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Dominance Concepts for Fehr-Schmidt Preferences

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Dominance concepts for Fehr-Schmidt preferences*

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Abstract

Many diverse problems in economics can only be reasonably explained by assuming that people have social preferences, i.e., in addition to their own payoffs they are altruistic towards those who are poorer and envious towards those who are richer. How do people with social preferences choose among alternative income distributions? The aim of our paper is to answer this question in the context of the Fehr-Schmidt (1999) preferences. The classical notions of first and second order stochastic dominance are not useful in this case. However, a fairly natural set of conditions that are a modification of the concepts of first and second order stochastic dominance and generalized Lorenz dominance turn out to successfully answer the question posed. We also introduce weak FS dominance, which is particularly suited to the linear form of Fehr-Schmidt preferences.

Keywords: Fehr-Schmidt preferences; first order Fehr-Schmidt dominance; second order Fehr-Schmidt dominance; weak Fehr-Schmidt dominance; strong Fehr-Schmidt dominance; Fehr-Schmidt-Lorenz dominance.

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1. Introduction

Choice over alternative income distributions lies at the heart of models in many areas of economics, particularly in the literature on public economics, welfare economics and taxation. The recent empirical literature on redistribution strongly supports the role of *social preferences* in experiments where people are asked to choose among alternative income distributions; see, for instance, Ackert et al. (2007), Bolton and Ockenfels (2006) and Tyran and Sausgruber (2006). These papers establish that voters often choose policies that reflect social concerns such as equity/fairness over purely selfish considerations.

Dominance concepts such as *first* and *second order stochastic dominance*[1] and *generalized Lorenz dominance*[2] play a critical role in economics. The approach in the literature is to begin with a purely statistical definition of a dominance concept and then to establish its economic relevance (or soundness) by relating it to the relevant decision theory. Suppose that we have a set of income classes $\mathbf{Y} = \{y_1 < y_2 < ... < y_n\}$. Consider two cumulative distributions, P, Q, over the set of incomes; so that under the distribution, P, P_i is the proportion of individuals with incomes less than, or equal to, y_i (and similarly for Q). Then P *first order stochastically dominates* Q (written, $P \succsim_1 Q$) if P lies everywhere below Q. Similarly P *second order stochastically dominates* Q (written, $P \succsim_2 Q$) if the cumulative distribution, $\widetilde{P}$, of P lies everywhere below the cumulative distribution, $\widetilde{Q}$, of Q and the mean of P is no less than that of Q.

Consider an individual with non-decreasing utility function, $u(y)$. An individual with such a utility function is said to have *selfish preferences* because utility accrues only from one's own income or consumption.[3] In discussing the standard dominance concepts, one might wonder why should someone with selfish preferences care about the entire income distribution? This is typically motivated by assuming that the individual operates behind a *veil of ignorance*: The individual does not know which income class he will lie in. He takes $p_i = P_i - P_{i-1}$ to be the probability that he will fall in income class i, hence, he cares about the entire distribution. Under this interpretation, every expected utility maximizer who has selfish preferences and a non-decreasing utility function will prefer P over Q if, and only if, P first order stochastically dominates Q. Similarly, every expected utility maximizer who has selfish preferences and a concave utility function will prefer P over Q if, and only if, P second order stochastically dominates Q. This establishes the soundness of the purely statistical concepts of first and second order stochastic dominance.

[1] See, Hanoch and Levy (1969) and Rothschild and Stiglitz (1970, 1971).

[2] See, for instance, Shorrocks, 1983, who built on earlier work by Kolm (1969) and Atkinson (1970). See, also the book length treatment in Lambert (2001).

[3] In this paper we are not concerned with the moral or ethical connotations that may arise with the use of terms such as selfish preferences or social preferences. We use these terms purely to separate the two different kinds of preferences.

Although, in principle, neoclassical economics is not restricted to selfish preferences, it is almost always the case that neoclassical economists restrict themselves to selfish preferences.[4] A more general form of the utility function for individual i, $i = 1, 2, ..., n$ is $u(y_i, \mathbf{y}_{-i})$, where $\mathbf{y}_{-i}$ is the vector of incomes in all income classes other than the i^{th} income class. Such a utility function represents an individual who may be said to have *social preferences* or *other-regarding preferences.* When individuals have social preferences they care, in addition to their own consumption bundle, for the consumption bundles of others. In particular, they may exhibit *altruism*, *pride* and *envy*. Both casual observation and an enormous amount of rigorous research has shown that people have some form of *social preferences.*[5]

Consider an individual who has social preferences and a level of income $y_j \in \mathbf{Y} = \{y_1 < y_2 < ... < y_n\}$. The central question of our paper is as follows. Given any two distributions P and Q over $\mathbf{Y}$, which distribution does this individual prefer? The individual knows his own income, y_j, with certainty. Thus, the question does not require the choice to be made behind a veil of ignorance.

This question assumes even greater importance if, as we will show, the traditional concepts of first and second order dominance and generalized Lorenz dominance are no longer suitable for social preferences. There is a range of models of social preferences that explain various aspects of social preferences. Before we answer the question that we have posed, we need to ask which model of social preferences is most appropriate for the question? We now address this issue.

Among competing models of social preferences, the Fehr-Schmidt model (Fehr and Schmidt 1999), henceforth the FS model, has been particularly influential. In the FS model, individuals care about their own payoffs (as in models of selfish preferences) but they also derive disutility from those who are richer (*envy*) and from those who are poorer (*altruism*). Thus the FS model is sometimes referred to as a model of *inequity aversion.*

The FS model successfully explains the results of many experimental games. These include, for instance, dictator games, ultimatum games, gift exchange games and public good games. By contrast, selfish preferences are unable to explain these experimental results.[6] The FS model also aids understanding in many diverse areas such as contract theory and political economy.[7]

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[4]There are, of course, exceptions. For instance models of *keeping up with the Jonses* that rely on relaxing the assumption of selfish preferences.

[5]See, for instance, Fehr and Fischbacher (2002), and Fehr and Schmidt (2006). The interested reader can read various aspects of the issues in the handbook by Kolm and Ythier (2006) and the references cited therein.

[6]For explanations of the results from the experimental games using models of social preferences, see, for instance, Fehr and Schmidt (2006).

[7]See for instance, Fehr and Schmidt (2006), Fehr et al, (2007), and Dhami and al-Nowaihi (2010a, 2010b).

In their experimental results on voting over redistribution that involved choosing alternative income distributions, Tyran and Sausgruber (2006) conclude that the FS model predicts much better than a model with selfish preferences. In addition, for the three income classes in their experiments, the FS model provides, in their words, "strikingly accurate predictions for individual voting in all three income classes." The econometric results of Ackert et al. (2007), based on their experimental data, lend further support to the FS model in the context of redistributive taxation. The estimated coefficients of altruism and envy in the FS model are statistically significant and, as expected, negative in sign.[8] Furthermore, there has been progress in the theoretical literature on applying FS preferences to issues of redistribution.[9] For these reasons, we choose the FS model as our model of social preferences.

Two concerns about the experimental evidence on fairness and reciprocity have been expressed.[10] (1) The student population may have biased the results towards social preferences. (2) What happens in the lab may not be representative of the real world. Both concerns have been shown to be unfounded by a large number of empirical studies. It is now a fairly robust finding that students exhibit weaker social preferences than the non-student population and students with more pronounced social preferences do not self-select themselves into experiments.[11] Furthermore, there is a close connection between the parameters of social preferences in experiments and in the real world, when the experiments are conducted with the non-student population. Thus, subjects who exhibit more cooperative behavior in the lab also exhibit greater cooperation in the real world.[12]

In FS preferences, on account of altruism, one derives disutility from the incomes of those who are poorer. By contrast, some contexts or frames may be suggestive of competition among individuals, e.g., a race or a tournament among workers in an organization. In such cases, one may derive *pride* (and positive utility) in coming out ahead of others. The literature on happiness economics also assumes that individuals take pride in coming out ahead of others; see Hopkins (2008). Although our primary focus is on the FS model, our model can easily deal with the case where pride rather than altruism is dominant, as we shall indicate.

The research agenda for the paper is as follows. We show that the classical concepts of first and second order stochastic dominance are unsuitable for FS preferences. Our first

[8] When voters vote over redistribution, which is often anonymous and takes place in large groups of people, models pf *type-based reciprocity* and *intention-based reciprocity* are not suitable. Hence, we do not disucss them here. See, for instance, Fehr and Schmidt (2006).

[9] See, for instance, Dhami and al-Nowaihi (2010a, 2010b).

[10] See, for instance, Levitt and List (2007).

[11] For a representative study from a large number of studies supporting this assertion, see Falk et al. (2011).

[12] For two representative studies among a large group of such studies, see Carpenter et al. (2011) and Fehr and Leibbrandt (2011).

set of questions ask which concepts of dominance are underpinned by FS preferences? In the standard model with selfish preferences there is a natural link between *second order stochastic dominance* and *generalized Lorenz dominance* that is of great importance in the work on income distribution. Therefore, our second set of questions address the link between second order dominance concepts that are suitable for FS preferences and concepts of Lorenz dominance that are suitable for FS preferences. It turns out that, for the first set of questions, a discrete formulation facilitates the derivation of the results with minimum technicality. However, a continuous formulation is indispensable when we consider our second set of questions about Lorenz dominance, for reasons that we shall explain below.

The structure of the model and some preliminary results are described in Section 2. Section 3 helps fix the ideas behind our answers to the two questions that we pose above through a series of illustrative examples. In Section 4 we analyze our first set of questions. We propose two main dominance concepts that are suitable for FS preferences: *first and second order Fehr-Schmidt dominance*. We also propose a third concept: *Weak Fehr-Schmidt dominance*. This is particularly suited to the linear form of FS preferences that is prevalent in applied work. For each new concept we introduce strict versions as well. We establish the soundness of these statistical concepts by relating them to a preference of one distribution to another for an individual who has FS preferences. We also establish the interconnections between the various dominance concepts we propose.

Section 6 addresses our second question. We introduce a slight strengthening of the concept of second order FS dominance that we call as *strong FS dominance*. We also introduce a sense in which a modification of generalized Lorenz dominance makes intuitive sense under FS preferences and call it *FS Lorenz dominance*. We show that strong FS dominance is equivalent to FS Lorenz dominance. Finally section 7 concludes.

2. The Model

The purpose of this section is to set up our model, give a formal definition of Fehr-Schmidt preferences, express these preferences in a convenient form and provide some useful intermediate results.

2.1. Notation and some preliminaries

Consider a society with N members. Income, y, of any member belongs to the set of possible incomes $\mathbf{Y} = \{y_1 < y_2 < ... < y_n\}$.

Let $p_i \geq 0$ be the proportion of individuals with income y_i, $i = 1, 2, ..., n$, $\sum_{i=1}^{i=n} p_i = 1$. The *cumulative probability distribution* is then given by $P_0 = 0$, $P_j = \sum_{i=1}^{i=j} p_i$. Let Π be the set of all such distributions over $\mathbf{Y}$. The *cumulative of the cumulative distribution* is given by $\widetilde{P}_0 = 0$, $\widetilde{P}_j = \sum_{i=1}^{i=j} P_i$, $j = 1, 2, ..., n$;[13] let $\widetilde{\Pi}$ be the set of all such distributions.

A total of $p_i N$ individuals have income y_i each, and $p_i N y_i$ is their total income. The proportion of individuals with incomes less than or equal to y_j is P_j; the total number of such individuals is $P_j N$. The total income accruing to the poorest $P_j N$ individuals is $N \sum_{i=1}^{i=j} p_i y_i$. The average, or mean, of $y_1, y_2, ..., y_n$ under the distribution $P \in \Pi$ is $\mu_P = \sum_{i=1}^{i=n} p_i y_i$.

Since we will be interested in the discrete case for our first set of questions, we need the analogue of the *integration by parts* formula. This is called the *summation by parts* formula, which we shall use extensively in the paper. For any sequences F, G (not necessarily distribution functions), the summation by parts formula is given by[14]

$$\sum_{i=1}^{i=n} F_i \left(G_i - G_{i-1}\right) = F_n G_n - F_0 G_0 - \sum_{i=0}^{i=n-1} G_i \left(F_{i+1} - F_i\right). \tag{2.1}$$

For some results it will be useful to restrict the income levels to be equally spaced.

Definition 1 *: We say that the income levels are equally spaced if for some positive real number, δ, $y_{i+1} - y_i = \delta$ for $i = 1, 2, ..., n-1$.*[15]

As our first application of (2.1), we have the following result.

