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Social Background Effects on School and Job Opportunities*

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Abstract

This paper proposes a theory on how students' social background affects school teaching and job opportunities. We study a set-up where students differ in ability and social background, and we analyse the interaction between a school and an employer. Students with disadvantaged background are penalised compared to other students: they receive less teaching and/or are less likely to be hired. A surprising result is that policy aiming to subsidise education for disadvantaged students might in fact decrease their job opportunities.

[Very Preliminary]

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1 Introduction

There is substantial evidence that individuals' social background influences their educational results and job opportunities[1]. This paper proposes a theoretical explanation for this by examining how school and employer behaviour changes according to the students' social background.

We consider a signalling game between a school and an employer where the students they deal with differ in ability and social background. The school prepares students for a final exam and wants the largest number of them to be hired. We assume that teaching improves the students' chance of obtaining a good grade but not their ability. On the other side, the employer wants to hire only high-ability students and observes the exam grade as a signal of ability.

We assume that students with advantaged social backgrounds are more likely to have higher ability. This assumption is crucial for our results and is supported by past research documenting that family and environmental factors are major predictors of cognitive skills (Cunha *et al.*, 2006, Carneiro and Heckman, 2003, Joshi and McCulloc, 2000). The idea is that given the same distribution of innate ability within populations with differing social backgrounds, an advantaged environment can help develop skills via parental and peer pressure.

Our results suggest that students from a disadvantaged social background receive less teaching and/or are hired with less chance. The intuition is the following. The employer prefers advantaged rather than disadvantaged and high-grade students as they are more likely to have high ability. To increase the hiring opportunities of disadvantaged students, the school may devote less teaching effort to disadvantaged and low-ability students, because this gives them less chance of obtaining a high grade, and thus the expected ability of disadvantaged and high-grade students increases. Despite that, the employer may still find it preferable to hire advantaged students. Thus disadvantaged students are penalised in school attainment and/or in job opportunities, and this clearly aggravates class differences.

[1] For some empirical evidence on the relationship between social background and educational attainment, see Haveman and Wolfe (1995) for a discussion, while Galindo-Rueda and Vignoles (2005) and Marcenaro-Gutierrez *et al.* (2007) give some recent contributions. For job opportunities, Glyn and Salverda (2000) and Berthoud and Blekesaune (2006) show how social background affects the chance of finding a job in OECD countries and UK, respectively.

Furthermore, these results are related to the phenomenon known as *grade inflation*, which takes place when good grades are awarded too easily. An interesting result is that the presence of grade inflation might help disadvantaged students to have the same job opportunities as advantaged students. The reason is that grade inflation diminishes the employer's expectations about students' ability. Since the school here devotes less teaching effort to disadvantaged rather than advantaged and low-ability students, the effect of grade inflation is stronger for advantaged students. This leads high-grade students to have the same job opportunities irrespective of their social backgrounds.

We then consider a government that subsidises the cost of teaching disadvantaged students. Such policy may diminish their chance of being hired. The reason is that more low-ability and disadvantaged students obtain a high grade, therefore the employer's expectations about the ability high-grade and disadvantaged students decrease.

The paper can be related to the literature on signalling models (Spence, 1973; for a survey of the literature, see Riley, 2001). In particular, the model presents a structure which is similar to Waldmann (1984), where a game between an employer and the job market takes place and the employees are "no players". In analogy, in our model a school and an employer interact and the students are "no players".

The paper is also related to the literature on grade inflation. Chan *et al.* (2005) proposes a signalling model where employers know only the students' grade but not the students' ability and the state of the world (that is, the proportion of talented students). This gives rise to an incentive to help some low ability students by giving them good grades. Indeed the labour market cannot fully distinguish whether this is due to a high grading standard or whether the school has a large proportion of talented students, and this in turn hampers the signal of good students. In Chan *et al.* (2005) differences in social background are not considered, which instead are central in our analysis.

Schwager (2008) examines the impact of grade inflation on the job market with students that differ in ability and social background. They are matched with firms which offer different kinds of job, according to the grade and the expected ability. Regardless the social background, it is possible that mediocre students receive a high grade caused by grade inflation. Also, the high-ability students from advantaged backgrounds may benefit from grade inflation since this shields them from the competition on the part of able and disadvantaged students. Compared to this analysis, we share the same assumptions on the distributions of ability with differing social backgrounds, but in our model disadvantaged students may benefit from the presence of inflation grade.

Finally, our paper is related to De Fraja (2005) who studies the provision of education when students differ in ability and social background. In the presence of asymmetric information (the government does not know the student's ability) and externalities (the public provision of education makes students acquire more education than they would acquire privately) the optimal provision of education is a second best result where high-ability and disadvantaged students receive more education than high-ability and advantaged students. Hence the introduction of reverse discrimination policies, like affirmative action[2], are justified on efficiency grounds, and the trade-off between equity and efficiency disappears. According to our results, a policy intervention is necessary in order to obtain the optimal provision of education in the presence of differences in social background, since a school caring about the employment of its students does not devote more teaching effort to disadvantaged rather than advantaged and high-ability students.

The remainder of the paper is organised as follows. The model is presented in Section 2. Section 3 examines the equilibria. Section 4 considers the government intervention. Section 5 concludes.

2 The model

We study the interaction between a single school and a single employer[3]. The interaction takes place since a number of students, with measure normalised to one, attends school and afterwards asks the employer for a job.

Students can have high (θ_H) or low (θ_L) ability. In addition to ability, stu-

[2]The term "affirmative action" refers to policies that attempt to increase the presence of individuals who belong to minorities in areas of employment and education. These policies generate controversy when they involve preferential selection on the basis of race, gender or ethnicity.

[3]For simplicity, we abstract from factors such as competition between schools and between employers.

dents can come from advantaged (a) or disadvantaged (d) social backgrounds, which is public information: this can be interpreted as a one-dimensional measure of family environment, peer groups[4] and neighbourhood. We denote as $\lambda \in [0, 1]$ the proportion of advantaged students. Let $p_a, p_d \in [0, 1]$ be the probability that an advantaged or disadvantaged student has high-ability, respectively. As we stressed in the introduction, we assume $p_a > p_d$.

2.1 Employer

The employer decides whether or not to hire a student and offers a single job type.

We define *students capacity* as the maximum number of students that may be hired and we denoted it as $\Phi \in [0, 1]$. For the sake of simplicity, we assume that Φ is exogenous and depends on the employer's production potential, that is the size and technology of his firm. As a consequence, neither the interaction between school and the employer nor the students' type can affect the employer's production potential.

