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Economic Value of Stock and Interest Rate Predictability in the UK

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Abstract

This paper examines asset return predictability by comparing the out-of-sample forecasting performance of both atheoretic and theory informed models of bond and stock returns. We evaluate forecasting performance using standard statistical criterion, together with a less frequently used decision-based criterion. In particular, for an investor seeking to optimally allocate her portfolio between bonds and stocks, we examine the impact parameter uncertainty and predictability in returns have on how the investor optimally allocates. We use a weekly dataset on UK Treasury Bill rates and the FTSE All-Share Index over the period 1997 to 2007. Our results suggest that in the context of investment decision making under an economic value criterion, the investor gains from not only assuming predictability but by modelling the bond and stock returns together.

Keywords: density forecasting, decision-based forecast evaluation, interest rate and stock return models, predictability and parameter uncertainty.
JEL Classifications: C32, C53, E43, E47

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1 Introduction

Evidence of predictability in asset returns has been reported by a number of studies including Campbell (1987), Fama and French (1988a,b and 1989), Kandel and Stambaugh (1996) and Ang and Bekaert (2007). They show variables including the dividend yield and term structure variables, have predictive power for the stock return. This overturning the long standing view held up until the 1970s in financial economics, that returns are not predictable. Most of this evidence is based on studies that assess predictability from a statistical standpoint, using measures like the significance of estimated coefficients, the explanatory power of the regressors and the RMSEs of forecasts.

However, recent research argues that conventional statistical forecast evaluation criteria, usually based on some measure of the forecasts error, may be inappropriate. Instead, it would be more appropriate to evaluate forecast accuracy using profitability, given firms use forecasts to increase profits, Leitch and Tanner (1991). Further, Granger and Pesaran (2000) and Pesaran and Skouras (2004) argue that forecasts should be evaluated in the decision making context for which they are intended. These studies advocate the use of decision-based forecast evaluation[1], where forecasts are judged in terms of their economic value to the user, rather than in terms of forecast errors.

This paper first examines the impact of predictability in bond and stock returns, together with the effect of parameter uncertainty upon how an investor optimally allocates her portfolio. Second, we consider if there is any economic value to the investor of bond and stock return predictability.

Authors including West, Edison and Cho (1993), Pesaran and Timmermann (1995), Xia (2001), Brooks and Persand (2003), Avramov (2002), Boudry and Gray (2003), and Marquering and Verbeek (2004) have previously considered the economic value of predictability in returns within an asset allocation framework. Barberis (2000) considers how asset return predictability affects optimal portfolio choice for long horizon investors, if this allocation differs with the investment horizon and further the impact on allocation when parameter uncertainty[2] is incorporated[3]. Barberis defines no predictability as the investor assuming that stock returns are *i.i.d.* and predictability as him believing that a single lagged dividend yield term has predictive power for stock returns. In both cases bond returns are assumed constant. Predictability has the effect of making stocks look less risky and parameter uncertainty makes them look more so. Barberis demonstrates that the investment horizon may not be irrelevant if returns are predictable. Further, even with parameter uncertainty there is sufficient predictability of returns, such that investors allocate significantly more to stocks the longer the horizon and that those who ignore parameter uncertainty over allocate to stocks by a considerable amount.

[1] We may also refer to economic value measures, these are the same as decision-based measures.

[2] Earlier studies by Klein and Bawa (1976), and Kandel and Stambaugh (1996) demonstrate the importance of parameter uncertainty in asset allocation.

[3] He uses monthly US data for two assets: T-bills and the stock index to examine the potential horizon effects under buy-and-hold and dynamic optimal rebalancing strategies, in discrete time for an investor with power utility over terminal wealth.

Recent studies that examine the predictive power of theory informed models under decision-based criteria for exchange rates include Abhyankar, Sarno and Valente (2005, henceforth ASV) and Garratt and Lee (2009, GL). Both find evidence of economic value to exchange rate predictability, in that the realised terminal wealth of an investor who assumes predictability is higher than that of the investor who assumes no predictability. For interest rates, Della Corte, Sarno and Thornton (2008, DST) assess the validity of the Expectations Hypothesis (EH) of the term structure of interest rates, to find that on the basis of statistical tests the EH is rejected, but from an economic value perspective favourable support is found.

The results reported by ASV, DST and GL illustrate that the forecasting performance of models can be significantly different depending on whether statistical or decision-based evaluation techniques are used. To re-iterate the point made in Hall et al (2010), under statistical measures atheoretic models like the random walk are difficult to beat. But under economic value methods encouraging evidence in favour of predictability, as captured by theory informed models, is found. The studies described here bring to our attention several key factors including the importance of predictability and parameter uncertainty in asset allocation, generating density forecasts to capture the risk as well as the return of the asset and the economic value to the investor of these forecasts.

The contributions of this paper are empirical. To my knowledge we are the first to model both bond and stock returns, separately and jointly, and evaluate their predictability in an asset allocation setting using economic value. Chordia et al (2005, pp. 87) argue that "A negative information shock in stocks often causes a "flight to quality" as investors substitute safe assets for risky assets". Further, "when stocks are expected to show weakness, investment funds often flow to the perceived haven of the bond market, with that shift usually going into reverse when, .., equities start to strengthen." Party (2001, cited in Chorida et al (2005))[4]. Both of these statements highlight the dynamic relationship that exists between bond and stock markets, this supports the need to model them together and try to capture these interactions. That is, allow for the possibility that the variables of one market have explanatory power for the variables of the other.

In brief, we compute the optimal portfolio allocation for a buy-and-hold investor with power utility over terminal wealth using weekly UK data during 1997 week 10 to 2007 week 19 for two assets, the 1-month T-bill and the FTSE All-Share Index. We extend the work of Barberis by allowing for the possibility of predictability in bond returns too and further model the bond and stock returns jointly. Here under predictability the investor assumes past values of the asset returns together with key stock and term structure variables, like the dividend yield and interest rate spreads have explanatory power. We consider a set of four models that assume varying degrees of bond and stock return predictability, all under a VAR framework. We examine the impact predictability and parameter uncertainty have on how the investor optimally allocates her portfolio. Both statistical and decision-based criteria are used to evaluate the out-of-sample forecasting performance of the models, to

[4] Full reference is John Party, The Wall Street Journal, 1st August 2001, pp. C1.

ascertain if indeed there is economic value to bond and stock return predictability.

Our results do suggest that in the context of investment decision making under an economic value criterion, the investor allocates differently when she assumes predictability to an investor who assumes that returns are not predictable. Moreover, she gains from not only assuming predictability in both returns, but by modelling the bond and stock returns jointly.

The setup of this paper is as follows Section 2 details how we model the interest rates and stocks, the investment decision and the framework used to evaluate the economic value of predictability when parameter uncertainty is both ignored and accounted for. Section 3 describes the dataset, the estimated models and provides a statistical evaluation of the forecasting performance of each model. In Section 4 we judge the models' forecasting performance by comparing the realised end-of-period wealth generated under each and Section 5 concludes.

2 Optimal Allocation, Parameter Uncertainty & Predictability

We examine how a utility-maximising investor allocates her portfolio between 1-month T-bills and the FTSE All-Share Index. That is, between the stock market and risk-free bonds. We consider if there are gains in utility for an investor, who employs a theory informed model to forecast interest rates and stock returns, in comparison to one who believes that the returns are not predictable. Here we describe the models estimated when we first ignore T-bill and stock return predictability and then when we consider predictability. Further, we introduce how we measure the economic value of interest rates and stock returns under predictability and parameter uncertainty.

When considering the predictability in interest rates we look to the EH of the term structure of interest rates. It suggests that a n-period long rate is given by a weighted average of current and future expected short m-period rates over n periods with the addition of a time invariant term premium. A further formulation of the EH describes the yield spread by expected changes in the future short rate. If the yields share a common stochastic trend then $(q-1)$ cointegrating vectors should exist, as implied by stationary bivariate spreads, in a set of q non-stationary yields. Assuming that yields are difference stationary and that there exists a cointegrating relationship between the n- and m-period yields, then there exists a Wold representation which can be approximated by a VAR(p) model that describes the change in the m-period rate and the spread between the n- and m-period yields using past changes and spreads, see Campbell and Shiller (1991)[5]. Here we use this VAR model, that embeds the cointegration implied by the EH to explain the term structure and in turn forecast the yields. As such, we proceed assuming that if the

[5] Numerous tests of the EH have been carried out using various datasets and testing methods, with the evidence in support of the EH being somewhat mixed, see Campbell and Shiller (1991); Taylor (1992); Cuthbertson (1996); Cuthbertson, Hayes & Nitzsche (1996, 2003); Longstaff (2000); Sarno, Thornton and Valente (2007).

investor believes bill returns are predictable she uses past yield changes and spreads to forecast future returns. For the stock returns we follow previous studies including Kandel and Stambaugh (1996) and Barberis (2000), and use the dividend yield to examine stock return predictability. Such that if the investor believes stock returns are predictable she uses dividend yields to forecast future returns.

2.1 Modelling Interest Rates and Stocks

Let r_t^s be the return on the FTSE All-Share Index in week t, $r_t^{(1)}$ be the return on a 1-month T-bill, both returns are continuously compounded monthly returns. dy_t is the dividend yield, the change in the 1-month T-bill rate $\Delta r_t^{(1)} = r_t^{(1)} - r_{t-1}^{(1)}$ and the spread between a n- and 1-month rate $s_t^{(n,1)} = r_t^{(n)} - r_t^{(1)}$ for $n = 3, 6, 12$. We refer to r_t^s and dy_t as the stock variables, and $\Delta r_t^{(1)}$ and $s_t^{(n,1)}$ as the bond (or term structure, TS) variables. In order to determine how the investor should optimally allocate her portfolio she requires forecasts of $r_t^{(1)}$ and r_t^s. We consider four alternative models from which the investor could derive these forecasts, generally each model can be summarised by the following VAR(p)

$$\mathbf{x}_t = \mu + \sum_{i=1}^{p} \mathbf{B}_i \mathbf{x}_{t-i} + \epsilon_t \tag{1}$$

where $\mathbf{x}_t$ is a $(q \times 1)$ vector of variables, $\mathbf{B}_i$ is a $(q \times q)$ matrix of parameters, β is a $(q \times 1)$ vector of intercepts and ϵ_t is assumed to be a $(q \times 1)$ vector containing *i.i.d* serially uncorrelated errors with zero means and a positive definite covariance matrix $\mathbf{\Sigma}$. The exact composition of $\mathbf{x}_t$ will depend upon the assumption made regarding predictability, as detailed below.

The VAR framework enables one to examine how predictability affects portfolio allocation by changing the variables in the VAR. We propose four models for predicting the returns on the T-bill and stock index, each incorporating varying degrees of predictability: Barberis Non Predictability (BNP), Barberis Predictability (BP), Individual VARs (IV) and the Joint VAR (JV) model.

The Barberis Non Predictability and Predictability models are named so, since they are in the spirit of those estimated by Barberis (2000). These models assume that the risk-free T-bill rate $r^{(1)}$ is constant[6] and allow only for the possibility of predictability in stock returns. Under the assumption of no predictability as in the BNP model, there are no predictor variables in the VAR, the stock index returns are assumed to be *i.i.d.* such that $r_t^s = \mu + \epsilon_t$, i.e. a drift term plus a random error term. Hence $\mathbf{x}_t = r_t^s$ and $\mathbf{B}_i = 0$.

However, under the assumption of predictability as in the BP model, the dividend yield is included in the VAR, with $\mathbf{x}_t = (r_t^s, z_t')'$, $z_t = (z_{1,t}, ..., z_{n,t})'$ and $\mathbf{x}_t = \mu + \mathbf{B}\mathbf{z}_{t-1} + \epsilon_t$. Such that z_t is a vector containing explanatory variables for the stock index return, i.e. the dividend yield. Hence the first equation of the VAR specifies the expected stock index return as a function of the dividend yield, and the second equation specifies the stochastic evolution of the dividend yield.

[6] The T-bill rate is assumed constant at the last value of the estimation sample, such that in the first recursion it is fixed at its 2004 week 18 value.

Further, it is possible to relax this assumption of a constant T-bill rate and allow for predictability in both T-bill and stock returns, we do this in two ways. First, using the IV model, where the predictability of T-bill and stock returns are described separately by two VARs (IV-BOND and IV-STOCK). The form of $\mathbf{x}_t$ for the bond returns and the stock returns are given by $\mathbf{x}_t^{(1)} = \left(\Delta r_t^{(1)}, s_t^{(12,1)}, s_t^{(6,1)}, s_t^{(3,1)}\right)'$ and $\mathbf{x}_t^s = (r_t^s, dy_t)'$ respectively. Second, using the JV model, where the predictability of the bill and stock returns are modelled jointly within a single system, here $\mathbf{x}_t = \left(r_t^s, dy_t, \Delta r_t^{(1)}, s_t^{(12,1)}, s_t^{(6,1)}, s_t^{(3,1)}\right)'$.

By modelling the predictability of T-bill and stock returns in these two ways allows us to test whether it is beneficial to the investor, in terms of wealth gains, to model the two returns jointly. In that, by allowing for interactions and feedbacks to exist between the bond and stock market, will the investor who uses the JV model to generate forecasts of the T-bill rate and the return on stocks achieve a higher wealth? Each of these four models are estimated when the parameter uncertainty, which is the uncertainty about the true values of the model's parameters, is both ignored and accounted for[7].

