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Charitable Giving and Optimal Public Policy in a Competitive Equilibrium with Multiple Equilibria.

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Abstract

In a competitive-equilibrium analysis of giving to charity, we show that strategic complementarity between individual giving and aggregate giving can lead to multiple equilibria. This provides a possible explanation for observed heterogeneity in giving. It is possible, but not necessary, that at a low equilibrium in giving (LE), an increase in subsidy reduces giving (perverse comparative statics) while at a high equilibrium (HE) the comparative statics are normal (subsidies promote giving). The perverse comparative statics at LE preclude using subsidies to move the economy to HE. We show how temporary direct government grants can engineer a permanent move from LE to HE. Once HE is established, the optimal mix of private and public giving is determined using a welfare analysis. We show that the Nash non-cooperative outcome is virtually identical to the competitive-equilibrium, even for relatively small numbers of givers. The competitive-equilibrium approach is more tractable and plausible, and more general because it does not rely on a symmetric equilibrium. We also show how our results are applicable to redistributive and public good contexts.

Keywords: Multiple equilibria; aggregate strategic substitutes and complements; competitive and non-cooperative equilibria; direct grants; charitable redistribution; voluntary contributions to public goods; optimal mix of public and private giving.

JEL Classification: D6, H2, H4.

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1. Introduction

Charitable donations is a significant economic activity. For instance, in 2003, for the USA, 89 percent of all households gave to charity with the average gift being \$1620 per annum, which gives an aggregate total of about \$100 billion per annum; see, Mayr et al. (2009). The economics of charity has elicited notable scholarship but it relies on the following two essential assumptions. Our aim is to relax both these assumptions.

1. First, individual giving to charity is determined by a *non-cooperative game*, typically focussing on a *symmetric Nash equilibrium*. It is not entirely convincing or plausible that when someone decides to contribute, say £10, to the Red Cross, they are engaged in a strategic game in charitable contributions with respect to all other givers. Certainly, no evidence of this seems to have been provided. Indeed, if there is a large number of givers, most being small and dispersed, then the notion of a *competitive equilibrium in charitable giving* seems worth exploring.

2. Second, the analysis typically focusses on a *unique equilibrium* in giving. Multiple equilibria would seem to be endemic in many important economic phenomena, however, they are typically ruled out by assumption. In particular, in a charitable context, it is quite conceivable that the uncoordinated giving of a large number of dispersed small givers gives rise to multiple equilibria, depending on the beliefs held by the givers.

Before we give an overview of our paper in section 2, below, we summarize our seven main contributions. First, we show that *aggregate strategic complementarity* is a necessary condition for multiple equilibria in a *competitive equilibrium* in charitable contributions. Strategic complementarity at the individual level is sufficient but not necessary to ensure aggregate strategic complementarity. Second, we show that even for relatively small numbers of contributors, the competitive solution is similar to the non-cooperative Nash equilibrium. Third, *multiple equilibria* provide a possible explanation of heterogeneity in charitable giving. It is possible that at a *low equilibrium* in individual and aggregate giving, the comparative statics are *perverse* (i.e., subsidies reduce giving), while the *high equilibrium* exhibits *normal* comparative statics (i.e., subsidies promote giving). In our fourth contribution, we show how, using temporary direct grants, a policy maker can *engineer a move* from the low to the high equilibrium. Now consider the case where comparative statics at the low equilibrium are *normal* and those at the high equilibrium are *perverse*. In our fifth contribution, we show that in this case, the government can do better than simply encouraging subsidy-induced giving at the low equilibrium. Indeed, once the government successfully engineers a move to the high equilibrium using temporary direct grants, the perverse comparative statics at the high equilibrium ensure that a reduction in subsides will induce even greater private giving. Sixth, by carrying out a welfare analysis, we give conditions that specify the *optimal mix* of public contributions and private contributions to charity. Seventh, we show our results are applicable to *redistributive* and *public goods* contexts.

Section 2 gives an overview of our paper. Section 3 describes the general theoretical model. Section 4 gives two illustrative examples of voluntary private contributions to redistribution and public good provision, respectively. Section 5 derives the equilibrium of the model and its comparative static results. Section 6 examines multiple equilibria in aggregate giving. Section 7 performs a welfare analysis and characterizes the normatively optimal public policy. Section 8 provides an explicit solution and numerical analysis of the two examples of section 4. Section 9 explores the implications of giving when givers play a symmetric non-cooperative Nash equilibrium and illustrates the robustness of our results. Section 10 concludes. Most proofs are collected in the appendix.

2. Overview

We first highlight some stylized facts, S1-S5, associated with philanthropic activity.[1]

- S1 *There is substantial heterogeneity in giving between countries.* As a percentage of GDP, for 1995-2000, non-religious philanthropic activity was in excess of 4% for the Netherlands and Sweden; 3-4% for Norway and Tanzania; 2-3% for France, UK, USA; and less than 0.5% for India, Brazil, and Poland; see Salamon et al. (2004).

- S2 *Individual private donors are the largest contributors.* For US data, for 2002, individuals accounted for 76.3 percent of the total charitable contributions. Other givers are: foundations (11.2%), bequests (7.5%), corporations (5.1%); see Andreoni (2006).

- S3 *Government direct grants are significant.* For non-US data, governments are typically the single most important contributors to charities. On average, in the developed countries, charities receive close to half their total budget directly as grants from the government, while the average for developing countries is about 21.6 percent.[2]

- S4 *Contributions to charity are typically tax deductible.* For instance, charitable deductions range from up to 50% in the US and 17-29% for Canada.

- S5 *Seed money, leadership contributions and direct grants are effective in stimulating charitable giving.* Evidence suggests that in addition to private giving, *direct grants*

[1] Andreoni (2006) deals with several of these facts in greater detail.

[2] See the Johns Hopkins Comparative Nonprofit Sector Project (http://www.jhu.edu/~cnp/).

in the form of seed money or leadership contributions (which precede private giving) made by governments, foundations, the national lottery (as in the UK), or exceptionally rich individuals etc. are efficacious.[3]

2.1. Setting up a bare bones model to explain the basic intuition

Suppose that n givers to a charity contribute respective amounts $g_1, g_2, ..., g_n$. Let G be the total contributions received by the charity. The utility function of the i^{th} giver, u^i, is[4]

$$u^i(c_i, g_i, G),\ \ i = 1, 2, ..., n, \tag{2.1}$$

where c_i is own private consumption, and g_i is own contribution to charity. Two interpretations could be given to G in (2.1). G could reflect pure altruism on the part of individuals (see Example 4.1, below) or G could be the aggregate level of public goods (see Example 4.2, below). The term g_i reflects *warm glow* or *prestige* from the act of giving (also known as *impure altruism*). There is overwhelming experimental/field evidence and growing neuroeconomic evidence that justifies such a formulation.[5]

The budget constraint of individual giver, i, is

$$c_i + (1 - s)\, g_i \leq (1 - t)\, m_i + \tau_i, \tag{2.2}$$

where m_i is individual i's exogenous income, s is the per-unit subsidy to charitable giving (fact S4), t is the income tax rate, and τ_i is *individual-specific* receipt of money from the charity (it is, of course, possible that some individuals only contribute, but receive nothing from the charity).[6]

The government has a balanced budget such that tax revenues ($\sum_{i=1}^{n} tm_i$), net of the total cost of subsidies to charity ($\sum_{i=1}^{n} sg_i$), are contributed as a direct government grant, D, to the charity (fact S3).

Aggregate contributions to charity, the sum of public and private contributions, are,

$$G = D + \sum\nolimits_{i=1}^{n} g_i. \tag{2.3}$$

The Government chooses its instruments s, t, D to maximize a social welfare function

$$U\left(u^1, u^2, ..., u^n\right), \tag{2.4}$$

[3] See, for instance, Karlan and List (2007), Potters et al (2007), and Rondeau and List (2008).

[4] Superscripts on names of *functions*, such as u, will range over individuals while subscripts will indicate partial differentiation.

[5] For the evidence on altruism, see Andreoni (2006). For experimental evidence on warm glow preferences, see Andreoni (1993, 2006), Palfrey and Prisbrey (1997) and Andreoni and Miller (2002). For the neuroeconomic evidence see, for instance, Harbaugh et al. (2007), Moll et al. (2006), and for a survey of the neuroeconomic evidence see Mayr et al. (2009).

[6] We could have also allowed for direct redistributive grants from the government to the public. These can be easily accommodated in our model, but add nothing to the results, while complicating the exposition.

which is followed by the public's choice of c_i, g_i.[7]

We assume that u^i is strictly increasing in c_i, so the budget constraint (2.2) binds:

$$c_i = (1-t)\, m_i + \tau_i - (1-s)\, g_i,\ i = 1, 2, ..., n. \tag{2.5}$$

We derive two useful representations of utility. Substituting from (2.5) into (2.1):

$$U^i\,(g_i, G, s, t) = u^i((1-t)\, m_i + \tau_i - (1-s)\, g_i, g_i, G),\ i = 1, 2, ..., n. \tag{2.6}$$

Let G_{-i} be the sum of givings by all individuals other than individual i, $G_{-i} = \sum_{j \neq i}^{n} g_j$. Then (2.3) can be written as $G = D + g_i + G_{-i}$, and the utility function of individual i, (2.6), can be written as

$$\widetilde{U}^i\,(g_i, G_{-i}, s, t, D) = U^i\,(g_i, D + g_i + G_{-i}, s, t)\,,\ i = 1, 2, ..., n. \tag{2.7}$$

In representations, (2.6) and (2.7), the main difference that we want to focus on is that, in the former, utility depends on aggregate contributions, G, while in the latter it depends on the contributions of all others, G_{-i}.

2.2. Equilibrium: Strategic or competitive?

There are at least two possible methods to determine the endogenous variables of the model. Both use a two-stage game to model behavior. In the first stage, in both methods, the government chooses s, t and D so as to maximize (2.4), correctly anticipating the behavior of the givers in the second stage. The two methods differ, however, in the behavior of givers in the second stage, which we now describe.

1. *Strategic behavior*. In the second stage, each giver, i, chooses his/her giving, g_i, so as to maximize his/her utility, $\widetilde{U}^i$, conditional on total giving by other givers, G_{-i} (see (2.7)). For tractability, existing mainstream models restrict attention to a symmetric Nash equilibrium. The strategic approach is not unambiguously justified, as the following considerations illustrate.
   (a) There is a great deal of evidence that the predictions of a Nash equilibrium are often, and systematically, violated; see, Camerer (2003).[8] Several behavioral alternatives include: quantal response equilibrium, level-k models, models of noisy introspection etc. A discussion of these is beyond the scope of this paper.
   (b) Large donors may behave in a strategic manner (say, to establish green credentials) but their share in the total contributions is small (5.1% for the US).

[7] Hence, D has the nature of seed money or leadership contributions (fact S5).

[8] For a more recent list of readings, the reader may consult, for instance, Vincent Crawford's website at: http://dss.ucsd.edu/~vcrawfor/.

(c) It stretches credulity to believe that individuals contributing relatively small amounts to charities are playing a strategic non-cooperative game vis-a-vis all other contributors. The literature does not provide any evidence for this assumption.

2. *Competitive equilibrium*: In this view, which we subscribe to, for reasons 1(a),1(b),1(c), there is a very large number of contributors who are individually very small (fact S2). In a competitive markets view, any g_i is sufficiently small compared to G, so that each giver i takes total giving, G, as exogenous (see (2.6)).

2.2.1. The convergence of the Nash equilibrium to the competitive equilibrium

Although we choose the competitive equilibrium view, however, to facilitate comparison with the existing literature, we also compute the *symmetric Nash equilibrium strategies*. Our calibration results suggest that even for relatively small numbers of contributors to charity (around 500, which should be satisfied for many charities), the competitive and the Nash equilibrium outcomes are very similar in terms of individual and aggregate giving.[9] In this sense, the two approaches are complementary.

However, the strategic approach has the shortcoming that, for reasons of tractability, it is typically restricted to a *symmetric Nash equilibrium* only. On the other hand, the symmetric and non-symmetric *competitive equilibria* are equally tractable, and allow for the exploration of much richer results.

2.3. Existence and some implications of multiple equilibria

It is entirely plausible that coordination problems among diverse and numerous small contributors give rise to *multiple equilibria*. Suppose that individual private giving, g_i, and aggregate giving (or the size of the charity), G, are *strategic complements*.[10] Thus, the marginal utility of contributing an extra unit of g_i is increasing in the level of G. Now, if one conjectures that G will be high (low), then one is also induced to contribute a larger (smaller) amount, g_i. Hence, there could be several, self fulfilling, rational expectations equilibria. In some equilibria, giving is *high* (HE), while in other equilibria, giving is *low* (LE). One could rank multiple equilibria by the amount of individual and aggregate giving and also possibly socially rank the equilibria. There are two immediate implications.

1. Multiple equilibria potentially explain *heterogeneity in giving* (fact S1). Among identical societies some can get stuck at HE, while others can be stuck at LE.

[9] The result is reminscent of the Cournot outcome approximating the competitive equilibrium outcome when the number of firms becomes large.

[10] In terms of (2.6), g_i, G are strategic complements (strategic substitutes), iff $\frac{\partial^2 U^i}{\partial g_i \partial G} \geq 0 \ \ (\leq 0)$.

2. At some equilibria, one might obtain *perverse comparative static results* (i.e., an *increase* in subsidy, s, *reduces* contributions). At other equilibria, the comparative static results could be *normal*, in the sense that contributions respond positively to incentives. Policy makers could, understandably, be interested in engineering a move from a *low equilibrium with perverse comparative statics* (LE-P) to a *high equilibrium with normal comparative statics* (HE-N), if such equilibria exist.

