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POPULATION AGEING, INEQUALITY AND THE POLITICAL ECONOMY OF PUBLIC EDUCATION

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Abstract

Population ageing has triggered concerns about the sustainability of public systems of education. The empirical evidence is still inconclusive, whereas some theoretical results present a somewhat optimistic view (Gradstein and Kaganovich, 2004; Levy, 2005). The present note re-examines the political economy of public education in an ageing society, using the classical median voter model. The normative analysis shows that elderly households introduce distortions that render political outcomes inefficient except in rare circumstances. It is then explained that the interplay among the political and financial consequences of ageing gives rise to a non-linear, and possibly non-monotonic (inverted-U shaped) relationship between spending per pupil and the share of childless households in the population. Income inequality is shown to play a crucial role of in the process, revealing that ageing has a stronger tendency towards underprovision in economies with high inequality. The implications for the empirical literature are discussed.

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1 Introduction

Population ageing is one of the most important demographic processes transforming the world since the end of World War II. Part of a new demographic transition western countries experienced after the post-war baby boom, ageing has triggered concerns about the sustainability of the Welfare State, especially, social security programs and, more recently, public systems of education. While much research effort has been devoted to the analysis of the former, contributions on the consequences over educaction policy are still scarce. This is rather surprising, as ageing raises important questions about the prospects of political support for public education: On the political side, a relative increase in the population of elderly households implies that a smaller share of voters receive direct benefits from education spending. In a democracy, one could expect ageing naturally leading to weaker political support for education. Nevertheless, on the financial side, a declining share of school-aged children lowers the *tax-price* of education spending (how much a unit of spending per pupil costs to tax-payers), which, *ceteris paribus*, makes voters willing to support higher levels of spending. The overall effect of population ageing over education spending is thus ambiguous and requires careful study.

What answers has the literature offered up to now? An increasingly large body of empirical evidence –which includes, among others, Rubinfeld (1977), Miller (1996), Poterba (1997), Ladd and Murray (2001), Evans et al. (2001), and, more recently, Baldson and Brunner (2004), Grob and Wolter (2005) and Fletcher and Kenny (2008)– has only provided mixed results and is thus inconclusive.

The flavour of the theoretical results on the issue is, instead, somewhat optimistic. From a positive perpsective, Gradstein and Kaganovich (2004) present a model where ageing is the consequence of increased longevity and has an overall positive impact on public education funding: Young adults realise education spending has an impact on the economy's future productivity and, thereby, on the returns they will obtain from their savings at retirement. The prospects of a longer retirement period, then, make them prefer higher education funding. When the share of physical capital in the production function is sufficiently high, that effect more than offsets the political impact of the larger share of elderly households a longer life-span implies. While this takes place in a dynamic economy across time, the analysis reveals that such a situation is compatible with the, apparently contradicting, cross-section negative relationship between the share of elderly households in a constituency and spending per pupil.

Another important theoretical contribution with implications for the ageing literature is found in Levy (2005). Levy analyses a state-of-the-art model of two-dimensional voting which enables her to study the political economy of public education when the government also provides (and households vote for) direct income redistribution. In that model, it is only when young households are a minority that the state provides educational services. In that case, the poor young are able to commit to policies that are less beneficial to them than the old poor can. Thereby they manage to form a winning coalition with the rich. In that coalition, the poor young give up full income redistribution in order to get some public education. The rich, in turn, accept positive educational transfers to avoid full income redistribution, which is aimed for by the largest minority –the old poor. If ageing entails that households with children become a minority then it would enhance public spending on education.[1]

The present note contributes to the literature by studying the political economy of public education in an ageing society, using the classical median voter model. I therefore give up the study of dynamics for a richer static framework[2] : In Gradstein and Kaganovich (2004), working adults

[1] From a normative perspective, Boldrin and Montes (2005) use an overlapping generations model to show that a simple intergenerational transfer system involving public education and pensions can restore efficiency when there are borrowing constraints to finance education. In their proposal, children borrow from the previous generation to finance their education, and pay the capitalised value of such debt back, once adults, in the form of reteriment pensions. Nevertheless, generations in Boldrin and Montes' contribution have a constant and identical size, and thus the model does not yield any conclusions about the effects of ageing on the system. An interesting theoretical exercise involves analysing their proposal in an ageing context to see how robust it is to such demographic phenomenon.

[2] The model is however able to capture particular dynamic aspects of the problem. Spending in education is beneficial for a young household because it is an investment that increases the future income of their children. Also, the demographic composition of the economy in my model can be derived from the dynamic demographic model used, for example, in Gradstein and Kaganovich (2003). Here, each share of the elderly can be seen as the result of a given value of their longevity parameter.

from the same cohort are identical, making the analysis silent about the role of contemporaneous inequality. Moreover, the ratio of children to tax-payers is constant, which leaves out an important aspect of the problem: the decreasing tax-price of education spending as the population of children shrinks. Likewise, I simplify the policy problem to one dimension in order to focus on the implications of the median voter framerwork. Which model best matches the data is of course an empirical question. In this respect, an advantage of the median voter framework is that it yields more specific predictions: the implemented policy is always that preferred by the median voter, while in Levy's model we only know it belongs to the Pareto-set of the members of the winning coalition. It therefore has no say about demographic changes that do not induce a change in the winning coalition.[3]

The analysis first reveals that elderly households introduce distortions that render political outcomes inefficient except in rare circumstances. Then, it is shown that the interplay among the political and financial consequences of ageing gives rise to a non-linear, and possibly non-monotonic (inverted-U shaped) relationship between spending per pupil and the share of childless households in the population. Finally, the crucial role of income inequality in the process is unconvered: the political effect is reinforced in economies with high inequality which thus have a stronger tendency towards underprovision. The last two results have direct implications for the empirical literature, which is discussed at the end of the paper.

