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Revealed preference tests under risk and uncertainty

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Abstract: Consider a finite data set where each observation consists of a bundle of contingent consumption chosen from a constraint set of contingent consumption bundles. We develop a general procedure for testing the consistency of such a data set with a broad class of models of choice under risk or uncertainty. Unlike previous tests, we do not require that the agent has a concave Bernoulli utility function.

Keywords: expected utility, rank dependent expected utility, maxmin expected utility, revealed preference

JEL classification numbers: C14, C60, D11, D12, D81

1. Introduction

Let $\mathcal{O} = \{(p^t, x^t)\}_{t \in \mathcal{T}}$ be a finite set of T elements, where $p^t \in R^{\bar{s}}_{++}$ and $x^t \in R^{\bar{s}}_{+}$. We interpret $\mathcal{O}$ as a set of observations, where x^t is the observed bundle of $\bar{s}$ goods chosen by an agent (the *demand bundle*) at the price vector p^t. A function $U : R^{\bar{s}}_{+} \to R$ is said to *rationalize* the set $\mathcal{O}$ if, at all $t \in \mathcal{T}$, x^t is the bundle that maximizes U in the budget set

$$B^t = \{x \in R^{\bar{s}}_{+} : p^t \cdot x \leqslant p^t \cdot x^t\}. \tag{1}$$

For any data set that is rationalizable by a locally non-satiated utility function, its revealed preference relations must satisfy a no-cycling condition called the generalized axiom of revealed preference (GARP). The famous theorem of Afriat (1967) shows that any data set that obeys GARP will in turn be rationalizable by a continuous, strictly increasing, and concave utility function. Afriat's result is very useful because it gives a nonparametric test of utility maximization that can be easily implemented in observational and experimental

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settings. It is known that GARP holds if and only if there is a solution to a set of linear inequalities constructed from the data; much applied work using Afriat's Theorem checks for GARP by checking for a solution to this linear program.[1]

It is both useful and natural to develop tests, similar to the one developed by Afriat, for alternative hypotheses on agent behavior. Our objective in this paper is to develop a procedure that is useful for testing models of choice under risk or uncertainty. Retaining the formal setting described in the previous paragraph, we can interpret $\bar{s}$ as the number of states of the world, with x^t a bundle of contingent consumption, and p^t the state prices faced by the agent. In a setting like this, we can ask what conditions on the data set are necessary and sufficient for it be consistent with an agent maximizing an expected utility (EU) function (in the case where probabilities are objective and known to both observer and agent and also in the case where it is subjective). This means that (for all observations t) the choice x^t maximizes the agent's expected utility, compared to other bundles in the budget set. Assuming that probability of state s is commonly known to be π_s, this involves recovering a Bernoulli utility function $u : R_+ \to R$, which we require to be increasing and continuous, such that, for each $t \in T$,

$$\sum_{s=1}^{\bar{s}} \pi_s u(x_s^t) \geqslant \sum_{s=1}^{\bar{s}} \pi_s u(x_s) \text{ for all } x \in B^t. \tag{2}$$

In the case where the state probabilities are subjective and not known to the observer, it would be necessary to recover both u and π_s such that (2) holds.

In fact, tests of this sort have already been developed by Varian (1983) and Green and Srivastava (1986). In these papers, the realization within each state is allowed to be multi-dimensional (in other words, it can be a bundle of goods rather than a monetary payoff), so it is even more general than our description above. As in Afriat's Theorem, the tests developed by these authors involve solving a set of inequalities that are derived from the data; there is consistency with EU maximization if and only if a solution to these inequalities exists.[2] More recently, Bayer *et al.* (2013) have developed similar tests for a broader class of models,

[1] For proofs of Afriat's Theorem, see Afriat (1967), Varian (1982), and Fostel *et al.* (2004). The term GARP is from Varian (1982); Afriat refers to the same property as *cyclical consistency.*

[2] More intuitive characterizations of expected utility maximization, closer in flavor to GARP, have been developed by Kubler *et al.* (2013, for objective EU) and Echenique and Saito (2013, for subjective EU). These papers require the concavity of the Bernoulli utility function and that there be one good in each state.

including maxmin expected utility and variational preferences. Common to all of these tests is the assumption that the Bernoulli utility function is concave, so risk aversion is assumed as part of the test. Furthermore, it is assumed that the budget set B^t has the classic linear form we defined earlier, so prices are linear and markets are complete.

Our contribution in this paper is to develop a testing procedure that has the following features: (i) it is potentially adaptable to test for different models of choice under uncertainty and not just the expected utility model; (ii) it is a 'pure' test of the model as such and does not require risk aversion or the concavity of the agent's objective function for its validity; and (iii) it is applicable to situations with more complex budgetary constraints and so can be employed even when there is market incompleteness or when there are non-convexities in the budget set because of non-linear pricing or other practices.[3] In the case of objective EU maximization, the test we develop takes the form of a linear program; a data set is consistent with this model if and only if there is a solution to a particular linear program. In the case of subjective EU, rank dependent EU, or maxmin EU, our test involves solving a finite set of bilinear inequalities that is constructed from the data. These problems are decidable, in the sense that there is a known algorithm that can determine in a finite number of steps whether or not a set of bilinear inequalities has a solution.

Nonlinear tests are not new to the revealed preference literature: for example, they appear in tests of weak separability (Varian, 1983), in tests of maxmin EU and other models developed in Bayer *et al.* (2013), and also in Brown and Matzkin's (1996) test of the Walrasian model of general equilibrium. The computational demands of solving these problems can in general be a serious obstacle to implementation, but some of these problems are computationally manageable if they possess certain special features and/or if the number of observations of each subject is small.[4] In the case of the tests that we develop, they simplify dramatically when there are just two states (though they remain nonlinear). The two state case, while special, is common in theoretical modeling and in laboratory experiments.

[3] For an extension Afriat's Theorem to nonlinear budget constraints, see Forges and Minelli (2009).

[4] It is not uncommon to perform tests on fewer than 20 observations. This is partly because revealed preference tests do not in general account for errors and these are unavoidable when there are many observations of the same subject.

