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Abstract
Recent evidence of increasing fertility rates in developed countries, offers support to the idea that, from the onset of early industrialisation to the present day, the dynamics of fertility can be represented by an N-shaped curve. An OLG model with parental investment in human capital can account for these observed movements in fertility rates during the different phases of demographic change. A demographic transition with declining fertility emerges at the intermediate phase, when parents engage on a child quantity-quality trade-off. At later stages however, the continuing process of economic growth generates sufficient resources so that households can rear more children while still providing the desirable amount of educational investment per child.

Keywords: Demographic transition; Fertility rebound; Human capital
JEL Classification: J11, O41

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1 Introduction

The analysis of the relation between economic development and demographic change has received considerable attention over the last three decades. The prevailing approach seems to favour the view that, since the onset of early industrialisation, population changes can be categorised into two broad, but largely distinct, stages. During the first stage, fertility rates and population growth increased drastically, while the second stage witnessed a demographic transition for which one of the major characteristics is the striking decrease in fertility rates (Galor 2005). Although the majority of existing theories have tended to focus mainly on the second stage of the aforementioned changes (e.g., Galor and Weil 1996; Iyigun 2000; Blackburn and Cipriani 2002; Bhattacharya and Chakraborty 2012; Varvarigos and Zakaria 2013), some seminal analyses have endeavoured to offer theoretical frameworks that account for all the distinct phases of demographic change. For example, Becker *et al.* (1990) constructed a theoretical model to show the existence of multiple development regimes in which the low (high) income equilibrium corresponds to high (low) fertility rates. Tamura (1996); Galor and Weil (2000); Lagerlöf (2003); and Strulik and Weisdorf (2008) have provided more complete accounts of the circumstances under which an economy can experience the transition from the early to the late stages of demo-economic outcomes.[1]

The process that commenced with the demographic transition, and continued during most of the 20th century, led to a marked decline in fertility. In fact, some developed countries experienced fertility rates that fell below replacement levels. To some extent, demographers viewed this as a worrying trend, considering that the related outcome of population ageing in these countries seemed to lay the foundations for a 'trap' in which the low fertility rates could persist (Goldstein *et al.* 2009). This is because the fraction of the population that belongs to their reproductive age declined as well, reinforcing the incidence of low fertility rates – an outcome that could lead to far-reaching consequences for the overall socio-economic environment of the countries that experienced such circumstances.

Nevertheless, more recent projections and empirical evidence are appeasing some of these concerns. Many researchers now agree that developed economies are entering a new phase of demographic change – a phase that is characterised by (once more) increasing fertility rates. Alkema *et al.* (2011) constructed a Bayesian projection model that they used to

[1] Tabata (2003) and Strulik (2008) also offer frameworks under which the dynamics of fertility can be traced along an inverted U-shaped curve.

forecast the trend in the Total Fertility Rate (TFR) for each country, up to the year 2100. With few exceptions, their projections indicate that the phase of increasing fertility rates among developed nations will most probably become a permanent characteristic during the coming decades, thus indicating that the fertility rebound reflects a change in trend rather than a mere temporary (or cyclical) change. A similar change in trend is suggested by Collins and Richards (2013) who use quantitative population models in order to project the evolution of fertility rates. In fact, recent empirical evidence suggests that the phase characterising the fertility rebound can already be observed in developed countries. Myrskylä *et al.* (2009) apply panel data estimation techniques for a sample of 37 countries from 1975 to 2005 and find that the relation between the Human Development Index (HDI) and the TFR is U-shaped. Particularly, there is a threshold level for the HDI below which the relation with the TFR is negative, whereas above it the relation with the TFR is positive. Despite the fact that per capita GDP is a major component of the HDI, Luci and Théveron (2010) argued that the results of Myrskylä *et al.* (2009) may not indicate the pure effect of economic development on the observed fertility rebound. For this reason, they use a panel data set of 30 OECD countries over the period that spans from 1980 to 2007, in order to estimate the effect of GDP per capita on the TFR. Their results, however, verify the idea that the decline in fertility is halted and fertility rates start increasing at higher stages of economic development, as they also find a U-shaped relation between the TFR and per capita GDP. It is important to note that, in both the aforementioned analyses, the significance of this type of non-monotonic effect is robust to adjustments in the TFR that are made to account for the fact that the timing of childbearing may shift to later stages of the reproductive age – an outcome that is conventionally used to account for subsequent reversals in fertility trends (Lee 2003). Goldstein *et al.* (2009) argue that "the trend of increasing TFR has not been limited to the countries with very low fertility, but took place across the developed world" (page 670). Among the possible explanations for this phenomenon, they include the improvement in economic conditions, such as the increase in GDP per capita. It is worth noting that while conventional wisdom would attribute this effect to immigration, the empirical analysis of Tromans *et al.* (2009) does not support this argument. Using the recent increase of the TFR in the United Kingdom, they show that a large part of the increase in the TFR is attributed to UK born women rather than foreign born ones, thus suggesting that immigration is not the only factor behind the fertility rebound.

The aforementioned evidence on the fertility rebound in developed economies, combined with the changes in fertility trends since the beginning of early industrialisation, offers credence to the idea that the dynamics of fertility, along the various stages of economic development, can be traced on an N-shaped curve. In the first stage, fertility rates increase; the second stage witnesses a demographic transition, characterised by declining fertility rates; and in the third stage, the trend is once more reversed as fertility rates are increasing. The purpose of this paper is to offer a theory that accounts for such fertility dynamics. Since the aforementioned evidence has shown that the 'tempo' effect and immigration are not the only drivers of the recent fertility rebound in developed economies, this theory will focus on the impact of increasing income that is associated with the economy's transition to higher stages of economic development.

