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The Design of the University System

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Abstract

This paper compares the organisation of the university sector under private provision with the structure which would be chosen by a welfare maximising government. It studies a general equilibrium model where universities carry out research and teach students. To attend university and earn higher incomes in the labour market, students pay a tuition fee. Each university chooses its tuition fee to maximise the amount of resources it can devote to research. Research bestows an externality on society because it increases labour market earnings. Government intervention needs to balance labour market efficiency considerations – which would tend to equalise the number of students attending each university – with considerations of efficiency on the production side, which suggest that the most productive universities should teach more students and carry out more research. We find that government concentrates research more that the private market would, but less than it would like to do if it had perfect information about the productivity of universities. It also allows fewer universities than would operate in a private system.

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1 Introduction.

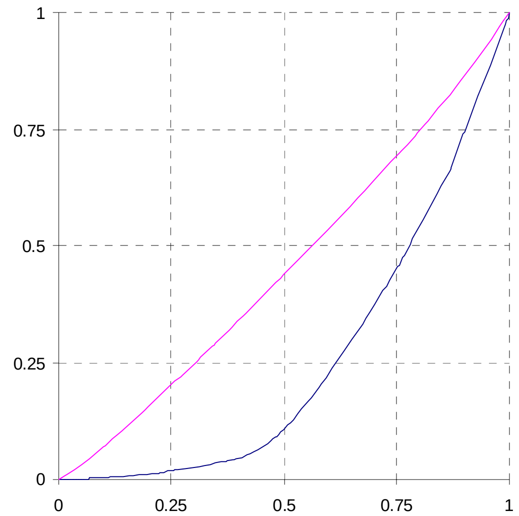
This paper studies the optimal organisation of the university sector. Theoretical analyses devoted to this topic are scarce, relative, for example, to public sector procurement, health care, and primary and secondary school systems. This is surprising, in view of the many peculiar features characterising the university sector, its importance for society, and of course its immediate relevance to the daily life of many researchers.

In the model of the paper, universities supply teaching and research. Students attend university to increase their labour market earnings. They differ in the overall utility they derive from attending university. They have preferences regarding which university they attend, for example because of mobility costs. Universities are the only institutions which can impart tertiary education, and their payoff is the amount of research that they do. The government[1] chooses the funding of each university to maximise a standard utilitarian welfare function.

Government intervention may be justified by the externality bestowed on society by university research, which affects the residual of the production function (Solow 1957), and therefore individuals' earnings. Our approach studies the *structure* of the funding and therefore sits between the macroeconomic study of the optimal aggregate amount of research and the microeconomic study of the optimal internal organisation of each individual university (as studied by Aghion *et al* 2008). In other words, we do not ask whether too much or too little research is done, and concentrate instead on the *distribution* of a given amount of total research and teaching among different universities, under the assumption that each individual university is carrying out its activities as efficiently as its resources and technology permit.

The gist of our results can be summarized by stating that the unfettered private market spreads research too thinly, and therefore the government would like to concentrate research and teaching in the most productive universities. This creates a tension. While the location of research is a matter of indifference, and so there is no harm in concentrating it in few institutions, the students' imperfect geographical mobility implies that the same is not true of teaching: concentrating teaching in some institutions prevents some students from attending university who would benefit from doing so. The tension is between concentrating teaching and research in the most productive universities only, which is efficient from the *cost* viewpoint, and ensuring that the students who

[1] We refer to the financing or regulatory agency, as "the government", although, conceivably, the views of an international organisation could also be influent for a developing country.



Cumulative proportion of research funding as a function of the cumulative proportion of total funding to state sector universities in UK (2009 lower line) and in Italy (2005; incentive proportion only, higher line) Source: HEFCE (UK), and MIUR (Italy).

Figure 1: Concentration of research in Italy and in the UK.

benefit most attend university, irrespective of their location, which is efficient from the *benefit* viewpoint. This tension is due to the link between teaching and research created by the university's budget constraint: teaching raises tuition fees and alumni donations with which universities fund their research. The government would like to sever this link. It, that is, would like to allocate the total amount of research exclusively to the most productive institutions, to allocate teaching according to the trade-off between benefits and costs in teaching, and to make all students contribute to research funding, including those taught at universities where no research is done. The government is however unable to do so unless it has perfect information about universities' productivity. Therefore the complementarity between teaching and research observed in practice is explained in our model by the preferences of the suppliers and the need of the government to provide incentives for universities to teach students, not by ad hoc assumptions on the technological characteristics of the university production function. As the universities have an information advantage vis-à-vis the government, they use the funding mechanism to increase the amount of research they do, and admit students only if there is a link with funding for research. This corresponds to the unfettered private market, where their research activities must be funded by tuition fees.

In practice, the structure of funding and organisational structure varies widely across different university systems. As an example, Figure 1 shows the very different concentration of research funding in UK and in Italy, two countries comparable in size and economic development. Research is clearly much more concentrated in the UK.[2] These huge differences call for a theoretical analysis of the organisation of the sector, to identify desirable direction for reform and explain difference in overall performance. Note that, while the UK system is generally considered "better" than the Italian one, it is not necessarily the case that concentration is *per se* preferable. It may affect the number of universities in the top 200 hundred in the Shanghai Jiao Tong University ranking (which includes 22 of the 129 British and five of the 57 Italian universities), but if the marginal cost of research is increasing, then concentration of funding may not be the best structure, and it may be more efficient instead to spread resources across institutions.

Both in the UK and in Italy, like in the rest of Europe, universities are government funded and regulated (for example in the tuition fees they can charge) to a very considerable extent. US states have instead a substantial private sector, alongside the state university system. Here as well we see a wide range of patterns; Figure 2 plots a measure of research intensity (the share of degrees awarded which are PhD's) against the average size of universities in each US state, both for the private sector and for the state university system. States where universities are more research intensive tend also to have universities with more students, and the effect is clearly stronger in the public sector universities.

This pattern is in line with the optimal funding mechanism we derive here, where the government ties research funding to the number of students enrolled. In this way more productive universities receive more funding and teach more students, and they do more research than they would in an the unfettered private market.

We find that the private and government designed systems differ not just in concentration of teaching and research, but also in the number and nature of universities. There are fewer universities in the government designed system than in an unfettered private system. While first best efficiency requires "teaching only" universities, the government is prevented in practice from establishing them by its information disadvantage: the system it designs shares with that emerging under unfettered private provision the feature that all universities do both teaching and research. Finally, "research only" universities can only exist

[2] For Italy, we plot only the "incentive" allocation. At the moment, only a small proportion of total funding is allocated in this way, the rest according to historic funding. The aim is gradually to increase the allocation based on teaching and research quality.

Figure 2: Research intensity and number of students.

in a private system where the government offers all institutions a lump sum subsidy, unrelated to teaching or research performance, and are sub-optimal.