Still for simplicity, we rule out uncertainty in the market where the employer operates and we assume that the students' ability determines the employer's profit entirely. In particular, each high-ability student yields a profit of $\nu > 0$ while each low-ability student yields a profit of -1. The choice of ν and -1 is to simplify the algebra: other normalisations would complicate the analysis without changing the results.

The assumption that a low-ability student gives negative profit can be interpreted in many ways: low-ability employees may have a marginal productivity which is lower than salary cost. In addition, the employer may want to lay off a low-ability employee but this action still comes at a cost, e.g. industrial disputes, wasted training costs and time, and so on.

The employer doesn't know the students' ability and observes the grade that a student obtains in a final exam[5] as a signal of it. The possible exam outcomes are a low (g_D) or a high (g_U) grade.

[4] Peer groups means that students learn better if they are in a group of abler students.

[5] The exams which we have in mind in the real world are the "Scholastic Assessment Test" in United States and the "National Curriculum Assessment" in United Kingdom. These exams are managed by the "Educational Testing Service" (Rourke and Ingram, 1991).

2.2 School

The school influences job opportunities by preparing students for the final exam[6], and learns the students' ability through their tests and assessments results.

The school obtains a benefit $\mu > 0$ for every hired student. The reason is that each student's employment increases its reputation as an effective institution for obtaining a job. Of course a school might pursue this objective in different ways, for example, by having a preference for some of their students: it may derive more benefit from increases in the job opportunities of their brightest pupils, or, vice versa, from increases in the job opportunities of their weakest pupils. To depict the interaction with the employer in the most general way we abstract from these differences.

The preparation for the exam requires resources: the quantity of teaching, the quality of buildings and classroom equipment and the teachers' effort. We refer to all these aspects as "teaching". In addition to teaching, the school can provide some students with "extra teaching", that is additional resources, extra tuition, trips and more facilities. We assume that, with teaching only, the student's probability of obtaining a high grade is $\eta \in (0, 1)$ if she has high ability and zero if she has low ability. With extra teaching, the student's probability of obtaining a high grade is 1 if she has high ability and $\eta \in (0, 1)$ if she has low ability[7]. The school bears a cost $c > 0$ for each student receiving extra teaching.

Hence the school's payoff is given by the difference between the total benefit and cost.

2.3 The game between the school and the employer

The interaction can be described as follows. Nature draws the student types. Then, each student[8] attends school and the school chooses whether to give

[6] Note that the school does not arrange the exam and hence it cannot manipulate the students' grades.

[7] The two events "high-ability with no extra teaching obtains a high grade" and "low-ability with extra teaching obtains a high grade" have the same probability to occur in order to simplify the algebra. To give to these two events a different probability would just complicate the analysis without adding any insight.

[8] For simplicity, we assumed away the influence of student's effort.

her extra teaching. At the end of school period, students take the final exam. Finally, each student applies for a job, and the employer decides whether to hire her.

3 Equilibria

We study the perfect Bayesian equilibria of this game. The choice of this equilibrium concept is motivated by the employer's missing information about students' ability.

A perfect Bayesian equilibrium is a combination of school and employer strategies and beliefs where both agents maximise their payoff. After observing a grade, the employer has a belief about the student type, conditional on all the information he has: student's social background and grade, and school strategy. Hence his belief must be consistent with the Bayes' rule. For each grade, the employer must maximise its expected profit, given his belief and the school strategy. In turn the school's strategy must maximise its expected payoff, given the employer's strategy[9]. Then, the students' capacity constraint requires that the number of hired students is at most Φ.

We start by making the following assumptions.

Assumption 1 $\Phi < \lambda(p_a + \eta\,(1 - p_a)) + (1 - \lambda)\,(p_d + \eta\,(1 - p_d)))$.

Assumption 2 $\mu > \max\left\{\frac{c}{\eta}, \frac{c}{1-\eta}\right\}$.

Assumption 1 says that the students' capacity is lower than the highest possible number of high-grade students. This assumption focuses the attention on the equilibria where social background plays a role in the school and employer's decisions. When this assumption does not hold, the employer may hire all the high-grade students irrespective of their social background. In other words, the individuals' social background would not affect their job opportunities. As we stressed in the introduction, the empirical evidence tells us in reality this is not the case, so we prefer to set this case aside.

Assumption 2 says that the school benefit of having a hired student is considerably higher than the cost of providing her with extra teaching. This

[9] Note that the school has perfect information of the student types.

rules out the possibility that a student would not receive extra teaching because, according to the school technology, it is too costly. Here we want to focus on the case where the school's response depends on the employer strategy completely, and disregard the role of school technology[10]. After presenting Assumption 1 and 2, we introduce the notations of the school and employer's actions:

- $x_{La}, x_{Ha}, x_{Ld}, x_{Hd} \in [0,1]$ are the probabilities that the school gives extra teaching to an advantaged and low or high-ability student and to a disadvantaged and low or high-ability student, respectively;

- $z_{Ua}, z_{Da}, z_{Ud}, z_{Dd} \in [0,1]$ are the probabilities that the employer hires an advantaged student with a high or low grade and a disadvantaged student with a high or low grade, respectively.

We then define the employer beliefs about the students ability. These are denoted by $\pi(\theta_z \mid g_j, p_i, x_{zi})$, where $z \in \{H, L\}$ is the ability level, $g_j, j \in \{U, D\}$ is the grade, $p_i, i \in \{a, d\}$ is the distribution of ability and x_{zi} is the school strategy.

Definition 1 *The employer's beliefs about the students' ability which are consistent with the Bayes' rule are* $\pi(\theta_H \mid g_j, p_i, x_{Hi}) = \frac{p_i x_{Hi}}{p_i x_{Hi} + \eta x_{Li}(1-p_i)}$, *and* $\pi(\theta_L \mid g_j, p_i, x_{Li}) = \frac{\eta x_{Li}(1-p_i)}{p_i x_{Hi} + \eta x_{Li}(1-p_i)}$.

