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INTERGENERATIONAL COMPLEMENTARITIES IN EDUCATION AND THE RELATIONSHIP BETWEEN GROWTH AND VOLATILITY

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Abstract

We construct an overlapping generations model in which parents vote on the tax rate that determines publicly provided education and offspring choose their effort in learning activities. The technology governing the accumulation of human capital allows these decisions to be strategic complements. In the presence of coordination failure, indeterminacy and, possibly, growth cycles emerge. In the absence of coordination failure, the economy moves along a uniquely determined balanced growth path. We argue that such structural differences can account for the negative correlation between volatility and growth.

Keywords: Human Capital, Economic Growth, Volatility

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1 Introduction

A relatively recent line of thought has identified the significance of coordination problems for the process of economic growth and development.[1] The view that growth-promoting factors, such as R&D, physical and human capital accumulation etc., are endogenously determined by individuals who respond optimally to various characteristics of the socio-economic environment, provides an idea on why coordination issues may prove important for the long-run development prospects: insofar as the actions by others affect a person's economic outcomes, one of the important characteristics that may affect that person's decisions and actions is her reflection of how others will decide and act. As it is well known from the seminal analysis of Cooper and John (1988), if individuals fail to coordinate their actions in an environment like the one described above, then multiple Pareto-ranked equilibria may emerge. If these actions are determinants of growth-promoting activities, then coordination failure implies that, at least a priori, it is equally likely for an economy to experience low or high growth prospects, depending on whether individuals are respectively pessimistic or optimistic when anticipating the actions of others.

The idea of multiple (low or high) growth paths evokes one of the most irrefutable facts of the world's economic development experience: in terms of both levels and growth rates, the world's per capita income distribution appears to be polarised and, with minor exceptions, some countries are permanently trapped in the lower quartiles of this distribution.[2] Indeed, some authors have built models in which persistent differences in long-term macroeconomic performance are attributed to the multiple equilibria that arise when there are failures of coordinating actions that are strategic complements (e.g., Redding, 1996; Palivos, 2001).[3] This line of research has, thus, sought to provide an explanation for the persistent differences in the world distribution of incomes and growth rates, which differs from the 'path dependence' hypothesis – a hypothesis that has been criticised on the

[1] For example, see the survey by Hoff (2001).
[2] For relevant evidence, see Quah (1997) and Canova (2004) among others.
[3] Strategic complementarity "implies that an increase in the action of all agents expect agent *i* increases the marginal return to agent *i*'s action" (Cooper and John, 1988; p. 445). Hence agent *i* will respond by raising her activity level.

basis that many industrialised countries did not happen to be rich at the initial stages of their development. Many of them were actually poor.[4]

The emergence of multiple growth equilibria, due to a combination of strategic complementarities and coordination failures, possesses an additional explanatory power when it comes to overall macroeconomic performance. In particular, it can also (partially) explain the incidence of growth volatility within an economy. In order to make the argument more transparent, consider an example where the accumulation of a growth promoting factor depends on the actions of two distinct groups of agents. In addition, suppose that these actions are strategic complements. If these groups fail to coordinate their actions, multiple equilibria may emerge depending of how each group expects the other to act. In a dynamic setting, as this process continues over time, there is nothing to preclude the possibility that in some periods agents may choose actions associated with high growth while in other periods they may choose actions associated with low growth. Periods of strong economic activity may be followed by periods of weak economic activity and vice versa, depending on how some agents expect others to behave and act. Of course, this is an explanation for the emergence of cycles which lies on structural characteristics of the economic environment – particularly, the intrinsic uncertainty that pertains complementary decisions by distinct (groups of) individuals – and it, thus, differs from the 'real business cycle' literature which views fluctuations as a natural outcome associated with the extrinsic uncertainty caused by the presence of exogenous shocks.[5]

The contribution of this paper is twofold. First, we identify a new type of strategic complementarity and coordination failure that may generate multiple growth equilibria. Second, given that the manner through which agents coordinate their actions is a structural characteristic of the economic environment, we are able to provide a novel explanation for the negative correlation between output growth and its volatility. We do this by considering an additional scenario where such coordination failures are not prevalent.

Our analysis builds upon an overlapping generations model in which the engine of growth is the accumulation of human capital. The actions of both young (offspring) and adults (parents) affect the formation of human capital. In particular, the parents vote on the

[4] Examples of path-dependent multiple equilibria are provided in the analyses of Azariadis and Drazen (1990), Galor and Zeira (1993), Ceroni (2001) and Chakraborty (2004) among others.
[5] A nice exposition of the 'real business cycle' literature and its fundamental ideas is provided by Stadler (1994).

tax rate that determines the revenues available to the government for the provision of public education, while the offspring get greater benefits from publicly provided education when the effort they devote towards learning activities is relatively high. The technology governing the evolution of human capital allows these decisions to be strategic complements. Specifically, the effort devoted by the young is an increasing function of the tax rate chosen by parents which, by itself, is an increasing function of the offspring's learning effort.

We begin with a scenario in which these actions are subject to coordination failure. This is a case where multiple equilibria emerge, including a set of equilibria in which both cohorts choose no provision (i.e., no effort by offspring and a zero tax rate chosen by parents), and sets of equilibria entailing positive effort by the young and a positive tax rate as chosen by the adult voters. Subsequently, we consider a scenario in which the distinct actions of these two cohorts are not beset by failures of coordination – a scenario which results in a uniquely determined set of equilibrium choices. Given that the growth indeterminacy apparent in the former scenario is inherently linked with the idea of growth cycles, we undertake a straightforward comparison between these two situations and find that the growth rate in the latter scenario is strictly higher that any of the multiple growth rates that could emerge in the former one. Consequently, we argue that there is a negative correlation between the growth rate of output and its volatility.[6]

Our result on the link between volatility and growth, together with the mechanism that drives it, finds support from existing empirical evidence. Particularly, the influential empirical study of Ramey and Ramey (1995) finds a strong negative correlation between economic volatility and average GDP growth. They also find that government spending volatility is negatively, and significantly, associated with the growth rate of GDP.