The remaining of the article is organised in five sections. The next one introduces the model. Section 3 studies the efficiency properties of democratic outcomes. Section 4 is devoted to a positive study of the impact of ageing on spending per pupil. The focus then moves towards income inequality in Section 5. Finally, Section 6 closes the paper by offering some concluding remarks and explaining promising directions for future research.

[3] Of course, predictions from her model are richer in other respects, as they involve consumption of public and private education *and* redistribution policy.

2 The model

Households and population ageing

A country, region or state is populated by a continuum of households with mass normalised to 1.[4] Households exogenously differ along two dimensions: income (which can also be interpreted as parental human capital) and the number of children at school age: a proportion $\gamma \in [0,1]$ of households are childless (or have children that are adults) and the rest have $\mu = 1$ children each.[5] That implies a ratio of children to the population of households (tax-payers) equal to $1-\gamma$. Household types are indexed with i; subscripts 0 and 1 denote households with zero and one children respectively. Ageing is the result of a rising life expectancy (or longevity), which raises the share of the elderly in the population, and thereby reduces the ratio of children to tax-payers.[6] The ageing process is specified as an exogenous increase in the share of childless households γ.[7] In what follows, I restrict attention to cases where households with children form a majority and control the political arena (i.e. where $\gamma < 0.5$).

Income is denoted by $y \in \left[\underline{y}, \overline{y}\right]$, and varies in the population according to the cummulative distribution functions $\Phi_1(y) \in [0,1]$ and $\Phi_0(y) \in [0,1]$. These are continuous, strictly positive on all their support and have densities $\phi_1(y) = \Phi_1'(y)$ and $\phi_0(y) = \Phi_0'(y)$. The aggregate distribution function can thus be written as:

$$\Phi(y) = \gamma \Phi_0(y) + (1-\gamma)\Phi_1(y)$$

[4] The argument is thus developed in a single-jurisdiction setting where the population cannot move to other jurisdictions. Interesting aspects of the problem emerge in multi-jurisdictional settings where households not only vote but also compete with each other for where -which school district- to live in. The decentralised case with mobility is certainly worth studying, but that exercise is left for future work.

[5] I set $\mu = 1$ to facilitate the exposition. As we shall see later on, the consequences of changing μ are straightforward to analyse.

[6] Ageing may also be the result of a decreasing fertility rate, which directly reduces the ratio of school-aged children to tax-payers.

[7] Note that such change can be motivated as the result of an increase in average (or expected) longevity. See, for example, Gradstein and Kaganovich (2004).

while total (and average) income is given by:

$$Y = \int_{\underline{y}}^{\overline{y}} y \left(\gamma \phi_0 \left(y \right) + \left(1 - \gamma \right) \phi_1 \left(y \right) \right) dy \tag{1}$$

Preferences

To facilitate welfare analysis, and following de Bartolome and Ross (2004), I choose a quasi-linear, money-metric utility function to represent households' preferences. Utility of young households depends on current numeraire consumption, x, and on the future income (human capital) of the offspring, y'. As they do not have children to educate, the utility of old households simply depends on current consumption:

$$u \left(x, e \right) = x + \delta_c y' = x + \delta_c h(e, y) \tag{2}$$

where $\delta_c \in \{0, 1\}$ indicates whether a household has a school-aged child ($\delta_c = 1$) or not. In equation (2), offspring future human capital (income) is a function of two inputs: school quality, e, and home inputs, as measured by household parental human capital: $h(e, y)$. The human capital production function is monotonically increasing in the both of its arguments and strictly concave; moreover, to ensure normality of demand for education, home and school inputs are assumed complements: i.e. $h_{ey} > 0$.[8]

Two points are worth noting. Firstly, the above specification of preferences implies that childless households align themselves with poor ones with children when voting over education policies. The hypothesis that the elderly, or more generally childless households, would vote following such narrow, self-centered motivations may seem a radical one. However, whilst it simplifies the analysis, this assumption does not alter the results. In particular, qualitative results also hold in a setting where old voters benefit from education relatively less than young ones, as the evidence suggests.[9] Secondly, the choice of a money-metric utility function has the advantage that it com-

[8] Subscriptis indicate derivatives.

[9] The empirical evidence in Rubinfeld (1977), Baldson and Brunner (2004) shows that the strength of preferences for education decline with age; that offered by Fletcher and Kenny (2008) lends specific support to a view of the political process where childless households ally with the poor. The theoretical model proposed by Levy (2005) adopts the same assumptiom.

pletely separates efficiency and equity considerations: because marginal utility of consumption is constant and equal to 1, aggregate welfare is invariant to the distribution of private consumption in the population; precisely for the same reason, this utiltity function makes the distribution of taxes across the population irrelevant for aggregate welfare.

School system and taxation

The state provides tuition-free public education to every child of school age. School quality depends on spending per pupil and is produced with a technology that exhibits constant returns to scale. Units of school quality are normalised so that each costs one unit of the numeraire. I impose the following (political) restriction on school policy: The government is commited to provide equal spending (and thus, in the current setting, quality) to all students.[10]

Education spending is financed by an economy-wide head tax. This provides a neater analysis, as it avoids the distortions that childless households induce being intertwined with those stemming from income taxation. Indirect utility is thus:

$$v(y, e, \tau) = y - \tau + \delta_c h(e, y) \tag{3}$$

Voting

The representation of the democratic process considered in this paper is majority voting: Voters choose among different spending-taxation alternatives in pairwise contests. As the government is required to balance its budget, a spending-taxation alternative is considered in the election if it is financially feasible, that is, if it satisfies the government budget constraint (GBC), given by:[11]

$$e = \frac{\tau}{(1-\gamma)\mu} = \tau\eta \tag{4}$$

Note $\eta = 1/\left(1-\gamma\right)\mu$ is the inverted tax price of education spending (how much spending per pupil the government can offer per unit of tax). I assume voters understand the relationship between

[10] This is probably the most common assumption in the literature (e.g. Epple and Romano, 1996; Martinez-Mora, 2006), though of course not the only one (e.g. De Fraja, 2002).