2.4. Engineering moves between multiple equilibria

Suppose that we have two, and only two, equilibria, LE-P and HE-N, but the economy is stuck at LE-P. Consider a policy maker who desires to engineer a move from LE-P to HE-N. How should the policy maker proceed? Clearly incentives for charitable giving, in the form of higher subsidies, s, will not work because of the perverse comparative static effects at LE-P. Suppose, instead, that the government gives a *temporary* direct grant, D, to the charity, financed by an income tax (levied at the rate t).[11] Let D exceed the level of aggregate contributions, G, at LE-P. This, as we shall see in greater detail below, leaves HE-N as the only feasible equilibrium.

Once the economy arrives at the equilibrium HE-N (where the comparative statics are *normal*), the government can withdraw the temporary direct grant, D, and successfully stimulate private giving through greater subsidies. This is welfare improving, because, unlike direct grants, increased private giving confers warm glow on the contributors. The applicability of these ideas is illustrated by examples where voluntary giving contributes towards public redistribution or towards public goods (see sections 4 and 8).[12]

2.5. A brief comparison with some other models of multiple equilibria

Our rationale for multiple equilibria in a competitive equilibrium is very different from Andreoni's (1998) strategic-Nash framework, which requires non-convexities in production. Unlike Andreoni (1998), we perform a welfare analysis in a general equilibrium model with a government budget constraint where the fiscal parameters (such as s, t, D) are optimally determined. Like us, Andreoni (1998) shows that policy can engineer moves between the various equilibria. However, in his model that requires the government to have the ability

[11] Empirical evidence shows that crowding-out of *private contributions* by direct government grants, if any, is quite small. See Andreoni (2006) for a discussion of the empirical evidence. It is also likely that some of the observed crowding-out is due to moral hazard issues on account of the fundraising activities by the charity. These issues lie beyond the scope of our paper.

[12] These results also provide another explanation for why *seed money* or *leadership donations* are often effective in improving the level of private charitable giving (fact S5). Seed money, leadership contributions and national lottery money play a role similar to the direct grant, D, in our framework. Existing models assume a unique equilibrium, hence, we provide an alternative explanation to this important phenomenon.

to levy taxes based on individual characteristics. We only need the weaker assumption that the government be able to observe the income of individuals.

The idea of engineering a move from a low equilibrium characterized by poverty traps to an equilibrium with greater prosperity is important in development economics.[13] Murphy et al. (1989) focus on the mechanism of aggregate demand spillovers arising from a coordinated increase in outputs in a range of imperfectly competitive industries. However, our mechanism for engineering moves between equilibria is very different.

Strategic complementarity is a necessary condition for multiple equilibria in the *non-cooperative Nash equilibrium framework* of Cooper and John (1988). We propose a natural extension of this condition to competitive equilibria, *aggregate strategic complementarity*, that we show to be the necessary condition for multiple equilibria in competitive contexts. Our new proposed concept is likely to have wide applicability.

3. Formal model

There are three main types of players in the economy, (1) *consumers*, (2) a *fiscal authority* (which we will simply refer to as the *Government*), and (3) *charities*. There are n consumers indexed by $i = 1, 2, ..., n$. Consumer i has an exogenously fixed income of $m_i \geq 0$ each period. The aggregate of all incomes is

$$M = \sum\nolimits_{i=1}^{n} m_i. \tag{3.1}$$

Assume that $M > 0$, i.e., at least some consumers have positive income.

3.1. Fiscal instruments

The government exercises the following three types of fiscal instruments. (i) An income tax on individual income, m_i, at the rate t, $0 \leq t < 1$. (ii) A subsidy to private giving to charity at the rate, s, $0 \leq s < 1$. (iii) Direct public contribution to charity, $D \geq 0$.

3.2. Consumers

The utility function of consumer i is given by

$$u^i \left(c_i, g_i, G\right). \tag{3.2}$$

The consumer derives utility from private consumption expenditure, c_i, from own giving to charity (*warm glow*)[14], g_i, and from the aggregate level of giving to charity, $G \geq 0$.

[13] The idea goes back to Rosenstein-Rodan (1943) and was formalized later on by Murphy, Shleifer and Vishny (1989).

[14] The introduction of a warm glow motive was suggested by Cornes and Sandler (1984) and Andreoni (1989, 1990). The presence of a warm glow term reflects the fact that individuals no longer consider their

Consumers derive utility from G either because G finances goods that are useful to the individual (selfish preferences), or because they are useful to others (altruistic preferences).

Remark 1 *(Notation): Superscripts on the utility function denote the identity of individual givers (e.g., u^i is the utility function of the i^{th} giver). On the other hand, subscripts are used to denote partial derivatives (e.g., $u_2^i = \partial u^i / \partial g_i$).*

The assumptions on preferences are quite standard. g_i, is bounded below by zero while c_i is bounded below by a constant, $\underline{c}_i \geq 0$ (possibly a subsistence level). We assume that u^i is a C^2 function (continuous, with continuous first and second partial derivatives) for $g_i > 0$ and $c_i > \underline{c}_i$. We also assume that $u_1^i > 0$, $u_{11}^i \leq 0$ (strictly positive but non-increasing marginal utility of consumption), $u_2^i \geq 0$, $u_3^i \geq 0$ (non-negative marginal utilities of own and aggregate giving) and $u_{22}^i \leq 0$ (concavity in own giving). In addition we make some technical assumptions. The first of these, (3.3), guarantees concavity in own giving of a transformed utility function (see (2.6), above).

$$(1-s)^2 u_{11}^i - 2(1-s) u_{12}^i + u_{22}^i < 0, \tag{3.3}$$

$$u_1^i \uparrow \infty \text{ as } c_i \downarrow \underline{c}_i. \tag{3.4}$$

We assume that either u^i is extended to the boundary, $g_i = 0$, as a continuous function or

$$u_2^i \uparrow \infty \text{ as } g_i \downarrow 0. \tag{3.5}$$

Examples that satisfy the above assumptions include $u = \sqrt{c} + g$ or $u = \ln(c-1) + \ln g$. The more complex examples 4.1 and 4.2 in section 4 also satisfy these assumptions.

The budget constraint of consumer i is given by

$$c_i + (1-s) g_i \leq (1-t) m_i + \tau_i. \tag{3.6}$$

The RHS of (3.6) is the after-tax income plus an individual-specific transfer, $\tau_i \geq 0$, received from the charity's aggregate revenue, G.[15] The LHS gives the two sources of expenditure, private consumption and the (net of subsidy) private charitable giving. Note that s and t are instruments of government, while τ_i is an instrument of the charity.

contributions to be perfect substitutes for the contributions of others. Hence, there is extra utility from one's own contribution, which mitigate the free rider problem arising from purely altruistic considerations, i.e., from a utility function of the form $u^i(c_i, G)$. It also obviously implies that government grants to charities do not crowd out private donations completely because the two are imperfect substitutes from the point of view of givers.

[15] This does not preclude the possibility that some individuals only contribute, but receive nothing from the charity.

Furthermore, we assume that each g_i is a small fraction of G, so that each consumer takes the aggregate G as given[16]. Similarly, and we believe quite realistically, consumer i takes τ_i as given. Thus, in making her decision to allocate after-tax income between c_i and g_i, the consumer takes as given m_i, τ_i, s, t, G (as in the theory of competitive markets) and maximizes u^i given in (3.2) subject to the budget constraint (3.6).

3.3. Government

Total income tax revenues equal $\sum_{i=1}^{n} tm_i = t\sum_{i=1}^{n} m_i = tM$. The proceeds of the income tax, tM, are used to finance subsidies on donations to charity, $s\sum_{i=1}^{n} g_i$, and on aggregate direct grants from the government to the charities, $D \geq 0$. Therefore, the (balanced) government budget constraint is

$$tM = D + s\sum_{i=1}^{n} g_i. \tag{3.7}$$

The Government chooses its instruments s, t, D to maximize a social welfare function

$$U = U\left(u^1, u^2, ..., u^n\right), \tag{3.8}$$

which is strictly increasing in the individual utility functions, $u^1, u^2, ..., u^n$.[17]

3.4. Charities

In order to focus on the simultaneous determinants of private giving and the influence of public policy, we assume that charities are passive players in the game[18]. They merely collect all donations from private consumers, $\sum_{i=1}^{n} g_i$, and from the government, D. The production function for charities is a simple linear function that converts the sum of all giving $D + \sum_{i=1}^{n} g_i$ to some aggregate output $G \geq 0$, so

$$G = D + \sum_{i=1}^{n} g_i. \tag{3.9}$$

[16] At this point, the reader might wish to recall the discussion in subsection 2.2. In Section 9 below, we show that the solutions under the two cases of *strategic* and *competitive* giving are very similar even for small numbers of givers. Our approach can, of course, be made completely rigorous by adopting an appropriate measure-theoretic formulation with a continuum of consumers. We have found this to considerably complicate our paper, without adding anything to either our conclusions or the literature on the measure-theoretic approach to economics.

[17] Note that the resulting social optimum is constrained by the available set of instruments $\{s, t, D\}$. In particular, an even better social optimum may be available if subsidies and taxes $\{s_i, t_i\}$ could be varied across individuals. We assume that this is either not desirable or not possible (although it is straightforward to extend our analysis to cope with the more general case).

[18] In actual practice, charities could be strategic players, using means such as bundling, marketing etc. to attract additional donations. Charities might also put in less effort to raise additional contributions in the presence of government grants. We abstract here from these issues. The interested reader can consult Andreoni (2006) for further details and references on these issues.

The charity can use G to finance transfers to individuals, $\sum_{i=1}^{n} \tau_i$ or to provide a public good.[19] More generally, our model allows for the case in which the charity uses an *exogenous* fraction of it's revenues to undertake individual-specific redistribution, $\sum_{i=1}^{n} \tau_i$, while using the balance $G - \sum_{i=1}^{n} \tau_i$ to finance provision of public goods.[20]

3.5. Sequence of moves

The government moves first to announce the parameters s, t, D. The charity (which is a passive player) moves simultaneously to announce the parameters $\tau_1, \tau_2, ..., \tau_n$. Each of the small and dispersed individual givers to charity moves next, taking as given s, t, D, τ_i as well as the aggregate contributions, G, exactly as in the theory of competitive markets (see the last paragraph in section 3.2).[21]

3.6. Some preliminary results

Since $u_1^i > 0$, the budget constraint (3.6) holds with equality. Hence, we can use it to eliminate c_i from (3.2). Letting $U^i(g_i, G; s, t)$ be the result, we have

$$U^i(g_i, G; s, t) = u^i((1-t)m_i + \tau_i - (1-s)g_i, g_i, G). \tag{3.10}$$

From (3.3) and (3.10) it follows that

$$U_{11}^i < 0. \tag{3.11}$$

The consumer's maximization problem can be restated as

$$\text{Maximize}_{\langle g_i | s,t,G \rangle}\ U^i(g_i, G; s, t) \ \text{ subject to } 0 \leq g_i \leq \frac{1}{1-s}\left[(1-t)m_i + \tau_i - \underline{c}_i\right]. \tag{3.12}$$

Proposition 1 : *Suppose $\underline{c}_i < (1-t)m_i + \tau_i$.*
(a) The consumer's maximization problem (3.12) has a unique solution, g_i^*.
(b) $0 \leq g_i^* < \frac{1}{1-s}\left[(1-t)m_i + \tau_i - \underline{c}_i\right]$.
(c) If, in addition, (3.5) holds, then $g_i^* > 0$.

[19] One example of each of these cases is given in Section 4. The competitive and symmetric Nash solution to both examples is given in section 8.

[20] An *endogenous* split into redistribution and public good provision will require specifying the charity's objective function in greater detail. Similar comments apply to an endogenous treatment of individual-specific redistribution $(\tau_1, \tau_2, ..., \tau_n)$. However, these issues lie beyond the scope of the paper.

[21] The reader might wonder how the charity could possibly announce $\tau_1, \tau_2, ..., \tau_n$ prior to observing the individual contributions, $g_1, g_2, ..., g_n$. One could also pose the same question with respect to the direct grants, D, made by the government. The government and the charity are assumed to be rational and forward looking. Since there are no stochastic stocks in the model, the beliefs of each of these two players, about $g_1, g_2, ..., g_n$, are always fulfilled in equilibrium. Technically, we have a perfect foresight rational expectations equilibrium.

We may use the example, $u = \ln(c-1) + \ln g$, to illustrate the idea behind the proof of Proposition 1. The problem is that the set on which u is defined, $\{(c,g) : c > 1, g > 0\}$ is not closed. Hence, its intersection with the budget set (to produce the opportunity set) is not a compact set. However, because $u_1 = \frac{1}{c-1} \uparrow \infty$ as $c \downarrow 1$, and since $u_2 = \frac{1}{g} \uparrow \infty$ as $g \downarrow 0$, the maximum of u can be bounded away from $c = 1$ and $g = 0$. This allows us to take the opportunity set to be compact with u continuous on the set $\{(c,g) : c > 1, g > 0\}$. Hence, the existence of a maximum.

