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**MEASURING CONVERGENCE OF THE  
NEW MEMBER COUNTRIES' EXCHANGE  
RATES TO THE EURO**

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# Measuring Convergence of the New Member Countries' Exchange Rates to the Euro

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## Abstract

We propose a common factor approach to analyse convergence, which we implement using principal components analysis. This technique has not been used to analyse convergence of time series but is shown to provide a useful new tool. We show how it is in many ways a more natural way of approaching the convergence debate. We apply these ideas to a dataset of bilateral Euro and US-Dollar exchange rates of the new member countries of the European Union. Our empirical application gives sensible results about the convergence process of the new member countries' exchange rates to the Euro.

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## **1. Introduction**

Despite the fact that convergence has been a popular theme in applied economics since the seminal papers of Barro (1991) and Barro and Sala-i-Martin (1992), the very notion of convergence quickly becomes problematic from an academic viewpoint when we try and formalise a framework to think about these issues. In the light of the continuous prominence of convergence issues in the academic and policy debate and the number of different definitions of convergence used, it would be useful to have a more universal framework to think about these issues. In this paper we consider some of the standard definitions of convergence and suggest an alternative way to analyse convergence based on a common factor framework. We show that a common factor approach to this problem is in many ways a more natural way to think about convergence than those conventionally used. We implement this framework using principal components analysis. We apply these ideas to a dataset of monthly bilateral Euro and US-Dollar exchange rates of the new member countries (NMC) of the European Union over the period January 1993 – December 2005. The common factor technique gives sensible results about the convergence process of the exchange rates of the NMC to the Euro. We argue that this common factor framework provides a useful new way of approaching the convergence debate.

The paper is structured as follows. Section 2 considers the notion of convergence and shows how a common factor approach is in many ways a more natural framework for defining convergence. Section 3 applies this technique to our dataset of monthly bilateral Euro and US-Dollar exchange rates of the NMC. It provides evidence of a general process of convergence to the Euro, which however is still a long way from being complete. Section 4 concludes.

## 2. Defining Convergence

While we have a clear idea regarding the importance of convergence as a pre-requisite for economic changes such as the formation of a monetary union and we have a clear intuitive understanding of what convergence means, it is surprisingly difficult to find a satisfactory formal definition of convergence. Hall, Robertson and Wickens (1997) consider a number of formal definitions of convergence which illustrate the difficulty here. So consider the pointwise convergence of two series  $X_t$  and  $Y_t$  which we might define as occurring when,

$$\lim_{t \rightarrow \infty} (X_t - \phi Y_t) = \alpha \quad 1.$$

where  $\alpha$  is a non stochastic constant which might often be required to be zero. This is a clear definition of convergence but it is unrealistically strong as it requires the two series to exactly move together in the limit. A more reasonable definition would be to think of stochastic convergence or convergence in expectations

$$\lim_{t \rightarrow \infty} E(X_t - \phi Y_t) = \alpha \quad 2.$$

This at first seems like a reasonable definition of convergence but the problem here is that it implies convergence in many quite unreasonable cases. For example if  $X$  and  $Y$  are both mean zero white noise processes then this definition would suggest that the two series are converged even though they have no relationship. If  $X$  and  $Y$  are non-stationary then sensible definitions may be offered through the notion of cointegration and the idea that convergence may limit the difference between the two series to a stationary difference either in the limit or over a given interval. However while this is a useful operational notion of convergence again it is limited by only being useful in the case of non-stationary series.

Here we propose a general measure of convergence which is based around the common factor representation of a group of series and which we believe more closely follows the basic conceptual idea which we have in mind when we talk about convergence. Consider a vector of 2 or more variables X which are determined by a set of factors F

$$x_{it} = \lambda_i f_{it} \quad 3.$$

Then we may give the following definition of when X are converged.

Definition 1: The set of variables X are converged when the general factor representation in (3) may be restricted to the single common factor model given by,

$$x_{it} = \lambda_i f_t + \varepsilon_{it} \quad 4.$$

and  $\lambda_i \neq 0$  for all  $i$

where  $\varepsilon_t$  are N specific factors.

