Accounting for the beneficial effects of environmental quality on life expectancy, they show that pollution abatement can provide an additional engine of long-run growth; at the same time, abatement eliminates the occurrence of periodic equilibria (endogenous fluctuations).

Subsequently, I modify the basic set-up so as to introduce two different technologies that are distinguished by their emission rate. Effectively, I endogenise the process through which entrepreneurs choose whether to employ either the more polluting or the less polluting technology in the production of their goods. Although the latter is more costly to implement, the presence of an environmental tax leads to an outcome whereby, as long as the economy's capital stock exceeds an endogenously derived threshold, entrepreneurs find optimal to use the cleaner technology. This outcome is the source of positive feedbacks that, under some parameter configurations, may lead to multiple, path-dependent equilibria for which a high (low) capital stock is associated with a low (high) stock of pollution. Despite the fact that, for a given emission rate, higher income is detrimental to environmental quality, it is the higher level of income that could induce entrepreneurs to employ cleaner technologies. The use of such technologies reduces the damaging environmental effect of production and, thus, promotes longevity. The latter induces a higher saving rate which, consequently, supports capital accumulation – hence, it guarantees that income levels are high enough to support the use of environmentally-friendly technologies.[4]

This is a significantly different explanation on the emergence of multiple equilibria – more importantly, an explanation whose foundations find unequivocal support from existing empirical evidence. For example, OECD (2007) reports results from a study which shows that environmentally related taxes encourage changes in production processes that are based on cleaner production techniques and environmental R&D. Existing studies also support the idea that higher GDP is positively associated with the promotion of new technologies that are directed towards environmental improvements (e.g., Komen *et al.*, 1997). Further discussion and support for these ideas is provided by Requate and Unold (2003) and Requate (2005) among others.

The remaining paper is structured as follows. In Section 2, I present a baseline set-up with the fundamental characteristics of the economy. The steady-state equilibrium and its

[4] Gutiérrez (2008) and Jouvet *et al.* (2010) also derive very interesting policy implications from OLG models in which pollution affects the health costs incurred by agents. Gutiérrez (2008) offers an appropriate design of income taxation that will eradicate the dynamic inefficiency that results from the significant health costs that pollution entails for the old generation. Jouvet *et al.* (2010) offer a very interesting, additional dimension in the analysis of the health effects from environmental degradation. In addition to the negative health externality of pollution, they also consider a positive externality emerging from the fact that lower life expectancy means that fewer people occupy the (fixed) available amount of land. They show that the taxation of private health expenditures internalises the congestion effect from higher longevity, thus it is an important policy element in the effort to achieve the social optimum.

analysis are presented in Section 3. Section 4 introduces the idea of environmental taxation and endogenous technology choice and shows how multiple equilibria can be attributed to the joint dynamics and two-way causal effects between pollution and capital accumulation. In Section 5, I conclude.

2 The Economy

Time takes the form of discrete periods which are indexed by $t = 0,1,2,...$. In each period, there are two groups of agents inhabiting the economy – 'workers/savers' and 'entrepreneurs'. At the beginning of a period, a mass of each group comes into existence. The sizes for both groups of agents are normalised to unity.

2.1 Entrepreneurs and Final Goods Production

Entrepreneurs live for only one period.[5] Each one is endowed with a technology that allows her to combine labour from workers, L_{it}, and capital from financial intermediaries, K_{it}, so as to produce a specific variety i of an intermediate product according to

$$y_{it} = BK_{it}^{\beta} L_{it}^{1-\beta}, \tag{1}$$

where $B > 0$ and $0 < \beta < 1$. Denote the wage per unit of labour by w_t and the rental price of capital by R_t. Furthermore, let us denote the price of an intermediate good by ϱ_{it}. Cost minimisation implies

$$w_t = m_t(1-\beta)BK_{it}^{\beta} L_{it}^{-\beta}, \tag{2}$$

and

$$R_t = m_t \beta BK_{it}^{\beta-1} L_{it}^{1-\beta}, \tag{3}$$

where m_t is the marginal cost of production – associated with the Lagrange multiplier of the cost minimisation problem. Using (2) and (3), we can write the entrepreneur's profits as

$$\pi_{it} = (\varrho_{it} - m_t) y_{it}. \tag{4}$$

[5] The assumption that different types of agents may face different lifespan (as it is the case with entrepreneurs and workers in this framework) is by no means a new one. See, for example, Chakraborty and Lahiri (2007).

The entrepreneur sells her product to firms who produce the economy's final consumption good. They do so by combining all the available varieties of intermediate products in a technology of the form

$$Y_t = \left(\int_0^1 y_{it}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}, \quad (5)$$

where $\sigma > 1$ is the elasticity of substitution between different varieties of intermediate inputs.

Let us assume that the final good is the numéraire and that firms producing it operate under perfect competition. Profit maximisation leads to the standard demand function

$$y_{it} = \varrho_{it}^{-\sigma} Y_t, \quad (6)$$

while the aggregate price index is given by

$$\overline{\varrho}_t = \left(\int_0^1 \varrho_{it}^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}. \quad (7)$$

Substituting (6) in (5), it is straightforward to establish that equation (7) leads to $\overline{\varrho}_t = 1$. Next, we want to find the price that maximises the entrepreneur's profits. Substituting (6) in (4) yields $\pi_{it} = (\varrho_{it} - m_t)\varrho_{it}^{-\sigma} Y_t$. Therefore, it can be easily established that

$$\varrho_{it} = \frac{\sigma}{\sigma - 1} m_t, \quad (8)$$

i.e., the price is set as a mark up over the marginal cost of production. Equation (8) also reveals a well-known outcome associated with monopolistic competition, i.e., the symmetric equilibrium. That is, $\varrho_{it} = \varrho_t \forall i$. Therefore, $K_{it} = K_t$, $L_{it} = L_t$ and $y_{it} = y_t$ for every i.

Combined with (7), these results imply that $\overline{\varrho}_t = \varrho_t = 1$. Substituting in (8), we derive

$$m_t = \frac{\sigma - 1}{\sigma}. \quad (9)$$

We can utilise the result in (9) so as to derive the equilibrium payments of labour and capital. These are derived as

$$w_t = \frac{\sigma - 1}{\sigma}(1 - \beta) B K_t^{\beta} L_t^{-\beta}, \quad (10)$$

and

$$R_t = \frac{\sigma - 1}{\sigma} \beta B K_t^{\beta - 1} L_t^{1-\beta}, \quad (11)$$

respectively.

Finally, notice that, by virtue of (5) and (1), the symmetric equilibrium implies that

$$Y_t = y_t = BK_t^{\beta} L_t^{1-\beta}, \tag{12}$$

while an entrepreneur's equilibrium profits are given by

$$\pi_t = \frac{1}{\sigma} y_t. \tag{13}$$

Recall that entrepreneurs live for only one period. Therefore, they do not save any of their income; instead, they consume their profits, given in (13), at the end of the period during which they are alive.[6]

2.2 Workers/Savers

In every period there are two cohorts of workers/savers inhabiting the economy – the young and the old. Each young worker is endowed with one unit of labour which she supplies inelastically to entrepreneurs in exchange for the market wage w_t. She then decides how much to consume and how much to save for retirement, given that, when old, she does not have any labour endowment and, therefore, any alternative source of income from which she could finance her future consumption needs.