The paper is organised as follows. We present the model in Section 2: the universities in Section 2.1, the students and the labour market in 2.2. In Section 3 we study a private university system unencumbered by government intervention, and in Section 4 derive the government optimal policy. Section 5 shows that the government policy can be implemented by offering a lump-sum grant which is higher the lower is the fee charged by universities to students. In Section 6 these two systems are compared against the common benchmark of the system a perfectly informed government would design. Finally, in Section 7 we argue that the analysis is robust to relaxation of some of the assumptions: we indicate how distributional concerns can be taken into account in Section 7.1; we introduce some student mobility in Section 7.2, the dependence of their willingness to pay on the research output of the university in Section 7.3, and we suggest, in Section 7.4 how a university's productivity could be determined endogenously. Section 8 is a brief conclusion. The proofs of all mathematical results are gathered in the Appendix.

2 The model

2.1 Universities.

We study an economy with a continuum of separated local education markets and a single economy-wide "global" labour market. The local education markets can be thought of as different towns or counties. In the global labour market there are two types of jobs, skilled and unskilled: skilled jobs require a university education. This is obtained in the local education markets, in each of which there is a single potential university, which has monopoly power:[3] it is available to all local residents, and only to them.[4] Potential universities differ in the value of an exogenously given[5] productivity parameter, $\theta \in \left(0, \bar{\theta}\right]$, with $\bar{\theta} > 1$. The distribution of θ in the economy follows a differentiable function $F(\theta)$, with density $f(\theta) = F'(\theta) > 0$ for $\theta \in \left(0, \bar{\theta}\right)$ which satisfies the following condition:

$$\frac{d}{d\theta}\left(\frac{1-F(\theta)}{f(\theta)}\right) < -1 + \left(\frac{2}{\theta}\frac{1-F(\theta)}{f(\theta)} + \frac{\theta}{\theta^a}\right). \tag{1}$$

Where $\theta^a = \frac{\int_\theta^{\bar{\theta}} \tilde{\theta} f(\tilde{\theta}) \mathrm{d}\tilde{\theta}}{1-F(\theta)}$ is the average type for types above θ. (1) imposes an upperbound on the slope of the hazard rate, and replaces the usual monotonicity constraint. It is satsfied, like the latter, for most commonly used distribution functions. Note that the RHS of (1) can have either sign and so (1) can be weaker or stronger than the monotonicity condition. The total measure of universities, $F\left(\bar{\theta}\right)$, is normalised without loss of generality to 1.

Universities can engage in research and teaching. To do so they must build lecture theatres, laboratories, libraries and so on, and employ "professors". The production function of a university of type θ is given by:

$$n = \frac{t+r}{\theta}, \tag{2}$$

[3] We do not consider competition among universities. This has also received limited attention in the literature: among the few contributions, in Del Rey (2001), universities choose the amount of research, and funding is positively related to the number of students. De Fraja and Iossa (2002) show that, if students are sufficiently mobile, competition among universities causes the emergence of "elite" institutions, which carry out more research and teach the best students. The link between competition and governance is analysed empirically in Aghion *et al* (2008).

[4] This is a simple way of capturing the assumption that students are not infinitely mobile: if they were, given constant returns to scale in production, there would be only one university, teaching *all* students, and carrying out *all* research. While this concentration may be optimal for some training centres, such as sporting academies, it clearly is not for tertiary education. Section 7.2 hints that allowing imperfect students mobility does not alter the qualitative features of the analysis.

[5] The stylised model of Section 7.4 illustrates how a university's value of θ can be determined endogenously as an equilibrium variable.

where $n > 0$ is the number of professors employed, $r \geqslant 0$ the amount of research carried out, and $t \geqslant 0$, the number of students taught. (2) implies that research and teaching both require professors, and that θ is a positive measure of productivity: a university with a higher θ needs fewer professors for a given amount of research and teaching.[6] The linear relationship with the number of professors implies that there are no economies of scale or scope, and that teaching and research are perfect substitutes as outputs.[7] All non-staff resources are assumed to be proportional to staff numbers, and therefore are fully captured by the parameter θ.

Universities receive income from students, who pay a tuition fee of $p \in \mathbb{R}$ each (negative if students are subsidised), and possibly from the government, in the form of a grant $g \in \mathbb{R}$ (which again can be negative, and therefore a tax). A university's budget constraint[8] is

$$pt + g - yn = 0, \tag{3}$$

where y is the salary paid to a professor, endogenously determined by a competitive labour market for skilled workers (see below).

We posit that the objective function of each university is the maximisation of the amount of research it does, interpreted here as "blue sky" research.[9] Universities view teaching as a source of income, used to fund research, not an activity that increases their utility directly. This tallies with the empirical regularities that universities are by and large managed by academics whose vocation is research, and that academics' rewards and progress are more closely linked to success in research than in teaching.[10] This of course does not mean

[6] The normalisation of $\frac{1}{\theta}$ as the number of staff needed to produce one combined unit of outputs will prove convenient when solving the government optimisation problem.

[7] Though there may be some complementarities, today's highly specialised nature of scientific research suggests that they are at best limited, and that resources, *viz* academic time, devoted to teaching are in fact resources subtracted to research.

[8] Other components of a university's budget constraint such as income from endowment, alumni donations, or state subsidies to tuition fees are included implicitly in (3). For example, p can be interpreted as including the net present values of future alumni donations. (3) captures the plain fact that if a university with a given endowment income and alumni donations wishes to increase its research expenditure, it must do so by increasing tuition revenues. This is an accepted explanation for the steep increase in tuition costs of the past decades in the US university sector (Ehrenberg 2007).

[9] Externally funded research or consultancy fees, can be treated as separate activities and we disregard them in what follows.

[10] Tuckman *et al* (1977) noted a long time ago that "outstanding teaching appears to yield a low rate of return" (p 697) and that "teaching and public service yield low compensation;

that universities do not care about teaching: a university may care passionately about the quality of its teaching, and indeed most do, but this is exactly in the same sense as a firm cares passionately about the quality of its products: both are means of furthering the organisation's objective, not ends in themselves.[11] Note also that in our set-up the concept of research can be plausibly extended: "research" can be defined as any academic activity which bestows an externality on society by benefitting individuals or organisations who cannot be made to pay for it.[12]

We also assume that there is an upper limit to the amount of research that can be carried out in each university: let $r_{\max}$ be this bound. This constraint is only necessary when the government has perfect information, and therefore we think of it as very "high", in a sense made precise in Assumption 1.

2.2 Students and the labour market.

Each local education market serves a population of potential students, with measure normalised to 1. Every potential student can attend university, obtain a degree, and subsequently work in the skilled labour market (which includes working as a professor), receiving income:

$$y(R), \tag{4}$$

publishing and administration carry much larger returns" (p 701); see also Hammond *et al* (1969). More recently, self-selection and the endogeneity of teaching loads make it more difficult to ascertain an independent impact of teaching on rewards (Euwals and Ward 2005, p 1663, and Golden *et al* 2009), and indeed, in his survey of the academic labour market, Ehrenberg (2004) does not report studies of the relationship between teaching and research and lifetime earnings.

[11] Similarly, it is commonly assumed that "non-profit" hospitals do in fact strive to maximise profit, which they do not distribute to owners as dividends, but use instead to further their aims, from excellence in research, to treating patients who are unable to pay (Danzon (1982) and Dranove and White (1994)). The link is in fact arguably stronger in universities, as typically good teaching attracts good students who both reduce the cost of teaching and may attract good staff improving research prowess (Rothschild and White (1995)).