[11] More precisely, the GBC should be written as $e \leq \tau\eta$. Nonetheless, it is immediate to prove that any alternative satisfying the GBC with inequality will always be beaten in an election. Put it differently, only the alternatives that satisfy the GBC with equality can be a political equilibrium of the model.

taxation and spending per pupil established by the GBC, which makes voting unidimensional. A voting equilibrium is a feasible alternative which constitutes a Condorcet winner, i.e. which is preferred by at least half the electorate in pairwise contests against all other feasible spending-taxation pairs.

3 Ageing and the inefficiency of democratic outcomes

In this Section, I investigate the role of childless households and population ageing in the efficiency properties of centralised democratic outcomes.

Political equilibrium

When voting is unidimensional and households' preferences over feasible spending-taxation pairs are single-peaked, existence of a unique political equilibrium directly follows from the classical median voter theorem (Black, 1948). Let us examine the single-peakedness of preferences in the current setting: childless households do not have children to care about their future income, and thus do not receive any benefits from education spending. Their utility is thus monotonically decreasing in the level of taxation and they prefer no taxes at all. Consider next households with children: decreasing marginal returns to education spending imply their preferences are single-peaked at the unique solution to their utility maximisation problem. Moreover, demand for public education spending is increasing in income, due to the complementarity between home and school inputs. Formally:

Lemma 1 *(i) Preferences over feasible spending-taxation pairs are single peaked.*

(ii) The most preferred spending-taxation pair of childless households is $(0, 0)$*; that of households with children is strictly increasing in income.*

Proof. The objective function of households with children in the voting problem is obtained by plugging the budget constraint (4) into the indirect utility function (3). Its strict concavity stems from the assumption of decreasing marginal returns to education spending. Of course, strict concavity along with a linear budget constraint implies existence of a unique solution to the optimisation problem, and that it is a global maximum. Childless voters, in turn, do not derive any benefit from education. Higher levels of spending per pupil, thus, only mean higher taxes for them and lower utility. Therefore, their preferences for spending have a single peak at zero. Finally, differentiating the FOC:

$$h_e(e, y) = 1 - \gamma \tag{5}$$

with respect to e and y, and rearranging one obtains:

$$\frac{de^*(y,\gamma)}{dy} = -\frac{h_{ey}(e^*, y)}{h_{ee}(e^*, y)}$$

Noting that $h_{ey} > 0$ completes the proof of the second part. ∎

Therefore, the median voter's most preferred spending-taxation pair is the unique equilibrium of the political process. That equilibrium is at the level of spending that equates her marginal rate of substitution between school quality and private consumption. That is to say, denoting the equilibrium spending level as $\widetilde{e}$, it is defined as:

$$\widetilde{e}\,(\widetilde{y}, \gamma) = e \;\text{ such that }\; h_e(e, \widetilde{y}) = 1 - \gamma \tag{6}$$

where $\widetilde{y}$ denotes median voter's income.

Next, lemma 2 provides the definition of the income of decisive voters.[12]

Lemma 2 *The median voter's level of income, denoted $\widetilde{y}$, is implicitly defined as*[13] *:*

$$1 - \Phi_1(\widetilde{y}) = \frac{1}{2\,(1-\gamma)} \tag{8}$$

Proof. The median voter is located at the median of the preference distribution across the whole population. Lemma 1 showed two things: (i) that all childless households have a strict preference for zero spending over any other alternative, and (ii) that the most preferred level of spending is increasing in income for households with children. These two results imply that the median

[12] Recall I restricted attention to cases where households with children form a majotity and dominate the elections. That is, cases where the median voter has children (and thus $\gamma < 0.5$).

[13] *The explicit definition is thus:*

$$\widetilde{y} = \Phi_1^{-1}\left(1 - \frac{1}{2\,(1-\gamma)}\right) \tag{7}$$

where Φ_1^{-1} is the inverse of the cumulative income distribution function.

voter's level of income is such that the mass of households with children whose income is weakly higher than $\widetilde{y}$ represents 50% of the total population. Equation (8) can be rewritten as:

$$(1-\gamma)\,(1-\Phi_1(\widetilde{y})) = \frac{1}{2}$$

the LHS of this equation is the product of the share of households with children and the proportion of them with income weakly above $\widetilde{y}$; hence, it is the share over the *total* population of young households with weakly higher income than $\widetilde{y}$. ■

This is a natural result: the median voter is located at the median of the preference distribution across the whole population. Given that public education is a normal good and that childless households prefer zero spending, median voter's income is such that the mass of households with children that are richer represent 50% of the *total* population. That is equivalent to a share $\frac{1}{2(1-\gamma)}$ of the population of households with children. Clearly, the median voter´s income is at the median of the income distribution of households with children if, and only if, $\gamma = 0$; and it is strictly below it for any $\gamma > 0$.

