Comparing the First-Best and Second-Best Provision of a Club Good: An Example[0]

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Abstract

Excludable and congestible shared goods - club goods (e.g., internet access facilities) - are more prevalent than Samuelsonian public goods. We construct an example to show that the optimal second-best provision level of a club good might exceed its first-best level. This is unlike the usual presumption with pure public goods. We argue that our finding arises because user charges can be levied on club goods; the government need not impose distortionary taxes on other goods to finance them. Thus, the only practical difference between the first and second best in a club economy is that informational constraints prevent the government achieving the right distribution of income in the latter.

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(Preliminary; not be be quoted directly without permission.)

1 Introduction

A large theoretical literature compares the first-best (F-B) and second-best (S-B) provision of a pure public good [e.g., Atkinson and Stern (1974), King (1986), Battina (1991), Wilson (1991a,b), Chang (2000).) and Gaube (2000)]. In the FB allocation, a planner uses unrestricted lump-sum taxation to achieve whatever allocation of private and shared goods it thinks fit, subject only to the economy's overall resource constraint In the SB, the planner is constrained to use distortionary means of financing the shared good and household-specific

[0] We are indebted to Myrna Wooders for providing us with a useful reference and to Marco Mariotti for supplying a copy of his forthcoming paper with Paola Manzini.

budget constraints operate. Although this literature is still inconclusive in general, the presumption - based on a series of special cases - is that there will be underprovision in the SB, both in terms of the level of the good and the fact that provision is taken to a point where the willingness to pay for the good at the margin exceeds its marginal cost. The basic intuition for this presumption is twofold: first, Pigou's (1947) argument that distortionary tax financing makes the total welfare cost of the public good exceed its production cost; second, the idea that the optimal level of a public good should be inversely related to its total cost.[1]

Empirically, there are few examples of pure public goods (defence and broadcasting being perhaps the notable exceptions). Most shared goods seem to be either excludable (as is even broadcasting) or/and congestible to some extent. Club goods are congestible and excludable shared goods [cf. Buchanan (1965)]. Archetypal examples include swimming pools, internet services and tolled trunk roads. This note examines whether the presumed relationship between FB and SB levels of provision for a pure public good carries over to club goods. We will demonstrate that, contrary to the case with pure public goods, the SB in a club economy is quite likely to be characterised by overprovision if distributional considerations predominate in the FB. This finding is very similar to Gaube's (2000) for a pure public good but, as we will argue below, for rather different reasons.

There are well known difficulties in modelling club goods [see, e.g., Fraser and Hollander (2001)]. Chief among these are the need to ensure self-selection of individuals who differ and the mutual consistency between their utilisation of the club and the level of quality which they perceive. Special problems arise when we seek to compare the FB and the SB. In the conventional analysis of a pure public good, it is usually assumed that the good is of unvarying quality, irrespective of the number of users, or the level of provision is taken to be synonymous with quality, with the focus simply on that level. However, club goods of a given type can differ in two dimensions: the quantity and the quality of provision (e.g., the size of the swimming pool and its average level of congestion). Thus, in comparing levels of provision in the FB and SB, we really need some notion of quality-adjusted quantity. In this paper, we will finesse this difficulty by focusing on the cases where FB and SB levels of quality coincide, leaving the comparison to be made only between the respective quantities of provision (and, of course, the numbers of users) in the two cases.

For our analysis, we will employ the Fraser-Hollander model of second-best club provision [cf.: Fraser and Hollander, Cornes and Sandler (1996), Fraser (2000)]. In this model, which builds on the approach of, e.g., Brito and Oakland (1980) and Fraser (1996) for excludable public goods, atomistic consumers confront a per visit price, facility size and conjectured quality for a club good. Taking these as parametric, they self-select to club membership or otherwise. In any Nash equilibrium which results, their simultaneous actions determine the level of club congestion, hence quality, which they confront. In turn, an

[1] See Gaube (2000) on this presumption.

entrepreneurial club good supplier can use the demand schedule, which it (correctly) anticipates will be generated by the consumers' joint actions, to determine the optimal price and level of facility provision which it should offer to fulfil its objectives.

2 The Model

Although there are many types of clubs in practice, we restrict attention to a single-club economy for simplicity.[2] Suppose there are N consumers, all having an identical utility function, $U[\cdot]$. This is defined over the quantity, x, of a private consumption good, visits to or use of a club good, v, and its quality, q. The private good is the numeraire and is a necessity. The club good is not a necessity and need not be demanded at low incomes. To make the analysis interesting, we focus on cases where individuals with sufficiently low incomes will choose not to consume the club good. We assume:

(A.1) U is strictly concave increasing in x, concave increasing in v and non-decreasing in q.