The conventional assumption is that  $f_t$  and  $\varepsilon_t$  are uncorrelated across all  $i$  and  $t$  and as Anderson (1963) pointed out this is unlikely to be true of time series data, which is the primary interest here. Geweke (1977) however generalised this model to produce the dynamic factor model in the following way. The assumption is made that  $f_t$  and  $\varepsilon_t$  are strictly indeterministic and covariance stationary, which of course allows them to have a constant, time invariant correlation structure. Then by Wold's (1938) theorem there exist two sets of white noise terms  $z$  and  $u_i$  such that

$$\lambda_i f_t = \sum_{s=0}^{\infty} a_{is} z_{t-s} = a_i^* z_t \quad 5.$$

and

$$\varepsilon_{it} = \sum_{s=0}^{\infty} b_{its} u_{it-s} = b_{it}^* u_{it} \quad 6.$$

hence

$$x_t = a^* z_t + b^* u_t \quad 7.$$

where  $b$  is a diagonal matrix and the variances of  $z$  and  $u$  are normalised to be unity. This is then the dynamic single factor model. This model is a straightforward representation of one notion of what we mean by convergence. All the elements of  $X$  are moving in a similar way although they do each have an idiosyncratic element, as the elements of  $b^*$  go to zero the common feature completely dominates the behaviour of  $x$  and variables move perfectly together.

Of course not all series will satisfy the conditions for the decomposition in (7) and so Geweke (1977) proposes a formal test of this structure based on the restrictions to the covariance structure of  $X$  implied by (7). This test works both for individual intervals in the frequency domain  $(\omega_1^j, \omega_2^j]$ ,  $j=1 \dots p$  and a joint test for all the intervals. Of course in the context of convergence the problem with this test is, that like many other tests, it may not detect a process of developing convergence as it is designed to detect complete convergence over the entire sample being tested.

To consider this process of gradual convergence we can return to the general factor model (3)

$$x_{it} = \lambda_i f_{it} \quad 8.$$

and define the factors to be orthogonal to each other. If the factors are then ordered so that the first factor is calculated to have the maximum explanatory power, the second factor has the next highest power and so on (as in a principal components analysis) then the notion of on going convergence becomes rather straightforward. Pointwise convergence, as defined above would imply that in the limit the first factor would be a complete explanation of X and so all the factors other than the first one would be zero. This would then collapse to the single factor model (7) where  $b=0$ . Convergence in expectations would imply that the expected value of all the factors except the first one would be zero and again in terms of (7) this would mean that the single factor model be accepted but the  $b$  would not be restricted to zero. However in a practical sense the usefulness of this approach becomes more obvious when we realise that there is a direct measure of the degree of convergence between the series in the form of the  $\%R^2$  of the first factor. This shows the % of the total variation of X which is explained by the first factor. Pointwise convergence would imply that this is 1 and in general the closer this is to 1 the more complete is convergence between the set of series. This then allows us to deal with the problem of using convergence in expectations. Consider the case of  $n$ , mean zero IID distributed series; the expectation of the difference between these series on a pairwise basis would be zero so they would all meet the condition for convergence in expectation, despite the fact that they are completely unrelated to each other. However in the factor representation the  $\%R^2$  for the first factor would be  $1/n$  as each factor would have equal explanatory power. The single factor model would be rejected and this would indicate that there was no common underlying driving force linking the  $n$  series together. If the series began to move together then the explanatory power on the first factor would rise and so this becomes a natural metric for the extent to which convergence has occurred.

Definition 2: Convergence is taking place between a vector of 2 or more series over any given period 1 to T if the %R<sup>2</sup> of the first principle component calculated over the period 1 to T/2 is less than the %R<sup>2</sup> of the first principal component calculated over the period T/2 to T.

This approach also works regardless of the Stationarity properties of the data. So in the I(1) case, if we have pairwise cointegration between the set of series so that between the n series there are n-1 cointegrating vectors then in ECM form the model may be written as

$$\gamma(L)(1-L)X_t = \Pi X_{t-1} + \nu + \varepsilon_t \quad 9.$$

where  $\Pi$  has rank n-1 and  $\nu$  is the deterministic component.

The moving average equivalent of this is

$$(1-L)X_t = C(L)(\varepsilon_t + \nu) \quad 10.$$

and the C matrix may be decomposed into

$$(1-L)X_t = C(1)(\varepsilon_t + \nu) + (1-L)C^*(L)(\varepsilon_t + \nu) \quad 11.$$

where C(1) has rank 1 and so there will be one common stochastic trend which is the dominant first factor in the factor representation. Asymptotically as the variance of this non stationary trend will dominate any stationary terms the %R<sup>2</sup> will go to one and convergence in expectation is clear. Over a small sample the size of the %R<sup>2</sup> will be an indicator of how important the common stochastic trend is relative to the noise in the series, again it becomes a direct measure of how much convergence has taken place. If any factor other than the first one shows signs of non Stationarity then this would imply less than n-1 cointegrating vectors and hence full pairwise cointegration would not exist.