The political equilibrium is thus reached in a situation where a coalition, with mass a half, formed by poor households with children and childless ones, and which prefers lower spending, opposes another coalition of voters, again with mass a half, made up by middle and high income households with children, and whose members prefer higher levels of funding per pupil. The result is similar to Epple and Romano´s (1996) *ends against the middle* property of political equilibria. In their paper, that property holds in an election over education policy in a context with private schooling. There, the poor –who vote for lower spending (and taxation) because of a higher marginal utility of private consumption– and the rich –who opt out of the public sector and thus do not benefit from government spending in education– ally themselves against the middle class that push for higher taxes and spending.[14]

[14] The income of decisive voters in an ageing economy could be further reduced by the presence of private schooling, as households opting out of the public system would shift their vote to the lower spending coalition. This would surely occur if it is the rich that opt for private schooling, but could not be the case if households in the private sector do not belong to the top tail of the income distribution (as in Martinez-Mora, 2006). On the

Efficient policy

Efficiency is defined against a utilitarian Social Welfare Function (SWF): the unweighted sum of household utilitities in society. In the current setting, this is exactly the same thing as maximising the net-of-costs aggregate benefit of education policy.[15] Hence, the objective function the government maximises with respect to e, is:

$$(1-\gamma)\,\mu\int_{\underline{y}}^{\overline{y}} h\,(e,y)\,\phi(y)dy - e\,(1-\gamma)\,\mu \tag{9}$$

which yields the following FOC:

$$\int_{\underline{y}}^{\overline{y}} h_e\,(e,y)\,\phi(y)dy = 1 \tag{10}$$

As usual, the FOC establishes that the average marginal rate of substitution between school quality and the numeraire *across beneficiaries* (households with children) must equal the marginal rate of transformation between those same goods, equal to one. This is a Samuelsonian condition for the efficient provision of a public good (or a publicly provided service), which takes account of the fact that only a subset of the population receives educational services. Of course, the efficient education policy does not depend on the share of childless households in the economy.

Let $MRS_a(e)$ be the average marginal rate of substitution across households with children at spending level e; $MRS_{mv}(e)$ denote the median voter's one and let $\left(\widetilde{e}^*, \widetilde{t}^*\right)$ be the equilibrium policy pair –i.e. the one preferred by the median voter:

Proposition 1 *(i) The political equilibrium is inefficient, unless the ratio of the average MRS across households with children and the median voter's one equals the inverted tax price:*

$$\frac{MRS_a(\widetilde{e}^*)}{MRS_{mv}(\widetilde{e}^*)} = \frac{1}{1-\gamma} = \eta \tag{11}$$

(ii) Overprovision (underprovision) results whenever the average marginal rate of substitution at the equilibrium level of spending, $MRS_a(\widetilde{e}^*)$, is smaller (greater) than one.

Proof. Divide (10) by (5). ■

other hand, as in the present framework, the tax price falls as households substitute public schools for private ones. Hence, the overall impact is again unclear. See Epple and Romano (1996), Nechyba (1999).

[15] To see why, note that with a quasi-linear utility function social welfare is invariant to the way taxation is distributed across the population.

Comparison of the efficient outcome to the political equilibrium reveals two sources of inefficiency of opposite sign: (i) The median voter's valuation of education spending will usually not coincide with the mean valuation of the beneficiaries. (ii) The tax price is below the production cost whenever the share of childless households in the population is positive. The former inefficiency source, which arises even with $\gamma = 0$, was already pointed out by Stiglitz (1974). As far as I am aware, the latter inefficiency has not been previously identified formally. It is however inherent to any situation where the government uses universal taxation to fund a service not every tax payer receives – indeed the case of most public services. Such inefficiency arises regardless of the tax instrument in use, even if the government uses a non-distortionary head tax. To conclude, it is important to remark that the tax-price inefficiency, as well as the divergence between median and median voter's income, widen with ageing.

4 The non-linear effect of ageing on spending

Demographic ageing alters the political equilibrium through two different mechanisms: a financial *tax-price effect* and a political *median voter effect*. The former –which corresponds to the RHS of equation (11)– swings the political equilibrium towards overprovision, for the tax price of a unit of school spending per pupil goes down when the proportion of children decreases. The latter effect –LHS of equation (11)– creates a tendency towards underprovision, as the income of decisive voters falls with population ageing.[16] [17] To shed light into the likely direction of those inefficiencies (i.e. into which effect dominates at different levels of γ), and into the way population ageing affects education policy and social welfare, in this Section, I study the consequences of an increase in the

[16] As a matter of fact, ageing could reverse the political situation and make childless households decisive. In the current setting, such situation would simply cause the collapse of the public education system.

[17] The political impact of ageing occurs because some households stop being direct beneficiaries of education spending; the financial aspect is related to a decrease in the ratio of children to tax payers (or more generally, to the tax base). If ageing did not alter the share of households with no children (e.g. because it simply consists of a reduction in the number of children per household and not in that of households having children), only the financial side of the problem would be relevant. In that case, ageing would clearly have a positive impact on spending per pupil. Likewise, if the ageing process only meant an increase in the share of households with no children, then a decrease in spending per pupil would unambiguously result, as only the political median voter effect would be at work.

mass of households with no children at school-age.