Lemma 1 *: Suppose incomes are equally spaced (Definition 1). Let $P, Q \in \Pi$. Then $\mu_P = y_n - \delta \widetilde{P}_{n-1}$ and $\mu_Q = y_n - \delta \widetilde{Q}_{n-1}$. Hence, $\mu_P \lesseqqgtr \mu_Q$ if, and only if, $\widetilde{P}_{n-1} \gtreqqless \widetilde{Q}_{n-1}$.*

Proof of Lemma 1: Applying (2.1) with $F = y$ and $G = P$, and using the facts that $P_0 = 0$ and $P_n = 1$, we get $\mu_P = \sum_{i=1}^{i=n} p_i y_i = \sum_{i=1}^{i=n} \left(P_i - P_{i-1}\right) y_i = y_n P_n - y_0 P_0 - \sum_{i=0}^{i=n-1} P_i \left(y_{i+1} - y_i\right) = y_n - \delta \sum_{i=0}^{i=n-1} P_i = y_n - \delta \sum_{i=0}^{i=n-1} P_i = y_n - \delta \widetilde{P}_{n-1}$. Similarly, $\mu_Q = y_n - \delta \widetilde{Q}_{n-1}$. Hence, $\mu_P \lesseqqgtr \mu_Q$ if, and only if, $\widetilde{P}_{n-1} \gtreqqless \widetilde{Q}_{n-1}$. ■.

Notation: Suppose that the utility function $u(y)$ is non-decreasing in income, y. The class of all such utility functions is denoted by $\mathbf{u}$.

We now state two obvious facts.

[13] This is important for concepts related to second order stochastic dominance.

[14] To check that (2.1) is correct, simply expand both sides; details available from the authors. Summation by parts can, of course, take other, equivalent forms. But (2.1) is the most useful for us. Note that, by convention, if F_0 is not defined, then it is set to zero. Likewise for G_0.

[15] This is not restrictive because we can always introduce extra income levels, each with probability zero, to achieve equal spacing. For example, suppose $y_1 = 5$, $y_2 = 7$, $y_3 = 11$, which are not equally spaced, and $P_1 = \frac{1}{3}$, $P_2 = \frac{2}{3}$. Consider $y_1 = 5$, $y_2 = 7$, $y_3 = 9$, $y_4 = 11$, which are equally spaced, and $Q_1 = \frac{1}{3}$, $Q_2 = \frac{2}{3}$, $Q_3 = \frac{2}{3}$. Both P and Q describe the same reality.

Result 1 *: (a) For $u \in \mathbf{u}$, for $i < j$, $u(y_j) - u(y_i) \geq 0$ and, if u is strictly increasing, then $u(y_j) - u(y_i) > 0$,*
(b) If incomes are equally spaced (Definition 1) then, for concave u, $[u(y_{i+1}) - u(y_i)] - [u(y_{i+2}) - u(y_{i+1})] \geq 0$. If u is strictly concave, then $[u(y_{i+1}) - u(y_i)] - [u(y_{i+2}) - u(y_{i+1})] > 0$.

2.2. Fehr-Schmidt social preferences when incomes are discrete

In its most general (discrete) form, the *Fehr-Schmidt utility function* U of an individual with income $y_j \in \mathbf{Y}$ is defined below.[16]

Definition 2 *(General form of FS utility): Consider the income distribution, P and $u \in \mathbf{u}$. The general form of the Fehr-Schmidt utility function (FS) for an individual with income $y_j \in \mathbf{Y}$ is given by*

$$U(y_j, P) = u(y_j) - \beta \sum_{i=1}^{j-1} p_i [u(y_j) - u(y_i)] - \alpha \sum_{k=j+1}^{n} p_k [u(y_k) - u(y_j)], \ \alpha \geq 0, 0 \leq \beta < 1. \tag{2.2}$$

For an individual with classical textbook *selfish* preferences, $\alpha = \beta = 0$, so

$$U(y, P) = u(y), \tag{2.3}$$

while for a fair individual $\alpha \neq 0$ or $\beta \neq 0$ or both. However, typically, one observes empirically that $\beta < \alpha$; see Fehr and Schmidt (1999).

From (2.2), a fair individual derives utility from 'own payoff' just like an individual with selfish preferences (first term). But in addition he derives disutility from two sources: From payoff relative to those where inequality is *advantageous* (second term) and payoff relative to those where inequality is *disadvantageous* (third term). We shall call the disutility arising from these two terms, respectively, as *advantageous inequity* and *disadvantageous inequity*; they capture respectively, *altruism* and *envy*. Notice that in FS-preferences, inequality is *self-centered*, i.e., the individual uses own payoff as a reference point with which everyone else is compared to. From (2.2), β is bounded above by 1 because $\beta > 1$ would imply that an individual could increase utility by merely destroying own payoff, which is counterfactual.

A second version, predominant in applied work, is the *linear version*, in which $u(y) = y$. The suitability of this version is not merely a matter of convenience. Fehr and Schmidt (1999) show that it fits well the data from experimental games.

[16] Neilson (2006) gives an axiomatization.

Definition 3 *(Linear version): The linear version of the Fehr-Schmidt utility function for an individual with income y_j is given by*

$$U(y_j, P) = y_j - \beta \sum_{i=1}^{j-1} p_i (y_j - y_i) - \alpha \sum_{k=j+1}^{n} p_k (y_k - y_j), \ \alpha \geq 0, 0 \leq \beta < 1. \tag{2.4}$$

The following is an immediate consequence of Definition 2 obtained by collecting terms corresponding to each $u(y_i)$, $i = 1, 2, ..., n$.[17]

Lemma 2 *: The Fehr-Schmidt utility function for an individual with income y_j (2.2) can be written in the following equivalent form:*

$$U(y_j, P) = \omega_j u(y_j) + \beta \sum_{i=1}^{j-1} p_i u(y_i) - \alpha \sum_{k=j+1}^{n} p_k u(y_k), \tag{2.5}$$

where

$$\omega_j = [1 - \beta P_{j-1} + \alpha (1 - P_j)] > 0, \ \alpha \geq 0, 0 \leq \beta < 1, \ u \in \mathbf{u}. \tag{2.6}$$

Remark 1 *: Suppose that u is increasing, $\alpha > 0$ and $\beta > 0$.*
(a) From (2.2) or, equivalently, from (2.5) and (2.6) we see that $U(y_j, P)$ is increasing in the utilities of the relatively poor ($i < j$) and decreasing in the utilities of the relatively rich ($k > j$). The former reduces advantageous inequity and the latter increases disadvantageous inequity.
(b) Since $\alpha > 0$, $0 < \beta < 1$ and $0 \leq P_{j-1} \leq P_j \leq 1$, it follows from (2.6) that $\omega_j > 0$. Hence, from (2.5), we see that an increase in y_j (keeping all other incomes fixed and the order of incomes fixed) increases $U(y_j, P)$.
(c) Begin with an income distribution $P \in \Pi$, under which an individual has the Fehr-Schmidt preferences given in (2.2) and has income y_j. Consider two income levels, y_s, y_t, in the support of P such that $y_s < y_t < y_j$ and $p_t > 0$. Suppose that we obtain the distribution Q from P by transferring a fraction Δ of individuals from the income class y_t to y_s then $U(y_j, P) - U(y_j, Q) = -\beta\Delta(u(y_t) - u(y_s)) < 0$. Thus individuals with FS preferences dislike 'rich to poor transfers' among people poorer than them. Now consider two income levels in the support of P, y_l, y_m such that $y_j < y_l < y_m$ and $p_m > 0$. Now obtain a new distribution $P\prime$ by transferring a fraction Δ of individuals from the income class y_m to y_l then $U(y_j, P) - U(y_j, P\prime) = \alpha\Delta(u(y_m) - u(y_l)) > 0$. Thus, individuals with FS preferences like 'rich to poor transfers' among people richer than them.

From (2.5), using the summation by parts formula in (2.1), we get the following corollary that is of fundamental importance for us.

[17] Details available from the authors.

Corollary 1 : *Let $u \in \mathbf{u}, P, Q \in \Pi$ and $\Delta u_k = u(y_{k+1}) - u(y_k) > 0$. Then $U(y_j, P) - U(y_j, Q)$ can be expressed in two alternative forms.*

$$(a)\ \beta \sum_{i=1}^{i=j-1} (Q_i - P_i)\Delta u_i + \alpha \sum_{k=j}^{k=n-1} (P_k - Q_k)\Delta u_k, \tag{2.7}$$

$$(b)\ \alpha\left(\widetilde{P}_{n-1} - \widetilde{Q}_{n-1}\right)\Delta u_{n-1} + \left(\widetilde{Q}_{j-1} - \widetilde{P}_{j-1}\right)(\alpha\Delta u_j + \beta\Delta u_{j-1})$$
$$+\beta \sum_{i=1}^{i=j-2}\left(\widetilde{P}_i - \widetilde{Q}_i\right)(\Delta u_{i+1} - \Delta u_i) + \alpha \sum_{k=j}^{k=n-2}\left(\widetilde{Q}_k - \widetilde{P}_k\right)(\Delta u_{k+1} - \Delta u_k). \tag{2.8}$$

Proof of *Corollary 1*: We give an outline of the proof.

(a) From (2.5) and (2.6) we get

$$U(y_j, P) = [1 - \beta P_{j-1} + \alpha(1 - P_j)]\, u(y_j) + \beta \sum_{i=1}^{j-1} p_i u(y_i) - \alpha \sum_{k=j+1}^{n} p_k u(y_k). \tag{2.9}$$

Replace p_i with $P_i - P_{i-1}$ and likewise for p_k. Apply (2.1) to $\sum_{i=1}^{j-1}(P_i - P_{i-1})\, u(y_i)$ to get $(P_{j-1})\, u(y_{j-1}) - \sum_{i=0}^{j-2} P_i \Delta u_i$ (recall that $P_0 = 0$). Likewise, $\sum_{k=j+1}^{n} p_k u(y_k) = u(y_n) - P_j u(y_j) - \sum_{k=j}^{n-1} P_k \Delta u_k$ (recall that $P_n = 1$). Substitute in (2.9) to get $U(y_j, P) = [1 - \beta P_{j-1} + \alpha(1 - P_j)]\, u(y_j) + \beta\left\{(P_{j-1})\, u(y_{j-1}) - \sum_{i=0}^{j-2} P_i \Delta u_i\right\} - \alpha\left\{u(y_n) - P_j u(y_j) - \sum_{k=j}^{n-1} P_k \Delta u_k\right\}$. Do the same for $U(y_j, Q)$. Subtract, to get the required result.

(b) Replace P_i in (2.7) by $\widetilde{P}_i - \widetilde{P}_{i-1}$. Do likewise for P_k, Q_i and Q_k. Collect terms. Apply (2.1) to $\sum_{i=0}^{j-1}\left[\left(\widetilde{Q}_i - \widetilde{P}_i\right) - \left(\widetilde{Q}_{i-1} - \widetilde{P}_{i-1}\right)\right]\Delta u_i$ with $F_i = \Delta u_i$ and $G_i = \left(\widetilde{Q}_i - \widetilde{P}_i\right) - \left(\widetilde{Q}_{i-1} - \widetilde{P}_{i-1}\right)$, recalling that $\widetilde{P}_0 = \widetilde{Q}_0 = 0$.

Do the same to $\sum_{k=j}^{n-1}\left[\left(\widetilde{P}_k - \widetilde{Q}_k\right) - \left(\widetilde{P}_{k-1} - \widetilde{Q}_{k-1}\right)\right]\Delta u_k$. Substitute back, collect terms and simplify to get the required result. ■

2.3. Pride or downward envy

In FS preferences, see (2.2), $\beta > 0$ captures altruism or compassion towards individuals who are poorer. Hopkins (2008) differentiates between the sign of β in two different literatures. In behavioral economics, where models of social preferences have gained increasing acceptance, $\beta > 0$. But in happiness economics, $\beta < 0$ is allowed for. Hopkins (2008) quotes a working paper version from Daniel Friedman to refer to $\beta < 0$ as 'pride', 'competitiveness' or 'downward envy'. For the case $\beta < 0$, (2.2) becomes:

$$U(y_j, P) = u(y_j) - \beta \sum_{i=1}^{j-1} p_i [u(y_j) - u(y_i)] - \alpha \sum_{k=j+1}^{n} p_k [u(y_k) - u(y_j)],\ \alpha \geq 0, \beta < 0. \tag{2.10}$$

Lemma 2 and Corollary 1 continue to apply but with $\beta < 0$.

3. Motivation

Subsection 3.1 gives definitions for the standard concepts of first and second order stochastic dominance and the related propositions. It ends with an example (Example 1) of two distributions, P and Q, where P strictly first order and strictly second dominates Q. Yet under Fehr-Schmidt preferences Q can be strictly preferred to P. Subsection 3.2 then gives an example that motivates the definitions of first order, second order and weak Fehr-Schmidt dominance, to be proposed later in the paper. Finally, subsection 3.3 offers a discussion of Lorenz dominance.

3.1. First and second order stochastic dominance are not appropriate for FS preferences

In this subsection, we consider individuals with purely *selfish preferences* (see the introduction for this concept) who follow expected utility theory. For any two distributions $P, Q \in \Pi$, when does such an individual who has the utility function, u, prefer P to Q, behind a veil of ignorance?[18] We now discuss two well known answers to this question; see Rothschild and Stiglitz (1970). The proofs are simple and standard in economics, so we omit them.[19]

Definition 4 *(First order stochastic dominance): Let $P, Q \in \Pi$. Then P first order stochastically dominates Q ($P \succsim_1 Q$) if $P_i \leq Q_i$ for $i = 1, 2, ..., n-1$. If, in addition, the inequality is strict for some i then P strictly first order stochastically dominates Q ($P \succ_1 Q$).*

The next proposition establishes the soundness of first order dominance.

