University of Leicester

Department of Economics

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Wojciech Charemza, University of Leicester

Carlos Díaz, University of Leicester

Svetlana Makarova, University College London

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EX-POST INFLATION FORECAST UNCERTAINTY AND SKEW NORMAL DISTRIBUTION: 'BACK FROM THE FUTURE' APPROACH

WOJCIECH CHAREMZA*, CARLOS DÍAZ* AND SVETLANA MAKAROVA**

* University of Leicester, UK

** University College London, UK

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ABSTRACT

Empirical evaluation of macroeconomic uncertainty and its use for probabilistic forecasting are investigated. New indicators of forecast uncertainty, which either include or exclude effects of macroeconomic policy, are developed. These indicators are derived from the weighted skew normal distribution proposed in this paper, which parameters are interpretable in relation to monetary policy outcomes and actions. This distribution is fitted to forecast errors, obtained recursively, of annual inflation recorded monthly for 38 countries. Forecast uncertainty term structure is evaluated for U.K. and U.S. using new indicators and compared with earlier results. This paper has supplementary material.

1. INTRODUCTION

Although the concept of uncertainty has been widely used in macroeconomics, there is not much agreement regarding its exact meaning. On the one hand, there is a substantial development in the literature on uncertainty understood in the Knightian sense, that is as the *ex-post* unobservable phenomenon (see e.g. Bloom, 2009, Baker, Bloom and Davis, 2013**,** Tuckett *et al.*, 2014, Jurado, Ludvigson and Ng, forthcoming). On the other hand, the concept of uncertainty has also been used in the non-Knightian sense in relation to particular macroeconomic indicators like inflation and output growth, where the uncertainty can be checked *ex-post* (e.g. Zarnowitz and Lambros, 1987, Bomberger, 1996, Giordani and Söderlind, 2003, and a huge literature which follows).

In this paper, we concentrate on the non-Knightian uncertainty and, in particular, on the relationship between the *ex-post* and *ex-ante* uncertainties. The *ex-post* uncertainty is usually expressed by the past forecast errors (the greater is variation of these errors, the greater is uncertainty. In this paper we think of uncertainty in the sense of distribution, characterized by a whole set of parameters, and not only dispersion. Experts formulate the ex-ante uncertainty on the basis of some indicators describing the future. The experts express uncertainty either directly, by formulating probabilities regarding future realisations or indirectly, if the number of experts come up with different projections. The concepts of *ex-post* and *ex-ante* uncertainties are in fact similar; the forecasters err because they did not have the perfect knowledge of the future at the moment of making the forecast, and the experts in their beliefs regarding *ex-ante* uncertainty are also 'uncertain' for the very same reason. In a stationary world, if there is no policy intervention undertaken on the basis of evaluations of the future, the *ex-post* and *ex-ante* uncertainties should coincide in that sense that their distributions should match. However, this is rarely, if ever, the case. The discrepancy has been first noticed while comparing the probabilistic forecast formulated on the basis of *a priori* beliefs regarding uncertainty, usually made at some central banks (see e.g., Dowd's, 2007, critique of the Bank of England probabilistic forecasts). With the increase in availability of panels of forecasters' data, e.g. the Survey of Professional Forecasters (SPF) for the U.S., the Euro area and a number of other countries, the problem becomes more evident. In particular, Clements (2014) noticed substantial discrepancies between the uncertainty measures resulting from SPF for U.S. and that derived on the basis of forecast errors (for earlier results see e.g. Zarnowitz and Lambros, 1987, Giordani and Söderlind, 2003, Engelberg, Manski and Williams, 2009, and Rich and Tracy, 2010). Numerous explanations have been offered regarding the (in)adequacy of assessing the *ex-ante* uncertainty with the use of panels of forecasts (see e.g. Lahiri and Sheng, 2010; Patton and Timmermann, 2010) and the overall quality of SFP results (e.g. Bowles *et al.*, 2007; Andrade and Le Bihan, 2013).

We claim that the *ex-post* and *ex-ante* uncertainties might indeed differ, even if measured properly. The difference comes from the fact that economic policy decisions undertaken on the basis of *ex-ante* uncertainty indicators might, in turn, affect the distribution of the *ex-post* uncertainty. With this in mind, we introduce a new statistical distribution, called the *weighted skew normal* distribution (WSN), which parameters can be interpreted as containing information on the effects of policy decisions on the uncertainty. By estimating these parameters, it is possible to retrieve this information and, consequently, compute a measure net of it which, conceptually, is analogous to *ex-ante* uncertainty.

Section 2 of the paper defines main concepts related to the *ex-post* and *ex-ante* uncertainties. The WSN distribution is introduced in Section 3 in the context of inflation forecast uncertainty. In Section 4 we propose a new measure of uncertainty, analogous to that of *ex-ante* uncertainty, developed from the WSN. In Section 5 we apply the WSN distribution to approximate the empirical distributions of inflation forecast errors (scaled for the predictable variance) for 38 countries and use for constructing term structure measures of inflation forecasts based on *ex-post* approximation of *ex-ante* uncertainty. The empirical results confirm the proposed interpretation of the WSN parameters and also show that, in a large number of cases, it fits better to the data than some alternative skew normal distributions. For U.K., recursively estimated parameters of the WSN track main changes in monetary policy. Section 6 concludes. Additional results and, in particular, the derivation of the density and moment generating functions of WSN, are given in the Supplementary Materials.

2. MOTIVATION*: EX-POST* AND *EX-ANTE* UNCERTAINTY

We define *ex-post* net forecast uncertainty, that is net of all information publically available at the time of making the forecast, $t-h$, for time *t*, as the root mean square error (RMSE) of the random variable $U_{t,h}$ given by:

$$U_{t,h} = \frac{Z_t - \mu_{t|t-h}}{\sigma_{t|t-h}} \sigma_{t,h} \sim D_P(\mu_{t,h}, \sigma^2_{t,h}) \quad , \tag{1}$$

where Z_t is the observed macroeconomic phenomenon (inflation, in our case), $\mu_{t|t-h}$ is its forecast made with the use of publically available information at time $t-h$ and known to all agents (referred later as the baseline forecast), $\sigma_{t|t-h}$ is a forecast of conditional standard deviation, also made at time $t-h$ with the use of the same information as for $\mu_{t|t-h}$, $\mu_{t,h}$ and $\sigma_{t,h}$ are respectively mean and unconditional standard deviation of forecast errors, observed at time *t*. $D_P(\cdot,\cdot)$ denotes *ex-post* distribution with finite first three moments, not necessarily symmetric.[1] Further on we assume that $\mu_{t|t-h}$ and $\sigma_{t|t-h}$ are efficiently estimated, usually by econometric methods, so that the model uncertainty problem is not considered here (for a recent development on model uncertainty see e.g. Garratt, *et al.*, 2009),. Later in the text we refer to observations on $U_{t,h}$ as *U*-uncertainties.

As information on $U_{t,h}$ is known at time *t* and not at the moment the forecast is made, that is at $t-h$, the decision-makers are usually interested in the *ex-ante* measure of uncertainty, which might be known at time $t-h$. We define the *ex-ante* uncertainty as the standard deviation $\sigma_{A,t|t-h}$ of the random variable $U_{t|t-h}$:

$$U_{t|t-h} = \frac{Z_{t|t-h} - \mu_{t|t-h}}{\sigma_{t|t-h}} \sigma_{A,t,h} \sim D_A(0, \sigma^2_{A,t,h}) \quad . \tag{2}$$

As the variable $Z_{t|t-h}$ is not observable, $U_{t|t-h}$ is not observable as well. Hence, the problem arises with obtaining the reliable estimate of $\sigma_{A,t,h}$. Unlike $\mu_{t|t-h}$ and $\sigma_{t|t-h}$, the estimates of

[1] The explained conditional variance, e.g. by a GARCH/ARCH process, is often regarded as a component of uncertainty (see e.g. Fountas, Karanasos and Kim, 2006). In Supplementary Materials we include results obtained under the assumption that such variance contributes to uncertainty that is for crude, forecast errors, not scaled by $\sigma_{t|t-h}$.

$\sigma_{A,t,h}$ from publically available data are often regarded by the practitioners as unreliable (unless it is realistically assumed that $\sigma_{A,t,h} = \sigma_{t|t-h}$, that is that entire uncertainty can be explained autoregressively). The practitioners tend to estimate $\sigma_{A,t,h}$ using subjective judgements based on information that are not publically available and/or applying their loss functions.

The estimates of $\sigma_{A,t,h}$ have been–often obtained using surveys of forecasts like SPF, either as a standard deviation of individual point forecasts, or in a more elaborate way (see e.g. Giordani and Söderlind, 2003; Wallis, 2005; Clements, 2006; Peng and Yang, 2008; Lahiri, Peng and Sheng, 2014). This gives a truly *ex-ante* measure, based on transparent and intuitive assumptions. However, it also has some disadvantages. It is costly and creates obvious difficulties in completing a competent panel of experts. Another problem can arise from the fact that the participants in the panel of forecasters usually use similar sets of information and similar forecasting techniques. This makes a cross section of forecasts, either deterministic or probabilistic, being close to each other, with small inter-forecasts variability. Another problem seems to be the inability of the forecaster to express statements in an unbiased way. This is supported by evidence based on the outcomes of psychological experiments, suggesting that an unbiased probabilistic assessment might not be possible (see e.g. Bolger and Harvey, 1995; Soll and Klaymen; 2004; Hansson, Juslin and Winman, 2008; So, 2013).

We are assuming, rather pessimistically, that relevant information on $\sigma_{A,t,h}$ available at time $t-h$, is either not publically available or not reliable. However, at time $t-h$ there is information on the distribution of past forecast errors. Under stationarity of $U_{t,h}$ in (1), parameters of its distribution should be the same at time *t* as they were at time $t-h$ which gives rise to the widespread in central banks practice of estimating *ex-ante* uncertainty by *ex-post* uncertainty. Indeed, the distributions of $U_{t|t-h}$ and $U_{t,h}$ coincide if $\mu_{t,h} = 0$ and $\sigma^2_{t,h} = \sigma^2_{A,t,h}$, that is, if the dispersion of the *ex-post* forecast errors is the same as the ex-ante uncertainty. This, however, is rarely the case, as noticed by Dowd (2007), who compared dispersion of the Bank of England fan charts with the dispersion of historically observed forecast errors, and by Clements (2014), who did a similar comparison using the U.S. SPF's data. They have attributed the discrepancy to the fact that the forecasters in SPF were at times either under- or overconfident in assessing uncertainty. We argue that the *ex-post* and *ex-ante* uncertainty might indeed differ even if measured properly. The reason for this is that the economic decision makers act on the basis of private information they could have regarding Z_t, which are not available to the researcher, but available (perceived) to the decision makers altering, as a result, the distribution of $U_{t,h}$. This enables identification of the effects of the policy action by examining *U*-uncertainties and, in particular, by evaluating parameters of its distribution, as in (1), which can be interpreted as reflecting the impact of economic policy onto forecast uncertainty. With some additional assumptions, we will also formulate an alternative measure of *ex-ante* uncertainty.

3. MODEL OF *EX-POST* INFLATION UNCERTAINTY

In this paper, we concentrate on the uncertainty of inflation. We assume that inflation, π_t, is a random variable that can be split into two parts: predictable and unpredictable from the publically available information regarding the past. However, the component unpredictable from the past can still be forecastable (improved) by methods other than those of time series analysis ('fine tuning', or experts' corrections based on some sort of inside information, or subjective weighting of information). We follow central banks' tradition of two-stage probabilistic forecasting, which consists of, first, conducting past-related econometric forecast using publically available data and then assessing the uncertainty relatively to this forecast. (see e.g. Pinheiro and Esteves, 2012). Consequently, considering *h*-step ahead predictions, we decompose π_t as:

$$\pi_t = \hat{\pi}_{t|t-h} + e_{t|t-h} \quad , \tag{3}$$

where $\hat{\pi}_{t|t-h}$ is the baseline point forecast, usually obtained from time series econometric model and $e_{t|t-h}$ are the baseline forecast errors.

According to (1), we define the *ex-post* net forecast uncertainty as the RMSE of $U_{t,h} = \left(\sigma_{t,h} / \sqrt{\hat{\sigma}^2_{t|t-h}}\right) e_{t|t-h}$ where $\hat{\sigma}^2_{t|t-h}$ is an *h*-step ahead forecast of the conditional variance and $\sigma_{t,h}$ is the unconditional standard deviation of $e_{t|t-h}$.

As we retrieve the distributional characteristics of the *U*-uncertainties from a historical series of observations on $U_{t|t-h}$ obtained by the 'pseudo-out of sample' way, we assume that the series constitutes a stationary third-order ergodic process. It essentially implies that there are no structural breaks and outliers in the baseline forecast, and the ARCH/GARCH process is properly specified. As further on we consider $U_{t|t-h}$ separately for each forecast horizon, we simplify the notation and drop the subscripts, so that $U \overset{def}{=} U_{t|t-h}$.

Let us consider the following specification of *U*:

$$U = X + \alpha \cdot Y \cdot I_{Y > \bar{m}} + \beta \cdot Y \cdot I_{Y < \bar{k}} \quad , \qquad (X,Y) \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 \end{bmatrix} \right), \tag{4}$$

where $I_{\{\bullet\}}$ is the indicator function of a set $\{\bullet\}$, $\alpha \in \mathbb{R}$, $\beta \in \mathbb{R}$, $\bar{m} \in \mathbb{R}$, $\bar{k} \in \mathbb{R}$ and $-1 < \rho < 1$. Random variable *U* as given by (4), defines a family of distributions that we name the *weighted skew-normal* and abbreviate as $\mathrm{WSN}_\sigma\left(\alpha, \beta, \bar{m}, \bar{k}, \rho\right)$.