The derivative of equilibrium spending with respect to the share of childless households writes:

$$\frac{d\widetilde{e}^*}{d\gamma} = \frac{de^*(\widetilde{y},\gamma)}{dy}\frac{d\widetilde{y}(\gamma)}{d\gamma} + \frac{de^*(\widetilde{y},\gamma)}{d\gamma} \tag{12}$$

The first term in the RHS of the equation catptures the *political* side of the problem. Specifically, by putting more households at the bottom of the policy-preference distribution, ageing reduces the income of the median voter, thus resulting in weaker support for public spending. The second term refers to the *financial* side of the question: a marginal increase in the share of childless households (or, more generally, a decrease in the ratio of children to tax-payers) implies a reduction in the tax price that, ceteris paribus, will yield higher spending per pupil. Let us first analyse the effect of ageing on the income of the decisive voters; the next lemma shows that it is non-linear:

Lemma 3 *(i) Population ageing reduces the income of decisive voters in a non-linear fashion and according to:*

$$\frac{d\widetilde{y}}{d\gamma} = -\frac{1}{2(1-\gamma)^2}\frac{1}{\phi_c(\widetilde{y}(\gamma))} < 0 \tag{13}$$

(ii) That reduction occurs at an increasing (decreasing) rate for any income distribution, or part thereof, that satisfies:

$$1 + \frac{\phi_c'(\widetilde{y})}{4(1-\gamma)\phi_c(\widetilde{y})^2} > (<)0 \tag{14}$$

Proof. Recall $\widetilde{y}$ is defined by (8). Use the implicit function theorem to obtain the first derivative of median voter income with respect to the share of childless households in the population. Then derive that again with respect to γ to obtain:

$$\frac{d^2\widetilde{y}}{d\gamma^2} = -\frac{1}{\phi_c(\widetilde{y})(1-\gamma)^3}\left[1 + \frac{\phi_c'(\widetilde{y})}{4(1-\gamma)\phi_c(\widetilde{y})^2}\right] \tag{15}$$

Clearly, this derivative is negative whenever $\phi_c'(\widetilde{y}) \geq 0$, that is to say with a uniform or monotonically increasing income distribution function, or, in general, whenever the term in square brakets is positive. ■

Population ageing depresses the income of decisive voters. After a marginal increase in γ, it falls by an amount equal to the increase in the proportion of households *with children* that form the higher spending *coalition*, $\frac{1}{2(1-\gamma)^2}$, divided by the density of households with children at the decisive voter's income, $\phi_c(\widetilde{y})$. As equation (13) reveals, the median voter effect is non-linear and it usually intensifies as the ageing process advances. To see why, note the first term in the RHS of equation (13) increases with γ at an increasing rate. That is the natural consequence of a shrinking population of households with children (see Figure 1). That effect is moderated by the density of households with children at the median voter's income: it is amplified where that density is smaller but it might even be reversed within ranges of the income distribution where the probability density function is decreasing and sufficiently steep.

With a uniform distribution, the term in square brackets in equation (15) equals 1 so that the median voter effect always intensifies with γ. Clearly, this is also true for monotonically increasing income distributions. Regarding non-monotonic ones, the median voter effect strengthens with γ at ranges where the income distribution is increasing, and even at some intervals where it is decreasing but not steep enough. Figure 2 shows the relationship among the share of childless households and median voter income for three lognormal distribution functions, with different degrees of inequality. In all these examples the median voter effect intensifies at higher levels of γ.[18]

Proposition 2 *(i) Ageing has a non-linear, and possibly non-monotonic, relationship with the level of spending per pupil:*

$$\frac{d\widetilde{e}^*(\widetilde{y}(\gamma),\gamma)}{d\gamma} = \frac{-h_{ey}(\widetilde{e}^*,\widetilde{y})\frac{d\widetilde{y}}{d\gamma} - 1}{h_{ee}(\widetilde{e}^*,y)} \tag{16}$$

(ii) Spending per pupil increases (falls) with ageing whenever:

$$-h_{ey}(\widetilde{e}^*,\widetilde{y})\frac{d\widetilde{y}}{d\gamma} < (>) 1 \tag{17}$$

Proof. Applying the implicit function theorem to the FOC:

$$h_e\left(e^*(y,\gamma),y\right) - (1-\gamma) = 0 \tag{18}$$

[18] Along the formal analysis, I will be using three computational examples to illustrate the results. Income distributions are lognormal with identical mean (of the associated normal distribution) parameter: $\mu = 3.5$, and spread parameter $\sigma_A = 0.3$, $\sigma_B = 0.7$ and $\sigma_C = 1.1$. I set a minimum level of income different from zero ($y_{\min} = 5$). The implied median income is equal to $y_{med} = 38.12$, and the means are, respectively, $y^A_{mean} = 39.64$, $y^B_{mean} = 47.31$, and $y^C_{mean} = 65.64$. These yield standard deviations of 10.63, 33.64, and 93.03, and mean to median income ratios of 1.04, 1.24 and 1.72, respectively. See footnote 20 for further details.

it is straightforward to obtain de^*/dy and $de^*/d\gamma$ and rewrite (12) as (16) which is also non-linear. Non-monotonicity will characterise the relationship under study whenever $-h_{ey}(\widetilde{e}^*, \widetilde{y})\frac{d\widetilde{y}}{d\gamma}-1$ changes sign for some $\gamma \in (0, 0.5)$. To prove (ii) note that given the strict concavity of the human capital production function, the overall impact on the level of spending only depends on the sign of the numerator of (16). ■

While the RHS of (17) is fixed at unity, the LHS depends on the income elasticity of education demand and on the marginal impact of γ over the income of median voters.[19] The larger that elasticity the stronger the impact of the falling political support in the actual level of spending per pupil. The previous results reveal that a rise in the proportion of childless households has a non-linear and possibly non-monotonic impact over funding per pupil. Not only because two opposing forces are at work, but also because the median voter effect is in general non-linear. Indeed, the examples plot in Figures 2 and 3 confirm that the ageing process may increase spending per pupil at the initial stages of the process, whilst leading to stark reductions as the process deepens, something which implies an inverted U-shaped relationship among γ and e.

5 The role of income inequality

The next proposition shows that income inequality among households with children affects the impact of ageing over public education. Restricting attention to uniform and lognormal income distributions, it reveals that population ageing has more pernicious effects in more unequal democracies.