In the special case of exchange rates (which is the main subject of this paper) this factor representation also offers a further possible insight. Here we will be considering the convergence of a set of currencies to the Euro. The natural thing to investigate then is the movement in the bilateral exchange rates for the set of countries we are interested in and the Euro against some third currency such as the dollar. When convergence has occurred we would expect all the countries to be moving with the Euro against the dollar and so we would find the dominant first factor as outlined above. However we can also consider a factor model of the bilateral Euro rates for each of the countries, in this case convergence would imply that these rates would simply be constant and pointwise convergence would imply that this is exact. Convergence in expectations would then imply that the fluctuation around this constant would be mean zero. A Factor model of these fluctuations would then tell us if the individual countries had any pattern in terms of groups or clubs which have common effects. If each country was simply fluctuating independently around its constant exchange rate then we should see a set of reasonably equally important factors with a largely random weighting matrix  $\lambda$ .

### **3. Empirical application of the common factor approach**

In this section, we implement the common factor technique using principal components analysis. We apply this to a dataset of monthly bilateral Euro and US-Dollar exchange rates of the NMC of the European Union over the period January 1993 – December 2005. The data were taken from the International Financial Statistics of the IMF. The ‘Euro rates’ use the DM as the reference currency until 1998 which has been linked to the Euro from 1999. Lithuania has been excluded from the sample because its fixed Dollar rate over the period

April 1994 – January 2002 precludes a principal components analysis for much of the investigation period.<sup>1</sup>

We begin by briefly reporting the result that unsurprisingly there is not a single factor underlying the principle components over the whole or our data period. Amongst our 9 new member country rates against the dollar we would expect to find 8 cointegrating vectors for the existence of one common stochastic trend. In fact the standard Johansen Trace tests suggests no more than 3 cointegrating vectors. The Geweke Test Also resoundingly rejects the single factor representation over the complete sample. The test statistic is 260.4 (the 5% critical value is approximately 124). So there is clearly not complete convergence over the whole period. We then proceed to investigate the existence of a process of ongoing convergence.

As we show in section 2, the  $\%R^2$  of the first principal component (PC) gives us a direct measure of the degree of convergence, so if in the factor model of the bilateral Euro rates the  $\%R^2$  are relatively constant over consecutive time periods, this means the NMC currencies move closely together with the Euro. As in our exchange rate model we have ten PCs, complete convergence would imply a  $\%R^2$  of 1/10 of each component, so if we find that the  $\%R^2$  of the first PC for the Euro rates exceeds 1/10 in the starting period, then a fall of the  $\%R^2$  over time toward 1/10 would imply gradual convergence of the national currencies of the NMC to the Euro.

Figure 1 shows the  $\%R^2$  of the first PC of the Euro and the Dollar exchange rates of the NMC for thirteen consecutive years.<sup>2</sup> Overall, we observe that the  $\%R^2$  for the Euro rates is always

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<sup>1</sup> For the periods in which all data are available the results for the samples with and without Lithuania were essentially identical, so excluding Lithuania is unlikely to affect the results.

below that for the Dollar rates, which indicates that the currencies of the NMC have generally moved closer together with the Euro than with the Dollar. Up to 1998 the  $\%R^2$  for the Euro and the Dollar rates are however relatively close to each other and no clear time profile emerges, although there does appear to be a slight downward trend with respect to the Euro rates. From 1999, the difference becomes more pronounced, and over the period 1993-2005 as a whole we observe an upward trend of the  $\%R^2$  for the Dollar rates and a downward trend for the Euro rates. This suggests that the NMC start have become closer to the Euro over time while diverging from the Dollar. Further inspection of the results reveals that convergence is particularly pronounced when the Euro is relatively strong, such as in 1999 on the introduction of fixed conversion rates and 2002 on the introduction of the Euro notes and coins, if perhaps only because the Euro was supported by self-fulfilling expectations.

The generally opposite movements of the  $\%R^2$  for the Dollar rates and the  $\%R^2$  for the Euro rates is illustrated even more clearly when we look at the time profile of their difference in Figure 2. A similar picture emerges when we consider longer consecutive periods such as four-year windows, as depicted in Figure 3.