As we will show below in the proof of Proposition 2, if Assumption 1 and 2 hold the equilibrium will be one of three types, which we label *high-employment*, *middle-employment* and *low-employment equilibrium.*

Definition 2 *A high-employment equilibrium is an equilibrium in which the school strategy is* $x_{Ha} = x_{Hd} = x_{La} = x_{Ld} = 1$, *while the employer strategy is* $z_{Ua} = 1, z_{Ud} = \frac{\Phi - \lambda(p_a + \eta(1-p_a))}{(1-\lambda)(p_d + \eta(1-p_d))} \in (0,1), z_{Da} = z_{Dd} = 0.$

Definition 3 *A middle-employment equilibrium is an equilibrium in which the school strategy is* $x_{Ha} = x_{Hd} = x_{La} = 1, x_{Ld} = \frac{p_d}{(1-p_d)} \frac{\nu}{\eta} \in (0,1)$, *while the employer strategy is* $z_{Ua} = 1, z_{Ud} = \min\left\{\frac{c}{\mu\eta}, \frac{\Phi - \lambda(p_a + \eta(1-p_a))}{(1-\lambda)(p_d(1+\nu))}\right\} \in (0,1)$, $z_{Da} = z_{Dd} = 0.$

[10] To relax this assumption would allow us to compare schools with different technology. This investigation can be interesting and may be considered for future work.

Definition 4 *A low-employment equilibrium is an equilibrium in which the school strategy is* $x_{Ha} = x_{Hd} = 1, x_{La} = \frac{p_a}{(1-p_a)} \frac{\nu}{\eta} \in (0,1)\,, x_{Ld} = \frac{p_d}{(1-p_d)} \frac{\nu}{\eta} \in (0,1)$, *while the employer strategy is* $z_{Ua} = z_{Ud} = \min\left\{ \frac{c}{\mu\eta}, \frac{\Phi}{(\lambda p_a + (1-\lambda) p_d)(1+\nu)} \right\} \in (0,1)\,,$ $z_{Da} = z_{Dd} = 0$.

In the *high-employment equilibrium* the employer hires Φ student (i.e., as many students as he can according to the students' capacity) while the school provides every student with extra teaching. Here the employer obtains a positive expected profit from high-grade students from both advantaged and disadvantaged backgrounds. Thus, his optimal strategy is to hire all of them, but Assumption 1 impedes this possibility. Then the employer needs to choose between these two types. He will hire all the advantaged students, since they give a higher expected profit, and the disadvantaged students will be hired only for the remaining students' capacity.

In the *middle-employment equilibrium*, the employer does not want to hire all the disadvantaged and high-grade students[11]. In turn, the school provides extra teaching to a *lower* number of disadvantaged and low-ability students. This strategy increases the probability that a disadvantaged and high-grade student has high ability and thus it increases her chance to be hired.

In the *low-employment equilibrium*, the employer does not hire all the high-grade students from both advantaged and disadvantaged backgrounds, but is indifferent between hiring one of these two types. Like in the previous equilibrium, the school provides extra teaching to fewer disadvantaged than advantaged and low-ability students.

Note that the employer never hires a low-grade student: indeed all the high-ability students receive extra-teaching in each equilibrium, and hence all of them will obtain a high grade with probability one. Thus a low-grade student has low ability with probability one.

3.1 Benchmark case: one social background

In this section we assume no differences in social background. This allow us to highlight the role of other characteristics, such as the probability η and the

[11] Given this strategy, the total of hired students might be higher or lower than the students' capacity, and in the former case clearly this is Φ again.

distribution of ability, and the school and employer technology.

Note that the probability η can be interpreted as an inverse measure of educational standard. The educational standard measures the level of difficulty at school, how hard is to obtain a high grade[12]. Indeed as η increases, obtaining a high grade becomes "easier" for some students. Thus the higher η, the lower the educational standard[13].

For concreteness, we consider a population of disadvantaged students[14], i.e., $\lambda = 0$. The following proposition shows which equilibrium takes place according to the value of p_d.

Proposition 1 *Let Assumptions 1 and 2 hold and $\lambda = 0$. The high-employment equilibrium occurs if $p_d \geq \frac{\eta}{\nu+\eta}$; the middle/low-employment equilibrium occurs if $p_d < \frac{\eta}{\nu+\eta}$.*

Proof. See Appendix. ■

Figure 1 illustrates Proposition 1. Assumption 1 holds below the downward-sloping area.

The equilibrium which takes place depends on a combination of p_d and η. If the educational standard is high (η low) and p_d is high, the equilibrium is *high-employment.* If η is high and p_d is low, the *middle/low-employment equilibrium* occurs. If both are high (or vice versa), which equilibrium takes place depends on which effect prevails.

The upward-sloping line represents the points where $p_d = \frac{\eta}{\nu+\eta}$. As the profit ν increases, the employer hires more students and the threshold shifts down. As the educational standard decreases (η high), the amount of low-ability and high-grade students increases. Therefore the employer hires less students and the threshold shifts up.

[12]The literature on educational standards examines the criteria adopted by schools in evaluating students. The theoretical frameworks on educational standards are provided by Costrell (1994, 1997) and Betts (1998). In the context of educational standards, the issue of social background has been introduced by Himmler and Schwager (2007), who show that a school with a large proportion of disadvantaged students applies less demanding standards since its students have less incentives to graduate.

[13]The educational standard can be employed as a tool for welfare analysis. A normative extension of our set-up can be considered for future work.

[14]Note that, if $\lambda = 0$ the *middle* or the *low-employment equilibrium* are equivalent for a disadvantaged student.

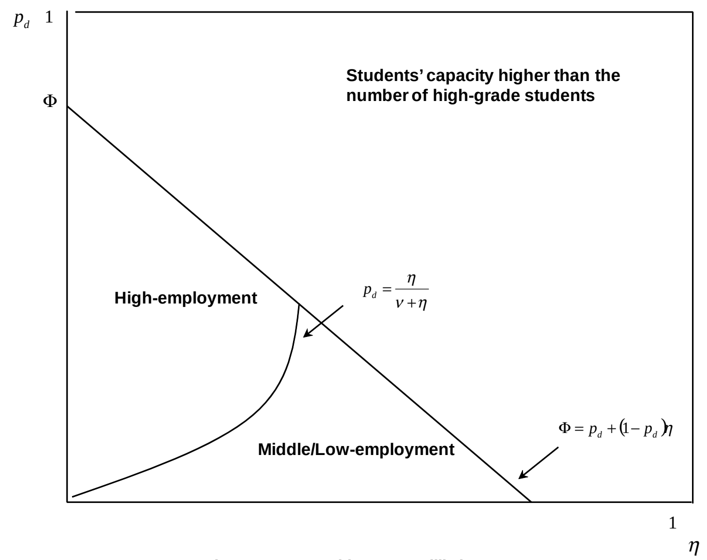

Figure 1: Proposition 1. Equilibria

Finally, some considerations are necessary about the school strategy. Note that in the *high-employment equilibrium*, the school gives extra-teaching to all students even though some of them will not be hired. This happens because of Assumption 2, which ensures a very high school benefit from a student's employment. The cost of teaching a student who will not be hired is much smaller than the benefit loss incurred from a non-hired student who did not receive extra teaching.