Since g_i^* is unique, we can write it as a function of the parameters that are exogenous to the consumer's maximizing problem (3.12). In particular, we write $g_i^*(s,t,G)$ explicitly as a function of the tax rate, t, the subsidy rate, s, and aggregate giving, G.[22]

A simple calculation shows that Proposition 1(b) implies the following:

$$tM \leq tM + (1-s)\sum\nolimits_{i=1}^{n} g_i^*(s,t,G) < M + \sum\nolimits_{i=1}^{n} \tau_i - \sum\nolimits_{i=1}^{n} \underline{c}_i. \tag{3.13}$$

On the left hand side of "<" in (3.13) we have the total amount paid in taxes and donations to charity (net of subsidy). On the right hand side of "<" we have total after-tax income minus total expenditure on subsistence consumption. Clearly, the former cannot exceed the latter. However, from (3.4) it follows that optimal consumption must be strictly higher than subsistence consumption. Hence, the strict inequality.

Proposition 2 : *Suppose* $g_i^* > 0$. *Let* $c_i^* = (1-t)m_i + \tau_i - (1-s)g_i^*$. *Then, at* g_i^*, c_i^*, (a) $U_1^i = 0$. (b) $(1-s)u_1^i = u_2^i$. (c) $\frac{\partial g_i^*}{\partial G} = \frac{u_{23}^i - (1-s)u_{13}^i}{-(1-s)^2 u_{11}^i + 2(1-s)u_{12}^i - u_{22}^i}$. (d) $\frac{dg_i^*}{ds} = \frac{u_1^i + g_i^*\left[u_{12}^i - (1-s)u_{11}^i\right]}{2(1-s)u_{12}^i - (1-s)^2 u_{11}^i - u_{22}^i}$. (e) $\frac{\partial g_i^*}{\partial t} = \frac{m_i\left[(1-s)u_{11}^i - u_{12}^i\right]}{2(1-s)u_{12}^i - u_{22}^i - (1-s)^2 u_{11}^i}$.

3.7. Strategic complements and strategic substitutes

Following Bulow, Geanakoplos and Klemperer (1985), strategic complements and strategic substitutes can be defined as follows.

Definition 1 : *(Strategic complements and substitutes)* g_i *and* G *are strategic complements (substitutes) if, and only if,* $\frac{\partial^2 U^i}{\partial g_i \partial G} > 0 \quad (\leq 0)$.

Thus, g_i and G are strategic complements (respectively, substitutes) if the marginal utility to individual i of making an extra unit of contribution, g_i, increases (respectively, decreases) with an increase in aggregate contributions, G.

Lemma 1 : g_i *and* G *are strategic complements (substitutes) if, and only if,*

$$u_{23}^i - (1-s)u_{13}^i > 0 \quad (\leq 0),$$

[22] We have suppressed other parameters such as τ_i and m_i to improve readability.

Lemma 2 *: g_i and G are strategic complements (substitutes) if, and only if,*

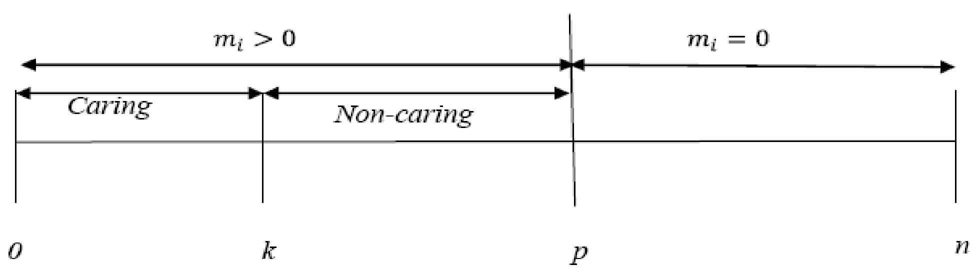
$$\frac{\partial g_i^*}{\partial G} > 0 \ \ (\leq 0) \, .$$

4. Two illustrative examples

In this section we present two illustrative examples of the general theoretical model. In Example 4.1, charitable contributions provide income to consumers who, otherwise, have no income.[23] In Example 4.2, charitable contributions finance public good provision. Section 8 provides an explicit solution and numerical analysis of these two examples. And in section 9 we will compute explicitly the multiple symmetric Nash equilibria in giving for these examples and contrast them with the competitive equilibria.

4.1. Example 1: Charitable contributions as public redistribution

We consider an economy where some consumers have no income. Their consumption expenditure is financed entirely by either charitable donations, g_i, made by other 'caring' consumers with positive income and/or by tax financed direct government grants, D (which now have the interpretation of social welfare payments). This is illustrated in Figure 4.1. The assumptions are as follows.



1. There are n consumers. Of these, p consumers, $0 < p < n$, and indexed by $i = 1, 2, ..., p$, have positive income ($m_i > 0$). The other $n - p$ consumers, indexed by $i = p + 1$, $p + 2, ..., n$, have no income ($m_i = 0$). All incomes are publicly observable.

2. The aggregate of all donations to charity (private and public), G, is divided equally among the consumers with no income; so each recipient receives $\tau_i = \tau = \frac{G}{n-p}$.

[23] Hence, in a straightforward but more expositionally demanding exercise to our paper, private charitable contributions supplement the public, redistributive, activities of the government (which are excluded in our paper).

3. Of the p consumers with positive incomes, k, $0 < k \leq p$, care about the plight of those with no income. Each of these *caring* consumers has the utility function

$$u^i = \ln c_i + a_i g_i G,\ a_i > 0,\ i = 1, ..., k, \tag{4.1}$$

where the following technical condition holds:

$$\frac{1}{a_i G} < \frac{1-t}{1-s} m_i,\ i = 1, ..., k. \tag{4.2}$$

4. The other $p - k$ consumers have positive income but do not care about those with no income. The utility function of the latter two groups of consumers (the non-caring with positive income and those with no income) is given by

$$u^i = \ln c_i;\ i = k+1, k+2, ..., p, p+1, ..., n. \tag{4.3}$$

From Definition 1, it follows that g_i, G are *strategic complements in* (4.1).

4.2. Example 2: Voluntary contributions to a public good

Individuals often voluntarily contribute to, and directly use, several kinds of public goods such as health services and education[24]. Suppose that the utility function of consumer i, $i = 1, 2, ..., n$ is given by

$$u^i = u^i (c_i, g_i, G) = (1 - a_i) \ln \left(c_i - \frac{b_i}{G} \right) + a_i \ln g_i, \tag{4.4}$$

where

$$0 < a_i < 1,\ b_i > 0,\ \frac{b_i}{G} < (1-t) m_i. \tag{4.5}$$

Condition (4.5) guarantees that consumer i has enough disposable income, $(1 - t)m_i$, to sustain a level of private consumption expenditure, c_i, greater than $\frac{b_i}{G}$ and also a positive level of donation to charity, g_i. It is straightforward to check that $u_1^i > 0$, $u_2^i > 0$, $u_3^i > 0$.

This example can be given the following interpretation. Private (voluntary) contributions to public goods, $\sum_{i=1}^{n} g_i$, plus public contribution, D, financed from income taxation, provide the necessary infrastructure for private consumption, c_i. An increase in aggregate expenditure on infrastructure, $G = D + \sum_{i=1}^{n} g_i$, leads to a higher level of utility for a given level of c_i. Using Definition 1 and (4.4), g_i, G are *strategic complements*.

[24] For the US, education, health and human services account for the greatest proportion of private giving after religion; see Table 3 in Andreoni (2006).

5. Equilibrium giving and public policy

Let us begin with an analogy of competitive markets in an exchange economy. Suppose there are n consumers who have vectors of initial endowments $\boldsymbol{\omega}_1, \boldsymbol{\omega}_2, ..., \boldsymbol{\omega}_n$. Let the price vector be $\mathbf{p}$. Denote the utility maximizing demand vector of the i^{th} consumer by $\mathbf{x}_i(\mathbf{p}, \boldsymbol{\omega}_i)$. The aggregate demand vector is then $\sum_{i=1}^{n} \mathbf{x}_i(\mathbf{p}, \boldsymbol{\omega}_i)$. The aggregate supply is, of course, $\sum_{i=1}^{n} \boldsymbol{\omega}_i$. There is no guarantee that at an arbitrary price vector, $\mathbf{p}$, aggregate desire to consume, $\sum_{i=1}^{n} \mathbf{x}_i(\mathbf{p}, \boldsymbol{\omega}_i)$, will be equal to aggregate supply, $\sum_{i=1}^{n} \boldsymbol{\omega}_i$. A competitive equilibrium price vector is a price vector, $\mathbf{p}^*$, that equates the two:

$$\sum_{i=1}^{n} \mathbf{x}_i(\mathbf{p}^*, \boldsymbol{\omega}_i) = \sum_{i=1}^{n} \boldsymbol{\omega}_i.$$

The competitive equilibrium in charitable donations is determined in an analogous manner, which we turn to, below.

5.1. The aggregate desire to give to charity

When making their charity decision, consumers take as given aggregate donations to charity, G, and determine their optimal charitable contributions, $g_i^*(s,t,G)$. Just as the aggregate demand in competitive markets need not equal actual aggregate supply for all price vectors, the aggregate of all *desired* donations, $D + \sum_{i=1}^{n} g_i^*(s,t,G)$, need not equal *actual* aggregate contributions, G. Therefore, we introduce a new function, F, which represents the aggregate of all desires (public and private) to give to charity:

$$F = D + \sum_{i=1}^{n} g_i^*(s,t,G). \tag{5.1}$$

We substitute $D = tM - s\sum_{i=1}^{n} g_i^*(s,t,G)$ from (3.7) into (5.1) to get

$$F(s,t,G) = tM + (1-s)\sum_{i=1}^{n} g_i^*(s,t,G). \tag{5.2}$$

From (3.13), (5.2) it follows that

$$0 \leq F(s,t,G) < M + \sum_{i=1}^{n} \tau_i - \sum_{i=1}^{n} \underline{c}_i. \tag{5.3}$$

To reduce the length of formulae, let

$$F_{\max} = M + \sum_{i=1}^{n} \tau_i - \sum_{i=1}^{n} \underline{c}_i, \tag{5.4}$$

then (5.3) becomes

$$0 \leq F(s,t,G) < F_{\max}. \tag{5.5}$$

Recalling that individuals take s,t as given as this stage, hence, without loss of generality, we may view $F(s,t,G)$ as a mapping from $[0, F_{\max}]$ to $[0, F_{\max}]$.

The above discussion suggests the following definition.

Definition 2 *: By the aggregate desire to give we mean the mapping,* $F(s,t,G):[0,F_{\max}]\rightarrow[0,F_{\max}]$*, defined by*

$$F(s,t,G)=tM+(1-s)\sum\nolimits_{i=1}^{n}g_i^*(s,t,G).$$

From Definition 2, we get that

$$F_G=\frac{\partial F}{\partial G}=(1-s)\sum\nolimits_{i=1}^{n}\frac{\partial g_i^*}{\partial G}. \tag{5.6}$$

In general, g_i and G could be strategic complements for consumer i but strategic substitutes for consumer j, $j\neq i$. So, we might wish to ask if in some *aggregate* sense, g and G are strategic complements or substitutes. Lemma 2 and (5.6) suggest the following definition.

Definition 3 *: g and G are aggregate strategic complements (substitutes) if, and only if,*

$$\sum\nolimits_{i=1}^{n}\frac{\partial g_i^*}{\partial G}>0\quad(\leq 0).$$

From definition 3, strategic complementarity (or substitutability) for *all* individuals is *sufficient but not necessary* for aggregate strategic complementarity (or substitutability).

Lemma 3 *: g and G are aggregate strategic complements (substitutes) if, and only if,*

$$F_G>0\quad(\leq 0).$$

5.2. Competitive Equilibria

Definition 4 *(Competitive equilibrium in giving): The economy is in a competitive equilibrium if, and only if, the aggregate of all desires to donate to charity, F, equals the aggregate of all donations, G, i.e., $G^*\in[0,F_{\max}]$ is an equilibrium if, and only if,*

$$G^*=F(s,t,G^*).$$

Because we are interested in multiple equilibria, we need the following definition.

Definition 5 *(Isolated equilibrium): An equilibrium, G^*, is isolated if there is a neighborhood of G^* in which it is the only equilibrium.*

Proposition 3 *: (a) An equilibrium, $G^*\in[0,F_{\max}]$, exists and satisfies $0\leq G^*<F_{\max}$.*
(b) If $F_G<1$ (and, in particular, if g and G are aggregate strategic substitutes, i.e., $F_G\leq 0$), then an equilibrium, G^*, is unique.
(c) If $[F_G]_{G^*}\neq 1$, then G^* is an isolated equilibrium.

Proof of Proposition 3: (a) Recall that in a competitive equilibrium, $G^* = F(s,t,G^*)$. Let

$$H(s,t,G) = G - F(s,t,G).$$

If $F(s,t,0) = 0$ then, clearly, 0 is an equilibrium. Suppose $F(s,t,0) > 0$, then

$$H(s,t,0) = -F(s,t,0) < 0.$$

From (5.5), $F(s,t,G) < F_{\max}$, hence,

$$H(s,t,F_{\max}) = F_{\max} - F(s,t,F_{\max}) > 0.$$

Since $H(s,t,G)$ is continuous, it follows that $H(s,t,G^*) = 0$ for some $G^* \in [0, F_{\max})$, i.e.,

$$G^* = F(s,t,G^*) \text{ and } 0 \leq G^* < F_{\max}.$$

(b) From the definition of $H(s,t,G)$, we get

$$\frac{\partial H}{\partial G} = 1 - \frac{\partial F}{\partial G}.$$

If $\frac{\partial F}{\partial G} < 1$, then $\frac{\partial H}{\partial G} > 0$ for all possible values of G. Thus, in this case, the equilibrium is unique. In particular, if g and G are aggregate strategic substitutes then, by Lemma 3, $\frac{\partial F}{\partial G} \leq 0$ and, hence, the equilibrium is unique.