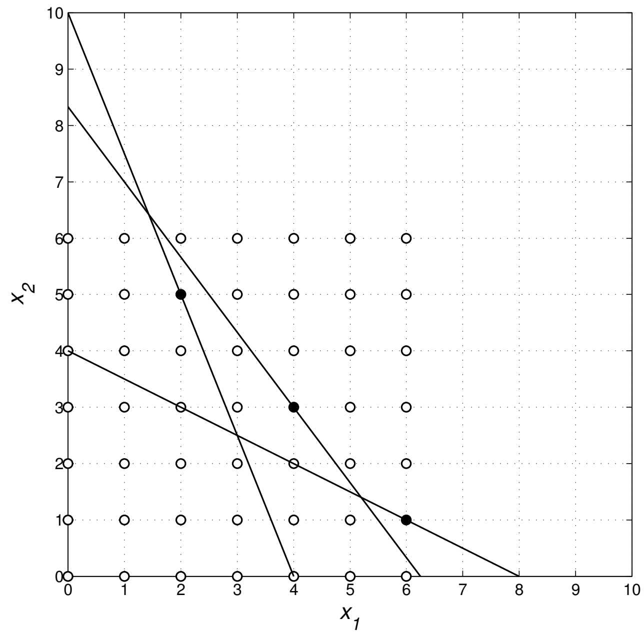

Figure 1: Example 1

Brief description of the test and its implementation

Given the data $\mathcal{O} = \{(p^t, x^t)\}_{t \in \mathcal{T}}$, we define the set

$$\mathcal{X} = \{x' \in R_+ : \text{there is } x^t \text{ such that } x_s^t = x'\} \cup \{0\}. \tag{3}$$

Besides zero, $\mathcal{X}$ consists of those levels of consumption that were chosen at some observation and at some state. Since $\mathcal{O}$ is finite, so is $\mathcal{X}$, and its product $\mathcal{L} = \mathcal{X}^{\bar{s}}$ forms a finite grid of points in $R_+^{\bar{s}}$; in formal terms, $\mathcal{L}$ is a finite lattice. For example, consider the data set depicted in Figure 1, where $x^1 = (2,5)$ at $p^1 = (5,2)$, $x^2 = (6,1)$ at $p^2 = (1,2)$, and $x^3 = (4,3)$ at $p^3 = (4,3)$. In this case, $\mathcal{X} = \{0,1,2,3,4,5,6\}$ and the lattice $\mathcal{L}$ consists of the points depicted with $\circ$.

Suppose we would like to test whether the data set is consistent with expected utility maximization with objective probabilities $\{\pi_s\}_{s=1}^{\bar{s}}$ that are known to us. Clearly, a *necessary* condition for this to hold is that we can find a set of numbers $\{u(r)\}_{r \in \mathcal{X}}$ with the following properties: (i) $u(r'') > u(r')$ whenever $r'' > r'$, and (ii) for all $t \in T$,

$$\sum_{s=1}^{\bar{s}} \pi_s \bar{u}(x_s^t) \geqslant \sum_{s=1}^{\bar{s}} \pi_s \bar{u}(x_s) \text{ for all } x \in B^t \cap \mathcal{L}, \tag{4}$$

with the inequality strict whenever $x \in B^t \cap \mathcal{L}$ and x is in the interior of B^t. (In this case, B^t is given by (1).) Notice that because $\mathcal{X}$ is finite, the existence or otherwise of $\{\bar{u}(r)\}_{r\in\mathcal{X}}$ with properties (i) and (ii) can be straightforwardly ascertained by solving a family of linear inequalities. Our main result says that if a solution can be found, then there is a continuous and strictly increasing utility function $u : R_+ \to R$ that extends $\{\bar{u}(r)\}_{r\in\mathcal{X}}$ and satisfies (2).

Returning to the example depicted in Figure 1, suppose we know that $\pi_1 = \pi_2 = 0.5$. Our test requires that we find $\bar{u}(r)$, for $r = 0, 1, 2, ..., 6$, such that the expected utility of the chosen bundle (x_1^t, x_2^t) is greater than that of the lattice points within the corresponding budget set B^t. One could check that these requirements are satisfied for $\bar{u}(r) = r$, for $r = 0, 1, ..., 6$, so we conclude that the data set is consistent with expected utility maximization.

A general description of the testing procedure we have just outlined, together with a proof of its validity, can be found in Section 2. In Section 3, we show how this procedure can applied to test for different models of choice behavior, including EU-maximization and also the maximization of rank dependent EU and maxmin EU. As an illustration of how these tests can be used, we implement them on a data set obtained from the portfolio choice experiment in Choi *et al.* (2007). In this experiment, each subject was asked to purchase Arrow-Debreu securities under different budget constraints. There were two states of the world and it is commonly known that each state occurred with probability 1/2. We tested these subjects for utility-maximization and for EU-maximization with a concave Bernoulli utility function, using the standard tests. We then tested the same subjects for EU-maximization and for rank dependent EU-maximization, using the tests we have developed. While most subjects exhibited behavior that is consistent (or close to consistent) with utility-maximization, pass rates for all the different models of EU-maximization were very much lower. However, a significant number of subjects did display behavior broadly consistent with EU-maximization, though virtually none were consistent with EU-maximization once concavity was imposed.

2. Testing the model on a lattice

We assume that there is a finite set of states, denoted by $S = \{1, 2, ..., \bar{s}\}$. The contingent consumption space is $R_+^{\bar{s}}$; for a typical consumption bundle $x \in R_+^{\bar{s}}$, the sth entry, x_s, specifies the consumption level in state s. We assume that there are T observations in the data set $\mathcal{O}$, where $\mathcal{O} = \{(x^t, B^t)\}_{t=1}^T$. This means that the agent is observed to choose the bundle x^t from the set $B^t \subset R_+^{\bar{s}}$. We assume that B^t is compact and that $x^t \in \partial B^t$, where ∂B^t denotes the *upper boundary* of B^t. An element $y \in B^t$ is in ∂B^t if there is no $x \in B^t$ such that $x > x^t$. The most important example of B^t is the standard budget set when markets are complete, i.e., when B^t is given by (1), with $p^t \gg 0$ the vector of state prices. We also allow for the market to be incomplete. Suppose that the agent's contingent consumption is achieved through a portfolio of securities and that the asset prices do not admit arbitrage; then the budget set is compact since there is $p^t \gg 0$ such that

$$B^t = \{x \in R_+^s : p^t \cdot x \leqslant p^t \cdot x^t\} \cap \{Z + \omega\},$$

where Z is the span of the assets available to the agent and ω is his endowment of contingent consumption. Note that the budget set B^t and the contingent consumption bundle x^t will both be known to the observer so long as he can observe the asset prices and the agent's holding of securities, knows the asset payoffs in every state and the agent's endowment of contingent consumption, ω.