3.2 General case: differences in social background

Now we consider $\lambda \in (0, 1)$. The following proposition shows which equilibrium takes place according to the values of p_d and p_a.

Proposition 2 *Let Assumptions 1 and 2 hold. The high-employment equilibrium occurs if* $p_a \geq \frac{\eta}{\nu+\eta}$ *and* $p_d \geq \frac{\eta}{\nu+\eta}$*; the middle-employment equilibrium occurs if* $p_a \geq \frac{\eta}{\nu+\eta}$ *and* $p_d < \frac{\eta}{\nu+\eta}$*; the low-employment equilibrium occurs if* $p_a < \frac{\eta}{\nu+\eta}$ *and* $p_d < \frac{\eta}{\nu+\eta}$.

Proof. See Appendix. ■

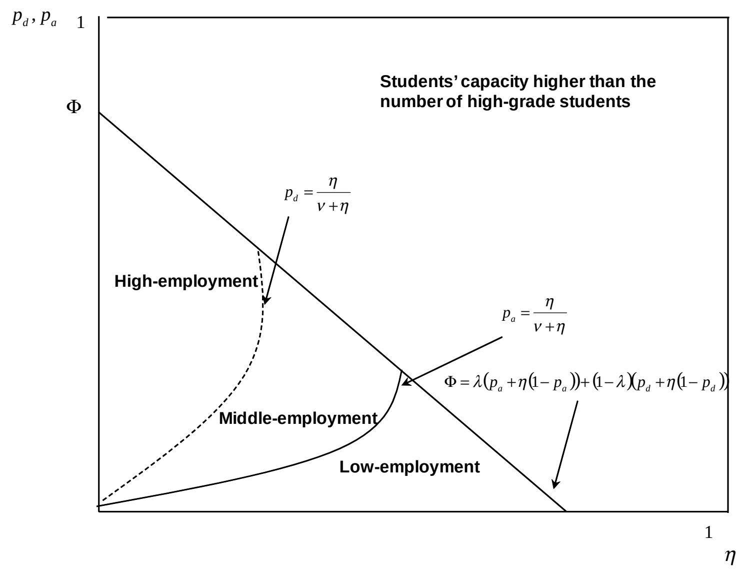

Figure 2: Proposition 2. Equilibria with differences in social background

Figure 2 illustrates Proposition 2. The dashed and the continuous upward-sloping lines represent the points where $p_d = \frac{\eta}{\nu+\eta}$ and $p_a = \frac{\eta}{\nu+\eta}$, respectively.

To interpret Proposition 2, begin by looking at the key assumption, $p_a > p_d$. This makes the employer obtain a higher expected payoff by hiring advantaged students, given the same school strategy for every student. However this may not happen if the school gives extra teaching to a lower proportion of disadvantaged than advantaged and low-ability students, since this would increase the expected quality of the disadvantaged and high-grade students.

When both p_d and p_a are higher than $\frac{\eta}{\nu+\eta}$ (*high-employment equilibrium*), the school maximises the amount of hired students by providing every student with extra teaching, since the employer thinks that a high grade student very likely has high ability, irrespective of her social background. In the other two cases, the school maximises the amount of hired disadvantaged students by giving less extra teaching to low-ability and disadvantaged compared to advantaged students.

In particular, when p_a is higher and p_d is lower than $\frac{\eta}{\nu+\eta}$ (*middle-employment equilibrium*), the employer still prefers advantaged rather than disadvantaged and high-grade students. When both p_d and p_a are lower than $\frac{\eta}{\nu+\eta}$ (*low-*

employment equilibrium), the employer is indifferent between hiring an advantaged or a disadvantaged and high-grade student.

Proposition 2 shows that disadvantaged students are penalised compared to advantaged students in each possible case: they may receive less teaching, or be hired with lower probability to the employer, or both. In particular the *high* and *middle-employment equilibrium*, where disadvantaged students are penalised in the job market, exacerbate differences in social class.

These results suggest some interesting considerations about grade inflation. In our model, this situation is depicted where the educational standard is low (η is high), since more low-ability students obtain a high grade. We can easily observe that when η is high we are very likely to be in the *low-employment equilibrium*, where the employer is indifferent between hiring advantaged or disadvantaged and high-grade students. Indeed the presence of grade inflation diminishes the employer's expectations about students' ability. Since the school devotes less teaching effort to disadvantaged rather than advantaged and low-ability students, the grade inflation effect is stronger for advantaged students, leading to the *low-employment equilibrium*. Therefore grade inflation may have some positive effect by helping disadvantaged students to have the same job opportunities as advantaged students. This is in contrast with other results on grade inflation (Schwager, 2008), where the job opportunities of high-ability and disadvantaged students are penalised by the grade inflation of low-ability and advantaged students.

Finally, this result can be linked to the analysis of efficient provision of education. De Fraja (2005) shows that, in the presence of differences in social background, the optimal provision of education requires that disadvantaged and high-ability students receive *more* education than high-ability and advantaged students. According to *Proposition 2*, a school caring about the employment of its students does not devote more teaching effort to disadvantaged rather than advantaged and high-ability students. Therefore a policy intervention would be necessary to reach an efficient level of education.

3.3 Analysis of equilibria

In this section we study the properties of our equilibria. The following corollary shows some comparative statics results.

Corollary 1 *A decrease in the educational standard (an increase in η) diminishes the employment opportunities and the provision of extra teaching; an increase in the employer's profit ν increases the provision of extra teaching; an increase in the proportion of advantaged students λ increases the employment opportunities for disadvantaged students.*

Proof. See Appendix. ■

An increase in η makes the number of high-grade students increase. Thus their probability of being hired diminishes. In turn this makes the probability of receiving extra teaching diminish.

An increase in ν leads to more employment opportunities, hence the school gives extra teaching to more low-ability students.