I construct an overlapping generations model in which households care about both the size of their family and the human capital of their children. Both child-rearing and the parental investment on education entail costs that are measured in units of output. The government collects taxes and uses them to finance the provision of public services, the various effects of which can be manifested as either complements or substitutes to private education investment. At low levels of development, households find optimal not to invest any of their income towards their children's education; therefore, a rise in disposable income leads to an increase in the number of children raised. As the economy grows, there is a critical level of development at which parents start investing in the education of each of their children. The high return to educational investment, coupled with the relatively limited resources at the disposal of each household, lead to a quantity-quality trade-off. Parental investment in human capital occurs at the expense of family size; thus, fertility rates decline. Nevertheless, as per capita income grows even further, families are not constrained by such trade-offs. Instead, disposable income is high enough so that fertility rates increase, while parents can still invest resources towards their children's education.

With regard to the existing literature, this paper is related to the analyses that have examined the joint determination of economic and demographic outcomes on the basis of models that introduce choices for fertility and parental investment towards each child's education. In addition to the analyses that have already been mentioned, other influential papers on this strand of literature are those by Kalemli-Ozcan (2002); Hazan and Berdugo (2002); de la Croix and Doepke (2004, 2009); Moav (2005); and Galor and Mountford (2008) among others.[2] Contrary to these analyses, I consider the case where the total costs associated with having children (i.e., both rearing and education) are measured in units of output rather than units of time. This approach is justified by the fact that the direct pecuniary costs of child rearing represent a considerable fraction of a household's total income. For instance, Lino (2012) estimates that for a family with two children, the direct child-related expenditures (per each child) on items such as shelter, food, clothing, health care, child care, education, entertainment and other personal items account for roughly 20% of a household's income. While it is not my intention to downgrade the importance of the opportunity costs of raising a child (stemming from the time/effort devoted by parents), these statistics are a testament to the significance of the direct expenses associated with child-rearing. Therefore, their consideration certainly merits more attention, especially since the majority of analyses in the economic growth-demography nexus have mainly focused on the time costs of raising the household's offspring. As it turns out, the focus on pecuniary expenditures facilitates an otherwise standard model in generating a fertility rebound at higher stages of economic development. This is because the process of economic growth generates enough resources so that a household's decision to invest in education need not necessarily be associated with the need to raise fewer children.

The exposition of the remaining analysis is as follows. In Section 2, I provide a detailed description of the basic economic set-up. Section 3 considers the household's (lifetime) utility maximisation problem. In Section 4, I analyse the dynamics of human capital accumulation while Section 5 analyses the dynamics of fertility. Section 6 concludes.

2 The Economy

Time is discrete and indexed by t. The economy is populated by overlapping generations of households that have a lifespan of two periods – *childhood* and *adulthood*. During childhood, individuals are reared by their parents and receive education that determines the stock of human capital, or *effective* labour, that will be available to them when they become adults. During adulthood, they receive a salary by offering their labour to perfectly competitive firms. These firms produce units of the economy's consumption good by utilising effective labour under a linear production technology. This technology implies that the wage per unit

[2] For empirical evidence on the quantity-quality trade-off, see Black *et al.* (2005) and Becker *et al.* (2010) among others.

of effective labour is constant over time. Henceforth, I denote the (constant) wage by $\omega > 0$ and assume that labour income is subject to a tax rate $\tau \in (0,1)$. Adult households decide how to allocate their after-tax (disposable) income between consumption, child-rearing, and education expenditures per child.

Consider a household that begins adulthood during period t and suppose that the members of the household wish to bear n_t children. Rearing each child entails a fixed cost of $q > 0$ units of output. Furthermore, parents may wish to spend resources towards the education of each of their offspring. Denoting the amount of education expenditures per child by x_t, we can write the household's budget constraint as

$$c_t = (1-\tau)\omega h_t - n_t(q + x_t), \tag{1}$$

where c_t denotes consumption and h_t is the stock of human capital, i.e., the variable that ultimately determines the amount of effective labour available to each household.

As noted earlier, parents can affect each child's human capital by devoting resources towards their education. Particularly, given that each parent devotes x_t units of output per child, human capital will be

$$h_{t+1} = \iota z_t + \lambda m_t x_t, \tag{2}$$

where $\iota, \lambda > 0$. Shortly, we shall see that the terms z_t and m_t represent functions that incorporate the effect of public spending towards activities that support human capital accumulation and the efficiency of labour. The government devotes g_t units of output towards these activities, an amount financed by the collected tax revenues according to a balanced budget rule. Denoting the population of adult households by N_t, the government's budget can be formally written as

$$g_t = \tau \omega h_t N_t. \tag{3}$$

In order to avoid the complication that arises from scale effects, I am going to assume that the benefit of publicly provided services on human capital depends on the amount of public spending per household. Particularly, it is assumed that $z_t = Z(g_t / N_t)$ and $m_t = M(g_t / N_t)$ such that $z'(\cdot) > 0$ and $M'(\cdot) > 0$ respectively. Henceforth, I will employ the specific functional forms