One deviation of this model from the standard overlapping generations setting (Diamond, 1965) is the idea that survival to maturity is not certain. Instead, I follow Chakraborty (2004) by assuming that survival is determined by the realisation of a mortality shock. Specifically, I assume that a young worker will survive to maturity with probability $\psi_t \in (0,1)$ whereas with probability $1-\psi_t$ she dies prematurely and cannot enjoy any activities when old. Provided that only agents who survive are able enjoy utility in both periods, their lifetime utility is given by[7]

$$U^t = \ln c_t^t + \psi_t \ln c_{t+1}^t. \tag{14}$$

[6] The assumption of monopolistic competition is a useful device that allows strictly positive entrepreneurial profits. The importance will become apparent in Section 4 where the possible choice of a cleaner technology by entrepreneurs is a costly one.
[7] In the utility function, a superscript indicates the period where the agent is born while the subscript indicates the period in which the actual activity takes place.

Each agent maximises her lifetime utility subject to the constraints for consumption during youth and old age. Denoting saving by s_t, the gross rate of interest on deposits by r_{t+1}, and given that $\varrho_{it} = \varrho_t = 1$, these constraints are given by $c_t^t = w_t - s_t$ and $c_{t+1}^t = r_{t+1} s_t$ respectively.

2.3 Financial Intermediaries

Financial intermediaries undertake the task of channelling capital from households to entrepreneurs. Specifically, they accept deposits by young workers and, in return, they offer the gross rate of return r_{t+1} per unit of deposited income. They transform these saving deposits into capital by accessing a technology that transforms one unit of time-t output into $q > 0$ units of time-$t+1$ capital which they supply to intermediate good suppliers at a rental cost of R_{t+1} per unit.[8]

Following others (e.g., Chakraborty, 2004; Tang and Zhang, 2007), I appeal to the idea that the young deposit their saving to a mutual fund which promises to provide retirement income, provided that the depositor survives to old age. Otherwise, the income of those who die is shared equally among surviving members of the mutual fund. In view of this, and the assumption that financial intermediaries operate under perfect competition, we have

$$\psi_t r_{t+1} = qR_{t+1}, \tag{15}$$

which implies that their costs (i.e., the total return to all surviving savers) must be equal to their revenues (i.e., the revenues they receive from entrepreneurs who rent capital).

2.4 Life Expectancy

I assume that life expectancy, captured by the probability of survival ψ_t, is endogenous.[9] In particular, it takes the form of a function

$$\psi_t = \Psi(x_t), \tag{16}$$

[8] We may think of q as the efficiency of the economy (in general) or of the financial sector (in particular) in successfully transforming resources into productive capital.

[9] The expected life span of a worker born in t is $2\psi_t + (1-\psi_t) = 1 + \psi_t$. For this reason, I will be making use of such terms as 'life expectancy', 'longevity' and 'survival probability' interchangeably.

where x_t is a variable that describes the health profile of an agent. Following Chakraborty (2004), the function in (16) satisfies $\Psi' > 0$, $\Psi'' < 0$, $\Psi(0) = 0$, $\Psi(\infty) = \lambda \in (0,1)$, $\Psi'(0) = \varphi > 0$ and $\Psi'(\infty) = 0$. It is also straightforward to establish that $\Psi(x_t) > \Psi'(x_t)x_t$ $\forall x_t > 0$ holds. Indicatively, a functional form that satisfies all these properties is $\Psi(x_t) = \frac{\lambda x_t}{1 + x_t}$, with $0 < \lambda < 1$ and $\lambda = \varphi$.

The idea of endogenous longevity is captured by the variable x_t for which I assume that it is related to average income per capita, $\overline{Y}_t$, and pollution, denoted μ_t, according to $x_t = X(\overline{Y}_t, \mu_t)$. This satisfies $X_{\overline{Y}_t} > 0$ and $X_{\mu_t} < 0$. I restrict my attention to a specific functional form which is

$$x_t = \frac{\overline{Y}_t}{\mu_t}. \tag{17}$$

Substitution of (17) in (16) yields

$$\psi_t = \Psi\left(\frac{\overline{Y}_t}{\mu_t}\right), \tag{18}$$

such that $\Psi_{\overline{Y}_t} > 0$ and $\Psi_{\mu_t} < 0$. Given $\Psi(x_t) > \Psi'(x_t)x_t$ it also follows that $\Psi_{\overline{Y}_t\overline{Y}_t} < 0$ and $\Psi_{\mu_t\mu_t} > 0$. Once more, all these properties are satisfied with the function $\Psi(x_t) = \frac{\lambda x_t}{1 + x_t}$ which, after substituting (17), becomes $\Psi(\cdot) = \frac{\lambda \overline{Y}_t}{\overline{Y}_t + \mu_t}$.

There is ample evidence to suggest that as economies develop and the population gets more educated, more people are likely to adopt a lifestyle which is conducive to their overall health status (e.g., Ross and Wu, 1995; Smith, 1999). The positive effect of $\overline{Y}_t$ on longevity, an assumption that has been widely used in the existing literature (e.g., Cervelati and Sunde, 2005; Cipriani and Makris, 2007) is meant to capture this idea. Furthermore, the negative effect of pollution on longevity conforms to empirical results (e.g., Pimentel *et al.*, 1998; Donohoe, 2003, Lacasaña *et al.*, 2005) which unequivocally show that heavily polluted environments have profoundly adverse effects on the health characteristics of the population.

2.5 Pollution

I assume that environmental degradation is a by-product of entrepreneurs' production activities. I model the dynamics of the pollution stock so as to capture changes in environmental quality over time. Specifically, I denote the stock of pollution by μ_t and I assume that it evolves according to

$$\mu_{t+1} = \eta\mu_t + P_t. \tag{19}$$

The parameter $\eta \in (0,1)$ is the rate of residual pollution. In particular, higher values of η point to the nature's reduced capacity in mitigating the cumulative impact of the current on the future pollution stock. The variable P_t is the flow of pollution which determines the degrading impact of economic activity on environmental quality. Hence, it is related to total entrepreneurial production – and, by virtue of (12), output – according to $P_t = py_t$, where $p > 0$ is an indicator of how 'dirty' the manufacturing process is. That is, it determines how many pollutant emissions are released into the environment per unit of intermediate good produced.

Using $P_t = py_t$ in (19) yields

$$\mu_{t+1} = \eta\mu_t + py_t, \tag{20}$$

an expression demonstrates how economic activity, combined with the existing level of pollution, contributes further to the decay of the natural environment. Effectively, it does this by adding to the future pollution stock.

3 Equilibrium

Taking account of the fundamental relationships in the economy, we can describe its temporary equilibrium with

Definition 1. *The temporary equilibrium of the economy is a set of quantities* $\{c_t^{t-1}, c_t^t, c_{t+1}^t, s_t, L_t, y_t, \overline{Y}_t, Y_t, \pi_t, \psi_t, P_t, \mu_t, K_t, K_{t+1}\}$ *and prices* $\{\varrho_t, w_t, R_t, R_{t+1}, r_{t+1}\}$ *such that:*

(i) *Given* w_t, ψ_t, r_{t+1} *and* μ_t, *the quantities* c_t^t, c_{t+1}^t *and* s_t *solve the optimisation problem of a worker born at time* t;

(ii) *Given* ϱ_{it} *, final goods firms choose quantities for* y_{it} *so as to maximise profits;*

(iii) *Given* w_t *and* R_t *, all entrepreneurs choose, symmetrically, quantities for* L_t *and* K_t *as well as the price* ϱ_t *in order to maximise profits;*

(iv) *The labour market clears, i.e.,* $L_t = 1$ *;*

(v) *The goods market clears, i.e.,* $Y_t = c_t^t + \psi_{t-1} c_t^{t-1} + s_t + \pi_t$ *;*

(vi) *The financial market clears, i.e.,* $\psi_t r_{t+1} = q R_{t+1}$ *;*

The optimisation problem of a young worker leads to a solution for saving given by

$$s_t = \frac{\psi_t}{1+\psi_t} w_t . \tag{21}$$

A person that does not possess any working abilities when old will find optimal to save a fraction of her labour income for retirement. The possibility of premature death induces the person to modify her saving behaviour in response to variations in life expectancy. Specifically, an increase in the probability of survival raises the (expected) marginal utility of consumption when old. To restore the equilibrium, the marginal utility of her consumption when young must increase as well – something that the agent can achieve by saving more and consuming less during the first period of her lifetime.