[12] Thus for example universities may subsidise doctoral supervision, or offer scholarship and financial aid to students from deprived backgrounds: these activities are undertaken by universities because they increase their payoff, even though – by definition – they do not generate enough revenue to cover their cost. They generate benefits to (parts of) society, for example by increasing future research activities or enhancing diversity and offering role-models to able individuals in deprived neighbourhoods, and so they fit this extended definition of "research". Further outputs have been suggested, such as the transfer of knowledge (Johnes *et al* 2005), or the production of human capital (Rothschild and White 1995). These simply extend the concept of teaching.

or obtain basic education only, which guarantees an unskilled job, with income

$$y(R) - \Delta, \tag{5}$$

where $\Delta > 0$ denotes therefore the salary premium earned by graduates. $R \geqslant 0$ measures the "state of technology in the society", the value of all research undertaken in the university sector. R is defined as follows: if $r(\theta)$ is the *average*[13] amount of research carried out by the universities of type θ, then:

$$R = \int_0^{\bar{\theta}} r(\theta) f(\theta) \, \mathrm{d}\theta. \tag{6}$$

The positive externality of research implies that $y'(R) > 0$: workers – both skilled and unskilled – are more productive if more research is carried out in society.[14]

(4) and (5) imply that the labour markets are "global": the earnings associated to a job depend only on the global variable R, not on where the workers have obtained education[15] or on the location of the job.

Students incur an effort cost if they go to university. This is denoted[16] by $a \in [a_{\min}, a_{\max})$, and its value is distributed among the potential students in each local market according to a distribution function $\Phi(a)$ with density $\phi(a) = \Phi'(a)$ and monotonic hazard rate $\frac{d}{da}\left(\frac{\Phi(a)}{\phi(a)}\right) > 0$.

While basic education is available in each local labour markets at no cost, attending university requires that the potential university becomes active. If the type θ university does so, it sets a tuition fee a tuition fee $p(\theta)$ which students must pay to attend university. A potential student of type a takes this tuition fee and the labour market rewards, $y(R)$ and Δ, as fixed. She chooses to attend university and work in the skilled labour market if and only

[13] In the equilibrium we consider, all type θ universities make identical choices, and so $r(\theta)$ and $t(\theta)$ are the amount of research and the number of students in each type θ university. Moreover, both $r(\theta)$ and $t(\theta)$ turn out to be monotonic, and therefore (Riemann) integrable.

[14] Note that the number of graduates does not affect earnings. This may be due to perfect international labour mobility and is a simplification relative to De Fraja and Valbonesi (2008), where the labour market income functions (4) and (5) also include the total number of graduates as argument. All the results regarding the structure of the university system obtained in De Fraja and Valbonesi (2008) carry through to the simplified version of the model.

[15] See Section 7.3, for a suggestion of how to include the "quality" of education in the workers' utility.

[16] The restriction to linear cost is simply a normalisation of the measure of the cost relative to the distribution function $\Phi(a)$. As shown in De Fraja and Valbonesi (2008) including a function $c(a)$ changes nothing in the analysis, provided that $-\frac{\phi(a)}{\Phi(a)} < \frac{c''(a)}{c'(a)}$ for every $a \in (a_{\min}, a_{\max})$. See also Section 7.3 below.

if[17] her earnings, net of tuition fees, and reduced by the cost of effort while at university, exceed the income obtained from the unskilled labour market:

$$y(R) - p(\theta) - a \geqslant y(R) - \Delta.$$

Therefore there is a threshold value of a, given by $\Delta - p(\theta)$, such that only individuals with a equal or lower than $\Delta - p(\theta)$ attend university and work in the skilled labour market.

To ensure that only some of the potential students attend university, we posit $\Delta \in (a_{\min}, a_{\max})$: that is, with a zero tuition fee, some students would like to go to university, others would not.

Note that, for simplicity, a denotes only the cost of attending university. The observation that labour market rewards are higher for higher "ability" individuals can be easily captured by positing a correlation between the effort cost of attending university and the effort cost of working: low a individuals receive the same salary but enjoy higher utility than high a individuals.[18]

Another simplification is that the students' utility, and hence their willingness to pay for a university education, does not depend on the type of the university attended. Section 7.3 suggests a stylised but plausible way of relaxing this restriction, which would also add realism to the tuition fee structure of the university sector. We do not introduce this in the main part of the paper, because doing so would complicate the algebra considerably.

We study the steady state of a dynamic model where new research carried out in a period balances the reduction of research stock due to obsolescence, and where students tuition is financed either by parents or by loans secured on their future income. In the absence of perfect capital markets, differential borrowing cost among households can be included in the cost of attending university, a, which would then measure a combination of the utility cost of effort and the interest payments on loans financing university attendance. Interesting on its own right, the steady-state analysis isalso a necessary first step in the study of a fuller dynamic model, where the accumulated stock of knowledge, not just current research, affects labour market earnings, and where there are intergenerational transfers.

[17]We maintain throughout, for the sake of definiteness, the assumption that indifferent students attend university: since they have measure 0 in $[a_{\min}, a_{\max}]$, this entails no loss of generality.

[18]Allowing earnings also to depend on a has no qualitative effects on the analysis of the potential students' choices, as long as, naturally, the effect is stronger in the skilled labour market, but would unnecessarily complicate the analysis, because the university salary would in this case not be straightforward to calculate.

3 Equilibrium with no government

In this section we study the benchmark where the university sector is private, and government intervention is limited to at most a lump sum subsidy or tax, independent of anything a university does. The number of students taught by a university of type θ which becomes active and chooses tuition fee $p(\theta)$ is:

$$t(\theta) = \Phi(\Delta - p(\theta)). \tag{7}$$

Universities charge the same price to all students, for example, because they cannot observe their type, and have no instruments to provide incentives for truth-telling. Universities do not "select" students: they never exclude students who are willing to enrol at the current fee. Gary-Bobo and Trannoy (2008) show that if students have imperfect information about their ability, research maximising universities do select students.

Let us define the function $z_k : [0,1] \longrightarrow \mathbb{R}$, with $k \in [0,1]$, which plays a key role in the rest of the analysis:

$$z_k(t) = \Phi^{-1}(t) + \frac{kt}{\phi(\Phi^{-1}(t))}. \tag{8}$$

z_k can be called the adjusted marginal teaching cost. To interpret it, note that if a university wants to add marginal students in measure dt to those already enrolled, it needs to enrol students whose cost of attendance is $\Phi^{-1}(t)$, and so the fee must be adjusted to entice them. When $k = 1$, the second component of the RHS of (8) is the lost fee income due to the fact that already enrolled students also pay the lower tuition fee. The number of the inframarginal students who benefit from the fee reduction, relative to the number of newly enrolled type $\Phi^{-1}(t)$ students, is the hazard rate, $\frac{\Phi(\Phi^{-1}(t))}{\phi(\Phi^{-1}(t))}$, which equals $\frac{t}{\phi(\Phi^{-1}(t))}$. This component of the marginal cost is offset when there is a social benefit to the fact that attendance to university becomes cheaper. We can take $k \in [0,1]$, as an inverse measure of the social benefit of the fee reduction: when $k = 0$, the lower revenues of the university are exactly compensated by the lower fees paid by the students, and the marginal cost of an extra student is unaffected by the fact that the university suffers a loss of revenue loss. In the intermediate case, $k \in (0,1)$, the university's revenue loss is only partly compensated by gains elsewhere; as we see in the next Section, this happens when the decision maker strictly prefers the universities to be funded by students fees than by general taxation, for example because of distributional, deadweight or administrative costs of taxation.