Let $\{\phi(\cdot, t)\}_{t=1}^T$ be a collection of functions, where $\phi(\cdot, t) : R_+^{\bar{s}} \to R$ is increasing in all its arguments. The data set $\mathcal{O} = \{(x^t, B^t)\}_{t=1}^T$ is said to be *rationalizable by* $\{\phi(\cdot, t)\}_{t \in T}$ if there exists a continuous and increasing function $u : R_+ \to R_+$ (which we shall call the *Bernoulli utility function*) such that

$$\phi(\mathbf{u}(x^t), t) \geqslant \phi(\mathbf{u}(x), t) \text{ for all } x \in B^t, \tag{5}$$

where $\mathbf{u}(x) = (u(x_1), u(x_2), ..., u(x_{\bar{s}}))$. In other words, there is some Bernoulli utility function u under which x^t is an optimal choice in B^t, assuming that the agent is maximizing $\phi(\mathbf{u}(x), t)$. Many of the basic models of choice under risk and uncertainty can be described within this framework, with different models leading to different functional forms for $\phi(\cdot, t)$. We shall explore some of these models later; for now, it suffices to point out, as a basic example, that expected utility is captured by this form.

Example: Suppose that both the observer and the agent knows that the probability of state s in observation t is $\pi_s^t > 0$. If the agent is maximizing expected utility, then

$$\phi(u_1, u_2, ..., u_{\bar{s}}, t) = \sum_{s=1}^{\bar{s}} \pi_s^t u_s \tag{6}$$

so (5) requires that

$$\sum_{s=1}^{\bar{s}} \pi_s^t u(x_s^t) \geqslant \sum_{s=1}^{\bar{s}} \pi_s^t u(x_s) \text{ for all } x \in B^t, \tag{7}$$

i.e., the expected utility of the bundle x^t is greater than that of any other bundle in B^t. When there exists a continuous and increasing function u such that (7) holds, we say that the data set is *EU-rationalizable with probability weights* $(\pi^t)_{t=1}^T$, where $\pi^t = (\pi_1^t, \pi_2^t, ..., \pi_{\bar{s}}^t)$.

If $\mathcal{O}$ is rationalizable by $\{\phi(\cdot, t)\}_{t \in T}$ then, since the objective function $\phi(\mathbf{u}(\cdot), t)$ is strongly increasing in x, we must have

$$\phi(\mathbf{u}(x^t), t) \geqslant \phi(\mathbf{u}(x), t) \text{ for all } x \in \underline{B}^t \tag{8}$$

where $\underline{B}^t = \{y \in R_+^{\bar{s}} : y \leqslant x \text{ for some } x \in B^t\}$. Furthermore, the inequality in (8) is strict whenever $x \in \underline{B}^t \backslash \partial \underline{B}^t$ (where $\partial \underline{B}^t$ refers to the upper boundary of $\underline{B}^t$). We define

$$\mathcal{X} = \{x' \in R_+ : \text{there is } x^t \text{ such that } x_s^t = x'\} \cup \{0\}.$$

Besides zero, $\mathcal{X}$ consists of those levels of consumption that were chosen at some observation and at some state. Since the data set is finite, so is $\mathcal{X}$. Given $\mathcal{X}$, we may construct $\mathcal{L} = \mathcal{X}^{\bar{s}}$, which consists of a finite grid of points in $R_+^{\bar{s}}$; in formal terms, $\mathcal{L}$ is a finite lattice. Let $\bar{u} : \mathcal{X} \to R_+$ be the restriction of the Bernoulli utility function u to $\mathcal{X}$. Given our observations, the following must hold:

$$\phi(\bar{\mathbf{u}}(x^t), t) \geqslant \phi(\bar{\mathbf{u}}(x), t) \text{ for all } x \in \underline{B}^t \cap \mathcal{L} \text{ and} \tag{9}$$

$$\phi(\bar{\mathbf{u}}(x^t), t) > \phi(\bar{\mathbf{u}}(x), t) \text{ for all } x \in \left(\underline{B}^t \backslash \partial \underline{B}^t\right) \cap \mathcal{L}, \tag{10}$$

where $\bar{\mathbf{u}}(x) = (\bar{u}(x_1), \bar{u}(x_2), ..., \bar{u}(x_{\bar{s}}))$. Our main theorem says that the converse is also true.

THEOREM 1. *Suppose that for some data set $\mathcal{O} = \{(x^t, B^t)\}_{t=1}^T$ and collection of strongly increasing and continuous functions $\{\phi(\cdot, t)\}_{t=1}^T$, there is an increasing function $\bar{u} : \mathcal{X} \to R_+$ that satisfy the conditions (9) and (10). Then there is an increasing and continuous function $u : R_+ \to R_+$ that extends $\bar{u}$ and guarantees the rationalizability of $\mathcal{O}$ by $\{\phi(\cdot, t)\}_{t=1}^T$.*

To proof this result, we use the following lemma.

Lemma 1. *Let* $\{C^t\}_{t=1}^T$ *be a finite collection of constraint sets in* $R_+^{\bar{s}}$ *that are compact and downward closed (i.e., if* $x \in C^t$ *then so is* $y \in R_+^{\bar{s}}$ *such that* $y < x$*) and let* $\{\phi(\cdot, t)\}_{t=1}^T$ *be a collection of strongly increasing and continuous functions. Suppose that there is a finite set* $\mathcal{X}$ *of* R_+*, an increasing function* $\bar{u} : \mathcal{X} \to R_+$*, and* $\{M^t\}_{t \in T}$ *such that the following holds:*

$$M^t \geqslant \phi(\bar{\mathbf{u}}(x), t) \text{ for all } x \in C^t \cap \mathcal{L} \text{ and} \tag{11}$$

$$M^t > \phi(\bar{\mathbf{u}}(x), t) \text{ for all } x \in (C^t \backslash \partial C^t) \cap \mathcal{L}, \tag{12}$$

where $\mathcal{L} = \mathcal{X}^{\bar{s}}$ *and* $\bar{\mathbf{u}}(x) = (\bar{u}(x_1), \bar{u}(x_2), ..., \bar{u}(x_{\bar{s}}))$*. Then there is a continuous and increasing function* $u : R_+ \to R_+$ *that extends* $\bar{u}$ *such that*

$$M^t \geqslant \phi(\mathbf{u}(x), t) \text{ for all } x \in C^t \text{ and} \tag{13}$$

$$\text{if } x \in C^t \text{ and } M^t = \phi(\mathbf{u}(x), t), \text{ then } x \in \partial C^t \cap \mathcal{L} \text{ and } M^t = \phi(\bar{\mathbf{u}}(x), t). \tag{14}$$

Remark: The property (14) needs some explanation. Conditions (11) and (12) allow for the possibility that $M^t = \phi(\bar{\mathbf{u}}(x'), t)$ for some $x' \in \partial C^t \cap \mathcal{L}$; we denote the set of points in $\partial C^t \cap \mathcal{L}$ with this property by X'. Clearly any extension u will preserve this property, i.e., $M^t = \phi(\mathbf{u}(x'), t)$ for all $x' \in X'$. Property (14) says that we can choose u such that for all $x \in C^t \backslash X'$, we have $M^t > \phi(\bar{\mathbf{u}}(x), t)$.