In order to provide interpretation of parameters in (4), it should be first noted that the random variable *X* contains two elements usually regarded as characteristics (or types) of uncertainty: ontological uncertainty, related to the purely random (unpredictable in mean) nature of future inflation, and epistemic uncertainty, related to fragmentary and incomplete knowledge of the forecaster (for general discussion of these concepts see e.g. Walker *et al.*, 2003, and for application in inflation forecasting context see Kowalczyk, 2013).[2] The epistemic element in *X* can, in fact, be predictable, e.g. by experts (private forecasters) who base their judgements on the analysis of non-quantitative data, expected effects of current

[2] Walker *et al.* (2003) and Kowalczyk (2013) talk of variability uncertainty rather than of ontological uncertainty. Walker's *et al.* classification has been criticised for incompleteness and tautology (Norton, Brown and Mysiak, 2006). Other definitions and classifications, also often criticised, are frequently used in different sciences. It is also important to acknowledge the relative and time-varying nature of ontological uncertainty in economics (see e.g. Lane and Maxfield, 2005).

political decisions, apply their own loss functions, *etc*..[3] That is why we refer to *X* as to *quasi-uncertainty*. These private forecasters act like the SPF panellists, with the difference being that they have common knowledge of the baseline forecast, so that their forecasts are formulated conditionally and in relation to $\hat{\pi}_{t|t-h}$ rather than unconditionally. Because of that we regard them as improvers rather than forecasters. Another difference is that their outcomes are not observable by the researcher (there is no data on private forecasts).

To what extent these private forecasts represented by *Y* are 'educated', or accurate, is expressed by the correlation coefficient ρ between *X* and *Y*. If either *X* is totally unpredictable (that is, if quasi-uncertainty becomes fully ontological) or if the private forecasters are ignorant, then $\rho = 0$. The higher is the value of ρ, the more epistemic becomes *X* and/or the private forecasters become more competent. For this reason we refer to *Y* as to imperfect knowledge (the knowledge becomes perfect if $\rho = 1$). It is reasonable to assume that the variances of *X* and *Y* are identical. This assumption is grounded within the conjecture that, in the absence of epistemic element in quasi-uncertainties, disagreement between the improvers has the same variability as ontological uncertainty. In another words, and relating to Clements (2014) terminology, we assume that the private forecasters are confident, that is are neither over- nor underconfident. Therefore, we will denote both variances as σ^2. We also assume that the improvers' forecasts cannot be negatively related to quasi-uncertainties *X*, that is $0 \le \rho < 1$.

If $\alpha = \beta = 0$, that is if the distributions of *U* and *X* are identical then the *ex-post* and *ex-ante* uncertainties, as defined in (1) and (2) coincide. However, signals from private forecasters are inputs for possible monetary policy actions and this might cause the distribution of *U*-uncertainties to differ from that of *X*, which will be the case if either α or/and β differs from zero. There is no need to assume anything specific regarding this policy except for the fact that it is based on private forecasts and that the policy undertaken at time $t-h$ might affect inflation at time *t*. The model requires the rather strong assumption that the baseline forecast $\hat{\pi}_{t|t-h}$ does not stimulate monetary policy outcomes and that the monetary authorities react to information passed to them through *Y* only. In another words, the baseline forecast $\hat{\pi}_{t|t-h}$ is assumed to be a policy-neutral part of inflation. This assumption can be easily relaxed in a theoretical model. However, its relaxation creates a number of practical and numerical problems, and the analysis of its consequences is left for further studies.

Further four parameters of the model (4), that is $\bar{m}$, $\bar{k}$, α, and β, can be interpreted in the light of actions and outcomes of some inflation-affecting policy. The parameters $\bar{m}$ and $\bar{k}$ denote respectively 'upper' and 'lower' thresholds for imperfect knowledge *Y*. If these thresholds are breached, it gives a signal to the policy makers regarding the necessity of undertaking an anti-inflationary decision (if $\bar{m}$ is breached from below) or pro-inflationary (if $\bar{k}$ is breached from above). Further on, α and β describe the actual outcomes of these decisions. The parameter α tells to what extent anti-inflationary decisions undertaken on the basis of private forecasters' signals are transmitted into the change in inflationary uncertainties and β tells the same for output- stimulating-decisions. Rational behaviour of the policymakers implies that $\alpha \le 0$, $\beta \le 0$, $\bar{m} \ge 0$, $\bar{k} \le 0$.

[3] Our concept of private forecasters is different than the concept of private sector forecasters (see e.g. Frenkel, Rülke and Zimmermann (2013). In our convention *private* means that signals produced by private forecasters are available only for certain recipients and not publically.

For operational simplicity, it is convenient to normalize WSN in such way that σ=1 and define U^* as:

$$U^* = \frac{U}{\sigma} \sim \mathrm{WSN}_1(\alpha, \beta, m, k, \rho) \quad , \tag{5}$$

where $m = \bar{m}/\sigma$ and $k = \bar{k}/\sigma$. The probability density function (*pdf*) of U^* is given by:

$$\begin{aligned} f_{WSN_1}(t) = {} & \frac{1}{\sqrt{A_\alpha}} \varphi\left(\frac{t}{\sqrt{A_\alpha}}\right) \Phi\left(\frac{B_\alpha t - m A_\alpha}{\sqrt{A_\alpha (1-\rho^2)}}\right) + \frac{1}{\sqrt{A_\beta}} \varphi\left(\frac{t}{\sqrt{A_\beta}}\right) \Phi\left(\frac{-B_\beta t + k A_\beta}{\sqrt{A_\beta (1-\rho^2)}}\right) \\ & + \varphi(t) \cdot \left[\Phi\left(\frac{m - \rho t}{\sqrt{1-\rho^2}}\right) - \Phi\left(\frac{k - \rho t}{\sqrt{1-\rho^2}}\right) \right] \quad , \end{aligned} \tag{6}$$

where φ and Φ denote respectively the density and cumulative distribution functions of the standard normal distribution, and:

$$A_\tau = 1 + 2\tau\rho + \tau^2, \qquad B_\tau = \tau + \rho \quad .$$

If, in (4), $\alpha = -2\rho$ and $\beta = m = 0$, the distribution of U coincides with the Azzalini (1985, 1986) skew-normal SN(ξ) distribution with *pdf* $f_{\mathrm{SN}}(t;\xi) = 2\varphi(t)\Phi(\xi t)$, where $\xi = \frac{-\rho}{\sqrt{1-\rho^2}}$. It follows from (6) that the *pdf* of the weighted skew-normal distribution $\mathrm{WSN}_1(\alpha, \beta, m, k, \rho)$ can be interpreted as a weighted sum of *pdf*'s of two Azzalini-type skew normal densities with different ξ's and a *pdf* of the conditional distribution $\left.\frac{X}{\sigma}\right| k \le \frac{Y}{\sigma} \le m$; hence the name for the distribution. Basic characteristics and properties of WSN distribution, generalisations and moment generating function are given in the Supplementary Materials, which also contain plots of density functions and moments of some specifications of WSN distributions which are of interest here.

It immediately follows from (4) that the WSN distribution is symmetric only if $\alpha = \beta = 0$ or if $\bar{k} = -\bar{m}$ and $\alpha = \beta$; otherwise it is asymmetric. This is in line with the consensus that distributions of inflation uncertainties might be skewed (for recent advances see Demetrescu and Wang, 2012). In Supplementary Materials we discuss different types of asymmetry of the WSN distribution. The type of skew distribution usually applied in central banks for constructing fan charts is the two-piece skew normal (see e.g. Wallis, 2004). The presence of skewness in the distribution of inflation uncertainties is described by Wallis (2004) as: 'the degree of skewness shows their collective assessment of the balance of risks on the upside and downside of the forecast'. We argue that skewness might result from widely defined asymmetries in monetary policy actions and outcomes. Our interpretation of WSN distribution implies that, under the assumption of normal distributions of quasi-uncertainties *X* and imperfect knowledge *Y*, inflation uncertainties become skewed if either the outcomes of anti- and pro-inflationary policy differ from each other and/or thresholds defining the expected inflationary 'danger zones' are not symmetric.

4. *EX-POST* MEASURE OF *EX-ANTE* UNCERTAINTY

For given parameters of (4), the *ex-post* measure of uncertainty is given by $RMSE_U$:

$$RMSE_U = \sqrt{var(U) + bias^2(U)} = \sqrt{E(U^2)} \quad .$$

Reconsidering the differences between the *ex-post* and *ex-ante* uncertainty distributions, as defined by (1) and (2) with the use of the terminology introduced in Section 3 it can be stated that the distributions of the theoretical *ex-post* and *ex-ante* uncertainties should be identical if the policy makers' knowledge of D_A does not affect D_P. Discussion in Section 3 above indicates that this might not be the case, as in time $t-h$ the decision makers act towards reducing the uncertainty using some information from private forecasters which is also contained in D_A, that is some information that constitutes epistemic uncertainty. More precisely, this is information available to the improvers, but not to the econometric modellers who delivered the baseline forecast $\hat{\pi}_{t|t-h}$. Evidently, if $\alpha = \beta = 0$, D_A is coincides with D_P. If there is an epistemic uncertainty which becomes knowledge to the private forecasters and policy makers' action is efficient to a degree, then $RMSE_U$ should usually be smaller than $\sigma_{A,t,h}$.

We suggest deriving a measure that approximates *ex-ante* uncertainty with the use of the parameters of WSN distribution. As this, in fact, this is based on the *ex-post* rather than *ex-ante* approach, we refer to it as the *quasi ex-ante* uncertainty. The ontological uncertainty, which is the non-predictable component in *U*, can partially be extracted as:

$$V = U - E(X\,|\,Y) = U - \rho Y = X - \rho Y + \alpha \cdot Y \cdot I_{Y>\bar{m}} + \beta \cdot Y \cdot I_{Y<\bar{k}} \quad . \tag{7}$$

Although *V* does not contain the epistemic element of *X*, it might be contaminated by it through possible policy outcomes. Further in the text we refer to *V* as *V-uncertainties*. Distribution of *V* is also related to WSN, as:

$$\frac{1}{\sigma\sqrt{1-\rho^2}} V \sim \mathrm{WSN}_1\left(\frac{\alpha}{\sqrt{1-\rho^2}}, \frac{\beta}{\sqrt{1-\rho^2}}, \frac{\bar{m}}{\sigma}, \frac{\bar{k}}{\sigma}, 0\right) \quad .$$

Measure of *V*-uncertainty, which can be regarded as analogous to $\sigma_{A,t,h}$ in (2), is the standard deviation of *V*, that is σ_V. It can be interpreted as a measure of uncertainty under the assumption that the uncertainty in *X* was entirely ontological and therefore not affected by policy action. Comparing $RMSE_U$ and σ_V could provide an idea of the influence that policy decisions might have on the distribution of inflation forecasts.

A straightforward way of doing this is by evaluating the *uncertainty ratio* UR, that is the ratio of squares of σ_V to the squares of $RMSE_U$. Properties of UR are discussed the Supplementary Materials. In particular, it is shown that it can be expressed as:

$$\mathrm{UR} \overset{def}{=} \frac{\sigma_V^2}{RMSE_U^2} = 1 + 2\rho \frac{-(\alpha D_m + \beta D_k) - \rho/2}{RMSE_{U^*}^2} - \frac{\left[E(U^*)\right]^2}{RMSE_{U^*}^2} \quad , \tag{8}$$

where U^* is defined by (5). In particular, $E(U^*) = \alpha \cdot \varphi(m) - \beta \cdot \varphi(k)$, and

$$D_a = \int_{|a|}^{+\infty} t^2 \varphi(t)dt = 1 - \Phi(|a|) + |a|\varphi(a) \quad .$$

UR is equal to unity, if $\rho = 0$ and $bias^2(U) = 0$ or $\rho = -2\left[(\alpha D_m + \beta D_k) + bias^2(U)\right]$. Note that UR does not depend on σ, but rather on the ratios $m = \bar{m} / \sigma$ and $k = \bar{k} / \sigma$.

The deviation of UR from unity represents the effect of the epistemic element on uncertainty (through ρ) and the effect of monetary policy (through $\alpha D_m + \beta D_k$), where α and β reflect the marginal intensity of monetary policy actions, and D_m, D_k reflect the frequency of such actions. Let us define the compound (monetary policy) strength as $S = |\alpha| D_m + |\beta| D_k$; then the formula (8) for UR can be re-written as:

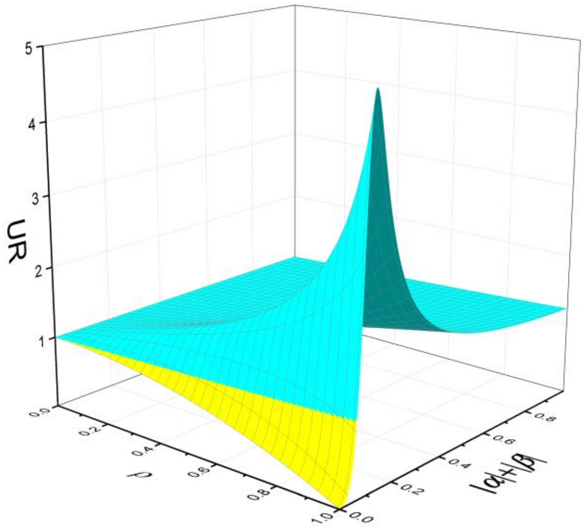
$$\text{UR} = \frac{\sigma_V^2}{RMSE_U^2} = 1 + \frac{2\rho S - \rho^2 - 2(\alpha\varphi(m) - \beta\varphi(k))^2}{1 - 2\rho S + W_{m,k} \cdot S^2} \quad , \qquad (9)$$

where $W_{m,k} = \left[D_m \varphi^2(k) + D_k \varphi^2(m)\right] / \left[D_m \varphi(k) + D_k \varphi(m)\right]^2$.

It is clear from (9) that (for $\alpha < 0, \beta < 0$) $\text{UR}\big|_{S=0} = 1 - \rho^2$. When bias of U is bounded, then $\text{UR}(\rho) \xrightarrow[S \to \infty]{} 1$ for fixed ρ. As a function of S and ρ, UR is (conditionally) unimodal for fixed $\rho > 0$.