Lemma 1 *(i)* $z_k(t)$ *is strictly increasing for every* $k \in (0,1]$. *(ii) For every* $k \in [0,1]$, $z_k(0) = a_{\min}$. *(iii) Let* $k_1 > k_0$*; then, for every* $t > 0$*, we have* $z_{k_1}(t) > z_{k_0}(t)$ *and* $z'_{k_1}(t) > z'_{k_0}(t)$.

The proof of all the results in the paper is in the Appendix. We can now establish our first result.

Proposition 1 *The university of type* θ *sets the following tuition fee:*

$$p(\theta) = \Delta - \Phi^{-1}\left(z_1^{-1}\left(\Delta - \frac{y(R)}{\theta}\right)\right), \tag{9}$$

and enrols

$$t(\theta) = z_1^{-1}\left(\Delta - \frac{y(R)}{\theta}\right) \tag{10}$$

students, provided the above is non-negative.

The number of students is set in (10) at a level such that the revenue which can be extracted from an additional student – her additional income Δ –, reduced by $z_1(t(\theta))$ – the lost revenue due to the fee reduction necessary to induce this additional student – equals the university's cost of teaching this student, $\frac{y(R)}{\theta}$. The university acts as a profit maximising monopolist, and uses the profit to finance research.

The next Corollary gives the relationship between productivity and size and fees.

Corollary 1 $\frac{dt}{d\theta} > 0$, $\frac{dp}{d\theta} < 0$, $\frac{dr}{d\theta} > 0$, *and* $\frac{dn}{d\theta} > 0$.

More productive universities do therefore teach more students, charge a lower price to attract them, employ more academics and carry out more research. These relationships between size output and efficiency are often observed in empirical studies,[19] and sometime attributed to complementarities between teaching and research (Becker 1975 and 1979): teachers (respectively researchers) are thought to be more productive if they also do some research

[19] Cohn *et al* (1989) study a sample of 1887 US colleges and universities in 1981-82, through a multi-product cost approach: they find economies of scale and scope. De Groot *et al* (1991) Glass *et al* (1995) and Dundar and Lewis (1995) extended Cohn's research focusing respectively on the sensitivity of the cost functions estimates to different output measures, on flexible cost function, and on the departmental production function. For stochastic and non-stochastic frontier analyses, see for example Izadi *et al* (2002) and McMillan and Chan (2006) and the references cited in these papers.

(respectively teaching). The empirical evidence for these complementarities is flimsy (Hattie and Marsh 1996). Corollary 1 shows that they need not necessarily imply that there are economies of scale and scope in the technology, but they could be due instead to an underlying unobserved parameter: universities which employ more staff have lower measured unit costs, both in teaching and in research, even though each university employs the same linear technology with no economies of scale and scope.

The above analysis clearly holds if the university of type θ is able to operate. We turn next to the question of which universities are in fact active. For university of type θ to be active, it must be that, at its optimal number of students, it can make positive revenues to pay for its research. This is the case if:

$$r(\theta) = (p(t)\, t(\theta) + g)\,\frac{\theta}{y(R)} - t(\theta) \geqslant 0. \tag{11}$$

Note that, for an active university, the optimal number of students is independent of the grant g. This has the following immediate consequence.

Corollary 2 *If $g \geqslant 0$, a university of type θ enrols students if and only if*

$$\theta > \frac{y(R)}{\Delta - a_{\min}}. \tag{12}$$

The interpretation of (12) is natural: the teaching cost of a student is $\frac{y(R)}{\theta}$. For the university to want to teach a strictly positive measure of students, it must be worth for at least the students with the lowest a to pay for this cost, and the willingness to pay for tuition of this student is the increase in her labour market earnings as a consequence of her having a degree, Δ, reduced by the utility cost of attending university, $a_{\min}$.

Consider research now. Clearly, if $g = 0$, a university can do research if and only if it can make positive revenues from its teaching. If $g > 0$, universities with type lower than the RHS in (12) can do some research by enrolling no students and spending all their grant on research: they are research only institutions, but they are the least productive among the active universities. Conversely, if $g < 0$, then some universities are prevented from becoming active which would raise enough tuition fees to pay their teaching costs, and the smallest universities teach a strictly positive number of students and do no research.

$t(\theta)$, $p(\theta)$ and $r(\theta)$ of course depend on R, which we need to derive to close the model. R is:

$$R = \int_{\frac{y(R)}{\Delta - a_{\min}}}^{\bar{\theta}} r(\theta)\, f(\theta)\, \mathrm{d}\theta, \tag{13}$$

where $r(\theta)$ is given in (11) $p(\theta)$ and $t(\theta)$ in Proposition 1.

4 Government intervention.

We now assume that the government intervenes actively in the higher education sector. We do not address the issue of actual *ownership* of universities. Anecdotal evidence suggests that there is little systematic difference in efficiency or in objective function between private and public universities. We simply assume that the government imposes constraints on the university sector, and that these constraints are the same for public and for private universities.

The crucial assumption in the paper is that universities have both a trade-off between teaching and research which leans more towards research than the government's, and an informational advantage over the government. This tallies with our perception of universities, be they public or private. Formally, the government maximises total utility in society, and universities know their own θ and how much r they carry out, while the government cannot observe either.[20]

The government designs the university policy.[21] In our model, this is simply a pair of functions $\{p(t), g(t)\}$, offered to all potential universities: the government commits to linking the number of students enrolled, t, with the tuition fee a university is allowed to charge, $p(t)$, and with the lump sum grant awarded to the university, $g(t)$ (both $p(t)$ and $g(t)$ can be negative). Faced with this policy, each university can choose freely the number of students it enrols, receiving the corresponding government grant $g(t)$, and charging the corresponding tuition fee $p(t)$. The government's grant to universities is funded by general taxation; to keep things simple, we model taxation as a lump sum tax h, the same for all individuals. Naturally, this is constrained not to exceed the income from the unskilled labour market: $h \leqslant y(R) - \Delta$. In the rest of the paper we assume that this constraint is in fact slack; that is we assume, plausibly, that the total tax needed to finance the preferred level of tertiary education is not so high as to require more than the aggregate income that would be obtained with no university sector, when all workers are unskilled. Raising one unit of resources in tax has an exogenously given cost $(1 + \lambda) > 1$. As Section 7.1 shows, in addition to the standard administrative and distortionary costs of taxation, λ also captures the government's preference for redistribution.

[20] Some research is of course *ex-ante* observable and *ex-post* measurable. Our assumption of asymmetric information requires only that at least some research is neither. This is highly plausible, uncertainty being the essence of research activities, where a large (unobserved) research effort may well lead to no results, and conversely, serendipity and luck may produce huge returns at little cost.