(c) If $[F_G]_{G^*} \neq 1$ then $[H_G]_{G^*} \neq 0$ and, hence, G^* is an isolated solution of $H(s,t,G) = 0$.■

Figure 5.1 illustrates the results in Proposition 3. Three possible shapes of the function $H(s,t,G)$ are shown. Along the curve AED, the condition for *uniqueness*, $F_G < 1$, holds, and we have a unique equilibrium at E. Along the two paths, ABCD and AHD, $F_G > 1$. In this case, we could have a *unique equilibrium* (as in the case of curve AHD) or *multiple equilibria* (as in the case of curve ABCD); see also our examples 4.1 and 4.2.[25]

5.3. Equilibrium analysis

We now investigate how aggregate equilibrium giving to charity, G, responds to the policy instruments, s, t. We therefore consider an equilibrium, G^*, at which $F_G \neq 1$. By Proposition 3c, such an equilibrium is isolated. We can then regard G^* as a C^1 function, $G^*(s,t)$, of s and t in that neighborhood (this is a special case of the implicit function theorem).

Proposition 4 gives some comparative static results for an isolated equilibrium.

[25] In Cooper and John (1998), individual team members choose their effort levels, in a non-cooperative Nash equilibrium, given the effort levels of other team members. As in Proposition 3 the strategic complements turns out to be a necessary condition for multiple equilibria. It is interesting that the same condition is crucial in the strategic and the non-strategic settings.

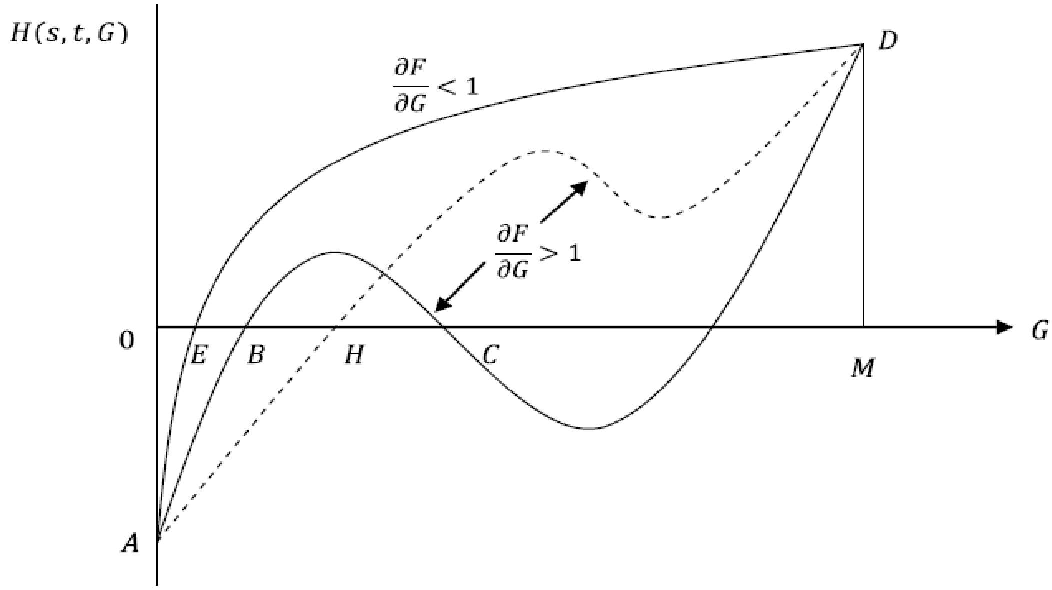


Figure 5.1: Unique and multiple equilibria.

Proposition 4 *: Let G^* be an equilibrium at which $F_G \neq 1$. Then G^* is isolated and (a) $G_s^*(s,t) = \frac{F_s}{1-F_G}$, (b) $G_t^*(s,t) = \frac{F_t}{1-F_G}$, (c) $G_{tt}^*(s,t) = \frac{(F_{tt}+F_{tG}G_t)(1-F_G)+F_t(F_{tG}+F_{GG}G_t)}{(1-F_G)^2}$.*

We now formally define the critical concepts of *normal* and *perverse* comparative statics.

Definition 6 *(Normal and perverse incentives): Comparative statics are* normal *if an increase in subsidy to private charitable giving increases aggregate contributions, i.e., $G_s^* > 0$. Comparative statics are* perverse *if an increase in subsidy to private charitable giving reduces aggregate contributions, i.e., $G_s^* < 0$.*

Proposition 5 *: Let G^* be an equilibrium at which $F_G \neq 1$. Then*
(a) Let $F_s > 0$.
(i) If $F_G < 1$, then $G_s^*(s,t) > 0$. (ii) In particular, if g, G are aggregate strategic substitutes, then $G_s^*(s,t) > 0$. (iii) If $F_G > 1$, then $G_s^*(s,t) < 0$.
(b) Let $F_t > 0$.
(i) If $F_G < 1$, then $G_t^*(s,t) > 0$. (ii) In particular, if g, G are aggregate strategic substitutes, then $G_t^*(s,t) > 0$. (iii) If $F_G > 1$, then $G_t^*(s,t) < 0$.

Proposition 6 *: Let G^* be an equilibrium at which $F_G \neq 1$. Then*
(a) Let $F_s < 0$.
(i) If $F_G < 1$, then $G_s^*(s,t) < 0$. (ii) In particular, if g_i, G are aggregate strategic substitutes, then $G_s^*(s,t) < 0$. (iii) If $F_G > 1$, then $G_s^*(s,t) > 0$.

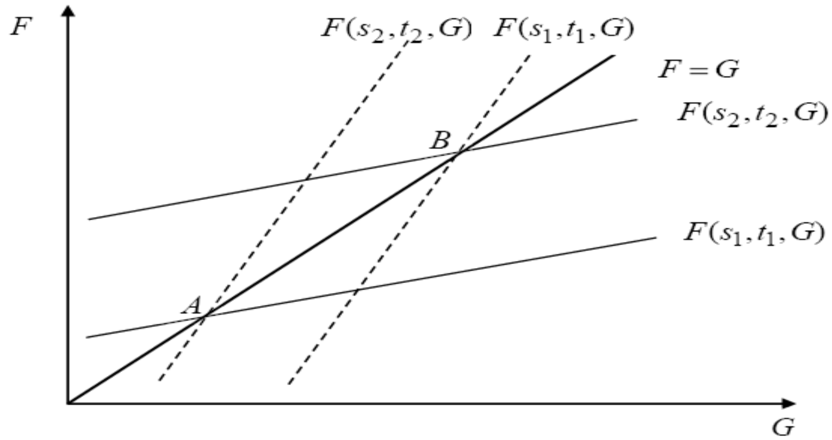


Figure 5.2: $F_s > 0$, $s_1 < s_2$. $0 < F_G < 1$ (continuous, light lines). $F_G > 1$ (dashed lines)

(b) Let $F_t < 0$.
(i) If $F_G < 1$, *then* $G_t^*(s,t) < 0$. *(ii) In particular, if* g_i, G *are aggregate strategic substitutes, then* $G_t^*(s,t) < 0$. *(iii) If* $F_G > 1$, *then* $G_t^*(s,t) > 0$.

5.4. Normal and perverse comparative statics

Figure 5.2 illustrates the two cases of *normal* and *perverse* comparative statics in Proposition 5(a). In Figure 5.2, $F_s > 0$ and $s_1 < s_2$. The 45^o line, $F = G$, is shown as the dark line. The case $0 < F_G < 1$ is illustrated by the two, thin, continuous straight lines, while the other case, $F_G > 1$ is shown by the two dashed lines. Thus, in each case, from Lemma 3, g and G are *aggregate strategic complements*. Figure 5.2 shows the outcome arising from an increase in subsidy from s_1 to s_2, financed by an increased in taxes from t_1 to t_2.

A. *Normal comparative statics* ($G_s^* > 0$): Figure 5.2 illustrates Proposition 5(ai) for the case $F_s > 0$, $0 < F_G < 1$ (although only $F_G < 1$ is required) by an upward shift of the continuous, light, curve $F(s_1, t_1, G)$ to $F(s_2, t_2, G)$. The equilibrium moves from A to B. Hence, the optimal level of aggregate giving, G, is increasing in the subsidy to individual giving, i.e., $G_s^* > 0$.

B. *Perverse comparative statics* ($G_s^* < 0$). This case is shown in Figure 5.2, by an upward shift of the dashed curve $F(s_1, t_1, G)$ to $F(s_2, t_2, G)$, which assumes $F_s > 0, F_G > 1$, as required in Proposition 5(aiii). The equilibrium moves from B to A. Equilibrium aggregate contributions, G, *decrease* as the price of giving reduces (larger s). Because $F_G > 1$, consumers over-react to an increase in G. Thus, paradoxically, G needs to *fall* in order to ensure equilibrium in the market for charity.[26] The

[26] To aid intuition, imagine an upward sloping demand curve that cuts the supply curve from below.

surprising effects of the tax reforms of the 1980's on charitable giving in the US is a potential example of perverse comparative statics; see, Clotfelter (1990). The tax reforms increased the price of giving by reducing the tax preference for charitable donations. Contrary to the predictions, charitable contributions continued to rise in the following years.

6. Equilibrium analysis with multiple equilibria

We now turn our attention to multiple equilibria. Recall, from Proposition 3(b), that if g, G are *aggregate strategic substitutes* (Definition 3), then the competitive equilibrium is unique. Therefore, we assume here that g, G are *aggregate strategic complements*. Hence, from Lemma 3, $F_G > 0$. We assume that $F_s > 0$, $s_1 < s_2$. We distinguish between two main cases here, $F_{GG} < 0$ and $F_{GG} > 0$.

6.1. Engineering moves between equilibria ($F_s > 0, F_t > 0,\ F_G > 0, F_{GG} < 0$): A heuristic discussion

Suppose that the government's objective is to move the economy from a low equilibrium with perverse comparative statics to a high equilibrium with normal comparative statics.[27] We now discuss this critical question that is central to our paper.

Suppose that the situation is as depicted in Figure 6.1. We begin with the policy parameters of the government, s, t_1, D_1. The government levies income tax at the rate t_1 in order to (i) grant a per unit subsidy, s, to private giving, and (ii) finance a direct grant, D_1, to the charity. Using (5.1), the aggregate desire to give to the charity is

$$F\left(s, t_1, G\right) = D_1 + \sum\nolimits_{i=1}^{n} g_i^*\left(s, t_1, G\right)$$

Figure 6.1 plots the locus $F(s, t_1, G)$; given the assumptions, it is increasing and concave. There are two equilibria, a low equilibrium, $G^-(s, t_1)$, and a high equilibrium, $G^+(s, t_1)$ shown respectively as the points 'a' and 'd'. Using, Definition 4, at $G^-(s, t_1)$,

$$G^-(s, t_1) = D_1 + \sum\nolimits_{i=1}^{n} g_i(s, t_1, G^-(s, t_1)) = F\left(s, t_1, G^-\right).$$

An analogous equation holds for $G^+(s, t_1)$. Since $F_G > 1$ at $G^-(s, t_1)$, by Proposition 5(aiii), $G_s^- < 0$ (*perverse* comparative statics). The economy can get stuck at $G^-(s, t_1)$.

Following an increase in income, we get the perverse result that price (and quantity) will fall. In demand theory this case requires atypical assumptions such as *giffen goods*. By contrast, in the charity context such cases arise naturally; aggregate strategic complementarity being a necessary condition. Recall that $G^* = F\left(s, t, G^*\right)$ is the condition for a competitive equilibrium. It can be checked that when $F_s > 0$, $F_G > 1$ an increase in s and a fall in G moves in the direction of restoring equilibrium.

[27] We consider these issues more formally in the context of Examples 4.1 and 4.2 in Section 8, below.

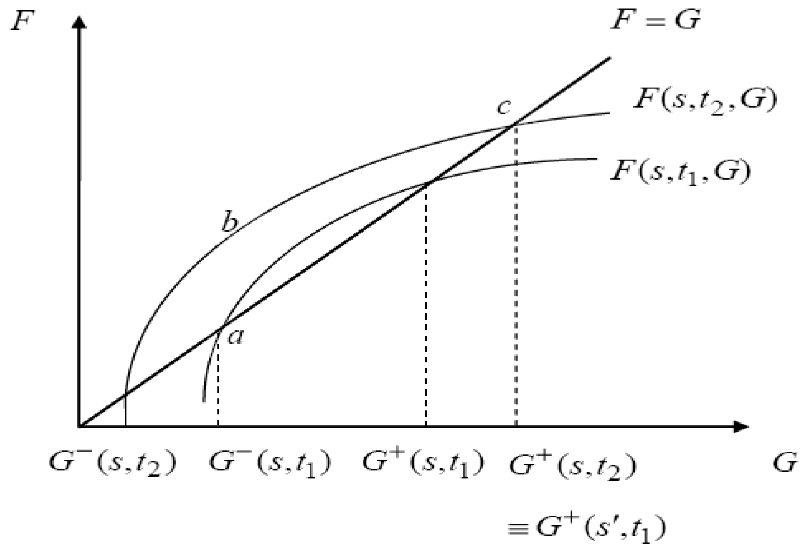


Figure 6.1: **Multiple equilibria when** $F_t > 0,\ F_G > 0, F_{GG} < 0,\ t_1 < t_2$.