An increase in λ has two contrasting effects in a *high-employment equilibrium*: the amount of disadvantaged and high-grade students diminishes and the employment opportunities for disadvantaged students are lowered. With the first effect, the probability of a disadvantaged and high-grade student being hired increases, while it diminishes with the second one. Nevertheless, the first effect more than offsets the second effect. The reason is the following: a decrease in the amount of disadvantaged and high-grade students increases the students' capacity relative to disadvantaged students ($\frac{\Phi}{(1-\lambda)(p_d+\eta(1-p_d))}$) with more intensity than it diminishes the relative employment placements ($-\frac{\lambda(p_a+\eta(1-p_a))}{(1-\lambda)(p_d+\eta(1-p_d))}$). Therefore a higher proportion of advantaged students may increase the job opportunities of disadvantaged students.

4 Subsidising disadvantaged students

In many countries, governments spend substantial resources to fight unequal educational outcomes[15]. We can depict such an intervention by considering a government that cannot observe the student's ability and subsidises c for all disadvantaged students. We examine the problem from a partial equilib-

[15] To cite some example, in the United States, recent measures of funding education for disadvantaged have been considered the "No Child Left Behind Act" of 2001, and in the "American Recovery and Reinvestment Act" of 2009. In the United Kingdom, the Education Manteinance Allowance (EMA) funds low-income students who decide to keep studying after 16.

rium perspective, in the sense that no government taxation is integrated into education subsidies.

The following proposition shows the policy results.

Proposition 3 *Let us assume that the government funds c for every disadvantaged student receiving extra-teaching:*

(i) if $p_a \geq \frac{\eta}{\nu+\eta}$ *and* $p_d \geq \frac{\eta}{\nu+\eta}$*, the high-employment equilibrium takes place (as before);*

(ii) if $p_a \geq \frac{\eta}{\nu+\eta}$ *and* $p_d < \frac{\eta}{\nu+\eta}$*, the school strategy is* $x_{Ha} = x_{La} = 1$, $x_{Hd}, x_{Ld} \in (0, 1)$*, while the employer strategy is* $z_{Ua} = 1, z_{Da} = z_{Ud} = z_{Dd} = 0$*;*

(iii) if $p_a < \frac{\eta}{\nu+\eta}$ *and* $p_d < \frac{\eta}{\nu+\eta}$*, the school strategy is* $x_{Ha} = 1, x_{La} = \frac{p_a}{(1-p_a)} \frac{\nu}{\eta}$, $x_{Hd}, x_{Ld} \in (0, 1)$*, while the employer strategy is* $z_{Ua} = \min\left\{\frac{c}{\mu\eta}, \frac{\Phi}{\lambda p_a(1+\nu)}\right\}, z_{Da} = z_{Ud} = z_{Dd} = 0$*.*

Proof. See Appendix. ■

In case (i) ($p_a \geq \frac{\eta}{\nu+\eta}$ and $p_d \geq \frac{\eta}{\nu+\eta}$), the occurring equilibrium is still *high-employment*, thus the policy does not have any effect whatsoever, since the school would have given extra teaching to every student even if no policy was applied. In the cases (ii) and (iii), the employer never hires a disadvantaged student, therefore the policy might worsen her hiring opportunities.

The reason is intuitive. The school has no costs to provide extra-teaching to disadvantaged students, so the employer does not believe their grade is a signal of their ability if p_d is low. Providing only high-ability students with extra teaching is not a credible strategy, as *ex post* the school would give it even to low-ability students.

This analysis suggests care should be taken in policy choices, since the attempt to improve the schooling attainment of disadvantaged students might in fact diminish their job opportunities.

5 Concluding remarks

This paper examines how social background affects school's teaching and an employer's recruitment. We analysed the interaction between a school and an employer when students attend school and then apply for a job. Our results suggest that disadvantaged students are penalised compared to advantaged students, as they receive less teaching and/or are less likely to be hired.

The policy considerations can be extended in many directions. The government might impose some restriction on the employer strategy in order to favour disadvantaged students, like in the case of affirmative action. For instance, the employer might be forced to hire a certain number of disadvantaged students. In welfare analysis a policy can be considered where the educational standard (η) is set to maximize welfare.

Furthermore, the framework can be developed in several ways, two of which we discuss briefly. First, it seems natural to consider different schools for each social group by taking into account differences in quality of teaching. Second, it would be interesting to examine this framework alongside different generations for explaining segregation or inequality. The analysis of an extended model regarding these expansions is left for future work.

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Appendix

Proof of Proposition 1 and 2

The proof follows *Proposition 2*. By setting $\lambda = 0$ we obtain the proof of *Proposition 1.*

Case 1. $p_a \geq \frac{\eta}{\nu+\eta}, p_d \geq \frac{\eta}{\nu+\eta}$

Employer. The employer strategy is $z_{Ua}=1$; $z_{Da}=0$, $z_{Ud}=\frac{\Phi-\lambda(p_a+\eta(1-p_a))}{(1-\lambda)(p_d+\eta(1-p_d))}$; $z_{Dd}=0$. The employer's beliefs for advantaged students are $\pi\left(\theta_H \mid g_U, a\right)=\frac{p_a}{p_a+\eta(1-p_a)}$ and $\pi\left(\theta_L \mid g_U, a\right)=\frac{\eta(1-p_a)}{p_a+\eta(1-p_a)}$, if the student has a high grade and $\pi\left(\theta_H \mid g_D, a\right)=0$ and $\pi\left(\theta_L \mid g_D, a\right)=1$ if the student has a low grade. Thus the expected profits[16] for hiring an advantaged and high-grade student is $\Pi_{Ua}^E=\frac{p_a}{p_a+\eta(1-p_a)}\nu-\frac{\eta(1-p_a)}{p_a+\eta(1-p_a)}$. This must be $\frac{p_a}{p_a+\eta(1-p_a)}\nu-\frac{\eta(1-p_a)}{p_a+\eta(1-p_a)} \geq 0$ and, after few passages, $p_a \geq \frac{\eta}{\nu+\eta}$. The expected profits for hiring and not hiring an advantaged and low-grade student are $\Pi_{Da}^E=-1$ and $\Pi_{Da}^N=0$, respectively, thus $\Pi_{Da}^E<\Pi_{Da}^N$.