Proposition 1 *: Let $P, Q \in \Pi$. Then $P \succsim_1 Q$ if, and only if, for any $u \in \mathbf{u}$,*

$$\sum_{i=1}^{i=n} p_i u(y_i) \geq \sum_{i=1}^{i=n} q_i u(y_i), \tag{3.1}$$

and $P \succ_1 Q$ if, and only if, the inequality in (3.1) is strict for all strictly increasing $u \in \mathbf{u}$.

Choosing $u(y) = y$ as a candidate function in Proposition 1 we get that if $P \succsim_1 Q$ ($P \succ_1 Q$) then $\mu_P \geq \mu_Q$ ($\mu_P > \mu_Q$).

[18] Under the distribution P, behind the veil of ignorance, the individual assigns a probability p_i of receiving the income level y_i, $i = 1, ..., n$.

[19] For a textbook treatment, see Mas-Colell et al. (1995).

Definition 5 *(Second Order Stochastic dominance): Suppose incomes are equally spaced (Definition 1).*[20] *Let* $P, Q \in \Pi$. *Then* P *second order stochastically dominates* Q ($P \succsim_2 Q$) *if* $\widetilde{P}_i \leq \widetilde{Q}_i$ *for* $i = 1, 2, ..., n-1$. *If, in addition, one of these inequalities is strict then* P *strictly second order stochastically dominates* Q ($P \succ_2 Q$).

The next proposition establishes the soundness of second order dominance.

Proposition 2 *: Let* $P, Q \in \Pi$. *Then* $P \succsim_2 Q$ *if, and only if, for any concave* $u \in \mathbf{u}$,

$$\sum_{i=1}^{i=n} p_i u(y_i) \geq \sum_{i=1}^{i=n} q_i u(y_i), \tag{3.2}$$

and $P \succ_2 Q$ *if, and only if, the inequality in (3.2) is strict for all strictly increasing strictly concave* $u \in \mathbf{u}$.

Example 1 *: Consider three income levels,* $0, 25, 50$ *and two distributions,* P, Q *defined by the table below.*

Table-I

y	0	25	50
p	0	$\frac{1}{3}$	$\frac{2}{3}$
q	$\frac{1}{6}$	ε	$\frac{5}{6} - \varepsilon$
P	0	$\frac{1}{3}$	1
Q	$\frac{1}{6}$	$\frac{1}{6} + \varepsilon$	1
$\widetilde{P}$	0	$\frac{1}{3}$	$\frac{4}{3}$
$\widetilde{Q}$	$\frac{1}{6}$	$\frac{1}{3} + \varepsilon$	$\frac{4}{3} + \varepsilon$

From Table-I it can be easily seen that $P \succ_1 Q$ *and* $P \succ_2 Q$ ($\mu_P = \frac{125}{3} > \frac{100}{3} = \mu_Q$). *For the individual with income* $y_2 = 25$, *we can find*

$$U(25, Q) - U(25, P) = \left(\varepsilon - \frac{1}{6}\right)(\alpha u(50) - u(25)) + \frac{\beta}{6} u(0) - \frac{\beta}{6} u(25) \tag{3.3}$$

It is easy to construct examples where an individual with Fehr-Schmidt preferences strictly prefers Q *to* P. *For example, let* $\varepsilon = \frac{1}{3}$ *and take* u *to be any strictly increasing own utility function. Take* $\alpha = 1$ *and* $\beta = 0$. *Consider an individual with income* 25. *Then, from (3.3) we get* $U(25, Q) - U(25, P) = \frac{u(50) - u(25)}{6} > 0$.
Consider now some implications of pride ($\beta < 0$) *rather than altruism* ($\beta > 0$). *Let* u *be any strictly increasing own utility function. Take* $\alpha = 1$ *and* $\varepsilon = \frac{1}{6}$. *Then* $U(25, Q) - U(25, P) = -\frac{\beta}{6}(u(50) - u(25))$. *Clearly the choice between* P, Q *now hinges solely on*

[20] Often 'equal spacing' is not stated as part of the definition of second order stochastic dominance. But then it has to be stated in the propositions. We find it more convenient to state in the definition. And, of course, it is not needed in the continuous case (see section 6, below).

the sign of β. An individual with altruism prefers P to Q but an individual with pride prefers Q to P. This example makes the more general point that an individual with pride will always prefer the dominated distribution.

3.2. Dominance concepts for FS preferences

We now consider an example that illustrates the main ideas behind several results in the paper. Consider the data given in Table-II below. We are interested in the individual with income of $y_3 = 50$ who has FS preferences.

Table-II

y	0	25	50	75	100
$p(y)$	0	$\frac{1}{3}$	$\frac{1}{3}+2\epsilon$	$\frac{1}{3}-2\epsilon$	0
$q(y)$	ϵ	$\frac{1}{3}-\epsilon$	$\frac{1}{3}$	$\frac{1}{3}-\epsilon$	ϵ
$P(y)$	0	$\frac{1}{3}$	$\frac{2}{3}+2\epsilon$	1	1
$Q(y)$	ϵ	$\frac{1}{3}$	$\frac{2}{3}$	$1-\epsilon$	1
$\widetilde{P}(y)$	0	$\frac{1}{3}$	$1+2\epsilon$	$2+2\epsilon$	$3+2\epsilon$
$\widetilde{Q}(y)$	ϵ	$\frac{1}{3}+\epsilon$	$1+\epsilon$	2	3

Below income $y_3 = 50$, distribution Q moves some richer individuals to a poorer income level and above income $y_3 = 50$ it move some poorer individuals to a richer income level. Thus, using Remark 1, we would expect such an individual to prefer the distribution P to the distribution Q. What inferences can we draw from this unambiguous preference for P? An examination of Table-II reveals that the following restrictions apply

$$P_i \leq Q_i,\ P_k \geq Q_k;\ i < 3,\ k \geq 3. \tag{3.4}$$

$$\widetilde{P}_i \leq \widetilde{Q}_i,\ \widetilde{P}_k \geq \widetilde{Q}_k;\ i < 3,\ k \geq 3. \tag{3.5}$$

From (3.4), neither distribution first order stochastically dominates the other, and, from (3.5), neither distribution second order stochastically dominates the other. Thus, a-priori, one cannot predict which distribution would be preferred by an individual with selfish preferences. But we shall see, these restrictions are ideal to determine which of the distributions is preferred by an individual with FS preferences whose income is $y_3 = 50$. One can make the following simple calculations using (2.5), (2.6).

$$U(50, P) = \left[1 - \frac{\beta}{3} + \alpha\left(\frac{1}{3} - 2\epsilon\right)\right] u(50) + \frac{\beta}{3} u(25) - \alpha\left(\frac{1}{3} - 2\epsilon\right) u(75),$$

$$U(50, Q) = \left(1 - \frac{\beta}{3} + \frac{\alpha}{3}\right) u(50) + \beta\epsilon u(0) + \beta\left(\frac{1}{3} - \epsilon\right) u(25) - \alpha\left(\frac{1}{3} - \epsilon\right) u(75) - \alpha\epsilon u(100).$$

Therefore, for any $u \in \mathbf{u}$,

$$U(50, P) - U(50, Q) = \beta\epsilon\left[u(25) - u(0)\right] + \epsilon\alpha\left[u(75) + u(100) - 2u(50)\right] > 0,$$

as claimed. We did not need to invoke the concavity of the utility function for this example because it satisfies (3.4), in which case it is sufficient that $u \in \mathbf{u}$.

In the case of first and second stochastic dominance (Definitions 4, 5) the dominant distribution, P, has a mean that is no lower than the dominated distribution, Q. This need not be the case under FS dominance. From the data given in Table-II,

$$\mu_Q - \mu_P = 50\epsilon > 0.$$

Thus, the dominant distribution in this case, P, has a lower mean. The reason is that, for $\alpha > 0$, a reduction in the mean can be associated with a reduction in disadvantageous inequality for an individual with income y_j without affecting that individual's income. This could happen by destroying some of the wealth of individuals with higher income than y_j (but still leaving them with incomes no less than y_j).[21] This is supported by the evidence where experimental subjects prefer a smaller cake which is is more equitably distributed.[22]

The conditions in (3.4), (3.5) are examples of the more general conditions that we give in the formal development of the theory, below. We call these conditions, respectively, *first order FS dominance* and *second order FS dominance*. We denote the binary relations corresponding to these dominance concepts, respectively, by $\succeq_{FS1_{y_j}}$ and $\succeq_{FS2_{y_j}}$ where y_j is the income of the individual who compares the two distributions. Thus, in the examples above we have $P \succeq_{FS1_{y_j}} Q$ and $P \succeq_{FS2_{y_j}} Q$.

Notice from the data given in Table-II that

$$\sum_{i=1}^{2} p_i \left(y_3 - y_i\right) \leq \sum_{i=1}^{2} q_i \left(y_3 - y_i\right), \tag{3.6}$$

$$\sum_{k=4}^{5} p_k \left(y_k - y_3\right) \leq \sum_{k=4}^{5} q_k \left(y_k - y_3\right). \tag{3.7}$$

Consider now the linear form of FS preferences given in Definition 2. It is obvious that when (3.6), (3.7) hold, then an individual with income y_3 will prefer the distribution P to Q. Conditions (3.6), (3.7) are examples of what we shall call as *weak FS dominance* in the general case, below. We denote the binary relation based on weak FS dominance by $\succeq_{WFS_{y_j}}$.

These examples enable us to anticipate the first set of results in the paper. We will, formally, show the following:

1. $P \succeq_{FS1_{y_j}} Q \Leftrightarrow U\left(y_j, P\right) \geq U\left(y_j, Q\right)$ for all non-decreasing own utility functions, u, all $\alpha \geq 0$ and all $\beta \in [0, 1)$,

[21] Technically, this is because of the presence of the term $\alpha\left(\widetilde{P}_{n-1} - \widetilde{Q}_{n-1}\right)\Delta u_{n-1}$ in (2.8).

[22] See, for instance, Ackert et al. (2007), Bolton and Ockenfels (2006) and Tyran and Sausgruber (2006).

2. $P \succeq_{FS2_{y_j}} Q \Leftrightarrow U(y_j, P) \geq U(y_j, Q)$ for all non-decreasing, concave, own utility functions, u, all $\alpha \geq 0$ and all $\beta \in [0, 1)$ when incomes are equally spaced.

We also show that for the linear form of FS preferences given in Definition 2,

1. $P \succeq_{WFS_{y_j}} Q \Leftrightarrow U(y_j, P) \geq U(y_j, Q)$ for all $\alpha \geq 0$ and all $\beta \in [0, 1)$,

2. $P \succeq_{FS1_{y_j}} Q \Rightarrow P \succeq_{WFS_{y_j}} Q$,

3. and, when incomes are equally spaced, $P \succeq_{FS2_{y_j}} Q \Rightarrow P \succeq_{WFS_{y_j}} Q$.

We shall also establish strict analogues of the above results.

A second set of results in our paper derive the appropriate notion of Lorenz dominance for the case of FS preferences. We motivate these results in the next subsection.

3.3. Lorenz dominance and FS preferences

The *Lorenz curve* plots the cumulative total income against the cumulative totals of the recipients of this income. When the Lorenz curve corresponding to P lies everywhere above the Lorenz curve for Q we say that P *Lorenz dominates* Q. For reasons that will become clear below, a continuous distribution is more satisfactory for the development of a Lorenz curve. Suppose that incomes are distributed over $[0, 1]$.[23]

A more useful concept is that of generalized Lorenz dominance because it can often rank distributions that cannot be ranked under Lorenz dominance but the converse is false. For any distribution P, the *generalized Lorenz curve* is found by multiplying the mean of P, μ_P, with the Lorenz curve of P. When the generalized Lorenz curve for P lies everywhere above that of another distribution, Q, we write this as $P \succsim_L Q$. When individuals have selfish preferences, follow expected utility, and choose behind a veil of ignorance, it is well known that the concept of *second order stochastic dominance* and *generalized Lorenz dominance* are very closely related. Under these conditions, Kolm (1969) and Shorrocks (1983) showed that $P \succsim_2 Q \Leftrightarrow P \succsim_L Q$. This result is enormously important for theoretical and applied research, particularly in income inequality.

Subsection 3.1 showed that the concept of second order stochastic dominance is not suitable for FS preferences. For Example 1 we showed that $P \succsim_2 Q$, yet $U(y_j, P) < U(y_j, Q)$. Thus, it is immediate that $P \succsim_L Q$ does not imply that an individual with FS preferences will prefer the distribution P to Q.