Figure 1 plots UR for the symmetric case, where $\alpha = \beta < 0$, $\sigma^2 = 1$, $\bar{m} / \sigma = -\bar{k} / \sigma = 1$, and for different values of ρ, against $|\alpha| + |\beta| = S / D_1$, representing the normalized strength of forecast-induced monetary policy. For the very low strength of the private forecast-induced monetary policy, the UR is smaller than one (yellow area on the graph) and is decreasing with the increase in ρ; that is when the degree of imperfect knowledge is increasing. In this case variance of *U* increases in relation to the variance of *V*. If, on the other hand, the epistemic element is more utilised in the monetary policy, so that its private forecast-induced strength increases up to the point where UR reaches its maximum, inflation uncertainty represented by MSE of *U* decreases in relation to the variance of quasi *ex-ante* uncertainty *V*.

Figure 1: UR for the case where $\sigma^2 = 1$, $\alpha = \beta$, $\bar{m} = -\bar{k} = 1$ and for different values of ρ . Values of UR smaller than one are in a lighter shade (yellow)



It is shown in Supplementary Materials, Part S1.2, that the maximum of UR for a given ρ and $k = -m$ is

$$\mathrm{UR}_{\max}(\rho) = 1 + \frac{4\rho}{\rho(1-4D)/D + 2\sqrt{2(1-\rho^2)/D + \rho^2/(4D^2)}} \quad ,$$

where $D = D_m = D_k$, achieved when $\alpha = \beta = -\left(\rho + \sqrt{8D(1-\rho^2)+\rho^2}\right)/(4D)$. Therefore, the compound strength *S* which maximizes the ratio of the *ex-post* to *ex-ante* uncertainty, is

$$S_{\max}(\rho) = \left(\rho + \sqrt{8D(1-\rho^2)+\rho^2}\right)/2 \ .$$

The ratio of UR to $\mathrm{UR}_{\max}$, called the *normalized uncertainty ratio*, NUR, can be interpreted as a compound, albeit symptomatic, measure of the effects of monetary policy. If NUR=1, then, $\mathrm{UR}=\mathrm{UR}_{\max}$ and the parameters α and β are set at such level that further reduction of the *ex-post* uncertainty expressed by $RMSE_U^2$ by changing α and β is not possible. The smaller is NUR, the greater might be the potential 'room for improvement' regarding the possibility of utilizing the epistemic elements in *X* for reducing the uncertainty.

5. EMPIRICAL RESULTS

In order to evaluate the practical relevance of our results, we have checked how the WSN distribution fits to the empirical data on inflation uncertainties and how it can be used further on for the evaluation of forecast term structure. We have started by computing point forecasts of the CPI (headline) annual inflation measured monthly, recovering $\hat{\pi}_{t|t-h}$ in (3). Clearly, the problem of choosing the appropriate model for obtaining point forecasts $\hat{\pi}_{t|t-h}$ is an important one, as using the wrong model might lead to misrepresentation of the distribution of uncertainties. It is a topic on its own, being extensively discussed in the literature, especially in relation to the effects of recent crisis and post-crisis periods. Experience of the forecasting inflation in U.S. suggests some predictive advantage of single-equation models over the multivariate model, with some evidence favouring autoregressive (Clark and Ravazzolo, 2014), and unobserved components (Stock and Watson, 2007, 2010) models above the others. Advantage of simple univariate models for forecasting, over more complex models for sample sizes smaller than 500 have been confirmed by Constantini and Kunst (2011). Additional complication here is in deciding about the type of volatility. Results of Groen, Paap and Ravazzolo (2013) and Clark (2011) for U.S. show slight superiority of the stochastic volatility models over the more conventional ARCH models. However, for European countries there is some evidence, also rather weak, of the superiority of autoregressive-ARCH models over other models (see e.g. Bjørnland *et al*., 2012, Buelens, 2012). Most likely, forecasts of all these models are to be beaten by the forecasts derived through model averaging (Koop and Korobilis, 2012). In the light of such vastly different conclusions we have decided, for the sake of illustrative simplicity, to resort to a simple autoregressive model with GARCH residuals.

Following Stock and Watson (2007) and many others, we have computed pseudo out of sample forecast errors for 38 countries; that is for 32 OECD countries, 5 BRICS countries (Brazil, China, India, South Africa and the Russian Federation) and Indonesia. The data series for different countries are of various lengths and end at January or February 2013. The longest series, starting in January 1949, is for Canada (770 observations), and two shortest are for Estonia (182 observations) and China (242 observations). The raw CPI data can be downloaded from http://stats.oecd.org/. It is conjectured that if these countries conducted some effective monetary policy, it might in turn affect the distribution of uncertainties. Although members of the European Monetary Union do not have autonomous monetary policy since the creation of the Eurozone, nevertheless decisions of European Central Bank affect inflation uncertainty in individual countries. In our approach, it is not relevant how the monetary policy decisions are made; their effect on uncertainties is what we consider.

For each country, data on *U*-uncertainties have been recovered in the following way. Point forecasts (that is, the estimates of $\hat{\pi}_{t|t-h}$ and $\hat{\sigma}^2_{t|t-h}$) have been made recursively using the estimated autoregressive integrated moving average model (*ARIMA*) for the first recursion period and then by updating it by one observation at a time and re-estimating the model. We have used GARCH(1,1) process in residuals, which allows us to remove variance predictability from the forecast errors. Orders of integration of the series have been initially identified using the battery of tests; namely the traditional GLS-detrended and optimal point unit root tests (see Ng and Perron, 2001, and Perron and Qu, 2007), and tests allowing for the presence of the structural breaks under the null and alternative (see Carrion-i-Silvestre, Kim and Perron, 2009). As the test gave overwhelmingly consistent results with that of Gómez and Maravall (1998), we have decided to use the automated differencing and lag polynomials selection procedure by Gómez and Maravall (1998) based on minimisation of the Bayesian Information Criteria (BIC). Models have been estimated by the quasi-maximum likelihood (QML) method.[4] Forecasts have been made for up to 24 periods (months) ahead. For each country, we have started the recursions using the first 20% of observations if the number of these observations was greater than 80; otherwise we have used first 80 observations. These forecasts have not been adjusted or manipulated. As a result, we have obtained reasonably long series of forecasts for

[4] For consistency and robustness of the QML method for the case of asymmetric and non-normally distributed errors see e.g. Francq and Zakoïan (2012). The codes are in GAUSS and the computations were made mainly using ALICE High Performance Computing Facility at the University of Leicester. The FANPAC package, by Ronald Schoenberg, has been adopted for estimation and forecasting the GARCH model.

different forecasts horizons, and then forecast errors, with the maximum number of sample observations for Canada (613) and the smallest for Estonia (99).

With the use of data on observed inflation, π_t, we have recovered forecasts errors as $e_{t|t-h} = \pi_t - \hat{\pi}_{t|t-h}$ and then the estimated *U*-uncertainties as $u_{t|t-h} = \left(\sigma_{t,h} / \hat{\sigma}_{t|t-h}\right) e_{t|t-h}$, where $\sigma_{t,h}$ is the unconditional standard deviation of $e_{t|t-h}$ and $\hat{\sigma}_{t|t-h}$ is the GARCH estimate of the conditional standard deviation. The rationale for using dispersion measures of forecast errors for the evaluation of uncertainty can be supported by the fact that, by using them, we are able to confirm earlier result of Demertzis and Hughes Hallett (2007) regarding the negative relationship between central banks' transparency and uncertainty. Demertzis and Hughes Hallett (2007) used data on quarterly inflation for 8 countries and the Euro since early 1990's till 2001 and data on indices of political and economic transparency published by Eijffinger and Geraats (2006). For these 8 observations they found a significant negative linear correlation between variance of inflation and the transparency indices. We have computed Spearman's rank correlations between the 2010 central banks' transparency index by Dincer and Eichengreen (2014) and the mean-square error of *U*-uncertainties. There are 23 data points (countries) for which data are available for both Dincer and Eichengreen index of transparency and *U*-uncertainties (excluding the Euro countries): Brazil, Canada, Chile, China, Czech Republic, Denmark, Hungary, Iceland, India, Indonesia, Israel, Japan, Korea, Mexico, Norway, Poland, Russia, South Africa, Sweden, Switzerland, Turkey, U.K. and U.S. For the entire sample of 23 countries, we have found evidence of negative correlation for all 24 forecast horizons, but only 9 of them are significant at the 10% level. However, if we reduce the sample to 15 inflation targeting countries, that is Brazil, Canada, Chile, Czech Republic, Hungary, Iceland, Indonesia, Israel, Korea, Mexico, Norway, Poland, Sweden, Turkey and U.K., we have a stronger result. In this case for 23 out of 24 forecast horizons the rank correlation coefficients are significant at the 5% level. However we have not found any negative relationship between the measures of central banks' independence and RMSE of *U*-uncertainties. We have used here both Dincer and Eichengreen (2014) and Bodea and Hicks (2015) measures of central banks' independence and for each of them, and all forecast horizons, to no avail. This seems to be in line with finding of Cargill (2013) that central banks' independence measures cannot be directly used for econometric modelling. However, later in this section we discuss some symptoms of the relationship of the central banks' independence measures with the UR's.

With the use of $u_{t|t-h}$, we have estimated the parameters of the WSN distribution (4). In order to reduce the computational burden we have assumed that the decision thresholds are fixed (relatively to σ) and identical for all countries so that $m = \bar{m} / \sigma = -k = -\bar{k} / \sigma = 1$ (that is, the thresholds are equal to one standard deviation of the uncertainties) and the correlation coefficient $\rho = 0.75$, that is that the level of imperfect knowledge and quality of forecasts made by the private forecasters is reasonably high. Possible consequences of misspecification resulting from imposition of such restrictions are discussed in the Supplementary Materials. Hence, we are left with three parameters to be estimated: α, β and σ. Computations also have been repeated for different thresholds and correlation coefficients and the results seem to be relatively robust to changes of these parameters.

Maximum likelihood estimation of parameters of various types of skew normal distributions, albeit formally straightforward as the density functions are expressed in closed form, is often numerically awkward, with possible bias and convergence problems (see e.g. Pewsey, 2000, Monti, 2003).[5] For this reason we have decided to apply the minimum distance estimators (*MDE's*) rather than maximum likelihood. Appropriately defined *MDE*'s are asymptotically efficient and asymptotically equivalent to the maximum likelihood estimators (see Basu, Shioya and Park, 2011).

The minimum distance criteria can be defined in different ways. In this paper we have used the Hellinger twice squared distance criterion (see e.g. Basu, Shioya and Park, 2011):

$$HD(d_n\ ,\ f_\theta) = 2\sum_{i=1}^{m}[d_n(i)^{1/2} - f_\theta(i)^{1/2}]^2 \quad ,$$

where n is the sample size, m is the number of disjoint intervals, $d_n(i)$ is the empirical frequency of data falling into the i^{th} interval and $f_\theta(i)$ is the corresponding theoretical probability for this interval. Properties of estimators based on Hellinger distances have been well researched in the context of other skew normal distributions (see Greco, 2011), and it is known that the estimates are reasonably robust to the presence of outliers, which might appear in a large sample of inflation forecast errors, especially for longer forecast horizons. Other distance measures belonging to the Cressie and Read (1984) family of power divergence disparities, have also been used leading to similar results.

We have obtained the estimates of the theoretical probabilities by simulation; that is, by Monte Carlo approximation of the theoretical probabilities. Details of this estimation procedure, called the *simulated minimum distance estimator*, *SMDE*, are given in Charemza *et. al* (2012); similar approach has been used by Dominicy and Veredas (2013). The version of *SMDE* applied here can be defined as:

$$\hat{\theta}_n^{SMDE} = \arg\min_{\theta\in\Theta}\left\{\lambda\left\{HD(d_n\ ,\ f_{r,\theta}\right\}_{r=1}^{R}\right\} \quad ,$$

where $f_{r,\theta}$ is the Monte Carlo approximation of the theoretical probabilities, f_θ, of a random variable obtained by generating r = 1,2,…,R replications (drawings) from a distribution with parameters $\theta = \{\alpha, \beta, \sigma\} \in \Theta \subset \mathbb{R}^3$, d_n denotes empirical density of sample of size n, and λ is an operator based on R replications, which deals with the problem of the 'noisy' criterion function (λ is the median, in this case).

We have compared the fit of WSN with two other distributions often used for modelling the distributions of inflation forecast errors, that is the two-piece skew normal distribution, TPN, and generalized three-parameters beta distribution, GB. TPN distribution has frequently been used for constructing fan charts of inflation (for its statistical properties see John, 1982 and Kimber, 1985; for wider discussion and use in the context of fan-chart modelling see e.g. Tay and Wallis, 2000). Three-parameters GBN distribution has been used for U.S. by Engelberg, Manski and Williams (2009) for approximation of the empirical distribution of SPF forecasts, and by Clements (2014) for interpolation of the histograms representing SPF probabilistic forecasts (that is, *ex-ante* uncertainty); for other economic applications and generalisations see McDonald and Xu (1995). Table 1 shows the frequency of cases the Hellinger MD statistic favours the particular distribution for all 38 countries (that is, reports the smallest distance for a particular country) for forecast horizons equal to 1, 4, 8, 12 and 24, and also the values of Hellinger MD statistics obtained for U.K. and U.S..

[5] We have also attempted to estimate the parameters by the maximum likelihood method. The results, however, have not been reliable due to convergence problems.