[6] A paper by Glomm and Ravikumar (1995) also shows that endogenously determined spending can result in equilibrium indeterminacy and, possibly, cycles. However, there are significant differences between their analysis and ours. Firstly, the mechanism leading to their result is different as it rests on the ideas that, (i) the young generation's education effort depends on the expectation of the future tax rate that will be chosen by the same generation when it becomes old, and (ii) the chosen tax rate depends on aggregate human capital due to the fact that the 'warm glow' argument in the utility function is introduced with CRRA coefficient which is different in comparison to the one attached to the remaining utility arguments; therefore, their result is not due to strategic complementarities in the decision making process of two distinct cohorts of agents. Put differently, we find multiple equilibria even with simple functional forms that imply uniqueness in their model. Secondly, when their parameter values allow multiple equilibria, they find an inverse relationship between the tax rate and income. In contrast, our model shows that the 'high growth' equilibrium actually corresponds to the relatively high tax rate. Finally, they do not examine the relationship between growth and volatility as we do in this paper.

The rest of the paper is structured as follows. In Section 2, we present the general set-up of our model. Section 3 solves and analyses the case with coordination failure while Section 4 does the same for the cases where coordination failure is absent. In Section 5, we analyse and discuss the implications for the relationship between volatility and growth. Section 6 concludes.

2 The Basic Structure

We consider an overlapping generations economy in which time is discrete and indexed by $t \in \mathbb{N}$. Each period, a cohort of unit mass is born. Agents within the cohort are identical and live for two periods. They are 'young' (or 'offspring') in the first period of their lifetime and 'old adults' (or 'parents') in the second one.[7] The young are endowed with one unit of time which they can allocate between activities that augment their human capital (e.g., formal schooling) and leisure. The old are also endowed with a unit of time which, combined with their human capital (determining knowledge, efficiency and expertise), they supply inelastically to firms in exchange for the prevailing market wage. Adults are also 'voters' in the sense that they cast a vote on their preferred marginal tax rate that the government imposes on their labour income. Their disposable income (i.e., the residual after taxation) finances their consumption. The revenues collected by the government are utilised so as to finance activities that support the qualitative characteristics of education (e.g., the quality of schools/colleges/universities, scholarships, research and teaching support etc.) and, therefore, promote the formation of human capital. The government abides by a balanced-budget rule each period.

There is single, perishable consumption good in the economy. It is produced and supplied by perfectly competitive firms who employ efficient labour so as to produce Y_t units of output according to

$$Y_t = AH_t, \quad A > 0, \tag{1}$$

where H_t is the aggregate stock of human capital. It also corresponds to the economy's available units of efficient labour because agents (whose large population is normalised to

[7] We implicitly assume that, just like an amoeba, each adult reproduces asexually and gives birth to one offspring.

one) supply one unit of 'raw' time each. Profit maximisation implies that the equilibrium market wage per unit of (efficient) labour is constant at $w_t = A \ \forall t$.

An agent born in period t enjoys utility over her whole lifetime according to

$$u^t = \theta \ln(1-e_t) + \psi \ln(c_{t+1}) + (1-\theta-\psi)\ln(w_{t+2}h_{t+2}), \quad \theta, \psi \in (0,1), \tag{2}$$

where e_t denotes schooling effort when young and c_{t+1} denotes consumption when old.[8] We implicitly assume that children's consumption is incorporated into the consumption of parents. The last term of the utility function indicates that parents are imperfectly altruistic towards their offspring. Specifically, a parent gets satisfaction by observing her offspring's realised income. This is meant to capture the idea that parents care about their offspring's future prospects and social status (both being enhanced through more advanced knowledge and/or increased income).

We assume that, when young, a person can pick up a fraction $v \in (0,1)$ of the existing (average) level of human capital H_t without effort. This may happen, for example, through some type of home tutoring or by simple observation. The government provides goods and services that increase the potential human capital that a person can acquire. In particular, by providing one unit of output per young person, the government can increase her potential human capital by $\varphi > 0$ units. Nevertheless, the young person must provide resources, which take the form of effort (or foregone leisure) in order to benefit from the government's offer of education. Specifically, by devoting a fraction e_t of her potential leisure time, she can assimilate a fraction $p(e_t) \in [0,1]$ ($p' > 0$) of the publicly provided human capital. Denoting the public expenditures per person by g_t, our assumptions imply that human capital is formed according to $h_{t+1} = vH_t + \varphi g_t p(e_t)$. Given that each young person has one unit of available time, the remaining analysis shall utilise the specific functional form $p(e_t) = e_t$. Therefore,[9]

$$h_{t+1} = vH_t + \varphi g_t e_t . \tag{3}$$

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[8] The superscript t on the left-hand side indicates the time of birth of the generation enjoying utility through this function. A similar notation applies to other functions below.

[9] This technology for the accumulation of human capital shares common features with de Gregorio and Kim (2000) and Ceroni (2001) among others. However, none of these have combined both effort (by offspring) and endogenously determined resources (by voters and the public sector) as complementary inputs within the same type of technology. Glomm and Ravikumar (1992, 1995) include both types of inputs in the formation of human capital, but they also assume that each input is essential for a positive stock of human capital.