Figure 4 shows the country-specific weights of the first PC. From section 2 we would expect the weights for the Dollar exchange rate to become closer over time to corroborate the finding of divergence of the NMC currencies from the Dollar, while increasing convergence to the Euro would imply a more random distribution of the weights for the Euro rates over time. Figures 4a - 4d indeed provide supportive evidence. The standard deviation of the weights may serve as a crude additional measure of the spread or randomness of the weights, and

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<sup>2</sup> We present the results in graphical form for ease of comparison. Full results are available upon request.

comparing the first (1993-96) and last (2001-05) period, the standard deviation of the Dollar rates falls from 0.76% to 0.05%, while that of the Euro rates increases from 0.49% to 0.70%.

While the weights for the Euro exchange rates for the NMC as a whole become more random over time, the fluctuations of the currencies of Latvia, Malta and Slovenia around the Euro have become more similar. Figure 5a presents the  $\%R^2$  of the first PC of a principal components analysis conducted for Latvia, Malta and Slovenia only, and Figure 5b presents the respective results for the group of the remaining countries. Figures 6a and 6b show the  $\%R^2$  as scaled by the average variance of each group's exchange rates against the Euro. This suggests that as a sub group Latvia, Malta and Slovenia are much closer to convergence with the Euro area than the other country Block. This is particularly interesting as, at the time of writing this version of the paper (Jan 2007), Slovenia has just joined the Euro area.

#### **4. Conclusion**

In this note we have proposed a new technique to analyse convergence based on a common factor framework. We have shown how this is in many ways a more natural way to think about convergence than standard definitions used in the work to date. We have implemented the common factor approach using principal components analysis, which we have applied to a data set of monthly bilateral Euro and US-Dollar exchange rates of the new member countries of the European Union over the period 1993-2005. The empirical application gives sensible results about the convergence process of the exchange rates of the new member countries to the Euro. Using the US-Dollar as the reference currency, the exchange rates of the new member countries appear to build a group which has common effects, which implies a process of divergence from the Dollar. By contrast, we can observe a general process of convergence to the Euro but it is still a long way from being complete. We conclude that the

common factor framework to measuring convergence provides a useful new way of approaching the convergence debate. Application to other convergence issues seems a fruitful avenue for further research.