[21] Beath *et al* (2005) take the funding mechanism as given and concentrate on the effects on incentives for teaching and research quality of different public funding schemes.

To determine the government's optimal policy, we take the standard revelation approach. The government asks each university to report its own type, and commits to imposing a vector of variables as functions of the reported type, which the university must adhere to: by the revelation principle, the government cannot improve on the payoff it can obtain by restricting its choices to the set of mechanisms such that no university has an incentive to mis-report its type. With this perspective, a policy is a triple, $\{t(\theta), p(\theta), g(\theta)\}_{\theta\in(0,\bar{\theta}]}$, the number of students, the tuition fee and the government grant as a function of the reported type. The employment at university of type θ is given by $n(\theta) = \frac{p(\theta)t(\theta)+g(\theta)}{y(R)}$. We include *both* t and p as policy variables, thus allowing, potentially, the number of students enrolled in a university to be different from the number of individuals who, given the tuition fee, would prefer to graduate. Clearly, it cannot exceed it, and so we must impose the constraint

$$\Phi^{-1}(t(\theta)) \leqslant \Delta - p(\theta), \qquad \theta \in \left[\underline{\theta}, \bar{\theta}\right]. \tag{14}$$

(14) says that the type of the marginal student must be no greater than the type of the student who is indifferent between going and not going to university. As we show below, (14) is in fact binding at the government's optimal policy: it cannot happen that the number of university places needs to be rationed by non-price methods. Intuitively, this is so because the shadow cost of public funds exceeds 1, and so it is always preferable for the government to raise funds through tuition fees than through taxes.

To set up the government's problem as an optimal control in a suitable way, we introduce the auxiliary variable R, the total amount of research, subject to definitional constraints (6). $\underline{\theta}$, the cut-off point type of university, such that those above operate, those below do not, is also determined endogenously, as the "initial time" (Leonard and van Long 1992, p 222 ff). Finally, $r(\theta)$ too is treated as a variable chosen by the government, subject, as explained above, to the incentive compatibility constraint that all universities prefer to reveal their type truthfully. We derive this constraint in Proposition 2. Note first that the utility of a type θ university who has reported type θ is

$$r(\theta) = [p(\theta)t(\theta) + g(\theta)] \frac{\theta}{y(R)} - t(\theta). \tag{15}$$

Proposition 2 *Let $\underline{\theta}$ be the least productive active university. The incentive compatibility constraint and the participation constraint are given, for $\theta \in \left[\underline{\theta}, \bar{\theta}\right]$,* *by:*

$$\dot{r}(\theta) = \frac{p(\theta)\,t(\theta) + g(\theta)}{y(R)}, \qquad r(\underline{\theta}) = 0, \qquad r(\bar{\theta}) \text{ free,} \tag{16}$$

$$\dot{t}(\theta) \geqslant 0. \tag{17}$$

We denote with a dot over a variable its derivative with respect to θ. A further constraint is that each university satisfies its budget constraint. Substitute (2) into (3), to obtain:

$$\frac{y(R)}{\theta}\left(r(\theta) + t(\theta)\right) - g(\theta) - p(\theta)\,t(\theta) = 0, \qquad \theta \in \left[\underline{\theta}, \bar{\theta}\right]. \tag{18}$$

Further, the number of students in a local market must be non-negative

$$t(\theta) \geqslant 0, \qquad \theta \in \left[\underline{\theta}, \bar{\theta}\right]. \tag{19}$$

$t(\theta)$ cannot exceed 1 either: since $\Delta < a_{\max}$, if the number of students from a local education market were 1, then the total utility form individuals in that market could be increased simply by stopping the students with the highest cost of effort from attending university, and so a situation were there are some $\theta \in \left[0, \bar{\theta}\right]$ were $t(\theta) = 1$ cannot happen at the optimum, and the constraint $t(\theta) \leqslant 1$ can be omitted.

In the jargon of optimal control analysis, the problem can be written as a free initial time optimal control problem with R as a parameter; the integral constraint (6) is re-written as a state constraint (Leonard and van Long, 1992, p 190), with $r_0(\theta)$ as an auxiliary variable:

$$\dot{r}_0(\theta) = r(\theta)\,f(\theta), \qquad r_0(\underline{\theta}) = 0, \qquad r_0(\bar{\theta}) = R; \tag{20}$$

The instruments described and the constraints derived, we can finally present the government's problem. This is the maximisation of a welfare function made up of three components. First, the total after tax income of the population. Second the disutility costs borne by those who attend university. And third, the non-monetary value of research, which we measure by ωR, with $\omega \geqslant 0$:[22] ω is a parameter of the government payoff function, and captures aspects such as the national pride at the award of Nobel prizes, Fields Medals, and other formal or informal recognition.

[22] Formally this is identical to including a proportion ω of the universities payoff in the computation of social welfare, just as a share of a regulated firm's profit is typically included.

Proposition 3 *The government's problem is:*

$$\max_{\substack{p(\theta),t(\theta),r(\theta),\\ g(\theta),R,\underline{\theta}}} \int_{\underline{\theta}}^{\bar{\theta}} \left((\Delta - p(\theta))\, t(\theta) - \int_{a_{\min}}^{\Phi^{-1}(t(\theta))} a\phi(a)\,\mathrm{d}a - (1+\lambda)\, g(\theta) \right) f(\theta)\,\mathrm{d}\theta$$

$$+ y(R) - \Delta F(\underline{\theta}) + \omega R, \tag{21}$$

s.t. (14), (16), (17), (18), (19) and (20)

To derive the payoff function intuitively, note that the total utility of the potential students in a type θ local education market is given by

$$(y(R) - \Delta - h) + (\Delta - p(\theta))\, t(\theta) - \int_{a_{\min}}^{\Phi^{-1}(t(\theta))} a\phi(a)\,\mathrm{d}a. \tag{22}$$

This has a natural interpretation: all potential students receive after tax income $y(R) - \Delta - h$ at least; of the potential students, $t(\theta)$ do go to university, and receive an *additional* income, net of tuition fee, equal to $\Delta - p(\theta)$: this is the second term in (22). The aggregate disutility cost of attending university is given by the last term in (22). Integrate over θ and add ωR, the direct benefit of research, to obtain (21).