$$z_t = Z\left(\frac{g_t}{N_t}\right) = \left(q\frac{g_t}{N_t}\right)^{\eta}, \quad \eta \in (0,1), \tag{4}$$

and

$$m_t = M\left(\frac{g_t}{N_t}\right) = \left[(1-q)\frac{g_t}{N_t}\right]^{\varepsilon}, \quad \varepsilon \in (0,1), \tag{5}$$

where $q \in (0,1)$ is a parameter that determines the allocation of expenditures among different types of public services. Substituting (3)-(5) in Equation (2), we can write the dynamics of human capital according to

$$h_{t+1} = \varphi h_t^{\eta} + \psi h_t^{\varepsilon} x_t, \tag{6}$$

where $\varphi \equiv \iota(q\tau\omega)^{\eta}$ and $\psi \equiv \lambda(\tau\omega)^{\varepsilon}(1-q)^{\varepsilon}$ are composite parameter terms.

Note that the ideas embedded in (2), (4) and (5) allow us to consider the general case where the overall impact of public services entails both complementary and substitute effects to the private resources devoted towards human capital improvements. Such a scenario is actually quite intuitive. Whereas services on public education can be thought as a reasonable substitute for the resources that the private sector dedicates to the accumulation of human capital, other forms of public infrastructure investment, such as public health, public transport, law and order etc., can support private investment towards activities that improve the efficiency of labour. My framework allows the manifestation of both possibilities concerning the overall effect of public services.

At this point, it is worth commenting on the technical role of introducing productive public spending in the model. As it must be evident from Equations (3)-(6), including public services as an input to the education technology is a convenient device to generate dynamics in the formation of human capital, i.e., a direct relation between h_t and h_{t+1}. However, the same outcome is possible without the need to resort to the idea of productive public spending. For instance, I could have assumed that the dynamics of human capital are derived directly from Equation (6), a scenario for which the effect of h_t would capture intergenerational externalities in the formation of human capital, in the same manner as in the large majority of overlapping generations models with education – too numerous to mention here but the reader may consult Chapter 5.2 of de la Croix and Michel (2002). Furthermore, the presence of the parameter $\varphi > 0$ would allow some basic knowledge to be available, even in the absence of parental investment in education. It goes without saying that my subsequent results would remain intact.

Another issue that should be noted is that the specification of the human capital technology in (6) will be consistent with a stationary solution for the human capital stock, as I shall establish shortly. Nevertheless, this property of the model is not crucial for the determination of my results. Later it will become clear that the dynamics of fertility remain identical even if one sets $\varepsilon = 1$ in (6), thus creating the conditions that allow an ever increasing stock of human capital and, consequently, growth in the long-run.

The lifetime utility of the household is given by

$$u_t = \gamma \ln(c_t) + (1-\gamma)\left[\beta \ln(n_t) + \theta \ln(n_t h_{t+1})\right], \tag{7}$$

where $\gamma \in (0,1)$ and $\beta, \theta > 0$ are preference parameters. In addition to the utility accruing from the consumption of goods, households enjoy utility by the children they bear and raise over their lifetime. In this context however, children are not only valued *per se* but also in terms of their human capital, i.e., households also enjoy greater felicity by more educated children – an (imperfectly) altruistic motive that may capture the idea that parents care about their children's human capital because this improves their future prospects.[3]

3 The Household's Problem

Households make their choices so as to maximise their lifetime utility in (7), subject to the constraints in Equations (2) and (6). In order to solve this problem, we can substitute these constraints in (7) and maximise with respect to n_t and x_t. The respective first order conditions are given by

$$\frac{\gamma(q + x_t)}{(1-\tau)\omega h_t - n_t(q + x_t)} \geq \frac{(1-\gamma)(\beta+\theta)}{n_t}, \quad n_t \geq 0, \tag{8}$$

and

$$\frac{\gamma n_t}{(1-\tau)\omega h_t - n_t(q + x_t)} \geq \frac{(1-\gamma)\theta\psi h_t^{\varepsilon}}{\varphi h_t^{\eta} + \psi h_t^{\varepsilon} x_t}, \quad x_t \geq 0. \tag{9}$$

[3] Rewriting the second part of the utility function as $(1-\gamma)\left[(\beta+\theta)\ln(n_t) + \theta \ln(h_{t+1})\right]$, we can see that the formulation in (7) implies that the utility weight on the number of children each household gives birth to is higher than the utility weight attached to human capital per child. This assumption is essential for the existence of an equilibrium with an interior solution for n_t. The same technical condition has been used by Moav (2005) and de la Croix and Doepke (2009) among others.

The expressions in (8) and (9) offer some familiar conditions according to which the marginal benefit from each activity must be equal to the corresponding marginal cost – both expressed in terms of utility. The marginal utility cost in both cases is associated with the loss of consumption that results from the increase in the resources required to raise and educate the household's offspring. The marginal utility benefit stems from the idea that parents enjoy raising children, as well as supporting their education.