(A.2) Consumers exogenously given incomes $m \in \left[\underline{M}, \overline{M}\right]$ have an absolutely continuous density, $dF(m)$. This density is known to the government or any other club supplier, but they cannot identify the income of a given person for tax or price discrimination purposes.

As we assume exogenous income, there are no incentive effects associated with providing and financing the club good. We specialise $F(m)$ presently.

We also assume that a club's quality is increasing in its facility size, y, and decreasing in its aggregate utilisation, V. Thus,

(A.3) $\partial q(y;V)/\partial y \equiv q_1(y;V) > 0; \partial q(y;V)/\partial V \equiv q_2(y;V) < 0$

The facility size will simply be measured by the expenditure on the club: one unit of expenditure purchases one unit of "facility."

To make the analysis tractable, suppose further that the quality function is homogeneous of degree zero in y and V:

(A.4) $q(y;V) \equiv q(y/V); q' > 0$

(A.4) is the form most commonly utilised in the literature. When the quality function is of this form, quality depends solely on the level of facility provision per use of the club. It is well known that this is the only form for which the FB "toll," if levied, would result in the club breaking even (Kolm, 1974; Mohring and Harwitz, 1962). Here, the FB "toll" is club members' identical marginal willingness to pay for a marginal visit by foregoing private consumption and must equal the value of the quality degradation that marginal visit imposes on club users.

Finally, we will restrict attention to the two families of utility functions for which optimal quality provision in the club is independent of the income distribution if (A.4) holds [Fraser (2000)]:

[2] Even the most sophisticated comparisons of FB and SB provision of pure public goods that allow for many private goods [e.g., Gaube (2000)] consider only one public good.

(A.5) Either all consumers have utility function (a) $U(x,v,q) \equiv u(x,vq)$, or all have utility function (b) $U(x,v,q) \equiv u(x,ve^{q/k})$, for some scalar $k > 0$.

For brevity, we actually only consider explicitly utility functions of the form (A.5)(a). But, it is worth stressing our results extend to utilities (A.5)(b).

The First Best

In the FB, the government has full information about consumers' incomes. Thus, it is able to pool resources to achieve any distribution of private and club good consumption, hence welfare, it thinks fit, subject to the economy's overall endowment. As everyone has the same utility function, in the FB an utilitarian government equalises all consumers' utilities. It chooses the levels of club provision and private good consumption which maximises utility with everyone treated equally.

Denote facility provision per use of (visit to) the club by p - i.e., $p \equiv y/V$. Then, given (A.4), $q = q(p)$. We will adopt the normalisation $q(0) = 0$. Let $\overline{m}$ denote mean income. Suppose (A.5)(a) holds. The FB problem can now be stated as:

$$\max_{p,v} u[\overline{m} - pv, vq(p)] \tag{1}$$

Using (*) to indicate the FB, the two first-order conditions (FOC) characterising an FB optimum are:

$$\left.\begin{array}{c} -u_1[\overline{m} - p^*v^*, v^*q(p^*)]p^* + u_2[\overline{m} - p^*v^*, v^*q(p^*)]q(p^*) \le 0 \\ v^* \ge 0 \end{array}\right\} \text{(CS)} \tag{2}$$

$$\left.\begin{array}{c} -u_1[\overline{m} - p^*v^*, v^*q(p^*)] + u_2[\overline{m} - p^*v^*, v^*q(p^*)]q'(p^*) \le 0 \\ p^* \ge 0 \end{array}\right\} \text{(CS)} \tag{3}$$