Subsidies to private giving are ineffective in moving the economy to the high equilibrium, $G^+(s,t_1)$, because $G_s^- < 0$.[28] However, because $F_G < 1$ at the high equilibrium, by Proposition 5(ai), $G_s^+ > 0$ (*normal* comparative statics). With the aim of moving the economy from the low equilibrium with perverse comparative statics, 'a', to the high equilibrium with normal comparative statics, 'c', consider an alternative policy.

6.1.1. Moving the economy from 'a' to 'c'.

Suppose that the Government makes a direct grant to the charity, $D_2 > D_1$ financed by an income tax levied at the rate $t_2 > t_1$ such that $D_2 > G^-(s,t_1)$. The policy parameters of the government now are s, t_2, D_2. From (5.1), the aggregate desire to give to the charity is

$$F\left(s,t_2,G\right) = D_2 + \sum\nolimits_{i=1}^{n} g_i^*\left(s,t_2,G\right) > G^-(s,t_1) \tag{6.1}$$

In terms of Figure 6.1, the economy is now on the $F(s,t_2,G)$ locus. Since it is assumed in this section that $F_t > 0$, it follows that the $F(.)$ locus has shifts upwards. The inequality in (6.1) follows from the fact that $D_2 > G^-(s,t_1)$ and our assumption that $g_i \geq 0$ for all i. Since the economy is now on the $F(s,t_2,G)$ locus and $G \geq G^-(s,t_1) > G^-(s,t_2)$ the sole candidate for equilibrium is the *high* equilibrium, $G^+(s,t_2) > G^+(s,t_1)$.[29]

[28] An increase in subsidies will, therefore, simply lower $G^-(s,t_1)$ further (see the dashed lines in Figure 5.2).

[29] Our analysis is an equilibrium analysis without any structural dynamics, hence, it is not suitable to answer questions about the transition dynamics between different equilibria. Rather, as in the literature, we have a perfect foresight rational expectations model in which the economy jumps from equilibrium

Once $G^+(s,t_2)$ is established, each individual will make her private giving decision conditional on $G^+(s,t_2)$, thereby, raising her own giving to a higher level such that equilibrium beliefs about $G^+(t_2)$ become self-fulfilling. At this equilibrium,

$$G^+(s,t_2) = D_2 + \sum\nolimits_{i=1}^{n} g_i(s,t_2,G^+(s,t_2)).$$

6.1.2. Optimal public policy at the new equilibrium

So what should *optimal public policy* look like when the high equilibrium, $G^+(s,t_2)$, is established?[30] From a welfare point of view, the two sources of receipts for the charity, direct government grants, D, and individual contributions to charity, g_i, are not identical. The reason is that substantial field, experimental and neuroeconomic evidence supports the assertion that private charitable giving is a source of *warm glow*, but direct government grants are not. Hence, it would be welfare improving to replace government grants by an equivalent amount of private giving, if possible.

Towards this end, stimulating private giving at the low equilibrium (at 'a') was not possible because of perverse comparative statics ($G_s^- < 0$). However, at the high equilibrium, 'c', the comparative statics are normal ($G_s^+ > 0$). For this reason, the incremental direct public grant, $D_2 - D_1$, could be withdrawn and replaced by an equivalent 'warm glow backed' contribution by the private sector. This, as we shall see below in Section 8, is the case for Example 1 of Section 4.

Optimal policy, therefore, uses both fiscal instruments, s,t, simultaneously, as follows. The income tax rate is lowered from t_2 to t_1 because the additional taxes are no longer needed to finance the increased direct government grant, $D_2 - D_1$. Subsidies are efficacious at the high equilibrium (because $G_s^+ > 0$). Hence, subsidies are simultaneously increased from s to s', where s' is chosen such that it induces an additional amount of private giving that exactly offsets the fall in public giving $D_2 - D_1$. In conjunction, this ensures that

$$G^+(s,t_2) \equiv G^+(s',t_1) = D_1 + \sum\nolimits_{i=1}^{n} g_i(s',t_1,G^+(s',t_1)).$$

Hence, an economy stuck at a low equilibrium at $G^-(s,t_1)$ is moved to a high equilibrium $G^+(s,t_2) \equiv G^+(s',t_1)$. Furthermore, temporary direct government grants can be welfare improving[31].

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a to *c*. One could use some adjustment process to study transition dynamics, e.g., the economy could jump from point *a* to *b* when direct government grants increase from D_1 to D_2 and then follow the path along the locus $F(s,t_2,G)$ to the equilibrium point, *c*. However, these adjustment processes are typically ad-hoc, and, so, we do not use them.

[30] The discussion here is heuristic, however, these issues are formally discussed in section 7, below.

[31] We do not consider direct grants by non-governmental institutions, such as the national lottery in the UK or private charitable trusts and organizations. However, such grants can be accomodated in our framework. These will, in principle, perform a role that is similar to direct government grants except that non-governmental organisations cannot levy taxes.

Garrett and Rhine (2007) report that in the US, the growth in private charitable giving over time has been paralleled by similar growth in expenditure by various levels of government. In particular, between 1965 and 2005, the most rapid growth in private giving (g_i in our set up) has been in charities associated with health, education and social services. These are precisely the areas in which there has been large increases in direct government expenditure (D in our set up). The fact that the Government has responded to an increased demand for these services by increasing the direct grant, rather than by increasing the subsidy to private giving, is consistent with our explanation.

6.2. Increasing and convex desire to contribute ($F_s > 0, F_t > 0,\ F_G > 0, F_{GG} > 0$)

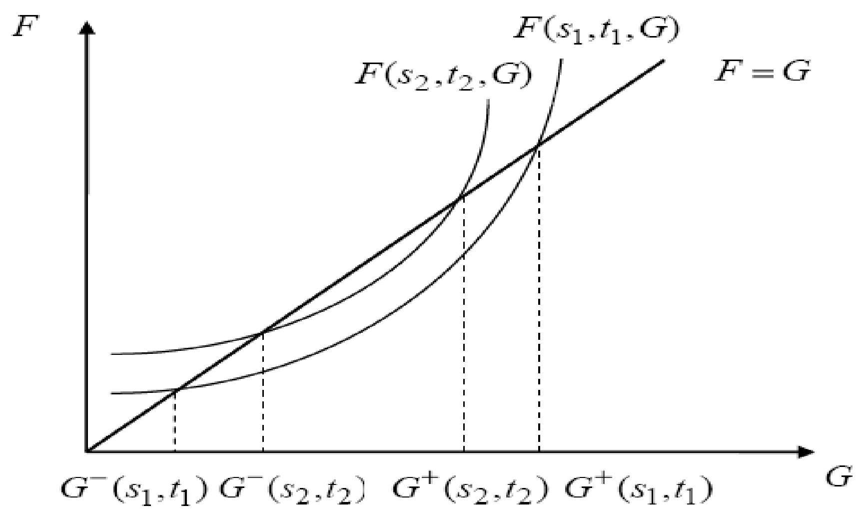

Figure 6.2: The Case $F_s > 0,\ F_G > 0, F_{GG} > 0,\ s_1 < s_2$.

Figure 6.2 shows the case of increasing and convex desire to contribute. Suppose that the original government policy parameters are s_1, t_1 and the economy is on the $F(s_1, t_1, G)$ locus. Furthermore, suppose that the economy is stuck at the low equilibrium, $G^-(s_1, t_1)$. Notice that at the low equilibrium, $F_G < 1$, hence, using Proposition 5(ai), $G_s^- > 0$. In other words, and unlike the situation in section 6.1, the comparative statics at the low equilibrium are normal. Hence, an increase in tax exemptions, from s_1 to s_2, financed by an increase in taxes from t_1 to t_2 would move the economy to the next, higher, equilibrium, $G^-(s_2, t_2)$ on the $F(s_2, t_2, G)$ locus.

However, for whatever reasons, the government might wish to do even better, i.e., move the economy to a high equilibrium, G^+. We consider, next, this possibility in Figure 6.3.

The analysis is similar to that in section 6.1, so we will be very brief. In Figure 6.3, suppose that the initial government policy parameters are s, t_1, D_1. The economy is on

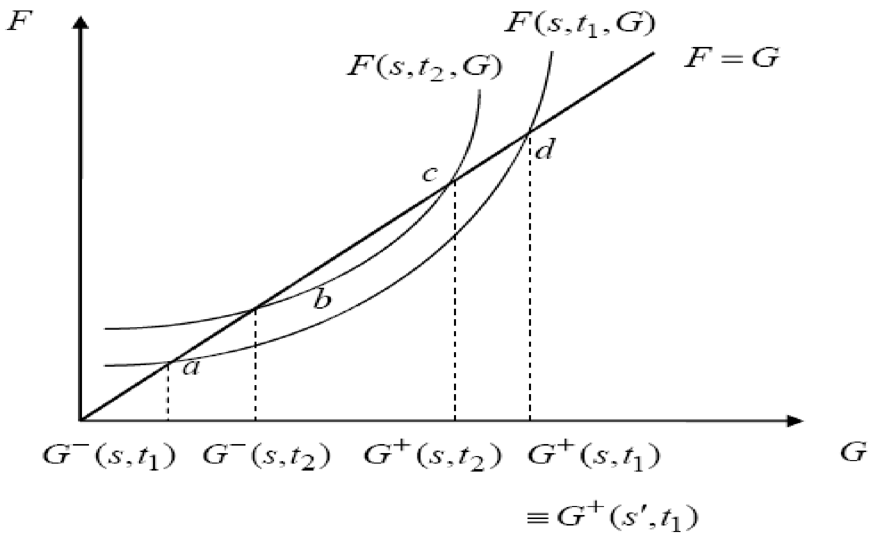

Figure 6.3: Multiple equilibria when $F_t > 0$, $F_G > 0, F_{GG} > 0$, $t_1 < t_2$.

the $F(s, t_1, G)$ locus. Suppose that it is stuck at the low equilibrium, $G^{-}(s, t_1)$, at point 'a', and the policymakers wish to move to a high equilibrium, point 'd'.

Consider a temporary increase in the direct government grant to charity from D_1 to D_2, financed by raising taxes at the rate $t_2 > t_1 \geq 0$ and $D_2 > G^{-}(s, t_2)$. The economy is now on the $F(s, t_2, G)$ locus which is higher because $F_t > 0$. Therefore, the aggregate desire to give to charity is $D_2 + \sum_{i=1}^{n} g_i(s, t_2, G) > G^{-}(s, t_2)$. Hence, the only possible candidate for equilibrium on the $F(s, t_2, G)$ locus is the *high* equilibrium, $G^{+}(s, t_2)$, point 'c', where the comparative statics are *perverse* (because $F_G > 1$).[32]

Once the high equilibrium, $G^{+}(s, t_2)$, is established then the government can withdraw its additional direct grant, $D_2 - D_1$ and eliminate the additional tax that was needed to finance it, hence, the tax rate reduces from t_2 to it's original level t_1. Since the comparative statics at the equilibrium $G^{+}(s, t_2)$ are perverse i.e., $G_s^{+}(s, t_2) < 0$, the subsidy is *reduced* from s to s', such that at point 'd', the final position of the economy,

$$G^{+}(s, t_1) \equiv G^{+}(s', t_1) = D_1 + \sum_{i=1}^{n} g_i(s', t_1, G^{+}(s', t_1)).$$

7. Welfare analysis

Substituting $g_i(s, t, G^*(s, t))$ and $G^*(s, t)$ in the individual utility function (3.10) gives the consumer's indirect utility function

$$v^i(s, t) = u^i\left[(1 - t) m_i + \tau_i - (1 - s) g_i(s, t, G^*(s, t)), g_i(s, t, G^*(s, t)), G^*(s, t)\right]. \quad (7.1)$$

[32] Our earlier comments on transition dynamics apply here as well. It is in the light of those comments that point b in Figure 6.3 could be interpreted.

Differentiating (7.1), implicitly, using Proposition 2(b), or appealing to the envelope theorem, gives

$$v_s^i = u_1^i g_i + u_3^i G_s^*, \tag{7.2}$$

$$v_t^i = -u_1^i m_i + u_3^i G_t^*, \tag{7.3}$$

where, the partial derivatives G_s, G_t are given by Proposition 4(a),(b), respectively.

Substituting the indirect utility function from (7.1) into (3.8) we get the government's indirect (social) utility function

$$V(s,t) = U\left(v^1(s,t), v^2(s,t), ..., v^n(s,t)\right). \tag{7.4}$$

Differentiating (7.4), using the chain rule, and (7.2), (7.3), we get

$$V_s = \sum\nolimits_{i=1}^{n} U_i u_1^i g_i + G_s^* \sum\nolimits_{i=1}^{n} U_i u_3^i, \tag{7.5}$$

$$V_t = -\sum\nolimits_{i=1}^{n} U_i u_1^i m_i + G_t^* \sum\nolimits_{i=1}^{n} U_i u_3^i, \tag{7.6}$$

where subscripts on the utility functions, U, u denote appropriate partial derivatives.

Propositions 7, 8 and 9, below, derive the optimal mix between private and public giving to charity in different cases. These propositions are used extensively in Section 8 and are crucial in determining the optimal public policy at different equilibria.

Proposition 7, below, implies that in an optimum where $G_t^* \leq 0$, no government intervention is needed, *warm glow* and/or *altruism* suffice to maximize social welfare.

Proposition 7 : *If $G_t^* \leq 0$ then $s = t = 0$. Since $t = 0$ in this case, income tax revenues are zero, thus, there are no government grants (i.e., $D = 0$), and all giving to charity is private giving. Conversely, if at a social optimum $t > 0$, then $G_t^* > 0$ (but $G_t^* > 0$ does not necessarily imply that $t > 0$).*

We now show, in Proposition 8, that if subsidies are effective then no direct government grant is needed. This is because when *private* donations replace an identical amount of *public* donations, welfare improves on account of the warm glow received by private givers.