We can express (8) as an equality and solve it to get

$$n_t(q+x_t)=\frac{(1-\gamma)(\beta+\theta)}{\gamma+(1-\gamma)(\beta+\theta)}(1-\tau)\omega h_t\,. \tag{10}$$

According to Equation (10), a household will dedicate a fixed fraction of disposable labour income in order to finance the total costs associated with having children – costs that include both rearing and education. This fraction corresponds to the relative weight attached to the utility that parents enjoy from their offspring. Next, we can substitute (8) in (9) and express the latter as an equality. Solving this, we get

$$x_t=\frac{(1-\gamma)\theta}{\gamma+(1-\gamma)(\beta+\theta)}\frac{(1-\tau)\omega h_t}{n_t}-\frac{\varphi}{\psi}h_t^{\eta-\varepsilon}\,. \tag{11}$$

Equation (11) reveals that the amount of resources that parents spend for the education of each child has two components. With regard to the first component, the fraction of disposable income devoted for total education expenditures is associated with the relative weight attached to the utility accruing from the number of children, when these are measured in effective terms (i.e., augmented by each child's human capital). Naturally, the educational resources per child are negatively related to the total number of children raised by the household. As for the second component, it reveals that the substitutability associated with public services reduces the incentive to provide private resources towards each child's education, thus reducing the private spending on education per child. This effect is mitigated by the fact that public services also entail complementary effects, meaning that higher public spending may also tend to increase the return to private investment in education.

The system of equations in (10) and (11) can be solved simultaneously to yield the solutions for private education expenditures per child and fertility. These solutions are given by

$$x_t=\chi(h_t)=\max\left\{0,\frac{1}{\beta}\left[\theta q-(\theta+\beta)\frac{\varphi}{\psi}h_t^{\eta-\varepsilon}\right]\right\}, \tag{12}$$

and

$$n_t = \nu(h_t) = \begin{cases} \dfrac{(1-\gamma)(\beta+\theta)}{\gamma+(1-\gamma)(\beta+\theta)} \dfrac{(1-\tau)\omega h_t}{q} & \text{if} \quad x_t = 0 \\ \\ \dfrac{(1-\gamma)\beta}{\gamma+(1-\gamma)(\beta+\theta)} \dfrac{(1-\tau)\omega\psi h_t^{1+\varepsilon-\eta}}{q\psi h_t^{\varepsilon-\eta} - \varphi} & \text{if} \quad x_t > 0 \end{cases}, \tag{13}$$

respectively. These two results can facilitate us in analysing and understanding the dynamics of demo-economic development. The following section will focus on the evolution of human capital, whereas the dynamics of population growth will be formally analysed in a subsequent part of the paper.

4 Economic Dynamics

A closer look at the result in (12) reveals that there are circumstances under which parents may find optimal not to invest any resources towards the education of their offspring. The underlying cause for this possibility lies on the fact that as long as $\iota > 0 \Rightarrow \varphi > 0$, each child will still be endowed with units of efficient labour, due to the presence of human capital-enhancing public services, even though parents may not invest any private resources towards their education. In order to keep the analysis consistent with the existing literature and the empirical evidence on the matter, it is natural to focus attention to the case where the decision not to invest in the children's education materialises at low levels of economic development. Henceforth, I will be assuming that the condition $\varepsilon > \eta$ holds. Given this, when the stock of human capital is relatively low, the utility cost of foregone consumption outweighs the utility benefit of educating the children and increasing their efficiency. Nevertheless, when the stock of human capital is relatively high, the complementary effect of public services becomes strong enough to guarantee that the return to private investment in education is sufficiently high to compensate parents for the utility loss due to decreased consumption. The main message from this discussion can be summarised in

Lemma 1. *Assuming that $\varepsilon > \eta$ holds, there exists a threshold $\tilde{h} \equiv \left[\dfrac{(\theta+\beta)\varphi}{\theta\psi q}\right]^{\frac{1}{\varepsilon-\eta}}$ such that*

$$x_t = \chi(h_t) = \begin{cases} 0 & \text{if} \quad h_t \le \tilde{h} \\ [\theta q - (\theta+\beta)(\varphi/\psi)h_t^{\eta-\varepsilon}]/\beta & \text{if} \quad h_t > \tilde{h} \end{cases}. \tag{14}$$

Proof. From Equation (12), we can see that $\chi(\tilde{h}) = 0$ and $\chi'(h_t) = \frac{(\varepsilon-\eta)(\theta+\beta)\varphi}{\beta\psi} h_t^{\eta-\varepsilon-1} > 0$. Therefore, $x_t > 0$ if and only if $h_t > \tilde{h}$. Furthermore, the non-negativity constraint implies that $x_t = 0 \;\; \forall \;\; h_t \le \tilde{h}$. □

The outcome summarised in Lemma 1 allows us to combine Equations (6) and (14) in order to write the dynamics of human capital as follows:

$$h_{t+1} = F(h_t) = \begin{cases} \varphi h_t^{\eta} & \text{for} \quad h_t \le \tilde{h} \\ (\theta/\beta)(\psi q h_t^{\varepsilon} - \varphi h_t^{\eta}) & \text{for} \quad h_t > \tilde{h} \end{cases}. \tag{15}$$

Using (15), we can characterise the dynamics of human capital through

Proposition 1. *Assume that* $\psi q > \frac{\theta+\beta}{\theta}\varphi^{(1-\varepsilon)/(1-\eta)}$ *holds. Then, for any* $h_0 > 0$, *the economy will converge to an asymptotically stable steady state* h^*, *such that* $h^* > \tilde{h}$.

Proof. See the Appendix. □

The dynamics of human capital are illustrated in the phase diagram of Figure 1. The return to human capital investment is high enough so that the economy will eventually exceed the threshold that governs the households' decision to devote private resources for the education of their children. This outcome supports the formation of human capital and leads to a (relatively) high steady state equilibrium h^*.