The government finances the provision of goods and services towards education by utilising its total revenues from labour income taxation $\tau_t w_t H_t$.[10] Given that there is a unit mass of young agents, spending per person corresponds to

$$g_t = \tau_t w_t H_t . \tag{4}$$

All adults are liable to income taxation. Therefore, they will meet their consumption needs out of their disposable income. Thus,

$$c_{t+1} = (1-\tau_{t+1}) w_{t+1} h_{t+1} . \tag{5}$$

As indicated earlier, the electorate is comprised by the adults who cast a vote on their preferred tax rate. Therefore, the problem of an agent born it t is to choose e_t, c_{t+1} and τ_{t+1} so as to maximise (2) subject to (3), (4) and (5), taking H_t, H_{t+1}, w_t, w_{t+1} and w_{t+2} as given. Equivalently, we can substitute (3)-(5) in (2) and write lifetime utility as

$$\begin{aligned} u^t &= \theta \ln(1-e_t) + \psi \ln[(1-\tau_{t+1}) w_{t+1} (vH_t + \varphi e_t \tau_t w_t H_t)] \\ &\quad + (1-\theta-\psi) \ln[w_{t+2} (vH_{t+1} + \varphi e_{t+1} \tau_{t+1} w_{t+1} H_{t+1})] \end{aligned} . \tag{6}$$

Now, the problem can be expressed as follows: the agent chooses e_t and τ_{t+1} so as to maximise (2) subject to $0 \le e_t \le 1$, $0 \le \tau_{t+1} \le 1$ and taking H_t, H_{t+1}, w_t, w_{t+1} and w_{t+2} as given. Of course, given that individuals are identical, the tax rate chosen by the representative parent is the one that will prevail in a democratic regime. Later, it shall become clear that the optimal choice for e_t varies with the existing tax rate τ_t, while the choice for τ_{t+1} varies with the offspring's chosen effort e_{t+1}. For this reason, we shall consider two different scenarios concerning the sequence of events governing these choices.

[10] We assume a linear effect for g_t to guarantee an equilibrium with ongoing output growth. We could have assumed a more general specification $p(e_t) = e_t^{\beta}$ ($0 < \beta < 1$) for the effort element in the formation of human capital. However, doing so would render analytical solutions impossible (in the presence of the term vH_t) without altering the qualitative nature of our results and their implications. Recall that when an agent chooses e_t, she takes the effect of g_t as given, because this term is determined by old agents. Therefore, we can consider $\varphi g_t e_t$ as a composite input, in the same manner as we do for efficient labour in models with endogenously determined accumulation of human capital, without worrying about implications of maximisation under increasing returns.

3 Simultaneous Choices

In this case, we can describe the dynamic equilibrium of the economy through

Definition 1. *Given an initial stock of human capital* $h_0 > 0$*, the economy's dynamic equilibrium is determined by sequences of quantities* $\{c_t, e_t, Y_t, g_t, \tau_t, h_{t+1}, H_{t+1}\}_{t=0}^{\infty}$ *and prices* $\{w_t\}_{t=0}^{\infty}$ *such that, for* $t \geq 0$*:*

(i) *the adults choose* τ_t *so as to maximise utility, taking* e_t *as given;*

(ii) *the young choose* e_t *so as to maximise utility, taking* τ_t *as given;*

(iii) *both cohorts take aggregate quantities and prices as given when maximising utility;*

(iv) *firms choose* H_t *so as to maximise profits;*

(v) $h_t = H_t$*;*

(vi) *the sequences* $\{Y_t\}_{t=0}^{\infty}$, $\{h_{t+1}\}_{t=0}^{\infty}$, $\{g_t\}_{t=0}^{\infty}$ *and* $\{c_t\}_{t=0}^{\infty}$ *are determined by (1), (3), (4) and (5) respectively.*

It is straightforward to check that the FOC associated with the problem can be eventually written as

$$\frac{\theta}{1-e_t} \geq \frac{\psi\varphi\tau_t w_t H_t}{vH_t + \varphi\tau_t w_t H_t e_t}, \quad e_t \geq 0, \tag{7}$$

and

$$\frac{\psi}{1-\tau_{t+1}} \geq \frac{(1-\theta-\psi)\varphi w_{t+1} H_{t+1} e_{t+1}}{vH_{t+1} + \varphi\tau_{t+1} w_{t+1} H_{t+1} e_{t+1}}, \quad \tau_{t+1} \geq 0, \tag{8}$$

with complementary slackness in both (7) and (8). Notice that we can use (8) to infer the tax rate that will be chosen by adults who were born in period $t-1$ (that is, the parents of agents born in period t). Solving (7) for e_t, (8) for τ_{t+1}, expressing the latter one period backwards and manipulating algebraically, eventually yield

$$e_t = \max\left\{0, \frac{1}{\theta+\psi}\left(\psi - \frac{\theta\gamma}{\tau_t}\right)\right\}, \tag{9}$$

and

$$\tau_t = \max\left\{0, \frac{1}{1-\theta}\left(1-\theta-\psi-\frac{\psi\gamma}{e_t}\right)\right\}, \tag{10}$$

where $\gamma = v / \varphi A$. Given that $\tau_t, e_t \in [0,1]$, we can summarise the solutions in (9) and (10) as

$$e_t \begin{cases} > 0 & \text{if} \quad \tau_t > \theta\gamma / \psi \\ \\ = 0 & \text{otherwise} \end{cases}, \tag{11}$$

and

$$\tau_t \begin{cases} > 0 & \text{if} \quad e_t > \psi\gamma / (1-\theta-\psi) \\ \\ = 0 & \text{otherwise} \end{cases}. \tag{12}$$

These results merit some discussion. First of all, we see that corner solutions are possible and, as a result, $e_t = \tau_t = 0$ is an equilibrium. Evidently, this is due to the effect of the composite term γ which stems, mainly, from the presence of the parameter v. The intuition is that, as long as $v > 0$, the marginal utility has a finite upper bound for zero values of e_t or τ_t. Therefore, such choices are possible due to the fact that utility may become monotonically decreasing in these arguments. Secondly, additional interior solutions with both e_t and τ_t being positive are possible as well, meaning that the model may actually admit multiple equilibria. The underlying cause of multiple equilibria in this framework can be clarified through