The equilibrium condition $L_t = 1$ implies that $K_t = K_t / L_t = k_t$ and $Y_t = y_t = Bk_t^\beta$ $\forall t$. Therefore, using (10) and $k_{t+1} = qs_t$, equation (21) becomes[10]

$$k_{t+1} = q\Theta \frac{\psi_t}{1+\psi_t} k_t^\beta , \tag{22}$$

where $\Theta = (1-\beta)B(\sigma-1)/\sigma$. Furthermore, we can use $\overline{Y}_t = Y_t = y_t$ together with (18), and substitute in (22) to derive the dynamics of capital accumulation according to

$$k_{t+1} = q\Theta \frac{\Psi\left(\frac{Bk_t^\beta}{\mu_t}\right)}{1+\Psi\left(\frac{Bk_t^\beta}{\mu_t}\right)} k_t^\beta \equiv \mathrm{K}(k_t, \mu_t) . \tag{23}$$

Using $L_t = 1$, $K_t = k_t$ in (12) and substituting in equation (20) yields

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[10] Production results in full depreciation of capital.

$$\mu_{t+1} = \eta\mu_t + pBk_t^{\beta} \equiv \mathrm{M}(k_t, \mu_t), \tag{24}$$

which represents the dynamics of the pollution stock. Given the above, the economy's dynamic equilibrium is formally described through

Definition 2. *For* $k_0, \mu_0 > 0$*, the dynamic equilibrium is a sequence of temporary equilibria that satisfy*

(i) $k_{t+1} = \mathrm{K}(k_t, \mu_t)$;

(ii) $\mu_{t+1} = \mathrm{M}(k_t, \mu_t)$.

The economy's long-run equilibrium – that is, its steady-state – is the solution to the planar system of difference equations for the stock of capital per worker and the stock of pollution. Formally, the steady-state equilibrium is a pair $(\hat{k}, \hat{\mu})$ that satisfies $\hat{k} = \mathrm{K}(\hat{k}, \hat{\mu})$ and $\hat{\mu} = \mathrm{M}(\hat{k}, \hat{\mu})$. To obtain it, we use $k_{t+1} = k_t = \hat{k}$ and $\mu_{t+1} = \mu_t = \hat{\mu}$ in equations (23) and (24). Solving (24) for $\hat{\mu}$ yields $\hat{\mu} = \frac{pB}{1-\eta}\hat{k}^{\beta}$. Substituting this solution in (23), and solving for $\hat{k}$, gives

$$\hat{k} = \left(q\Theta \frac{\Psi\left(\frac{1-\eta}{p}\right)}{1+\Psi\left(\frac{1-\eta}{p}\right)} \right)^{\frac{1}{1-\beta}}, \tag{25}$$

therefore, the steady-state equilibrium for the pollution stock is

$$\hat{\mu} = \frac{pB}{1-\eta}\left(q\Theta \frac{\Psi\left(\frac{1-\eta}{p}\right)}{1+\Psi\left(\frac{1-\eta}{p}\right)} \right)^{\frac{\beta}{1-\beta}}. \tag{26}$$

The foregoing analysis provides analytical and explicit solutions for the steady-state values of capital per worker and pollution. Notice that, as long as economic activity is responsible for environmental degradation, the model generates a unique steady-state equilibrium. Multiple equilibria cannot emerge because the dynamics of pollution and capital accumulation do not entail positive feedbacks in a bi-directional manner: while the natural environment is beneficial for the economy (due to its health benefits), economic activity is detrimental for the environment. Given that Section 4 will provide significantly different results and implications, it is proper to summarise the existing result through

Remark 1. *When the emission rate* p *is fixed, the effect of pollution on life expectancy is not sufficient to generate multiple equilibria. Under any parameter configuration, the dynamics converge to a unique steady-state equilibrium.*

Prior to examining the economic implications of varying some structural parameters, we need to determine whether this equilibrium is stable. As it turns out, an additional restriction on the relative share of capital is sufficient to guarantee the stability of the equilibrium. Formally, this is established in

Lemma 1. *Suppose that* $\beta \leq \frac{1}{2}$. *Then the equilibrium pair* $(\hat{k}, \hat{\mu})$, *with* $\hat{k}, \hat{\mu} > 0$, *is locally stable.*

Proof. See the Appendix. ■

Thus, as long as the share of capital on national income is not high enough, the steady-state equilibrium is non-trivial in the sense that the dynamics starting from any pair of initial values $(k_0 > 0, \mu_0 > 0)$, in the neighbourhood of $(\hat{k}, \hat{\mu})$, will converge to $k_\infty = \hat{k}$ and $\mu_\infty = \hat{\mu}$. Notice that, although it may appear as a limiting scenario, the restriction $\beta \leq \frac{1}{2}$ is supported by numerous empirical estimates who conclude that the relative share of capital income is significantly below 50% (e.g., Poterba, 1998; Gollin, 2002).

The equilibrium can be illustrated by means of the phase diagram in Figure 1. The PS locus is derived from points that satisfy $\mu_t = \mathrm{M}(k_t, \mu_t)$. From (24) we get $\mu_t = \frac{pB}{1-\eta} k_t^\beta \equiv G(k_t)$ such that $G' > 0$, $G(0) = 0$ and $G(\infty) \to \infty$. The CS locus is derived from points that satisfy $k_t = \mathrm{K}(k_t, \mu_t)$. Using $k_{t+1} = k_t$ in (23) and rearranging yields

$\dfrac{k_t^{1-\beta}}{\Psi\left(\dfrac{Bk_t^{\beta}}{\mu_t}\right)(q\Theta - k_t^{1-\beta})} = 1$ or, alternatively, $\Phi(k_t, \mu_t) = 1$. First, we can check that

$\Phi_{\mu_t} = \dfrac{k_t^{1-\beta}}{(q\Theta - k_t^{1-\beta})} \dfrac{\Psi'}{[\Psi(\cdot)]^2} \dfrac{Bk_t^{\beta}}{\mu_t^2} > 0$. The next step is to analyse the derivative,

$\Phi_{k_t} = \dfrac{(1-\beta)k_t^{-\beta}}{\Psi(\cdot)(q\Theta - k_t^{1-\beta})} + \dfrac{k_t^{1-\beta}(1-\beta)k_t^{-\beta}}{\Psi(\cdot)(q\Theta - k_t^{1-\beta})^2} - \dfrac{k_t^{1-\beta}\Psi'}{[\Psi(\cdot)]^2(q\Theta - k_t^{1-\beta})} \dfrac{\beta Bk_t^{\beta-1}}{\mu_t}$ or (after factorisation)