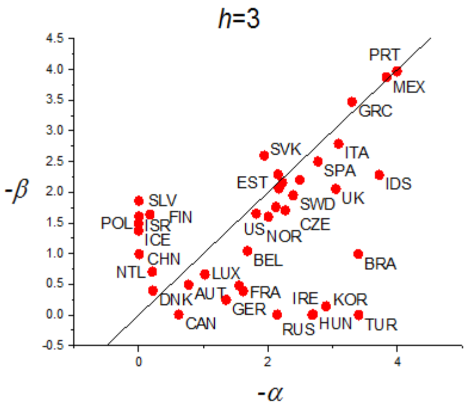
Table 1: Minimum distance characteristics of fitted distributions to *U*-uncertainties for 38 countries

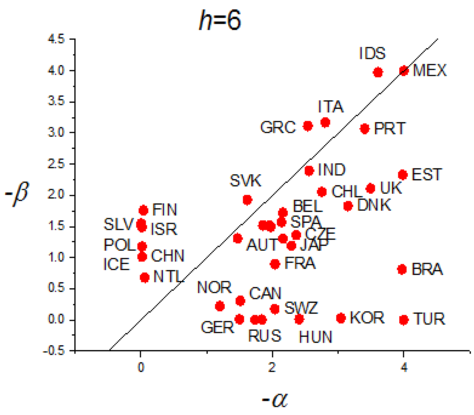
	Frequencies of cases in which the MD measure was the smallest for 38 countries			MD's for U.K.			MD's for U.S.		
Forecast horizon	WSN	TPN	GB	WSN	TPN	GB	WSN	TPN	GB
1	0.42	0.53	0.05	274.08	81.54	115.32	34.23	32.60	63.87
4	0.45	0.53	0.02	98.80	390.20	160.84	40.34	42.06	46.80
8	0.45	0.50	0.05	74.46	245.39	287.53	24.12	32.45	34.41
12	0.47	0.42	0.11	88.71	240.84	306.15	5.77	29.64	181.75
24	0.39	0.37	0.24	55.59	149.47	203.57	90.05	99.68	226.78

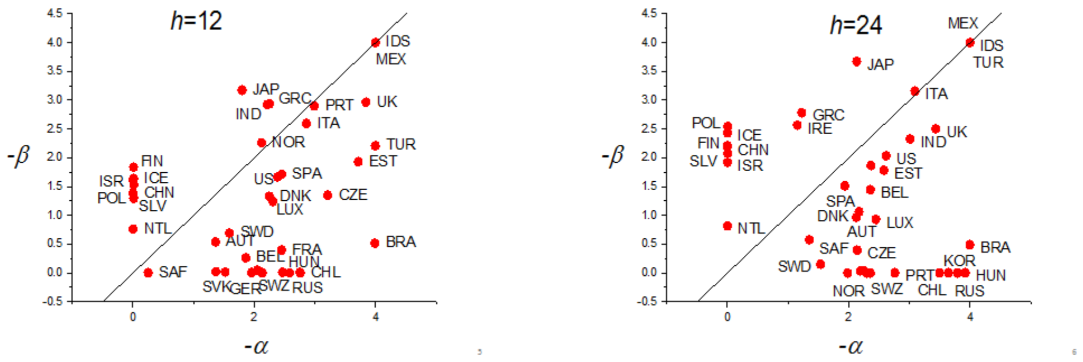
Results given in Table 1 show that fit of the generalized beta distribution is in most cases worse than that of WSN and TPN. Generally, TPN fits better to the uncertainties for shorter horizons, while WSN is better for the medium and longer horizons. For U.K. and U.S., WSN is nearly always better than TPN, but the differences in fit are often minimal.

Figure 2 depicts a comparison between estimated α and β parameters (multiplied by -1, for the clarity of the graphs) for the forecasts horizons h = 3, 6, 12 and 24. Deviations from the 45-degree line downwards indicate the dominance of the forecast-induced anti-inflationary influence on uncertainty (that is $-\alpha > -\beta$) and *vice versa*. Labels abbreviating particular countries, (not all are printed, for the sake of clarity), are explained in the Appendix.

Figure 2: Estimated α and β parameters. Each point represents the estimated $-\alpha$ and $-\beta$ for forecast horizons h=3, 6, 12, and 24. Country symbols are explained in Appendix.





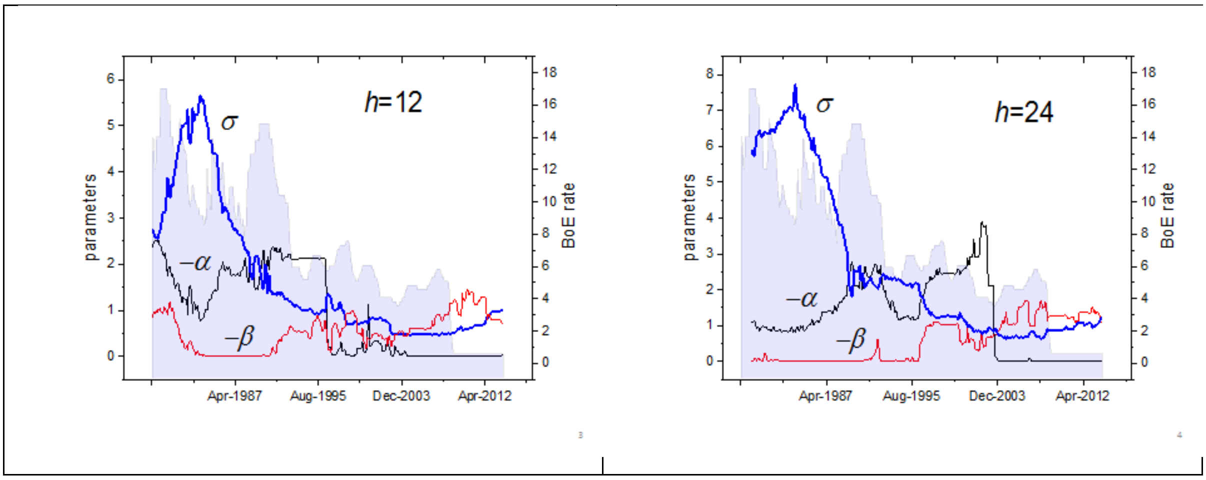


Interpretation here has to be done with caution. If, for instance, for a given country, $-\alpha > -\beta$, it means that the forecast signals delivered by the private forecasters stimulate the anti-inflationary policy more strongly than output-stimulating policy. Also, the bigger are negative values of α and β, that is, the higher are the points along the 45-degree line; the greater is deviation from normality of the distribution of ex-post uncertainties. In line with the proposed interpretation of WSN, this represents effects of private forecast-induced monetary policy on uncertainty. Values of α and β which are close to zero do not necessarily mean that the distribution of forecast errors is close to normality as the U-uncertainties are obtained from forecast errors through scaling by conditional standard deviation.

The general conclusions derived from Figure 2 are: (i) The non-leading large Euro countries which were strongly affected by the economic crisis of 2008, Greece, Italy and Portugal, are predominantly high up along the 45 degrees line, indicating strong dependence of the inflation uncertainty on anti-inflationary and output-stimulating effect of the decisions of the European Central Bank for these countries. For these countries, the effects are close to being symmetric, with Greece gradually moving, with an increase in the forecast horizon, into the output-stimulating zone, with Portugal moving the other way. (ii) For longer horizons, Japan is among the countries with the highest $-\beta$, which is markedly greater than its $-\alpha$. This might reflect the preference to output-stimulating and anti-deflationary policy. (iii) For all forecast horizons, Turkey is among the countries with the highest $-\alpha$'s. With $-\beta$ close to zero, this might indicate stronger preference to anti-inflationary, rather than output-stimulating, reaction to forecasts signals. However, with an increase in the forecast horizon, $-\beta$ increases, bringing Turkey, for the 2-years horizon, close to the situation of more balanced reaction.

In order to evaluate changes in α, β and σ in time, and check whether their dynamics reflects changes in economic policy over time, we have estimated the parameters of the WSN distribution in a rolling window, by recursively adding one observation and dropping the last one. Figure 3 presents the results for the U.K. for *h*=12 and *h*=24, where the window of length 120 has been used with the first window covering the period from September 1968 to August 1978 for *h*=12 and September 1969 to August 1979 for *h*=24. The shaded background depicts the Bank of England interest rate measured on the right-hand side axis.

Figure 3: Recursive estimates of the WSN parameters $-\alpha$, $-\beta$ and σ for U.K..



For the one-year uncertainty, $-\alpha$ has been higher than $-\beta$ until December 1996, after which $-\beta$ started to rise and $-\alpha$ first rapidly fell, and then stabilize at a low level (that is, close to zero). This change of sign of the difference between α and β resulted in a change in skewness of the *U*-uncertainties distribution from negative to positive and coincides with more strenuous output-stimulating policy of the Bank of England, expressed by a tendency to reduce the base rate. In particular a similar switch between the strength of α and β can be observed for the 2-years uncertainty; in which case, however, it happened much later, in August 2003. Our interpretation, however, cannot be applied to the periods from November 2003 to August 2004 and then from August 2006 to July 2007, where an increase in $-\beta$, which should indicate the effect of output-stimulating policy, coincides with the temporary increase in the Bank of England base rate. This requires further investigation.

We have also looked for a possible relationship between the UR's and the measures of central banks' independence. The Spearman's rank correlation coefficient between the central banks' independence measures (see Dincer and Eichengreen, 2014; Bodea and Hicks, 2015) and the UR's for 23 countries with independent central banks listed earlier, computed for each forecast horizon, are predominantly negative and insignificant. However, we have found some mild evidence of a nonlinear relationship between the UR's and central banks independence measures. This can be explained by the fact that, according to Figure 1, with the increase in the compound monetary policy strength the UR's are initially increasing, up to the point of $\mathrm{UR}_{\max}$, and then decreasing. This implies possible different signs of the relationships between the central banks' independence measures and the UR's. This is illustrated by Figure 4.

Figure 4a plots the UR's for 23 countries with independent central banks against the compound monetary policy strength for the forecast horizon of 12. The red (solid) line represents a cut of the surface shown at Figure 1 at $\rho = 0.75$. This is the value of ρ assumed in the empirical estimation. The dotted lines represent fitted values of the empirical UR's with the corresponding compound strengths as the argument is each sub-sample, that is for the values of the compound strength being respectively below and above of that of $\mathrm{UR}_{\max}$. The red line represents the theoretical UR's, computed as in Figure 1 for $\rho = 0.75$, and for the output symmetric case, that is for $\alpha = \beta$ that is with no bias due to asymmetry, as $E(U^*) = 0$.

Figure 4: Theoretical and empirical UR's and central banks' independence measures, Forecast horizon =12. Country symbols are explained in Appendix

Figure 4a: Theoretical and empirical UR's

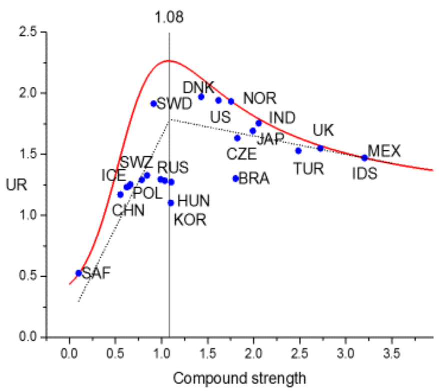


Figure 4b: Theoretical UR's and LVAW, Dincer and Eichengreen (2014)

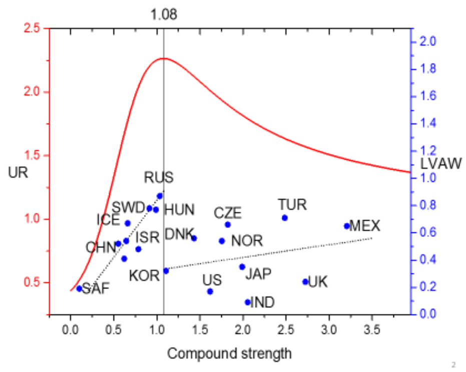
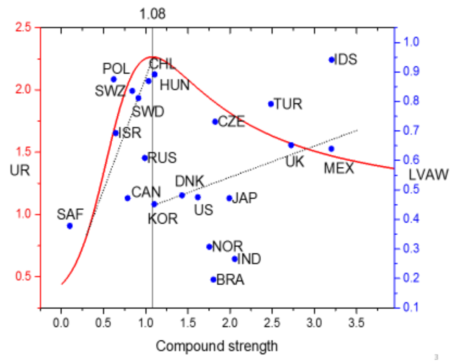


Figure 4c: Theoretical UR's and LVAW, Bodea and Hicks (2015)



Equation (8) shows that the empirical values of UR cannot be greater than the values at the solid line. Possible bias, due to the fact that $\hat{\alpha} \neq \hat{\beta}$, increases denominator in (8) and places the empirical UR's below the solid line. It can be noticed that the mass of the empirical UR's is to the right of UR_{max}. There are 13 empirical UR's with the compound strength greater than 1.08, which is the strength of UR_{max}, and only 10 UR's with the smaller strength. Also, the average strength of empirical UR's is 1.58, which is markedly higher than that for UR_{max}. Below the point of 1.08, the UR's increase with the increase in the compound strength, and decrease after that point. Figures 4b and 4c plot two of the central banks' independence measures, namely the LWAW (weighted index) of Dincer and Eichengreen (2014) and analogous weighted index, also denoted by LVAW, by Bodea and Hicks (2015) against the compound strengths of the corresponding empirical UR's. For these measures the latest data available have been used, that is for 2010. For the points below the threshold of 1.08 and, separately, above these threshold, there is positive, albeit insignificant at 10% significance level, correlation between the central bank's independence measures and the compound strength. This confirms the positive correlation of the central banks' independence measures and empirical UR's for the values with the compound strength being below 1.08, and negative otherwise.

For all central banks' independence measures and all forecast horizons, such evidence is rather weak. Table 2 below gives the aggregated results of a simple split test for the sample of 15 inflation targeting countries. We assume that we have a weak confirmation of the split relationship between the empirical UR's and central banks transparency measures if, for a given forecast horizon, their rank correlation coefficient is positive for the compound strength being below $\mathrm{UR}_{\max}$, and negative otherwise. We have a semi-strong confirmation if one of these correlations is significant at 10% significance level, and a strong confirmation, if both correlations are significant. We have applied it for LVAU, LVAW, CBIU and CBIV measures of Dincer and Eichengreen (2014) and LVAU and LVAW measures of Bodea and Hicks (2015).

Table 2: Split test results for empirical UR's and central banks' independence measures for 15 countries with inflation targeting and 24 forecast horizons

	Dincer and Eichengreen (2014)				Bodea and Hicks (2015)	
	LVAU	LVAW	CBIU	CBIW	LVAU	LVAW
Weak test	20	19	19	19	3	3
Semi-strong test	7	10	10	10	0	1
Strong test	1	0	1	0	0	1

Table 2 suggests a possible split relationship between UR's and the degree of central banks' independence, For the Dincer and Eichengreen measures, the signs of Spearman's rank correlations are consistent with the expected ones in 19/20 out of 24 cases. However this evidence is very weak and rarely significant. If it, nevertheless, exists, it would provide evidence that the central banks' independence contributes positively towards reduction in forecast uncertainty if the strength of the monetary policy is not greater than that for the $\mathrm{UR}_{\max}$; otherwise the policy is too strong and the contribution is negative. This is in line with current findings that a high degree of central banks' independence can sometimes be sub-optimal (see e.g. Hefeker and Zimmer, 2012; Hielscher and Markwardt, 2012).