Lemma 1. *The choices made by parents and offspring are strategic complements.*

Proof. Using (9) and (10), it is straightforward to verify that $\partial e_t / \partial \tau_t$ and $\partial \tau_t / \partial e_t$ are both positive. ■

Our model is, thus, able to generate a type of strategic complementarities similar to that suggested by Cooper and John (1988). A higher activity by one cohort of agents induces the other cohort to increase its activity as well. Once more, the presence of the parameter v (which implies a positive γ) is responsible for these effects. We can clearly see that when

$v=0$ ($\gamma=0$), both solutions become invariant to each other. The intuition is that, for $v=0$, the marginal utilities of both e_t and τ_t depend only on the relative weights of the utility arguments that they ultimately affect. When $v>0$, however, the marginal utility of e_t (τ_t) is increasing in τ_t (e_t). Following increases in these variables, individuals will restore the equilibrium by taking the appropriate action so as to reduce their marginal utility – something they can do with an increase in e_t (τ_t). In terms of intuition, a higher tax rate implies an increase in publicly provided education, therefore an increase in the benefits from devoting effort towards human capital accumulation. Similarly, a greater education effort by the young increases their parents' marginal utility benefit of foregoing consumption and choosing a higher tax rate, a benefit that is due to the presence of the 'warm-glow' element in their preferences.

At this point, we shall derive the interior equilibrium of the model. Substitute (10) in (9) and manipulate algebraically to derive the quadratic equation

$$(1-\theta-\psi)(\theta+\psi)e_t^2-[\gamma\psi^2+(1-\theta-\psi)(\psi-\gamma\theta)]e_t+\gamma\psi^2=0, \tag{13}$$

whose solution is given by

$$e_t=\begin{cases} e_L=\dfrac{\gamma\psi^2+(1-\theta-\psi)(\psi-\gamma\theta)-\sqrt{[\gamma\psi^2+(1-\theta-\psi)(\psi-\gamma\theta)]^2-4\gamma(1-\theta-\psi)(\theta+\psi)\psi^2}}{2(1-\theta-\psi)(\theta+\psi)} \\ \\ e_H=\dfrac{\gamma\psi^2+(1-\theta-\psi)(\psi-\gamma\theta)+\sqrt{[\gamma\psi^2+(1-\theta-\psi)(\psi-\gamma\theta)]^2-4\gamma(1-\theta-\psi)(\theta+\psi)\psi^2}}{2(1-\theta-\psi)(\theta+\psi)} \end{cases}. \tag{14}$$

Similarly, we can substitute (9) in (10) so as to get the quadratic equation

$$(1-\theta)\psi\tau_t^2-[(1-\theta-\psi)(\psi+\gamma\theta)-\gamma\psi^2]\tau_t+\gamma\theta(1-\theta-\psi)=0, \tag{15}$$

whose solution can be written as

$$\tau_t=\begin{cases} \tau_L=\dfrac{(1-\theta-\psi)(\psi+\gamma\theta)-\gamma\psi^2-\sqrt{[(1-\theta-\psi)(\psi+\gamma\theta)-\gamma\psi^2]^2-4\gamma(1-\theta)\psi(1-\theta-\psi)\theta}}{2(1-\theta)\psi} \\ \\ \tau_H=\dfrac{(1-\theta-\psi)(\psi+\gamma\theta)-\gamma\psi^2+\sqrt{[(1-\theta-\psi)(\psi+\gamma\theta)-\gamma\psi^2]^2-4\gamma(1-\theta)\psi(1-\theta-\psi)\theta}}{2(1-\theta)\psi} \end{cases}. \tag{16}$$

The results in (14) and (16) show that it is possible to get interior equilibria in addition to the corner solution. We formalise this result with

Proposition 1. *As long as the roots of (13) and (15) are real, there exist three sets of pure strategy equilibria. These are* $\{0,0\}$, $\{e_L, \tau_L\}$ *and* $\{e_H, \tau_H\}$ *where, for* $x = e, \tau$, *it is* $x_H > x_L > 0$.

Proof. Our previous analysis, together with $\partial e_t / \partial \tau_t > 0$ and $\partial \tau_t / \partial e_t > 0$ suffice as a formal proof. ■

Given the complexity of the solutions in (14) and (16), in conjunction with the already identified importance of the composite term γ, we are going to impose a parameter restriction that will allow us to characterise the interior solutions in an analytical manner but without diverging our focus from the presence and importance of multiple equilibria (and without any significant loss in terms of generality). In particular, for the remaining analysis we are going to assume that $\theta = \psi = 1/3$. Under this restriction, the reaction functions in (9) and (10) become

$$e_t = \max\left\{0, \frac{1}{2}\left(1 - \frac{\gamma}{\tau_t}\right)\right\}, \ \tau_t = \max\left\{0, \frac{1}{2}\left(1 - \frac{\gamma}{e_t}\right)\right\}, \tag{17}$$

while the results in (14) and (16) simplify to

$$e_t = \begin{cases} e_L = \dfrac{1 - \sqrt{1 - 8\gamma}}{4} \\ \\ e_H = \dfrac{1 + \sqrt{1 - 8\gamma}}{4} \end{cases}, \tag{18}$$

and

$$\tau_t = \begin{cases} \tau_L = \dfrac{1 - \sqrt{1 - 8\gamma}}{4} \\ \\ \tau_H = \dfrac{1 + \sqrt{1 - 8\gamma}}{4} \end{cases}, \tag{19}$$

respectively. As long as $\gamma < 1/8$ (henceforth, a condition that we assume to hold) these solutions satisfy $0 < \gamma < e_L < e_H < 1$ and $0 < \gamma < \tau_L < \tau_H < 1$.