$\Phi_{k_t} = \dfrac{k_t^{-\beta}}{\Psi(\cdot)(q\Theta - k_t^{1-\beta})}\left[(1-\beta) + \dfrac{(1-\beta)k_t^{1-\beta}}{(q\Theta - k_t^{1-\beta})} - \beta \dfrac{\Psi'}{\Psi(\cdot)} \dfrac{Bk_t^{\beta}}{\mu_t}\right]$. We know that $\dfrac{\Psi(x_t)}{x_t} > \Psi'(x_t)$ therefore $\dfrac{1}{x_t} > \dfrac{\Psi'(x_t)}{\Psi(x_t)}$. If we replace $\dfrac{1}{x_t}$ for $\dfrac{\Psi'(x_t)}{\Psi(x_t)}$ in the third term of the expression inside brackets, and then add the first term of the same expression, we get $1 - \beta - \dfrac{\beta}{x_t} \dfrac{Bk_t^{\beta}}{\mu_t}$.[11] After substituting (17) and $\bar{Y}_t = y_t = Bk_t^{\beta}$, this expression becomes $1 - 2\beta$ which is non-negative given that $\beta \leq \dfrac{1}{2}$ holds by assumption. However, if this expression is non-negative when using $\dfrac{1}{x_t}$ then it is certainly positive when using $\dfrac{\Psi'(x_t)}{\Psi(x_t)} < \dfrac{1}{x_t}$. Consequently, $\Phi_{k_t} > 0$ and equation (23) defines a function $\mu_t \equiv J(k_t)$ such that $J' = -\dfrac{\Phi_{k_t}}{\Phi_{\mu_t}} < 0$. In addition, $\mu_t = 0$ implies $\Psi(\cdot) = \lambda$ and $k_t = \left(q\Theta \dfrac{\lambda}{1+\lambda}\right)^{\frac{1}{1-\beta}}$ while $\mu_t \to \infty$ implies $\Psi(\cdot) = 0$ and $k_t = 0$. The construction of the diagram is completed by observing that $\mathrm{K}_{\mu_t} < 0$ (see the Appendix) and $\mathrm{M}_{k_t} > 0$. These imply that above (below) the CS schedule we have $k_{t+1} < k_t$ ($k_{t+1} > k_t$) and on the left (right) of the PS schedule we have $\mu_{t+1} < \mu_t$ ($\mu_{t+1} > \mu_t$).

[11] The second term $\dfrac{(1-\beta)k_t^{1-\beta}}{(q\Theta - k_t^{1-\beta})}$ is obviously positive.

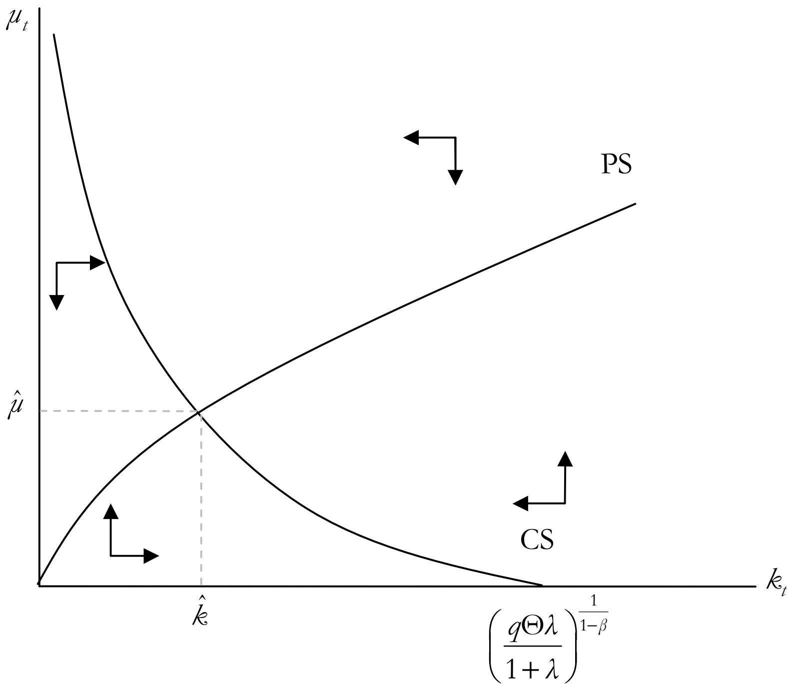

Figure 1. The phase diagram

3.1 Some Comparative Statics

This part of the paper is devoted to the analysis of the equilibrium effects resulting from variations in the model's structural parameters. These effects are summarised in

Proposition 1. *An economy with increased emissions and reduced natural absorption capacity will have lower income and higher pollution. More productive technologies and a stronger degree of competition are associated with higher income and higher pollution.*

Proof. See the Appendix. ■

An economy that employs more polluting manufacturing techniques (i.e., higher p) and/or possesses a limited absorption capacity (i.e., a higher rate of residual pollution η) will experience a deterioration of environmental quality. This worsens the health profile of the population, leads to lower life expectancy and acts as an incentive to reduce saving. The process of capital accumulation is impeded and causes a decline in production and, therefore, income. The latter effect imposes some reduction in overall pollutant emissions which is not strong enough, however, to counteract the increase in pollution resulting from the higher rate of emissions per unit manufactured goods. Eventually, the economy will settle down to a new long-run equilibrium with lower income and higher pollution.

An improvement in the productivity of the manufacturing process (i.e., higher B) will promote aggregate savings due to the rise in wages, while an improvement in the efficiency of the financial sector (i.e., higher q) will improve the process of capital formation for given amounts of saving . Both result in greater accumulation of capital which stimulates economic activity. The rise in aggregate production has two conflicting effects on the prospects of longevity. On the one hand, it has a direct benefit due to the rise in output. On the other hand, the stimulated activity implies that more pollutants are emitted in the environment. As it turns out, these two conflicting effects will cancel each other out and, eventually, the economy will settle to a new long-run equilibrium with higher income and more pollution.

Finally, a higher degree of competition, captured by a higher elasticity of substitution σ, will stimulate saving and capital formation because it increases the share of output accruing to workers in the form of salaries. For a given emission rate, the accumulation of capital will stimulate production, thus leading to an increase in the stock of pollution.

4 Threshold Effects and the Income-Pollution Nexus

So far, the analysis has been based on the assumption that, in terms of its environmental impact, there is only one entrepreneurial technology that emits p units of pollution per unit of production. The purpose of this Section is to relax this assumption. Particularly, I shall assume that there are different technologies which can be distinguished according to their 'dirtiness' – i.e., their environmental repercussions. Furthermore, I shall treat the choice between these different technologies as being endogenous. Without loss of generality, the following analysis is focused in the symmetric equilibrium.

Suppose that entrepreneurs have a choice between technologies that differ purely in terms of their emission indicator p_t. To simplify matters, let us assume that the choice is between two such technologies: the dirtier technology emits $p_t = \overline{p}$ pollutants per unit of production while the cleaner technology emits $p_t = \underline{p}$ units of pollution per unit of production. Naturally, I assume that $\overline{p} > \underline{p}$. Entrepreneurs pay a 'penalty' (e.g., some type of environmental tax) which is related to the impact of their activities on environmental quality. Specifically, they pay a fraction $\gamma(p_t)$ of their profits. Following the analysis of Section 2.1, this assumption implies that an entrepreneur's after-tax variable profits (a fixed cost will be introduced shortly) are given by $[1-\gamma(p_t)](\varrho_t - m_t)y_t$. I assume that $\gamma'(p_t) > 0$, meaning that employing the more polluting technology results in a higher penalty.[12] Furthermore, I assume that the proceeds from taxation are used to finance public sector consumption, g_t, according to a balanced budget rule $g_t = \gamma(p_t)\pi_t$.

Although this set-up renders the choice of a cleaner technology to be economically appealing, such a choice is not straightforward as entrepreneurs will also incur a fixed cost for its implementation. To formalise the argument, assume that entrepreneurs face a cost $e(p_t)$ of implementing a particular technology. Moreover, assume that this cost satisfies

$$e(p_t) = \begin{cases} 0, \text{ if } p_t = \overline{p} \\ \\ \varepsilon, \text{ if } p_t = \underline{p} \end{cases}, \tag{27}$$

where $\varepsilon > 0$.