In time T the buy-and-hold investor faces the problem of how to optimally allocate her wealth over a H month investment horizon between 1-month T-bills and the FTSE All-Share Index, where these two assets yield the continuously compounded returns $r_T^{(1)}$ and r_T^s respectively.

With an initial wealth of $W_T = 1$ and ω being defined as the proportion of initial wealth allocated to bonds[8], the end-of-horizon wealth is given by

$$W_{T+H} = \omega \exp\left(\sum_{i=1}^{H} r_{T+i-1}^{(1)}\right) + (1-\omega)\exp\left(\sum_{i=1}^{H} r_{T+(i-1)}^{s}\right) \tag{2}$$

Further, risk aversion can be incorporated into the investor's decision making, by assuming that the utility gained from the end-of-horizon wealth follows that given by a constant relative risk-aversion (CRRA) power utility function.

$$\upsilon(W) = \frac{W^{1-A}}{1-A} \tag{3}$$

where A is the coefficient of risk aversion. The optimisation problem faced by the investor in T is

$$\max_{\omega} E_T \left\{\upsilon(W_{T+H}(\omega)) \mid \Omega_T\right\} \tag{4}$$

where the investor computes the expectation above conditional upon the information set available at T. Fundamental to this optimisation problem is the distribution the investor employs to evaluate this expectation. The distribution used depends upon whether the investor assumes predictability in bond and stock returns. To ascertain the influence of predictability on allocation decisions, a comparison between the allocations of an investor

[7] We differentiate between when the model is estimated subject to stochastic uncertainty only, and when it is estimated subject to stochastic and parameter uncertainty by denoting them as BNP, BP, IV, JV and BNPPU, BPPU, IVPU, JVPU respectively.

[8] Under the BNP and BP models the T-bill return is assumed to be constant, such that $\sum_{i=1}^{H} r_{T+(i-1)}^{(1)} = H.\bar{r}^{(1)}$.

who ignores predictability, to that of one who takes it into account can be made. This will now be discussed in greater detail below.

2.2 The Predictive Density Function

In this section we discuss the approach taken to estimate the density function in the case where parameter uncertainty is not considered and when it is. The form of the density $P(\mathbf{X}_{T+1,H} \mid \mathbf{X}_T)$ is determined by the types of uncertainty surrounding the forecasts, and how the function is characterised and estimated. Here we follow the method proposed by Garratt, Lee, Pesaran and Shin (2003 and 2006, GLPS) and GL, which takes a classical view of the Bayesian approach[9] to calculating the density function. This involves approximations of certain probabilities of interest, thereby avoiding the need for priors. We will now provide a summary of the methods described in GLPS and GL.

To evaluate each investment decision over the investment horizon, the investor needs the probability density function of the forecast values of the 1-month rate and the stock return. Following GL, $\mathbf{x}_t = (x_{1t}, x_{2t}, ..., x_{qt})'$ is a $q \times 1$ vector of q variables (including at least $r_t^{(1)}$ and r_t^s), and $\mathbf{X}_T = (\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_T)'$ is a $q \times T$ vector containing the observations 1 to T of the q variables. Since forecasts of the variables are required, the conditional probability density function $P(\mathbf{X}_{T+1,H} \mid \mathbf{X}_T)$ is of interest, this predictive density function gives the probability density function of $\mathbf{X}_{T+1,H} = (\mathbf{x}_{T+1}, \mathbf{x}_{T+2}, ..., \mathbf{x}_{T+H})'$ conditional on $\mathbf{X}_T$.

When the investor ignores parameter uncertainty, she calculates the expectation of the distribution of returns conditional on the fixed parameter values $\widehat{\theta}$. So the investor's problem to solve is

$$\max_{\omega} \left\{ E_T \upsilon \left(W_{T+H}(\omega) \right) = \int \upsilon \left(W_{T+H}(\omega) \right) .P \left(\mathbf{X}_{T+1,H} \mid \mathbf{X}_T, \widehat{\theta} \right) d\mathbf{X}_{T+1,H} \right\} \tag{5}$$

However, if the investor incorporates parameter uncertainty then the predictive density for the returns is conditional on the observed data only, given by

$$P(\mathbf{X}_{T+1,H} \mid \mathbf{X}_T) = \int P \left(\mathbf{X}_{T+1,H} \mid \mathbf{X}_T, \widehat{\theta} \right) P(\theta \mid \mathbf{X}_T) \, d\theta \tag{6}$$

The posterior probability of θ, denoted $P(\theta \mid \mathbf{X}_T)$ gives the uncertainty about the parameters given the observed data. Now the investor acknowledges that θ has a distribution conditional on $\mathbf{X}_T$. So the investor's problem to solve under parameter uncertainty is

$$\max_{\omega} \left\{ E_T \upsilon \left(W_{T+H}(\omega) \right) = \int \upsilon \left(W_{T+H}(\omega) \right) .P(\mathbf{X}_{T+1,H} \mid \mathbf{X}_T) \, d\mathbf{X}_{T+1,H} \right\} \tag{7}$$

The posterior density $P(\theta \mid \mathbf{X}_T)$ in equation (6) is proportionate to the prior on θ and the likelihood function i.e. $P(\theta).P(\mathbf{X}_T \mid \theta)$. GLPS and GL suggest that in the case where meaningful priors exist are difficult to obtain, approximations of key probabilities

[9] Kandel and Stambaugh (1996), Barberis and ASV use a fully Bayesian approach to estimate the density function, through the construction of a posterior distribution and using priors for the parameters.

needed to estimate the predictive density $P\left(\mathbf{X}_{T+1,H} \mid \mathbf{X}_T\right)$ can be used. They assume for the posterior probability of θ

$$\theta \mid \mathbf{X}_T \overset{\omega}{\sim} N\left(\widehat{\theta}_T, T^{-1}\widehat{\mathbf{V}}_\theta\right) \tag{8}$$

where $\widehat{\theta}_T$ is the maximum likelihood estimate of the true parameter value of θ, and $T^{-1}\widehat{\mathbf{V}}_\theta$ is the asymtotic covariance matrix of $\widehat{\theta}_T$ i.e. of the estimated parameters.

In this exercise we consider stochastic and parameter uncertainty, the uncertainty associated with the model and the estimated model parameters respectively. We appreciate that interest rates and stock returns can be modelled under various assumptions, and thus model the two returns in four different ways: BNP, BP, IV and JV models as described above, which can all be summarised by equation (1).

For each of these models, through stochastic simulation techniques, an estimate of the probability density function of the forecasts can be computed. Given that these simulations provide an estimate of the predictive densities $P\left(\mathbf{X}_{T+1,H} \mid \mathbf{X}_T, \widehat{\theta}\right)$ when parameter uncertainty is ignored and $P\left(\mathbf{X}_{T+1,H} \mid \mathbf{X}_T\right)$ when it is considered, it is now possible to evaluate $E_T\left(\upsilon\left(W_{T+H}\right) \mid \Omega_T\right)$ for a range of portfolio weights ω. That is, $\upsilon\left(W_{T+H}\left(\omega\right)\right)$ is computed $\widetilde{R}$ times for each value of ω. Then the mean across these $\widetilde{R}$ replications is calculated, from which the investor chooses the weight ω that maximises the expected utility $E_T\upsilon\left(W_{T+H}\left(\omega\right)\right)$. Here ω takes values 0, 0.01,...,0.99,1, where $\omega = 0$ suggests all should be allocated to bills, equally $\omega = 1$ suggests that all should be allocated to stocks. The weight is between 0 and 1, so we do not allow for short selling. Details of the estimation procedure, how the computations are carried out and the method by which the errors are calculated [10] are provided in Appendix A.

3 Modelling the UK Treasury Bill Rates and the FTSE All-Share Index

3.1 Returns Data

In this study we use weekly observations on the continuously compounded monthly returns for both the 1-month T-bill[11] $r_t^{(1)}$ and the FTSE All-Share Index[12] r_t^s, and the dividend yield dy_t for the UK. These variables together with $\Delta r_t^{(1)}$, $s_t^{(3,1)}$, $s_t^{(6,1)}$ and $s_t^{(12,1)}$ are used in the analysis, refer to the Data Appendix for the definitions, sources and transformations conducted. The entire sample period is from 1997 week 10 to 2007 week 19 (532 observations). Figures 1 and 2 plot the monthly stock return, the dividend yield, the monthly bill return in levels and first differences, and the three spreads over the entire sample. The

[10] The errors can be drawn using either parametric or non-parametric methods (see GLPS (2006) p166-168), here parameteric methods are utilised where the errors are assumed to be $i.i.dN\left(0, \mathbf{\Sigma}\right)$ serially uncorrelated white noise errors.

[11] The estimated yield curve data is used as opposed to actual T-bill data here, because data was unavailable during some periods of our sample. However, we are satisfied that the data used here is a fair reflection of what the investor would get, should she want to undertake an investment in T-bills.

[12] We use the FTSE All-Share Index since it gives a broad portfolio of stocks.

monthly stock return takes an average value of 0.59% compared with 0.41% for the T-bill, with a minimum and maximum of -17.72 to 15.32% and 0.26 to 0.60% respectively over the whole sample. This corresponds to what we would expect, average returns from the stock market tend to be higher, but there is a risk of making a loss. The return from the 1-month T-bill has a general downward trend up until the end of 2004, before increasing until the end of the sample. The annual dividend yield takes an average value of 2.86%, although there are some persistent deviations, the dividend yield exhibits mean reversion. The yield difference and spreads display mean reverting behaviour which is consistent with a stationary process.

The four models are each estimated over the period 1997 week 10 to 2004 week 18 (374 observations) and then recursively at weekly intervals through to 1997 week 10 to 2005 week 18 (427 observations), giving 54 recursions in total. For each recursion we generate h-step ahead out-of-sample forecasts[13] for $h = 1, 2, ..., H, ...$ and the investment horizon $H = 3, 6, 12, 18$ and 24 months. So for the first recursion we forecast over the period 2004 week 19 to 2006 week 18 and for the last recursion 2005 week 19 to 2007 week 19. For each recursion the investor will use his generated forecasts to determine the optimal allocation of his portfolio. Hence in this exercise we will have 54 allocation decisions for each A and H, with which to compare the allocations and utility gains under each model without and with parameter uncertainty.

3.2 Estimating the Models

Here we describe how we estimate the four models and present the estimated regression results for the first recursion[14] over 1997 week 10 to 2004 week 18. We begin by employing the ADF, PP and KPSS unit root tests to determine the order of integration of r_t^s, dy_t, $r_t^{(1)}$, $s_t^{(3,1)}$, $s_t^{(6,1)}$ and $s_t^{(12,1)}$ over the entire sample period, see Table 1. All three tests indicate that r_t^s and the spreads are found to be stationary in levels and $r_t^{(1)}$ is difference stationary. As for dy_t the unit root tests suggest it is non-stationary, but given the test statistics are close to their respective critical values and the series exhibits mean reversion we treat, like in previous studies, the dividend yield as stationary.

The optimal lag length for the IV and JV models is chosen by estimating a set of VAR(p) with $p = 0, 1, ..., 12$ for each model over 1997 week 10 to 2004 week 18. The optimal lag length is that which minimises the Schwarz Information lag selection criteria, as well as satisfying the diagnostic checks, in particular the model's residuals should be free of serial correlation at the 5% level. Based on this, the lag length chosen was five for the IV-STOCK model, six for the IV-BOND and JV models. Tables 2 to 8 summarise the estimates with the diagnostics of the BNP, BP, IV and JV models.

[13] We denote the investment horizon H in months since $r_t^{(1)}$ and r_t^s are monthly returns. However, the data has a weekly frequency, so when we refer to the 'h-step' ahead forecasts each 'step' is a week.

[14] Estimates of each model for the first recursion only are provided, to give an overall impression of the in-sample predictability. At the forecasting stage the models are estimated recursively.

Comparing the estimated BP model to the BNP model, Table 2 to 3, there is a small gain in explanatory power by allowing for predictability in stock returns through the inclusion of a single lagged dividend yield term. Further, all coefficients in the estimated BP model are significantly different from zero. Moving from the BP to the IV-STOCK model, Table 3 to 4, allows for past values of both r^s and dy to influence current values. A substantial gain in explanatory power for stock returns is observed. All the coefficients are jointly significant, which suggests there are gains from relaxing the assumptions of no and limited predictability made under the BNP and BP models. For each equation in the IV-BOND model, Tables 5 and 6, the TS[15] variables are jointly significant. The JV model, Table 7 and 8, is a generalisation of the individual VARs, allowing for feedbacks between the two markets. In terms of explanatory power as indicated by $\overline{R}^2$, the gains from modelling the two returns together are small. However, the stock variables are jointly significant in all the TS equations, but the TS variables are jointly significant in the TS equations only. This implies that causality exists from the stock market variables to the TS variables, which provides support in favour of modelling the two markets together.

The diagnostics are satisfactory, there is indication of some serial correlation in the stock equations of the BNP and BP models, but we want to replicate those estimated in Barberis. In the IV and JV models we do not have serial correlation at the 5% level and the explanatory power of the models is quite high. Rejection of the nulls that the regression residuals are homoskedastic and normal is not surprising given that we are using financial data. But we follow the assumptions made by the literature that also utilise such data.