The pure strategy equilibria are illustrated in Figure 1. It depicts the reaction curves, which are derived from equation (17). For $e_t, \tau_t \leq \gamma$ they move along the axes, while for $e_t, \tau_t > \gamma$ the reaction functions are increasing and concave.

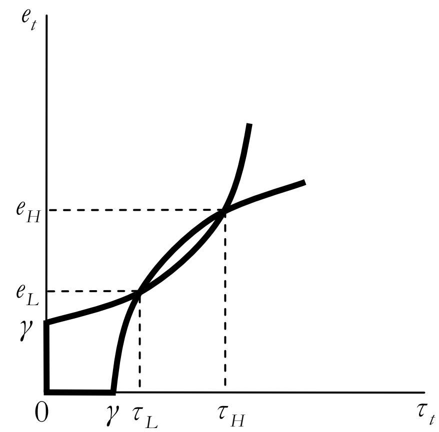

Figure 1

The next step of our analysis is to examine whether the multiplicity of equilibria rests upon the presence of a coordination failure in the decision making process by the young and the old. Formally, we can analyse this issue through

Lemma 2. *The three set of equilibria are ranked in the Pareto sense.*

Proof. Consider the utility of the old adult/parent during period t. Using (2), it can be written as

$$u^{t-1}(e_t, \tau_t) = \Psi^{t-1} + \ln(1-\tau_t) + \ln(v + \omega \tau_t e_t),$$

where $\omega = \varphi A$ and $\Psi^{t-1} = \ln(1-e_{t-1}) + 2\ln[AH_{t-1}(v + \omega \tau_{t-1} e_{t-1})]$. We can also write the utility of the young adult/offspring during period t as

$$u^t(e_t, \tau_t) = \Xi^t + \ln(1-e_t) + \ln(v + \omega \tau_t e_t),$$

where $\Xi^t = \ln(1-\tau_{t+1}) + 2\ln[AH_{t+1}(v + \omega \tau_{t+1} e_{t+1})]$. Using the results in (18) and (19) we get

$$1-\tau_L = \frac{3+\sqrt{1-8\gamma}}{4}, \quad 1-\tau_H = \frac{3-\sqrt{1-8\gamma}}{4},$$

$$1-e_L=\frac{3+\sqrt{1-8\gamma}}{4},\ 1-e_H=\frac{3-\sqrt{1-8\gamma}}{4},$$

and

$$\tau_L e_L=\frac{(1-\sqrt{1-8\gamma})^2}{16},\ \tau_H e_H=\frac{(1+\sqrt{1-8\gamma})^2}{16}.$$

Taking account of these results, and for given Ψ^{t-1} and Ξ^t (recall that for an individual agent, the aggregate stock of human capital is taken as given for any $t \geq 0$), we can show that $u^{t-1}(e_H,\tau_H) > u^{t-1}(e_L,\tau_L)$ and $u^t(e_H,\tau_H) > u^t(e_L,\tau_L)$ as long as

$$\ln\left(\frac{3+\sqrt{1-8\gamma}}{4}\right)+\ln\left[v+\omega\frac{(1-\sqrt{1-8\gamma})^2}{16}\right]<\ln\left(\frac{3-\sqrt{1-8\gamma}}{4}\right)+\ln\left[v+\omega\frac{(1+\sqrt{1-8\gamma})^2}{16}\right],$$

holds or, equivalently,

$$\frac{3+\sqrt{1-8\gamma}}{3-\sqrt{1-8\gamma}}<\frac{16v+\omega(1+\sqrt{1-8\gamma})^2}{16v+\omega(1-\sqrt{1-8\gamma})^2}.$$

After some extensive algebra, the last expression reduces to

$$\gamma<1/2,$$

which holds. Thus, $u^{t-1}(e_H,\tau_H) > u^{t-1}(e_L,\tau_L)$ and $u^t(e_H,\tau_H) > u^t(e_L,\tau_L)$ hold simultaneously. With this result in mind, it is sufficient to show that $u^{t-1}(e_L,\tau_L) > u^{t-1}(0,0)$ and $u^t(e_L,\tau_L) > u^t(0,0)$ so as to prove that the equilibria are Pareto ranked. Both these inequalities are satisfied as long as

$$\ln\left(\frac{3+\sqrt{1-8\gamma}}{4}\right)+\ln\left[v+\omega\frac{(1-\sqrt{1-8\gamma})^2}{16}\right]>\ln(1)+\ln(v)\Rightarrow$$

$$\frac{3+\sqrt{1-8\gamma}}{4}>\frac{16v}{16v+\omega(1-\sqrt{1-8\gamma})^2},$$

holds. Some algebraic manipulation can reduce this expression to

$$(1-6\gamma)^2>(1-8\gamma)(1-2\gamma)^2\Rightarrow$$

$$0>-32\gamma^3,$$

which, of course, holds with a positive γ. In conclusion, $u^j(0,0) < u^j(e_L,\tau_L) < u^j(e_H,\tau_H)$ for $j=t-1,t$ and for every $t \geq 0$. ■