Since the choice of technology is discrete, it can be separated from a firm's other choices.[13] Hence, I am going to solve the entrepreneurs' optimisation problem using two distinct steps. In the first step, they choose the amount of capital and labour they employ and the price of their product, for any technology described by p_t. In the second step, they choose the technology they will implement by comparing their utility (which corresponds to their after-tax profitability) in each case, taking account of the results from the first step of the optimisation procedure. In the symmetric equilibrium, and given our preceding assumptions and equation (13), the entrepreneur will enjoy utility, v_t, according to

$$v_t = \begin{cases} [1-\gamma(\overline{p})]\dfrac{1}{\sigma}y_t, \text{ if } p_t = \overline{p} \\ \\ [1-\gamma(\underline{p})]\left(\dfrac{1}{\sigma}y_t - \varepsilon\right), \text{ if } p_t = \underline{p} \end{cases}. \tag{28}$$

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[12] As an example, we may think of the Republic of Ireland's Vehicle Registration Tax (VRT) whose rates are lower for cars that emit less carbon dioxides per kilometre.

[13] See the recent paper by Chen *et al.* (2009) for a similar application of this concept.

Given $L_t = 1$ and $K_t = k_t$ by combining (12) and (28) we can conclude that the technology choice is endogenous. In particular, we can infer $p_t = p(k_t)$ such that

$$p(k_t) = \begin{cases} \overline{p}, \text{ if } k_t < \tilde{k} \\ \\ \underline{p}, \text{ if } k_t \geq \tilde{k} \end{cases}, \tag{29}$$

where

$$\tilde{k} = \left\{ \frac{[1-\gamma(\underline{p})]\varepsilon\sigma}{B[\gamma(\overline{p}) - \gamma(\underline{p})]} \right\}^{\frac{1}{\beta}}. \tag{30}$$

The emergence of an endogenous threshold in the process of economic development could be a potential source for some very important implications. Above all, it is related to the possibility of multiple, non-trivial steady-state equilibria which imply that long-run outcomes concerning prosperity, environmental conditions and longevity are path-dependent. We can begin formalising these issues by observing that the preceding results, combined with the dynamics of the pollution stock in (24), imply that the PS locus will now take the form of a discontinuous curve. In particular, this is generated by the function $G(k_t) = \frac{p(k_t)B}{1-\eta} k_t^{\beta}$ for which the expressions in (29) and (30) indicate that $\lim_{k_t \to \tilde{k}^-} G(k_t) > \lim_{k_t \to \tilde{k}^+} G(k_t)$. These observations generate the result that I demonstrate in

Lemma 2. *Suppose that* $\lim_{k_t \to \tilde{k}^-} G(k_t) > J(\tilde{k}) > \lim_{k_t \to \tilde{k}^+} G(k_t)$ *. Then, there exist two pairs of locally stable steady-state equilibria,* $(\hat{k}^1, \hat{\mu}^1)$ *and* $(\hat{k}^2, \hat{\mu}^2)$*, such that* $\hat{k}^2 > \tilde{k} > \hat{k}^1$ *and* $\hat{\mu}^2 < \hat{\mu}^1$.

Proof. Let us begin with the values for capital intensity that satisfy $k_t < \tilde{k}$. Given $\lim_{k_t \to \tilde{k}^-} G(k_t) > J(\tilde{k})$, $0 = G(0) < J(0) \to \infty$, $G'(\cdot) > 0$ and $J'(\cdot) < 0$, a steady-state equilibrium $(\hat{k}^1, \hat{\mu}^1)$ with $\hat{k}^1 < \tilde{k}$ exists. Analogously, for values of capital intensity that satisfy $k_t \geq \tilde{k}$ we have $\lim_{k_t \to \tilde{k}^+} G(k_t) < J(\tilde{k})$, $G\left(\left(\frac{q\Theta\lambda}{1+\lambda}\right)^{1/(1-\beta)}\right) > 0 > J\left(\left(\frac{q\Theta\lambda}{1+\lambda}\right)^{1/(1-\beta)}\right) = 0$, $G'(\cdot) > 0$ and

$J'(\cdot)<0$. Therefore, a steady-state equilibrium $(\hat{k}^2,\hat{\mu}^2)$ with $\hat{k}^2>\tilde{k}$ exists. Of course, $J'(\cdot)<0$ and $\hat{k}^2>\hat{k}^1$ imply that $J(\hat{k}^2)<J(\hat{k}^1)\Rightarrow\hat{\mu}^2<\hat{\mu}^1$. Analytically, these pairs of equilibria are given by

$$\hat{k}^1=\left(q\Theta\frac{\Psi\left(\frac{1-\eta}{\overline{p}}\right)}{1+\Psi\left(\frac{1-\eta}{\overline{p}}\right)}\right)^{\frac{1}{1-\beta}},\ \hat{\mu}^1=\frac{\overline{p}B}{1-\eta}\left(q\Theta\frac{\Psi\left(\frac{1-\eta}{\overline{p}}\right)}{1+\Psi\left(\frac{1-\eta}{\overline{p}}\right)}\right)^{\frac{\beta}{1-\beta}},$$

and

$$\hat{k}^2=\left(q\Theta\frac{\Psi\left(\frac{1-\eta}{\underline{p}}\right)}{1+\Psi\left(\frac{1-\eta}{\underline{p}}\right)}\right)^{\frac{1}{1-\beta}},\ \hat{\mu}^2=\frac{\underline{p}B}{1-\eta}\left(q\Theta\frac{\Psi\left(\frac{1-\eta}{\underline{p}}\right)}{1+\Psi\left(\frac{1-\eta}{\underline{p}}\right)}\right)^{\frac{\beta}{1-\beta}}.$$

By appealing to Proposition 1, we can readily verify that $\hat{k}^2>\hat{k}^1$ and $\hat{\mu}^2<\hat{\mu}^1$, while the local stability of these equilibria can be inferred from Lemma 1. ■

At this point, we can make a direct comparison with the results of the baseline model, as this was presented in Section 3. This is done through

Remark 2. *A varying and endogenously determined emission rate p_t is necessary for the effect of pollution on life expectancy to generate multiple equilibria. Depending on initial capital endowments, the dynamics may converge to different pairs of steady-state equilibria.*

In Figure 2, I illustrate the scenario which is formally derived in Lemma 2. Given that all other parameters determining the CS schedule are unchanged, the increase in equilibrium income necessitates an improvement in survival prospects brought forward by improvements in environmental quality. The drop in the emission rate is indeed sufficient enough to guarantee that pollutant emissions are lower, even though production is higher for

$\hat{k}^2 > \hat{k}^1$. If parameter values satisfy $\hat{k}^1 < \tilde{k} < \hat{k}^2$ then the determination of the long-run equilibrium is path-dependent: the economy's history, in terms of capital endowment, will determine its stationary, long-run equilibrium. We can summarise these implications through

Proposition 2. *As long as multiple equilibria exist, the economy achieving the high-income equilibrium will enjoy better environmental quality and higher longevity in comparison to the economy achieving the low-income equilibrium.*

Proof. It follows from Lemma 2 and the properties of the survival probability function, given in (18). ■

One of the irrefutable facts of the economic development experience is that the world's per capita income distribution appears to be polarised between low- and high-income countries – an observation that has been labelled as 'club-convergence' (e.g., Quah, 1997; Canova, 2004). Another persistent feature in the data is the strong, positive correlation between per capita GDP and life expectancy (e.g., Fogel, 1994; Jamison *et al.*, 2005). The emergence of multiple equilibria in this framework can account for these facts and, in addition, it illustrates another important point: although environmental decay is a by-product of economic activity, when multiple equilibria exist it is actually the more developed country that enjoys improved environmental quality and, therefore, higher life expectancy.