In our second set of results we propose a new notion of Lorenz dominance that we call *FS Lorenz dominance*. When distribution P *FS Lorenz dominates* distribution Q for

[23] We also provide the continuous versions of first and second order FS dominance as well as weak FS dominance, below.

an individual with income $y \in [0,1]$, who has FS preferences, we write it as $P \succsim_{FSL_y} Q$. We establish that this concept is related to a slight strengthening of second order FS dominance that we call *strong FS dominance* (denoted by the binary relation $\succeq_{SFS_y}$). For an individual with income y who has FS preferences, strong FS dominance is identical to second order FS dominance but in addition it requires that $P(y) = Q(y)$. We will show, formally, that $P \succeq_{SFS_y} Q \Leftrightarrow P \succsim_{FSL_y} Q$.

Strong FS dominance ($\succeq_{SFS_y}$) is identical to second order FS dominance ($\succeq_{FS2_y}$) except, possibly, at the income level y. Hence, we can use the $\succeq_{FS2_y}$ relation to build intuition about the shapes of the Lorenz curves implied by the relation $\succeq_{SFS_y}$. When $P \succeq_{FS2_y} Q$ then P second order stochastically dominates Q below the income level y and is second order stochastically dominated by Q above the income level y. Hence, we would expect the Lorenz curve for P to lie above the Lorenz curve of Q below the income level y and for the converse to happen above the income level y. In other words, the preferred distribution P has relatively lower inequality below the individual's own income level y but relatively higher income inequality above the income y. That such a shape of the Lorenz curve is implied by the statement $P \succsim_{FSL_y} Q$ is formally confirmed below.

4. Fehr-Schmidt dominance

We seek to answer the question: Given two distributions $P, Q \in \Pi$, which would an individual with income $y_j \in \mathbf{Y}$ prefer? As with the standard approach to first and second order stochastic dominance, we first give purely statistical definitions (Definitions 6-8, below), then we establish their relevance to economics by relating them to the relevant economic model (here, the Fehr-Schmidt model); see Propositions 3-11, below.

4.1. First order Fehr-Schmidt dominance

First order FS_{y_j} dominance that we propose below, captures one sense in which an income distribution, P, is relatively preferred by an individual with income y_j.

Definition 6 *(First order Fehr-Schmidt dominance): Consider an individual with income $y_j \in \mathbf{Y}$. Let $P, Q \in \Pi$. Then P first order FS_{y_j} dominates Q ($P \succeq_{FS1_{y_j}} Q$) if*
(a) $P_i \leq Q_i$ for each $i = 1, 2, ..., j-1$, and
(b) $P_k \geq Q_k$ for each $k = j, j+1, ..., n-1$.
If, in addition, one of these inequalities is strict, then we say that P strictly first order FS_{y_j} dominates Q ($P \succ_{FS1_{y_j}} Q$).

In Figure 4.1 we have found it convenient to show the case $P \succeq_{FS1_y} Q$ when incomes are distributed continuously.[24] The following proposition establishes the economic foundations

[24] The continuous version of Definition 6 is given in Definition 12 below.

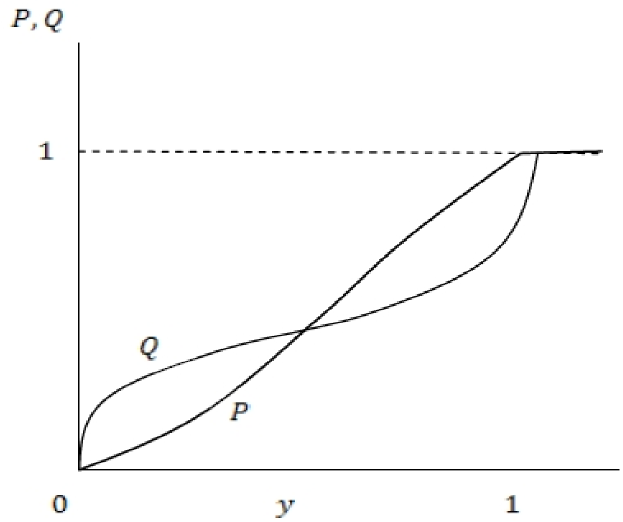

Figure 4.1: P first order FS$_y$ dominates Q

of first order Fehr-Schmidt dominance from the point of view of a decision maker with the FS utility function, U (Definition 2).

Proposition 3 *: Consider an individual with income $y_j \in \mathbf{Y}$. Let $P, Q \in \Pi$. Then P first order FS$_{y_j}$ dominates Q ($P \succeq_{FS1_{y_j}} Q$) if, and only if, $U(y_j, P) \geq U(y_j, Q)$ for all $u \in \mathbf{u}$, $\alpha \geq 0$ and $\beta \in [0, 1)$.*

Proof of Proposition 3: Suppose $P \succeq_{FS1_{y_j}} Q$, $y_j \in [0, 1]$, $u \in \mathbf{u}$, $\alpha \geq 0$ and $\beta \in [0, 1)$. From (2.7) and Definition 6, it follows that $U(y_j, P) \geq U(y_j, Q)$. This establishes the "only if" part.

Now to the "if" part. Let $P, Q \in \Pi$. Assume that $U(y_j, P) \geq U(y_j, Q)$ for all $u \in \mathbf{u}$, $\alpha \geq 0$ and $\beta \in [0, 1)$. We establish the result in two steps.

1. $P_i \leq Q_i$ for each $i = 1, 2, ..., j - 1$. *Proof*: Suppose not. Then $P_i > Q_i$ for some $i \in \{1, 2, ..., j - 1\}$. Hence, $j > 1$. Let $\alpha = 0$, $\beta = 0.5$. Let u be the utility function[25] $u(y) = 0$, $y \leq y_i$, $u(y) = 2$, $y > y_i$. From (2.7) we then get $U(y_j, P) - U(y_j, Q) = Q_i - P_i < 0$, which cannot be since $U(y_j, Q) \leq U(y_j, P)$.

2. $P_k \geq Q_k$ for each $k = j, j + 1, ..., n - 1$. *Proof*: Suppose not. Then $P_k < Q_k$ for some $k \in \{j, j + 1, ..., n - 1\}$. Hence, $j < n$. Let $\alpha = 1$, $\beta = 0$. Let u be the utility function $u(y) = 0$, $y \leq y_k$, $u(y) = 1$, $y > y_k$. From (2.7) we then get $U(y_j, P) - U(y_j, Q) = P_k - Q_k < 0$, which cannot be since $U(y_j, P) \geq U(y_j, Q)$. ■.

[25] Many other choices are also possible.

We now state the analogue of Proposition 3 when we have first order FS dominance in the strict sense.

Proposition 4 *Consider an individual with income $y_j \in \mathbf{Y}$. Let $P, Q \in \Pi$.*
(a) Suppose that $U(y_j, P) \geq U(y_j, Q)$ for all $u \in \mathbf{u}$, $\alpha \geq 0$ and $\beta \in [0,1)$, and suppose that $U(y_j, P) > U(y_j, Q)$ for some $u \in \mathbf{u}$, $\alpha \geq 0$ and $\beta \in [0,1)$, then P strictly first order FS_{y_j} dominates Q ($P \succ_{FS1_{y_j}} Q$).
(b) Suppose that P first order FS_{y_j} dominates Q ($P \succeq_{FS1_{y_j}} Q$). Let $u \in \mathbf{u}$. Suppose one of the following holds.
(bi) $\beta > 0$ and for some $i \in \{1, 2, ..., j-1\}$, $P_i < Q_i$ and $u(y_i) < u(y_{i+1})$, or
(bii) $\alpha > 0$ and for some $k \in \{j, j+1, ..., n-1\}$, $P_k > Q_k$ and $u(y_k) < u(y_{k+1})$.
Then $U(y_j, P) > U(y_j, Q)$.

Proof of Proposition 4:

(a) From Proposition 3, it follows that $P \succeq_{FS1_y} Q$. Hence, $P_i \leq Q_i$ for each $i = 1, 2, ..., j-1$ and $P_k \geq Q_k$ for each $k = j, j+1, ..., n-1$. Hence, if $P \succ_{FS1_y} Q$ did not hold, then we would have $P_i \leq Q_i$ for each $i = 1, 2, ..., j-1$ and $P_k \geq Q_k$ for each $k = j, j+1, ..., n-1$. From (2.7) we would then get $U(y_j, P) = U(y_j, Q)$, contrary to assumption. Hence, $P \succ_{FS1_y} Q$.

(b) From (2.7) we have

$$U(y_j, P) - U(y_j, Q) = \beta \sum_{i=1}^{i=j-1} (Q_i - P_i) \Delta u_i + \alpha \sum_{k=j}^{k=n-1} (P_k - Q_k) \Delta u_k, \qquad (4.1)$$

where, from Definition 6, $P_i \leq Q_i$ for each $i = 1, 2, ..., j-1$ and $P_k \geq Q_k$ for each $k = j, j+1, ..., n-1$; and where either (bi) or (bii) holds. From (4.1) it then follows that $U(y_j, P) > U(y_j, Q)$. ■.

4.2. Second order Fehr-Schmidt dominance

When individuals have FS preferences, if neither P nor Q first order FS dominate the other for an individual with income y_j, then we need further restrictions on the utility function, u to compare the distributions. We turn to this issue here.

Definition 7 *(Second order Fehr-Schmidt dominance): Let $P, Q \in \Pi$, $y_j \in \mathbf{Y}$. Then P second order FS_{y_j} dominates Q ($P \succeq_{FS2_{y_j}} Q$) if*
(a) Incomes are equally spaced (Definition 1),
(b) $\widetilde{P}_i \leq \widetilde{Q}_i$ for each $i = 1, 2, ..., j-1$ and
(c) $\widetilde{P}_k \geq \widetilde{Q}_k$ for each $k = j, j+1, ..., n-1$.
If, in addition, one of the inequalities in (b) or (c) is strict, then we say that P strictly second order FS_{y_j} dominates Q ($P \succ_{FS2_{y_j}} Q$).

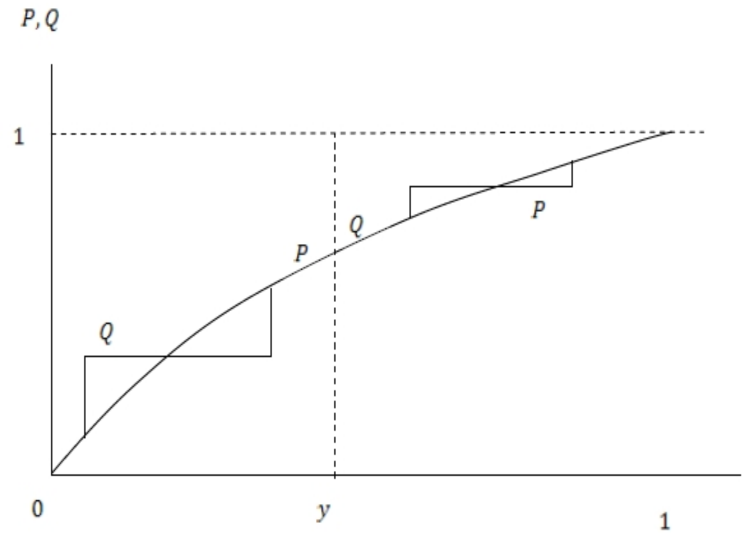

Figure 4.2: P second order FS dominates Q

Figure 4.2 gives an illustration of the case $P \succeq_{FS2_y} Q$ when incomes are continuously distributed.[26] Below the income level y, Figure 4.2 is intended to take a mean preserving spread of the distribution P to obtain the distribution Q and the converse takes place above the income level y.

Remark 2 *: From Definition 7, we get that* $P \succeq_{FS2_{y_j}} Q \Rightarrow \widetilde{P}_{n-1} \geq \widetilde{Q}_{n-1}$. *From Lemma 1 we get that* $\mu_P \leq \mu_Q$. *Contrast this to the converse restriction under second order stochastic dominance under selfish preferences,* $\mu_P \geq \mu_Q$.

Proposition 5 *: Let* $P, Q \in \Pi$, $y_j \in \mathbf{Y}$. *Let* P *second order* FS_{y_j} *dominate* Q *(*$P \succeq_{FS2_{y_j}} Q$*). Then* $U(y_j, P) \geq U(y_j, Q)$ *for all* $\alpha \geq 0$, $\beta \in [0, 1)$ *and all concave* $u \in \mathbf{u}$.

Proof of Proposition 5: Immediate from Result 1, Definition 7 and (2.8) of Corollary 1. ■.

The following Proposition establishes a partial converse of Proposition 5.

Proposition 6 *: Let* $P, Q \in \Pi$ *and* $y_j \in \mathbf{Y}$. *Suppose incomes are equally spaced (Definition 1). Suppose* $\widetilde{P}_{j-1} = \widetilde{Q}_{j-1}$.[27] *If* $U(y_j, P) \geq U(y_j, Q)$ *for all* $\alpha \geq 0$, *all* $\beta \in [0, 1)$ *and all concave* $u \in \mathbf{u}$, *then* P *second order* FS_{y_j} *dominates* Q *(*$P \succeq_{FS2_{y_j}} Q$*).*

[26] For the continuous analogue of Definition 7, see Definition 13.

[27] This condition is required to get rid of the term $\left(\widetilde{Q}_{j-1} - \widetilde{P}_{j-1}\right)(\alpha \Delta u_j + \beta \Delta u_{j-1})$ in (2.8).