At an interior solution, the FOCs reduce to

$$p^*q'(p^*) = q(p^*) \tag{4}$$

This identifies the unique p^* if (A.5)(a) holds. It is also the unique p^* for which the quality provision per unit of expenditure is maximised. Note also from (2) that, if $v^* = 0$,

$$-u_1(\overline{m}, 0)p^* + u_2(\overline{m}, 0)q(p^*) \le 0 \tag{5}$$

If the club good is normal, when (5) holds with equality it identifies a unique mean income, m^* say, below which $v^* = 0$ and above which $v^* > 0$. The FB level of facility provision is then given by

$$y^* = Np^*v^* \tag{6}$$

The Second Best

In the SB, the government does not know each household's income and cannot redistribute between them. It can only fix the quality provision of the club good and, by using the revenues derived from a break-even per visit toll levied on the facility when consumers self-select, the overall level of provision. This situation is superficially similar to the SB with a pure public good analysed in the literature. In the latter (epitomised by Gaube (2000)), the government sets the welfare maximising level of the public good and tax rate(s) on private commodities, subject to breaking even, given optimal choices by consumers. Because of the non-excludability of a pure public good and the consequent preference revelation problem, the government cannot charge directly for it in the SB. Instead, it must be financed by distortionary tax(es) on other goods. Conversely, because of the excludability of a club good, it can be charged for directly, mitigating both the free rider problem and the need to impose distortionary taxes on other goods[3]. Thus, the SB nature of the government's problem in supplying a club good lies mainly in the fact that it is unable to levy unrestricted lump sum taxes and thereby obtain the "right" distribution of income. We will see that this means that, when everyone is identical and there are no distributional concerns, the FB and SB coincide in our club model, unlike in the case with a pure public good.

Suppose now the government announces a toll p which it uses to finance the quality provision per visit. It can be shown [Fraser (2000)] that, given (A.4) and (A.5)(a), it will choose the FB p, p^*, in the SB. A household with income m will then choose its club utilisation to maximise utility, solving

$$\underset{v}{Max}: u[m - p^* v; vq(p^*)] \tag{7}$$

and the resulting FOC (with ** indicating SB magnitudes)[4]

$$\left.\begin{array}{c} -u_1[m - p^* v^{**}; v^{**} q(p^*)]p^* + u_2[m - p^* v^{**}; v^{**} q(p^*)]q(p^*) \leq 0 \\ v^{**} \geq 0 \end{array}\right\} (CS) \tag{8}$$

Notice from (8) that the consumer with income m will not buy the club good if

$$-u_1(m; 0)p^* + u_2(m; 0)q(p^*) \leq 0 \tag{9}$$

By inspection of (5) and (9) holding with equality, it is obvious that the m which leaves a consumer indifferent between making visits and otherwise in the

[3] As the club is modelled as a luxury good, the government will not impose distortionary taxes on private goods to finance it.

[4] In general, an household will choose the v^{**} which maximises its utility and then choose to be a club user if, at that v^{**}, it obtains utility at least as great as from spending all its income on the private good. If utility takes a different form from those in (A.5), an household might get utility which is less than that from consuming the private good alone at a $v > 0$ which satisfies the counterpart of (8)(i) with equality. See Fraser and Hollander (2001) for such intricacies.

SB is precisely the level of mean income which leaves the government indifferent between providing the club good and not in the FB. I.e., denoting the m which solves (9) with equality by m^{**}, we have $m^{**} = m^{*}$. It is also immediately apparent from this comparison that if everyone is identical, thus everyone has mean income $m = \overline{m}$, then each would choose the FB level of club good consumption, v^{*}, in the SB. In that event, we would have $y^{*} = y^{**}$.[5]

Returning to the case of non-identical individuals, the penultimate observation suggests why we might then expect $y^{*} < y^{**}$. Even if $\overline{m} \leq m^{*} = m^{**}$, thus $y^{*} = 0$, we will have $y^{**} > 0$ provided there exists some consumer(s) with income(s) $m > m^{**}$. Of course, the comparison is only non-trivial if $\overline{m} > m^{**}$ - i.e., if mean income is sufficiently high for the government to wish to supply the club good. We assume this from hereon.

If (9) holds with equality for some $m^{**} \in \left[\underline{M}, \overline{M}\right]$, it can be shown that all with income $m > m^{**}$ will use the club and those with $m < m^{**}$ will not. Club users will satisfy (8)(i) with equality. We can invert this to obtain their optimal club usage, $v^{**}(m; p^{*})$. The break-even SB level of club facility provision will then be

$$y^{**} = N p^{*} \int_{m^{**}}^{\overline{M}} v^{**}(m; p^{*}) dF(m) \tag{10}$$

3 An Example

To compare y^{*} given by (6) with y^{**} given by (10), we will specialise the utility and distribution functions further.[6] Suppose the utility function takes the following form (an extension of the linear expenditure system to allow for zero club consumption):

(A.6) $u(x; vq) = (x - \overline{x})^{(1-\gamma)}(vq + \varepsilon)^{\gamma}$, for scalars $\overline{x}, \varepsilon > 0$, $1 > \gamma > 0$, $\overline{x} < \underline{M}$.