Proof: We shall prove this result by induction on the dimension of the space containing the constraint sets. It is trivial to check that the claim is true if $\bar{s} = 1$. In this case, $\mathcal{L}$ consists of a finite set of points on R_+ and each C^t is a closed interval with 0 as its lowest point. Now let us suppose that the claim holds for $\bar{s} = m$ and we shall prove it for $\bar{s} = m+1$. If, for each t, there is an increasing and continuous utility function $u^t : R_+ \to R_+$ extending $\bar{u}$ such that (13) and (14) hold, then the the same conditions will hold for the increasing and continuous function $u = \min_{t \in T} u^t$. So we can focus our attention on constructing u^t for a single constraint set C^t.

Suppose $\mathcal{X} = \{0, r^1, r^2, r^3, ..., r^I\}$, with $r^0 = 0 < r^i < r^{i+1}$, for $i = 1, 2, ..., I-1$. Let $\bar{r} = \max\{r \in R_+ : (r, 0, 0, ..., 0) \in C^t\}$ and suppose that $(r^i, 0, 0, ..., 0) \in C^t$ if and only if $i \leqslant N$. Consider the collection of sets of the form $D^i = \{y \in R_+^m : (r^i, y) \in C^t\}$ (for

$i = 1, 2, ..., N$); this is a finite collection of compact and downward closed sets in R_+^m. By the induction hypothesis there is a function $u^* : R_+ \to R_+$ that extends $\bar{u}$ such that

$$M^t \geqslant \phi(\bar{u}(r^i), \mathbf{u}^*(y), t) \text{ for all } (r^i, y) \in C^t \text{ and} \tag{15}$$

$$\text{if } (r^i, y) \in C^t \text{ and } M^t = \phi(\bar{u}(r^i), \mathbf{u}^*(y), t), \text{ then } (r^i, y) \in \partial C^t \cap \mathcal{L} \text{ and } M^t = \phi(\bar{\mathbf{u}}(r^i, y), t). \tag{16}$$

For each $r \in [0, \bar{r}]$, define

$$U(r) = \{u \leqslant u^*(r) : \max\{\phi(u, \mathbf{u}^*(y), t) : (r, y) \in C^t\} \leqslant M^t\}.$$

This set is nonempty; indeed $\bar{u}(r^k) = u^*(r^k) \in U(r)$, where r^k is the largest element in $\mathcal{X}$ that is weakly smaller than r. This is because, if $(r, y) \in C^t$ then so is (r^k, y), and (15) guarantees that $\phi(\bar{u}(r^k), \mathbf{u}^*(y), t) \leqslant M^t$. The downward closedness of C^t also guarantees that $U(r) \subseteq U(r')$ whenever $r < r'$. Now define $\tilde{u}(r) = \sup U(r)$; the function $\tilde{u}$ has a number of significant properties. (i) For $r \in \mathcal{X}$, $\tilde{u}(r) = u^*(r) = \bar{u}(r)$ (by the induction hypothesis). (ii) $\tilde{u}$ is a nondecreasing function since U is nondecreasing. (iii) $\tilde{u}(r) > \bar{u}(r^k)$ if $r > r^k$ and the latter is largest element in $\mathcal{X}$ smaller than r. If instead, $\tilde{u}(r) = \bar{u}(r^k)$, then the compactness of C^t guarantees that there is $\hat{y}$ such that $\phi(\bar{u}(r^k), \mathbf{u}^*(\hat{y}), t) = M^t$, with $(r, \hat{y}) \in C^t$. Consequently, $(r^k, \hat{y}) \in C^t$ and, since $\bar{u}(r^k) = u^*(r^k)$, we have $\phi(\mathbf{u}^*(r^k, \hat{y}), t) = M^t$. This can only occur if $(r^k, \hat{y}) \in \partial C^t \cap \mathcal{L}$ (because of (16)), but it is clear that $(r^k, \hat{y}) \notin \partial C^t$ since $(r^k, \hat{y}) < (r, \hat{y})$. (iv) If $r_n \to r^i \in \mathcal{X}$, then $\tilde{u}(r_n) \to u^*(r^i)$. Suppose to the contrary, that the limit is $\hat{u} < u^*(r^i) = \bar{u}(r^i)$. We can assume, without loss of generality, that $\tilde{u}(r_n) < u^*(r_n)$. By the compactness of C^t, there is $(r_n, y_n) \in C^t$ such that $\phi(\tilde{u}(r_n), \mathbf{u}^*(y_n), t) = M^t$. This leads to $\phi(\hat{u}, \mathbf{u}^*(y'), t) = M^t$, where y' is an accumulation point of y_n and $(r^i, y') \in C^t$. But since ϕ is strictly increasing, we obtain $\phi(u^*(r^i), \mathbf{u}^*(y'), t) > M^t$, which contradicts (15).

Given the properties of $\tilde{u}$, we can find a continuous and increasing function u^t such that u^t extends $\bar{u}$, $u^t(r) < u^*(r)$ for all $r \in R_+ \backslash \mathcal{X}$ and $u^t(r) < \tilde{u}(r) \leqslant u^*(r)$ for all $r \in [0, \bar{r}] \backslash \mathcal{X}$. Then the conditions (13) and (14) are satisfied for C^t. QED

Proof of Theorem 1: This follows immediately from Lemma 1 if we set $C^t = \underline{B}^t$, and $M^t = \phi(\bar{\mathbf{u}}(x^t), t)$. If $\bar{u}$ obeys conditions (9) and (10) then it obeys conditions (11) and (12). The rationalizability of $\mathcal{O}$ by $\{\phi(\cdot, t)\}_{t \in T}$ then follows from (13). QED

3. Applications of our testing procedure

Theorem 1 can be used to test different models of choice under risk and uncertainty, with each model requiring different functional forms for $\phi(\cdot, t)$.