3.3 Statistical Forecast Evaluation

The root mean squared error (RMSE) provides a statistical evaluation of the out-of-sample forecasting performance of each model. Table 9 gives the RMSEs of the bond and stock return forecasts, for the forecast horizons $H = 1, 3, 6, 12, 18$ and 24 months for each model, without and with parameter uncertainty being considered. Table 10 reports the ratio of the RMSEs for each model to the benchmark model. A value of the ratio greater than one indicates that the RMSE of the model is lower than that of the benchmark. The benchmark taken is the BNP model which assumes $r_t^{(1)}$ is constant and $r_t^s = \mu + \epsilon_t$, since it assumes no predictability a comparison can be made with the other models which assume varying degrees of predictability.

The RMSEs for forecasts of the bond returns indicate that only at $H = 1$ do the JV and JVPU models beat the benchmark. The BNP, BP, BNPPU and BPPU models that make the strong assumption that $r_{T+H}^{(1)}$ is constant, outperform the other more theory informed models at each horizon under this criteria. However, it can be seen that the differences in the RMSEs amongst the models are small. These results broadly correspond to those found in the exchange rate forecasting literature, as summarised in ASV and GL. Which in general find sophisticated theory informed models are outperformed by a simple

[15] TS is used to denote the term structure. The TS variables are $\Delta r_t^{(1)}, s_t^{(3,1)}, s_t^{(6,1)}$ and $s_t^{(12,1)}$.

random walk.

With the stock returns, the RMSEs show that there is not a single model that performs consistently well over all horizons. The JV and JVPU models perform the best at $H = 1, 3$ and 12, whereas the BP and BPPU models perform well at $H = 6, 18$ and 24. These results suggest some gain in terms of forecasting performance from incorporating predictability when modelling stock returns.

When comparing the size of the RMSEs of the two returns, there is greater variance in the r_t^s forecasts than the $r_t^{(1)}$ forecasts. This is not surprising since stock returns are more volatile and thus more difficult to predict. In general, the RMSEs increase up until $H = 6$ and 12 before decreasing. This suggests that the RMSEs for both the returns are non-monotonic, i.e. they oscillate in relative value and do not just increase with H. Although the RMSEs for both the returns are non-monotonic, the rates at which the two are changing across the horizons are different. Over the shorter horizon, the rate at which the RMSEs for $r_t^{(1)}$ increase is smaller than the rate at which the RMSE for r_t^s increases. But over the longer horizon the rate at which the RMSE for $r_t^{(1)}$ decreases is greater.

This statistical evaluation provides an indication of the forecasting performance of each model. But does not provide a clear indication of how these models perform in an investment decision making context, i.e. in terms of the economic value of the gains from the models' forecasts.

4 Effects on Allocation

We now examine the implications for optimal allocations when the returns are either *i.i.d.* or predictable, where the degree of predictability is varied and parameter uncertainty is both ignored and accounted for. In the case where parameters are assumed fixed the maximisation problem is given by equation (5) and under parameter uncertainty it is given by (7). Figures 3 to 7 give the optimal allocations to bonds, $100\omega\%$, at each investment horizon $H = 3, 6, 12, 18, 24$ months, for each model and for the levels of risk aversion $A = 2, 5$ and 10, $A = 10$ is the highest level of risk aversion. The models are estimated first over 1997 week 10 to 2004 week 18, the optimal weights are calculated from the forecasts generated from each estimated model. Then moving forward one week this is repeated, re-calculating expected wealth and utility to find the optimal weight for this new augmented sample. This is repeated for each recursion, giving results for 54 recursions over the total evaluation period 2004 week 19 to 2007 week 19. The plots are based on the optimal allocation averaged over the 54 recursions for a particular A, H and model.

Figure 3 gives the optimal allocation under each model, when parameter uncertainty is ignored, here allocations are conditional on the fixed parameter values estimated. A risk aversion effect is evident for all the models, where the investor allocates more to bonds at all horizons the more risk averse she is. Further, under the BP, IV and JV models the difference in the allocation to bonds under each A increases with H, with differences of up to 65% being observed for an investor with $A = 2$ compared with $A = 10$. This suggests that the allocation to bonds for a longer horizon investor greatly depends on how risk averse they are.

It can be seen that the investment horizon is also important in determining how the investor allocates. In the absence of horizon effects, the short horizon investor allocates no differently than a long horizon investor. With horizon effects there is a difference between the allocations of a short and long horizon investor, such that the 'allocation curve' which we define as describing for a particular A how the investor allocates over H, has a slope. Further, this curve may have a positive or negative slope, if the slope is positive then the investor allocates more to bonds as H increases. Here strong horizon effects are present under all models. In general, we find as H increases under the BNP and BP models the investor allocates more to bonds for all A. This is true for $A = 5$ and 10 under the IV model, but for $A = 2$ the allocation to stocks increases with H. Equally, under the JV model for $A = 10$ the investor increases her allocation to bonds with H, for $A = 5$ she increases the allocation to stocks over the medium horizon before increasing the allocation to bonds in the longer horizon, whereas with $A = 2$ the investor increase her allocation to stocks with H.

In short, horizon effects are present. But the extent of the effect the investment horizon has on the allocation depends upon the predictability assumptions the investor makes. That is, which model she believes to be true and her level of risk aversion.

We will now try and provide an explanation for these allocation results by first considering the effects of predictability (ignoring parameter uncertainty) and then the effects of parameter uncertainty.

4.1 Predictability Effects

In this exercise we consider four different models for forecasting interest rates and stock returns. The atheoretic BNP and BP models assume no predictability in regard to bond returns. Further, the BNP model assumes no variables are able to predict the stock return. However, the BP model relaxes this assumption allowing for some predictability in stock returns. On the opposite end of the spectrum, the theory informed IV and JV models not only assume predictability, but as in the case of the JV model allow for the possibility of feedbacks amongst the stock and term structure variables.

These models reflect opposing views of whether bond and stock returns are predictable, and further have a varying degree of predictability which increases as we move from the BNP to BP to IV to JV model. If the investor assumes no predictability then she believes in the BNP model. Conversely, if she assumes predictability she may believe in the BP, IV or JV model depending on the extent of the predictability assumed. Ultimately, how the investor allocates is determined by which model she believes to be a true depiction of reality.

From Figure 3 it can be seen that the BNP model allocates the most to bonds, followed by the BP, the IV and then the JV model at each A and H. Where the JV model allocates the most to stocks. The difference in allocation to bonds in some cases is over 70% amongst the models, e.g. $H = 24$ and $A = 2$ the BNP model allocates 77% more to bonds than the JV model.

Under no predictability, which is similar to assuming the stock returns follow a random walk process, the variance of the cumulative log returns distribution $\sigma^2 \to \infty$, i.e. the variance continues to grow with the horizon. Whereas, when the return is modelled as a stationary process, as is the case under predictability, then $\sigma^2 \to$ *long run mean* i.e. mean reversion of the variance of returns. In which case, stocks appear less risky in the long run and are more attractive to long horizon investors, Fama and French (1988).

Under the BNP model we find horizon effects, where the investor allocates more to bonds as H increases. Under the assumption that log returns are independently and identically normally distributed (assumption of normality is not necessary for this to hold) the mean and variance of the cumulative log returns distribution grows proportionally with the investment horizon[16] i.e. $H\mu$ and $H\sigma^2$. For the risk averse investor with power utility function, although return per unit of variance is the same as H increases, the higher return is coupled with higher risk in absolute terms and since the investor is risk averse she allocates less to stocks as H increases.

With predictability the investor recognises that rather than the returns being *i.i.d.* they may be predictable, as is the case under the BP, IV and JV models. Now returns are no longer independent, but the distribution of future returns is conditional on the current and past values of the explanatory variables. In which case the mean and the variance of the returns no longer grow linearly. Barberis highlights that under predictability the variance of cumulative log stock returns may grow slower than linearly with H, such that stocks appear comparatively less risky at longer horizons, resulting in higher allocations to stocks as H increase.

With the BP model however, we find that it is the allocation to bonds that increases with H. A possible explanation for this is that although we are now incorporating predictability the gain in terms of explanatory power for stocks returns are small, $\overline{R}^2$ increases from 0% under the BNP model to just over 2% under the BP model, so the increase in predictability is not sufficient for the investor to increase her allocation to stocks with the horizon.

The bond returns are also modelled[17] under the IV and JV models. So now both returns will be subject to future uncertainty and ultimately the optimal allocation hinges on how risky bonds look relative to stocks. With the IV model the investor allocates more to stocks at all horizons than the BNP and BP models, i.e. allocation curve shifts down for all A. This can be attributed to two factors, firstly bond returns now look relatively

[16] $r_{t,t+H} = r_{t+1} + r_{t+2} + ... + r_{t+H} \Longrightarrow E(r_{t,t+H}) = E(r_{t+1}) + E(r_{t+2}) + ... + E(r_{t+H}) = H\mu$, where each return has the same mean (identically distributed) and returns are independent in that one return does not contain information about the other returns. Further, $var(r_{t,t+H}) = var(r_{t+1}) + var(r_{t+2}) + ... + var(r_{t+H}) = H\sigma^2$, where the returns are uncorrelated so there is no covariance term and all the variances are equal (identically distributed).

[17] Note when bond returns are modelled too, the variance of cumulative log bond returns may also grow less than linearly with H. So now bond and stock returns may both be subject to these predictability effects.

more risky than they did under the BNP and BP models since the return is no longer known with certainty. Secondly, stock predictability under the IV model has increased dramatically, from 2% under the BP model to nearly 70%. Both of these factors make stocks look more attractive.

Predictability increases further under the JV model, we expect an increase in the allocation to the asset that has gained most from the increase in predictability. An increase in the allocation to stocks at each H in comparison to the IV model is observed. Thus stock returns appear to have gained more from modelling the returns jointly, so that they appear less risky and the investor is more willing to hold them. For $A = 2$ stock return predictability dominates as the investor increases the amount allocated to stocks as H increases. For $A = 5$ stock return predictability dominates until $H = 12$, then bond return predictability dominates such that the investor allocates more to bonds. For $A = 10$ bond return predictability dominates as the investor increases allocation to bonds with H.

Under the varying degrees of predictability that each model assumes, how the increased predictability alters the optimal allocation depends, firstly on which return (bond or stock) benefits more from the predictability effect[18]. Secondly, how risk averse the investor is. As we move from the BNP to JV model the investor allocates more to stocks at each H, so the allocation curves shifts down. This could be because the investor is able to predict stocks better as we move from the BNP to the JV model, so she is prepared to allocate more to stocks at every horizon for each A. But most evidently for $A = 2$, when moving from BNP through to JV the slope of the allocation curve changes. For the IV and JV models the investor is prepared to allocate substantially more to stocks at longer horizons, which could be attributed to σ^2 growing less than linearly combined with the investor not being very risk averse. Whereas for $A = 10$ the investor is very risk averse and increases her allocation to bonds with H.

4.2 Parameter Uncertainty Effects

Figures 4 to 7 compare the allocations under each model when parameter uncertainty is ignored to that when it is considered. Incorporating parameter uncertainty has the effect of increasing the variance of the distribution of cumulative returns. Further, the variance increases faster than linearly with H in the case of *i.i.d.* returns, when this additional uncertainty is accounted for. This increase in the variance serves to make the asset seem riskier at longer horizons.

When the investor believes in the BNP model we indeed find that the allocation to stocks is reduced by 0 to 2% with parameter uncertainty, the effects are small over the horizons considered. For the BP model this additional uncertainty increases the allocation to bonds by up to 7%, with the effect of parameter uncertainty decreasing as the investor becomes more risk averse.

[18] The predictability effect results in the variance of cumulative log returns to grow less than linearly, making the asset appear less risky at longer horizons.

Under the IV and JV models the bond returns are also being modelled, such that they too are subject to parameter uncertainty. Now bonds look riskier than they did under the BNP and BP models, so the optimal allocation hinges on which asset is affected by parameter uncertainty more and hence the riskiness of bonds relative to stocks.

Parameter uncertainty under the IV model has the effect of increasing the allocation to stocks by 3 to 10% in the short to medium horizon for $A = 2$ and 5, the increase is smaller for $A = 10$, before the allocation to bonds increases in the longer horizon to levels similar to those when parameter uncertainty is ignored. Here we find that the impact of this uncertainty is different for each A, where the more risk averse the investor is, the less willing she is to hold more stocks. Allocations emerge as being non-monotonic over H, because the investor does not simply increase her allocation to stocks with the horizon, but the slope of the curve actually changes over H. Over the short to medium horizon it appears that the effect of parameter uncertainty is greater on bond returns than stock returns. That is, the variance of the cumulative stock returns is less than that of bonds, $\sigma^2_{rs} < \sigma^2_{r1}$, making stocks look less risky and more being allocated to them. But over the longer horizon the converse seems true, such that stocks look riskier and the optimal allocation is equal to that when parameter uncertainty is ignored.