Further results for all 38 countries, and also for selected groups of countries, disaggregated and aggregated and ordered according to individual measures proposed here are available on request. Below, as illustration~~,~~ we present some results for U.K. and U.S. Table 3 shows basic characteristics of uncertainty obtained for U.K. and U.S. for selected forecast horizons. It gives, respectively, the forecast horizon, 'theoretical' uncertainty computed analogously to that of Clements (2014), that is assuming an *iid* month-to-month inflation uncertainty, $RMSE_U$ and σ_V, normalized as ratios of their values for a given *h* to that of the last forecast horizon ($h = 24$), UR, that is the ratio of σ_V^2 to $RMSE_U^2$, see (8), and NUR, that is the ratio of UR to $\mathrm{UR}_{\max}$. Such layout enables comparison with Clements's (2014) results for *ex-ante* and *ex-post* uncertainty obtained for U.S. and indicates similarity of the computed $RMSE_U$ and σ_V measures with Clements' (2014) *ex-ante* and *ex-post* uncertainties. The comparison is, nevertheless, limited, as the Clements' results are related to inflation forecasts for the end of each year only, due to the way the U.S. SPF's , are constructed, and our forecasts are made for each month of each year within the forecast time span.

Table 3 Forecast uncertainty measures for U.K. and U.S.

h	Theor	U.K. normalised $RMSE_U$	U.K. normalised σ_V	U.K. UR	U.K. NUR	U.S. normalised $RMSE_U$	U.S. normalised σ_V	U.S. UR	U.S. NUR
3	0.11	0.44	0.45	1.73	0.76	0.40	0.43	2.14	0.95
6	0.28	0.63	0.63	1.63	0.72	0.49	0.52	2.10	0.93
9	0.50	0.74	0.72	1.57	0.69	0.60	0.63	2.03	0.90
12	0.75	0.90	0.88	1.55	0.68	0.71	0.73	1.94	0.86
15	0.91	0.91	0.89	1.57	0.69	0.82	0.82	1.85	0.82
18	0.98	0.95	0.93	1.59	0.70	0.92	0.92	1.80	0.79
21	1.00	1.06	1.03	1.55	0.69	0.99	0.99	1.80	0.79
24	1.00	1.00	1.00	1.63	0.72	1.00	1.00	1.85	0.82

Clements' *ex-ante* uncertainty is always greater than the corresponding *ex-post* uncertainty (see Clements, 2014, Table 1) and similarly, for all forecast horizons, $\sigma_V > RMSE_U$ as UR>1. However, our interpretation of this is different. Clements (2014) comments that the fact that the *ex-ante* uncertainty is greater that *ex-post* uncertainty is an indication of underconfidence of the forecasters. In our approach, the fact that pseudo *ex-ante* uncertainty, expressed by σ_V, is greater than the *ex-post* uncertainty expressed by $RMSE_U$, is interpreted as a positive effect of the monetary policy undertaken on the basis of information available to the improvers. The term structure of UR, which is the ratio of the squares of pseudo ex-ante to ex-post uncertainty, is different that the term structure of Clements' ratio of the *ex-ante* to *ex-post* uncertainty measures. With the increase in forecast horizon the Clements' ratio is increasing while, in our case, for both U.K. and U.S, UR is decreasing. This, in line with our interpretation, indicates diminishing effects of monetary policy in reducing inflation uncertainty with the increase in the forecast horizon. The comparison of the normalized $RMSE_U$ and σ_V with the theoretical uncertainty leads to conclusions in line that that obtained by Clements (2014). For longer horizons, the normalized measures of *V*-uncertainty are smaller than the corresponding theoretical uncertainty, suggesting that the private forecasters utilize some valuable additional information leading to a reduction of uncertainty. Clements has formulated similar conclusion in relation to *ex-post* uncertainty measure derived from SPF's.

For U.K., the ratio of UR to $\mathrm{UR}_{\max}$ remains relatively stable for all forecast horizons. It is, however, lower that such ratio for U.S., which shows a slight tendency to decrease with an increase in forecast horizon. It provides a symptomatic evidence for a better effectiveness of the U.S. policy, rather than U.K., regarding reducing uncertainty, as their combination of the policy related parameters, α and β, are closer to the theoretically most effective one.

6. CONCLUSIONS

We have found out that the *ex-post* forecast errors of inflation might tell us more than just by how much the forecasters err. Firstly, and rather evidently, they create a convenient base for measuring *ex-post* uncertainty. Measures of *ex-post* uncertainty could be of interest to economic agents who do not have an influence on economic policy and who do not really care of what is epistemic for the modelers and what is not. However, if the weighed skew normal distribution proposed in this paper is fitted to the forecast errors (after removing possible predictability in variance), footprints of economic policy, its preferences and outcomes can be identified. Consequently, we suggest a *quasi-ex-ante* measure that might be used as an alternative (or substitute) to *ex-ante* uncertainty measures derived from surveys of forecasts. Our measure is defined in the *back to the future* fashion as it requires the knowledge of *ex-post* forecast errors and parameters of the weighted skew-normal distribution fitted to them. This measure, which is, to an extent, free from the potentially predictable epistemic element (and, consequently, of the effects of policy decisions), could be of interest to the policy makers, who does not want the picture of uncertainty be blurred by outcomes of their own decisions. Instead, they could rather be interested in answering the question of 'what would the uncertainty be if we do not carry out the policy we actually want to implement?'. The comparison of both measures: *quasi-ex ante* and *ex-post* can be useful as an indicator of a possible room for improvement regarding further reduction in uncertainty.

APPENDIX: Country symbols

AUT	Austria	FRA	France	JAP	Japan	SLV	Slovenia
BEL	Belgium	GER	Germany	KOR	Korea	SAF	South Africa
BRA	Brazil	GRC	Greece	LUX	Luxembourg	SPA	Spain
CAN	Canada	HUN	Hungary	MEX	Mexico	SWD	Sweden
CHL	Chile	ICE	Iceland	NTL	Netherlands	SWZ	Switzerland
CHN	China	IND	India	NOR	Norway	TUR	Turkey
CZE	Czech Rep	IDS	Indonesia	POL	Poland	UK	United Kingdom
DNK	Denmark	IRE	Ireland	PRT	Portugal	US	United States
EST	Estonia	ISR	Israel	RUS	Russia		
FIN	Finland	ITA	Italy	SVK	Slovak Rep		

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SUPPLEMENTARY MATERIALS

Part S1: Derivations of the density and moment generating functions of the WSN distribution, Uncertainty Ratio (UR) with discussion of their properties

Part S2: Graphical representation of some WSN distributions (with various constraints imposed on the parameters)

Part S3: More detailed empirical results for U.K and U.S. This part contains, in particular: the WSN parameters' estimates for all forecast horizons, and the term structure of the compound strength measure.

Part S4: Details about data and computer programs which are available on request.

Part S1: Weighted skew normal distribution (WSN), Uncertainty Ratio (UR) and their properties

I. Weighted skew normal distribution and its properties

General notation

For a random variable Y and a real number a, notation $I_{Y>a}$ (or $I_{Y<a}$) denotes an indicator of the event $\{Y>a\}$ (correspondingly $\{Y<a\}$), that is equal to unity if $Y>a$ and zero otherwise.

Definition 1S. Let *X* and *Y* constitute a bivariate normal random variable such as:

$$(X,Y) \sim N\left(\begin{bmatrix}\mu_X \\ \mu_Y\end{bmatrix}, \begin{bmatrix}\sigma_X^2 & \rho\sigma_X\sigma_Y \\ \rho\sigma_X\sigma_Y & \sigma_Y^2\end{bmatrix}\right), \text{ with } |\rho|<1 \quad , \tag{S.1}$$

and random variable U is defined as

$$U = X + \alpha\cdot Y\cdot I_{Y>\bar{m}} + \beta\cdot Y\cdot I_{Y<\bar{k}} \;, \quad \text{where } \alpha,\beta,\bar{k}<\bar{m}\in\mathbb{R} \;. \tag{S.2}$$

We call the distribution of U defined by (S.1)-(S.2) as *weighted skew normal* and denote it as $U \sim \mathrm{WSN}_{\sigma_X,\sigma_Y}^{(\mu_X,\mu_Y)}(\alpha,\beta,\bar{m},\bar{k},\rho)$.

Definition 2S. A weighted skew normal variable U^* with $\mu_X=\mu_Y=0$ and $\sigma_X=\sigma_Y=1$ is called *standard weighted skew normal* and is denoted as $U^* \sim \mathrm{WSN}_1(\alpha,\beta,\bar{m},\bar{k},\rho)$.

Proposition 1S. The probability density function (*pdf*) of the standard weighted skew normal distribution $U^* \sim \mathrm{WSN}_1(\alpha,\beta,m,k,\rho)$ is given by:

$$\begin{aligned} f_{\mathrm{WSN}_1}(t) &= \frac{1}{\sqrt{A_\alpha}}\varphi\left(\frac{t}{\sqrt{A_\alpha}}\right)\Phi\left(\frac{B_\alpha t - mA_\alpha}{\sqrt{A_\alpha(1-\rho^2)}}\right) + \frac{1}{\sqrt{A_\beta}}\varphi\left(\frac{t}{\sqrt{A_\beta}}\right)\Phi\left(\frac{-B_\beta t + kA_\beta}{\sqrt{A_\beta(1-\rho^2)}}\right) \\ &\quad + \varphi(t)\cdot\left[\Phi\left(\frac{m-\rho t}{\sqrt{1-\rho^2}}\right) - \Phi\left(\frac{k-\rho t}{\sqrt{1-\rho^2}}\right)\right] \quad , \end{aligned} \tag{S.3}$$

where φ and Φ denote respectively the density and cumulative distribution functions of the standard normal distribution, $A_\tau = A(\tau) = 1+2\tau\rho+\tau^2$, and $B_\tau = B(\tau) = \tau+\rho$.

Proof. The cumulative distribution function (*cdf*) F_{WSN_1} of U^* can be obtained by integrating normal bivariate *pdf* with zero means, unit variances and correlation coefficient of ρ over three disjoint areas as follows: $F_{WSN_1}(t) = \int_{-\infty}^{t-\alpha m} dx \int_{m}^{(t-x)/\alpha} + \int_{-\infty}^{t-\beta k} dx \int_{(t-x)/\beta}^{k} + \int_{-\infty}^{t} dx \int_{k}^{m}$. Taking the first derivative $dF_{WSN_1}(t)/dt$ complete the proof.■

It immediately follows from Proposition 1S that f_{WSN_1} can be interpreted as a weighted sum of three *pdf*'s as $f_{\mathrm{WSN}_1}(t) = \alpha_1 f_1(t) + \alpha_2 f_2(t) + \alpha_3 f_3(t)$, where $\alpha_1 = \Phi(-m)$, $\alpha_2 = \Phi(k)$, $\alpha_3 = \Phi(m) - \Phi(k)$ and f_i ($i = 1,2,3$) are three corresponding consecutive components of *pdf* in (S.3). The *pdf* f_3 is a *pdf* of conditional variable $\left(X_0 \middle| k \le Y_0 \le m\right)$. Relations between f_1, f_2 and skew normal distribution are as follows. Simple Azzalini (1985, 1986) skew normal distribution $SN(\lambda, \omega)$ can be defined by its *pdf* as $f_{SN}(t; \lambda, \omega) = \dfrac{2}{\omega} \varphi(t/\omega) \Phi(\lambda x/\omega)$. Hence, for $m = k = 0$ and $\alpha = -2\rho$ functions f_1 and f_2 reduce to *pdf*'s of $SN(\lambda_1, \omega_\alpha)$ and $SN(\lambda_2, \omega_\beta)$ with $\lambda_{1,2} = \mp \rho / \sqrt{1-\rho^2}$ and $\omega_\tau = \sqrt{A_\tau}$ ($\tau = \alpha, \beta$). This representation allows for another interpretation of Azzalini skew normal distribution, as $U^{SN} \sim \mathrm{WSN}_1(-2\rho, 0, 0, 0, \rho)$, or $U^{SN} = X_0 - 2\rho Y_0 \cdot I_{Y_0 > 0}$, where $X_0, Y_0 \sim N(0,1)$ and $corr(X_0, Y_0) = \rho$.

Representation $f_{\mathrm{WSN}_1}(t) = \alpha_1 f_1(t) + \alpha_2 f_2(t) + \alpha_3 f_3(t)$ can be now interpreted as a weighted sum of conditional *pdf* of $\left(X_0 \middle| k \le Y_0 \le m\right)$ and two *pdf*'s that, under some restrictions on parameters, coincide with that of Azzalini skew normal (hence the name of the WSN distribution).