3.1 Testing for objective expected utility

Suppose that for a data set $\mathcal{O} = \{(x^t, B^t)\}_{t=1}^T$, there are $\ell + 1$ elements in $\mathcal{X}$; we denote the typical element in $\mathcal{X}$ by r^i, with $r^0 = 0$ and $r^{i-1} < r^i$ for $i = 1, 2, ..., \ell$. We wish to check whether $\mathcal{O}$ is EU-rationalizable with probability weights $\{\pi^t\}_{t=1}^T$, in the sense defined in the Example in the previous section. By Theorem 1, EU-rationalizability holds if and only if there is a collection of real numbers $\{\bar{u}(r)\}_{r\in\mathcal{X}}$ such that

$$0 \leqslant \bar{u}(r^{i-1}) < \bar{u}(r^i) \text{ for } i = 1, 2, ..., \ell \tag{17}$$

and the inequalities (9) and (10) hold, where $\phi(\cdot, t)$ is defined by (6). This is a linear program and it is both *solvable* (in the sense that there is an algorithm that can decide within a known number of steps whether or not a solution exists) and computationally feasible.

While Theorem 1 guarantees that there is a continuous function u that extends $\bar{u} : \mathcal{X} \to R$ when the required conditions are satisfied, this function is not necessarily smooth. For example, suppose that it is commonly known that states 1 and 2 occur with equal probability and we observe the agent choosing the bundle $(1, 1)$ at price (p_1, p_2), with $p_1 \neq p_2$. It is trivial to check that this observation is EU-rationalizable in our sense. In fact, one could even find a concave $u : R_+ \to R$ for which $(1, 1)$ maximizes expected utility. However, any continuous and increasing function u that EU-rationalizes the data *cannot* be smooth. This is because, if it is smooth and given that the two states are equiprobable, the slope of the indifference curve at $(1, 1)$ must equal 1; thus it will not be tangential to the budget line and will not be a local optimum.

Note also that the utility function guaranteed by Theorem 1 need not be a concave function. Consider the example given in Figure 2 and suppose that $\pi_1 = \pi_2 = 1/2$. In this case, $\mathcal{X} = \{0, 1, 2, 7\}$, and one could check that (9) and (10) are satisfied, where $\phi(\cdot, t)$ is defined by (6), if $\bar{u}(0) = 0$, $\bar{u}(1) = 2$, $\bar{u}(2) = 3$, and $\bar{u}(7) = 6$. Thus we know that the

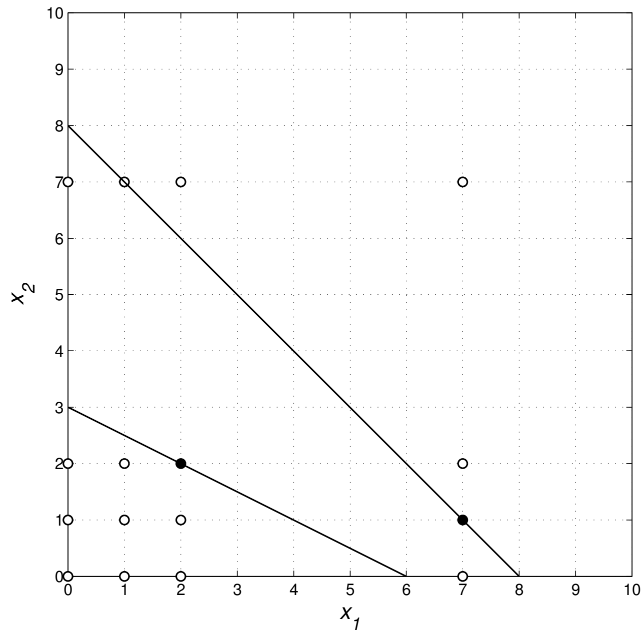

Figure 2: Example 2

data is consistent with EU-maximization. However any Bernoulli utility function that EU-rationalizes the data cannot be concave. Indeed, since $(3,1)$ is strictly within the budget set when $(2,2)$ was chosen, $2u(2) > u(1)+u(3)$. By the concavity of u, $u(3)-u(2) \geqslant u(7)-u(6)$ and thus we obtain $u(6)+u(2) > u(7)+u(1)$, contradicting the optimality of $(1,7)$.

3.2 Testing for subjective expected utility (SEU)

We now consider a setting where no objective probabilities could be attached to each state. The data set $\mathcal{O} = \{(x^t, B^t)\}_{t=1}^T$ is said to be SEU-rationalizable if there is $\pi = (\pi_1, \pi_2, ..., \pi_{\bar{s}}) \gg 0$ and an increasing function $u : R_+ \to R$ such that, for all $t = 1, 2, ..., T$

$$\sum_{s=1}^{\bar{s}} \pi_s u(x_s^t) \geqslant \sum_{s=1}^{\bar{s}} \pi_s u(x_s) \text{ for all } x \in B^t.$$

In other words, at every observation t, the agent is acting as though he attributes a probability of π_s to state s and is maximizing expected utility. In this case, ϕ is independent of t, with $\phi(\mathbf{u}) = \sum_{s=1}^{\bar{s}} \pi_s u_s$. The conditions (9) and (10) can be written as

$$\sum_{s=1}^{\bar{s}} \pi_s \bar{u}(x_s^t) \geqslant \sum_{s=1}^{\bar{s}} \pi_s \bar{u}(x_s) \text{ for all } x \in \underline{B}^t \cap \mathcal{L} \text{ and} \tag{18}$$

$$\sum_{s=1}^{\bar{s}} \pi_s \bar{u}(x_s^t) > \sum_{s=1}^{\bar{s}} \pi_s \bar{u}(x_s) \text{ for all } x \in \left(\underline{B}^t \backslash \partial \underline{B}^t\right) \cap \mathcal{L}. \tag{19}$$

In other words, a necessary and sufficient condition for SEU-rationalizability is that we can find real numbers $\{\pi_s\}_{s=1}^{\bar{s}}$ and $\{u(r)\}_{r\in\mathcal{X}}$ such that $\pi_s > 0$ for all $s \in S$, $\sum_{s=1}^{\bar{s}} \pi_s = 1$, and (17), (18), and (19) are satisfied. This set of conditions form a finite system of bilinear inequalities. The Tarski-Seidenberg Theorem tells us that such systems are decidable.