The proof to Proposition 1 (see the Appendix) also reveals the outcome that transpires when the condition $\psi q > \frac{\theta+\beta}{\theta}\varphi^{(1-\varepsilon)/(1-\eta)}$ is not satisfied. In the case where

$\psi q < \dfrac{\theta+\beta}{\theta}\varphi^{(1-\varepsilon)/(1-\eta)}$, it is $\breve{h} < \tilde{h}$ and $F'(\breve{h}) = \eta \in (0,1)$. Consequently, $\breve{h}$ will emerge as a stable steady state, at least for some range on the domain of h_t. Given that this equilibrium is associated with $x_t = 0$, whereas my purpose is to examine a scenario for which a transition from $x_t = 0$ to $x_t > 0$ will occur, I rule out this possibility when I analyse the dynamics of fertility in the following section.

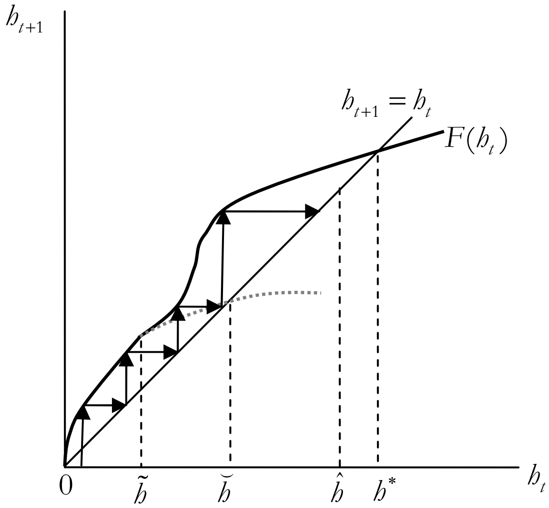

Figure 1. The dynamics of human capital

5 Fertility Dynamics

The purpose of this section is to trace the dynamics of fertility along the process of economic development. I shall begin the analysis by using the results in (13) in order to examine how fertility varies with the stock of human capital. This analysis leads to

Lemma 2. *Consider $n_t = v(h_t)$. It is straightforward to establish that*

i. *When $x_t = 0$, then $v'(h_t) > 0$;*

ii. *When $x_t > 0$, then there exists $\hat{h} \equiv \left[\dfrac{(1+\varepsilon-\eta)\varphi}{\psi q}\right]^{\frac{1}{\varepsilon-\eta}}$ such that*

$$v'(h_t)\begin{cases} <0 & for \quad h_t < \hat{h} \\ \\ >0 & for \quad h_t > \hat{h} \end{cases}.$$

Proof. See the Appendix. □

It will be useful to make a comparison between the threshold values $\tilde{h}$ and $\hat{h}$. This exercise is undertaken in

Lemma 3. *As long as* $\frac{(1+\varepsilon-\eta)\theta}{\theta+\beta}>1$ *holds, it is* $\hat{h}>\tilde{h}$.

Proof. The condition $\hat{h}>\tilde{h}$ implies $\left[\frac{(1+\varepsilon-\eta)\varphi}{q\psi}\right]^{\frac{1}{\varepsilon-\eta}}>\left[\frac{(\theta+\beta)\varphi}{\theta q\psi}\right]^{\frac{1}{\varepsilon-\eta}}$ which is indeed true for $(1+\varepsilon-\eta)\theta/(\theta+\beta)>1$. □

Now, we can gather the previous results in order to understand the qualitative impact of the human capital stock on fertility. This is summarised in

Proposition 2. *Consider* $n_t = v(h_t)$. *Then*

$$v'(h_t)\begin{cases} >0 & for \quad h_t < \tilde{h} \\ \\ <0 & for \quad \tilde{h} < h_t < \hat{h} \\ \\ >0 & for \quad h_t > \hat{h} \end{cases}. \tag{16}$$

Proof. It follows from Lemmas 2 and 3. □

My objective is to analyse an economy that goes through all the stages of the possible demographic changes, as it converges to the long-run equilibrium that is characterised by h^*. With this in mind, let us consider

Lemma 4. *Assume that* $q\psi > \max\left\{\left(\frac{\beta}{\theta}\right)^{\frac{\varepsilon-\eta}{1-\eta}} \frac{(1+\varepsilon-\eta)\varphi^{(1-\varepsilon)/(1-\eta)}}{(\varepsilon-\eta)^{(\varepsilon-\eta)/(1-\eta)}}, \frac{(\theta+\beta)\varphi^{(1-\varepsilon)/(1-\eta)}}{\theta}\right\}$ *holds. Then* $h^* > \hat{h}$.