Proposition 3 *Consider two economies L and H (acronyms for low and high inequality) identical except for the income distribution function of households with children, Φ_L and Φ_H.*

1. Let Φ_L and Φ_H be uniform distributions with identical mean and median income and suppose $y^L_{max} - y^L_{min} < y^H_{max} - y^H_{min}$. Then, the median voter effect is stronger in H than in L:

$$\left(\frac{d\widetilde{y}(\gamma)}{d\gamma}\right)_L < \left(\frac{d\widetilde{y}(\gamma)}{d\gamma}\right)_H$$

and the median voter's income is lower in H than in L for all $\gamma \in (0, 0.5]$:

$$\widetilde{y}_L(\gamma) > \widetilde{y}_H(\gamma), \ \forall \gamma \in (0, 0.5]$$

[19] In particular, under my specification of preferences, it depends on the degree of complementarity between home and school inputs.

2. Let Φ_L and Φ_H be lognomal distribution functions such that the mean and standard deviation parameters of the associated normal distributions satisfy: $\mu_L = \mu_H$ and $\sigma_L < \sigma_H$. Then, the median voter effect is stronger in H than in L:

$$\left(\frac{d\widetilde{y}(\gamma)}{d\gamma}\right)_L < \left(\frac{d\widetilde{y}(\gamma)}{d\gamma}\right)_H$$

and the median voter's income is lower in H than in L for all $\gamma \in (0, 0.5)$:

$$\widetilde{y}_L(\gamma) > \widetilde{y}_H(\gamma), \ \forall \gamma \in (0, 0.5]$$

Proof. 1. For a uniform distribution, inequality increases with the difference between the maximum and minimum level of income. Note also that the density $\phi_c(y) = \frac{1}{\overline{y} - \underline{y}}$ is constant over the support and it decreases with the degree of income inequality. Hence, as it can be checked in equation (13) $d\widetilde{y}/d\gamma$ increases when $\overline{y} - \underline{y}$ does.

2. Note first that for $\gamma \in (0, 0.5)$ the definition of median voter income implies the relevant range of the income distribution is precisely that below median income. A property of the lognormal cumulative distribution function is invoked for this proof: Consider two lognormal distribution functions Φ_L and Φ_H with $\mu_L = \mu_H$ and with $\sigma_L < \sigma_L$. Then Φ_L First Order Stochastically Dominates Φ_H for y below the median and viceversa. That is to say, in the relevant range:

$$\Phi_L(y) < \Phi_H(y), \ \forall y < y_{median}$$

which in turn entails:

$$\Phi_L^{-1}\left(1 - \frac{1}{2(1-\gamma)}\right) > \Phi_H^{-1}\left(1 - \frac{1}{2(1-\gamma)}\right), \forall \gamma \in (0, 0.5)$$

and therefore:

$$\widetilde{y}_L > \widetilde{y}_H, \ \forall \gamma \in (0, 0.5)$$

■

In order to go back to an equilibrium situation after a *marginal* increase in γ, some initially decisive households with children must join the higher spending coalition. Hence, the income of the median voter falls. Increased income inequality implies a lower density around the income of the median voter with both uniform and lognormal distributions. Thereby, an (equally small) increase in the share of old households triggers a greater decrease in the income of decisive voters (Figure 2) and in the level of spending per pupil (Figure 3).

Figure 3 plots equilibrium spending per pupil against the share of childless households.[20] Several remarks are in place: (i) In examples A and B (with low and medium inequality), the relationship among spending per pupil and the proportion of childless households is non-monotonic and has an inverted-U shape. (ii) In these cases, the tax price effect dominates for low levels of γ, which makes spending per pupil to increase with ageing within a range $(0, \gamma^*)$. (iii) This effect is more intense and holds for a larger range of γ, the less inequality there is. (iv) Further increases in the proportion of elderly households, make the political median voter effect dominate and reduce funding per student. Declines in spending are increasingly larger, and become stark once the median voter is below the mode of the lognormal distribution. (iv) In example C, an economy with high income inequality, the tax price effect never dominates. Instead, the waning political support for public education funding makes spending per pupil fall with ageing at all stages of the process.

Before ending this Section, it is worth calling the reader's attention towards a caveat of the analysis. The model above adopts the neutral assumption that population ageing does not change Φ_1. But this of course needs not be the case in the real world. The ageing process might alter the income distribution of households with children. It is therefore relevant to ask: what happens if the income distribution of households reaching old ages is different from the initial Φ_1? To investigate this issue, let now the income distribution depend on γ: $\Phi_1(y; \mu, \sigma, \gamma)$.

---

[20] In these examples, I adopt a constant returns to scale Cobb-Douglas human capital production function; the induced utility function is thus written as:

$$u(x, e) = x + \delta_c h(e, y) = x + \delta_c e^{\alpha} y^{1-\alpha} \tag{19}$$

The median voter´s most preferred policy pair is then:

$$e = \widetilde{y} \left(\frac{A\alpha}{1-\gamma} \right)^{\frac{1}{1-\alpha}} ; \, t = \frac{e}{\eta}$$

The income distributions are lognormal with parameters set at the levels used in the examples in figure 2 (see footnote **??**). The technology parameter α, which accounts for the share of school inputs in the generation of future income, is set at 0.06. This generates a level of spending which is roughly 5% of total income.

Proposition 4 *If a marginal increase in γ rises (reduces) the value of the cumulative income distribution function of young households at the initial median voter income, i.e. if $\frac{\partial \Phi_1(\widetilde{y};\mu,\sigma;\gamma)}{\partial \gamma} >$ $(<)\, 0$, then the median voter effect intensifies (falls) by*

$$\frac{\frac{\partial \Phi_1(\widetilde{y};\mu,\sigma;\gamma)}{\partial \gamma}}{\phi_y^1\left(\widetilde{y},\gamma\right)}$$

Proof. Let the income distribution function depend on the share of childless households: $\Phi_1\left(y;\mu,\sigma;\gamma\right)$.