The effect of parameter uncertainty is most apparent under the JV model, with allocations to stocks increasing by up to 4% for $A = 2$, and by the same margin for $A = 5$ over the short to medium horizon before the allocation to bonds increases over the longer horizon by 9 to 13%. The changes in allocation to bonds for $A = 10$ over the investment horizon are similar to those observed for $A = 5$, but of a smaller magnitude. Again allocations are non-monotonic for $A = 5$ and 10, in that after $H = 12$ the parameter uncertainty risk is less for bonds, thus making them appear more attractive.

To explain the non-monotonic allocations that arise under parameter uncertainty, we consider how the variances about the distribution of future predicted returns evolve over the forecast horizon. In this case it is reasonable to expect the RMSEs and the variances to be closely related, such thatwe use the RMSEs as an indication of how the variances of the forecasts evolve[19]. Recall Tables 9 and 10, the non-monotonic RMSEs imply that the variances of the forecasts are also non-monotonic[20]. This suggests that the variance about the forecasts contracts and expands with H, so under parameter uncertainty the asset will appear more risky at some horizons than at others. Further, the variances of the two returns oscillate at different rates, such that the effects of parameter uncertainty will be different at different H, so at some horizons stocks will appear more risky than bonds and at others less. This non-monotonicity combined with the fact that the variances of the two returns expand and contract at different rates could provide an explanation for the impact of parameter uncertainty observed here.

[19] Since the σ^2 gives the dispersion about the mean of the distribution and the RMSE measures the dispersion about the actual value of a variable. Then the mean of the distribution will equal the actual value if the distribution is unbiased, thus the RMSE will equal the σ^2 of the forecast.

[20] Which as Hall and Hendry (1988, pp. 256-7) argue may not be so surprising, since non-monotonic model standard errors may result in non-monotonic total standard errors.

We can see that as the investor becomes more risk averse, she is less prepared to allocate more to stocks when parameter uncertainty is incorporated. Further, she is prepared to allocate more to stocks under parameter uncertainty over the short to medium horizon, but not at the longer horizons. Boudry and Gray (2003, BG) also find "negative horizon effects", where the investor allocates more to bonds at longer horizons. This is contrary to Barberis, who finds that parameter uncertainty reduces not eliminates the positive horizon effects. BG argue that their model contains more predictor variables that require estimating than Barberis', which introduces a significant degree of parameter uncertainty. Thus the perceived riskiness of stocks grows faster than linearly with H and allocation to stocks decreases. Further, they state that this negative horizon effect may be intensified by the fact that the investment is buy-and-hold, whereby the consequence of inaccurately judging the level of predictability is more severe when the investor is locked-in for long horizons.

In short, predictability has the effect of making the assets appear less risky at longer H, while parameter uncertainty makes the asset look more risky. The final allocation depends on which effect dominates for that asset. Additionally, since we consider two assets-bonds and stocks, which of the two emerges as the less riskier.

4.3 Economic Evaluation of Forecasts

The RMSE is a statistical measure of forecast accuracy, here we focus on assessing forecast performance using the economic value to an investor. An economic evaluation of the forecast performance of each model is reported in Tables 11 to 13 [21]. We compute the end-of-period wealth that the risk averse investor would have achieved over 2004 week 19 to 2007 week 19 had she allocated her portfolio as suggested by the optimal weights of each model for a particular A and H. The optimal weight ω is calculated by solving the utility maximisation problem[22]. These realised wealths are averaged over 54 recursions and then ranked in descending order so the performance of each model can be compared.

Apart from the four models described above, under which we both ignore parameter uncertainty and incorporate it to derive the optimal allocations, we also introduce three passive 'lazy' strategies. Under the lazy strategies the investor makes no attempt to model or predict the returns, but instead either invests (1) all in bonds (AB), (2) all in stocks (AS) or (3) half in bonds and half in stocks (HH). The top position is always occupied by the lazy 'all in stocks' strategy. Although it should be noted that during the forecast horizon 2004 week 19 to 2007 week 19 over which this evaluation of the models is made, the UK stock market was buoyant which explains the success of this strategy here. Hence during times of market growth investing 'all in bonds' would yield the lowest realised wealth. Looking to positions 2 to 10, the success of the JV models (without and with parameter uncertainty) is clear, with it occupying 2nd and 3rd place for almost all

[21] Like ASV and GL our measure of economic value is based on wealth.

[22] The optimal weight is determined by the forecasts from the model. These weights are then combined with actual/realised returns to give the realised end-of-horizon wealth.

A and H. The IV models come mostly 4th and 5th, followed by the BP models and then the BNP models.

What emerges from these results is that the success of the model in terms of profitability appears to be closely related to the level of predictability the investor assumes. Whereby the more theory informed IV and JV models consistently outperform the more restricted BP models and the atheoretic BNP models. Broadly speaking these results are not sensitive to the investment horizon or level of risk aversion. This provides evidence not only in favour predictability, but of modelling the two returns jointly as under the JV models rather than separately, when we use economic value as a means to evaluate forecasts.

5 Conclusion

For a utility maximising investor, we compare how the optimal allocations differ under a set of atheoretic and theory informed models, and how it differs when the investor incorporates parameter uncertainty to when she ignores it. Further, we evaluate the economic value of the out-of-sample forecasts of bond and stock returns generated under each of these models. The investment decision is whether to invest in bonds or stocks, this is examined in a framework that both ignores parameter uncertainty and explicitly allows for it.

The key innovation here is that we model both returns by using the EH to model the interest rate and the dividend yield to model the stock returns, and then evaluate interest rate and stock predictability in an economic value framework. Under the assumption of bond return predictability, we first model the bond and stock returns separately with their predictor variables. Then secondly model the two returns jointly with all the predictor variables. This joint modelling framework allows for the possibility of stock variables to influence the term structure variables and vice versa. Over the sample investigated here we find evidence to suggest that an investor seeking to optimally allocate her wealth between UK bonds and stocks is better off, in terms of higher end-of-horizon wealth, by assuming predictability in returns and further modelling both returns together, than an investor who assumes no predictability.

We find the effect of predictability on the optimal allocation is considerable, where the optimal weights under predictability of returns are in some cases greatly different to those under no predictability. In particular, the predictability in the bond and stock returns led to more being allocated to stocks at each horizon, and under the IV and JV models for $A = 2$ the investor increases the allocation to stocks with the horizon. These findings lend support to the predictive ability of the stock and term structure models considered here, and to modelling both returns jointly. The effect of parameter uncertainty is not large over the investment horizon considered here. Although Barberis reports significant effects of parameter uncertainty on the optimal allocation these are prominent at longer horizons, he considers horizons up to 10 years. At our comparatively shorter horizons of up to 2 years, the magnitude of the impact is of similar proportions to those reported by

Barberis.

Using a statistical evaluation criterion i.e. RMSEs, the BNP and BP models outperform the models that assume predictability at almost all horizons when forecasting bond returns. However, when forecasting stocks returns there is not a single model that outperforms the others, the JV models perform well over the shorter horizons and the BNP and BP models over the longer horizons. In general, under this statistical criterion the Barberis models which assume no or limited predictability forecast well. Conversely, when an economic value approach is used these Barberis models are the worst performing and are outperformed by the theory based models. So we observe that the results from the two differing evaluation techniques do not entirely coincide, where the model that achieves the lowest RMSE is not necessarily the one that will maximise realised wealths. It is apparent from this that models and their forecasts need to be evaluated using appropriate criteria. Here we want to know how to optimally allocate the portfolio, so it is necessary to incorporate the investor's feelings about risk and to consider the distribution about the predicted returns, in which case the RMSE seems inadequate for this purpose.

The results show evidence of economic value to bond and stock return predictability. As we increase the degree of predictability assumed in the model, when moving from the BNP model right through to the JV model, there are increasing gains in terms of economic value to the investor. Since the end-of-horizon wealth gained by the investor who assumes bond and stock return predictability is greater, than one who assumes they are not predictable. With the investor who assumes the highest level of predictability here as given by the JV model, achieving the greatest end-of-horizon wealth.

To conclude we find further evidence to that reported by Abhyankar et al (2005), Della Corte et al (2008), and Garratt and Lee (2009) amongst others, which highlights the importance of having an evaluation criterion that reflects the purpose for which the forecasts are intended. Our results suggest that in the context of investment decision making under an economic value criterion, the investor gains from not only assuming predictability, but by modelling the bond and stock returns together.

Data Appendix

Here we provide details of the source, definitions of the data together with any transformations conducted.

Bond data : $r_t^{(n)}$

- Source: Bank of England, www.bankofengland.co.uk/statistics/yieldcurve/archive.htm
- Definition: Nominal government spot interest rate for n months, obtained from 'UK Nominal Spot Curve' data at the short end is used, the curve is estimated using gilt and gilt repos rates.
- Transformation: Using $R_t^{(n)}$ the annualised 1-, 3-, 6- and 12-month rates, Wednesday observations, to construct the n-month rate expressed as a monthly rate $r_t^{(n)} = \ln\left[1 + \left(R_t^{(n)}/100\right)\right]^{1/12}$.

Stock data : r_t^s

- Source: Datastream, mneumonic=DSR1
- Datastream Definition: Data on FTSE All-Share Return Index RI_t, where RI_t = return index on day t.
- Transformation: Using RI_t, Wednesday observations, compute the continuously compounded monthly return $r_t^s = \ln\left[RI_{t+4}/RI_t\right]$.

Dividend yield : dy_t

- Source: Datastream, mneumonic=DY
- Datastream Definition: "For sectors, dy is derived by calculating the total dividend amount for a sector and expressing it as a percentage of the total market value for the constituents of that sector."
- Although this definition makes reference to DY on day t, a plot of the series shows that the dividend yield is between 2 and 4%, these magnitudes would suggest this is an annual measure.
- Use the Wednesday observations of this series.

Appendix A: Using stochastic simulation to compute density forecasts based on the VAR model

From equation (1), we can denote the maximum likelihood estimates of the model parameters $\widehat{\theta} = \left(\widehat{\mu}, \hat{\mathbf{B}}_i, \hat{\mathbf{\Sigma}}\right)$, for $i = 1$ to p. In the absence of parameter uncertainty, the investor assumes there is no uncertainty about the model parameters and they are fixed at the estimated values, then the model is iterated forward to produce the point estimates of the h-step ahead forecasts, conditional on the observed data $\mathbf{X}_T$ and the estimated parameter values $\widehat{\theta}$

$$\hat{\mathbf{x}}_{T+h} = \widehat{\mu} + \sum_{i=1}^{p} \hat{\mathbf{B}}_i \hat{\mathbf{x}}_{T+h-i} \tag{9}$$

for $h = 1, 2, ..., H, ...$ Using the initial values of the variables $\mathbf{x}_{T,}\mathbf{x}_{T-1},..., \mathbf{x}_{T-p+1,}$ these forecasts are produced recursively. A detailed discussion of how, through stochastic simulation techniques, an estimate of the probability density function of the forecasts can be obtained is given in GLPS and GL (2009), here we provide a summary. Using the methods described in GL and GLPS all of the steps below are conducted for each of the four interest rate and stock models in turn.

Predictive density considering stochastic uncertainty only: $P\left(\mathbf{X}_{T+1,H} \mid \mathbf{X}_T, \widehat{\theta}\right)$

1. Using the estimated model parameters $\widehat{\theta}$, forecasts of the bill and stock returns are generated $r_{T+h}^{(\widetilde{r})}$ for $h = 1, ..., H$.and $\widetilde{r} = 1, ..., \widetilde{R}$. Here $\widetilde{R} = 50,000$.

2. From the above forecasts, values of $W_{T+H}^{(\widetilde{r},\omega)}$ can be calculated for each replication, where $\omega = 0, ..., 1$ increasing in steps of 0.01. So for each value of H i.e. forecast horizon we have $\widetilde{R} \times 101$ values of $W_{T+H}^{(\widetilde{r},\omega)}$, where $H =$ 3, 6,12,18 and 24 months.

3. These wealths are used to calculate utility as given by the CRRA definition, $\upsilon_{T+H}^{(\widetilde{r},\omega,A)}$ where $A = 2, 5$ and 10. For the given values of ω, A and H the expected utility is given by averaging across the replications as follows

$$E_T \upsilon\left(W_{T+H}\right) = \frac{1}{\widetilde{R}} \sum_{\widetilde{r}=1}^{\widetilde{R}} \upsilon_{T+H}^{(\widetilde{r},\omega,A)}$$

4. Hence for a given investment horizon H and level of risk A, the investor selects that portfolio weight which maximises expected utility.

Predictive density considering stochastic and parameter uncertainty: $P\left(\mathbf{X}_{T+1,H} \mid \mathbf{X}_T\right)$

1. Using $\widehat{\theta}$ in-sample values of $\mathbf{x}_T$ are simulated $\widetilde{H}$ times, where $t = 1, ..., T$ and $\widetilde{h} = 1, ..., \widetilde{H}$. Here $\widetilde{R} = 1000$ and $\widetilde{H} = 2000$.

2. Using each of these $\widetilde{H}$ 'histories' of $\mathbf{x}_T$ estimate the model. This yields $\widetilde{H}$ sets of parameter estimates $\widehat{\theta}^{(\widetilde{h})}$, one for each history generated.

3. For each history compute $\widetilde{R}$ replications of the h-step ahead point forecasts of $\mathbf{x}_T$, where $\widetilde{r} = 1, ..., \widetilde{R}$.