The employer's beliefs for disadvantaged students are $\pi\left(\theta_H \mid g_U, d\right)=\frac{p_d}{p_d+\eta(1-p_d)}$ and $\pi\left(\theta_L \mid g_U, d\right)=\frac{\eta(1-p_d)}{p_d+\eta(1-p_d)}$ if the student has a high grade and $\pi\left(\theta_H \mid g_D, d\right)=0$ and $\pi\left(\theta_L \mid g_D, d\right)=1$ if the student has a low grade. The expected profit for hiring one disadvantaged and high-grade student is $\Pi_{Ud}^E=\frac{p_d}{p_d+\eta(1-p_d)}\nu-\frac{\eta(1-p_d)}{p_d+\eta(1-p_d)}$. This must be $\frac{p_a}{p_a+\eta(1-p_a)}\nu-\frac{\eta(1-p_a)}{p_a+\eta(1-p_a)} \geq 0$ and, after few passages, $p_d \geq \frac{\eta}{\nu+\eta}$. The expected profits for hiring and not hiring a disadvantaged and low-grade student are $\Pi_{Dd}^E=-1$ and $\Pi_{Dd}^N=0$, respectively, thus $\Pi_{Dd}^E<\Pi_{Dd}^N$.

Then the employer needs to compare the expected profit obtained by high grade students with different social background[17]: this is $\Pi_{Ua}^E>\Pi_{Ud}^E$, as $p_a>p_d$. As a consequence,the employer admits all the advantaged and high-grade students and the disadvantaged ones only for the remainder of the students' capacity. Given the restrictions on the students' capacity, the number of hired disadvantaged and high grade ones is $\frac{\Phi-\lambda(p_a+\eta(1-p_a))}{(1-\lambda)(p_d+\eta(1-p_d))}$.

School. The school strategy is $x_{La}=1; x_{Ha}=1, x_{Ld}=1; x_{Hd}=1$. The expected payoffs for giving or not giving extra teaching to an advantaged and high-ability student are $\Pi_{Ha}^T=\mu-c$ and $\Pi_{Ha}^{NT}=\mu\eta$, respectively. This must be $\Pi_{Ha}^T>\Pi_{Ha}^{NT}$, that is $\mu-c \geq \mu\eta$, and therefore $\mu \geq \frac{c}{1-\eta}$. The expected payoffs for giving or not giving extra teaching to an advantaged and low-ability student are $\Pi_{La}^T=\mu\eta-c$ and $\Pi_{Ha}^{NT}=0$, respectively. This must be $\Pi_{La}^T \geq \Pi_{La}^{NT}$,

[16]The superscript of the university's expected profit indicates the strategy performed by the employer, where E indicates "to admit" and N not. The subscript specifies the student's grade, where U indicates a high grade and D a low grade, while a and d indicates the student's social background.

[17]This is not necessary for low-grade students as none of them are admitted.

that is $\mu\eta - c \geq 0$, and therefore $\mu \geq \frac{c}{\eta}$.

The expected payoffs for giving or not giving extra teaching to a disadvantaged and high-ability student are $\Pi_{Hd}^{T} = \mu z_{Ud} - c$ and $\Pi_{Hd}^{NT} = \mu\eta z_{Ud}$, respectively. This must be $\Pi_{Hd}^{T} \geq \Pi_{Hd}^{NT}$, that is $\mu z_{Ud} - c \geq \mu z_{Ud}\eta$, and therefore $\mu \geq \frac{c}{z_{Ud}(1-\eta)}$. The expected payoffs for giving or not giving extra teaching to a disadvantaged and high-ability student are $\Pi_{Ld}^{T} = \mu\eta z_{Ud} - c$ and $\Pi_{Hd}^{NT} = 0$, respectively. This must be $\Pi_{Ld}^{T} \geq \Pi_{Ld}^{NT}$, that is $\mu\eta z_{Ud} - c \geq 0$, and therefore $\mu \geq \frac{c}{z_{Ud}\eta}$.

Demand constraint. The total number of hired students is:

$$\lambda(p_a + \eta\,(1 - p_a)) + (1 - \lambda)\,(p_d + \eta\,(1 - p_d))\,\frac{\Phi - \lambda(p_a + \eta\,(1 - p_a))}{(1 - \lambda)\,(p_d + \eta\,(1 - p_d))} \equiv \Phi.$$

Case 2. $p_a \geq \frac{\eta}{\nu+\eta}, p_d < \frac{\eta}{\nu+\eta}$

As $p_a \geq \frac{\eta}{\nu+\eta}$, the employer and school strategy for advantaged students does not change compared to the previous case.

Employer. The employer strategy is $z_{Ua} = 1$; $z_{Da} = 0$, $z_{Ud} = \min\left\{\frac{c}{\mu\eta}, \frac{\Phi-\lambda(p_a+\eta(1-p_a))}{(1-\lambda)(p_d(1+\nu))}\right\}$; $z_{Dd} = 0$. The employer's beliefs for disadvantaged students are $\pi\left(\theta_H \mid g_U, d\right) = \frac{p_d}{p_d+\eta x_{Ld}(1-p_d)}$ and $\pi\left(\theta_L \mid g_U, d\right) = \frac{\eta x_{Ld}(1-p_d)}{p_d+\eta x_{Ld}(1-p_d)}$, if the student has a high grade and $\pi\left(\theta_H \mid g_D, d\right) = 0$ and $\pi\left(\theta_L \mid g_D, d\right) = 1$ if the student has a low grade. Thus the expected profit for hiring an advantaged and high-grade student is $\Pi_{Ud}^{E} = \frac{p_d}{p_d+\eta x_{Ld}(1-p_d)}\nu - \frac{\eta x_{Ld}(1-p_d)}{p_d+\eta x_{Ld}(1-p_d)}$. This must be $\frac{p_d}{p_d+\eta x_{Ld}(1-p_d)}\nu - \frac{\eta x_{Ld}(1-p_d)}{p_d+\eta x_{Ld}(1-p_d)} = 0$ and, after few passages, $x_{Ld} = \frac{p_d}{(1-p_d)}\frac{\nu}{\eta}$. To be a probability, then $\frac{p_d}{(1-p_d)}\frac{\nu}{\eta} < 1$, by which $p_d < \frac{\eta}{\nu+\eta}$. The expected profits for hiring and not hiring a disadvantaged and low-grade student are $\Pi_{Dd}^{E} = -1$ and $\Pi_{Dd}^{N} = 0$, respectively, thus $\Pi_{Dd}^{E} < \Pi_{Dd}^{N}$.

Then the employer needs to compare the expected profit obtained by high grade students with different social background: this is $\Pi_{Ua}^{E} > \Pi_{Ud}^{E}$, as $\Pi_{Ua}^{E} > 0$, while $\Pi_{Ud}^{E} = 0$.