3.3 Testing for rank dependent expected utility (RDEU)

We now return to a setting where there is an objective probability $\pi_s > 0$ attached to state s that is known to both the agent and the observer. We would like to test whether the agent's behavior is consistent with maximizing a rank dependent expected utility (RDEU) function. Given a vector x, we can rank the entries of x from the smallest to the largest, with ties broken by the rank of the state. We denote by $r(x,s)$, the rank of x_s in x. For example, if there are five states and $x = (1,4,4,3,5)$, we have $r(x,1) = 1$, $r(x,2) = 3$, $r(x,3) = 4$, $r(x,4) = 2$, and $r(x,5) = 5$. A rank dependent expected utility function gives to the bundle x the utility

$$V(x,\pi) = \sum_{s=1}^{\bar{s}} \rho(x,s,\pi) u(x_s) \tag{20}$$

where $u : R_+ \to R$ is an increasing and continuous function,

$$\rho(x,s,\pi) = g\left(\textstyle\sum_{\{s':r(x,s')\leqslant r(x,s)\}} \pi_{s'}\right) - g\left(\textstyle\sum_{\{s':r(x,s')<r(x,s)\}} \pi_{s'}\right), \tag{21}$$

and $g : [0,1] \to R$ is an increasing and continuous function. (If $\{s' : r(x,s') < r(x,s)\}$ is empty, we let $g\left(\sum_{\{s':r(x,s')<r(x,s)\}} \pi_{s'}\right) = g(0)$.) If g is the identity function (or, more generally when g is affine), we simply recover the expected utility model. When it is nonlinear, the function g distorts the cumulative distribution of the lottery x, so that an agent maximizing RDEU can behave as though the probability he attaches to a state depends on the relative attractiveness of the outcome in that state. Since u is increasing, $\rho(x,s,\pi) = \rho(\mathbf{u}(x),s,\pi)$. It follows that we can write $V(x,\pi) = \phi(\mathbf{u}(x),\pi)$, where for any vector $\mathbf{u} = (u_1, u_2, ..., u_S)$,

$$\phi(\mathbf{u},\pi) = \sum_{s=1}^{\bar{s}} \rho(\mathbf{u},s,\pi) u_s. \tag{22}$$

The function ϕ is a continuous and strictly increasing in $\mathbf{u}$.

Suppose we wish to check whether $\mathcal{O} = \{(x^t, B^t)\}_{t=1}^T$ is RDEU-rationalizable with probability weights $\{\pi^t\}_{t=1}^T$. (Recall that $\pi^t \in R_{++}^{\bar{s}}$ gives the objective probability weights attached to each state at observation t.) RDEU-rationalizability holds if and only if there are increasing functions $g : [0,] \to R$ and $u : R_+ \to R$ such that, for each $t \in T$, $V(x^t, \pi^t) \geqslant V(x, \pi^t)$ for all $x \in B^t$, where V is given by (20). To develop a necessary and sufficient test for this property, we first define the set

$$\Gamma = \left\{\gamma : \text{there is } s \in S,\ t \in T, \text{ and } x \in \mathcal{L} \text{ such that } \gamma = \textstyle\sum_{\{s' : r(x,s') \leqslant r(x,s)\}} \pi_{s'}^t\right\} \cup \{0\}.$$

Note that the set Γ is a finite subset of $[0,1]$ and includes both 0 and 1. We may denote the elements of Γ by γ^j, where $\gamma^{j-1} < \gamma^j$, with $\gamma^0 = 0$ and $\gamma^{\bar{m}} = 1$ (so Γ has $\bar{m}+1$ elements).

If $\mathcal{O}$ is RDEU-rationalizable, there must be increasing functions $\bar{g} : \Gamma \to R$ and $\bar{u} : \mathcal{X} \to R$ such that

$$\sum_{s=1}^{\bar{s}} \bar{\rho}(x^t, s, \pi^t)\bar{u}(x_s^t) \geqslant \sum_{s=1}^{\bar{s}} \bar{\rho}(x, s, \pi^t)u(x_s) \text{ for all } x \in \underline{B}^t \cap \mathcal{L} \text{ and} \tag{23}$$

$$\sum_{s=1}^{\bar{s}} \bar{\rho}(x^t, s, \pi^t)\bar{u}(x_s^t) > \sum_{s=1}^{\bar{s}} \bar{\rho}(x, s, \pi^t)\bar{u}(x_s) \text{ for all } x \in \left(\underline{B}^t \backslash \partial \underline{B}^t\right) \cap \mathcal{L}, \tag{24}$$

where

$$\bar{\rho}(x, s, \pi) = \bar{g}\left(\textstyle\sum_{\{s' : r(x,s') \leqslant r(x,s)\}} \pi_{s'}\right) - \bar{g}\left(\textstyle\sum_{\{s' : r(x,s') < r(x,s)\}} \pi_{s'}\right). \tag{25}$$

This is clear since we can simply take $\bar{g}$ and $\bar{u}$ to be the restriction of g and u respectively. Conversely, suppose there are increasing functions $\bar{g} : \Gamma \to R$ and $\bar{u} : \mathcal{X} \to R$ such that (23), (24), and (25) are satisfied, and let $g : [0,1] \to R$ be *any* continuous and increasing extension of $\bar{g}$. Defining $\phi(\mathbf{u}, \pi)$ by (22), the properties (23) and (24) may be re-written as

$$\phi(\mathbf{u}(x^t), \pi^t) \geqslant \phi(\mathbf{u}(x), \pi^t) \text{ for all } x \in \underline{B}^t \cap \mathcal{L} \text{ and}$$

$$\phi(\mathbf{u}(x^t), \pi^t) > \phi(\mathbf{u}(x), \pi^t) \text{ for all } x \in \left(\underline{B}^t \backslash \partial \underline{B}^t\right) \cap \mathcal{L}.$$

By Theorem 1, these properties guarantee that there exists $u : R_+ \to R$ that extends $\bar{u}$ such that the data set $\mathcal{O}$ can be rationalized by $V(x, \pi) = \phi(\mathbf{u}(x), \pi)$.

To recap, we have shown that $\mathcal{O} = \{(x^t, B^t)\}_{t=1}^T$ is RDEU-rationalizable with probability weights $\{\pi^t\}_{t=1}^T$ if and only if there exist real numbers $\{\bar{g}(\gamma)\}_{\gamma \in \Gamma}$ and $\{\bar{u}(r)\}_{r \in \mathcal{X}}$ that satisfy

$$\bar{g}(\gamma^{j-1}) < \bar{g}(\gamma^j) \text{ for } j = 1, 2, ..., \bar{m}, \tag{26}$$

(17), (23), (24), and (25). As in the test for SEU-rationalizability, this test involves finding a solution to a finite set of bilinear inequalities (with unknowns $\{\bar{u}(r)\}_{r\in\mathcal{X}}$ and $\{\bar{g}(\gamma)\}_{\gamma\in\Gamma}$).