4. Repeat steps 2-4 from the stochastic uncertainty only method above for each history and its corresponding set of $\widetilde{R}$ simulated futures. Such that $\widetilde{H}$ sets of $E_T \upsilon \left(W_{T+H}\right)$ are calculated for given values of ω, A and H, where the aim is to select the portfolio weight that maximises $E_T \upsilon \left(W_{T+H}\right)$ for a given investment horizon H and level of risk A, for each history $\widetilde{H}$.

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Figure 1: Stock Return and Dividend Yield 1997 to 2007

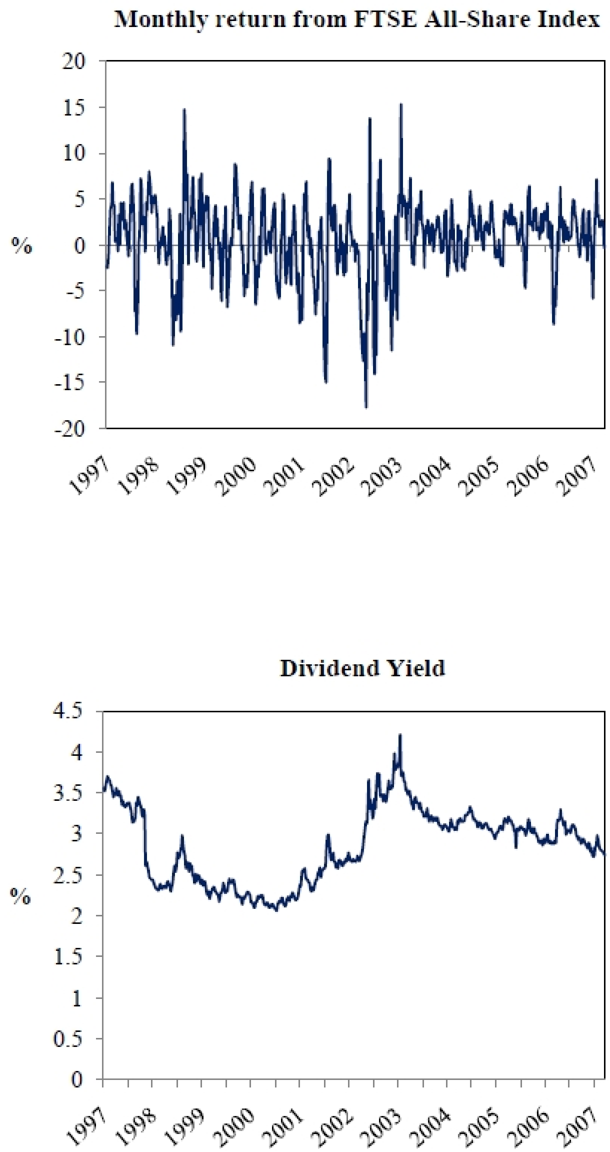

Figure 2: Bond Return, Changes and Spreads 1997 to 2007

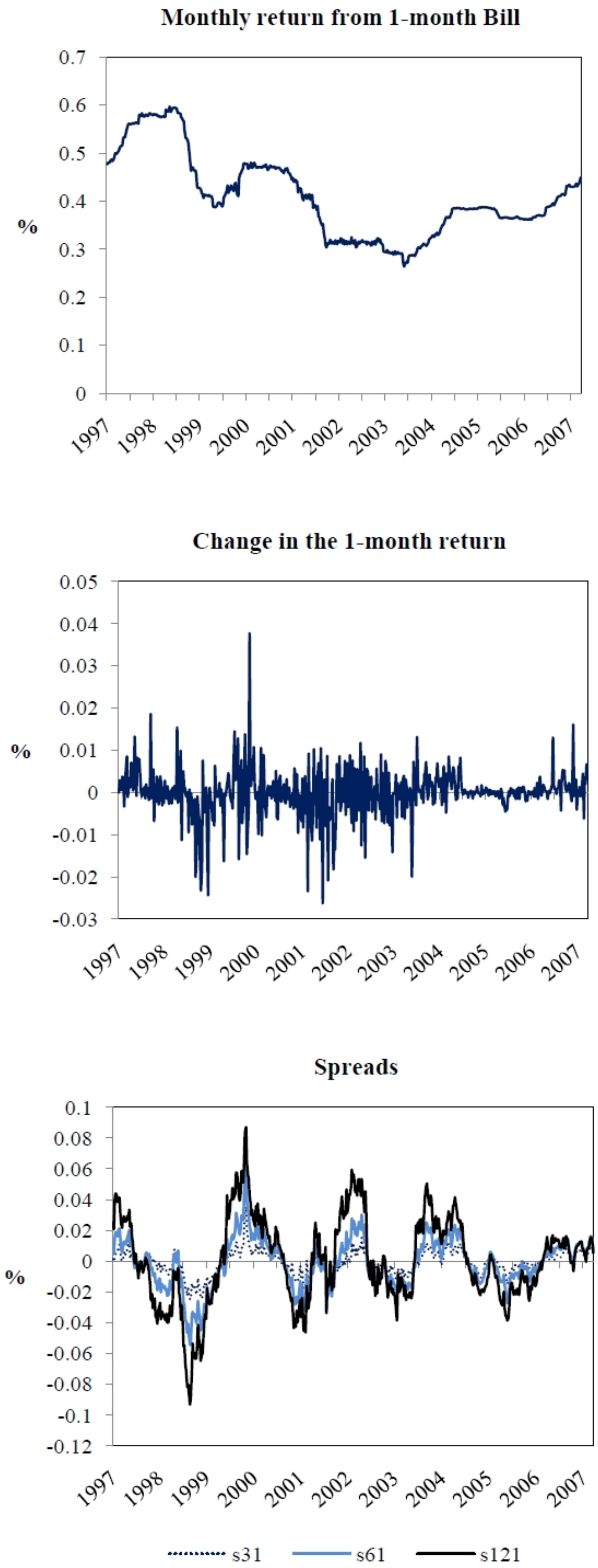

Figure 3: Effect of Predictability ignoring Parameter Uncertainty

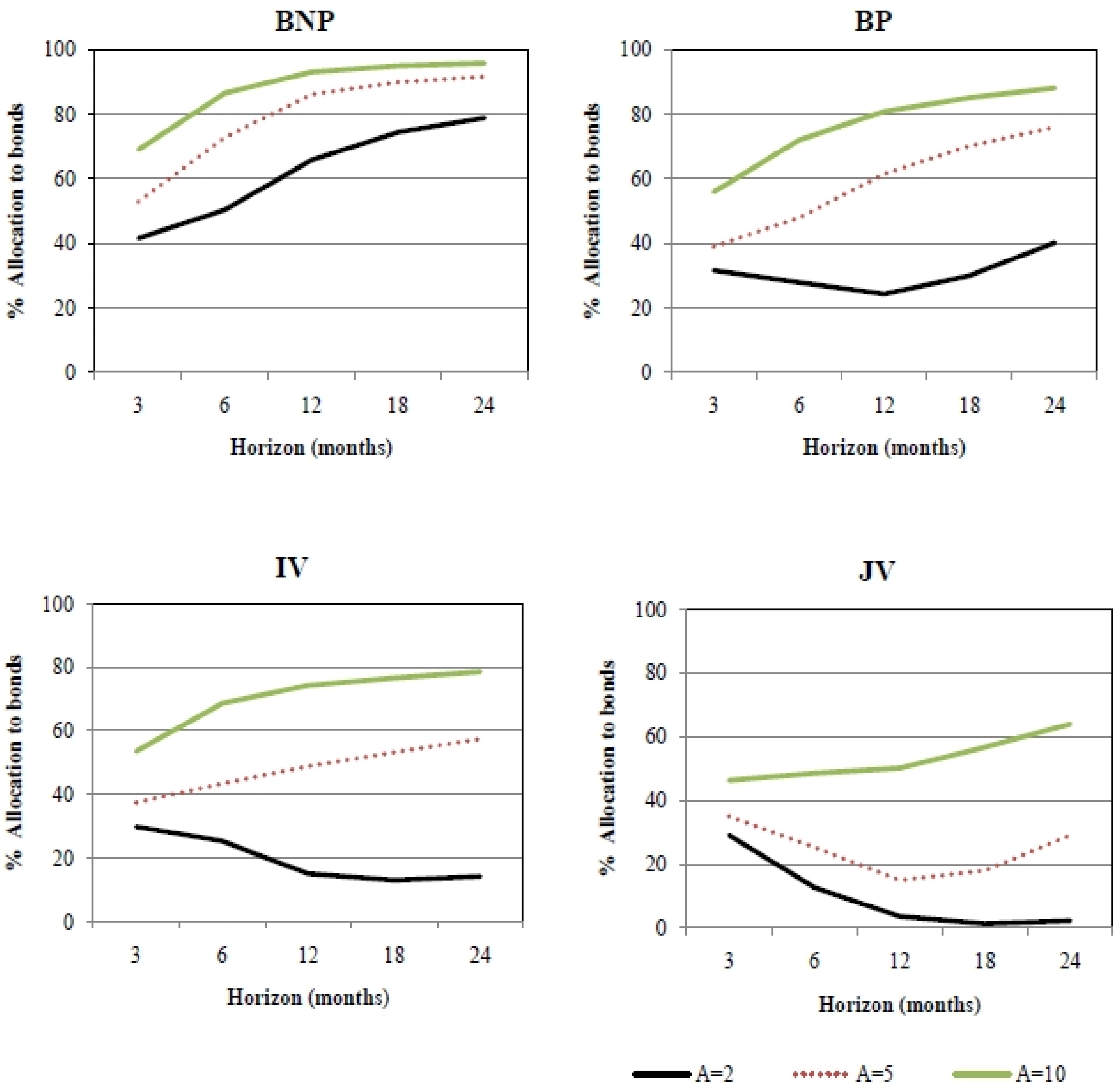

Figure 4: Allocation under the BNP Model Without (solid line) and With (dotted line) Parameter Uncertainty

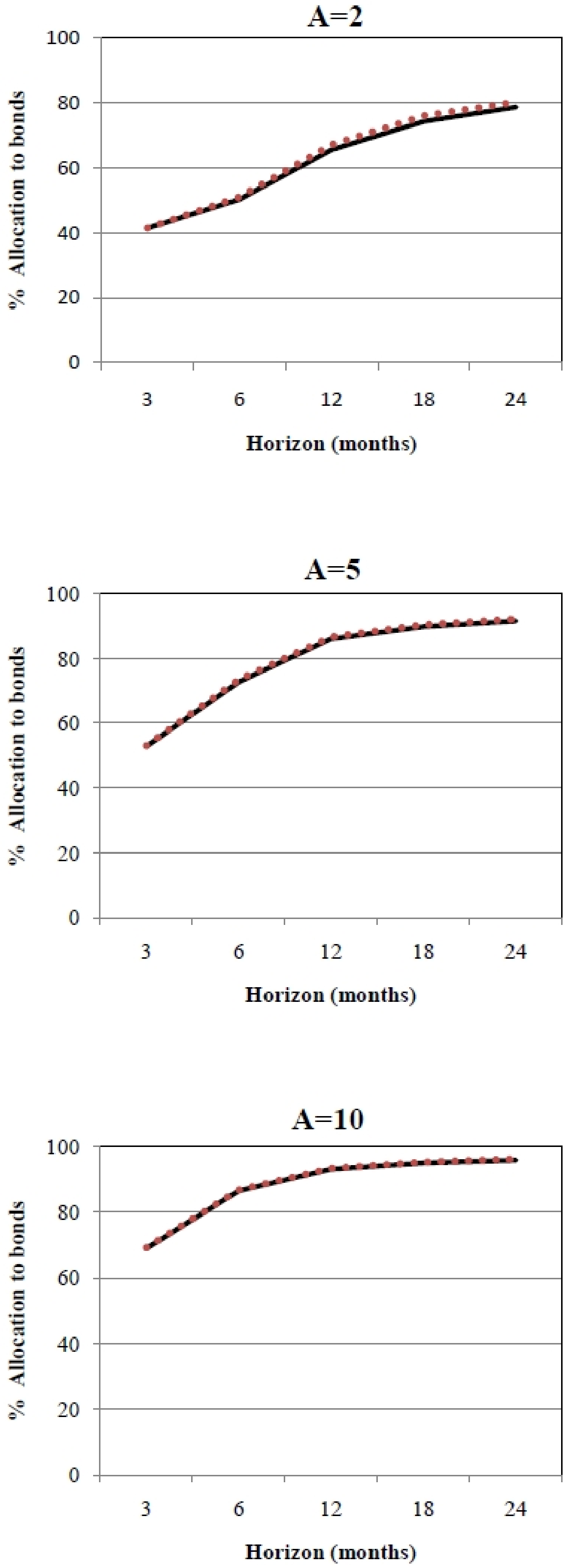

Figure 5: Allocation under the BP Model Without (solid line) and With (dotted line) Parameter Uncertainty