Proof of Proposition 6: Since $\widetilde{P}_{j-1} = \widetilde{Q}_{j-1}$ we get, from (2.8) of Corollary 1, that

$$U(y_j, P) - U(y_j, Q) = \alpha \left(\widetilde{P}_{n-1} - \widetilde{Q}_{n-1} \right) \Delta u_{n-1}$$
$$+\beta \sum_{i=1}^{i=j-2} \left(\widetilde{P}_i - \widetilde{Q}_i \right) (\Delta u_{i+1} - \Delta u_i) + \alpha \sum_{k=j}^{k=n-2} \left(\widetilde{Q}_k - \widetilde{P}_k \right) (\Delta u_{k+1} - \Delta u_k). \tag{4.2}$$

We carry out the proof in three steps.

1. $\widetilde{P}_{n-1} \geq \widetilde{Q}_{n-1}$.
   Proof: Consider the utility function $u(y) = y$, which is clearly concave. Set $\alpha = 1$. Then (4.2) gives $U(y_j, P) - U(y_j, Q) = \left(\widetilde{P}_{n-1} - \widetilde{Q}_{n-1} \right) (y_n - y_{n-1})$. Since $y_n > y_{n-1}$ and $U(y_j, P) \geq U(y_j, Q)$, it follows that $\widetilde{P}_{n-1} \geq \widetilde{Q}_{n-1}$.

2. $\widetilde{P}_i \leq \widetilde{Q}_i$ for each $i = 1, 2, ..., j-1$.
   Proof: Suppose not. Then $\widetilde{Q}_i < \widetilde{P}_i$ for some $i \in \{1, 2, ..., j-1\}$. Since $\widetilde{P}_{j-1} = \widetilde{Q}_{j-1}$, we must have $i < j - 1$. Set $\beta = \frac{1}{2}$ and define concave u as follows.

$$\begin{aligned} u(y) &= y,\ y \leq y_{i+1}, \\ u(y) &= y_{i+1},\ y \geq y_{i+1}. \end{aligned} \tag{4.3}$$

   Since incomes are equally spaced (Definition 1) it follows from, (4.2) and (4.3), that $U(y_j, P) - U(y_j, Q) = \frac{1}{2} \left(\widetilde{Q}_i - \widetilde{P}_i \right) (y_{i+1} - y_i) < 0$, which cannot be, since $U(y_j, P) \geq U(y_j, Q)$.

3. $\widetilde{P}_k \geq \widetilde{Q}_k$ for each $k = j, j+1, ..., n-1$.
   Proof: Suppose not. Then $\widetilde{P}_k < \widetilde{Q}_k$ for some $k \in \{j, j, ..., n-1\}$. Since $\widetilde{P}_{n-1} \geq \widetilde{Q}_{n-1}$ (proved above), it follows that $k < n - 1$. Let $\alpha = 1$ and $\beta = 0$. Define concave u as follows.

$$\begin{aligned} u(y) &= y,\ y \leq y_{k+1}, \\ u(y) &= y_{k+1},\ y \geq y_{k+1}. \end{aligned} \tag{4.4}$$

   Then (4.2) and (4.4) give $U(y_j, P) - U(y_j, Q) = \left(\widetilde{P}_k - \widetilde{Q}_k \right) (y_{k+1} - y_k) < 0$, which cannot be since $U(y_j, P) - U(y_j, Q) \geq 0$. ■.

Strict analogues of Propositions 5 and 6 are also possible.

Proposition 7 : *Consider an individual with income $y_j \in \mathbf{Y}$.*
(a) Suppose that P strictly second order FS_{y_j} dominates Q ($P \succ_{FS2_{y_j}} Q$). Let $\alpha > 0$, $\beta > 0$ and let $u \in \mathbf{u}$ be strictly increasing and strictly concave. Then $U(y_j, P) > U(y_j, Q)$.

(b) Assume incomes are equally spaced (Definition 1). Assume $\widetilde{P}_{j-1} = \widetilde{Q}_{j-1}$. *Suppose that* $U(y_j, P) \geq U(y_j, Q)$ *for all* $\alpha \geq 0$, *all* $\beta \in [0,1)$ *and all concave* $u \in \mathbf{u}$. *Suppose that, in addition,* $U(y_j, P) > U(y_j, Q)$ *for some* $\alpha \geq 0$, *some* $\beta \in [0,1)$ *and some concave* $u \in \mathbf{u}$. *Then* P *strictly second order* FS_{y_j} *dominates* Q ($P \succ_{FS2_{y_j}} Q$).

Proof of Proposition 7:

(a) The result immediately follows from (2.8) of Corollary 1 and Definition 7.

(b) From Proposition 6, it follows that $P \succeq_{FS2_{y_j}} Q$. It then follows that each term on the right hand side of (2.8) of Corollary 1 is non-negative. If P did not strictly second order FS_{y_j} dominate Q then we would get $U(y_j, P) = U(y_j, Q)$ for all $\alpha \geq 0$, all $\beta \in [0,1)$ and all concave u. This is not the case. Hence, $P \succ_{FS2_{y_j}} Q$. ■.

4.3. Weak Fehr-Schmidt dominance

Definition 8, immediately below, proposes a concept of dominance *weak Fehr-Schmidt dominance* that is suited to the linear form of Fehr-Schmidt preferences (Definition 3). A formal justification is given by Proposition 8, further below. It is strictly weaker than both first order Fehr-Schmidt dominance and second order Fehr-Schmidt dominance (Propositions 9 and 11, below).

Definition 8 *(Weak Fehr-Schmidt dominance): Let* $y_j \in \mathbf{Y}$ *and* $P, Q \in \Pi$. *Then* P *weakly* FS_{y_j} *dominates* Q ($P \succeq_{WFS_{y_j}} Q$), *if*

$$\sum_{i=1}^{j-1} p_i (y_j - y_i) \leq \sum_{i=1}^{j-1} q_i (y_j - y_i), \tag{4.5}$$

$$\sum_{k=j+1}^{n} p_k (y_k - y_j) \leq \sum_{k=j+1}^{n} q_k (y_k - y_j). \tag{4.6}$$

If, in addition, at least one of these inequalities is strict, then P *strictly weakly* FS_{y_j} *dominates* Q ($P \succ_{WFS_{y_j}} Q$).[28]

The following proposition is clear from Definitions 3 and 8.

Proposition 8 *: Consider the linear version of Fehr-Schmidt utility, (2.4) of Definition 3. Let* $y_j \in \mathbf{Y}$ *and* $P, Q \in \Pi$. *Then* $P \succeq_{WFS_{y_j}} Q$ *(Definition 8) if, and only if,* $U(y_j, P) \geq U(y_j, Q)$ *for all* $\alpha \geq 0$ *and all* $\beta \in [0,1)$. *Furthermore,* $P \succ_{WFS_{y_j}} Q$ *if, and only if,* $U(y_j, P) \geq U(y_j, Q)$ *for all* $\alpha \geq 0$ *and all* $\beta \in [0,1)$ *and* $U(y_j, P) > U(y_j, Q)$ *for some* $\alpha \geq 0$ *or some* $\beta \in [0,1)$.

[28] One may use the alternative terminology: "*P weakly* FS_{y_j} *dominates Q in the strict sense*".

Proposition 9 : *Let $y_j \in \mathbf{Y}$ and $P, Q \in \Pi$. If P first order FS_{y_j} dominates Q, then P weakly FS_{y_j} dominates Q.*

Proof of Proposition 9: Let $y_j \in \mathbf{Y}$ and $P, Q \in \Pi$. Suppose P first order FS_{y_j} dominates Q. From Proposition 3 it then follows that $U(y_j, P) \geq U(y_j, Q)$ for all $u \in \mathbf{u}$, $\alpha \geq 0$ and $\beta \in [0, 1)$. In particular, it follows that $U(y_j, P) \geq U(y_j, Q)$ for $u(y) = y$, all $\alpha \geq 0$ and all $\beta \in [0, 1)$. From Proposition 8 it then follows that $P \succeq_{WFS_{y_j}} Q$. ■.

Proposition 10 : *Let $y_j \in \mathbf{Y}$ and $P, Q \in \Pi$. If P strictly first order FS_{y_j} dominates Q, then P strictly weakly FS_{y_j} dominates Q.*

Proof of Proposition 10: Let $y_j \in \mathbf{Y}$ and $P, Q \in \Pi$. Suppose P strictly first order FS_{y_j} dominates Q. Hence, P first order FS_{y_j} dominates Q. From Proposition 9 it follows that P weakly FS_{y_j} dominates Q. From Proposition 8 it follows that for $u(y) = y$, $U(y_j, Q) \leq U(y_j, P)$ for all $\alpha \geq 0$ and all $\beta \in [0, 1)$. Since P strictly first order FS_{y_j} dominates Q, it also follows that (a) and (b) of Definition 6 hold, with one of them being strict. If (a) is strict, choose $\beta = \frac{1}{2}$. Take $u(y) = y$, then $u(y_i) = y_i < y_{i+1} = u(y_{i+1})$. Hence, condition (bi) of Proposition 4 holds and, hence, $U(y_j, P) > U(y_j, Q)$. If (b) of Definition 6 is strict, choose $\alpha = 1$. Take $u(y) = y$, then $u(y_k) = y_k < y_{k+1} = u(y_{k+1})$. Hence, condition (bii) of Proposition 4 holds and, hence, again, $U(y_j, P) > U(y_j, Q)$. From Proposition 8 it then follows that P strictly weakly FS_{y_j} dominates Q. ■.

Proposition 11 : *Let $y_j \in \mathbf{Y}$ and $P, Q \in \Pi$. If P second order FS_{y_j} dominates Q, then P weakly FS_{y_j} dominates Q.*

Proof of Proposition 11: Let $P, Q \in \Pi$, $y_j \in \mathbf{Y}$. Suppose incomes are equally spaced (Definition 1). Let P second order FS_{y_j} dominate Q ($P \succeq_{FS2_{y_j}} Q$). From Proposition 5 it then follows that $U(y_j, P) \geq U(y_j, Q)$ for all $\alpha \geq 0$, all $\beta \in [0, 1)$ and all concave u. In particular, it follows that $U(y_j, P) \geq U(y_j, Q)$ for $u(y) = y$, all $\alpha \geq 0$ and all $\beta \in [0, 1)$. From Proposition 8 it then follows that $P \succeq_{WFS_{y_j}} Q$. ■.

The converses of Propositions 9, 11 do not hold because weak FS_{y_j} dominance applies only to the class of linear functions while first and second order FS_{y_j} dominance hold for all $u \in \mathbf{u}$.

Remark 3 : *Although Proposition 10 is a strict version of Proposition 9, there is no strict version of Proposition 11. The reason is that such a proposition would require the own utility function, u, to be strictly concave. But this is not possible for the linear version of Fehr-Schmidt preferences (Definition 3).*

5. A note on dominance concepts when individuals have pride rather than altruism

Although our main focus is on Fehr-Schmidt preferences ($\beta \geq 0$), here we briefly consider the consequences of $\beta < 0$ for our dominance concepts (see section 2.3, above). Due to the similarity of the proofs with the case of $\beta \geq 0$ we give the proof only for the first proposition below (Proposition 12). In the context of stochastic dominance, the main insights when preferences exhibit pride (see (2.10)) are twofold.

1. The traditional concepts of stochastic dominance, first and second order dominance, suffice.

2. Individuals with preferences in (2.10) prefer the distribution that is stochastically *dominated*.

In our first proposition we show that an individual with the utility function $U(y_j, P)$ with $\beta < 0$ would prefer a distribution that is first order stochastically dominated by another.

Proposition 12 : *Consider an individual with income $y_j \in \mathbf{Y}$. Let $P, Q \in \Pi$. Then P first order stochastically dominates Q ($P \succeq_1 Q$) if, and only if, $U(y_j, P) \leq U(y_j, Q)$ for all $u \in \mathbf{u}$, all $\alpha \geq 0$ and all $\beta < 0$.*

Proof of Proposition 12 : Suppose $P \succeq_1 Q$, $y_j \in [0,1]$, $u \in \mathbf{u}$, $\alpha \geq 0$ and $\beta < 0$. From Definition 4 we have $P_i \leq Q_i$ for $i = 1, 2, ..., n$. From (2.7) it follows that $U(y_j, P) \leq U(y_j, Q)$. This establishes the "only if" part.

Now to the "if" part. Let $P, Q \in \Pi$. Assume that $U(y_j, P) \leq U(y_j, Q)$ for all $u \in \mathbf{u}$, $\alpha \geq 0$ and $\beta < 0$. We establish the result in two steps.

1. $P_i \leq Q_i$ for each $i = 1, 2, ..., j-1$. *Proof*: Suppose not. Then $P_i > Q_i$ for some $i \in \{1, 2, ..., j-1\}$. Hence, $j > 1$. Let $\alpha = 0$, $\beta = -0.5$. Let u be the utility function $u(y) = 0$, $y \leq y_i$, $u(y) = 2$, $y > y_i$. From (2.7) we then get $U(y_j, P) - U(y_j, Q) = P_i - Q_i > 0$, which cannot be since $U(y_j, P) \leq U(y_j, Q)$.