4.1 Mechanism and Intuition

This paper seeks to provide a novel mechanism leading to the emergence of multiple equilibria in the joint determination of capital accumulation, life expectancy and pollution. In the existing literature, this result has been attributed to the complementary nature of economic activity and environmental quality: the latter improves the former through improved life expectancy, while the former improves the latter because additively separable environmental effects allow pollution abatement to be more pronounced than actual pollution (see Mariani *et al.*, 2010).

The intuition and the mechanisms involved in my paper are quite different. In addition to the positive impact of reduced pollution for health and economic activity – exemplified by the negative slope of the CS schedule – another crucial aspect of my analysis is that economic activity may, to some extent, also facilitate environmental improvements because it provides the incentive to entrepreneurs for implementing cleaner production methods in response to environmental policy.

Effectively, what my framework shows is that the costs of pollution in terms of increased mortality are not sufficient to guarantee multiple equilibria when, for a given emission rate, output growth entails net environmental costs. For path-dependent equilibria to emerge in this framework, a crucial aspect is the endogeneity of the emission rate – in particular, the way that the emission-to-output ratio is optimally chosen in response to environmental taxation (see Remarks 1 and 2). The intuition for this mechanism can be clarified as follows. Environmental taxation induces entrepreneurs to incur the (fixed) cost of implementing a cleaner technology (i.e., one with a lower emission rate). Nevertheless, this incentive materialises only when the economy's capital stock exceeds some (endogenously derived) threshold. When this happens, the implementation of cleaner production techniques ameliorates the quality of the environment and, consequently, improves life expectancy. The latter effect supports saving, capital accumulation and economic growth. Thus, the economy retains the economic resources that are necessary in rendering the choice of cleaner technologies optimal.

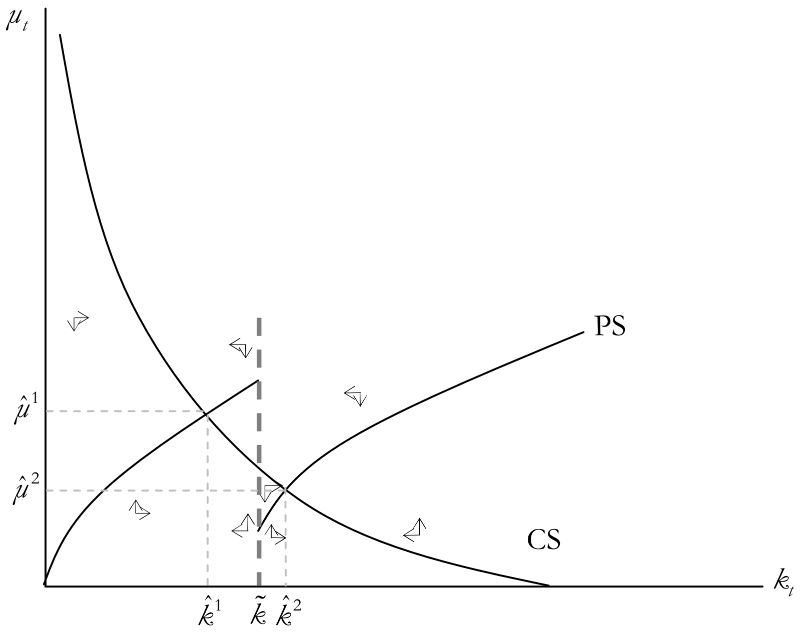

Figure 2. Threshold effects

5 Conclusion

In this paper, I have constructed a model in which the dynamics of pollution and capital accumulation interact and are, therefore, jointly determined. Although capital accumulation is responsible for the built-up of more pollutants, the latter reduce capital formation due to their detrimental effect on life expectancy and, therefore, saving behaviour. While the baseline setting results in a unique stationary equilibrium, I have shown that the introduction of environmental taxation and endogenous technology choice (the different technologies classified according to their emission rate) can lead to the emergence of multiple equilibria. When this happens, the lower stock of pollution is associated with a higher capital stock and higher longevity. The mechanism that is emphasised in my paper comes through the interactions of environmental taxation and technology choice (in terms of the emission rate) and how these, together with endogenous longevity, can impinge on the joint dynamics of pollution and capital accumulation.

Admittedly, when it comes to the trade-off between generality and clarity of the mechanisms involved, I have opted for the latter. I believe that there is enough justification for this approach. First of all, the driving forces behind the main results, as well as their intuition, are so transparent that it is perhaps unlikely that more general functional forms or an expanded set of available entrepreneurial technologies would do anything other than blurring the clarity of intuition. Secondly, the decision to abscond from the discussion of some other important issues on the pollution-economic growth nexus, such as the environmental Kuznets curve (EKC), should not be mistakenly viewed as a conscious effort to demote their importance. On the contrary, it was the understanding that the importance (as well as the highly contested nature) of issues such as the EKC hypothesis probably merit an analysis of their own.

In order to make my argument more transparent, let me return to the simplest version of the model, as this is presented in Section 3 and illustrated in Figure 1. Obviously, the (stable) equilibrium represented diagrammatically in Figure 1 can allow us to choose different combinations for the initial stocks of pollution and physical capital and examine when the resulting transitional dynamics are in accordance with the EKC and when they are not. I believe that such a discussion would move the attention away from the particular issue of interest in my analysis – the issue of threshold effects on the joint determination of capital accumulation, environmental quality and life expectancy. What is more, it would not do justice to an important topic whose proper analysis and discussion requires a distinct consideration and discussion of its own. Perhaps, the current framework can provide the basis for such analysis, as part of future research work.

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Appendix

Proof of Lemma 1

The Jacobian matrix associated with the dynamical system of (23) and (24) is

$$\begin{pmatrix} \mathrm{K}_{k_t}(\hat{k},\hat{\mu}) & \mathrm{K}_{\mu_t}(\hat{k},\hat{\mu}) \\ \mathrm{M}_{k_t}(\hat{k},\hat{\mu}) & \mathrm{M}_{\mu_t}(\hat{k},\hat{\mu}) \end{pmatrix}.$$

The trace and the determinant are given by $T = \mathrm{K}_{k_t}(\hat{k},\hat{\mu}) + \mathrm{M}_{\mu_t}(\hat{k},\hat{\mu})$ and $D = \mathrm{K}_{k_t}(\hat{k},\hat{\mu})\mathrm{M}_{\mu_t}(\hat{k},\hat{\mu}) - \mathrm{K}_{\mu_t}(\hat{k},\hat{\mu})\mathrm{M}_{k_t}(\hat{k},\hat{\mu})$ respectively. It is well known that the stability of the equilibrium is established when the conditions $(1+D-T)(1+D+T)>0$ and $|D|<1$ hold simultaneously.