Proposition 2S. Moment generating function (MGF) of $U^* \sim \mathrm{WSN}_1(\alpha, \beta, m, k, \rho)$ is given by:

$$R_{\mathrm{WSN}_1}(u) = e^{\frac{u^2}{2} A_\beta} \Phi(k - B_\beta u) + e^{\frac{u^2}{2}} \cdot \left[\Phi(m - \rho u) - \Phi(k - \rho u)\right] + e^{\frac{u^2}{2} A_\alpha} \Phi(B_\alpha u - m), \qquad \text{(S.4)}$$

Proof. By definition,

$$R_{WSN_1}(u) = E\left(e^{u \cdot U^*}\right) = \frac{1}{2\pi\sqrt{1-\rho^2}} \int_{-\infty}^{+\infty} dx \left[\int_{-\infty}^{k} e^{u[x + \beta \cdot y]} + \int_{k}^{m} e^{ux} + \int_{m}^{+\infty} e^{u[x + \alpha \cdot y]} \right] \cdot e^{-\frac{x^2 - 2\rho xy + y^2}{2(1-\rho^2)}} dy .$$

Changing order of integration in each of the integrals above and noting that MGF of standard normal distribution is $e^{u^2/2}$, complete the proof.■

Corollary. Moment generating function R_{WSN} of $U \sim \text{WSN}_{\sigma_X,\sigma_Y}^{(\mu_X,\mu_Y)}(\alpha,\beta,\bar{m},\bar{k},\rho)$ is given by:

$$R_{\text{WSN}}(u) = e^{u\mu_X}\left\{ e^{\alpha\mu_Y u+\frac{(u\sigma_X)^2}{2}A_{\alpha\frac{\sigma_Y}{\sigma_X}}}\Phi(B_{\alpha\frac{\sigma_Y}{\sigma_X}}u\sigma_X - \frac{\bar{m}-\mu_Y}{\sigma_Y}) + e^{\beta\mu_Y u+\frac{(u\sigma_X)^2}{2}A_{\beta\frac{\sigma_Y}{\sigma_X}}}\Phi(\frac{\bar{k}-\mu_Y}{\sigma_Y} - B_{\beta\frac{\sigma_Y}{\sigma_X}}u\sigma_X)\right.$$
$$\left. + e^{\frac{(u\sigma_X)^2}{2}}\cdot\left[\Phi(\frac{\bar{m}-\mu_Y}{\sigma_Y}-\rho u\sigma_X)-\Phi(\frac{\bar{k}-\mu_Y}{\sigma_Y}-\rho u\sigma_X)\right]\right\}$$

Proof. It follows from the representation of $U \sim \text{WSN}_{\sigma_X,\sigma_Y}^{(\mu_X,\mu_Y)}(\alpha,\beta,\bar{m},\bar{k},\rho)$ *via* $U^* \sim \text{WSN}_1\left(\alpha\frac{\sigma_Y}{\sigma_X},\beta\frac{\sigma_Y}{\sigma_X},\frac{\bar{m}-\mu_Y}{\sigma_Y},\frac{\bar{k}-\mu_Y}{\sigma_Y},\rho\right)$ as $U = \sigma_X U^* + \mu_X + \mu_Y\cdot\left(\alpha I_{Y_0>m} + \beta I_{Y_0<k}\right)$, where Y_0 is standard normal. ■

Proposition 3S. Let R_{WSN_1} be a MGF given by (S.4), then

$$R'_{\text{WSN}_1}(0) = \alpha\cdot\varphi(m) - \beta\cdot\varphi(k);$$

$$R''_{\text{WSN}_1}(0) = A_\alpha + [1-A_\alpha]\Phi(m) + \left[B_\alpha^2-\rho^2\right]m\varphi(m) + \left[A_\beta - 1\right]\Phi(k) - \left[B_\beta^2-\rho^2\right]k\varphi(k);$$

$$R^{(3)}_{\text{WSN}_1}(0) = \varphi(m)\cdot\left\{B_\alpha\cdot\left[3A_\alpha + B_\alpha^2\left(m^2-1\right)\right] - \rho\cdot\left[3+\rho^2\left(m^2-1\right)\right]\right\} +$$
$$\varphi(k)\cdot\left\{-B_\beta\cdot\left[3A_\beta + B_\beta^2\left(k^2-1\right)\right] + \rho\cdot\left[3+\rho^2\left(k^2-1\right)\right]\right\} \quad ;$$

$$R^{(4)}_{\text{WSN}_1} = 3\cdot\left\{A_\alpha^2 + \Phi(m)\cdot\left[1-A_\alpha^2\right]\right\} + m\cdot\varphi(m)\cdot\left\{\left(3-m^2\right)\cdot\left(\rho^4 - B_\alpha^4\right) + 6\cdot\left(A_\alpha B_\alpha^2 - \rho^2\right)\right\}$$
$$+3\cdot\Phi(k)\cdot\left[A_\beta^2-1\right] - k\cdot\varphi(k)\left\{\left(3-k^2\right)\cdot\left(\rho^4-B_\beta^4\right) + 6\cdot\left(A_\beta B_\beta^2-\rho^2\right)\right\} \quad .$$

Proof. Substituting Taylor expansions of $e^{\frac{au^2}{2}}$ and $\Phi(bu+c)$ into $g_{a,b,c}(u) = e^{\frac{au^2}{2}}\Phi(bu+c)$ yields:

$$g_{a,b,c}(u) = \Phi(c) + b\varphi(c)\cdot u + \frac{1}{2}\left[a\cdot\Phi(c) - b^2c\cdot\varphi(c)\right]\cdot u^2 + \frac{1}{3!}\left[3a + b^2(c^2-1)\right]b\varphi(c)\cdot u^3$$
$$+\frac{1}{4!}\left[3a^2\Phi(c) - 6ab^2c\varphi(c) + b^4c(3-c^2)\varphi(c)\right]\cdot u^4 + \ldots$$

Bearing in mind that MGF in (S.4) can be expressed via $g_{a,b,c}$ as:

$$R_{\text{WSN}_1}(u) = g_{A_\alpha,B_\alpha,(-m)}(u) + g_{A_\beta,(-B_\beta),k}(u) + g_{1,(-\rho),m}(u) - g_{1,(-\rho),k}(u) \quad , \qquad \text{(S.5)}$$

taking derivative of the both sides of (S.5) and substituting corresponding derivatives of $g_{a,b,c}$ at zero, complete the proof. ■

Note. For $m>0$ and $k<0$ it is convenient to simplify expression for $R''_{\mathrm{WSN}_1}(0)$ as:

$$R''_{\mathrm{WSN}_1}(0)=1+C_\alpha D_m+C_\beta D_k \quad , \tag{S.6}$$

where

$$C_\tau=\tau(\tau+2\rho) \text{ and } D_a=1-\Phi(|a|)+|a|\varphi(a)=\int_{|a|}^{+\infty}t^2\varphi(t)dt \quad . \tag{S.7}$$

II. Uncertainty ratio (UR) and its properties

Definition 3S. Let $U \sim \mathrm{WSN}_{\sigma,\sigma}^{(0,0)}(\alpha,\beta,\bar{m},\bar{k},\rho)$ be defined by (S.1)-(S.2) and $V=U-E\left(X\middle|Y\right)=U-\rho Y$. Let also $RMSE_U=\sqrt{\mathrm{var}(U)+E^2(U)}$ and $\sigma_V=\sqrt{\mathrm{var}(V)}$.

We will define *uncertainty ratio* of U and V, as

$$\mathrm{UR}=\frac{\sigma_V^2}{RMSE_U^2} \quad .$$

Properties of uncertainty ratio UR and its useful re-parameterisations are summarised in the following proposition.

Proposition 4S. Properties of UR.

1) Noting that $EU=\sigma EU^*=\sigma\left[\alpha\varphi(m)-\beta\varphi(k)\right]$ and

$$\mathrm{var}(V)=\mathrm{var}(U)+\rho^2\sigma^2-2\rho E\left(X+\alpha\cdot Y\cdot I_{Y>\bar{m}}+\beta\cdot Y\cdot I_{Y<\bar{k}}\right)Y=\sigma^2\,\mathrm{var}(U^*)-\rho^2\sigma^2$$
$$-2\rho\sigma^2\left(\alpha D_m+\beta D_k\right)$$

yields the following representation:

$$\mathrm{UR}=1-2\rho\frac{\alpha D_m+\beta D_k+\rho/2}{\mathrm{var}(U^*)+\left[EU^*\right]^2}-\frac{\left[EU^*\right]^2}{E\left[U^*\right]^2} \quad , \tag{S.8}$$

where D_m, D_k are defined by (S.7).

2) For $m>0$ and $k<0$, by applying Proposition 3A and (S.6)-(S.7) to U^* we get:

$\mathrm{var}(U^*)+\left[EU^*\right]^2=1+C_\alpha D_m+C_\beta D_k=1-2\rho S+\alpha^2D_m+\beta^2D_k$, where $S=-\alpha D_m-\beta D_k$.

This, together with (S.8), gives another convenient expression for UR:

$$\mathrm{UR}=1+2\rho\frac{S-\rho/2}{1-2\rho S+\alpha^2D_m+\beta^2D_k}-\frac{\left[\alpha\varphi(m)-\beta\varphi(k)\right]^2}{E\left[(U^*)\right]^2} \quad . \tag{S.9}$$

3) Derivation of maximum of UR (for fixed ρ) in a symmetric case of $m=-k$.

Denote: $D=D_m=D_k$, $\varphi=\varphi(m)=\varphi(k)$, $t=\left(S-\rho/2\right)/D=-(\alpha+\beta)-\dfrac{\rho}{2D}$. Hence (S.9) reduces to

$$\mathrm{UR}=1+2\rho D\frac{t}{1-2\rho(Dt+\rho/2)+(\alpha^2+\beta^2)D}-\frac{\left(\alpha-\beta\right)^2\varphi^2}{E\left[U^*\right]^2}. \tag{S.10}$$

Let $\tau=\alpha-\beta$, then

$$\alpha^2+\beta^2=\frac{1}{2}\left[(\alpha+\beta)^2+(\alpha-\beta)^2\right]=\frac{t^2}{2}+\frac{t\rho}{2D}+\frac{\rho^2}{8D^2}+\frac{\tau^2}{2}. \tag{S.11}$$

Substituting (S.11) into (S.10) we get

$$\mathrm{UR}=1+\frac{4\rho}{F(t)}-\frac{\tau^2\varphi^2}{E\left[(U^*)\right]^2} \tag{S.12}$$

where

$$F(t)=t+\frac{(1-\rho^2)\dfrac{2}{D}+\dfrac{\rho^2}{4D^2}+\tau^2}{t}+\frac{\rho}{D}(1-4D). \tag{S.13}$$

Note, that maximum of UR, $\mathrm{UR}_{\max}$, is achieved for minimum F when t>0. Consider the function $G(t)=t+\dfrac{A}{t}$, where t, A>0. Hence: $\arg\min G(t)=t_0=\sqrt{A}$ and $G_{\min}=G(t_0)=2\sqrt{A}$. Therefore, for F defined by (S.13), $A=(1-\rho^2)\dfrac{2}{D}+\dfrac{\rho^2}{4D^2}+\tau^2$ and for fixed $\tau>0$ we have:

$$F_{\min,\tau}=2\sqrt{(1-\rho^2)\frac{2}{D}+\frac{\rho^2}{4D^2}+\tau^2}+\frac{\rho}{D}(1-4D).$$

Obviously, $F_{\min,\tau}\geq F_{\min,\tau=0}$, and

$$F_{\min,\tau=0}=2\sqrt{(1-\rho^2)\frac{2}{D}+\frac{\rho^2}{4D^2}}+\frac{\rho}{D}(1-4D),$$

which means that F achieves its minimum when $\tau=0$, that is for $\alpha=\beta=\alpha_0$, and

$$t_0=\sqrt{(1-\rho^2)\frac{2}{D}+\frac{\rho^2}{4D^2}}.$$

Noting that $t=\left(S-\rho/2\right)/D=-(\alpha+\beta)-\dfrac{\rho}{2D}$, gives $\alpha_0=\dfrac{\rho}{4D}+\dfrac{1}{2}\sqrt{(1-\rho^2)\dfrac{2}{D}+\dfrac{\rho^2}{4D^2}}$.

Note also that $\min\left\{\dfrac{\tau^2\varphi^2}{E\left[U^*\right]^2}\right\}=0$ and is achieved at $\tau=0$, which, in combination with (S.13), gives, that

$$\mathrm{UR}_{\max}(\rho)=1+\frac{4\rho}{F_{\min}}=1+\frac{4\rho}{2\sqrt{(1-\rho^2)\dfrac{2}{D}+\dfrac{\rho^2}{4D^2}}+\dfrac{\rho}{D}(1-4D)}$$

and is achieved at $\alpha_0=\beta_0=-\left(\rho+\sqrt{8D(1-\rho^2)+\rho^2}\right)/(4D)$.■

Part S2: Graphical representation of some WSN distributions

S2.1: Some WSN density functions

Following the notation introduced in the main body of the paper, we denote by $\mathrm{WSN}_\sigma\left(\alpha, \beta, \bar{m}, \bar{k}, \rho\right)$ the weighted skew normal distribution as in (4). Within our approach this is a distribution of a random variable *U* representing *U*-uncertainties. For the sake of clarity of interpretation we also apply the following terminology:

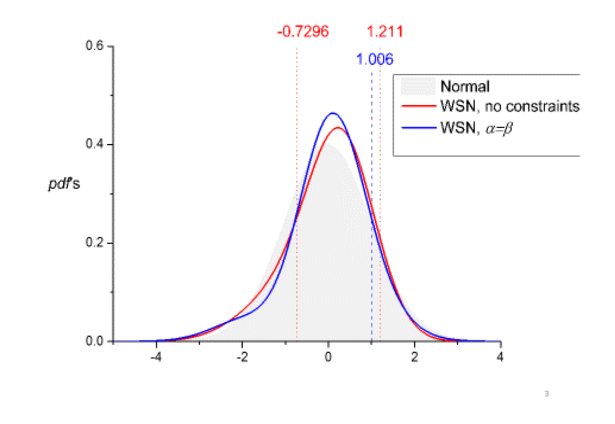
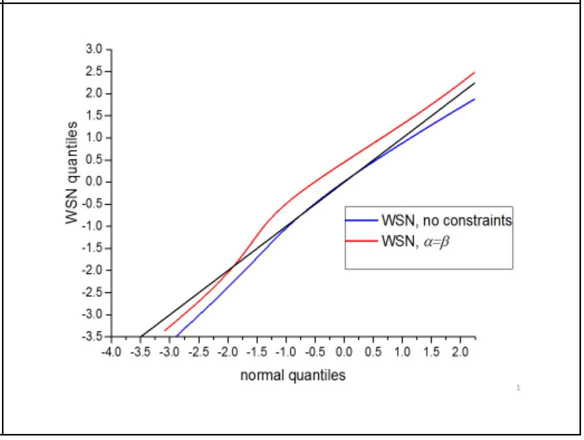
1. The WSN distribution is fully symmetric if $\alpha = \beta$ and $\bar{m} = -\bar{k}$ (in fact the distribution is also symmetric in the special case when $\alpha = \beta = 0$ irrespectively of $\bar{m}$ and $\bar{k}$. As in this case the distribution collapses to normal we do not discuss it here).
2. The WSN distribution is policy-output symmetric if $\alpha = \beta$. It is policy-output asymmetric if $\alpha \neq \beta$; in this case $\alpha < \beta$ indicates anti-inflationary policy and $\alpha > \beta$ pro-inflationary (output stimulating) policy.
3. The WSN distribution is policy-input symmetric if $\bar{m} = -\bar{k}$. and policy-output asymmetric if $\bar{m} \neq -\bar{k}$.