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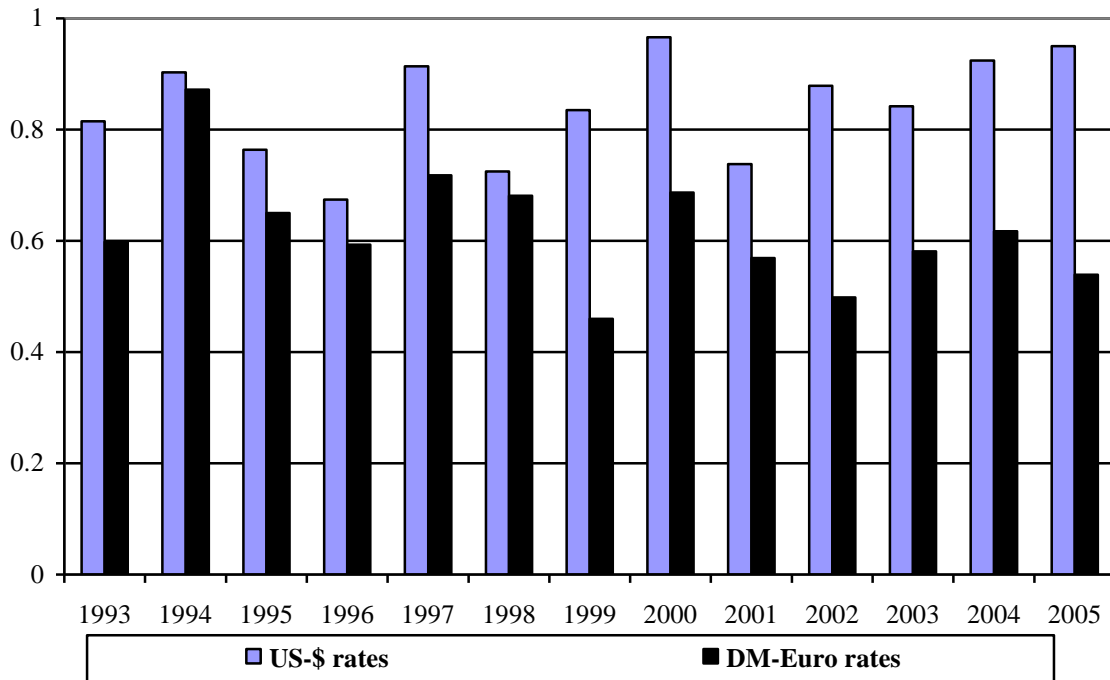
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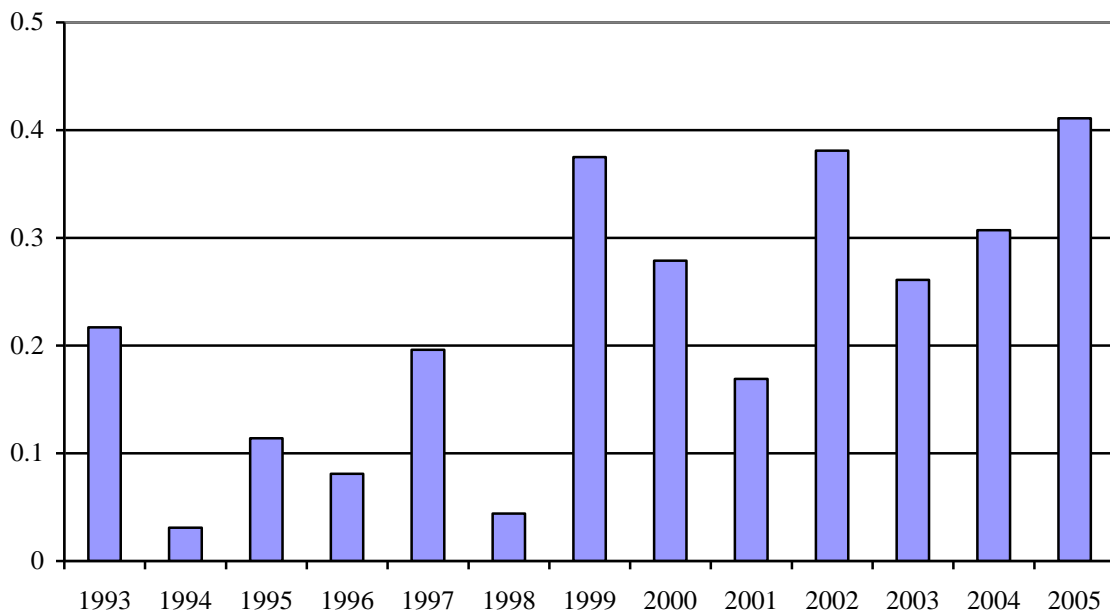
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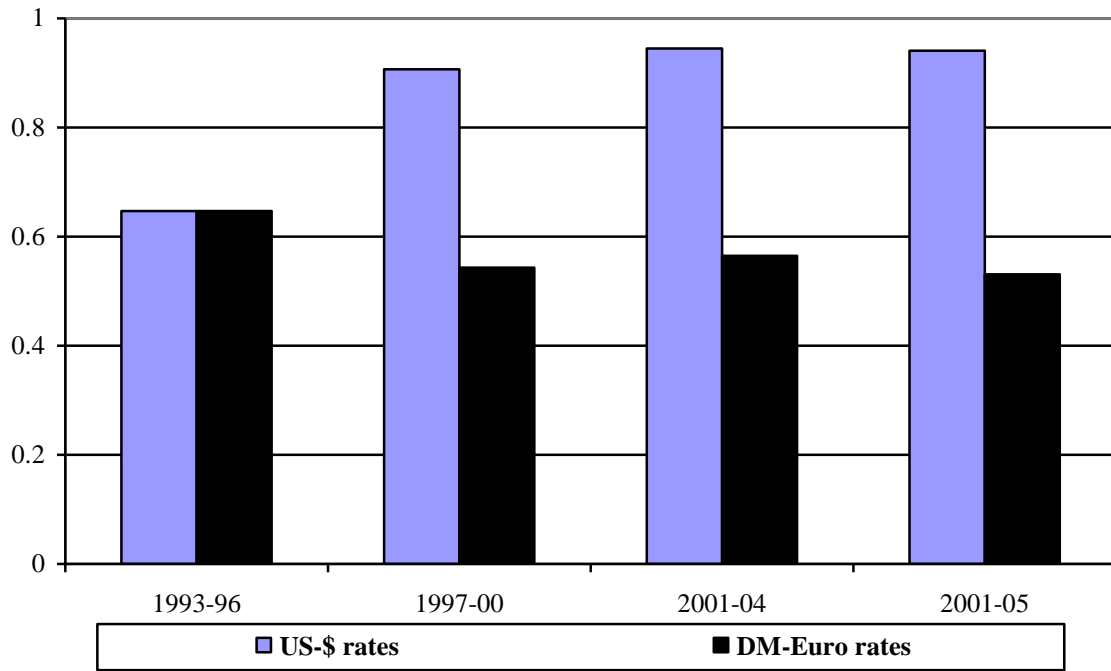
**Figure 1.** %R<sup>2</sup> of first principal component over individual years