Proposition 8 : *If at an optimum $G_s^* \geq 0$ then s attains its maximum value and the government makes no direct contributions to charity, i.e., $D = 0$. Conversely, if a social optimum involves positive government donations, i.e., $D > 0$ then, necessarily, $G_s^* < 0$.*

Proposition 9 : *If at a social optimum (i) $F_s \geq 0$ and $F_G < 1$, or if (ii) $F_s \leq 0$ and $F_G > 1$, then all charitable contributions come from individual private donations and $D = 0$.*

The intuition behind Proposition 9 can be seen from the results in Propositions 5, 6. In Proposition 9(i), for instance, the comparative statics are normal (and not perverse), hence, private giving can be encouraged through the use of subsidies. Since private giving leads to warm glow, and an improvement in welfare, it is optimal to generate all charitable giving through private giving, rather than by direct government grants. A similar intuition explains Proposition 9(ii).

8. Equilibrium outcomes for the examples in section 4

We now apply the general theory developed so far to the two examples of section 4.

8.1. Example 1: Charitable contributions as public redistribution

Consider the setup of the first example in subsection 4.1 that we now take as given. Let m be the aggregate income of the caring consumers (see Figure 4.1). Then, for $k < p$,

$$m = \sum\nolimits_{i=1}^{k} m_i < \sum\nolimits_{i=1}^{p} m_i = \sum\nolimits_{i=1}^{n} m_i = M. \tag{8.1}$$

And when all consumers who have positive income are caring (i.e., $k = p$), then

$$m = M. \tag{8.2}$$

Also, let

$$A = \sum\nolimits_{i=1}^{k} \frac{1}{a_i} > 0. \tag{8.3}$$

Proposition 10 summarizes the main results.

Proposition 10 : *(a) (Multiple equilibria) The only economically interesting cases occur when* $[m + t(M-m)]^2 > 4(1-s)A$*. In this case, we have two distinct, real, positive, equilibria* $0 < G^{-}(s,t) < G^{+}(s,t)$*. These are given by*

$$G^{\pm}(s,t) = \frac{1}{2}\left[m + t(M-m) \pm \sqrt{[m + t(M-m)]^2 - 4(1-s)A}\right].$$

(b) (Increasing and concave desire to contribute) The aggregate desire to give, $F(.)$*, (i) responds positively to subsides i.e.,* $F_s > 0$ *and, (ii) it is increasing and concave i.e.,* $F_G > 0$*,* $F_{GG} < 0$*. So we have the case depicted in Figure 6.1. Furthermore,*

$$G = G^{+} \Rightarrow F_G < 1;\; G = G^{-} \Rightarrow F_G > 1.$$

(c) (Perverse and normal comparative statics) The comparative statics with respect to the subsidy are perverse at the low equilibrium and normal at the high equilibrium, i.e., $G_s^{-} < 0$*,* $G_s^{+} > 0$*. For* $m < M$ *(and so* $k < p$*) the same holds for the comparative static result with respect to the tax rate, i.e.,* $G_t^{-} < 0$*,* $G_t^{+} > 0$*. For* $k = p$*,* $G_t^{\pm} = 0$*.*

The equilibria are as in Figure 6.1. Suppose an economy is at the low equilibrium, G^-, and also suppose that it is socially desirable to move it to the high equilibrium, G^+. How can this be done? The situation is identical to the one presented in section 6.1, and so we follow the same solution method.

From Proposition 10(b), due to perverse comparative statics at the low equilibrium, $G_t^- \leq 0$. Hence, from Proposition 7, it would appear that the best policy is no intervention i.e., $s = t = 0$, leaving the economy at the low equilibrium, G^-. However, an alternative policy is possible, as we shall now describe; see Figures 8.1, 8.2.

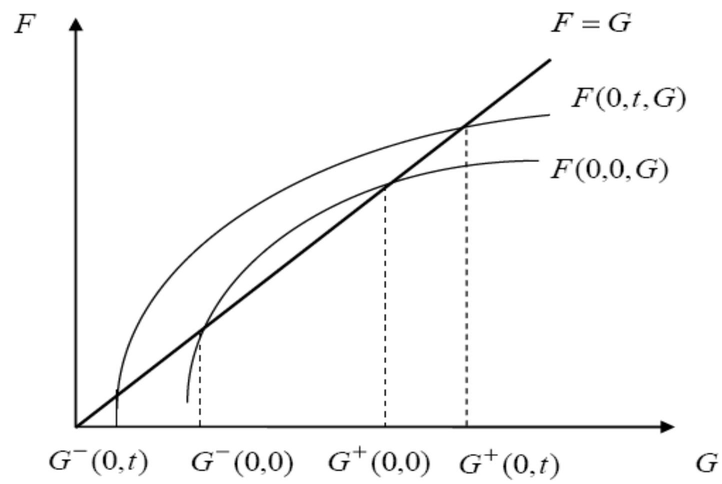

Figure 8.1: Multiple equilibria when $F_t > 0$, $F_G > 0$, $F_{GG} < 0$.

Set the tax rate t as $t = G^-(0,0)/M$. Since $G^-(0,0)$ is an equilibrium, hence, $0 < G^-(0,0) < M$. It follows that $0 < t < 1$ and, hence, it is feasible. Set s, D as follows: $s = 0$ and $D = tM = G^-(0,0)$.

Thus, the government gives a direct grant equal to G^- $(0,0)$ financed with an income tax (any grant $D > G^-(0,0)$ will also work). Since $g_i > 0$ for some i $(i = 1, 2, ..., k)$ it follows, from (3.9), that $G > D = G^-(0,0)$. Hence, because we are now on the $F(0,t,G)$ locus (see Figure 8.1) and the equilibrium aggregate donation $G > G^-(0,0) > G^-(0,t)$, the only possible candidate for equilibrium is

$$G = G^+(0,t). \tag{8.4}$$

Once the economy is at the high equilibrium, $G^+(0,t)$, the fiscal parameters, s,t can be adjusted to their socially optimum values. We now address this issue.

8.1.1. Socially optimal public policy at the new equilibrium

At the low equilibrium, $G^-(0,0)$, private individuals cannot be induced to make additional contributions because of the *perverse* comparative statics, $G_s^-(0,0) < 0$, $G_t^-(0,0) < 0$. In contrast, the comparative statics at the high equilibrium, $G^+(s,t)$, for any values of s,t are *normal*. We now illustrate the insights of subsection 6.1 for the concrete case of Example 4.1. Consider two cases.

1. If $k < p$, so that not all consumers with positive income are caring and so do not contribute to charity, then, $m < M$. Hence, from Proposition 10(c), $G_s^+ > 0$, $G_t^+ > 0$, i.e., the comparative static effects are reversed at the high equilibrium; see Figure 8.2. Depending on the parameter values, the optimal tax rate may be positive, in which case it can be found by setting $V_t = 0$ in (7.6). From Proposition 10(b), at the high equilibrium G^+, $F_s > 0$, and $F_G < 1$, so, Proposition 9 implies that $D = 0$. Thus, once the economy has moved from G^- to G^+, the direct grant from the government to the charity is phased out. In the new, socially optimal, equilibrium, all contributions to charity are exclusively private (because only private contributions are associated with warm glow) and all income tax revenue is used to subsidize private donations to charity.

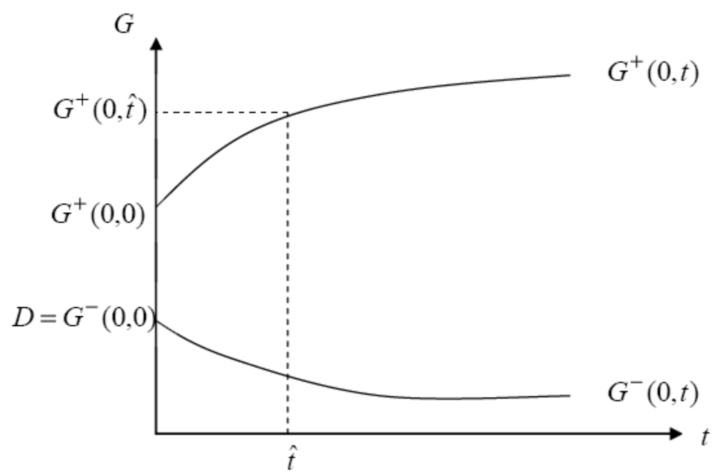

Figure 8.2: The response of equilibrium G to the tax rate.

2. If $k = p$, so that all consumers with positive income contribute to charity, then, from (8.2), $m = M$. Hence, from Proposition 10(c), $G_t^+ = 0$. It follows, from Proposition 7, that $s = t = 0$. Thus, once the (one-off) direct government grant to charity (financed by an income tax) has shifted the economy from the bad equilibrium, G^-, to the good equilibrium, G^+, no further government intervention is needed and the entire charitable contributions come from voluntary private contributions. So at the new optimum, equilibrium is described by $G^+(0,0) = \sum_{i=1}^{n} g_i(0,0,G^+(0,0))$.

8.1.2. A numerical illustration

We now illustrate the insights of the previous subsection by a numerical example. Consider an economy with $n = 1900$ consumers. Of these, 1000 do not have an income of their own and they are supported by charity. The other $p = 900$ consumers each has a positive income $m_i = 1$; $i = 1, 2, ..., 900$. Of these, $k = 450$ get a *warm glow* from giving to charity. Specifically, $a_i = 0.01$ (see the utility function in (4.1)) for $i = 1, 2, ..., 450$. The others do not derive warm glow from giving to charity, i.e., $a_i = 0$ for $i = 451, 452, ..., 1900$. Thus,

$$m = \sum\nolimits_{i=1}^{450} m_i = 450;\ M = \sum\nolimits_{i=1}^{900} m_i = \sum\nolimits_{i=1}^{1900} m_i = 900;\ A = \sum\nolimits_{i=1}^{450} \frac{1}{a_i} = 45000.$$

Initially, assume that $s = t = 0$. Then $G^-(0,0) = 150$, $G^+(0,0) = 300$. Suppose that the locus passing through G^- is considered to be socially inferior to that passing through G^+. How can government policy shift the economy onto the better locus?

Since there are perverse comparative statics at the low equilibrium, i.e., $G_s^- < 0$, $G_t^- < 0$, the best policy would appear to be no intervention: $s = t = 0$. However, there is an alternative. The government sets $s = 0$, $t = 1/6$. This raises a total tax revenue equal to $tM = 900/6 = 150$. Since $G_t^- < 0$, it follows that $G^-(0, 1/6) < 150$ at $t = 1/6$. The government makes a direct grant $D = G^-(0,0) = 150$ to the charity. Since $g_i > 0$, $i = 1, 2, ..., 450$, we must have $G(0, 1/6) = D + \sum\nolimits_{i=1}^{450} g_i(0, 1/6, G) > D = 150 = G^-(0,0) > G^-(0, 1/6)$. Hence, the only possible equilibrium is $G = G^+(0, 1/6)$. Once the economy is on the G^+ locus, s, t can be given their optimal values as shown in the previous subsection.

8.2. Example 2: Charitable contributions as voluntary contributions to a public good

We now consider the second example outlined in section 4.2, above, which the reader may consult before reading on. Define the constants B, C as:

$$B = \sum\nolimits_{i=1}^{n} a_i m_i + t \sum\nolimits_{i=1}^{n} (1 - a_i) m_i; C = \sum\nolimits_{i=1}^{n} a_i b_i \tag{8.5}$$

The main results for this example are listed in Proposition 11, below.

Proposition 11 *: (a) (Multiple equilibria) The only economically interesting cases occur when $B^2 > 4C$. In this case, we have two distinct real positive equilibria $0 < G^- < G^+$. These are given by*

$$G^{\pm} = \frac{1}{2}\left(B \pm \sqrt{B^2 - 4C}\right). \tag{8.6}$$

(b) (Increasing and concave desire to contribute) The aggregate desire to give, $F(.)$ (i) responds positively to taxes, i.e., $F_t > 0$, (ii) is unresponsive to subsidies, i.e., $F_s = 0$, and, (ii) it is increasing and concave, i.e., $F_G > 0$, $F_{GG} < 0$ (as in Figure 6.1). Furthermore,

$$G = G^+ \Rightarrow F_G < 1;\ G = G^- \Rightarrow F_G > 1.$$

(c) (Perverse and normal comparative statics) The comparative statics with respect to the income tax are perverse at the low equilibrium and normal at the high equilibrium, i.e., $G_t^- < 0$, $G_t^+ > 0$. Subsidies are ineffective in influencing aggregate giving, i.e., $G_s = 0$.

From Proposition 11, we know that the economy has two equilibria as in Figure 6.1.

1. The low equilibrium is characterized by low voluntary contributions to the public good, causing low aggregate spending on the public good infrastructure, G^-. From (4.4), to achieve any specific utility level, high private consumption expenditure is needed. From the budget constraint, (3.6), we see that less income can be contributed to the public good, perpetuating the low expenditure on infrastructure.

2. The high equilibrium is characterized by high contributions to the public good, causing high aggregate expenditure on infrastructure, G^+. In turn, this implies that relatively less private consumption expenditure is needed to reach any specific utility level. Hence, relatively more income is left over to donate to charity, perpetuating high expenditure on infrastructure.