Suppose also that the population distribution function is Pareto with $\overline{M} = 1$:

$$\text{(A.7)}\ F(m) = \begin{cases} 0; & m < \underline{M} \\ 1 - (\underline{M}/m)^{\alpha}; & m \geq \underline{M} \end{cases}$$

In (A.7), $\alpha > 0$ is a parameter and $\alpha > 2$ is required for the variance of income to be well-defined. The mean income is now given by $\overline{m} = \alpha\underline{M} = (\alpha - 1)$; thus $\alpha > 1$ is required for mean income to be well-defined.[7]

Assuming an interior solution, each consumer's optimal club usage in the FB can be shown to equal

[5] This coincidence of the FB and SB with identical individuals is derived here for the families of utility and congestion functions in (A.4) and (A.5)(a). It can be shown to hold for all well-behaved utility functions $U(x; v; q)$ if (A.4) holds.

[6] al-Nowaihi and Fraser (2001) consider general utility and distribution functions.

[7] See Degroot (1971) and Lambert (1993) on properties of the Pareto distribution.

$$
\begin{aligned}
v^* &= \frac{\gamma}{p^*}(\overline{m} - \overline{x}) - \frac{(1-\gamma)}{q(p^*)}\varepsilon \\
&= \frac{\gamma}{p^*}\left[\left(\frac{\alpha}{\alpha-1}\right)\underline{M} - \overline{x}\right] - \frac{(1-\gamma)}{q(p^*)}\varepsilon \equiv \frac{\gamma}{p^*}\left[\left(\frac{\alpha}{\alpha-1}\right)\underline{M} - m^*\right]
\end{aligned} \tag{11}
$$

where $v^* \geq 0$ requires

$$
\overline{m} \geq \overline{x} + \frac{(1-\gamma)p^*\varepsilon}{\gamma q(p^*)} \equiv m^* \tag{12}
$$

To make the problem interesting, we assume that a strict inequality holds in (12). The FB level of club provision is then

$$
y^* = Np^*v^* = \frac{\gamma N}{\alpha - 1}[\alpha\underline{M} - (\alpha - 1)m^*] \tag{13}
$$

In the SB, the consumer with income m solves the problem (7) to yield optimal club usage given by

$$
v(m) = \frac{\gamma}{p^*}(m - \overline{x}) - \frac{(1-\gamma)}{q(p^*)}\varepsilon = \frac{\gamma}{p^*}(m - m^*) \tag{14}
$$

(Again, $v(m) > 0$ if $m > m^*$.) Thus, the SB level of club provision is

$$
y^{**} = N\int_{m^*}^{\infty}\gamma(m - m^*)\alpha\underline{M}^{\alpha}m^{-(1-\alpha)}dm = N\gamma\underline{M}^{\alpha}m^{*(1-\alpha)}/(1-\alpha) \tag{15}
$$

Hence

$$
\begin{aligned}
y^{**} - y^* &= \tfrac{\gamma N}{\alpha-1}\underline{M}^{\alpha}m^{*(1-\alpha)} - \tfrac{\gamma N}{\alpha-1}\alpha\underline{M} - (\alpha-1)m^* \\
&= \left(\frac{\gamma N}{\alpha-1}\right)m^{*(1-\alpha)}\left[\underline{M}^{\alpha} - m^{*\alpha} + \alpha m^{*(\alpha-1)}(m^* - \underline{M})\right]
\end{aligned} \tag{16}
$$

To sign $y^{**} - y^*$, we will use the following theorem.

Theorem 1 $\alpha > (1 - z^{\alpha})/(1 - z)$ if $\alpha > 1, 1 > z > 0$.

Proof. Theorem 1 follows from two lemmas.

Lemma 1. Let $f(z) = 1 - z + z\ln z$, $1 > z > 0$. Then $f(z) > 0$.

Proof of Lemma 1. $f'(z) = -1 + 1 + \ln z = \ln z < 0$, $f(1) = 1 - 1 + 1\ln 1 = 0$.

$\Rightarrow f(z) > 0$ for all $z \in (0,1)$.

Lemma 2. Let $f(\alpha) = \alpha - (1 - z^{\alpha})/(1 - z)$, $\alpha > 1$, $1 > z > 0$. Then $f(\alpha) > 0$.