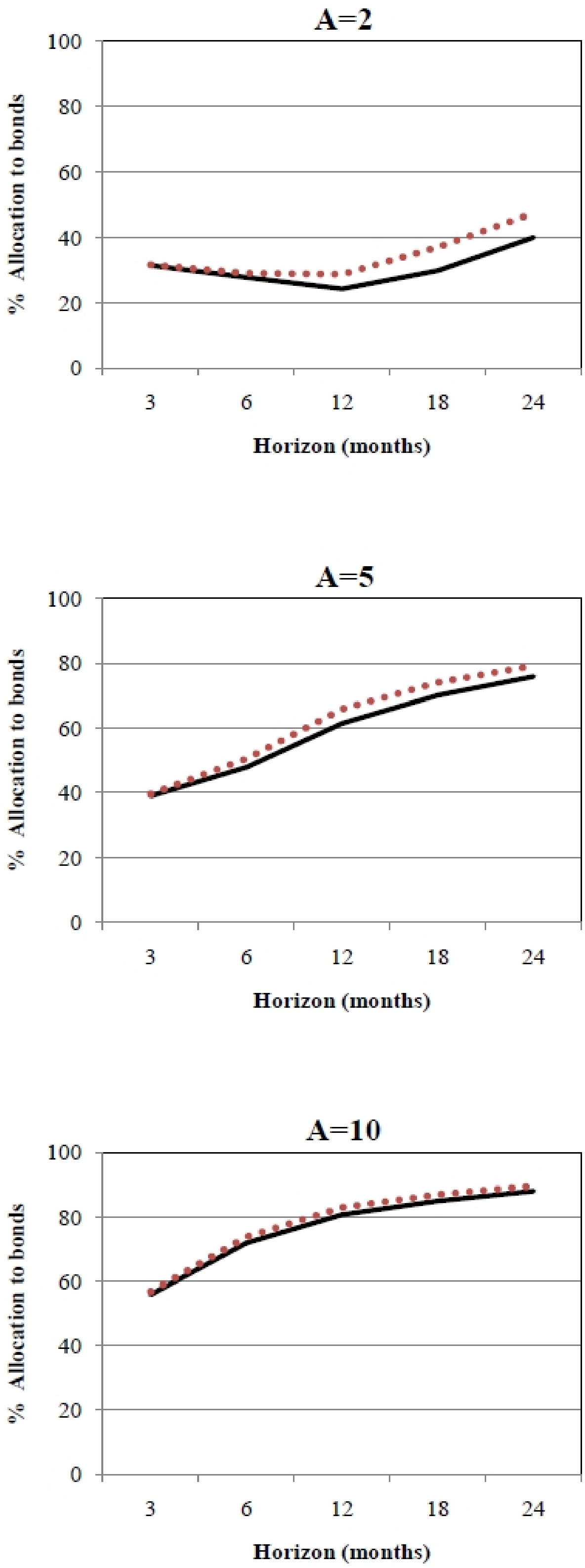

Figure 6: Allocation under the IV Model Without (solid line) and With (dotted line) Parameter Uncertainty

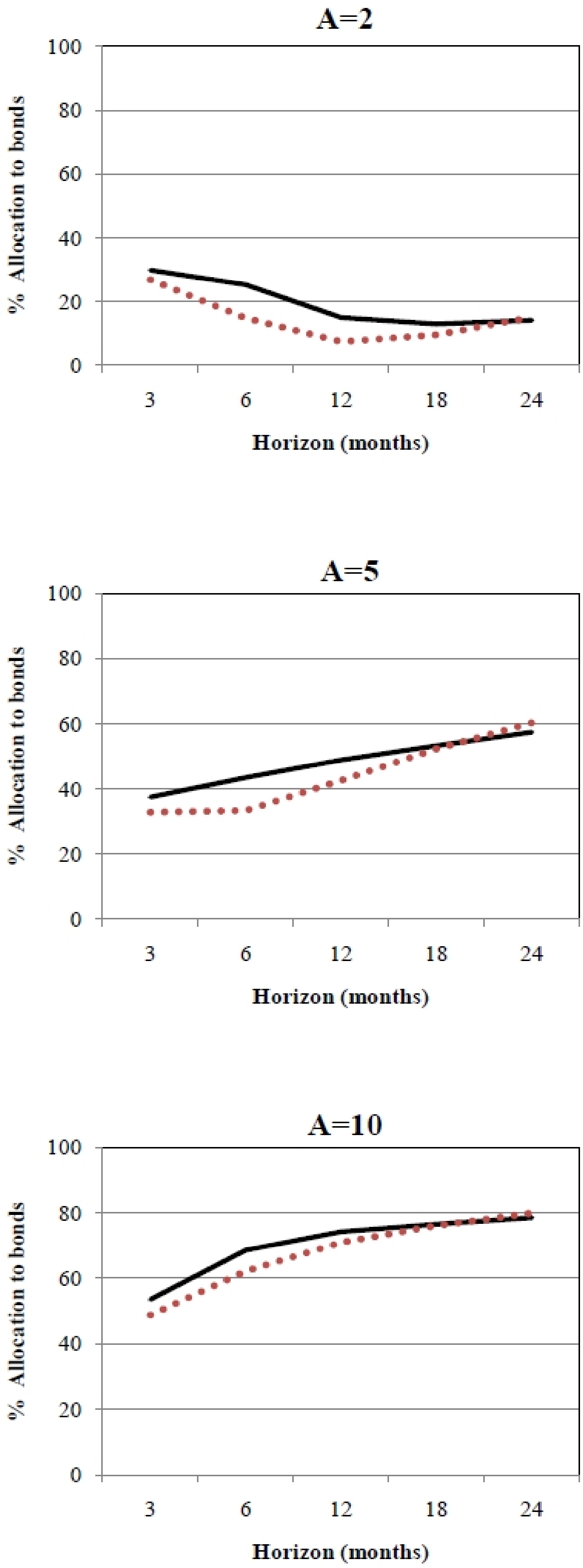

Figure 7: Allocation under the JV Model Without (solid line) and With (dotted line) Parameter Uncertainty

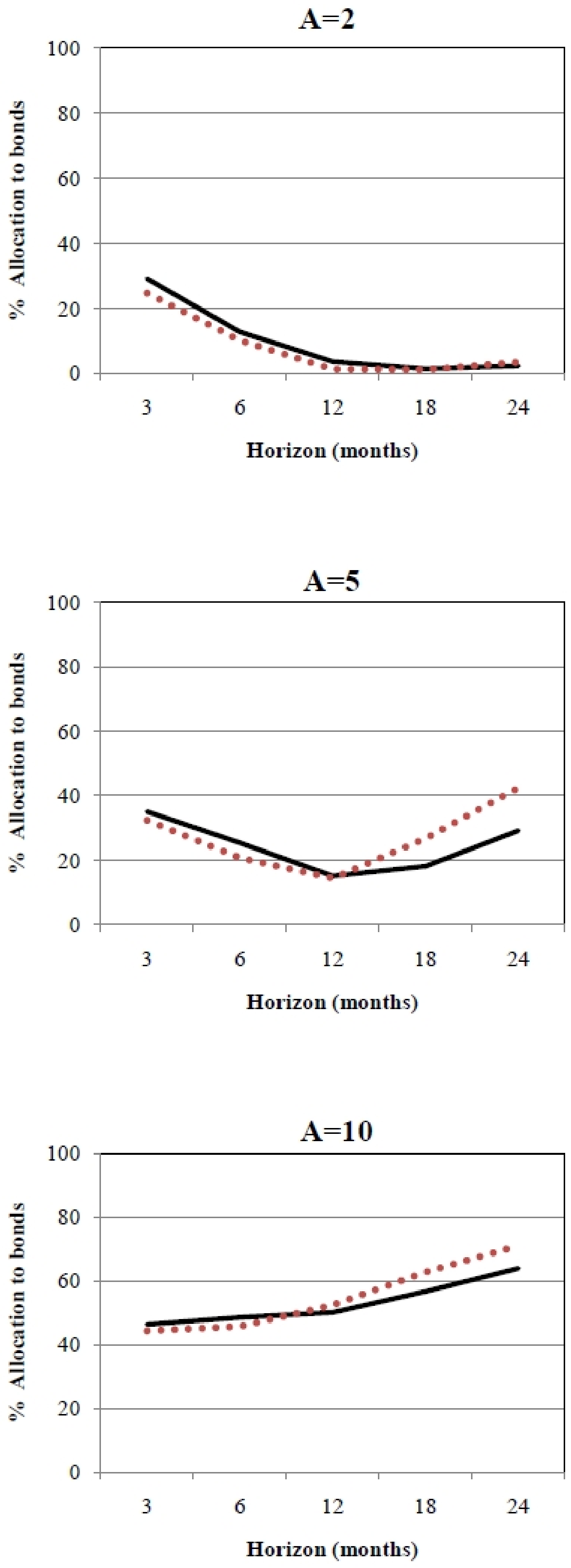

Table 1: Unit Root Tests

Variable	ADF t-statistic (lag length)	PP adj t-statistic (bandwidth)	KPSS LM statistic (bandwidth)
r_t^s	-5.002^{***} (0)	-9.150^{***} (22)	0.248 (9)
dy_t	-2.005 (1)	-2.107 (4)	0.799^{***} (17)
Δdy_t	-26.238^{***} (0)	-26.189^{***} (3)	0.170 (6)
r_t^1	-0.888 (0)	-1.184 (12)	1.568^{***} (18)
Δr_t^1	-21.686^{***} (0)	-23.129^{***} (12)	0.266^{***} (12)
$s_t^{12,1}$	-3.121^{**} (0)	-3.141^{**} (8)	0.114 (17)
$s_t^{6,1}$	-3.840^{***} (0)	-3.639^{***} (7)	0.128 (17)
$s_t^{3,1}$	-4.454^{***} (1)	-4.700^{***} (4)	0.141 (17)

Critical Values

	ADF Test	PP Test	KPSS Test
1% level	-3.445	-3.445	0.739
5% level	-2.868	-2.868	0.463
10% level	-2.570	-2.570	0.347

Notes: The ADF test statistics are computed using ADF regressions with an intercept and 'L' lagged first differences of the dependent variable. The order of augmentation in the Dickey-Fuller regressions are chosen using the Schwarz Information Criterion, with maximum lag length of 20. The bandwidth for both the PP and KPSS test was selected using the Newey-West (1994) method based on the Bartlett Kernel. The PP test statistics are calculated with an intercept only in the underlying DF regressions. Tests are performed on the entire sample 1997 week 10 to 2007 week 19. Null rejected at *** 1% level, ** 5% level, * 10% level of significance.

Table 2: Estimation of Barberis No Predictability (BNP) Model

Equation	r_t^s
μ	0.0026 (0.0024)
$\overline{R^2}$	0.000
$\widehat{\sigma}$	0.0471
$eq^n LL$	612.42
χ^2_N [2]	40.31^{***}
χ^2_{SC} [12]	246.35^{***}

Table 3: Estimation of Barberis Predictability (BP) Model

$Equation$	r_t^s	dy_t
μ	-0.0385^{***} (0.0132)	0.0004^{*} (0.0002)
dy_{t-1}	1.4813^{***} (0.4666)	0.9845^{***} (0.0083)
$\overline{R^2}$	0.0238	0.9743
$\widehat{\sigma}$	0.0466	0.0008
$eq^n LL$	615.44	2118.61
χ_N^2	53.83^{***}	2324.69^{***}
$\chi_H^2\,[1]$	1.22	20.11^{***}
$\chi_{SC}^2\,[12]$	240.30^{***}	18.70^{*}

Notes: Standard errors in parenthesis (.). The $\overline{R}^2$, standard error of the regression $(\widehat{\sigma})$,log likelihood of the equation (LL) presented, together with the chi-squared statistics for Breusch-Pagan Serial Correlation test (SC), the Jarque-Bera Test for Normality (N), Breusch-Pagan-Godfrey test for Heteroskedasticity (H). The BNP and BP models are estimated over 1997 week 10 to 2004 week 18 (364 observations), the BNP model assumes that $r_t^s = \mu + \epsilon_t$ and the BP model assumes that both r_t^s and dy_t are determined by a single lagged dividend yield term. Both assume that r_t^1 is a constant taken at the last value in the sample i.e. 2004 week 18. Null rejected at *** 1% level, ** 5% level, * 10% level of significance.