2. $P_k \leq Q_k$ for each $k = j, j+1, ..., n-1$. *Proof*: Suppose not. Then $P_k > Q_k$ for some $k \in \{j, j+1, ..., n-1\}$. Hence, $j < n$. Let $\alpha = 1$, $\beta < 0$. Let u be the utility function $u(y) = 0$, $y \leq y_k$, $u(y) = 1$, $y > y_k$. From (2.7) we then get $U(y_j, P) - U(y_j, Q) = P_k - Q_k > 0$, which cannot be since $U(y_j, P) \leq U(y_j, Q)$. $\blacksquare$.

We now state the analogous result for second order stochastic dominance. The proof uses techniques similar to those used for Propositions 5 and 6. Hence, it is omitted.

Proposition 13 : *Let* $P, Q \in \Pi$ *and* $y_j \in \mathbf{Y}$. *Suppose incomes are equally spaced (Definition 1). Suppose* $\widetilde{P}_{j-1} = \widetilde{Q}_{j-1}$.[29] *Then* $U(y_j, P) \geq U(y_j, Q)$ *for all* $\alpha \geq 0$, *all* $\beta < 0$ *and all concave* $u \in \mathbf{u}$ *if, and only if,* P *second order stochastically dominates* Q $(P \succeq_2 Q)$.

Strict analogues of Propositions 12, 13 can also be constructed. The standard dominance concepts suffice in this case too.

6. A continuous formulation and Lorenz dominance

So far, we have followed the discrete formulation of section 2. This has the advantage of simplicity and closeness to the laboratory situation where, typically, experimental subjects face discrete outcomes. However, a continuous formulation is more convenient for some purposes[30] and it is essential for the discussion of Lorenz dominance. A Lorenz curve can, of course, be drawn even in the discrete case, by joining up a set of point in a suitable way. For example, by a polygon or step function (see, for example, Lambert 2001). However, the equivalence of *generalized Lorenz dominance* to *second order stochastic dominance* in the traditional case with selfish preferences can only be demonstrated in a continuous formulation; see Shorrocks (1983) and Lambert (2001).

The reason for a continuous formulation is as follows. For any $p \in [0, 1]$, we first find an income level y such the proportion of the population that has at least the income y is the bottom $100p$ percent of the population. This is found from $F(y) = p$ (so $y = F^{-1}(p)$, if $F(y)$ were invertible, which need not be the case). In general, this is assured only if F is a continuous distribution. The Lorenz curve $L_F(p)$ under the distribution F then requires us to find the fraction of the total income of the bottom $100p$ percent of the population for all p. This is given by

$$L_F(p) = \frac{N \int_{x=-\infty}^{y} x dF(x)}{N \int_{x=-\infty}^{+\infty} x dF(x)} = \frac{\int_{x=-\infty}^{y} x dF(x)}{\mu_F}.$$

The generalized Lorenz curve is then given by

$$\Gamma(p) = \mu_F L_F(p)$$

The great advantage of generalized Lorenz dominance is that it can often rank distributions which cannot be ranked under Lorenz dominance. As an example of this, consider for instance, two distributions, F, G and the two cases given in Lambert (2001, p.51). (1) $\mu_F > \mu_G$, $L_F \leq L_G$ but $\Gamma_F \geq \Gamma_G$. (2) L_F and L_G cross but Γ_F and Γ_G do not cross. In

[29] This condition is required to get rid of the term $\left(\widetilde{Q}_{j-1} - \widetilde{P}_{j-1}\right)(\alpha \Delta u_j + \beta \Delta u_{j-1})$ in (2.8).

[30] For example, see Deaton (2003).

each case one can rank the distributions F, G by generalized Lorenz dominance but not by Lorenz dominance.

We propose a modified version of Lorenz dominance, which we call *Fehr-Schmidt-Lorenz dominance*, and show that it is equivalent to a slightly strengthened version of second order Fehr-Schmidt dominance, which we call *strong Fehr-Schmidt dominance*. See subsections 6.7, 6.8, below.

For ease of reference, subsection 6.2, below, gives some standard results on second order stochastic dominance and generalized Lorenz dominance.

Proofs are omitted when they are similar to their discrete counterparts.

6.1. Preliminaries

Let the lowest income be 0 and the highest income be 1. We have a continuum of individuals located on $[0,1]$. Let Π be the set of all *continuous* distribution, $P : [0,1] \to [0,1]$.[31] For $y \in [0,1]$, let $\widetilde{P}(y) = \int_{x=0}^{y} dP(x)$ be the cumulative of the cumulative distribution. Let $\mathbf{u}$ be the set of bounded, non-decreasing, right-continuous utility functions, $u : [0,1] \to \mathbb{R}$. Let $\mathbf{u}^1$ be the set of all continuous, non-decreasing utility functions, $u \in \mathbf{u}$, with right-sided derivative, u'. In particular, we have:

Result 2 *: (a) Let $u \in \mathbf{u}$ be concave or convex on some non-empty interval, then $u \in \mathbf{u}^1$ and $u' \geq 0$ in that interval. If u is strictly increasing, then $u' > 0$.*
(bi) If u is concave on some interval, then u' is non-increasing on that interval. If u is strictly concave, then u' is decreasing.[32]
(bii) If u is convex on some interval, then u' is non-decreasing on that interval. If u is strictly convex, then u' is increasing.

Let $F : [0,1] \to \mathbb{R}$ and $G : [0,1] \to \mathbb{R}$. If one of F or G is continuous and the other is monotonic, then the following integration by parts formula is valid.[33]

$$\int_{x=h}^{z} G(x)\, dF = F(z) G(z) - F(h) G(h) - \int_{x=h}^{z} F(x)\, dG(x), \tag{6.1}$$

where $h, z \in [0,1]$, and one of F or G is continuous and the other is monotonic.

Lemma 3 *: Let $P, Q \in \Pi$. Then $\mu_P = 1 - \widetilde{P}(1)$ and $\mu_Q = 1 - \widetilde{Q}(1)$. Hence, $\mu_P \lesseqqgtr \mu_Q$ if, and only if, $\widetilde{P}(1) \gtreqqless \widetilde{Q}(1)$.*

Proof of Lemma 3: Similar to that of Lemma 1, except that we use (6.1) in place of (2.1). ■.

[31] Hence, P is continuous, non-decreasing, $P(0) = 0$ and $P(1) = 1$.
[32] Note that we do not impose continuity of u'. Thus u'' may fail to exist at some points in the domain.
[33] See, for example, Widder (1947), Theorem 2, chapter 5, section 3, pp. 159/160. The theorem is stated for continuous or non-decreasing functions. But if F, say, is non-increasing, then we can simply apply (6.1) with F replaced by $-F$.

6.2. Some standard results

Definition 9 *(Second order stochastic dominance): Let $P, Q \in \Pi$. Then P second order stochastically dominates Q ($P \succeq_2 Q$) if $\widetilde{P}(x) \leq \widetilde{Q}(x)$ for all $x \in [0,1]$. If, in addition, $\widetilde{P}(x) < \widetilde{Q}(x)$ for some $x \in [0,1]$ then P strictly second order stochastically dominates Q ($P \succ_2 Q$).*

Prior to defining Lorenz curves and generalized Lorenz curves (Definition 10), we give a preliminary result.

Lemma 4 *: Let $F \in \Pi$. Let $y, z \in [0,1]$, $y < z$. Suppose $F(y) = F(z) = p \in [0,1]$. Then $\int_{x=0}^{y} xdF = \int_{x=0}^{z} xdF$.*

Proof of Lemma 4: We have $\int_{x=0}^{z} xdF = \int_{x=0}^{y} xdF + \int_{x=y}^{z} xdF$. Since $F(y) = F(z)$ and since F is non-decreasing, it follows that $dF = 0$ on $[y, z]$. Hence, $\int_{x=y}^{z} xdF = 0$. Hence, $\int_{x=0}^{z} xdF = \int_{x=0}^{y} xdF$. ■.

Definition 10 *(Lorenz and generalized Lorenz curves): Let $F \in \Pi$. Then the generalized Lorenz curve, Γ_F, of F is the mapping $\Gamma_F : [0,1] \rightarrow, \mathbb{R}$ defined as follows. Let $p \in [0,1]$. Let $F(y) = p$. Then $\Gamma_F(p) = \int_{x=0}^{y} xdF(x)$. The Lorenz curve of F is then Γ_F / μ_F.*

Remark 4 *: Definition 10 is sound for the following two reasons. First, since $F \in \Pi$ is a continuous distribution it follows that for any $p \in [0,1]$ there will be a $y \in [0,1]$ such that $F(y) = p$. Second, although such a y may not be unique that does not matter. For if $F(y) = F(z) = p$ then, by Lemma 4, $\int_{x=0}^{y} xdF = \int_{x=0}^{z} xdF$.*

Definition 11 *(Lorenz and generalized Lorenz dominance): Let $F, G \in \Pi$. Then F generalized Lorenz dominates G ($F \succeq_L G$) if $\Gamma_F(p) \geq \Gamma_G(p)$, for all $p \in [0,1]$ and if $\mu_F \geq \mu_G$.*[34]*. If, addition, $\Gamma_F(p) > \Gamma_G(p)$ for some $p \in [0,1]$ or if $\mu_F > \mu_G$, then F strictly generalized Lorenz dominates G ($F \succ_L G$). If $\mu_F = \mu_G$, then Lorenz dominance and generalized Lorenz dominance coincide.*

We now state some well known results due to Kolm (1969) and Shorrocks (1983).

Result 3 *(Equivalence of generalized Lorenz dominance and second order stochastic dominance): Let $F, G \in \Pi$. Then F (strictly) generalized Lorenz dominates G if, and only if, F (strictly) second order stochastically dominates G.*

[34] Often the condition $\mu_F \geq \mu_F$ is not included as part of the definition of generalized Lorenz dominance. But then it needs to be included in the statement of the relevant propositions. We found it slightly simpler to include $\mu_F \geq \mu_G$ as part of the definition of generalized Lorenz dominance.

We now state a well known result and offer a proof of a part of the result because we consider a slightly more general case.

Result 4 : *Let $F, G \in \Pi$. Then the following are equivalent:*
(a) F second order stochastically dominates G.
(b) F generalized Lorenz dominates G.
(c) An expected utility maximizer with any non-decreasing concave utility function prefers F to G.

Proof of Result 4: The proof of the equivalence of (a) and (c) that is usually given in textbooks is for the special case of equal means. So here we give a proof that does not require that. Let $U(F) = \int_{x=0}^{1} u(x)\,dF(x)$ and $U(G) = \int_{x=0}^{1} u(x)\,dG(x)$. Integrating by parts, twice, and using Lemma 3, we get[35]

$$U(F) - U(G) = (\mu_F - \mu_G)\,u'(1) - \int_{x=0}^{1} \left[\widetilde{G}(x) - \widetilde{F}(x)\right] du'(x). \tag{6.2}$$

Suppose (a) holds, so that $\widetilde{G}(x) \geq \widetilde{F}(x)$ for all $x \in [0,1]$. In particular, for $x = 1$ it follows form Lemma 3 that $\mu_F \geq \mu_G$. Let $u(x)$ be non-decreasing, so that $u' \geq 0$, and concave, so that u' is non-increasing. Then, from (6.2), we get $U(F) \geq U(G)$. Thus, (a) implies (c). Conversely, suppose (c) holds. First, choose $u(x) = x$, then $u'(x) = 1$ and, hence, (6.2) gives $U(F) - U(G) = \mu_F - \mu_G$. Hence, if $U(F) \geq U(G)$, then $\mu_F \geq \mu_G$. Next, suppose $\widetilde{G}(b) < \widetilde{F}(b)$ for some $a \in [0,1]$. Since, $\widetilde{G}(0) = \widetilde{F}(0)$ $(= 0)$, it follows that $b \in (0,1]$. Since $\widetilde{F}$ and $\widetilde{G}$ are continuous (because, by assumption, F and G are continuous), it follows that, for some $a \in [0,b)$, $\widetilde{G}(x) < \widetilde{F}(x)$ for all $x \in (a,b)$. Choose u to be continuous, linear below a, strictly concave between a and b and constant at b and above.[36] (6.2) then gives $U(F) - U(G) = -\int_{x=a}^{b} \left[\widetilde{G}(x) - \widetilde{F}(x)\right] du'(x) < 0$; which cannot be if $U(F) \geq U(G)$. Hence, (c) implies (a). ■.