School. The school strategy is $x_{La} = 1; x_{Ha} =; x_{Ld} = \frac{p_d}{(1-p_d)}\frac{\nu}{\eta}; x_{Hd} = 1$. The expected payoffs for giving or not giving extra teaching to a disadvantaged and high-ability student are $\Pi_{Hd}^{T} = \mu z_{Ud} - c$ and $\Pi_{Hd}^{NT} = \mu\eta z_{Ud}$, respectively. This must be $\Pi_{Hd}^{T} \geq \Pi_{Hd}^{NT}$, that is $\mu z_{Ud} - c \geq \mu\eta z_{Ud}$, and therefore $\mu \geq \frac{c}{z_{Ud}(1-\eta)}$. The expected payoffs for giving or not giving extra teaching to a disadvantaged and low-ability student are $\Pi_{Ld}^{T} = z_{Ud}\mu\eta - c$ and $\Pi_{Hd}^{NT} = 0$, respectively. This must be $\Pi_{Ld}^{T}=\Pi_{Ld}^{NT}$, that is $z_{Ud}\mu\eta-c=0,$ and therefore $z_{Ud}=\frac{c}{\mu\eta}$.

Demand constraint. The total number of hired students[18] is:

$$\lambda(p_a+\eta\,(1-p_a))+(1-\lambda)\,(p_d\,(1+\nu))\,\frac{c}{\mu\eta}\leq\Phi,$$

thus the students' capacity constraint implies $z_{Ud}=\min\left\{\frac{c}{\mu\eta},\frac{\Phi-\lambda(p_a+\eta(1-p_a))}{(1-\lambda)(p_d(1+\nu))}\right\}$.

Case 3. $p_a<\frac{\eta}{\nu+\eta},p_d<\frac{\eta}{\nu+\eta}$

As $p_d<\frac{\eta}{\nu+\eta}$, the employer and school strategy for disadvantaged students does not change compared to the previous case.

Employer. The employer strategy is $z_{Ua},z_{Ud}=\min\left\{\frac{c}{\mu\eta},\frac{\Phi}{(\lambda p_a+(1-\lambda)p_d)(1+\nu)}\right\}$; $z_{Da}=0$; $z_{Dd}=0$. The employer's beliefs for advantaged students are $\pi\left(\theta_H\mid g_U,a\right)=\frac{p_a}{p_a+\eta x_{La}(1-p_a)}$ and $\pi\left(\theta_L\mid g_U,a\right)=\frac{\eta x_{La}(1-p_a)}{p_a+\eta x_{La}(1-p_a)}$, if the student has a high grade and $\pi\left(\theta_H\mid g_D,a\right)=0$ and $\pi\left(\theta_L\mid g_D,a\right)=1$ if the student has a low grade. Thus the expected profit for hiring an advantaged and high-grade student is $\Pi_{Ua}^{E}=\frac{p_a}{p_a+\eta x_{La}(1-p_a)}\nu-\frac{\eta x_{La}(1-p_a)}{p_a+\eta x_{La}(1-p_a)}$. This must be $\frac{p_a}{p_a+\eta x_{La}(1-p_a)}\nu-\frac{\eta x_{La}(1-p_a)}{p_a+\eta x_{La}(1-p_a)}=0$ and, after few passages, $x_{La}=\frac{p_a}{(1-p_a)}\frac{\nu}{\eta}$. To be a probability, it is necessary that $\frac{p_a}{(1-p_a)}\frac{\nu}{\eta}<1$, by which $p_a<\frac{\eta}{\nu+\eta}$. The expected profits for hiring and not hiring an advantaged and low-grade student are $\Pi_{Da}^{E}=-1$ and $\Pi_{Da}^{N}=0$, respectively, thus $\Pi_{Da}^{E}<\Pi_{Da}^{N}$.

Then the employer needs to compare the expected profit obtained by high grade students with different social background: this is $\Pi_{Ua}^{E}=\Pi_{Ud}^{E}$, as both $\Pi_{Ua}^{E}=0$ and $\Pi_{Ud}^{E}=0$.

School. The school strategy is $x_{La}=\frac{p_a}{(1-p_a)}\frac{\nu}{\eta}$; $x_{Ha}=1$; $x_{Ld}=\frac{p_d}{(1-p_d)}\frac{\nu}{\eta}$; $x_{Hd}=1$. The expected payoffs for giving or not giving extra teaching to an advantaged and high-ability student are $\Pi_{Ha}^{T}=\mu z_{Ua}-c$ and $\Pi_{Ha}^{NT}=\mu\eta z_{Ua}$, respectively. This must be $\Pi_{Ha}^{T}\geq\Pi_{Ha}^{NT}$, that is $\mu z_{Ua}-c\geq\mu\eta z_{Ua}$, and therefore $\mu\geq\frac{c}{z_{Ua}(1-\eta)}$. The expected payoffs for giving or not giving extra teaching to a disadvantaged and low-ability student are $\Pi_{La}^{T}=\mu z_{Ua}\eta-c$ and $\Pi_{Hd}^{NT}=0$, respectively. This must be $\Pi_{La}^{T}=\Pi_{La}^{NT}$, that is $\mu z_{Ua}\eta-c=0,$ and therefore $z_{Ua}=\frac{c}{\mu\eta}$.

[18]Note that the number of disadvantaged and high-grade students in this equilibrium is $(1-\lambda)\,(p_d+\eta\,(1-p_d)\,x_{Ld})$, in this equilibrium $x_{Ld}=\frac{p_d}{(1-p_d)}\frac{\nu}{\eta}$, by substituting we obtain $(1-\lambda)\left(p_d+\eta\,(1-p_d)\,\frac{p_d}{(1-p_d)}\frac{\nu}{\eta}\right)$, which can be simplified in $(1-\lambda)\,(p_d\,(1+\nu))$.