From equation (23), we have

$$\mathrm{K}_{k_t}(\hat{k},\hat{\mu}) = q\Theta\left\{\beta\hat{k}^{\beta-1}\frac{\Psi(\cdot)}{1+\Psi(\cdot)} + \hat{k}^{\beta}\frac{\Psi'(\cdot)}{[1+\Psi(\cdot)]^2}\frac{\beta B\hat{k}^{\beta-1}}{\hat{\mu}}\right\} > 0 . \tag{A1}$$

Substituting (25) and (26) in (A1) yields

$$\mathrm{K}_{k_t}(\hat{k},\hat{\mu}) = \beta q\Theta\left\{(q\Theta)^{\frac{\beta-1}{1-\beta}}\left[\frac{\Psi(\cdot)}{1+\Psi(\cdot)}\right]^{\frac{\beta-1}{1-\beta}}\frac{\Psi(\cdot)}{1+\Psi(\cdot)} + \frac{\Psi'(\cdot)}{[1+\Psi(\cdot)]^2}\frac{B\hat{k}^{2\beta-1}}{\frac{pB}{1-\eta}\hat{k}^{\beta}}\right\} \Rightarrow$$

$$\mathrm{K}_{k_t}(\hat{k},\hat{\mu}) = \beta q\Theta\left\{\frac{1}{q\Theta} + \frac{\Psi'(\cdot)}{[1+\Psi(\cdot)]^2}\frac{(1-\eta)\hat{k}^{\beta-1}}{p}\right\} \Rightarrow$$

$$\mathrm{K}_{k_t}(\hat{k},\hat{\mu}) = \beta q\Theta\left\{\frac{1}{q\Theta} + \frac{\Psi'(\cdot)}{[1+\Psi(\cdot)]^2}\frac{1-\eta}{p}(q\Theta)^{\frac{\beta-1}{1-\beta}}\left[\frac{\Psi(\cdot)}{1+\Psi(\cdot)}\right]^{\frac{\beta-1}{1-\beta}}\right\} \Rightarrow$$

$$\mathrm{K}_{k_t}(\hat{k},\hat{\mu}) = \beta\left\{1+\frac{\Psi'(\cdot)}{\Psi(\cdot)[1+\Psi(\cdot)]}\frac{1-\eta}{p}\right\}. \qquad \text{(A2)}$$

Let us consider the expression

$$\beta\left(1+\frac{1}{\hat{x}}\frac{1-\eta}{p}\right). \qquad \text{(A3)}$$

In the steady-state we have $\hat{x} = \frac{\hat{y}}{\hat{\mu}} = \frac{B\hat{k}^{\beta}}{pB\hat{k}^{\beta}/(1-\eta)} = \frac{1-\eta}{p}$ therefore (A3) becomes 2β. Of course, $2\beta \leq 1$ given that $\beta \leq 1/2$ by assumption. Since $\frac{\Psi(\hat{x})}{\hat{x}} > \Psi'(\hat{x})$ also holds then $\frac{1}{\hat{x}} > \frac{\Psi'(\hat{x})}{\Psi(\hat{x})} > \frac{\Psi'(\hat{x})}{\Psi(\hat{x})[1+\Psi(\hat{x})]}$. Consequently, if (A3) cannot take a value above unity then, from (A2), it is certainly $0 < \mathrm{K}_{k_t}(\hat{k},\hat{\mu}) < 1$.

Using equation (24) we get $\mathrm{M}_{\mu_t}(\hat{k},\hat{\mu}) = \eta \in (0,1)$ which implies that $T = \eta + \mathrm{K}_{k_t}(\hat{k},\hat{\mu}) > 0$. Furthermore, we can use (23) and (24) to derive

$$\mathrm{M}_{k_t}(\hat{k},\hat{\mu}) = p\beta B\hat{k}^{\beta-1} > 0, \qquad \text{(A4)}$$

and

$$\mathrm{K}_{\mu_t}(\hat{k},\hat{\mu}) = q\Theta\hat{k}^{\beta}\frac{\Psi'(\cdot)}{[1+\Psi(\cdot)]^2}\left(-\frac{B\hat{k}^{\beta}}{\hat{\mu}^2}\right) < 0. \qquad \text{(A5)}$$

Thus, (A4) and (A5), combined with previous results, imply that $D = \eta\mathrm{K}_{k_t}(\hat{k},\hat{\mu}) - \mathrm{K}_{\mu_t}(\hat{k},\hat{\mu})\mathrm{M}_{k_t}(\hat{k},\hat{\mu}) > 0$ and $1+D+T > 0$. Additionally, we can derive

$$D-T+1 = \eta\mathrm{K}_{k_t}(\hat{k},\hat{\mu}) - \mathrm{K}_{\mu_t}(\hat{k},\hat{\mu})\mathrm{M}_{k_t}(\hat{k},\hat{\mu}) - \eta - \mathrm{K}_{k_t}(\hat{k},\hat{\mu}) + 1 \Rightarrow$$

$$D-T+1 = 1-\eta-(1-\eta)\mathrm{K}_{k_t}(\hat{k},\hat{\mu}) - \mathrm{K}_{\mu_t}(\hat{k},\hat{\mu})\mathrm{M}_{k_t}(\hat{k},\hat{\mu}) \Rightarrow$$

$$D-T+1 = (1-\eta)[1-\mathrm{K}_{k_t}(\hat{k},\hat{\mu})] - \mathrm{K}_{\mu_t}(\hat{k},\hat{\mu})\mathrm{M}_{k_t}(\hat{k},\hat{\mu}).$$

Given (A4), (A5) and $0 < \mathrm{K}_{k_t}(\hat{k},\hat{\mu}) < 1$, we have $D-T+1 > 0$ which means that $(D+T+1)(D-T+1) > 0$. Consequently, since $D > 0$, we need to show that $D < 1$ in order to establish the stability of the equilibrium.

Substitution of (26) in (A5) yields

$$\mathrm{K}_{\mu_t}(\hat{k},\hat{\mu}) = -q\Theta\frac{\Psi'(\cdot)}{[1+\Psi(\cdot)]^2}\frac{B\hat{k}^{2\beta}}{(pB)^2\hat{k}^{2\beta}/(1-\eta)^2} \Rightarrow$$

$$\mathrm{K}_{\mu_t}(\hat{k},\hat{\mu}) = \frac{-q\Theta(1-\eta)^2}{p^2B}\frac{\Psi'(\cdot)}{[1+\Psi(\cdot)]^2}. \tag{A6}$$

Using (25) in (A4) yields

$$\mathrm{M}_{k_t}(\hat{k},\hat{\mu}) = p\beta B(q\Theta)^{\frac{\beta-1}{1-\beta}}\left[\frac{\Psi(\cdot)}{1+\Psi(\cdot)}\right]^{\frac{\beta-1}{1-\beta}} \Rightarrow$$

$$\mathrm{M}_{k_t}(\hat{k},\hat{\mu}) = \frac{p\beta B}{q\Theta}\frac{1+\Psi(\cdot)}{\Psi(\cdot)}. \tag{A7}$$

Combining (A6) and (A7), we can derive

$$\mathrm{K}_{\mu_t}(\hat{k},\hat{\mu})\mathrm{M}_{k_t}(\hat{k},\hat{\mu}) = \frac{-q\Theta(1-\eta)^2}{p^2B}\frac{\Psi'(\cdot)}{[1+\Psi(\cdot)]^2}\frac{p\beta B}{q\Theta}\frac{1+\Psi(\cdot)}{\Psi(\cdot)} \Rightarrow$$

$$\mathrm{K}_{\mu_t}(\hat{k},\hat{\mu})\mathrm{M}_{k_t}(\hat{k},\hat{\mu}) = \frac{-\beta(1-\eta)^2}{p}\frac{\Psi'(\cdot)}{\Psi(\cdot)[1+\Psi(\cdot)]}. \tag{A8}$$