[35] In detail, $U(F) - U(G) = \int_{x=0}^{1} u(x)\,dF - \int_{x=0}^{1} u(x)\,dG = \int_{x=0}^{1} u(x)\,d(F-G) = u(1)[F(1) - G(1)] - u(0)[F(0) - G(0)] - \int_{x=0}^{1} [F(x) - G(x)]\,du(x) = -\int_{x=0}^{1} [F(x) - G(x)]\,du(x) = -\int_{x=0}^{1} u'(x)\,d\left[\widetilde{F}(x) - \widetilde{G}(x)\right] = -\left\{u'(1)\left[\widetilde{F}(1) - \widetilde{G}(1)\right] - u'(0)\left[\widetilde{F}(0) - \widetilde{G}(0)\right] - \int_{x=0}^{1} \left[\widetilde{F}(x) - \widetilde{G}(x)\right] du'(x)\right\} = u'(1)\left[\widetilde{G}(1) - \widetilde{F}(1)\right] - \int_{x=0}^{1} \left[\widetilde{G}(x) - \widetilde{F}(x)\right] du'(x) = (\mu_F - \mu_G)\,u'(1) - \int_{x=0}^{1} \left[\widetilde{G}(x) - \widetilde{F}(x)\right] du'(x)$.

[36] For example,

$$\begin{aligned} u(x) &= \left(1 - \frac{a}{b}\right) x + \frac{a^2}{2b}, x \leq a, \\ u(x) &= x - \frac{1}{2b} x^2, x \in [a,b], \\ u(x) &= \frac{1}{2} b, x \geq b. \end{aligned}$$

Result 5 : *Let $F, G \in \Pi$. Then the following are equivalent.*
(a) F strictly second order stochastically dominates G,
(b) F strictly generalized Lorenz dominates G,
(c) An expected utility maximizer with any strictly increasing strictly concave utility function strictly prefers F to G.

6.3. Fehr-Schmidt preferences: A continuous formulation

The continuous analogue of Fehr-Schmidt preferences ((2.2) in Definition 2) is

$$U(y,P) = u(y) - \beta \int_{x=0}^{y} [u(y) - u(x)]\, dP(x) - \alpha \int_{z=y}^{1} [u(z) - u(y)]\, dP(z), \qquad (6.3)$$

$\alpha \geq 0, 0 \leq \beta < 1$, $u \in \mathbf{u}$. The linear version of FS preferences is then

$$U(y,P) = y - \beta \int_{x=0}^{y} (y - x)\, dP(x) - \alpha \int_{z=y}^{1} (z - y)\, dP(z),\ \alpha \geq 0, 0 \leq \beta < 1. \qquad (6.4)$$

Using (6.1), and Lemma 3, we get the continuous analogues of (2.7) and (2.8) of Corollary 1:

$$\text{(a) } U(y,P) - U(y,Q) = \beta \int_{x=0}^{y} [Q(x) - P(x)]\, du(x) + \alpha \int_{z=y}^{1} [P(z) - Q(z)]\, du(z), \qquad (6.5)$$

where $\alpha \geq 0, 0 \leq \beta < 1$, $u \in \mathbf{u}$.

$$\text{(b) } U(y,P) - U(y,Q) = \alpha \left(\mu_Q - \mu_P\right) u'(1) + (\alpha + \beta) \left[\widetilde{Q}(y) - \widetilde{P}(y)\right] u'(y)$$
$$+ \beta \int_{x=a}^{y} \left[\widetilde{P}(x) - \widetilde{Q}(x)\right] du'(x) + \alpha \int_{z=y}^{1} \left[\widetilde{Q}(z) - \widetilde{P}(z)\right] du'(z), \qquad (6.6)$$

where $\alpha \geq 0, 0 \leq \beta < 1$, $u \in \mathbf{u}^1$.

6.4. First order Fehr-Schmidt dominance

Definition 12 *(First order Fehr-Schmidt dominance): Consider an individual with income $y \in [a,b]$. Let $P, Q \in \Pi$. Then P first order FS_y dominates Q ($P \succeq_{FS1_y} Q$) if the following conditions are met.*
(a) $P(x) \leq Q(x)$ for all $x \in [0,y]$,
(b) $P(z) \geq Q(z)$ for all $z \in [y,1]$.
If, in addition, (a) is strict for some $x \in [0,y]$ or (b) is strict for some $z \in [y,1]$, then we say that P strictly first order FS_y dominates Q ($P \succ_{FS1} Q$).

Proposition 14 : *Consider an individual with income $y \in [0,1]$. Let $P, Q \in \Pi$. Then P first order FS$_y$ dominates Q ($P \succeq_{FS1_y} Q$) if, and only if, $U(y,P) \geq U(y,Q)$ for all $u \in \mathbf{u}$, $\alpha \geq 0$ and $\beta \in [0,1)$.*

Proposition 15 : *Consider an individual with income $y \in [0,1]$. Let $P, Q \in \Pi$.*
(a) Suppose that $U(y,P) \geq U(y,Q)$ for all $u \in \mathbf{u}$, $\alpha \geq 0$ and $\beta \in [0,1)$, and suppose that $U(y,P) > U(y,Q)$ for some $u \in \mathbf{u}$, $\alpha \geq 0$ and $\beta \in [0,1)$, then P strictly first order FS$_{y_j}$ dominates Q ($P \succ_{FS1_y} Q$).
(b) Suppose that P first order FS$_y$ dominates Q ($P \succeq_{FS1_y} P$). Let $u \in \mathbf{u}$. Suppose one of the following holds.
(bi) $\beta > 0$ and for some $x \in [0,y]$, $P(x) < Q(x)$ and u is strictly increasing at x, or
(bii) $\alpha > 0$ and for some $z \in [y,1]$, $P(z) > Q(z)$ and u is strictly increasing at z.
Then $U(y,P) > U(y,Q)$.

6.5. Second order Fehr-Schmidt dominance

Definition 13 *(Second order Fehr-Schmidt dominance): Let $P, Q \in \Pi$, $y \in [0,1]$. Then P second order FS$_y$ dominates Q ($P \succeq_{FS2_y} Q$) if the following two conditions hold.*
(a) $\widetilde{P}(x) \leq \widetilde{Q}(x)$ for all $x \in [0,y]$,
(b) $\widetilde{P}(z) \geq \widetilde{Q}(z)$ for all $z \in [y,1]$.
If, in addition, (a) is strict for some $x \in [0,y]$ or some $z \in [y,1]$, then we say that P strictly second order FS$_y$ dominates Q ($P \succ_{FS2_y} Q$).[37]

Result 6 : *Using Lemma 3, condition (b) in Definition 13 implies that $\mu_Q \geq \mu_P$.*

Proposition 16 : *Let $P, Q \in \Pi$, $y \in [0,1]$. Let P second order FS$_y$ dominate Q ($P \succeq_{FS2_y} Q$). Then $U(y,P) \geq U(y,Q)$ for all $\alpha \geq 0$, $\beta \in [0,1)$ and all concave $u \in \mathbf{u}$.*

Proof of Proposition 16: Immediate from Results 2, 6, Definition 13 and (6.6). ■.

Proposition 17 : *Let $P, Q \in \Pi$ and $y \in [0,1]$. Suppose that $\widetilde{P}(y) = \widetilde{Q}(y)$. If $U(y,P) \geq U(y,Q)$ for all $\alpha \geq 0$, all $\beta \in [0,1)$ and all concave $u \in \mathbf{u}$ then P second order FS$_y$ dominates Q ($P \succeq_{FS2_y} Q$).*

[37] There is a superficial resemblance to *Markowitz stochastic dominance* (MSD) introduced by Levy and Levy (2002). In our terminology, *P Markowitz stochastically dominates Q* if $\widetilde{P}(x) \leq \widetilde{Q}(x)$ for all $x \in [0,y]$ and $\widetilde{P}(z) + \mu_P \geq \widetilde{Q}(z) + \mu_Q$ for all $z \in [y,1]$; from which it follows that $\mu_P \geq \mu_Q$. However, if $P \succeq_{FS2_y} Q$, then $\mu_P \leq \mu_Q$ (Result 6, below). Furthermore, the context is different. Levy and Levy consider prospect theory with selfish preferences, while we consider Fehr-Schmidt preferences. Moreover, Levy and Levy (2002) ignore non-linear transformation of probabilities, an essential component of prospect theory. Finally, even the relevance of MSD for prospect theory is in doubt. Levy and Levy assume the value function is concave for losses but convex for gains, which is exactly the opposite of the evidence, even according to their own data (see Wakker, 2003).

We now consider the strict version of Proposition 17.

Proposition 18 : *Consider an individual with income $y \in [0,1]$.*
(a) Suppose that P strictly second order FS$_y$ dominates Q ($P \succ_{FS2_y} Q$). Let $\alpha > 0$, $\beta > 0$ and let $u \in \mathbf{u}$ be strictly increasing and strictly concave. Then $U(y,P) > U(y,Q)$.
(b) Assume $\widetilde{P}(y) = \widetilde{Q}(y)$. Suppose that $U(y,P) \geq U(y,Q)$ for all $\alpha \geq 0$, all $\beta \in [0,1)$ and all concave $u \in \mathbf{u}$. Suppose that, in addition, $U(y,P) > U(y,Q)$ for some $\alpha \geq 0$, some $\beta \in [0,1)$ and some concave $u \in \mathbf{u}$. Then P strictly second order FS$_y$ dominates Q ($P \succ_{FS2_y} Q$).

6.6. Weak Fehr-Schmidt dominance

We introduced the concept of 'weak Fehr-Schmidt dominance' and its implications in section 4.3 for the discrete case. We now provide the analogous results for the continuous case.

Definition 14 *(Weak Fehr-Schmidt dominance): Let $y \in [0,1]$ and $P, Q \in \Pi$. Then P weakly FS$_y$ dominates Q ($P \succeq_{WFS_y} Q$), if*

$$\int_{x=0}^{y} (y-x)\, dP(x) \leq \int_{x=0}^{y} (y-x)\, dQ(x)\,, \tag{6.7}$$

$$\int_{z=y}^{1} (z-y)\, dP(z) \leq \int_{z=y}^{1} (z-y)\, dQ(z)\,. \tag{6.8}$$

If, in addition, at least one of these inequalities is strict, then P strictly weakly FS$_y$ dominates Q ($P \succ_{WFS_y} Q$).

Proposition 19 : *Consider the linear version of Fehr-Schmidt utility (2.4). Let $y \in [0,1]$ and $P, Q \in \Pi$. Then $P \succeq_{WFS_y} Q$ (see Definition 14) if, and only if, $U(y,P) \geq U(y,Q)$ for all $\alpha \geq 0$ and all $\beta \in [0,1)$. Furthermore, $P \succ_{WFS_y} Q$ if, and only if, $U(y,P) \geq U(y,Q)$ for all $\alpha \geq 0$ and all $\beta \in [0,1)$ and $U(y,P) > U(y,Q)$ for some $\alpha \geq 0$ or some $\beta \in [0,1)$.*

Proposition 20 : *Let $y \in [0,1]$ and $P, Q \in \Pi$. If P (strictly) first order FS$_y$ dominates Q, then P (strictly) weakly FS$_y$ dominates Q.*

Proposition 21 : *Let $y \in [0,1]$ and $P, Q \in \Pi$. If P second order FS$_y$ dominates Q, then P weakly FS$_y$ dominates Q.*

6.7. Strong Fehr-Schmidt dominance

We now give a dominance concept for FS preferences, strong FS_y dominance, that is suitable for establishing a link with an appropriate version of Lorenz dominance. It is identical to second order FS_y dominance except that we require $P(y) = Q(y)$, i.e., the proportion of individuals who are poorer relative to individual y are identical under the two comparison distributions.

Definition 15 *(Strong Fehr-Schmidt dominance): Let $P, Q \in \Pi$, $y \in [0,1]$. Then P strongly FS$_y$ dominates Q ($P \succeq_{SFS_y} Q$) if the following two conditions hold.*
(a) P second order FS$_y$ dominates Q ($P \succeq_{FS2_y} Q$),
(b) $P(y) = Q(y)$.
If, in addition, P strictly second order FS$_y$ dominates Q ($P \succ_{FS2_y} Q$), then we say that P strictly strongly FS$_y$ dominates Q ($P \succ_{SFS_y} Q$).

We now give the relation between first order FS dominance and strong FS dominance.

Proposition 22 *: Let $P, Q \in \Pi$, $y \in [0,1]$. Let P first order FS$_y$ dominate Q ($P \succeq_{FS1_y} Q$). Suppose that, in addition, $\widetilde{P}(y) = \widetilde{Q}(y)$. Then P strongly FS$_y$ dominates Q ($P \succeq_{SFS_y} Q$).*

Proof of Proposition 22: Since $P \succeq_{FS1_y} Q$, it follows (Definition 12a) that $\int_{t=a}^{x} P(t)\,dt \leq \int_{t=a}^{x} Q(t)\,dt$ for all $x \in [0,y]$. Hence, $\widetilde{P}(x) \leq \widetilde{Q}(x)$ for all $x \in [0,y]$. Since $\widetilde{P}(y) = \widetilde{Q}(y)$ it follows from Definition 12b that $\widetilde{P}(z) = \int_{t=0}^{z} P(t)\,dt = \int_{t=0}^{y} P(t)\,dt + \int_{t=y}^{z} P(t)\,dt = \widetilde{P}(y) + \int_{t=y}^{z} P(t)\,dt = \widetilde{Q}(y) + \int_{t=y}^{z} P(t)\,dt \geq \widetilde{Q}(y) + \int_{t=y}^{z} Q(t)\,dt = \int_{t=0}^{y} Q(t)\,dt + \int_{t=y}^{z} Q(t)\,dt = \int_{t=0}^{z} Q(t)\,dt = \widetilde{Q}(z)$ for all $z \in [y,1]$. Hence, $P \succeq_{FS2_y} Q$. It also follows from Definition 12 that $P(y) = Q(y)$. Hence $P \succeq_{SFS_y} Q$ (Definition 15). ■.