Proof. See the Appendix. □

The results in Proposition 2 and Lemmas 3 and 4 allow us to understand the movements in fertility, and therefore population growth, as the economy goes through different stages of the development process towards its convergence to the stationary equilibrium. From these results, it follows that for an initial stock of human capital that satisfies $h_0 < \tilde{h}$, the economy will initially exceed the threshold indicated by $\tilde{h}$ and will subsequently exceed the threshold indicated by $\hat{h}$ as well. Let us define time periods $\tilde{T}$ and $\hat{T}$ such that

$$h_t \begin{cases} < \tilde{h} & for \quad t = 0, \ldots \tilde{T} - 1 \\ > \tilde{h} & for \quad t = \tilde{T}, \ldots \end{cases}, \tag{17}$$

and

$$h_t \begin{cases} < \hat{h} & for \quad t = \tilde{T}, \ldots \hat{T} \\ > \hat{h} & for \quad t = \hat{T} + 1, \ldots \end{cases}, \tag{18}$$

where it should be noted that $\hat{T} > \tilde{T}$ holds by virtue of Proposition 1 and Lemma 3. Given these, a formal characterisation of the dynamics of fertility is possible through

Proposition 3. *There are three different stages of fertility dynamics. Fertility increases from* $t = 0$ *to* $t = \tilde{T} - 1$, *it declines from* $t = \tilde{T} - 1$ *to* $t = \hat{T}$, *and it increases again from* $t = \hat{T}$ *onwards.*

Proof. It follows from Equations (16)-(18). □

The dynamics of fertility/population growth are illustrated in Figure 2, where it is clear that they depict an N-shaped graph. The intuition is the following. At the first stage (corresponding to $h_t < \tilde{h}$), the return to the parental investment in education is so low that parents decide to spend the amount of income that they do not consume, entirely for child-rearing purposes. As disposable income grows, families have more resources so that they can rear more children (see Equation 10 for $x_t = 0$, where child-rearing absorbs a constant fraction of disposable income). Gradually however, the threshold defined by $\tilde{h}$ will be exceeded and the return to private education spending will be high enough to motivate households to dedicate part of their resources towards this purpose. A better explanation of the outcomes that transpire from this point onwards is possible if we use (13) and (14) to write child-rearing and total education expenditures as fractions of disposable income. That is

$$\frac{qn_t}{(1-\tau)\omega h_t} = \frac{(1-\gamma)\beta}{\gamma+(1-\gamma)(\beta+\theta)} \frac{q\psi h_t^{\varepsilon-\eta}}{q\psi h_t^{\varepsilon-\eta} - \varphi} = \delta(h_t), \tag{19}$$

and

$$\frac{n_t x_t}{(1-\tau)\omega h_t} = \frac{(1-\gamma)(\beta+\theta)}{\gamma+(1-\gamma)(\beta+\theta)} \frac{\left(\frac{\theta}{\beta+\theta} q\psi h_t^{\varepsilon-\eta} - \varphi\right)}{q\psi h_t^{\varepsilon-\eta} - \varphi} = \zeta(h_t), \tag{20}$$

from where we can easily check that $\delta(h_t) \in (0,1)$ and $\zeta(h_t) \in (0,1)$ for $h_t > \tilde{h}$. From (19) and (20), it follows that $\delta'(h_t) < 0$ and $\zeta'(h_t) > 0$, i.e., as the economy develops, parents devote a decreasing fraction of their income towards child-rearing and an increasing part of their income towards the education of their offspring. In fact, the return to education spending is so high during the second stage (corresponding to $\tilde{h} < h_t < \hat{h}$) that we observe what is effectively a quantity-quality trade-off. In other words, households actually reduce the number of children they rear in order to finance the optimal amount of education expenditures per child. Nevertheless, the economy continues to grow and eventually reaches the third stage (corresponding to $h_t > \hat{h}$). Now, disposable income is sufficiently high so that a quantity-quality trade-off is not necessary. In other words, the share of total income on child-rearing may be declining, but the increase in income is so pronounced that the overall amount available for raising children is higher. Households have enough resources to raise more children and still provide the desirable amount of education spending for each of them, as the economy converges to its long-run equilibrium.

Despite the fact that I employ an education technology that results in a stationary solution for human capital (see Proposition 1), the qualitative results of the model remain intact even under a different specification that permits long-run growth. For instance, setting $\varepsilon = 1$ in (6) will alter the dynamics of human capital in the sense that, once in exceeds the threshold defined by $\tilde{h}$, the economy will be able to sustain an equilibrium where the human capital stock grows without bound. This would be the only qualitative change of the model's equilibrium behaviour though. The interested reader can use $\varepsilon = 1$ in Lemmas 1-4 and Propositions 2-3 to verify that the results concerning the equilibrium characteristics of x_t and n_t, as well as the dynamics of fertility, remain qualitatively identical. Therefore, the fertility rebound at later stages of economic development is not a result that should be attributed to the stationarity of human capital and GDP per capita since the same result emerges under a set-up that allows growth in the long-run.

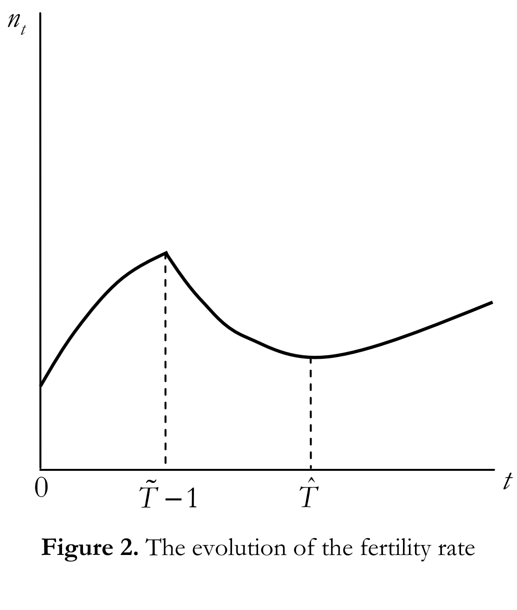

Figure 2. The evolution of the fertility rate

6 Conclusion

Recent empirical evidence suggests that those countries that have witnessed marked reductions in their fertility since the onset of demographic transition, now appear to experience a ‘fertility rebound’ with rising fertility rates. Among the various explanations for this reversal, empirical evidence suggests that economic development – in the past, the main engine behind the demographic transition – is now one of the driving forces behind this new phase of demographic change.