Proposition 4 describes the government's optimal policy. For given R and $N > 0$, define the function $\sigma(R, N; \theta)$ as

$$\sigma(R, N; \theta) = \Delta - \frac{y(R)}{\theta} + \zeta(R, N; \theta), \tag{23}$$

where,

$$\zeta(R, N; \theta) = \frac{(1 - F(\theta))\, y(R) - \left(y'(R) \left(\frac{1}{1+\lambda} - N \right) - \frac{\omega}{1+\lambda} \right) \int_{\theta}^{\bar{\theta}} \tilde{\theta} f\left(\tilde{\theta}\right) \mathrm{d}\tilde{\theta}}{f(\theta)\, \theta^2}$$

and let the vector $(R, \underline{\theta}, N)$ solve the following system of three equations in three unknowns:

$$a_{\min} = \sigma(R, N; \underline{\theta}), \tag{24}$$

$$R = \int_{\underline{\theta}}^{\bar{\theta}} \theta \int_{\underline{\theta}}^{\theta} \frac{z^{-1}_{\frac{\lambda}{1+\lambda}}\left(\sigma\left(R, N; \tilde{\theta}\right)\right)}{\tilde{\theta}^2} \mathrm{d}\tilde{\theta} f(\theta)\,\mathrm{d}\theta, \tag{25}$$

$$N = \int_{\underline{\theta}}^{\bar{\theta}} \left(\frac{z^{-1}_{\frac{\lambda}{1+\lambda}}(\sigma(R, N; \theta))}{\theta} + \int_{\underline{\theta}}^{\theta} \frac{z^{-1}_{\frac{\lambda}{1+\lambda}}\left(\sigma\left(R, N; \tilde{\theta}\right)\right)}{\tilde{\theta}^2} \mathrm{d}\tilde{\theta} \right) f(\theta)\,\mathrm{d}\theta. \tag{26}$$

We said before that we require $r_{\max}$ to be "high". Formally we now posit:

Assumption 1 $r_{\max} > \bar{\theta} \int_{\underline{\theta}}^{\bar{\theta}} \frac{1}{\tilde{\theta}^2} z_{\frac{\lambda}{1+\lambda}}^{-1} \left(\sigma \left(R, N; \tilde{\theta} \right) \right) \mathrm{d}\tilde{\theta}.$

This implies that at the solution, no university is constrained by the requirement $r(\theta) \leqslant r_{\max}$, and this constraint can be omitted from Problem (21).

Proposition 4 *Let Assumption 1 hold, and, for every* $\theta \in \left[\underline{\theta}, \bar{\theta} \right]$, *let*

$$t(\theta) = z_{\frac{\lambda}{1+\lambda}}^{-1} \left(\sigma (R, N; \theta) \right), \tag{27}$$

$$p(\theta) = \Delta - \Phi^{-1} \left(z_{\frac{\lambda}{1+\lambda}}^{-1} \left(\sigma (R, N; \theta) \right) \right), \tag{28}$$

$$r(\theta) = \theta \int_{\underline{\theta}}^{\theta} \frac{z_{\frac{\lambda}{1+\lambda}}^{-1} \left(\sigma \left(R, N; \tilde{\theta} \right) \right)}{\tilde{\theta}^2} \mathrm{d}\tilde{\theta}. \tag{29}$$

If the values of R *and* N *which obtain by substituting these in (25)-(26) satisfy*

$$\frac{y'(R)}{y(R)} \left(\frac{1}{1+\lambda} - N \right) + \frac{\omega}{1+\lambda} > \frac{1 - F(\theta)}{\int_{\theta}^{\bar{\theta}} \tilde{\theta} f\left(\tilde{\theta}\right) \mathrm{d}\tilde{\theta} + \frac{f(\theta)^2 \theta^2}{f'(\theta)\theta + 2f(\theta)}}, \tag{30}$$

then (24)-(29) solve Problem (21).

To interpret Proposition 4, note first of all that the optimal values of R, the total amount of research in society, and $\underline{\theta}$ the least productive university are given by (24)-(26). As shown in the Appendix, N is the total employment in academia. Given these "global" variables, (27) gives the number of students in university θ. This, as shown in the Appendix, is increasing, which says that more productive universities have more students. (29) shows that they also do more research. As we saw in Section 3, this was also the case with unfettered private provision. The explanation, however, is different in this case: with unfettered private provision, a more productive university needs to teach more students to be able to carry out more research. On the other hand, when the government controls the sector, it asks more productive universities to teach more students because they are more productive: precisely for the same reason it also asks them to carry out more research. This different angle is brought in starker relief in Section 6, which presents the case in which the government is not constrained by its information disadvantage, and is therefore able to separate the allocation of teaching and research.

5 Implementation: The design of the funding scheme.

In this Section we show how the above mechanism can be implemented in practice. Note that (18) must hold for every θ. Differentiate it with respect to

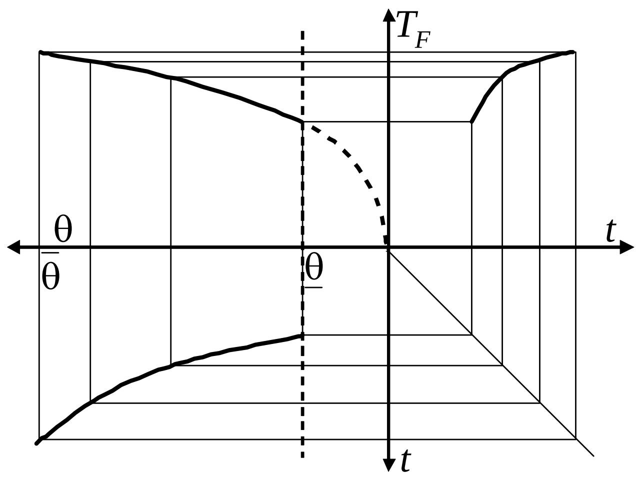


Figure 3: The implementation of the optimal policy.

θ:

$$\frac{y(R)}{\theta}\left(\dot{r}(\theta)+\dot{t}(\theta)\right)-\frac{y(R)}{\theta^2}\left(r(\theta)+t(\theta)\right)-\dot{g}(\theta)-\frac{d\left(p(\theta)\,t(\theta)\right)}{d\theta}=0.$$

Denote by $T_F(\theta)$ the total funding available to a type θuniversity. Substitute (16) and (18) into the above, to obtain:

$$\dot{T}_F(\theta)=\dot{g}(\theta)+\frac{d\left(p(\theta)\,t(\theta)\right)}{d\theta}=\frac{y(R)}{\theta}\dot{t}(\theta)>0. \tag{31}$$

From (31) we can draw the relationship between t and T_F, which we depict in the north-east quadrant of Figure 3. This is derived in the following way: the south-west quadrant illustrates the relationships between θ and the number of students $t(\theta)$, taught by a type θ university, given in (27). This is increasing. Also increasing, by (31), is the relationship between θ and the total funding available to a type θ university, $T_F(\theta)$, in the north-west quadrant. Joining the two via the 45 degree line in the south-east quadrant, we obtain, in the north-east quadrant, the relationship between the number of students and the total funding available to a university. If the government offers this relationship to all universities, allowing each to choose any point on the curve, then each university will select the combination of students given by (27) and total funding which will allow it carry out the amount of research given by (29). Note that $t(\theta)$ is increasing, and since the total funding also is (from (31)), then so is the relationship between number of students and total funding, as depicted.

The following Corollary illustrates that the relationship is concave, and therefore, as in Laffont and Tirole's (1993 pp 69-73) analysis of procurement contracts, the policy can be implemented by offering universities a menu of "subsidy per student"-"lump-sum grant" combination.

Corollary 3 *The relationship between T_F and t is concave.*

Therefore the government can simply offer all universities a menu of linear contracts, $g_T(p)$, where p, the tuition fee per student, is given by the slope of the tangent of the curve in the north-east quadrant, and g_T, the lump sum grant, is the ordinate of this tangent. Faced with this menu, each university will simply select the combination of funding and fee per student that corresponds to its own type. Concavity of the curve implies that universities which charge students less are "rewarded" with a larger lump-sum grant.