Suppose that the economy is at the low equilibrium, G^-, and that it is socially desirable to move the economy to the high equilibrium, G^+. How can this transition be achieved? From Proposition 11(c), we know that incentives in the form of a subsidy will not work because $G_s = 0$. From Proposition 11(c), $G_t^- < 0$, so, from Proposition 7, it would appear that, at the low equilibrium G^-, the best feasible policy is no intervention: $s = t = 0$, leaving the economy at the low equilibrium G^-. However, an alternative policy is possible, as we shall now describe; see Figures 8.1, 8.2. Set a tax rate t, given by

$$t = \frac{G^- (0, 0)}{M}. \tag{8.7}$$

Since $G^- (0, 0)$ is an equilibrium, $0 < G^- (0, 0) < M$. Hence, it follows, from Proposition 3(a), that $0 < t < 1$, which is a feasible tax rate. Set $s = 0$ and $D = tM = G^- (0, 0)$, i.e., the government gives a direct grant to public good provision equal to $G^- (0, 0)$, and financed from an income tax. Since $g_i > 0$ it follows, from (3.9), that $G > D = G^- (0, 0)$. Since we are now on the $F(0, t, G)$ locus (see Figure 8.1) and the equilibrium aggregate donation $G > G^- (0, 0) > G^- (0, t)$, the only possible equilibrium is $G = G^+ (0, t)$. Once the economy is on the high equilibrium, s, t can be adjusted to their socially optimal values, an issue that we now turn to.

8.2.1. Socially optimal public policy at the new equilibrium

From Proposition 11(b) the relevant graphs are as in Figures 6.1, 8.2 and 8.1. Once the economy has moved to the new equilibrium, G^+, the direct grant from the government towards the public good can be phased out. It is welfare improving to do so because of the normal comparative statics at the high equilibrium, and the fact that private giving confers warm glow, while an equivalent amount of direct grants does not.

In the low equilibrium, G^-, we have seen above that the optimal policy solution is $s = t = 0$. However, from Proposition 11(c), we know that at the high equilibrium, $G_t^+ > 0$, and so, the comparative static results are reversed from the low equilibrium. The optimal tax rate, which can be found from (7.6), balances the loss in private consumption against the gain arising from the additional amount of the public good. Also, from (7.5), $V_s > 0$, hence, it is welfare improving to provide additional subsidies. Thus, all tax revenues are used to finance subsidies on charitable donations. In the socially optimal solution at the high equilibrium, G^+, therefore, $s > 0$, $t > 0$ while $D = 0$ (see Proposition 8).

8.2.2. Numerical illustration

As a numerical illustration, consider an economy of $n = 50$ identical consumers, each with income $m_i = 1$. Choose $a_i = 0.1$ and $b_i = 0.8$ (the relevant utility function in given in (4.4)). Suppose that initially, $s = t = 0$. Then (8.5), (8.6) give $G^-(0,0) = 1$ and $G^+(0,0) = 4$, and the feasibility condition (4.5) is satisfied (see Figure 8.1).

Suppose that the economy is, initially, at the low equilibrium $G^-(0,0) = 1$. If the move to a high equilibrium is considered desirable, then, from (8.7) we get that the required tax rate, t, is

$$t = \frac{G^-(0,0)}{M} = \frac{1}{50} = 0.02, \tag{8.8}$$

and the feasibility condition (4.5) still holds at this tax rate. The government uses its entire tax revenue $tM = 1 = G^-(0,0)$ to make a contribution $D = G^-(0,0) = 1$ to the public good. Since $g_i^* > 0$, $G^*(0, 0.02) = D + \sum_{i=1}^{n} g_i^* > 1 = G^-(0,0)$. In terms of Figure 8.1, the economy is on the $F(0, 0.02, G)$ locus. Hence, the only candidate for equilibrium is $G = G^+(0, 0.02)$. Once the economy is on the high equilibrium, $G^+(0, 0.02)$, the policy parameters s, t can be adjusted to their socially optimal values. This will involve the phasing out of the direct grant. Once the final position of the, new, socially optimal, equilibrium is established, the government uses all tax revenues to subsidize voluntary contributions to the public good. The direct grant, here, is only a temporary measure to shift the economy from the low equilibrium to the high equilibrium.

9. Strategic giving

Under strategic giving, each giver, i, takes as given $G_{-i} = G - g_i$, the contributions of all others, and behaves strategically with respect to all others. One would expect that, as the number of individuals increases, the outcomes under the competitive and the strategic approaches should converge. Here, for our two examples in Section 8, we show that this is the case for even reasonably small number of givers.

9.1. Nash equilibria for Example 1 (public redistribution)

We first derive the *symmetric Nash equilibria* (SNE) for Example 1 in Section 8 (see also Figure 4.1). Suppose that all *caring* individuals with positive income are identical. Recall that there was no such symmetry restriction in the case of a competitive equilibrium; *in this sense competitive equilibria are less restricted*. Using the same parameter values as in the numerical example in subsection 8.1.2, we have $a_i = a = 0.01$, $m_i = m = 1$, $i = 1, 2, ..., 450$. For simplicity, we report the case $s = t = D = 0$. Other cases are similar.

Recalling that $G_{-i} = G - g_i$, the optimization problem of the i^{th} such individual is

$$\underset{\langle g_i | G_{-i} \rangle}{Maximize} \quad u^i = \ln c_i + 0.01 g_i \left(g_i + G_{-i} \right), \tag{9.1}$$

subject to the individual budget constraint (3.6). We have replaced total aggregate giving, G, by $g_i + G_{-i}$. The consumer now takes as given, the contribution of all others, G_{-i}. The first order condition to the maximization problem in (9.1) is

$$(1 - g_i)^{-1} = 0.01(G + g_i). \tag{9.2}$$

In a SNE, $g_i = kG$ where $k = 450$. Substituting $g_i = 450G$ in (9.2) we get the following quadratic equation in $G : 4.51G^2 - 2029.5G + 202500 = 0$. Solving for the two Nash equilibria G^{N-}, G^{N+} we get

$$G^{N-} = 149.33, \; G^{N+} = 300.66. \tag{9.3}$$

For non-strategic giving, the two equilibria, given in subsection 8.1.2, were $G^- = 150$, $G^+ = 300$. Comparing to (9.3), we find that the equilibria are virtually identical, as claimed earlier, although the number of givers, 450, is relatively small.

9.2. Nash equilibria for Example 2 (public goods)

We show here, analytically, the effect on the equilibrium magnitudes as $n \to \infty$ and also report simulation exercises for smaller values of n. Rewrite (4.4) as

$$u^i \left(c_i, g_i, G \right) = (1 - a_i) \ln \left(c_i - \frac{b_i/n}{G/n} \right) + a_i \ln g_i. \tag{9.4}$$

Define the constant $d^i = b_i/n$ and rewrite (9.4) as[33]

$$u^i\left(c_i, g_i, G\right) = (1-a_i)\ln\left(c_i - \frac{d^i}{G/n}\right) + a_i \ln g_i \equiv w^i\left(c_i, g_i, \frac{G}{n}\right). \tag{9.5}$$

Substituting out c_i in (9.5) using the budget constraint (3.6) we get

$$w^i\left((1-t)\,m_i - (1-s)\,g_i, g_i, \frac{G}{n}\right) = (1-a_i)\ln\left((1-t)\,m_i - (1-s)\,g_i - \frac{d^i}{G/n}\right) + a_i \ln g_i. \tag{9.6}$$

9.2.1. General case

Under non-strategic (competitive) giving, each consumer chooses her contribution, given the per capita aggregate contribution, G/n. Therefore, in a competitive market equilibrium, the problem of the i^{th} consumer is to maximize (9.6), given G. The first order condition is

$$-(1-s)w_1^i + w_2^i = 0. \tag{9.7}$$

Under strategic giving, on the other hand, each consumer chooses her contribution, given the contribution of all others, G_{-i}. Therefore, the problem of the consumer is to maximize (9.6) given G_{-i}. The first order condition is

$$-(1-s)w_1^i + w_2^i + \frac{1}{n}w_3^i = 0. \tag{9.8}$$

Comparing (9.7), (9.8) we see that, because w_3^i is bounded, as $n \to \infty$, the first order conditions for strategic and non-strategic giving coincide. Note that this result does not depend on any particular function form, hence, it is completely general.

It remains to show, as we did in subsection 9.1, that even for smaller n, the equilibria of the two models are reasonably close. To keep the exposition simple, and to ensure comparability with the literature, we focus on *symmetric equilibria* below. Set $d^i = d$, $a_i = a$ and $m_i = m$ for all i. Furthermore, to simplify the exposition we report the case $s = t = D = 0$.

9.2.2. Symmetric Nash equilibria (SNE)

Using (9.5), (9.8), the first order condition in the case of a SNE is

$$\frac{1-a}{m - g - \frac{nd}{G}}\left(\frac{nd}{G^2} - 1\right) + \frac{a}{g} = 0. \tag{9.9}$$

[33] Note that d^i is now a parameter of the model. This reformulation is necessary because as n increases, G increases and so effectively the parameter b^i in the original formulation falls relative to G. In other words, such a replication of the economy alters the underlying model, which is not admissible. The reformulation in terms of the parameter d^i, however, is not subject to this problem.

In a SNE, $g = \frac{G}{n}$. Substituting $G = ng$ in (9.9) we get the following quadratic equation for individual donation in g : $g^2 - amg + d\,(a - n^{-1}(1-a)) = 0$. Solving this equation, and using superscript 'N' for the Nash outcome, we get the two solutions for individual giving as

$$g^{N\pm} = \frac{1}{2}\left[ma \pm \sqrt{m^2a^2 - 4d\,(a - n^{-1}(1-a))}\right]. \tag{9.10}$$

9.2.3. Symmetric non-strategic (competitive) equilibria

In order to compare with a *symmetric Nash equilibrium*, we now compute the *symmetric competitive equilibrium*. In this case, the consumer takes as given the aggregate contributions, G. Using (9.5), (9.7), the first order condition is

$$\frac{(1-a)}{m - g - \frac{nd}{G}} = \frac{a}{g}. \tag{9.11}$$

In a symmetric equilibrium, $g = G/n$. Substituting $G = ng$ in (9.11) we get a quadratic equation in g : $g^2 - amg + ad = 0$, from which we get the two solutions in a competitive equilibrium, g^- and g^+,

$$g^{\pm} = \frac{1}{2}\left(am \pm \sqrt{a^2m^2 - 4ad}\right). \tag{9.12}$$

Clearly, from (9.10), (9.12), $g^{N+} \to g^+$ and $g^{N-} \to g^-$, as $n \to \infty$.

9.2.4. Simulations

In this subsection we address, numerically, the question of whether the results under strategic and non-strategic giving are similar for reasonably small number of givers? From (9.12), the equilibrium level of individual giving in the *symmetric non-strategic equilibrium* is independent of the number of givers, n. Substituting $m = 1$, $a = 0.1$, $b = 0.8$, $d = \frac{b}{50} = 0.016$ in (9.12), it can be checked that for any n

$$g^- = 0.2,\ g^+ = 0.8. \tag{9.13}$$

Using (9.10), in Table-I, below, we report the simulation results for optimal individual giving in the symmetric strategic case, $g^{N\pm}$, as n increases, using $m = 1$, $a = 0.1$, $d = 0.016$.

TABLE-I : Multiple Nash Equilibria as the Number of Givers Varies

n	100	500	1000	$10,000$
g^{N-}	0.0177	0.0195	0.0198	0.02
g^{N+}	0.0823	0.0805	0.0802	0.08

It is evident that even for relatively small values of n, the SNE, g^{N-} and g^{N+}, converge rapidly to the competitive equilibria, g^- and g^+, given in (9.13). These numbers seem to accord with the actual size of a typical individual donation relative to the budget of a charity. Furthermore, it allows one to conduct comparative static results that would be difficult in the case of strategic giving, even if restricted to the case of a symmetric Nash equilibrium.

10. Conclusions

Private philanthropic activity is much studied in economics and is of immense economic importance. The empirical evidence suggests that the vast bulk of giving activity is undertaken by very large numbers of diverse, dispersed and small givers. For givers of such nature, the competitive equilibrium model in economics would seem to be a natural fit. However, the existing literature has typically described giving activity within the ambit of a *symmetric Nash equilibrium* in strategic giving. Furthermore, the literature has, by and large, restricted itself to a *unique equilibrium* in giving. We relax both these features of the existing literature.

Coordination problems and multiple equilibria are the norm in situations of economic interest, but they are often ruled out by assumption. It is not surprising that uncoordinated individual giving might lead to the possibility of multiple equilibria in private giving. We show that a necessary condition for multiple equilibria is *aggregate strategic complementarity* between own-giving and aggregate-giving to a charity. Strategic complementarity at the level of each giver is sufficient but not necessary for aggregate strategic complementarity.

Once one allows for multiple equilibria that can potentially be ranked according to some social criteria, the following question arises naturally. If society is stuck at a *low* equilibrium, characterized by low levels of giving, can public policy help it to attain a *high* equilibrium? This is not a trivial question because our understanding of engineering moves between alternative equilibria is not very well developed. In the context of private philanthropic activity, we show that temporary direct government grants to charities allow for such engineering of moves between equilibria.[34] The significance of this result, to our mind, goes beyond the philanthropic context that we are interested in.

We also perform a welfare analysis and examine the optimality of alternative mixes of private and public contributions to charity. We show, for some parameter values, that additional incentives to giving, can reduce the aggregate of private contributions in equilibrium (*perverse comparative statics*). In such cases, it might be best to finance charitable giving, if required, by direct government grants financed through taxation. For other parameter values, however, giving to charity responds well to incentives (*normal comparative statics*). In this case, charitable giving should be entirely funded by private individual

[34]This result, in our model, also has the potential to explain the efficacy of *seed money*, *leadership contributions*, and direct grants by large donors, e.g., the National Lottery (as in the UK).

contributions, possibly subsidized through taxation. This is welfare improving in this case because private giving leads to warm glow, while direct government grants do not.