Table 4: Estimation of Individual VAR-STOCK (IV-STOCK) Model

Equation	r_t^s	dy_t
r_{t-1}^s	0.8488*** (0.0520)	−0.0021*** (0.0009)
r_{t-2}^s	0.0969* (0.0680)	0.00002 (0.0011)
r_{t-3}^s	0.0395 (0.0682)	0.0016* (0.0011)
r_{t-4}^s	−0.9627*** (0.0684)	−0.0253*** (0.0011)
r_{t-5}^s	0.6429*** (0.0800)	0.0174*** (0.0013)
dy_{t-1}	−5.2877** (2.7871)	0.7973*** (0.0464)
dy_{t-2}	−4.4200* (3.0747)	0.0130 (0.0512)
dy_{t-3}	2.4994 (3.0838)	−0.0089 (0.0514)
dy_{t-4}	25.6587*** (3.0723)	0.6554*** (0.0512)
dy_{t-5}	−18.0215*** (2.8277)	−0.4597*** (0.0471)
inpt	−0.0111* (0.0078)	0.00009 (0.0001)
$\overline{R}^2$	0.6835	0.9925
$\widehat{\sigma}$	0.0267	0.0004
$Fstat$	80.48***	4862.61***
$eq^n LL$	819.78	2330.93
Excl. r^s terms	118.68***	95.82***
Excl. dy terms	4.79***	22793.15***
χ^2_N [4]	377.63***	
χ^2_H [60]	273.46***	
χ^2_{SC} [4]	9.37*	

Notes: Standard errors in parenthesis (.). The $\overline{R}^2$, standard error of the regression $(\widehat{\sigma})$,log likelihood (LL) presented with the model diagnostic tests which are all carried out on the VAR residuals. No roots of the characteristic polynomial lie outside the unit circle, so the VAR is stable. Chi-squared statistics presented for: (N) the VAR Residual Normality Test (orthogonalization: residual correlation (Doornik-Hansen) this test statistic is not sensitive to the ordering or the scale of the variables) for the null that the residuals are multivariate normal, (H) the VAR Residual Heteroskedasticity Test (no cross terms, but the conclusion was the same when cross terms were included), and (SC) the VAR Residual Serial Correlation LM Test. "Excl ... terms" tests the joint significance of the excluded terms, we give the F-statistic of the Wald test of these restrictions. The IV model for the stock returns is estimated of order 5, over 1997 week 10 to 2004 week 18 (364 observations). Null rejected at *** 1% level, ** 5% level, * 10% level of significance.

Table 5: Estimation of Individual VAR-BOND (IV-BOND) Model

$Equation$	Δr_t^1	$s_t^{12,1}$	$s_t^{6,1}$	$s_t^{3,1}$
Δr_{t-1}^1	0.0261 (0.0641)	0.0807 (0.0924)	0.1005^{*} (0.0643)	0.0677^{**} (0.0403)
Δr_{t-2}^1	-0.1324^{**} (0.0642)	0.1024 (0.0926)	0.1262^{**} (0.0644)	0.0905^{**} (0.0404)
Δr_{t-3}^1	-0.0401 (0.0647)	0.0147 (0.0933)	0.0340 (0.0648)	0.0365 (0.0407)
Δr_{t-4}^1	0.0401 (0.0647)	0.1181 (0.0932)	0.1194^{**} (0.0648)	0.0698^{**} (0.0407)
Δr_{t-5}^1	-0.1556^{***} (0.0652)	0.2144^{**} (0.0940)	0.2256^{***} (0.0654)	0.1635^{***} (0.0410)
Δr_{t-6}^1	0.0637 (0.0503)	-0.0425 (0.0724)	-0.0015 (0.0504)	-0.0056 (0.0316)
$s_{t-1}^{12,1}$	0.2036 (0.1694)	0.9127^{***} (0.2442)	0.1289 (0.1698)	0.0512 (0.1065)
$s_{t-2}^{12,1}$	-0.0399 (0.2085)	0.2616 (0.3005)	0.0256 (0.2089)	-0.0231 (0.1311)
$s_{t-3}^{12,1}$	-0.1141 (0.2099)	-0.2798 (0.3025)	-0.2414 (0.2103)	-0.1303 (0.1320)
$s_{t-4}^{12,1}$	-0.1828 (0.2102)	0.5160^{**} (0.3029)	0.3192^{*} (0.2106)	0.1697^{*} (0.1321)
$s_{t-5}^{12,1}$	0.6021^{***} (0.2079)	-0.3520^{*} (0.2997)	-0.1617 (0.2084)	-0.0507 (0.1307)
$s_{t-6}^{12,1}$	-0.2516^{*} (0.1729)	0.0471 (0.2491)	-0.0724 (0.1732)	-0.1011 (0.1087)
$s_{t-1}^{6,1}$	-0.6234^{*} (0.4141)	-0.0628 (0.5969)	0.7338^{**} (0.4150)	0.0820 (0.2604)
$s_{t-2}^{6,1}$	0.0182 (0.5081)	-0.2724 (0.7323)	0.1476 (0.5091)	0.1774 (0.3195)
$s_{t-3}^{6,1}$	0.4275 (0.5102)	0.6366 (0.7353)	0.5034 (0.5113)	0.1914 (0.3208)
$s_{t-4}^{6,1}$	0.2740 (0.510)	-1.2991^{**} (0.7350)	-0.7821^{*} (0.5111)	-0.3879 (0.3206)
$s_{t-5}^{6,1}$	-1.4676^{***} (0.5076)	1.1729^{*} (0.7315)	0.6392 (0.5086)	0.2476 (0.3191)
$s_{t-6}^{6,1}$	0.3932 (0.4273)	-0.1773 (0.6158)	0.0961 (0.4282)	0.2071 (0.2687)

Table 6: Estimation of Individual VAR-BOND Model (continued)

$s^{3,1}_{t-1}$	1.1372^{***} (0.3693)	-0.0945 (0.5322)	-0.2179 (0.3700)	0.4756^{**} (0.2322)
$s^{3,1}_{t-2}$	-0.1628 (0.4582)	0.0026 (0.6604)	-0.1587 (0.4592)	-0.1316 (0.2881)
$s^{12,1}_{t-3}$	-0.2502 (0.4590)	-0.7137 (0.6615)	-0.5592 (0.4599)	-0.2200 (0.2886)
$s^{3,1}_{t-4}$	-0.0421 (0.4587)	1.2917^{**} (0.6613)	0.8299^{**} (0.4598)	0.4046^{*} (0.2885)
$s^{3,1}_{t-5}$	0.8163^{**} (0.4595)	-0.9184^{*} (0.6623)	-0.5250 (0.4605)	-0.1750 (0.2889)
$s^{3,1}_{t-6}$	0.2139 (0.3898)	-0.3190 (0.5617)	-0.3820 (0.3906)	-0.3364^{*} (0.2451)
$inpt$	-0.000001 (0.000004)	-0.000011^{**} (0.000005)	-0.000006^{*} (0.000004)	-0.000002 (0.000002)
$\overline{R^2}$	0.2582	0.9353	0.9015	0.8264
$\widehat{\sigma}$	0.0001	0.0001	0.0001	0.0000
$Fstat$	6.31^{***}	221.24^{***}	140.74^{***}	73.69^{***}
$eq^n LL$	3079.65	2945.51	3078.88	3249.95
Excl. Δr^1_t terms	2.12^{*}	1.17	1.87^{*}	2.38^{**}
Excl. $s^{12,1}_t$ terms	1.78	4.03^{***}	0.78	0.60
Excl. $s^{6,1}_t$ terms	1.82	0.89	1.12	0.48
Excl. $s^{3,1}_t$ terms	1.76	1.02	0.99	1.89^{*}
χ^2_N [8]	55.65^{***}			
χ^2_H [480]	730.43^{***}			
χ^2_{SC} [16]	11.53			

Notes: Standard errors in parenthesis (.). The $\overline{R}^2$, standard error of the regression $(\widehat{\sigma})$,log likelihood (LL) presented with VAR residual diagnostic tests and the test of restrictions as detailed before. No roots of the characteristic polynomial lie outside the unit circle, so the VAR is stable. The IV model for the bond returns is estimated of order 6, over 1997 week 10 to 2004 week 18 (364 observations). Null rejected at *** 1% level, ** 5% level, * 10% level of significance.

Table 7: Estimation of Joint VAR (JV) Model

$Equation$	r_t^s	dy_t	Δr_t^1	$s_t^{12,1}$	$s_t^{6,1}$	$s_t^{3,1}$
r_{t-1}^s	0.8437*** (0.0559)	−0.0016** (0.0009)	0.00003 (0.0001)	0.00002 (0.0002)	−0.00005 (0.0001)	−0.00009 (0.0001)
r_{t-2}^s	0.0955* (0.0733)	−0.00043 (0.0012)	−0.00013 (0.0002)	0.00005 (0.0002)	0.00006 (0.0002)	0.00009 (0.0001)
r_{t-3}^s	0.0280 (0.0722)	0.0009 (0.0012)	0.00019 (0.0002)	−0.0003 (0.0002)	−0.0002 (0.0002)	−0.0001 (0.0001)
r_{t-4}^s	−0.9675*** (0.0723)	−0.0253*** (0.0012)	−0.0002* (0.0002)	0.0011*** (0.0002)	0.0007*** (0.0002)	0.0003*** (0.0001)
r_{t-5}^s	0.7132*** (0.1187)	0.0217*** (0.0019)	0.0002 (0.0003)	−0.0004 (0.0003)	−0.0004* (0.0002)	−0.0002* (0.0002)
r_{t-6}^s	−0.1010 (0.1069)	−0.0059*** (0.0017)	−0.000087 (0.0002)	−0.00006 (0.0003)	0.000043 (0.0002)	0.00007 (0.0001)
dy_{t-1}	−3.4884 (3.4040)	0.8663*** (0.0550)	0.0039 (0.0071)	0.0041 (0.0098)	0.0016 (0.0069)	0.0018 (0.0044)
dy_{t-2}	−7.4658** (4.0558)	−0.0966* (0.0655)	−0.0050 (0.0085)	0.0029 (0.0116)	0.0028 (0.0082)	−0.0009 (0.0053)
dy_{t-3}	3.2377 (3.3330)	−0.0071 (0.0539)	0.0004 (0.0070)	−0.00001 (0.0096)	−0.0004 (0.0068)	0.0006 (0.0044)
dy_{t-4}	24.7579*** (3.3222)	0.6343*** (0.0537)	−0.0042 (0.0070)	−0.0176** (0.0095)	−0.0010* (0.0067)	−0.0029 (0.0043)
dy_{t-5}	−20.4026*** (4.0628)	−0.5584*** (0.0657)	0.0068 (0.0085)	0.0055 (0.0117)	0.0049 (0.0083)	0.0023 (0.0053)
dy_{t-6}	3.8254 (3.4294)	0.1579*** (0.0554)	−0.0013 (0.0072)	0.0033 (0.0098)	0.0003 (0.0070)	−0.0011 (0.0045)
Δr_{t-1}^1	44.3657* (31.22)	−0.1888 (0.5045)	0.0345 (0.0653)	0.0490 (0.0895)	0.0810 (0.0634)	0.0611 (0.0407)
Δr_{t-2}^1	−9.8129 (31.1126)	0.2307 (0.5028)	−0.1481** (0.0651)	0.1290* (0.0892)	0.1449** (0.0632)	0.1010*** (0.0406)
Δr_{t-3}^1	−20.7582 (31.4388)	−0.1499 (0.5080)	−0.0343 (0.0657)	0.0358 (0.0901)	0.0453 (0.0638)	0.0374 (0.0410)
Δr_{t-4}^1	−0.8345 (31.2922)	0.4512 (0.5057)	0.0326 (0.0654)	0.0999 (0.0897)	0.1080** (0.0635)	0.0652* (0.0408)
Δr_{t-5}^1	5.1664 (31.4400)	0.2952 (0.5081)	−0.1546*** (0.0658)	0.1807** (0.0901)	0.2059*** (0.0638)	0.1551*** (0.0410)
Δr_{t-6}^1	5.7750 (24.2492)	0.2479 (0.3919)	0.0682* (0.0507)	−0.0697 (0.0695)	−0.0183 (0.0492)	−0.0146 (0.0316)
$s_{t-1}^{12,1}$	−45.0026 (81.7993)	0.8657 (1.3213)	0.1970 (0.1710)	0.9253*** (0.2345)	0.1357 (0.1660)	0.0610 (0.1067)
$s_{t-2}^{12,1}$	−48.8576 (101.310)	−1.5758 (1.6371)	−0.0487 (0.2119)	0.1956 (0.2905)	−0.0151 (0.2056)	−0.0512 (0.1321)
$s_{t-3}^{12,1}$	91.6225 (101.889)	1.1484 (1.6465)	−0.0599 (0.2131)	−0.2590 (0.2921)	−0.2352 (0.2068)	−0.1281 (0.1329)
$s_{t-4}^{12,1}$	13.1789 (101.991)	1.0135 (1.6481)	−0.2623 (0.2133)	0.5374** (0.2924)	0.3419** (0.2070)	0.1874* (0.1333)
$s_{t-5}^{12,1}$	−20.9624 (100.933)	−1.5036 (1.6310)	0.6723*** (0.2111)	−0.3337 (0.2894)	−0.1568 (0.2049)	−0.0592 (0.1316)
$s_{t-6}^{12,1}$	−19.0208 (84.3141)	0.7271 (1.3625)	−0.3056** (0.1763)	0.1620 (0.2417)	−0.0084 (0.1711)	−0.0770 (0.110)

Table 8: Estimation of Joint VAR Model (continued)