6.8. Fehr-Schmidt-Lorenz dominance

We now propose a concept of Lorenz dominance, *FS Lorenz dominance*, that is appropriate for FS preferences. The relevant concepts for FS preferences are given from the perspective an individual with income $y \in [a,b]$ because inequity is self centered under FS preferences and we are interested in a positive perspective rather than a normative perspective using 'behind the veil' arguments (see discussion in the introduction).

Definition 16 *(Fehr-Schmidt-Lorenz dominance): Let $F, G \in \Pi$, $y \in [0,1]$ and $F(y) = G(y) = p_y \in [0,1]$. Then F Fehr-Schmidt-Lorenz dominates G ($F \succeq_{FSL_y} G$) if the following two conditions hold.*
(a) $\Gamma_F(p) \geq \Gamma_G(p)$, for all $p \in [0, p_y]$,

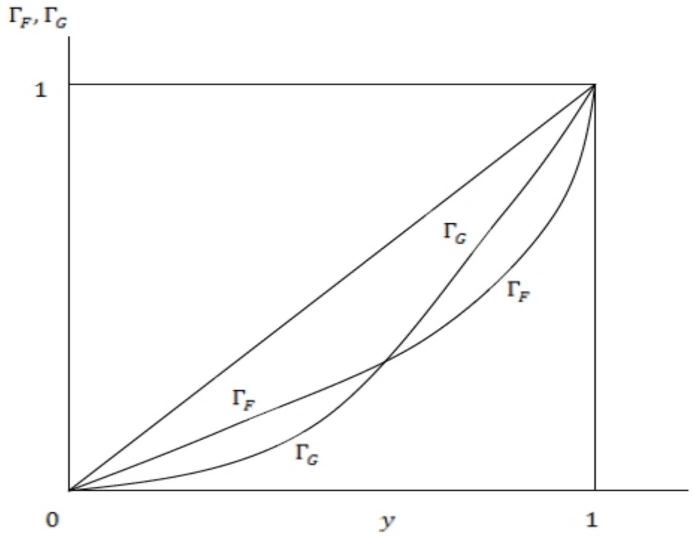

Figure 6.1: F strictly Fehr-Schmidt-Lorenz dominates G

(b) $\Gamma_F(p) \leq \Gamma_G(p)$*, for all* $p \in [p_y, 1]$.
If, in addition, (a) is strict for some $p \in [0, p_y]$ *or if (b) is strict for some* $p \in [p_y, 1]$*, then we say that* F *strictly Fehr-Schmidt-Lorenz dominates* G *(*$F \succ_{FSL_y} G$*).*

Figure 6.1 illustrates the case $F \succeq_{FSL_y} G$. We first give two intermediate Lemmas that help us establish, in Proposition 23, the link between strong FS dominance and FS Lorenz dominance.

Lemma 5 *: Let* $F, G \in \Pi, y \in [0,1]$*. Suppose* $F(y) = G(y) = p_y \in [0,1]$*, then*
(a) $\widetilde{F}(x) \leq \widetilde{G}(x)$ *for all* $x \in [0, y]$ *if, and only if,* $\Gamma_F(p) \geq \Gamma_G(p)$ *for all* $p \in [0, p_y]$*,*
(b) $\widetilde{F}(z) \geq \widetilde{G}(z)$ *for all* $z \in [y, 1]$ *if, and only if,* $\Gamma_F(p) \leq \Gamma_G(p)$ *for all* $p \in [p_y, 1]$*.*

Proof of Lemma 5: Let $p \in [0, p_y]$. Since F and G are continuous and non-decreasing, and since $F(y) = G(y) = p_y$, it follows that $F(w) = G(x) = p$ for some $w, x \in [0, y]$. And, conversely, if $p = F(w) = G(x)$ for some $w, x \in [0, y]$, then $p \in [0, p_y]$.

Suppose $F(w) = G(x) = p$ where $w, x \in [0, y]$ and $p \in [0, p_y]$. From Definition 10, we get that $\Gamma_F(p) = \int_{t=0}^{w} tdF$ and $\Gamma_G(p) = \int_{t=0}^{x} tdG$. Hence,

$$\Gamma_F(p) - \Gamma_G(p) = \int_{t=0}^{w} tdF - \int_{t=0}^{x} tdG.$$

Integration by parts gives

$$\begin{aligned}
\Gamma_F(p)-\Gamma_G(p) &= wF(w)-xG(x)+\int_{t=0}^{x}Gdt-\int_{t=a}^{w}Fdt \\
&= wF(w)-xG(x)+\int_{t=0}^{x}Gdt-\int_{t=0}^{x}Fdt-\int_{t=x}^{w}Fdt \\
&= \widetilde{G}(x)-\widetilde{F}(x)+wF(w)-xG(x)-\int_{t=x}^{w}Fdt \\
&= \widetilde{G}(x)-\widetilde{F}(x)+(w-x)F(w)-\int_{t=x}^{w}Fdt \text{ (since } F(w)=G(x)\text{)} \\
&\geq \widetilde{G}(x)-\widetilde{F}(x) \text{ for } w\geq x \text{ (since } F \text{ is non-decreasing).}
\end{aligned}$$

Similarly, for $x \geq w$ we get that

$$\begin{aligned}
\Gamma_F(p)-\Gamma_G(p) &= \widetilde{G}(x)-\widetilde{F}(x)+\int_{t=w}^{x}Fdt-(x-w)F(w) \\
&\geq \widetilde{G}(x)-\widetilde{F}(x).
\end{aligned}$$

Hence,

$$\Gamma_F(p)-\Gamma_G(p)\geq \widetilde{G}(x)-\widetilde{F}(x).$$

Similarly, one can show that

$$\Gamma_F(p)-\Gamma_G(p)\leq \widetilde{G}(w)-\widetilde{F}(w).$$

Hence,

$$\widetilde{G}(x)-\widetilde{F}(x)\leq \Gamma_F(p)-\Gamma_G(p)\leq \widetilde{G}(w)-\widetilde{F}(w).$$

Hence, $\widetilde{G}(x)-\widetilde{F}(x)\geq 0 \Rightarrow \Gamma_F(p)-\Gamma_G(p)\geq 0$ and $\Gamma_F(p)-\Gamma_G(p)\geq 0 \Rightarrow \widetilde{G}(w)-\widetilde{F}(w)\geq 0$. Thus, $\widetilde{G}(x)-\widetilde{F}(x)\geq 0$ for all $x\in[0,y] \Leftrightarrow \Gamma_F(p)-\Gamma_G(p)\geq 0$ for all $p\in[0,p_y]$.

This establishes part (a). The proof of part (b) is similar. ■.

We now state the strict version of Lemma 5.

Lemma 6 *: Let $F,G\in\Pi$, $y\in[0,1]$. Suppose $F(y)=G(y)=p_y\in[0,1]$, then*
(a) $\widetilde{G}(x)\geq\widetilde{F}(x)$ for all $x\in[0,y]$ and $\widetilde{G}(x)>\widetilde{F}(x)$ for some $x\in[0,y]$ if, and only if, $\Gamma_F(p)\geq\Gamma_G(p)$ for all $p\in[0,p_y]$ and $\Gamma_F(p)>\Gamma_G(p)$ for some $p\in[0,p_y]$,
(b) $\widetilde{F}(z)\geq\widetilde{G}(z)$ for all $z\in[y,1]$ and $\widetilde{F}(z)>\widetilde{G}(z)$ for some $z\in[y,1]$ if, and only if, $\Gamma_G(p)\geq\Gamma_F(p)$ for all $p\in[p_y,1]$ and $\Gamma_G(p)>\Gamma_F(p)$ for some $p\in[p_y,1]$.

Proof of Lemma 6: Similar to that of Lemma 5.

Proposition 23 *: Fehr-Schmidt-Lorenz dominance is equivalent to strong Fehr-Schmidt dominance and strict Fehr-Schmidt-Lorenz dominance is equivalent to strict strong Fehr-Schmidt dominance (Definitions 13, 16).*

Proof of Proposition 23: Follows from Definitions 13, 15, 16 and Lemmas 5, 6. ■.

We now show that our purely statistical concept of FS Lorenz dominance is sound by relating it to the appropriate decision theory, which in this case is FS preferences.

Proposition 24 *: Let $F, G \in \Pi$, $y \in [0,1]$. Suppose F Fehr-Schmidt-Lorenz dominates G ($F \succeq_{FSL_y} G$). Then $U(y,F) \geq U(y,G)$ for all $\alpha \geq 0$, $\beta \in [0,1)$ and all concave $u \in \mathbf{u}$.*

Proof of Proposition 24: Immediate from Propositions 16 and 23. ■.

Proposition 25 *: Let $F, G \in \Pi$, $y \in [0,1]$ and $F(y) = G(y)$. If $U(y,F) \geq U(y,G)$ for all $\alpha \geq 0$, $\beta \in [0,1)$ and all concave $u \in \mathbf{u}$, then F Fehr-Schmidt-Lorenz dominates G ($F \succeq_{FSL_y} G$).*

Proof of Proposition 25: Immediate from Propositions 17 and 23. ■.

As our final set of results we give the strict versions of Propositions 24, 25.

Proposition 26 *Let $F, G \in \Pi$, $y \in [0,1]$ and $F(y) = G(y)$.*
(a) Suppose that F strictly Fehr-Schmidt-Lorenz dominates G ($F \succ_{FSL_y} G$). Let $\alpha > 0$, $\beta > 0$, $y \in [0,1]$ and $u \in \mathbf{u}$ be strictly increasing and strictly concave. Then $U(y,F) > U(y,G)$.
(b) Suppose that $U(y,F) \geq U(y,G)$ for all concave $u \in \mathbf{u}$, all $\alpha \geq 0$ and all $\beta \in [0,1)$, and suppose that $U(y,F) > U(y,G)$ for some concave $u \in \mathbf{u}$, $\alpha \geq 0$ and $\beta \in [0,1)$, then F strictly Fehr-Schmidt-Lorenz dominates G ($F \succ_{FSL_y} G$).

Proof of Proposition 26: Immediate from Propositions 18 and 23. ■.

7. Conclusions

The question of which of two distributions is preferred by an economic agent is one of the fundamental questions in economics. The traditional literature has answered this question for an individual with *selfish preferences* who derives utility solely from his own consumption bundle. Furthermore, when choosing among two distributions, such an individual acts behind a veil of ignorance and uses expected utility theory.

The evidence shows that a large fraction of individuals have *social preferences*, i.e., they care not only about their own consumption (selfishness) but also about the consumption of those who are richer (envy) and those who are poorer (altruism or pride). The experimental literature shows that the model of social preferences due to Fehr and Schmidt (1999), that we called the FS model, fits quite well the evidence from a large number of games. In particular, the evidence on choosing among actual income distributions lends strong support to FS preferences. For the questions that we are interested in, other models of social preferences based on intention-based reciprocity and type-based reciprocity are less relevant.

In this paper we establish analogous results for social preferences to those that were established for selfish preferences in the seminal works of Diamond and Stiglitz (1970, 1971), Kolm (1969), Atkinson (1970) and Shorrocks (1983). The importance of having such tools for theory and applications in economics can hardly be understated.

We ask which of two distributions is preferred by an individual with FS preferences when he knows his own income level. Thus, we do not need 'behind the veil of ignorance' arguments to motivate our choice. We introduce the analogues of 1st and 2nd order stochastic dominance and call these *First and Second order Fehr-Schmidt dominance*, respectively. We also introduced a dominance concept, *weak Fehr-Schmidt dominance*, that is suited to the linear version of FS preferences. We introduce strict and non-strict versions for each dominance concept and establish their soundness in terms of the underlying FS preferences. We also establish the interrelations between these concepts.

A slightly stronger version of second order Fehr-Schmidt dominance, *strong Fehr-Schmidt dominance*, turns out to be equivalent to a modified version of generalized Lorenz dominance, *Fehr-Schmidt-Lorenz dominance*. This establishes the analogue of the classical relation between second order stochastic dominance and Lorenz dominance.

Looking forward, a number of applications that we did not pursue here, are possible. In models of redistribution one may ask which of two alternative income distributions does a *median voter*, who has FS preferences, prefer?[38] Political parties are also likely to be interested in the question of which distributions do their median constituents prefer? Organizations may be interested in the morale of their workers because, for example, a worker may care about his income relative to the incomes of other workers. Such organizations would naturally wish to find out how workers respond to alternative incentive schemes that induce alternative income distributions over the set of workers.

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[38] This of course, makes sense only if a median voter with FS preferences exist. See Dhami and al-Nowaihi (2010a) for the conditions that guarantee the existence of a Condorcet winner under FS preferences.

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