Suppose the economy is in an equilibrium with $\gamma = \gamma_0$, and $\widetilde{y} = \widetilde{y}\left(\gamma_0\right)$. Derive the median voter effect using equation (8) and the implicit function theorem to obtain:

$$\frac{d\widetilde{y}}{d\gamma} = -\frac{\dfrac{\partial \Phi_1\left(\widetilde{y};\mu,\sigma,\gamma\right)}{\partial \gamma} + \dfrac{1}{2\left(1-\gamma\right)^2}}{\phi_y^1\left(\widetilde{y},\gamma\right)}$$

■

In other words, if more (less) households with income above the median voter's become childless in relative terms, then the median voter effect intensifies (weakens), strenghthening the tendency of ageing towardsunderprovision (overprovision).

6 Discussion and final remarks

This paper uses the median voter model to take a new look at the impact of population ageing over education policy in centralised democracies. The focus is on the interplay of two opposing forces: the *tax-price effect* induced by a lower ratio of children to tax payers; and the *median-voter effect* which reduces her income when the share of childless households increases. It is first explained that childless households, and ageing, introduce distortions into democratic systems of education. On the one hand, the marginal benefit derived from education spending by decisive voters rarely coincides with the average marginal benefit. On the other hand, while the government funds education spending through universal taxation, only a part of society receives its benefits. In that case, even with non-distortionary head taxes, the tax price voters face is downwards biased with respect to production costs.

From a positive perspective, the analysis reveals that population ageing has a non-linear and possibly non-monotonic relationship with spending per pupil. Whether that demographic process tends to increase or depress funding per student depends on the degree of income inequality. Lower levels of inequality imply milder median-voter effects so that the tax-price effect, which creates a tendency towards overprovision, dominates. In economies with high income inequality, in turn, the median-voter effect of ageing is strong and easily dominates, which may lead the economy to a situation of underprovision of public education. These theoretical results yield two hypotheses to be tested empirically: the non-linear impact of the share of elderly households on spending, and the moderating effect of income inequality. Levy (2005) reveals that predictions of the median voter do not extend to elections on the level of redistribution and of education spending. Her results hinge on whether the elderly are in a minority or in a majority in the population. In the latter case, young households form part of the coalition in government and obtain more spending per pupil. Moreover, an increase in income inequality may in that situation enhance spending per pupil. Which model best matches the data should be subject to empirical scrutiny.

There are still other open questions on the issue, which make further theoretical efforts necessary. A particularly challenging problem is the analysis of the question in a multi-community setting. Given the recent development in the political economy literature, it is also worth to continue exploring richer models of the political process.