Notice also that it is straightforward to modify the test to include restrictions on the shape of g. For example, we may wish to test that $\mathcal{O}$ is RDEU-rationalizable with a convex function g. Then we need to specify that $\bar{g}$ obeys

$$\frac{\bar{g}(\gamma^j) - \bar{g}(\gamma^{j-1})}{\gamma^j - \gamma^{j-1}} \leqslant \frac{\bar{g}(\gamma^{j+1}) - \bar{g}(\gamma^j)}{\gamma^{j+1} - \gamma^j} \text{ for } j = 1, ..., \bar{m} - 1. \tag{27}$$

It is clear that this condition is necessary for the convexity of g. It is also sufficient for the extension of $\bar{g}$ to a convex and increasing function $g : [0,1] \to R$. Thus $\mathcal{O}$ is RDEU-rationalizable with probability weights $\{\pi^t\}_{t=1}^T$ and a convex function g if and only if there exist real numbers $\{\bar{g}(\gamma)\}_{\gamma\in\Gamma}$ and $\{\bar{u}(r)\}_{r\in\mathcal{X}}$ that satisfy (17), (23), (24), (25), (26), and (27).

We now turn to special case which is relevant to our application of this test in Section 4. Suppose there are two equiprobable states of the world. Then $\Gamma = \{0, 1/2, 1\}$. For a bundle (x_1, x_2), the weight attached to the state with the higher outcome is $\bar{\rho}_{**} = \bar{g}(1) - \bar{g}(1/2)$ and the weight attached to the lower outcome is $\bar{\rho}_* = \bar{g}(1/2) - \bar{g}(0)$. With no loss of generality, we may assume that $\bar{g}(0) = 0$ and $\bar{g}(1) = 1$, so then $\bar{\rho}_* = \bar{g}(1/2)$ and $\bar{\rho}_{**} = 1 - \bar{g}(1/2)$. In short, for a bundle $(x_1, x_2) \in \mathcal{L}$

$$\phi\left((\bar{u}(x_1), \bar{u}(x_2)), (1/2, 1/2)\right) = \bar{\rho}_* \bar{u}(x_1) + (1 - \bar{\rho}_*)\bar{u}(x_2) \text{ if } x_1 \leqslant x_2 \text{ and} \tag{28}$$

$$\phi\left((\bar{u}(x_1), \bar{u}(x_2)), (1/2, 1/2)\right) = \bar{\rho}_* \bar{u}(x_2) + (1 - \bar{\rho}_*)\bar{u}(x_1) \text{ if } x_2 > x_1. \tag{29}$$

In this case, a data set is RDEU-rationalizable if and only if we can find $\bar{\rho}_* \in (0,1)$ and $\{\bar{u}(r)\}_{r\in\mathcal{X}}$ such that (23) and (24) are satisfied. Note that this test is clearly more permissive than the test for objective expected utility; in the latter, we effectively require $\bar{\rho}_* = 1/2$ rather than allowing it to take some other value.

3.4 Testing for maxmin expected utility (MEU)

We now consider a setting where no objective probabilities could be attached to each state. An agent with maxmin expected utility behaves as though he evaluates each bundle $x \in R_+^{\bar{s}}$ using the formula

$$V(x) = \min_{\pi\in\Pi} \left\{ \sum_{s=1}^{\bar{s}} \pi_s u(x_s) \right\} \tag{30}$$

where $u : R_+ \to R$ is increasing and continuous and Π is a nonempty, closed, and convex set of probability weights. For our purpose, we shall make two more assumptions about Π. First, we assume that π_s is uniformly bounded away from zero in Π. Second, we assume that Π is the solution to a finite set of linear inequalities, i.e., there are vectors $a_n \in R^S$ and scalars c_n (for $n = 1, 2, ...N$) such that

$$\Pi = \bigcap_{n=1}^{N} \{\pi \in \triangle : a_n \cdot \pi \geqslant c_n\}, \tag{31}$$

where $\triangle = \{\pi \in R_+^{\bar{s}} : \sum_{s=1}^{\bar{s}} \pi_s = 1\}$. It is clear that $V(x) = \phi(\mathbf{u}(x))$, where $\phi(\mathbf{u}) = \min_{\pi \in \Pi} \sum_{s=1}^{\bar{s}} \pi_s u_s$. The function ϕ is continuous and strictly increasing (the latter because of our first assumption on Π). We denote by $\tilde{\pi}(\mathbf{u})$ the value of $\pi \in \Pi$ that minimizes $\sum_{s=1}^{\bar{s}} \pi_s u_s$. Therefore, we can write $\phi(\mathbf{u}) = \sum_{s=1}^{\bar{s}} \tilde{\pi}_s(\mathbf{u}) u_s$.

We wish to develop a test for the hypothesis that the data set $\mathcal{O} = \{(x^t, B^t)\}_{t=1}^{T}$ is rationalizable by a maxmin expected utility function, for a given Π defined by (31). This means finding $u : R_+ \to R$ such that for each $t \in T$, $V(x^t, \pi^t) \geqslant V(x, \pi^t)$ for all $x \in B^t$, where V is given by (30). By Theorem 1, it is necessary and sufficient to find an increasing function $\bar{u} : \mathcal{X} \to R$ such that (9) and (10) holds. In this context, those conditions take the following form:

$$\sum_{s=1}^{\bar{s}} \tilde{\pi}_s(\bar{\mathbf{u}}(x^t))\bar{u}(x_s^t) \geqslant \sum_{s=1}^{\bar{s}} \tilde{\pi}_s(\bar{\mathbf{u}}(x))\bar{u}(x_s) \quad \text{for all } x \in \underline{B}^t \cap \mathcal{L} \text{ and} \tag{32}$$

$$\sum_{s=1}^{\bar{s}} \tilde{\pi}_s(\bar{\mathbf{u}}(x^t))\bar{u}(x_s^t) > \sum_{s=1}^{\bar{s}} \tilde{\pi}_s(\bar{\mathbf{u}}(x))\bar{u}(x_s) \quad \text{for all } x \in \left(\underline{B}^t \backslash \partial \underline{B}^t\right) \cap \mathcal{L}. \tag{33}$$