Proof of Lemma 2. $f'(\alpha) = 1 + \frac{z^{\alpha}\ln z}{1-z}$

$f''(\alpha) = \frac{z^{\alpha}(\ln z)^2}{1-z} > 0$ for $1 > z > 0$

$f(1) = 1 - \frac{1-z}{1-z} = 0$

$f'(1) = 1 + \frac{z \ln z}{1-z} = \frac{1}{1-z}[1 - z - z\ln z] > 0$ by Lemma 1.

$\Rightarrow f(\alpha) > 0$ for all $\alpha > 1; 1 > z > 0$.

Theorem 1 follows from Lemma 2. ■

Theorem 1 and 16 now enable us to prove our central result.

Theorem 2 *If $u(x; vq) = (x - \overline{x})^{(1-\gamma)}(vq + \varepsilon)^{\gamma}$ and the population density is Pareto ((A.7)), then $y^{**} > y^{*}$.*

Proof. From (16),

$$y^{**} > y^{*} \Leftrightarrow \alpha > m^{*(1-\alpha)}(m^{*\alpha} - \underline{M}^{\alpha}) / (m^{*} - \underline{M}) = m^{*}\left(\frac{m^{*\alpha}}{m^{*\alpha}} - \frac{\underline{M}^{\alpha}}{m^{*\alpha}}\right) / (m^{*} - \underline{M})$$

$$= \left[1 - \left(\frac{\underline{M}}{m^{*}}\right)^{\alpha}\right] \Big/ \left[1 - \left(\frac{\underline{M}}{m^{*}}\right)\right] \equiv (1 - z^{\alpha})/(1 - z) \qquad (17)$$

letting $z \equiv \underline{M}/m^{*} < 1$. Were α integer-valued, $(1 - z^{\alpha})/(1 - z)$ would represent the sum to $\alpha - 1$ terms of a geometric progression with first term 1 and common ratio between successive terms of $z < 1$. It is then trivial to show that $\alpha > (1 - z^{\alpha})/(1 - z)$ for integer $\alpha \geq 2$. For other values of α, we can use Theorem 1. As $\alpha > 1$ is required for mean income to be defined and $\alpha > 2$ for the variance, we can conclude from (17) and Theorem 1 that $y^{**} > y^{*}$. ■

4 Discussion and Conclusion

Is it reasonable to believe that club goods will be underprovided in the second best, as is usually presumed with pure public goods? This paper shows, via an example, that this is unlikely to be so. Unlike pure public goods, club goods can be charged for directly. A government need not impose distortionary taxes on other goods to finance clubs. This means that, practically, the only important source of difference between the first best and the second best in a club economy is the government's inability to achieve the correct distribution of income in the latter due to informational constraints. In the SB, incomes differ and the relatively rich are the one who are more likely to buy the club good. The government has to fix the size of the club facility to satisfy their demand for it at the SB toll and quality. It is this need to meet the relatively high club demand by the relatively wealthy which results in "overprovision" in the SB compared with the FB. Unlike the case with a pure public good, the government cannot use the club good as a redistributive device because it cannot price discriminate (by assumption), not everyone uses it and those that do use different amounts. Thus, although our explanation for overprovision in the club SB hinge on distributional considerations as does Gaube's for pure public goods, our mechanisms are very different.

Our observations have been derived from a club model in which FB and SB club "tolls" and qualities coincide, leaving comparison only to be made between the facility sizes (and numbers of users) in the two cases. If we depart from these circumstances, it will still be possible for the government to finance the club good by user charges rather than distortionary taxes if it wishes. FB and SB "tolls", qualities and facility sizes will then differ in general, but these differences will again primarily reflect distributional considerations [Fraser and Hollander (2001)].

Note, finally, that our results can be regarded as complementary to those of Scotchmer (1985) and Manzini and Mariotti (M&M, 2001, forthcoming). They also find evidence of "excesses" in some aspects of club good provision. Scotchmer shows that the equilibrium number of firms which enter a market to supply a club facility will exceed the efficient number - there will be too many clubs. M&M study of a three-consumer non-cooperative game of club formation establishes a "tragedy of clubs": the possibility that there will be excess entry of members into a single club. Both these analyses consider identical consumers (with M&M's having market power) while we consider atomistic, heterogeneous, price- and quality-taking ones Unlike M&M's club, our SB club has too few members - it is the provision for them which is socially excessive.

5 References

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