Figure F.1 shows two density functions and the corresponding normal q-q plots for two $\mathrm{WSN}_\sigma\left(\alpha, \beta, \bar{m}, \bar{k}, 0.75\right)$ distributions and the Pearson's moment coefficient of skewness (that is, the normalized third central moment) equal to -0.5. One of the distributions does not have any constraints on α, β, $\bar{m}$, $\bar{k}$ and σ parameters, and the other has policy-output symmetricity constraint imposed, that is $\alpha = \beta$. Figure F.2 compares two $\mathrm{WSN}_\sigma\left(\alpha, \beta, \bar{m}, \bar{k}, 0.75\right)$ distributions, also with the skewness coefficients equal to -0.5 and with two different types of symmetricity constraints. One distribution is, as in F.1, policy-output symmetric, that is with $\alpha = \beta$ constraint and the other is policy-input symmetric, that is with the $\bar{m} = -\bar{k}$ constraint. For all distributions values of the parameters have been fixed at such levels that the unweighted sum of squares of the deviations of variance from unity and the coefficient of skewness from -0.5 is negligible. Values of the parameters are given in Table T.1 below.

Table T.1: Parameters of WSN distributions used for plotting *pdf*'s at Figures F.1 and F.2

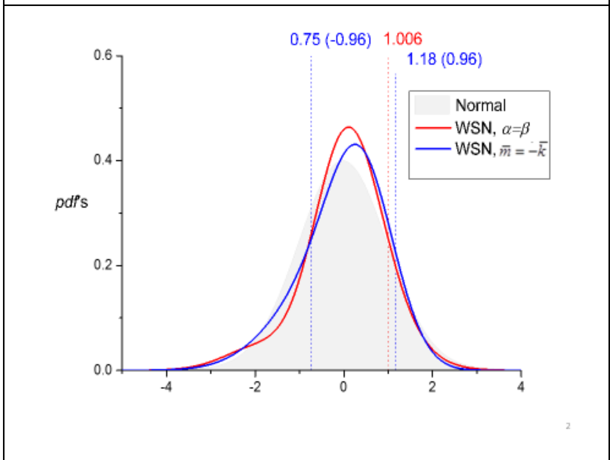
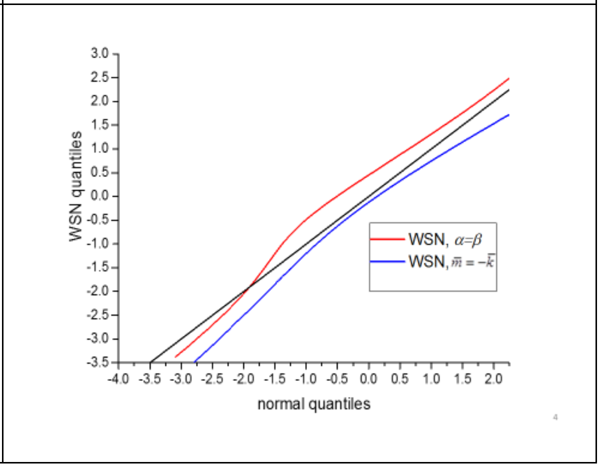
	α	β	m	K	ρ	σ
not-constrained	-1.670	-1.021	1.211	-0.7296	0.75	1.064
policy-output symmetric, constrained by $\alpha = \beta$	-1.639	-1.639	1.006	-6.272	0.75	0.9886
policy-input symmetric constrained by $m = -k$	-1.770	-0.9587	0.9616	-0.9616	0.75	1.032

Figure F.1: Comparison of *pdf'*s of the unconstrained and policy-output symmetric WSN distributions

*pdf'*s	normal q-q plot
	

Legend: Standard normal distribution is represented by a shaded contour. Vertical dashed and dotted lines indicate the estimated policy input thresholds ($\overline{m}$ and $\overline{k}$). For comparison, the *pdf*'s have been shifted by mean to zero.

Figure F.2: Comparison of *pdf'*s of the policy-output and policy input symmetric WSN distributions

*pdf'*s	normal q-q plot
	

Legend: Standard normal distribution is represented by a shaded contour. Vertical dashed and dotted lines indicate the estimated policy input thresholds ($\overline{m}$ and $\overline{k}$). For comparison, the *pdf*'s have been shifted by mean to zero. Because of this shift, the coordinates of $\overline{m}$ and $\overline{k}$ have changed. The original coordinates are given in brackets

Figures F.1 and F.2 illustrate the differences in shapes of the WSN *pdf*'s for particular sets of policy-input and -output constraints. The negative skewness implies the anti-inflationary asymmetry in effects of monetary policy. In terms of the parameters of the WSN distribution it means that either $\alpha < \beta$, and/or the upper thresholds is closer to zero than the upper one, and hence is more likely being breached ($\overline{m} < -\overline{k}$). Figure F.1 (and also the red line on Figure F.2) indicates that imposing symmetry constraints on the strength of the policy, that is $\alpha = \beta$, is causing the lower threshold $\overline{k}$ to disappear from the scale ($\overline{k} = -6.27$). In another words, if inflation uncertainties are WSN-distributed with a moderate skewness and policy-output symmetry, pro-inflationary policy either does not take place at all, or leave no traces in the uncertainty. The differences between the upper thresholds in the case of the unconstrained and policy-output constrained cases are not substantial, with the threshold for the constrained policy being slightly smaller. This suggest that interventions affecting uncertainty based on the signals that upper threshold is breached can be marginally more frequent in the policy-output symmetric case than in the corresponding fully asymmetric case. Similar conclusion can be drawn from Figure F.2, while comparing the symmetricity of policy input and output signals for distributions with identical variance and skewness. Here also the upper threshold $\bar{m}$ for the policy output-symmetric case is below that of the policy-input symmetric case.

The normal q-q plots represent pairs of quantiles of the normal and WSN distributions, with the deviations from the 45° line illustrating deviations from normality. It can be noticed that, although the skewness of the policy-input symmetric and policy-output symmetric distributions is the same, only the former distribution (that is where $\bar{m} = -\bar{k}$) shows a typical shape for a skewed distribution, with the WSN quantiles being consistently below the 45° line. Also, the q-q line for the unconstrained WSN shows a typical pattern for skewness. However, for the policy output-symmetric case, the pattern is mixed, indicating more substantial differences to the normal distribution for values which are not the tails.

The results discussed above suggest that imposing the policy-input symmetric restrictions can be more rational than imposing the policy output restrictions. The latter restrictions might give results which are difficult to interpret, due to the sensitivity of the policy input thresholds $\bar{m}$ and $\bar{k}$ to such restrictions, even if skewness is moderate. The input symmetric restrictions lead to results which are directly interpretable, by comparison of the magnitudes of α and β parameters. Also, setting the values of $\bar{m}$ and $\bar{k}$ can by justified more easily, for instance by imposing one standard deviation thresholds limits, that is by setting $\bar{m} = \sigma$ and $\bar{k} = -\sigma$.

S2.2: WSN density functions, imperfect knowledge and output-asymmetry

We have checked to what extent the arbitrary setting of the parameter ρ in (4), which describes the amount of imperfect knowledge of the private forecasters, might affect the accuracy of the analysis. Figure F.3 shows the density functions of the $\mathrm{WSN}_1\left(-2, 0, 1, -1, \rho\right)$ distribution, that is with policy-output anti-inflationary asymmetry and policy-input symmetry, where ρ changes from 0.05 to 0.95. Changes in ρ represent changes in the imperfect knowledge of the private forecasters from a nearly total ignorance ($\rho = 0.05$) to almost perfect knowledge ($\rho = 0.95$).

Figure F.3 indicates that the shapes of the distribution do not change much with ρ, that is are not strongly affected by possible misjudgements regarding the private forecasters' competence. Although the overall changes in the second and third moments for the entire span of ρ are substantial (variance of U changes from 2.28 for $\rho = 0.05$ to 0.85 for $\rho = 0.95$ and, for the same values of ρ, skewness changes from -0.98 to -0.34), the practical effects of a mild misjudgements are not substantial. For instance, 10% change in ρ from 0.6 to 0.7 for this rather extreme case of policy input asymmetry, alters the $RMSE_U$ from 1.27 to 1.22, which corresponds to a change in the uncertainty measured by $RMSE_U$ by about 4% (for definition of $RMSE_U$ see Section 4 of the paper).

Nonlinear pattern of the central moments of U can be observed while analysing evolution of the WSN distribution resulting from the changes in policy output asymmetry. Figure F.4 shows the *pdf*'s of the $\mathrm{WSN}_1(\alpha, 0, 1, -1, 0.75)$ distributions with the parameter α changing from -3.0 to 0.

Figure F.3: Comparison of *pdf*'s of $U \sim \mathrm{WSN}_1(-2, 0, 1, -1, \rho)$ with $\rho \in [0.05, 0.95]$

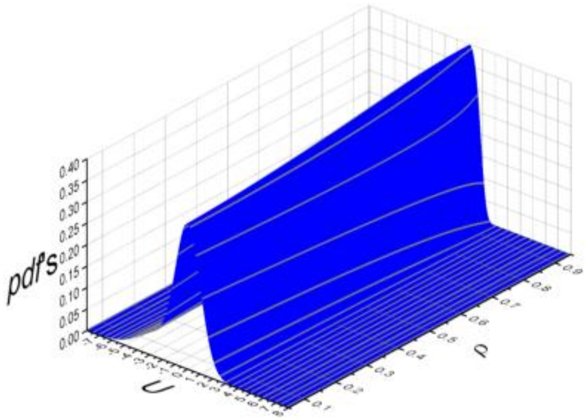
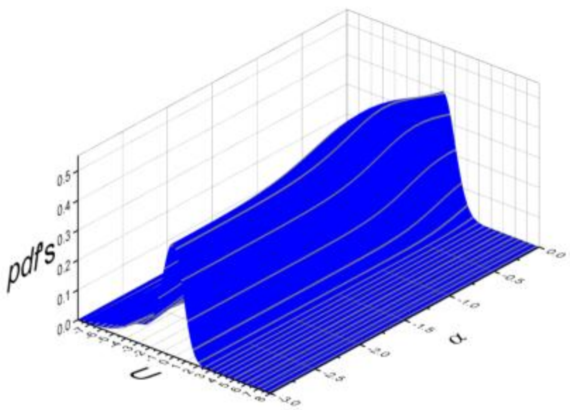


Figure F.4: Comparison of *pdf*'s of $U \sim \mathrm{WSN}_1(\alpha, 0, 1, -1, 0.75)$ with $\alpha \in [-3.0, 0]$



Variance of U is reaching its minimum for $\alpha = -0.9$; for $\alpha = 0$ the variance is equal to unity and for $\alpha = -3.0$ it is equal to 2.28. However, the bias is increasing monotonically, which gives the minimum of the $RMSE_U$ at $\alpha = -0.75$. In another words, for $\rho = 0.75$, symmetric unitary policy-input thresholds and purely output-asymmetric anti-inflationary policy, the optimal marginal strength minimizing the uncertainty is at $\alpha = -0.75$.[6]

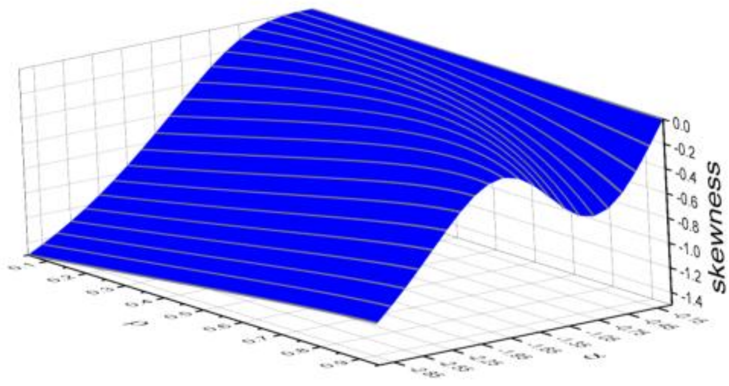
Figures F.5 illustrate the effects of changes in the imperfect knowledge, that is in ρ, and policy output-asymmetry, that is in α, on skewness and $RMSE_U$ of the WSN distributions and uncertainty measure. They show values and contours (isoquants) of the skewness

[6] More numerical results, and also a computer program for the analysis of moments of WSN distributions, written in GAUSS, are available on request.

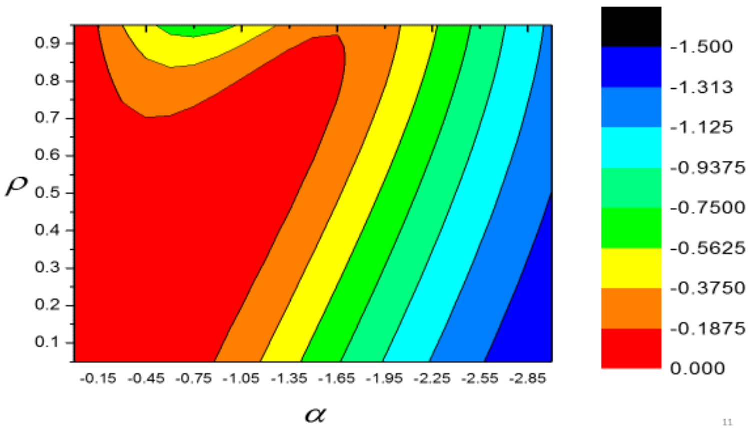
coefficient for $\mathrm{WSN}_1(\alpha, 0,1,-1,\rho)$ distribution with $\rho \in [0.05, 0.95]$ and $\alpha \in [-3.0, 0]$. These figures depict the nonlinearities reflected by the WSN distributions. Generally, low level of skewness is shown by WSN distributions for low α's (for $\alpha = 0$ the distribution is symmetric). The skewness is increasing with the increase in the policy-output asymmetry. However, substantial skewness is also evident for moderate policy-output asymmetry and a high level of imperfect knowledge. The uncertainty expressed by $RMSE_U$ is increasing with the increase in policy output asymmetry. In particular, minimum values of $RMSE_U$ are attained for moderate asymmetry.