6 The systems compared.

To interpret the solution obtained in Proposition 4, it is helpful to compare it with the policy the government would choose if, *ex-ante*, it knew perfectly the type of each university, or, equivalently, if it could *ex-post* measure precisely each university's research effort. This is referred to as the first best and is described it in the next proposition. Let R and N be given by:

$$N = r_{\max} \int_{\frac{1+\lambda}{1-(1+\lambda)N}\frac{y(R)}{y'(R)}}^{\bar{\theta}} \frac{f(\theta)}{\theta} \mathrm{d}\theta + \int_{\frac{y(R)}{\Delta - a_{\min}}}^{\bar{\theta}} \frac{z_{\frac{\lambda}{1+\lambda}}^{-1}\left(\Delta - \frac{y(R)}{\theta}\right)}{\theta^2} f(\theta)\,\mathrm{d}\theta, \tag{32}$$

$$R = \left(1 - F\left(\frac{1+\lambda}{1-(1+\lambda)N}\frac{y(R)}{y'(R)}\right)\right) r_{\max}. \tag{33}$$

Proposition 5 *If the government had perfect information it would choose:*

$$t(\theta) = z_{\frac{\lambda}{1+\lambda}}^{-1}\left(\Delta - \frac{y(R)}{\theta}\right), \tag{34}$$

$$p(\theta) = \Delta - \Phi^{-1}(t(\theta)), \tag{35}$$

$$r(\theta) = \begin{cases} r_{\max} & \textit{for} \quad \theta \geqslant \frac{1+\lambda}{1-(1+\lambda)N}\frac{y(R)}{y'(R)} \\ 0 & \textit{for} \quad \theta \in \left[\frac{y(R)}{\Delta - a_{\min}}, \frac{1+\lambda}{1-(1+\lambda)N}\frac{y(R)}{y'(R)}\right) \end{cases}. \tag{36}$$

The number of students, given in (34), has a similar expression as for the unfettered market case, (10), and for the case of asymmetric information (27). From the former it differs as $z_{\frac{\lambda}{1+\lambda}}$ replaces z_1, from the latter in that the argument of $z_{\frac{\lambda}{1+\lambda}}^{-1}$ does not include the information distortion terms. The two expressions are identical when $\theta = \bar{\theta}$ (efficiency at the top).The expression for university's amount of research, (36), on the other hand, is radically different from the corresponding expression for the case of asymmetric information, (29). While Assumption 1 ensures that the upper boundary on research is not binding in the asymmetric information case, this is not possible in this case, since the government does not need to provide incentives for research, but can simply command and control the activities of each university. It therefore allocates research to the most productive universities, its only constraint the technological upper bound.[23] Unlike the private market case and the case of imperfect information, with perfect information there are "teaching only" universities. Universities with θ in $\left[\frac{y(R)}{\Delta - a_{\min}}, \frac{1+\lambda}{1-(1+\lambda)N}\frac{y(R)}{y'(R)}\right)$ enrol students but carry out no research. From Proposition 5, we can determine the government subsidy $g(\theta)$:

$$g(\theta) = \begin{cases} \frac{r_{\max}}{\theta} y(R,t) - \frac{t(\theta)^2}{\phi(\Phi^{-1}(t))} & \text{for} \quad \theta \geqslant \frac{1+\lambda}{1-(1+\lambda)N}\frac{y(R)}{y'(R)} \\ -\frac{t(\theta)^2}{\phi(\Phi^{-1}(t))} & \text{for} \quad \theta \in \left[\frac{y(R)}{\Delta - a_{\min}}, \frac{1+\lambda}{1-(1+\lambda)N}\frac{y(R)}{y'(R)}\right) \end{cases}.$$

The subsidy is *negative* for the "teaching only" universities: they receive more in fees that they pay out in salaries, and their surplus is transferred to the high θ universities which do research. Students attending these universities pay for research carried out elsewhere. This does not happen in the unfettered private market, or with asymmetric information. In the latter case, if a university spends more than its revenues, the shortfall is funded only by the taxpayer.

Notice that in our model there is no natural welfare ranking of the total amount of research in the three regimes. This is because even though research does bestow an externality, it is not necessarily underprovided by an unfettered private university sector: whether or not it is depends in general on the balance between technology – how much the nation's scientific and cultural state affects the production of goods and services –, the direction research takes – how much of what researchers do spills over to the rest of society –, and on the subjective preferences of the government.[24] In other words, the first best value of R (the preferred amount of research when the government has perfect information)

[23] In Proposition 5, research can take two values only, $r_{\max}$ or 0. This follows from the hypothesis that marginal cost is 0 up to the exogenously given upper bound, and is $+\infty$ beyond it. With less extreme forms of decreasing returns to research expenditure, the "bang-bang" nature of the research policy would be tempered: the concentration of research in the most productive universities would remain, but different high productivity universities would do different amount of research.

[24] While a mathematical theorem may eventually help improve computer software used in designing robots, a chemical discovery may allow the development of more effective drugs, reducing the number of days lost due to illness, and advances in game theory may lead

may well be lower than the amount of research carried out in an unfettered private market. To see this, think of a situation where research has little social benefit: $y'(R)$, the effect on aggregate income, and ω, the non-monetary benefit of research, are both small. If Δ is sufficiently large, then students are willing to pay for university tuition, so universities can use the income raised to pay for their research and the total amount may exceed that which a welfare maximising government would want to choose.[25]

The following assumption allows us to abstract from the equity and efficiency effect of the *aggregate* amount of research and to concentrate on the more microeconomic aspect of the *distribution* of teaching and research across institutions.

Assumption 2 *$r_{\max}$ and ω are such that the equilibrium total amount of research, R, is the same in the three regimes considered.*

It is in general possible for the parameters $r_{\max}$ and ω to satisfy Assumption 2. To see this, let $\hat{R}$ be the amount of research in the unfettered private market. $\hat{R}$ is independent of both $r_{\max}$ and ω, as (13) shows. Therefore there exists a value of ω such that the aggregate amount of research chosen by the government under asymmetric information equals $\hat{R}$ (this is the value of ω such that the RHS in (25) equals $\hat{R}$). Finally, note that $r_{\max}$ appears only in the solution for the first best case, and, when it is such that the RHS in (33) is $\hat{R}$, then Assumption 2 holds.

Figure 4 illustrates the distribution of the total amount of research across universities in the three regimes, under Assumption 2, so that the total research is the same in all three. The three curves depict the amount of research as a function of the productivity of the university, θ. The solid one with private provision, the dashed curve in the case of government intervention with imperfect information, the dotted curve when the government can perfectly observe each university's productivity. The integral (with measure $f(\theta)$) of the three curves is the same, and so the dashed line is above the solid one for high θ and vice versa, as drawn. The dotted line is 0 below the threshold value of θ given in (36) (the "teaching only" institutions), and at its maximum above this threshold. The intuition is that, with perfect information, the government can

to improved incentive mechanisms used by organizations to select and motivate staff, other research activities could instead be viewed as an end in itself, academics indulging in their hobbies, with no expected current or long term benefit to society.