To complete the characterisation of the different equilibria, we need to address the issue of their stability. In other words, we need to consider whether small perturbations in the neighbourhood of each set of equilibrium choices will leave these choices unaffected or not. As it is known from the analysis of Cooper and John (1988), not all possible equilibria of a coordination game are unresponsive to such perturbations, as one of them may be locally unstable. In our model, such an equilibrium is represented by the point $\{e_{L,}\tau_L\}$. This becomes evident from the fact that $\partial e_L / \partial\gamma > 0$ and $\partial \tau_L / \partial\gamma > 0$ – results that are completely at odds with the nature of the reaction functions in (17). If anything, we would expect that both cohorts choose lower values when the composite parameter term γ is higher, as they actually do at $\{e_{H,}\tau_H\}$. Thus, the point $\{e_{L,}\tau_L\}$ represents nothing else but a threshold which, in conjunction with agents' expectations of how others will act, determines which of the two stable equilibria – i.e., $\{0,0\}$ or $\{e_{H,}\tau_H\}$ – will prevail. For example, consider that each cohort makes a choice x, where $x = e, \tau$. If one cohort expects the other to choose $x < x_L$ ($x > x_L$), then it will choose 0 (x_H). Anticipating this, the other cohort will choose $0 < x_L$ ($x_H > x_L$), thus verifying the initial expectation.[11]

3.1 Indeterminacy and Growth Volatility

As we have seen, when decisions by the young and the old within a given period are strategic complements, and in the presence of a coordination failure, the model can generate multiple equilibria. By itself, this is not such a surprising result for anyone who has some familiarity with the seminal analysis of Cooper and John (1988). Nonetheless, what is particularly interesting with our analysis is the idea of output growth indeterminacy that arises because any of the two sets of equilibria can prevail: for a given H_t, next period's human capital (which, in equilibrium, satisfies $h_{t+1} = H_{t+1}$) can take more than one possible values. The reason why indeterminacy in the determination of variables that affect the accumulation of

[11] Notice that this notion of instability differs from the one applied in variables that display an explicit dynamic pattern. More formally, let $e_t = f(\tau_t)$ and $\tau_t = \Phi(e_t)$ denote the reaction function of the children and the parents, respectively, when $e_t, \tau_t > 0$. A (Nash) equilibrium is said to be stable if, starting from any point in its neighbourhood, the adjustment process in which players take turns myopically playing a best response to each other's current strategies converges to the equilibrium. This requires that $f' < (\Phi^{-1})'$, which, using (17), is equivalent to $\tau > 1/4$. Hence, $\{e_L, \tau_L\}$ is unstable and $\{e_H, \tau_H\}$ is stable.

human capital and, therefore, economic growth may prove of particular importance is twofold.

Firstly, our paper belongs to the strand of literature that explains the stylised fact of 'club' convergence, without resorting to the problematic scenario in which growth/development paths depend on initial conditions or endowments – problematic in the sense that the suggestion that some countries are currently rich simply because they happened to be rich before does not appear to be historically accurate. Other analyses that arrive to similar conclusions, but under different settings, are those of Redding (1996) and Palivos (2001). In the former, strategic complementarities between workers and entrepreneurs imply that, over some range of parameter values, multiple growth equilibria may emerge. In the latter, the complementarities generated by the existence of family-size norms imply indeterminate fertility choices and, given the trade-off between child-rearing and educational attainment, multiple growth equilibria.

The second significant implication of indeterminacy is that the growth rate of output (which, given $h_t = H_t$, corresponds to the growth rate of human capital) may not settle down to a balanced growth path, instead its behaviour may display a periodic pattern. This is because there is no intertemporal element in each cohort's choices, as it is obvious from (17), therefore any equilibrium $\{0,0\}$ or $\{e_H, \tau_H\}$ may prevail during each distinct period $t \in [0, \infty)$. We can formalise this argument with

Proposition 2. *In the presence of strategic complementarities and coordination failure, the growth rate of output may not be balanced, instead it may be volatile as it is given by* $\hat{\eta}_t \equiv Y_{t+1} / Y_t = H_{t+1} / H_t = v + \omega \hat{e}_t \hat{\tau}_t$, *where* $\{\hat{e}_t, \hat{\tau}_t\} = \{0,0\}$ *or* $\{\hat{e}_t, \hat{\tau}_t\} = \{e_H, \tau_H\}$, *for* $t \geq 0$.

Proof. The previous analysis suffices as a formal proof. ■

The idea of cyclical growth is absent from the analysis of Redding (1996) because he employs a framework in which the economy terminates at the end of the second period, implying that interactions among agents occur only once: consequently, multiple equilibria cannot be considered as a sign of periodic fluctuations in economic activity. In this respect, our framework shares more similarities with the analysis of Palivos (2001) in the sense that both employ full-fledged dynamic settings which allow interactions between agents to occur at every distinct period. Like we do in this paper, he recognises that multiplicity and indeterminacy are sources of growth cycles. In a framework which is closer to ours, Glomm and Ravikumar (1995) also discuss the possibility of cycles due to the presence of multiple equilibria. In their model, these arise (under some parameter specifications) because the future tax rate, which depends on future income, affects current education decisions which, partially, determine future income due to the accumulation of human capital.

Notwithstanding these common equilibrium implications, our particular framework allows us to examine scenarios in which the choices between the two cohorts are sequential and entail some degree of commitment by one of them. This is something we do in the subsequent part of our paper. As we shall see at an even later part, a straightforward comparison between these two scenarios will allow us to draw additional and important conclusions from our theoretical structure.

4 Equilibrium with Commitment

In this part, we are going to reconsider our problem when the assumption of simultaneous choices is relaxed. In particular, we shall assume that the old decide on the tax rate first and, following this announcement, the young decide on their education effort.[12] Effectively, we shall assume that adults act as 'Stackelberg' leaders. Formally, we can describe the model's equilibrium through

Definition 2. *Given an initial stock of human capital* $h_0 > 0$*, the economy's dynamic equilibrium is determined by sequences of quantities* $\{c_t, e_t, Y_t, g_t, \tau_t, h_{t+1}, H_{t+1}\}_{t=0}^{\infty}$ *and prices* $\{w_t\}_{t=0}^{\infty}$ *such that, for* $t \geq 0$*:*

(i) *the adults choose* τ_t *so as to maximise utility, taking account that the education effort by the young is related to their decision concerning taxation, i.e.,* $e_t = f(\tau_t)$*;*

(ii) *parts (ii)-(vi) of Definition 1 hold.*

[12] We may think that this idea captures scenarios in which governments commit to the provision of a certain fraction of GDP or of tax revenue towards education spending. For example, Section 8 of Article XVI of the California state constitution, added by Proposition 98 of 1988, establishes a minimum funding level or guarantee for K–12 education and community colleges. See, among others, Leyden 2005.