Figure F.5: Skewness and $RMSE_U$ for $\mathrm{WSN}_1(\alpha, 0,1,-1,\rho)$; $\alpha \in [-3.0, 0]$, $\rho \in [0.05, 0.95]$

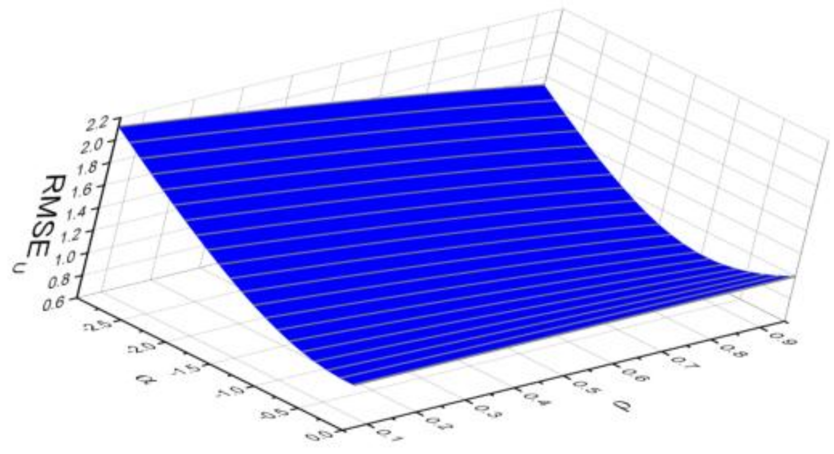
Coefficient of skewness



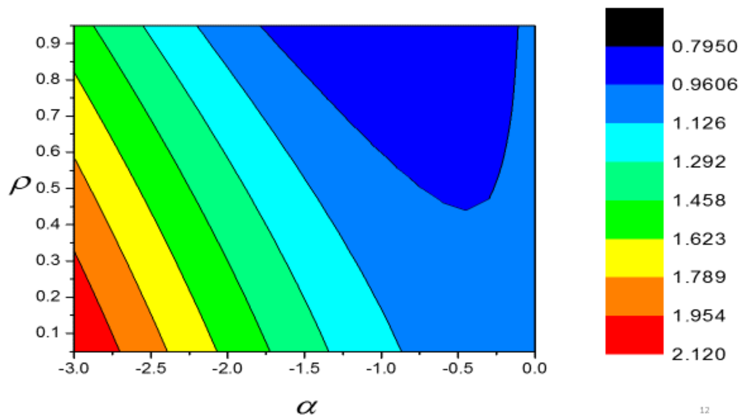
Contour plot for coefficient of skewness



$RMSE_U$



Contour plot for $RMSE_U$



Part S3: More detailed empirical results for U.K. and U.S.

In addition to the empirical results given in Section 5 of the paper, this part presents the following:

1. Parameter estimates of the WSN distribution fitted to GARCH-adjusted *U*-uncertainties for U.K. and U.S, for forecast horizons from 1 to 24.
2. Plot of the term structure of strength of GARCH-adjusted *U*-uncertainties for U.K. and U.S. for the forecast horizons from 1 to 24.
3. Table showing the term structure based on not adjusted for GARCH *U*-uncertainties, analogous to Table 2 in the paper, which is based on GARCH-adjusted *U*-uncertainties.

Table T2: Estimated WSN parameters (standard errors below the estimates)

	U.K.			U.S.		
for.hor	α	β	σ	α	β	σ
1	-0.85 1.35	-0.78 0.58	0.69 0.12	-1.81 0.67	-1.78 0.58	0.35 0.06
2	-2.94 1.19	-1.81 1.66	0.47 0.58	-0.28 0.90	-0.10 0.31	0.69 0.15
3	-3.05 0.54	-2.06 0.08	0.54 0.35	-1.82 0.68	-1.65 1.17	0.57 0.26
4	-3.04 0.00	-1.76 1.01	0.70 0.16	-2.24 0.51	-1.85 0.80	0.57 0.25
5	-3.39 0.91	-1.94 0.93	0.82 0.53	-2.10 0.07	-1.76 0.00	0.68 0.09
6	-3.49 0.59	-2.11 0.60	0.89 0.26	-1.97 0.17	-1.49 1.18	0.82 0.48
7	-3.65 0.40	-2.15 0.30	0.93 0.11	-1.91 0.04	-1.55 0.34	0.91 0.18
8	-3.65 0.10	-1.91 1.05	1.00 0.10	-1.99 0.22	-1.58 0.08	0.99 0.08
9	-3.80 0.65	-2.22 0.56	1.00 0.10	-2.12 0.39	-1.77 0.47	0.98 0.05
10	-3.70 0.07	-2.53 0.41	1.13 0.01	-2.17 0.29	-1.70 0.33	1.07 0.17
11	-3.63 0.15	-2.58 0.94	1.19 0.19	-2.33 0.22	-1.66 0.18	1.13 0.02
12	-3.84 0.51	-2.96 0.75	1.22 0.29	-2.38 0.95	-1.67 1.22	1.20 0.78
13	-3.49 0.42	-2.75 0.59	1.35 0.30	-2.39 0.97	-1.71 0.35	1.29 0.51
14	-3.59 0.29	-2.94 0.18	1.30 0.46	-2.26 0.57	-1.48 0.14	1.36 0.27
15	-3.74 0.19	-2.83 0.15	1.33 0.36	-2.63 1.24	-1.83 0.22	1.28 0.51

	U.K.			U.S.		
for.hor	α	β	σ	α	β	σ
16	-3.94 0.19	-2.67 0.65	1.37 0.23	-2.64 0.25	-1.78 0.07	1.35 0.29
17	-3.69 0.98	-2.82 0.18	1.45 0.00	-2.72 0.50	-2.12 1.12	1.33 0.38
18	-3.67 0.04	-2.71 0.55	1.56 0.35	-2.78 0.70	-2.16 0.25	1.34 0.34
19	-3.54 0.08	-2.87 0.46	1.65 0.36	-2.60 0.12	-2.18 0.32	1.41 0.12
20	-3.85 0.04	-2.94 0.70	1.69 0.24	-2.61 0.84	-2.16 0.24	1.41 0.11
21	-3.82 0.43	-2.94 0.32	1.71 0.17	-2.71 0.53	-2.30 0.19	1.53 0.26
22	-3.68 0.50	-2.96 0.74	1.86 0.31	-2.55 0.04	-2.16 0.77	1.58 0.59
23	-3.73 0.16	-2.52 0.36	1.96 0.11	-2.44 0.38	-2.03 0.36	1.78 0.05
24	-3.43 0.78	-2.50 1.19	2.03 0.34	-2.61 0.33	-2.03 0.36	1.78 0.06

It can be noticed that the largest standard errors of the estimates (relatively to the estimated parameters) are for the shortest forecast horizons, of 1-3 months. For the forecast horizons beyond of 3 months the accuracy of the parameters' estimates improves in the sense that the ratios of the estimates to their standard errors are, in nearly all cases, greater than two.

In Figure F.6 the compound strength is computed as: $Strength = |\alpha| \cdot D_m + |\beta| \cdot D_k$, where:

$$D_a = \int_{|a|}^{+\infty} t^2 \varphi(t)dt = 1 - \Phi(|a|) + |a|\varphi(a) \quad .$$

(see Section 4 in the paper). In the policy-input symmetric case, where $m = -k$ (and this is the assumption applied for empirical estimation), the multipliers D_a can be ignored. Figure F.6 shows some volatility in the compound strength for the short forecast horizon, and then indicates that the maximum strength can be for forecast of about 20 months ahead. For U.K. there is also a spike of the compound strength for the forecast horizon of 12 months.

Table T3 presents the term structure measures of forecast uncertainty based on crude forecast errors. Entries for Table T3 are computed analogously to that in Table 2 in the paper, with the difference being that *U*-uncertainties have not been GARCH-adjusted. Instead of computing *U*-uncertainties as in (2):

$$U_{t,h} = \frac{Z_t - \mu_{t|t-h}}{\sigma_{t|t-h}} \sigma_{t,h}$$

(for the explanation of symbols see Section 2), they are defined simply as unadjusted forecast errors, that is as:

$$U_{t,h} = Z_t - \mu_{t|t-h} \, .$$

Figure F.6: Term structure of the compound strength of forecast-induced monetary policies in U.K. and U.S. in reducing uncertainty

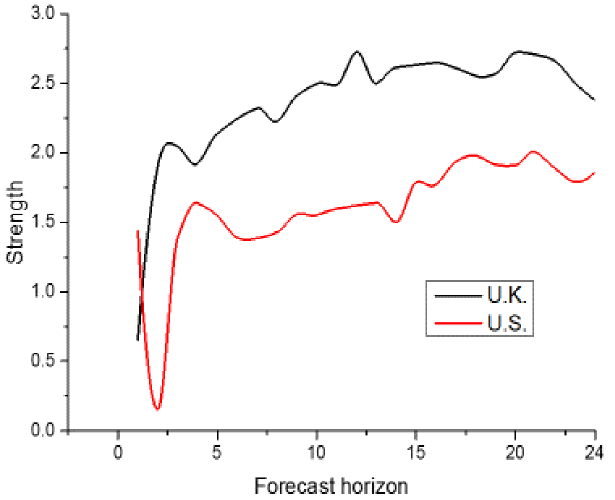


Table T3: Forecast uncertainty measures for U.K. and U.S for GARCH-unadjusted uncertainties.

		U.K.				U.S.			
for. hor	Theor	normalised $RMSE_u$	normalised σ_V	UR	NUR	normalised $RMSE_u$	normalised σ_V	UR	NUR
3	0.11	0.86	0.79	1.29	0.57	0.41	0.46	2.22	0.98
6	0.28	0.75	0.76	1.53	0.68	0.51	0.56	2.11	0.93
9	0.50	0.74	0.74	1.53	0.67	0.60	0.65	2.06	0.91
12	0.75	0.91	0.92	1.57	0.69	0.77	0.79	1.87	0.82
15	0.91	1.36	1.24	1.25	0.55	0.86	0.87	1.81	0.80
18	0.98	0.98	0.95	1.40	0.62	0.98	0.97	1.73	0.76
21	1.00	0.91	0.92	1.55	0.68	1.01	1.00	1.74	0.77
24	1.00	1.00	1.00	1.51	0.67	1.00	1.00	1.77	0.78

For the U.S., the differences between the unadjusted and GARCH-adjusted results are rather small. For the U.K., however, they are more evident. In particular, the unadjusted results are less smooth, with a difficult to interpret peak in uncertainty for the forecast horizon of 15 months. This peak is further transmitted to the estimates of UR and NUR. Lack of smoothness of the estimates can be further documented by the difficult to interpret term structure of strength of the forecast-induced monetary policy (this is not reported here but available on request) Also, the estimated uncertainty for short forecast horizons is substantially bigger than that for GARCH-adjusted uncertainties. The above points at a superiority of using the GARCH-adjusted uncertainties over the unadjusted ones for the analysis of the 'back from the future' policy effects.

Part S4: Data and programs available on request

As the computer programs and especially the data files in format ready for further processing are bulky (over 10MB of data), we do not enclose them with the Supplementary Materials. The data and programs are available from any of the authors on request. However, we are giving below a short description of the programs we use and data.

All computations have been done in APTECH GAUSS 14. Some of the procedures are ours (with all copyright restrictions), some are by authors whose names are known to us (in which cases we identify them by name and left the original copyright messages) and some are of an unknown authorship, accumulated by us over the years. We made every reasonable effort to identify the authors, but often this was not possible.

The main program used for processing data on uncertainties, CDM_ASSEMBLER, performs the following:

a. Reads data for 38 countries on 1 to 24 months ahead forecasts errors obtained in recursively updated windows. According to option selected, the data read can either be GARCH-adjusted, or not. Computations can be made for all countries, of for a selected group of countries.
b. Prints ARIMA and minimum distance estimation results.
c. Computes theoretical and empirical moments of *U*-and *V*-uncertainties.
d. Computes different variations of UR and NUR measures.
e. Constructs forecast term structure tables.
f. Prepares data for graphical representation of probabilistic forecasts based on *U*- and *V*-uncertainties (fan-charts).
g. Performs rank correlation analysis (using Spearman rho and Kendall tau coefficients with *p*-values obtained analytically and by bootstrapping) for various characteristics of *U*- and *V*-uncertainties and different central bank transparency and independency measures. It also performs different types of the split test, discussed in the paper.
h. Saves the outcomes to an Excel file in format easy for further plotting, processing and tabling.

It is accompanied by 114 data files in GAUSS `fmt` format. There are three files for each country, containing (1) GARCH adjusted and (3) non-adjusted forecasts errors or each country, and also (3) files with the results of ARIMA recursive estimation.

Additional files also available on request are:

1. Program and procedures for identification, estimation and forecasting of the seasonal ARIMA model, performing all operations in recursively updated windows and saving forecast errors.
2. Program and procedures for performing simulated minimum distance estimation (GRISHA).
3. Program for optimisation of the WSN parameters subject to a desired set of moments, used for preparation of data for making the *pdf* and q-q plots.

These programs are also in GAUSS, but they require additional libraries to be installed (FANPAC, CML, CO, MAXLIK). Except for the optimisation program, which runs in GAUSS for Windows, the ARIMA estimation and the minimum distance estimation programs run under LINUX at an HPC parallel computer and requires another assembler-type GAUSS Windows program for collecting the data and preparing them for CDM_ASSEMBLER.