Throughout we focus on equilibrium analysis. The dynamics of time paths from one equilibrium to another involve fundamental questions about the precise learning mechanisms to be used. Although progress on learning mechanisms is being made and in due course such mechanisms may enrich our model, currently such issues lie beyond the scope of this paper.

11. Appendix

Proof of Proposition 1: Suppose

$$\underline{c}_i < (1-t)\, m_i + \tau_i. \tag{11.1}$$

Let

$$c_i\,(g_i) = (1-t)\, m_i + \tau_i - (1-s)\, g_i, \tag{11.2}$$

then, from (3.10), we get

$$U^i\,(g_i, G; s, t) = u^i\,(c_i\,(g_i)\,, g_i, G)\, c_i\,(g_i)\,, \tag{11.3}$$

$$U^i_1\,(g_i, G; s, t) = u^i_2\,(c_i\,(g_i)\,, g_i, G) - (1-s)\, u^i_1\,(c_i\,(g_i)\,, g_i, G)\,, \tag{11.4}$$

From (11.2), note that, as $g_i \uparrow \frac{1}{1-s}\left[(1-t)\, m_i + \tau_i - \underline{c}_i\right]$, $c_i\,(g_i) \downarrow \underline{c}_i$ and, hence, $u^i_1\,(c_i\,(g_i)\,, g_i, G) \uparrow \infty$ (recall, (3.4)). On the other hand, since, by assumption, $u^i_{22} \leq 0$, u^i_2 remains bounded above as $g_i \uparrow \frac{1}{1-s}\left[(1-t)\, m_i + \tau_i - \underline{c}_i\right]$. Hence, from (11.4), it follows that $U^i_1 \downarrow -\infty$ as $g_i \uparrow \frac{1}{1-s}\left[(1-t)\, m_i + \tau_i - \underline{c}_i\right]$. Therefore, there must be a $\overline{g}_i < \frac{1}{1-s}\left[(1-t)\, m_i + \tau_i - \underline{c}_i\right]$ such that $U^i_1 < 0$ at $g_i = \overline{g}_i$. From (3.11) it then follows that $U^i_1 < 0$ for all $g_i \geq \overline{g}_i$. Thus, if there is a g^*_i that solves the consumers maximization problem (3.12), then, necessarily, $g^*_i \leq \overline{g}_i$. If u^i can be extended as a continuous function to the boundary, $g_i = 0$, then take $\underline{g}_i = 0$. Otherwise, using the assumptions that $u^i_{11} \leq 0$ and $u^i_2 \uparrow \infty$ as $g_i \downarrow 0$, we can show that there is a $\underline{g}_i > 0$ such that g^*_i (if it exists) must satisfy $g^*_i \geq \underline{g}_i$. U^i is continuous on the compact interval, $\left[\underline{g}_i, \overline{g}_i\right]$, and, hence, attains a maximum at some $g^*_i \in \left[\underline{g}_i, \overline{g}_i\right]$. But then g^*_i must be a global maximum. Furthermore, $0 \leq g^*_i < \frac{1}{1-s}\left[(1-t)\, m_i + \tau_i - \underline{c}_i\right]$ and, from (3.11), g^*_i is unique. This establishes parts (a) and (b). If, in addition, (3.5) holds, then a similar argument shows that $g^*_i > 0$. This establishes part (c). ■

Proof of Proposition 2: If $g^*_i > 0$, then, in the light of part (b) of Proposition 1, g^*_i is an interior maximum and part (a) follows. Part (b) follow as a simple consequence of (3.10). Appealing to the implicit function theorem, or differentiating the identity, $(1-s)\, u^i_1 = u^i_2$, establishes parts (c), (d) and (e). ■

Proof of Lemma 1: Follows from (3.10) and Definition 1. ■

Proof of Lemma 2: (a) First, suppose that $g_i^* > 0$. The result then follows from Lemma 1 and Proposition 1 d(iii).

(bi) Suppose that $\frac{\partial g_i^*}{\partial G} \geq 0$ (≤ 0) holds for all $g_i^* \geq 0$. Then, a fortiori, it holds for all $g_i^* > 0$. Hence, by (a), g_i and G are strategic complements (substitutes). (bii) Since u^i is C^2, g_i^* is C^1, i.e., $\frac{\partial g_i^*}{\partial G}$ is continuous. Now, suppose that g_i and G are strategic complements (substitutes). Then, by (a), $\frac{\partial g_i^*}{\partial G} \geq 0$ (≤ 0) holds for all $g_i^* > 0$. By the continuity of $\frac{\partial g_i^*}{\partial G}$, it follows that $\frac{\partial g_i^*}{\partial G} \geq 0$ (≤ 0) holds for $g_i^* \geq 0$. ■

Proof of Lemma 3: Follows from (5.6) and Lemma 2. ■

Proof of Proposition 4: Let G^* be an equilibrium at which $F_G \neq 1$. Then, from Proposition 3c, G^* is isolated. By Definition 4, $G^* = F(s, t, G^*)$. Differentiating this implicitly, and rearranging, gives the required results. ■

Proof of Propositions 5 and 6: Obvious from Proposition 4(a),(b), respectively, and Lemma 3. ■

Proof of Proposition 7: Let (s, t) maximize social welfare (7.4). We have assumed that $m_i \geq 0$, with some $m_i > 0$, $u_1^i > 0$, $u_3^i \geq 0$ and $U_i > 0$. If $G_t \leq 0$ then, from (7.6), $V_t < 0$ and it follows that necessarily, $s = t = 0$. The last statement in the proposition is simply the contrapositive of the first. ■

Proof of Proposition 8: Let (s, t) maximize social welfare (7.4). We have assumed that $g_i \geq 0$, with some $g_i > 0$, $u_1^i > 0$, $u_3^i \geq 0$ and $U_i > 0$. If $G_s \geq 0$ then the first order condition (7.5) implies that $V_s > 0$ and so, the subsidy s attains its maximum possible value. Recall from subsection 3.1 that, by assumption, the maximum value of s is bounded away from unity. The consequence of $s = 1$ is that, from (3.6), the price of giving is zero and so any individual with $u_2^i > 0$ would like to give an infinite amount to charity. Since individual private giving can be increased substantially by decreasing its price, it is best to channel all giving privately because of the additional benefit arising to each individual from warm glow and, therefore, $D = 0$. The last statement in the proposition is simply the contrapositive of the first. ■

Proof of Proposition 9: From Proposition 4(a), $G_s^*(s,t) = \frac{F_s}{1-F_G}$. So when $F_s \geq 0$ and $F_G < 1$, or if $F_s \leq 0$ and $F_G > 1$, we get $G_s^*(s,t) > 0$. Proposition 8 then implies that s attains its maximum value and direct government grants are zero. ■

Proof of Proposition 10: Maximizing (4.1) subject to (3.6) gives

$$g_i^*(s,t,G) = \frac{1-t}{1-s} m_i - \frac{1}{a_i G}; \quad i = 1, ..., k, \tag{11.5}$$

$$c_i^*(s,t,G) = \frac{1-s}{a_i G}; \quad i = 1, ..., k. \tag{11.6}$$

From (4.2) and (11.5) we see that $g_i^*(s,t,G) > 0$ and from (11.6) $c_i^*(s,t,G) > 0$. On the other hand, from (4.3) we get that

$$g_i^* \equiv 0; \quad i = k+1, k+2, ..., n. \tag{11.7}$$

From (5.2), (8.1) - (8.3), (11.5), the aggregate desire to give to charity is

$$F(s,t,G) = m + t(M-m) - \frac{1-s}{G}A, \tag{11.8}$$

$$\Rightarrow F_t = M - m \geq 0. \tag{11.9}$$

The inequality in (11.9) is strict for $m < M$, i.e., $k < p$. Direct differentiation of (11.8) proves part (b) of the proposition, i.e., $F_s > 0$, $F_G > 0$, $F_{GG} < 0$.

$$F_s = \frac{A}{G} > 0,\ F_G = (1-s)\frac{A}{G^2} > 0,\ F_{GG} = -2(1-s)\frac{A}{G^3} < 0. \tag{11.10}$$

From Definition 4 and (11.8), an equilibrium G, must satisfy the quadratic equation

$$G^2 - [m + t(M-m)]G + (1-s)A = 0. \tag{11.11}$$

The quadratic equation in (11.11) has the solutions

$$G^{\pm} = \frac{1}{2}\left[m + t(M-m) \pm \sqrt{[m + t(M-m)]^2 - 4(1-s)A}\right]. \tag{11.12}$$

For real roots we need $[m + t(M-m)]^2 \geq 4(1-s)A$. If $[m + t(M-m)]^2 = 4(1-s)A$ then, from (11.10), $F_G = 1$. From Proposition 4(a),(b) it would follow that G_s^*, G_t^* are undefined. Hence, the only economically interesting cases occur when,

$$[m + t(M-m)]^2 > 4(1-s)A, \tag{11.13}$$

in which case we have two distinct real positive equilibria

$$0 < G^- < G^+. \tag{11.14}$$

Using the fact that for real numbers a, b, $a > b$: $\sqrt{a-b} > \sqrt{a} - \sqrt{b}$, as well as (11.10) and (11.12) - (11.14), we get

$$G = G^+ \Rightarrow F_G < 1,\ G = G^- \Rightarrow F_G > 1. \tag{11.15}$$

From Proposition 4(a),(b), (11.9), (11.10), (11.15) we get

$$G_s^+ > 0, \quad G_s^- < 0, \tag{11.16}$$

$$G_t^+ > 0, \quad G_t^- < 0 \text{ (for } m < M, \text{ i.e., } k < p), \tag{11.17}$$

$$G_t^{\pm} = 0 \text{ (for } m = M, \text{ i.e., } k = p). \tag{11.18}$$

Proof of Proposition 11: Applying Proposition 2(b) to the utility function (4.4), and using the budget constraint (3.6), gives

$$g_i^*(s,t,G) = \frac{a_i}{1-s}\left[(1-t)\,m_i - \frac{b_i}{G}\right], \tag{11.19}$$

$$c_i^*(s,t,G) = (1-a_i)\left[(1-t)\,m_i - \frac{b_i}{G}\right] + \frac{b_i}{G}. \tag{11.20}$$

From (4.5) and (11.19), (11.20), we see that $g_i^*(s,t,G) > 0$ and $c_i^*(s,t,G) > \frac{b_i}{G}$. Furthermore, it is straightforward to verify that the second order conditions also hold. Hence, given s,t,G; $g_i^*(s,t,G)$, $c_i^*(s,t,G)$ maximize utility (4.4) subject to the budget constraint (3.6), and are unique.

Substituting from (11.19) into (5.2) the aggregate desire to give, $F(s,t,G)$ is:

$$F(s,t,G) = \sum\nolimits_{i=1}^{n} a_i m_i + t \sum\nolimits_{i=1}^{n} (1-a_i)\,m_i - \frac{1}{G}\sum\nolimits_{i=1}^{n} a_i b_i. \tag{11.21}$$

From (11.21) we get:

$$F_s = 0; F_t = \sum\nolimits_{i=1}^{n} (1-a_i)\,m_i > 0; F_G = \frac{1}{G^2}\sum\nolimits_{i=1}^{n} a_i b_i > 0; F_{GG} = -\frac{2}{G^3}\sum\nolimits_{i=1}^{n} a_i b_i < 0. \tag{11.22}$$

From (11.22) and Proposition 4(a), we get

$$G_s^* = \frac{F_s}{1-F_G} = 0. \tag{11.23}$$

From (11.22) and Proposition 9, it follows that, at a social optimum, $D = 0$, i.e., no direct grant from the government to the charity is involved. Giving to charity is entirely funded by private donations, which are subsidized from taxation if $s > 0$, $t > 0$.

To make further progress, we need to determine the equilibrium values of G. From (5.2), (11.21) and Definition 4, the equilibrium values of G are the solutions to the equation

$$G = \sum\nolimits_{i=1}^{n} a_i m_i + t \sum\nolimits_{i=1}^{n} (1-a_i)\,m_i - \frac{1}{G}\sum\nolimits_{i=1}^{n} a_i b_i. \tag{11.24}$$

Substituting (8.5) in (11.24) we get

$$G^2 - BG + C = 0, \tag{11.25}$$

with solutions

$$G^{\pm} = \frac{1}{2}\left[B \pm \sqrt{B^2 - 4C}\right]. \tag{11.26}$$

If $B^2 < 4C$, then no equilibrium exists. If $B^2 = 4C$ then a unique equilibrium exists, and is $G = \frac{B}{2} = \sqrt{C}$. But then, from (11.22), (8.5), $F_G = 1$. In this case, neither G_s nor G_t are defined, see Proposition 4(a),(b). Hence, the only interesting case is when $B^2 > 4C$.

In this case, (11.25) has two distinct real positive roots:

$$0 < G^- < G^+. \tag{11.27}$$

Using the fact that for real numbers $a > b > 0$: $\sqrt{a-b} > \sqrt{a} - \sqrt{b}$, as well as (11.22), (8.5) and (11.26) - (11.27), we get

$$G = G^+ \Rightarrow F_G < 1, \, G = G^- \Rightarrow F_G > 1. \tag{11.28}$$

From (11.22), (11.28) and Proposition 4(b), we get $G_t^+ > 0$, $G_t^- < 0$.

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