Given the above, our approach to the problem entails that we solve the problem of a young person in t so as to get her reaction function $e_t = f(\tau_t)$. The old adults of generation $t-1$ will take account of this when choosing their preferred tax, $\tilde{\tau}_t$, therefore the optimal education choice by the young will be $\tilde{e}_t = f(\tilde{\tau}_t)$.

Following our preceding analysis, we still restrict our attention to $\theta = \psi = 1/3$. Using this in (2) and maximising with respect to e_t yields

$$\frac{1}{1-e_t} = \frac{\varphi \tau_t w_t H_t}{v H_t + \varphi \tau_t w_t H_t e_t}. \tag{20}$$

Solving (20) for e_t gives

$$e_t = \frac{1}{2}\left(1 - \frac{v}{\varphi \tau_t w_t}\right). \tag{21}$$

Our next step is to substitute (21) into the $t-1$ variant of the utility function in (2), i.e., after expressing it in terms of u^{t-1}. Eventually, we get

$$\ln(1-e_{t-1}) + \ln[(1-\tau_t) w_t (v H_{t-1} + \varphi \tau_{t-1} w_{t-1} H_{t-1} e_{t-1})] + \ln\left[\frac{w_{t+1}}{2}(v H_t + \varphi \tau_t w_t H_t)\right]. \tag{22}$$

Maximising (22) with respect to τ_t yields

$$\frac{1}{1-\tau_t} = \frac{\varphi w_t H_t}{v H_t + \varphi \tau_t w_t H_t}. \tag{23}$$

Now, we can substitute $w_t = A$ and $\gamma = v / \varphi A$ in (23), and solve for τ_t to get

$$\tau_t = \frac{1-\gamma}{2} \equiv \tilde{\tau}. \tag{24}$$

Finally, substituting (24) in (21) gives us

$$e_t = \frac{1}{2}\left(\frac{1-3\gamma}{1-\gamma}\right) \equiv \tilde{e}. \tag{25}$$

As long as the previously imposed restriction $\gamma < 1/8$ still applies (which we assume that it does), these solutions satisfy $\gamma < \tilde{\tau} < 1$ and $\gamma < \tilde{e} < 1$ as required. Thus, we can present our next result in the form of

Proposition 3. *In the absence of coordination failure, the equilibrium is uniquely determined and the economy moves permanently along a balanced growth path. The equilibrium growth rate is equal to* $\tilde{\eta} \equiv Y_{t+1} / Y_t = H_{t+1} / H_t = v + \omega \tilde{\tau} \tilde{e}$, *for every* $t \geq 0$.

Proof. The analysis leading to the solutions in (24) and (25) suffices as a formal proof.

It is evident that, in this scenario, the possibility of growth cycles has disappeared. The reason for this result is because the equilibrium decisions by agents are now uniquely determined. In terms of intuition, the intrinsic uncertainty that pertained choices when these were made simultaneously has vanished. The old understand that an increase in the tax rate increases the willingness of the young to forego part of their leisure activities, simply because the benefits from doing so are higher. Consequently, they decide the tax rate that will induce children to provide the relatively high education effort that will satisfy their parents.

Once more, this result is not surprising by itself. Nonetheless, when compared to the previous scenario, it allows us to identify a novel fundamental mechanism for the link between volatility and growth. This is an issue to which we shall turn, after we examine what happens in a scenario where the offspring become 'leaders' and parents become 'followers' during the decision making process.

4.1 Commitment by the Young

Although this represents a less reasonable scenario, we shall briefly discuss the case where the young are the ones who commit to a certain effort towards learning activities. We do this purely as a means of illustrating the robustness of our main results to the different assumptions concerning the 'leaders' and 'followers' of the decision making process. In terms of a concrete real-life example, we may think of scholarships and/or tuition fee waivers that are provided on the basis of students' success on achieving some performance targets.

In this case, when the young choose e_t, they take account that $\tau_t = \Phi(e_t)$. Based on this, they choose their optimal learning effort $\tilde{e}_t$ which, subsequently, determines the chosen tax rate by adults through $\tilde{\tau}_t = \Phi(\tilde{e}_t)$.[13]

We can write (6) in terms of u^{t-1} (using $\theta = \psi = 1/3$), maximise with respect to τ_t and rearrange the result. Eventually, we obtain

$$\tau_t = \frac{1}{2}\left(1 - \frac{v}{\varphi e_t w_t}\right). \tag{26}$$

We can substitute (26) in (6) and maximise with respect to e_t. We get

$$e_t = \frac{1-\gamma}{2} \equiv \tilde{e}, \tag{27}$$

which, after substituting in (26), leads us to

$$\tau_t = \frac{1}{2}\left(\frac{1-3\gamma}{1-\gamma}\right) \equiv \tilde{\tau}. \tag{28}$$

The results in (27) and (28) indicate that the implication from Proposition 3 still applies, because the temporary equilibrium and, therefore, the growth rate of output are uniquely determined. In terms of intuition, the young understand that by foregoing some of their leisure will increase the adults' utility benefit from foregoing part of their consumption, in order to support a higher tax rate. As a result, they devote the amount of learning effort that will provide adults with the incentive to choose relatively high public spending on education.