This new evidence indicates that, from the beginning of early industrialisation to the present day, the dynamics of fertility can be traced along an N-shaped curve. In this paper, I presented a theory that accounts for this dynamic behaviour. The main theme of the analysis is that the child quantity-quality trade-off – one of the main explanations for the demographic transition in the economics literature – materialises in an intermediate phase where the joint effects of the high return to human capital investment and the income constraint faced by households, implies that parents can increase the investment towards their children’s education, only at the expense of the number of children they bear over their reproductive age. As incomes grow even further though, there is a new phase where parents can provide the desirable expenditures towards the education of their offspring without necessarily reducing the number of children they give birth to.

As it became evident from the main part of the paper, the model that underlines my theory is qualitative rather than quantitative. Furthermore, it was constructed with the purpose of offering analytical solutions that pinpoint the main mechanisms that characterise the equilibrium outcomes without blurring their intuition. Naturally, a more general framework that will be simulated numerically can be a fruitful avenue for future research. Nevertheless, even in this simple form, my model is able to draw attention to an outcome that, although is supported by recent evidence in demographic research, it has so far evaded the attention of the literature on the nexus between economic growth and the demography.

As a final note, I should emphasise that my theory formalises just one of a variety of possible explanations behind the fertility rebound in developed economies – the outcome that is ultimately responsible for the emergence of N-shaped fertility dynamics. This should not be viewed as a stance against other legitimate explanations for this phenomenon (e.g., the ‘tempo’ effect; immigration etc). These may offer important accounts behind the reversal of fertility trends in developed countries; therefore their exploration certainly merits formal analysis through future research work. In any case however, existing evidence (cited in the Introduction) reveals that the 'tempo' effect and immigration cannot fully account for the change in fertility trends in developed countries. Therefore, offering an additional or complementary explanation was an endeavour certainly worth undertaking.

Appendix

Proof of Proposition 1

Consider the case where $h_t \leq \tilde{h}$. According to (15), we have $F(h_t) = \varphi h_t^{\eta}$ where $F'(h_t) = \eta\varphi h_t^{\eta-1} > 0$, $F'(0) = \infty$ and $F''(h_t) = (\eta-1)\eta\varphi h_t^{\eta-2} < 0$. Furthermore, note that $\breve{h} = \varphi\breve{h}^{\eta} \Rightarrow \breve{h} = \varphi^{\frac{1}{1-\eta}}$. However, it is true that $\breve{h} > \tilde{h}$ given that $\psi q > \dfrac{\theta+\beta}{\theta}\varphi^{(1-\varepsilon)/(1-\eta)}$ holds by assumption. Consequently $F(h_t) > h_t \ \ \forall \ \ h_t \leq \tilde{h}$, meaning that the economy will not reach the steady state $\breve{h}$ because the dynamic behaviour of human capital will change when its stock exceeds $\tilde{h}$. According to (15), the transition equation becomes $F(h_t) = \dfrac{\theta}{\beta}(\psi q h_t^{\varepsilon} - \varphi h_t^{\eta})$ thereafter. Note that, by the definition of $\tilde{h}$

$$\tilde{h} = \left[\frac{(\theta+\beta)\varphi}{\theta\psi q}\right]^{\frac{1}{\varepsilon-\eta}} \Rightarrow$$

$$\theta\psi q\tilde{h}^{\varepsilon-\eta} = (\theta+\beta)\varphi \Rightarrow$$

$$\frac{\theta}{\beta}\psi q\tilde{h}^{\varepsilon} = \frac{\theta+\beta}{\beta}\varphi\tilde{h}^{\eta} \Rightarrow$$

$$\frac{\theta}{\beta}\psi q\tilde{h}^{\varepsilon} + \left(1 - \frac{\theta+\beta}{\beta}\right)\varphi\tilde{h}^{\eta} = \varphi\tilde{h}^{\eta} \Rightarrow$$

$$\frac{\theta}{\beta}(\psi q\tilde{h}^{\varepsilon} - \varphi\tilde{h}^{\eta}) = \varphi\tilde{h}^{\eta} \Rightarrow$$

$$\lim_{h_t \to \tilde{h}^{+}} F(h_t) = \lim_{h_t \to \tilde{h}^{-}} F(h_t).$$

It is,

$$F'(h_t) = \frac{\theta}{\beta}(\varepsilon\psi q h_t^{\varepsilon-1} - \eta\varphi h_t^{\eta-1}), \tag{A1}$$

so that $F'(h_t) > 0$ as long as

$$\varepsilon\psi q h_t^{\varepsilon-1} > \eta\varphi h_t^{\eta-1} \Rightarrow$$

$$h_t^{\varepsilon-\eta} > \frac{\eta\varphi}{\varepsilon\psi q} \Rightarrow$$

$$h_t > \left(\frac{\eta\varphi}{\varepsilon\psi q}\right)^{\frac{1}{\varepsilon-\eta}} \equiv \underset{\sim}{h},$$

which is true because $\tilde{h} > \underset{\sim}{h}$ for $\varepsilon > \eta$. Furthermore, given $\eta, \varepsilon \in (0,1)$, Equation (A1) reveals that $F'(\infty) = 0$. Therefore, for $h_t > \tilde{h}$, the transition graph will cross the 45^0 line at a point h^*, such that $h^* = F(h^*)$ and

$$F(h_t)\begin{cases} > h_t & \text{for} \quad \tilde{h} < h_t < h^* \\ \\ < h_t & \text{for} \quad h_t > h^* \end{cases}.$$