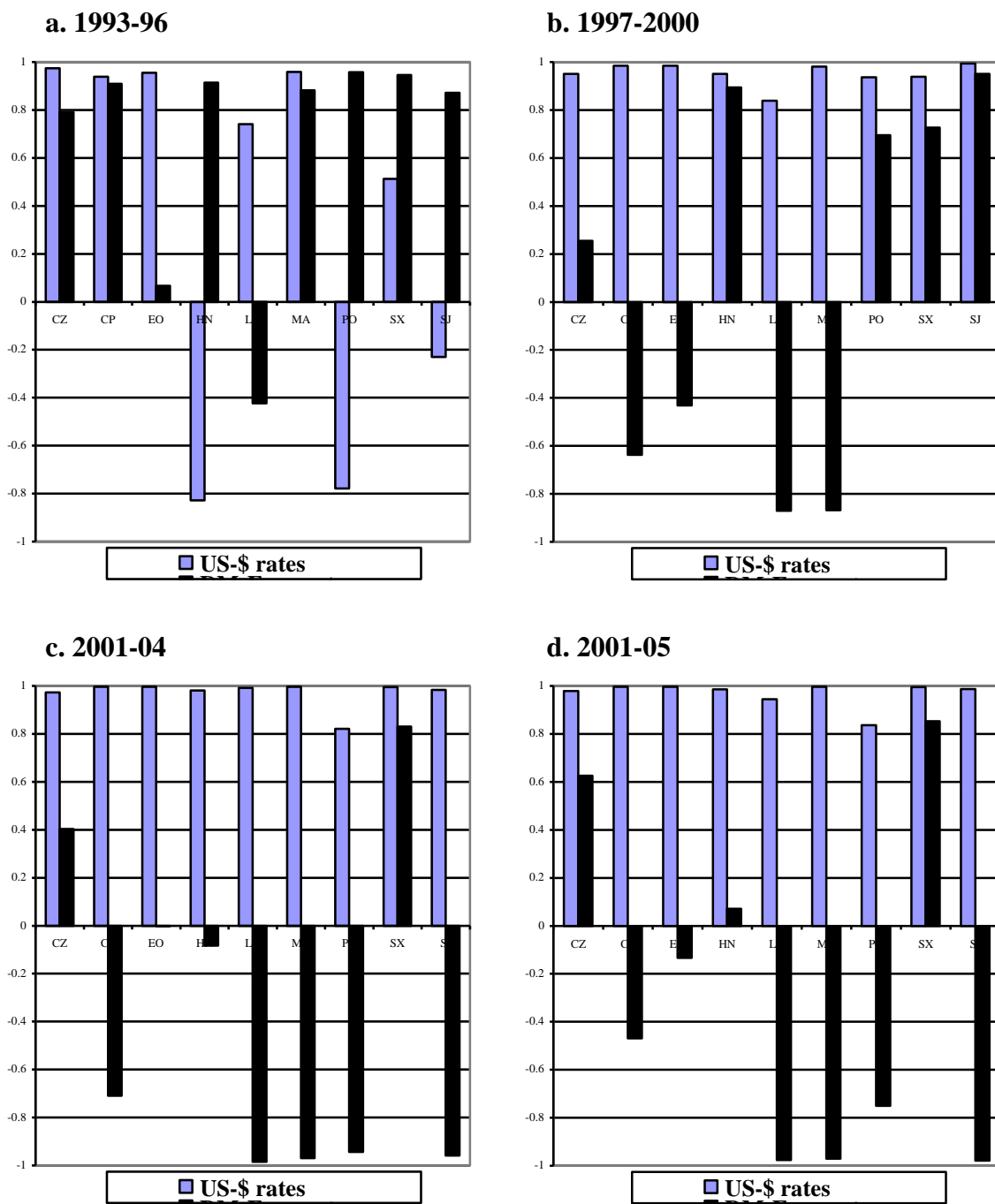
**Figure 2.** Difference between %R<sup>2</sup> of first principal component of the US-\$ and € exchange rates over individual years (%points)