5 Growth and Volatility

In the preceding analysis, we have considered two cases concerning the sequence of events that governs the choices made by old adults/voters and young agents – choices that relate to and affect the accumulation of human capital. As such, they have significant implications for the growth rate of output in the economy. When choices are made simultaneously, multiple equilibria (i.e., indeterminacy) emerge. These imply that the actual growth rate may display fluctuations over time depending on how each cohort of agents expects the other to behave

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13 The formal definition of the equilibrium is similar to Definition 2, the only difference being that part (i) should now read "*the young choose e_t so as to maximise utility, taking account that the tax rate chosen by adults is related to their decision concerning education effort, i.e., $\tau_t = \Phi(e_t)$*".

and act. When choices are made sequentially, the equilibrium is unique and, therefore, leads to a uniquely determined balanced growth path.

These results bring forth an important repercussion concerning the correlation between an economy's periodic behaviour and its long-term macroeconomic performance. Despite the fact that some early economists conjectured that temporary and long-term movements in economic activity are inherently linked, it is only recently that a growing body of literature considered the analysis of the fundamentals behind this link as a research question worth pursuing.[14] This strand of literature was further stimulated by an increasing number of empirical analyses (e.g., Ramey and Ramey, 1995; Martin and Rogers, 2000; Turnovsky and Chattopadhyay, 2003) showing that growth rates are significantly – and, mainly, inversely – correlated, on average, with proxies of their variability. Until recently, theoretical studies have explored this issue with the construction and solution of stochastic endogenous growth models – i.e., models in which (extrinsic) uncertainty is introduced through the incorporation of some RBC-type real and/or monetary shocks (e.g., Dotsey and Sarte, 2000; Varvarigos, 2007). These shocks have non-linear effects on the equilibrium growth rate, implying that their variability impinges on the average rate of output growth.

Our analysis can be viewed as providing an alternative suggestion – mainly, the idea that both differences in growth rates and the incidence of growth volatility are inherently linked to the structural characteristics of the economic environment. We outline the main implication from this idea in

Proposition 4. *There is a negative correlation between volatility and growth, in the sense that the growth rate of the economy that may undergo cyclical fluctuations is strictly lower than the growth rate of the economy in which such fluctuations are absent. That is,* $\hat{\eta}_t < \tilde{\eta} \quad \forall t$.

Proof. Given our analysis and results so far, it suffices to show that $e_H \tau_H < \tilde{e}\tilde{\tau}$. It is

$$e_H \tau_H = \left(\frac{1+\sqrt{1-8\gamma}}{4} \right)^2 = \frac{1+2\sqrt{1-8\gamma}+1-8\gamma}{16},$$

and

[14] See Schumpeter (1934) and Kaldor (1954), among others, for some early ideas on the relationship between economic growth and cyclical fluctuations.

$$\tilde{e}\tilde{\tau} = \frac{1}{2}\left(\frac{1-3\gamma}{1-\gamma}\right)\frac{(1-\gamma)}{2} = \frac{1-3\gamma}{4}.$$

Then, for $e_H \tau_H < \tilde{e}\tilde{\tau}$ we want

$$\frac{1+2\sqrt{1-8\gamma}+1-8\gamma}{16} < \frac{1-3\gamma}{4} \Rightarrow$$

$$1-8\gamma < 1-4\gamma+4\gamma^2,$$

a condition that is indeed true. Therefore, we conclude that $\hat{\eta}_t < \tilde{\eta}$. ■

Contrary to the existing literature, our model does not rely on exogenously introduced random shocks so as to generate growth cycles. Rather, it is the intrinsic uncertainty which is inherent in strategically complement decisions, when these are subjected to coordination failure, which is responsible for growth volatility.[15] When the structural characteristics of the economy render such failures absent, growth cycles disappear and the complementary actions by agents are conducive to the formation of human capital. Perhaps, it is because of this idea that our model, in comparison to the aforementioned literature, is able to make an even stronger claim: cyclical growth rates are not only lower on average, but also periodically (i.e., at any moment in time).

6 Conclusions

In the preceding analysis, we have sought to provide a novel explanation for the, empirically observed, negative correlation between volatility and growth. In particular, we argued that this may be due to the structural characteristics that govern choices by different cohorts of agents, when such choices are strategic complements and affect the accumulation of a growth promoting factor – in our case, human capital. Furthermore, our framework lies in the class of models that are able to explain convergence in 'growth clubs' without resorting

[15] A recent contribution by Wang and Wen (2006) also examines the relationship between endogenously driven cycles and growth. They do so in a completely different setting however. In their model, imperfectly competitive firms set prices one period in advance. Given that the decisions by firms within the industry are strategic complements, each one faces extrinsic uncertainty concerning its competitors' actions – uncertainty which can be self-fulfilling and, thus, lead to sunspot equilibria. They show that, under certain parameter restrictions, the mean growth rate of an economy perturbed by sunspot shocks is lower than the growth rate of an economy in which sunspot shocks do not emerge.

to the idea of differences in initial conditions. In terms of policy implication, our analysis suggests that a credible policy of commitment towards growth promoting factors (such as education in our particular framework) could lead to both an increase in output growth and, as an added benefit, a reduction in the incidence of aggregate variability.

Our model is stylised in a manner that facilitates it in admitting analytical solutions. Yet, we do not view this as a shortcoming but rather as a vehicle that allows it to benefit from clarity of intuition and tight focus on the mechanisms involved during the materialisation of the basic results. Of course, we do not want to argue against the idea that more general specifications for preferences and technologies will provide a more complete picture of the issue at hand. What we firmly believe, however, is that the main mechanisms of our paper will survive even under the most general set-up. In any case, such a general, numerically solved, setting may constitute a worth pursuing avenue for future work.

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