It follows that $F'(h^*) \in (0,1)$, allowing us to conclude that the fixed point h^* corresponds to a stable equilibrium. □

Proof of Lemma 2

The first part of Lemma 2 is easily proven after using (13) for $x_t = 0$, and showing that $v'(h_t) = \frac{(1-\gamma)(\beta+\theta)}{\gamma+(1-\gamma)(\beta+\theta)}\frac{(1-\tau)\omega}{q} > 0$. Next, we can consider the expression for fertility that corresponds to $x_t > 0$. First of all, notice that the denominator $q\psi h_t^{\varepsilon-\eta} - \varphi$ is positive for $h_t > \tilde{h}$. Calculating the derivative, we get

$$v'(h_t) = \frac{(1-\gamma)\beta}{\gamma+(1-\gamma)(\beta+\theta)}(1-\tau)\omega\psi\left[\frac{(1+\varepsilon-\eta)h_t^{\varepsilon-\eta}(q\psi h_t^{\varepsilon-\eta} - \varphi) - h_t^{1+\varepsilon-\eta}(\varepsilon-\eta)q\psi h_t^{\varepsilon-\eta-1}}{(q\psi h_t^{\varepsilon-\eta} - \varphi)^2}\right].$$

Obviously, the sign of the derivative will depend on the sign of the expression inside squared brackets. In particular, it will be $v'(h_t) > 0$ as long as

$$(1+\varepsilon-\eta)h_t^{\varepsilon-\eta}(q\psi h_t^{\varepsilon-\eta} - \varphi) - h_t^{1+\varepsilon-\eta}(\varepsilon-\eta)q\psi h_t^{\varepsilon-\eta-1} > 0 \Rightarrow$$

$$(1+\varepsilon-\eta)h_t^{2(\varepsilon-\eta)}q\psi-(1+\varepsilon-\eta)\varphi h_t^{\varepsilon-\eta}-(\varepsilon-\eta)h_t^{2(\varepsilon-\eta)}q\psi>0\Rightarrow$$

$$h_t^{2(\varepsilon-\eta)}q\psi-(1+\varepsilon-\eta)\varphi h_t^{\varepsilon-\eta}>0\Rightarrow$$

$$h_t^{\varepsilon-\eta}[h_t^{\varepsilon-\eta}q\psi-(1+\varepsilon-\eta)\varphi]>0\Rightarrow$$

$$h_t>\left[\frac{(1+\varepsilon-\eta)\varphi}{q\psi}\right]^{\frac{1}{\varepsilon-\eta}}\equiv\hat{h}.$$

Therefore, for $h_t<\hat{h}$ it is $v'(h_t)<0$. □

Proof of Lemma 4

Given $F(h^*)=h^*$ and $F(h_t)<h_t$ for $h_t>h^*$, it is sufficient to show that $F(\hat{h})>\hat{h}$. This condition corresponds to

$$\frac{\theta}{\beta}(\psi q\hat{h}^{\varepsilon}-\varphi\hat{h}^{\eta})>\hat{h}\Rightarrow$$

$$\frac{\theta}{\beta}(\psi q\hat{h}^{\varepsilon-\eta}-\varphi)>\hat{h}^{1-\eta}\Rightarrow$$

$$\psi q\hat{h}^{\varepsilon-\eta}-\varphi>\frac{\beta}{\theta}\hat{h}^{1-\eta}\Rightarrow$$

$$\psi q\left[\frac{(1+\varepsilon-\eta)\varphi}{q\psi}\right]-\varphi>\frac{\beta}{\theta}\left[\frac{(1+\varepsilon-\eta)\varphi}{q\psi}\right]^{\frac{1-\eta}{\varepsilon-\eta}}\Rightarrow$$

$$(\varepsilon-\eta)\varphi>\frac{\beta}{\theta}\left[\frac{(1+\varepsilon-\eta)\varphi}{q\psi}\right]^{\frac{1-\eta}{\varepsilon-\eta}}\Rightarrow$$

$$q\psi>\left(\frac{\beta}{\theta}\right)^{\frac{\varepsilon-\eta}{1-\eta}}\frac{(1+\varepsilon-\eta)\varphi^{(1-\varepsilon)/(1-\eta)}}{(\varepsilon-\eta)^{(\varepsilon-\eta)/(1-\eta)}}.$$

Together with the fact that $q\psi>\frac{(\theta+\beta)\varphi^{(1-\varepsilon)/(1-\eta)}}{\theta}$ holds by virtue of Proposition 1, then $q\psi>\max\left\{\left(\frac{\beta}{\theta}\right)^{\frac{\varepsilon-\eta}{1-\eta}}\frac{(1+\varepsilon-\eta)\varphi^{(1-\varepsilon)/(1-\eta)}}{(\varepsilon-\eta)^{(\varepsilon-\eta)/(1-\eta)}},\frac{(\theta+\beta)\varphi^{(1-\varepsilon)/(1-\eta)}}{\theta}\right\}$ is sufficient to establish that $h^*>\hat{h}$.

□

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