Next, we can combine (A2) and (A8) to derive the determinant

$$D = \eta\mathrm{K}_{k_t}(\hat{k},\hat{\mu}) - \mathrm{K}_{\mu_t}(\hat{k},\hat{\mu})\mathrm{M}_{k_t}(\hat{k},\hat{\mu}) \Rightarrow$$

$$D = \eta\beta + \eta\beta\frac{\Psi'(\cdot)}{\Psi(\cdot)[1+\Psi(\cdot)]}\frac{1-\eta}{p} + \frac{\beta(1-\eta)^2}{p}\frac{\Psi'(\cdot)}{\Psi(\cdot)[1+\Psi(\cdot)]} \Rightarrow$$

$$D = \beta\left\{\eta + \frac{\Psi'(\cdot)}{\Psi(\cdot)[1+\Psi(\cdot)]}\frac{1-\eta}{p}[\eta+(1-\eta)]\right\} \Rightarrow$$

$$D = \beta\left\{\eta + \frac{\Psi'(\cdot)}{\Psi(\cdot)[1+\Psi(\cdot)]}\frac{1-\eta}{p}\right\}. \tag{A9}$$

Now, consider the expression

$$\beta\left(\eta + \frac{1}{\hat{x}}\frac{1-\eta}{p}\right). \tag{A10}$$

In the steady-state we have $\hat{x} = \frac{1-\eta}{p}$. Substituting in (A10) yields $\beta(1+\eta) < 1$ because $\beta \leq 1/2$ and $0 < \eta < 1$. However, it is $\frac{1}{\hat{x}} > \frac{\Psi'(\hat{x})}{\Psi(\hat{x})} > \frac{\Psi'(\hat{x})}{\Psi(\hat{x})[1+\Psi(\hat{x})]}$ because $\frac{\Psi(\hat{x})}{\hat{x}} > \Psi'(\hat{x})$ holds by assumption. This implies that, if (A10) is below 1, then, given (A9), we can conclude that $D<1$ as well. Hence, we have proven that the equilibrium $\hat{k}, \hat{\mu}>0$ is locally stable. ■

Proof of Proposition 1

From (25) we can derive

$$\frac{\partial \hat{k}}{\partial p}=\frac{1}{1-\beta}\left[\frac{\Theta\Psi(\cdot)}{1+\Psi(\cdot)}\right]^{\frac{1}{1-\beta}-1}\frac{\Theta\Psi'(\cdot)}{[1+\Psi(\cdot)]^2}\left(-\frac{1-\eta}{p^2}\right)<0,$$

$$\frac{\partial \hat{k}}{\partial \eta}=\frac{1}{1-\beta}\left[\frac{\Theta\Psi(\cdot)}{1+\Psi(\cdot)}\right]^{\frac{1}{1-\beta}-1}\frac{\Theta\Psi'(\cdot)}{[1+\Psi(\cdot)]^2}\left(-\frac{1}{p}\right)<0,$$

$$\frac{\partial \hat{k}}{\partial \sigma}=\frac{1}{1-\beta}\left[\frac{\Theta\Psi(\cdot)}{1+\Psi(\cdot)}\right]^{\frac{1}{1-\beta}-1}\left[\frac{q(1-\beta)\Psi(\cdot)}{1+\Psi(\cdot)}\frac{1}{\sigma^2}\right]>0,$$

$$\frac{\partial \hat{k}}{\partial B}=\frac{1}{1-\beta}B^{\frac{1}{1-\beta}-1}\left[\frac{\sigma-1}{\sigma}\frac{q(1-\beta)\Psi(\cdot)}{1+\Psi(\cdot)}\right]^{\frac{1}{1-\beta}}>0,$$

and

$$\frac{\partial \hat{k}}{\partial q}=\frac{1}{1-\beta}q^{\frac{1}{1-\beta}-1}\left[\frac{\sigma-1}{\sigma}\frac{B(1-\beta)\Psi(\cdot)}{1+\Psi(\cdot)}\right]^{\frac{1}{1-\beta}}>0.$$

From (26) we have

$$\frac{\partial \hat{\mu}}{\partial B}=\frac{p}{1-\eta}\hat{k}^{\frac{\beta}{1-\beta}}+\frac{pB}{1-\eta}\frac{\beta}{1-\beta}\hat{k}^{\frac{\beta}{1-\beta}-1}\frac{\partial \hat{k}}{\partial B}>0,$$

$$\frac{\partial \hat{\mu}}{\partial q}=\frac{pB}{1-\eta}\frac{\beta}{1-\beta}\hat{k}^{\frac{\beta}{1-\beta}-1}\frac{\partial \hat{k}}{\partial q}>0,$$

$$\frac{\partial \hat{\mu}}{\partial \sigma}=\frac{pB}{1-\eta}\frac{\beta}{1-\beta}\hat{k}^{\frac{\beta}{1-\beta}-1}\frac{\partial \hat{k}}{\partial \sigma}>0,$$

$$\frac{\partial \hat{\mu}}{\partial p} = \frac{B}{1-\eta}\left[\frac{\Theta\Psi(\cdot)}{1+\Psi(\cdot)}\right]^{\frac{\beta}{1-\beta}} + \frac{pB}{1-\eta}\times$$

$$\frac{\beta}{1-\beta}\left[\frac{\Theta\Psi(\cdot)}{1+\Psi(\cdot)}\right]^{\frac{\beta}{1-\beta}-1} \frac{\Theta\Psi'(\cdot)}{[1+\Psi(\cdot)]^2}\left(-\frac{1-\eta}{p^2}\right), \tag{A11}$$

and

$$\frac{\partial \hat{\mu}}{\partial \eta} = \frac{Bp}{(1-\eta)^2}\left[\frac{\Theta\Psi(\cdot)}{1+\Psi(\cdot)}\right]^{\frac{\beta}{1-\beta}} + \frac{pB}{1-\eta}\times$$

$$\frac{\beta}{1-\beta}\left[\frac{\Theta\Psi(\cdot)}{1+\Psi(\cdot)}\right]^{\frac{\beta}{1-\beta}-1} \frac{\Theta\Psi'(\cdot)}{[1+\Psi(\cdot)]^2}\left(-\frac{1}{p}\right) \tag{A12}$$

After some manipulation, equations (A11) and (A12) can be written as

$$\frac{\partial \hat{\mu}}{\partial p} = \frac{B}{1-\eta}\left[\frac{\Theta\Psi(\cdot)}{1+\Psi(\cdot)}\right]^{\frac{\beta}{1-\beta}}\left[1-\frac{\beta}{1-\beta}\frac{1-\eta}{p}\frac{\Psi'(\cdot)}{\Psi(\cdot)[1+\Psi(\cdot)]}\right], \tag{A13}$$

and

$$\frac{\partial \hat{\mu}}{\partial \eta} = \frac{Bp}{(1-\eta)^2}\left[\frac{\Theta\Psi(\cdot)}{1+\Psi(\cdot)}\right]^{\frac{\beta}{1-\beta}}\left[1-\frac{\beta}{1-\beta}\frac{1-\eta}{p}\frac{\Psi'(\cdot)}{\Psi(\cdot)[1+\Psi(\cdot)]}\right], \tag{A14}$$

respectively. Now consider the expression $\frac{\beta}{1-\beta}\frac{1-\eta}{p}\frac{1}{\hat{x}}$ which, given $\hat{x} = \frac{1-\eta}{p}$, equals $\frac{\beta}{1-\beta} \leq 1$ because $\beta \leq 1/2$ holds. However, we know that $\frac{1}{\hat{x}} > \frac{\Psi'(\hat{x})}{\Psi(\hat{x})} > \frac{\Psi'(\hat{x})}{\Psi(\hat{x})[1+\Psi(\hat{x})]}$ holds by assumption. Taking account of equations (20) and (21), we conclude that $\frac{\partial \hat{\mu}}{\partial p} > 0$ and $\frac{\partial \hat{\mu}}{\partial \eta} > 0$. ■