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Figure 1

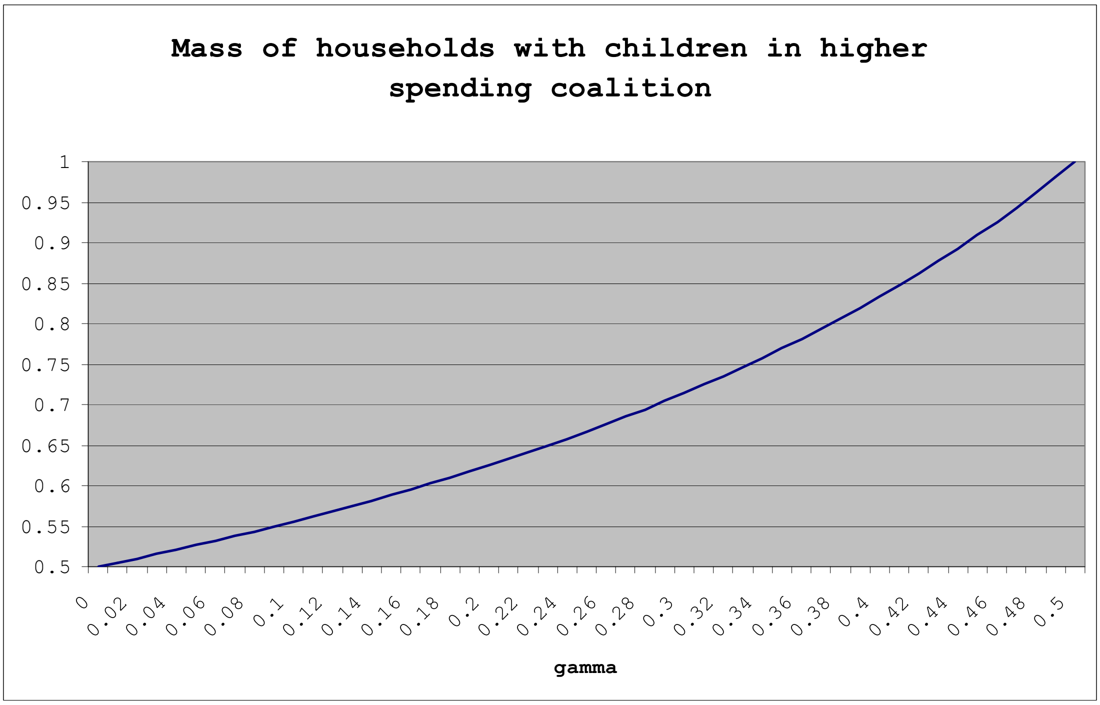


Figure 2

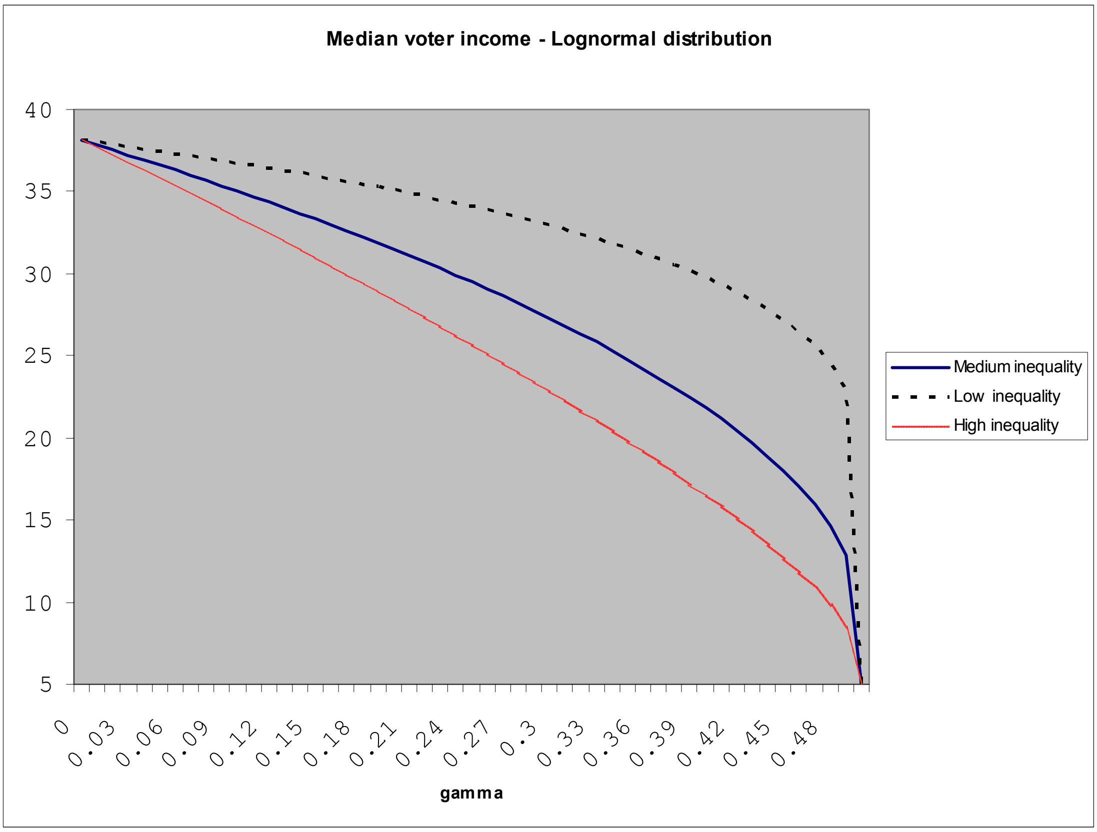
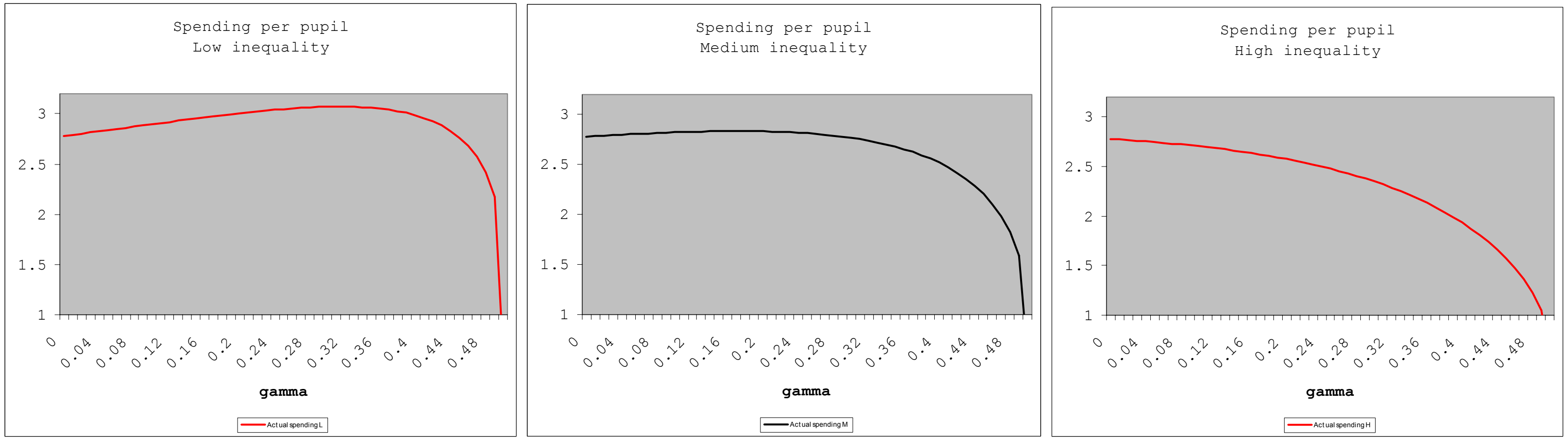


Figure 3


Spending per pupil
Low inequality
3
2.5
2
1.5
1
0
0.04
0.08
0.12
0.16
0.2
0.24
0.28
0.32
0.36
0.4
0.44
0.48
gamma
Spending per pupil
Medium inequality
3
2.5
2
1.5
1
0
0.04
0.08
0.12
0.16
0.2
0.24
0.28
0.32
0.36
0.4
0.44
0.48
gamma
Spending per pupil
High inequality
3
2.5
2
1.5
1
0
0.04
0.08
0.12
0.16
0.2
0.24
0.28
0.32
0.36
0.4
0.44
0.48
gamma