**Figure 3.** %R<sup>2</sup> of first principal component over four-year windows



**Figure 4.** Country-specific weights of first principal component over four-year windows

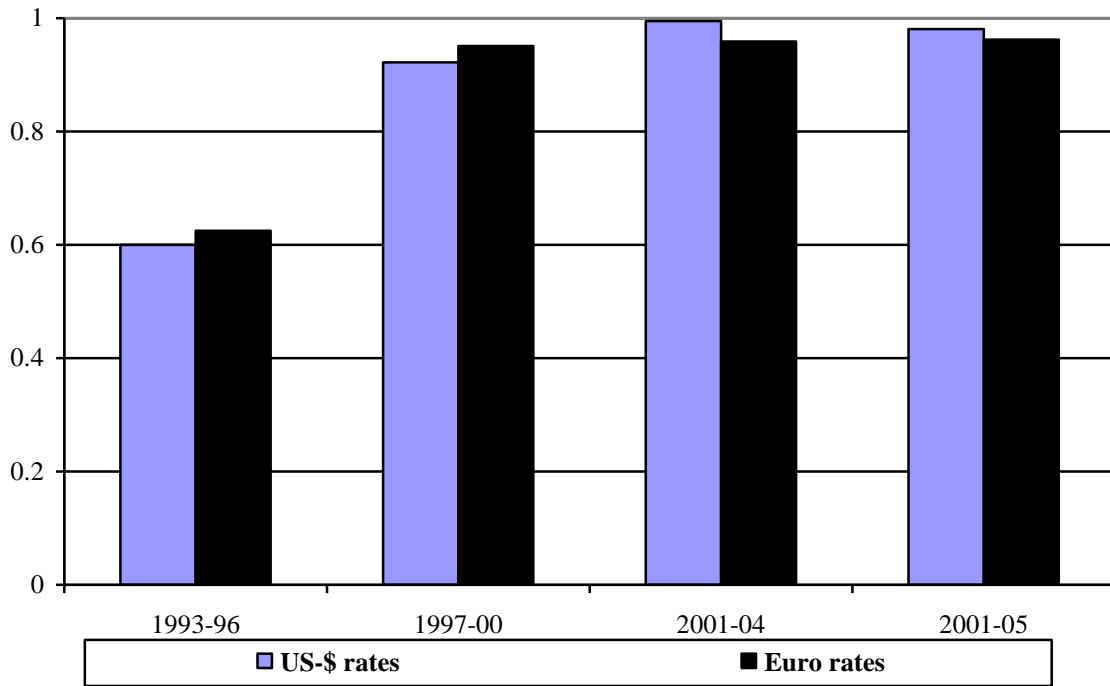


Note: The countries are denoted CP (Cyprus), CZ (Czech Republic), EO (Estonia), HN (Hungary), LV (Latvia), MA (Malta), PO (Poland), SX (Slovak Republic), SJ (Slovenia).