$s_{t-1}^{6,1}$	88.8499 (200.813)	−3.4790 (3.2450)	−0.6127* (0.4199)	−0.1499 (0.5758)	0.6944** (0.4076)	0.0589 (0.2619)
$s_{t-2}^{6,1}$	203.8494 (247.846)	8.8052** (4.0050)	0.0261 (0.5183)	−0.1192 (0.7106)	0.2311 (0.5030)	0.2267 (0.3233)
$s_{t-3}^{6,1}$	−238.2515 (248.877)	−5.2168* (4.0217)	0.3597 (0.5205)	0.3213 (0.7136)	0.3375 (0.5051)	0.1256 (0.3246)
$s_{t-4}^{6,1}$	−83.3880 (248.824)	−3.8379 (4.0208)	0.3672 (0.5203)	−1.0019* (0.7134)	−0.6166 (0.5050)	−0.3113 (0.3245)
$s_{t-5}^{6,1}$	82.6709 (246.359)	6.3508* (3.9810)	−1.5152*** (0.5152)	0.9559* (0.7064)	0.5057 (0.5000)	0.1922 (0.3213)
$s_{t-6}^{6,1}$	43.2428 (207.804)	−4.0780 (3.3580)	0.4916 (0.4346)	−0.4441 (0.5958)	−0.04545 (0.4218)	0.1585 (0.2710)
$s_{t-1}^{3,1}$	−15.1047 (180.825)	3.0517 (2.9220)	1.1146*** (0.3781)	0.0194 (0.5185)	−0.1664 (0.3670)	0.5014** (0.2358)
$s_{t-2}^{3,1}$	−343.980* (225.663)	−9.2975*** (3.6466)	−0.1809 (0.4719)	−0.0786 (0.6470)	−0.1952 (0.4580)	−0.1568 (0.2943)
$s_{t-3}^{3,1}$	212.502 (226.610)	4.7176* (3.6619)	−0.1935 (0.4739)	−0.2940 (0.6497)	−0.3398 (0.4599)	−0.1431 (0.2956)
$s_{t-4}^{3,1}$	120.809 (225.803)	3.9380 (3.6488)	−0.1221 (0.4722)	0.8971* (0.6474)	0.6008* (0.4583)	0.2985 (0.2945)
$s_{t-5}^{3,1}$	−70.8071 (224.02)	−6.7076** (3.620)	0.8540** (0.4685)	−0.7411 (0.6423)	−0.410 (0.4547)	−0.1196 (0.2922)
$s_{t-6}^{3,1}$	−44.6769 (189.193)	4.4755* (3.0572)	0.1246 (0.3956)	−0.0122 (0.5425)	−0.2115 (0.3840)	−0.2715 (0.2468)
$inpt$	−0.0119* (0.0085)	0.00008 (0.0001)	−0.00002 (0.00002)	0.00004* (0.00002)	0.00002 (0.00002)	0.000003 (0.00001)
$\overline{R}^2$	0.6710	0.9926	0.2611	0.9416	0.9080	0.8298
$\widehat{\sigma}$	0.0271	0.0004	0.00006	0.00008	0.00006	0.00004
$Fstat$	21.74	1360.44	4.59	165.01	101.34	50.58
$eq^n LL$	822.51	2336.48	3086.91	2971.09	3097.89	3260.17
Excl. r_t^s terms	82.35***	78.10***	1.03	4.61***	3.99***	2.64**
Excl. dy_t terms	4.40***	13821.12***	0.83	1.95*	1.20	0.31
Excl. Δr_t^1 terms	0.48	0.44	2.16**	1.18	2.9**	3.83***
Excl. $s_t^{12,1}$ terms	0.32	0.57	3.43***	19.33***	0.80	0.86
Excl. $s_t^{6,1}$ terms	0.37	1.16	3.15***	0.66	3.65***	1.17
Excl. $s_t^{3,1}$ terms	0.67	0.88	3.99***	0.52	1.32	1.47
Excl. *stock* terms	56.68***	9301.20***	2.30**	3.48***	2.61***	1.76**
Excl. *bond* terms	0.62	0.77	6.70***	333.43***	196.51***	89.49***
χ_N^2 [12]	295.87***					
χ_H^2 [1512]	2123.63***					
χ_{SC}^2 [36]	24.42*					

Notes: Standard errors in parenthesis (.). The $\overline{R}^2$, standard error of the regression $(\widehat{\sigma})$,log likelihood (LL) presented with VAR residual diagnostic tests and the test of restrictions as detailed before. No roots of the characteristic polynomial lie outside the unit circle, so the VAR is stable. The JV model is estimated of order 6, over 1997 week 10 to 2004 week 18 (364 observations). Null rejected at *** 1% level, ** 5% level, * 10% level of significance.

Table 9: Root Mean Squared Errors of Bond and Stock Returns

(a) Bond Returns $\left(r_t^1\right)$

Model	$H = 1$	$H = 3$	$H = 6$	$H = 12$	$H = 18$	$H = 24$
BNP	3.02E-06	3.43E-06	3.98E-06	3.44E-06	1.77E-06	2.56E-07
BP	3.02E-06	3.43E-06	3.98E-06	3.44E-06	1.77E-06	2.56E-07
IV	3.04E-06	3.55E-06	4.51E-06	5.03E-06	3.05E-06	1.33E-06
JV	3.00E-06	3.44E-06	4.35E-06	4.99E-06	3.15E-06	1.46E-06
BNPPU	3.02E-06	3.43E-06	3.98E-06	3.44E-06	1.77E-06	2.56E-07
BPPU	3.02E-06	3.43E-06	3.98E-06	3.44E-06	1.77E-06	2.56E-07
IVPU	3.05E-06	3.59E-06	4.61E-06	5.07E-06	3.00E-06	1.26E-06
JVPU	3.01E-06	3.47E-06	4.43E-06	4.99E-06	3.06E-06	1.34E-06

(b) Stock Returns $\left(r_t^s\right)$

Model	$H = 1$	$H = 3$	$H = 6$	$H = 12$	$H = 18$	$H = 24$
BNP	0.001760	0.001599	0.002343	0.002479	0.002334	0.001431
BP	0.001607	0.001497	0.002316	0.002490	0.002316	0.001431
IV	0.001559	0.001520	0.002333	0.002482	0.002327	0.001430
JV	0.001533	0.001420	0.002368	0.002468	0.002360	0.001466
BNPPU	0.001762	0.001601	0.002340	0.002483	0.002333	0.001431
BPPU	0.001615	0.001505	0.002318	0.002485	0.002320	0.001429
IVPU	0.001504	0.001495	0.002314	0.002486	0.002327	0.001436
JVPU	0.001495	0.001411	0.002375	0.002477	0.002365	0.001470

Notes: The RMSEs are computed for each model for the horizons H = 1, 3, 6, 12, 18, 24 months as follows $\sqrt{\frac{\sum_{i=1}^{54}(r_{T+H}-\widehat{r}_{T+H})_i}{54}}$ where r_{T+H} is the actual monthly return i.e. $r_t^1$and r_t^s, $\widehat{r}_{T+H}$ is the forecast and the difference between the two $\left(r_{T+H} - \widehat{r}_{T+H}\right)$ is computed for each recursion i, where there are 54 weekly recursions. The BNP, BP, IV and JV models are estimated subject to stochastic uncertainty only. The BNPPU, BPPU, IVPU and JVPU models consider parameter uncertainty too.

Table 10: Ratio of RMSEs of Returns

(a) Bond Returns $\left(r_t^1\right)$

Model	$H=1$	$H=3$	$H=6$	$H=12$	$H=18$	$H=24$
BNP	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
BP	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
IV	1.0073	1.0338	1.1339	1.4635	1.7241	5.2038
JV	0.9956	1.0033	1.0927	1.4508	1.7805	5.7062
BNPPU	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
BPPU	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
IVPU	1.0111	1.0456	1.1573	1.4734	1.6958	4.9097
JVPU	0.9986	1.0125	1.1118	1.4507	1.7309	5.2440

(b) Stock Returns $\left(r_t^s\right)$

Model	$H=1$	$H=3$	$H=6$	$H=12$	$H=18$	$H=24$
BNP	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
BP	0.9129	0.9361	0.9886	1.0043	0.9924	0.9999
IV	0.8859	0.9507	0.9959	1.0011	0.9970	0.9992
JV	0.8713	0.8883	1.0107	0.9953	1.0112	1.0247
BNPPU	1.0013	1.0013	0.9986	1.0016	0.9997	0.9999
BPPU	0.9174	0.9410	0.9894	1.0023	0.9942	0.9984
IVPU	0.8543	0.9349	0.9991	1.0028	0.9971	1.0031
JVPU	0.8495	0.8826	1.0136	0.9991	1.0135	1.0270

Notes: The above ratios are that of the RMSE for each model to the RMSE of the BNP model, which is taken as the benchmark.

Table 11: Realised Wealth under each Strategy for A=2

Position	$H = 3$	$H = 6$	$H = 12$	$H = 18$	$H = 24$
1st	AS 1.039024	AS 1.090983	AS 1.198814	AS 1.303596	AS 1.402846
2nd	JVPU 1.036934	JV 1.085841	JV 1.195839	JV 1.301244	JV 1.395425
3rd	IVPU 1.036653	JVPU 1.080824	JVPU 1.190218	JVPU 1.288895	JVPU 1.364283
4th	BPPU 1.036635	IVPU 1.078341	IV 1.179595	IV 1.279403	IV 1.359239
5th	JV 1.036284	IV 1.076682	IVPU 1.168318	IVPU 1.267401	IVPU 1.330711
6th	IV 1.036280	BP 1.075567	BP 1.166442	BP 1.242216	BPPU 1.280586
7th	BP 1.036424	BPPU 1.073885	BPPU 1.161321	BPPU 1.234480	BP 1.280391
8th	BNP 1.033901	BNP 1.06513	HH 1.122679	HH 1.186747	HH 1.248576
9th	BNPPU 1.033538	BNPPU 1.059189	BNPPU 1.106594	BNPPU 1.135350	BNPPU 1.162686
10th	HH 1.025256	HH 1.057076	BNP 1.106185	BNP 1.133146	BNP 1.158903
11th	AB 1.011487	AB 1.023170	AB 1.046544	AB 1.069898	AB 1.094306

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Notes: The realised wealths above are the end-of-investment horizon wealths that the investor would have achieved over 2004 week 19 to 2007 week 19 had he allocated according to the optimal weights for each model, A and H. These have been averaged over the 54 recursions. The realised wealths for each model are ranked in descending order, for a particular A and H. The table shows how the four models, BNP, BP, IV and JV without and with parameter uncertainty perform, together with the lazy strategies in terms of their achieved realised wealths. The actual realised wealths are given below the model code. The BNP, BP, IV and JV models are estimated subject to stochastic uncertainty only. The BNPPU, BPPU, IVPU and JVPU models consider parameter uncertainty too. The lazy strategies are 'AS' is the 'all in stocks', 'AB' is the 'all in 1-month' and 'HH' is half in stocks and half in bonds.

Table 12: Realised Wealth under each Strategy for A=5

Position	$H = 3$	$H = 6$	$H = 12$	$H = 18$	$H = 24$
1st	AS 1.039024	AS 1.090983	AS 1.198814	AS 1.303596	AS 1.402846
2nd	JVPU 1.036578	JV 1.076580	JV 1.182284	JV 1.266296	JV 1.311270
3rd	JV 1.036177	JVPU 1.073879	JVPU 1.172839	JVPU 1.247169	JVPU 1.290901
4th	BP 1.034746	IVPU 1.067545	IV 1.130203	IVPU 1.188850	HH 1.248576
5th	IV 1.034722	IV 1.065844	IVPU 1.129411	HH 1.186747	IVPU 1.226558
6th	BPPU 1.034379	BP 1.063308	HH 1.22679	IV 1.185716	IV 1.225723
7th	IVPU 1.034076	BPPU 1.061854	BP 1.110021	BP 1.144193	BPPU 1.170952
8th	BNP 1.031437	HH 1.057076	BPPU 1.108467	BPPU 1.143424	BP 1.169210
9th	BNPPU 1.031427	BNP 1.045392	BNPPU 1.071853	BNPPU 1.096009	BNPPU 1.121266
10th	HH 1.025256	BNPPU 1.044855	BNP 1.070828	BNP 1.094976	BNP 1.119881
11th	AB 1.011487	AB 1.023170	AB 1.046544	AB 1.069898	AB 1.094306

Notes: See those for Table (11).

Table 13: Realised Wealth under each Strategy for A=10

Position	$H = 3$	$H = 6$	$H = 12$	$H = 18$	$H = 24$
1st	AS 1.039024	AS 1.090983	AS 1.198814	AS 1.303596	AS 1.402846
2nd	JVPU 1.034242	JVPU 1.062446	JV 1.127786	HH 1.186747	HH
3rd	JV 1.033848	JV 1.061033	JVPU 1.127539	JVPU 1.174776	JV 1.204549
4th	IV 1.030925	HH 1.057076	HH 1.122670	JV 1.174361	JVPU 1.200687
5th	IVPU 1.030333	IVPU 1.050169	IVPU 1.090307	IVPU 1.131481	IVPU 1.161707
6th	BP 1.030233	IV 1.047795	IV 1.088793	IV 1.127939	IV 1.160499
7th	BPPU 1.030059	BP 1.045415	BP 1.078158	BP 1.107021	BPPU 1.132265
8th	BNP 1.025752	BPPU 1.044642	BPPU 1.077299	BPPU 1.106430	BP 1.131377
9th	BNPPU 1.025671	BNP 1.034226	BNPPU 1.059303	BNPPU 1.082984	BNPPU 1.107677
10th	HH 1.025256	BNPPU 1.034002	BNP 1.058743	BNP 1.082416	BNP 1.107207
11th	AB 1.011487	AB 1.023170	AB 1.046544	AB 1.069898	AB 1.094306

Notes: See those for Table (11).