Demand constraint. The total number of hired students[19] is:

$$\frac{c}{\mu\eta}\left(1+\nu\right)\left(\lambda p_a+\left(1-\lambda\right)p_d\right)\leq\Phi,$$

thus the students' capacity constraint implies $z_{Ud}=\min\left\{\frac{c}{\mu\eta},\frac{\Phi}{(\lambda p_a+(1-\lambda)p_d)(1+\nu)}\right\}$.■

Proof of corollary 1

High-employment equilibrium. Differentiation of $\frac{\Phi-\lambda(p_a+\eta(1-p_a))}{(1-\lambda)(p_d+\eta(1-p_d))}$ with respect to η, and λ yields $\frac{\partial}{\partial\eta}\frac{\Phi-\lambda(p_a+\eta(1-p_a))}{(1-\lambda)(p_d+\eta(1-p_d))}=\frac{\lambda}{\lambda-1}\frac{1-p_a}{p_d+\eta(1-p_d)}+\frac{(1-p_d)(\Phi-\lambda(p_a+\eta(1-p_a)))}{(\lambda-1)(p_d-\eta(p_d-1))^2}$ <0, and $\frac{\partial}{\partial\lambda}\frac{\Phi-\lambda(p_a+\eta(1-p_a))}{(1-\lambda)(p_d+\eta(1-p_d))}=\frac{\Phi-(p_a+\eta(1-p_a))}{(\lambda-1)^2(p_d+\eta(1-p_d))}>0$, respectively.

Middle-employment equilibrium. Differentiation of $\frac{c}{\mu\eta}$ with respect to η yields $\frac{\partial}{\partial\eta}\frac{c}{\mu\eta}=-\frac{c}{\mu\eta^2}<0$. Differentiation of $\frac{p_d}{(1-p_d)}\frac{\nu}{\eta}$ with respect to η and ν yields $\frac{\partial}{\partial\eta}\frac{p_d}{(1-p_d)}\frac{\nu}{\eta}=\frac{\nu p_d}{\eta^2(p_d-1)}<0$, and $\frac{\partial}{\partial\nu}\frac{p_d}{(1-p_d)}\frac{\nu}{\eta}=\frac{1}{\eta}\frac{p_d}{1-p_d}>0$, respectively.

Low-employment equilibrium. Differentiation of $\frac{p_d}{(1-p_d)}\frac{\nu}{\eta}$ with respect to η and ν yields $\frac{\partial}{\partial\eta}\frac{p_a}{(1-p_a)}\frac{\nu}{\eta}=\frac{\nu}{\eta^2}\frac{p_a}{p_a-1}<0$, and $\frac{\partial}{\partial\nu}\frac{p_a}{(1-p_a)}\frac{\nu}{\eta}=\frac{1}{\eta}\frac{p_a}{1-p_a}>0$, respectively.■

Proof of Proposition 3

The government subsidises c. Thus to provide disadvantaged students with extra teaching is weakly dominant. This does not changes anything in the case 1, as the school strategy was $x_{Ld}=1;x_{Hd}=1$.

Case 2. $p_a\geq\frac{\eta}{\nu+\eta},p_d<\frac{\eta}{\nu+\eta}$

For advantaged students we refer to the proof (case 2) of *Proposition 2*.

Employer. The employer strategy is $z_{Ua}=1;z_{Da}=0,z_{Ud}=0;z_{Dd}=0$. The employer's beliefs for disadvantaged students are $\pi\left(\theta_H\mid g_U,d\right)=\frac{x_{Hd}p_d}{x_{Hd}p_d+\eta x_{Ld}(1-p_d)}$ and $\pi\left(\theta_L\mid g_U,d\right)=\frac{\eta x_{Ld}(1-p_d)}{x_{Hd}p_d+\eta x_{Ld}(1-p_d)}$, if the student has a high grade and $\pi\left(\theta_H\mid g_D,d\right)=0$ and $\pi\left(\theta_L\mid g_D,d\right)=1$ if the student has a low grade. Thus the expected profit for hiring an advantaged and high-grade student is $\Pi^E_{Ud}=\frac{x_{Hd}p_d}{x_{Hd}p_d+\eta x_{Ld}(1-p_d)}\nu-$

[19] Note that the number of advantaged and high-grade students in this equilibrium is $\lambda\left(p_a+\eta\left(1-p_a\right)x_{La}\right)$, in this equilibrium $x_{La}=\frac{p_a}{(1-p_a)}\frac{\nu}{\eta}$, by substituting we obtain $\lambda\left(p_a+\eta\left(1-p_a\right)\frac{p_a}{(1-p_a)}\frac{\nu}{\eta}\right)$, which can be simplified in $\lambda\left(p_a\left(1+\nu\right)\right)$.

$\frac{\eta x_{Ld}(1-p_d)}{x_{Hd}p_d+\eta x_{Ld}(1-p_d)}$. This must be $\frac{x_{Hd}p_d}{x_{Hd}p_d+\eta x_{Ld}(1-p_d)}\nu - \frac{\eta x_{Ld}(1-p_d)}{x_{Hd}p_d+\eta x_{Ld}(1-p_d)} < 0$ and, after few passages, $p_d < \frac{\eta x_{Ld}}{\nu x_{Hd}+\eta x_{Ld}}$. The condition $\frac{\eta x_{Ld}}{\nu x_{Hd}+\eta x_{Ld}} \leq \frac{\eta}{\nu+\eta}$ is sufficient to have $p_d < \frac{\eta}{\nu+\eta}$. After few passages, this is verified if $x_{Hd} \geq x_{Ld}$, which always holds[20]. The expected profits for hiring and not hiring a disadvantaged and low-grade student are $\Pi^E_{Dd} = -1$ and $\Pi^N_{Dd} = 0$, respectively, thus $\Pi^E_{Dd} < \Pi^N_{Dd}$.

School. The school strategy is $x_{La} = 1; x_{Ha} = 1; x_{Ld} \in (0,1); x_{Hd} \in (0,1)$. The expected payoffs for giving or not giving extra teaching to a disadvantaged and high-ability student are $\Pi^T_{Hd} = 0$ and $\Pi^{NT}_{Hd} = 0$, respectively. This must be $\Pi^T_{Hd} = \Pi^{NT}_{Hd}$, and it is verified for every $x_{Hd} \in (0,1)$. The expected payoffs for giving or not giving extra teaching to a disadvantaged and low-ability student are $\Pi^T_{Ld} = 0$ and $\Pi^{NT}_{Hd} = 0$, respectively, and it is verified for every $x_{Hd} \in (0,1)$.

Demand constraint. It is verified as $\lambda(p_a + \eta(1-p_a)) < \Phi$.

Case 3. $p_a < \frac{\eta}{\nu+\eta}, p_d < \frac{\eta}{\nu+\eta}$

For advantaged students we refer to the proof (case 3) of *Proposition 2*. As $p_d < \frac{\eta}{\nu+\eta}$, the employer and school strategy for disadvantaged students does not change compared to the previous case.

[20] Since the payoff of giving extra teaching is higher for high-ability rather than low-ability students, $\mu > \eta\mu$.