For each $\hat{x} \in \mathcal{L}$, $\tilde{\pi}(\bar{\mathbf{u}}(\hat{x})$ minimizes $\sum_{s=1}^{\bar{s}} \pi_s \bar{u}(\hat{x}_s)$ subject to $\pi \in \Pi$. This minimization problem is linear and so it is necessary and sufficient that $\tilde{\pi}(\bar{\mathbf{u}}(\hat{x})$ obeys the Kuhn-Tucker conditions. The conditions require that there are $\lambda_n(\hat{x}) \in R$ for $n = 0, 1, 2, .., N$ such that

$$\bar{u}(\hat{x}_s) = \lambda_0(\hat{x}) + \sum_{n=1}^{N} \lambda_n(\hat{x}) a_{ns} \quad \text{for all } s \in S, \tag{34}$$

$$\lambda_n(\hat{x}) \geqslant 0 \quad \text{for all } n = 1, 2, ..., N, \tag{35}$$

$$\lambda_n(\hat{x})(a_n \cdot \tilde{\pi}(\bar{\mathbf{u}}(\hat{x})) - c_n) = 0 \quad \text{for all } n = 1, 2, ..., N, \text{ and} \tag{36}$$

$$\sum_{s=1}^{\bar{s}} \tilde{\pi}_s(\bar{\mathbf{u}}(\hat{x})) = 1 \quad \text{for all } n = 1, 2, ..., N. \tag{37}$$

(Note that λ_0 is the Kuhn-Tucker multiplier of the constraint $\sum_{s=1}^{\bar{s}} \pi_s = 1$ and λ_n is the Kuhn-Tucker multiplier of the constraint $a_n \cdot \pi \geqslant c_n$.) To sum up, the data set $\mathcal{O}$ is MEU-rationalizable with respect to the set of probability weights Π if and only if there are real numbers $\{\bar{u}(r)\}_{r \in \mathcal{X}}$ and, for each $\hat{x} \in \mathcal{L}$, $\{\tilde{\pi}_s(\bar{\mathbf{u}}(\hat{x}))\}_{s=1}^{\bar{s}}$ and $\{\lambda_n(\hat{x})\}_{n=0}^{N}$, such that (17) and (32) — (37) are satisfied. Notice that this is again a set of bilinear inequalities.

While this looks like a very complicated test, it simplifies dramatically when there are two states. In that case, we may assume without loss of generality we may assume that π_1 varies between $\pi_1^* \in (0,1)$ and $\pi_1^{**} \in (0,1)$, with $\pi_1^* < \pi^{**}$, so that $\Pi = \{(\pi_1, 1-\pi_1) : \pi_1^* \geqslant \pi_1 \geqslant \pi_1^{**}\}$. Then it is clear that $\phi(u_1, u_2) = \pi_1^* u_1 + (1-\pi_1^*) u_2$ if $u_1 \geqslant u_2$ and $\phi(u_1, u_2) = \pi_1^{**} u_1 + (1-\pi_1^{**}) u_2$ if $u_1 < u_2$. In other words, independently of the choice of $\bar{u}$, we know $\tilde{\pi}_s(\bar{u}(\hat{x}_1), \bar{u}(\hat{x}_2))$ (for $s = 1, 2$) for every $\hat{x} \in \mathcal{L}$. Thus all that needs to be done is to find $\{\bar{u}(r)\}_{r \in \mathcal{X}}$ that solve the linear conditions (17), (32), and (33); a solution exists if and only if the data set is MEU-rationalizable with respect to Π.

4. Implementation

We implement our tests using data from the portfolio choice experiment in Choi *et al.* (2007), which was performed on undergraduate subjects at the University of California, Berkeley. Each subject was asked to make consumption choices across 50 decision problems under risk. To be specific, each subject was asked to divide a budget between two Arrow-Debreu securities, with each security paying one token if the corresponding state was realized, and zero otherwise. In a symmetric treatment, each state of the world occurred with probability 1/2, and this was known to the subjects. Income was normalized to one, and the state prices were chosen at random and differed across subjects.[5] Further details on the experiment and the data can be found in Choi *et al.* (2007) and the data appendices.

Analysis[6]

The results are shown in Table 1. The first row of the table shows that across 50 decision problems, 12 out of 47 subjects were obey GARP and were thus rationalizable by a

[5] Subjecting different subjects to different choice problems was critical for identification in the econometric analysis in that paper. This variation in the data *across* subjects plays no role in our empirical analysis, which is maximally heterogeneous.

[6] Data and programs are available from the authors upon request.

	GARP	RDEU	EU	EU*
Drop 0	12/47 (26%)	2/47 (4%)	2/47 (4%)	0/47 (0%)
Drop 1	14/47 (30%)	7/47 (15%)	5/47 (11%)	0/47 (0%)
Drop 2	27/47 (57%)	9/47 (19%)	7/47 (15%)	2/47 (4%)
Drop 3	32/47 (68%)	12/47 (26%)	10/47 (21%)	2/47 (4%)

Table 1: Results

continuous and strongly monotone utility function, but none were consistent with expected utility maximization when concavity of the Bernoulli utility function was imposed (EU*).[7] When concavity is dropped, 2 of the 12 subjects obeying GARP were also rationalizable by expected utility (EU), and hence, by rank dependent expected utility (RDEU). The tests for GARP, EU, and EU* are computationally straightforward since they involve solving linear programs. The test for RDEU requires us to solve a set of inequalities that is bilinear in $\bar{u}$ and $\bar{\rho}_*$ (see the formula for ϕ in (28) and (29)). To get round this difficulty, we let $\bar{\rho}_*$ take different values in $(0,1)$ that are multiples of 0.01; for each value, we check for a solution to $\{\bar{u}(r)\}_{r\in\mathcal{X}}$ in the corresponding linear program. (In other words, we do a grid search for $\bar{\rho}_*$, using a grid width of 0.01.)

Since revealed preference tests are exact, it is useful to perform a sensitivity analysis that allows for some degree of flexibility. To this end, we examined subsets of the data from each subject. For example, if we exclude a single observation, then there are 50 subsets of the data from each subject, with each subset containing 49 observations; if one or more of these 50 subsets admits a rationalization, then we record that subject as rationalizable, allowing for a single exclusion. The results, after excluding up to 3 observations, are displayed in Table 1.[8]

Naturally, more subjects became rationalizable as we successively dropped observations. The pass rates for GARP increased fairly steeply, with nearly 68% of the sample becoming rationalizable by some preference once we allowed for the exclusion of three observations. Conditional on passing GARP, what emerges is that roughly one third of the subjects were

[7] We use the test formulated by Varian (1983) and Green and Srivastava (1986).

[8] If we exclude 2 and 3 observations, then there are 1,225 and 19,600 subsets of the data, respectively.

EU-rationalizable (and hence RDEU-rationalizable), which suggests that there is support for the EU-maximization hypothesis. The RDEU model provided enough flexibility to rationalize a few additional subjects that could not be explained by EU.[9] The pass rates remain low for EU*, i.e., for expected utility maximization with a concave Bernoulli utility function.

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[9] Note that while RDEU consistently outperforms EU by two subjects (for Drop 1, Drop 2, and Drop 3) these subjects were not the *same* subjects in each instance.