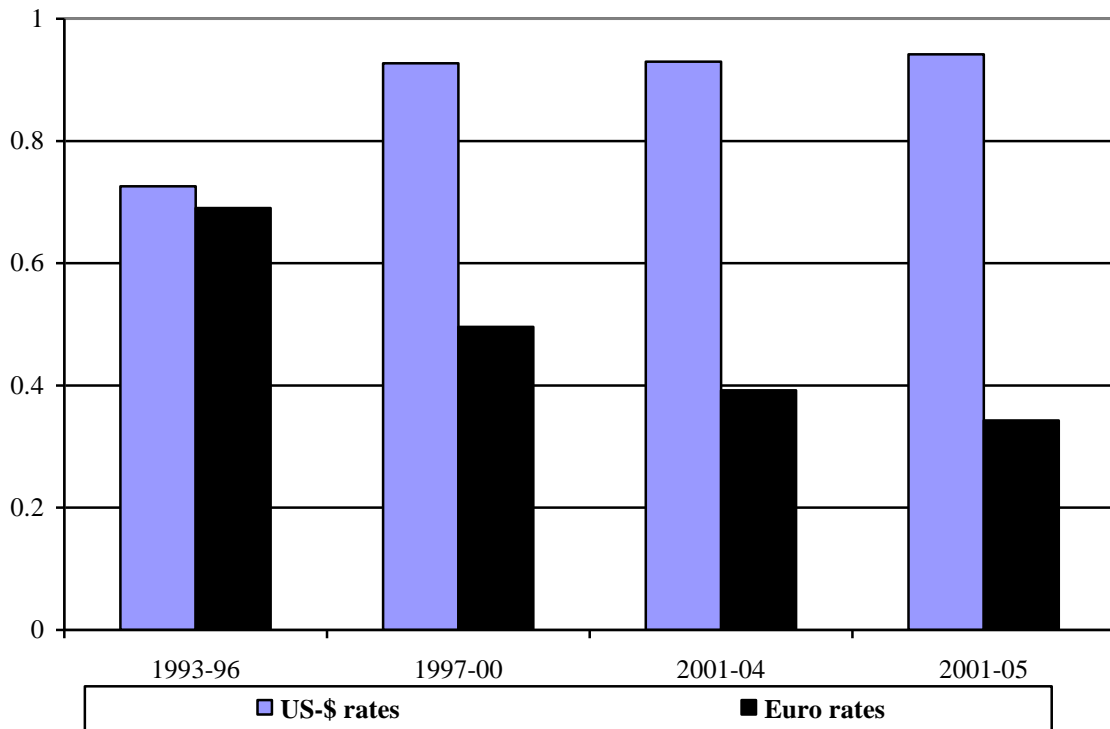


**Figure 5.** %R<sup>2</sup> of first principal component over four-year windows for country groups

**a. Latvia, Malta, Slovenia**

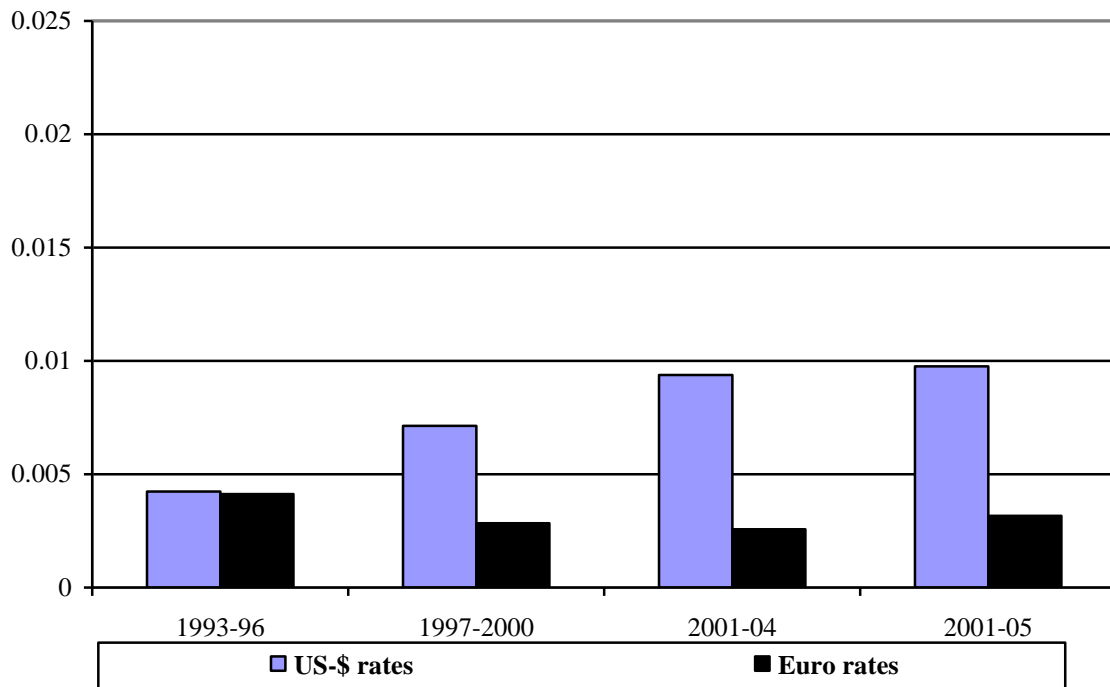


**b. Cyprus, Czech Republic, Estonia, Hungary, Poland, Slovak Republic**



**Figure 6.** Variance-scaled  $\%R^2$  of first principal component over four-year windows for country groups

**a. Latvia, Malta, Slovenia**



**b. Cyprus, Czech Republic, Estonia, Hungary, Poland, Slovak Republic**