[25] Formally, given the assumption that $w''(R) < 0$, $\frac{y(R)}{y'(R)}$ is increasing, and the optimal R is 0 if $\frac{1+\lambda}{1-(1+\lambda)N} \frac{y(0)}{y'(0)} > 1$, see (33).

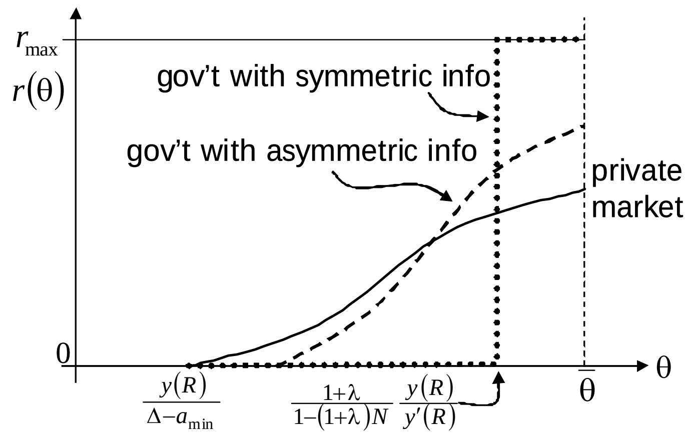


Figure 4: The amount of research with private and government provision.

separate teaching and research: the former is chosen on the basis of efficiency and equity considerations in teaching only, in each local market to the point where marginal benefits equal marginal costs. The total amount of research, R, is chosen at the optimally global level (see (33)), given by the condition that *global* marginal benefits equal *global* marginal costs. This total amount of research is allocated to the university sector in the most cost effective way, by asking the most productive universities to do as much research as they can.

Consider next the distribution and the number of students in the three regimes. The following summarises the comparison.

Corollary 4 *Let Assumptions 1 and 2 hold. Then the following hold:*

1. *The universities active with unfettered private provision are the same a perfectly informed government would allow to operate; fewer universities are active with asymmetric information.*

2. *With unfettered private provision, each active university has fewer students than it would have with a perfectly informed government.*

3. *Relative to perfect information, the government information disadvantage reduces the number of students at each university except the most productive.*

The diagram of Figure 5 illustrates. The Corollary implies that, compared with private provision, government intervention concentrates students in the most productive institutions: the higher (lower) productivity institutions have

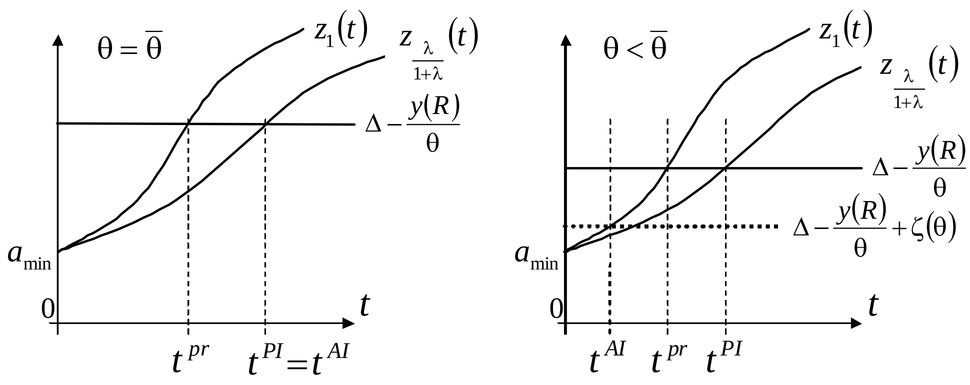


Figure 5: The Determination of the number of students.

more (fewer) students than they would in a private system. The horizontal axis measures the number of students. This, for university θ, is given by the intersection of the increasing line $z_{\frac{\lambda}{1+\lambda}}(t)$, under government provision, and $z_1(t)$ for the private market, with the appropriate horizontal line:

$$\Delta - \frac{y(R)}{\theta} \qquad \text{for the private market or with perfect information,}$$

$$\Delta - \frac{y(R)}{\theta} + \zeta(R, N; \theta) \qquad \text{with imperfect government information,}$$

The LHS of Figure 5 shows the determination of the number of students at the highest possible value of θ. Assumption 2 holds, and so the horizontal line $\Delta - \frac{y(R)}{\theta}$ is the same in all three regimes. The number of students is lower with private provision than with government intervention. This is so because, as Lemma 1 implies, when k increases from 0 to 1, the curve z swings anticlockwise around the point $(0, a_{\min})$, and since $1 = \lim_{\lambda \to +\infty} \frac{\lambda}{1+\lambda}$, the curve z is lower under government provision for every finite value of λ. In this case we have $t^{pr} < t^{AI} = t^{PI}$. This has a natural explanation: unlike private universities, the government receives some benefit from the fact that students pay lower fees. It therefore will want to push the number of students recruited beyond what a private university sector would do. How strong this effect is depends on the social cost of raising taxes to pay for students' tuition: if this is very high, then the overall government cost of enticing students to attend university becomes similar to the private universities' and the curve $z_{\frac{\lambda}{1+\lambda}}(t)$ draws closer to $z_1(t)$. When $\theta = \bar{\theta}$, we have $t^{AI} = t^{PI}$: this is the standard "efficiency at the top" result. The RHS of Figure 5 considers a value of θ lower than $\bar{\theta}$. For such θ, the horizontal curve is lower, for all three regimes, than with $\theta = \bar{\theta}$, but is “more lower” when the government has imperfect information, as depicted by the dotted horizontal line, because $\zeta(R, N; \theta)$ is 0 and increasing at $\theta = \bar{\theta}$. As the RHS diagram shows, for sufficiently low θ, we have that $t^{AI} < t^{pr}$ and both of them are smaller than t^{PI}: productive universities do more research than less productive ones both in an unfettered market and with the optimal government policy, but they recruit more students in the latter regime, as is roughly suggested by the data presented in Figure 2 in the introduction.

The intuition is the following. Research is cheaper in more productive universities, and so the government wants them to carry out more research. To give them the incentive to do so, it rewards them with a combination of a larger total income and a bigger number of students. A less productive university, which would like to receive the higher total income promised to a productive one, is thus deterred from claiming to have high productivity: if it did so, to receive a bigger grant, it would also have to recruit an increased number of students. This is costly, as they can only be recruited by charging them a lower fee. Since it is less productive, extra students are more expensive than they would be for a higher θ university, and the extra total income received for teaching more students is not sufficient to cover the cost of teaching them. The term $\zeta(\theta)$ can be interpreted as the information cost incurred by the government: except for the most productive university, the balance between student fees and lump-sum grant is inefficiently skewed towards student fees, so as to make less productive universities less willing to expand their student intake.

As θ decreases further, the intersection of the horizontal lines with the curves $z_1(t)$ and $z_{\frac{\lambda}{1+\lambda}}(t)$ move towards the vertical axis. The value of θ for which this intersection reaches the axis is the type of the least productive university that teaches any students: clearly this happens for the same value when provision is via an unfettered private market and in the first best, and for a lower value of θ when the government has imperfect information: fewer universities are active in this case.

Figure 6 sketches the relationship between θ and the number of students, in the three regimes, analogously to Figure 4 for research. The dotted line, depicting the perfect information case, coincides with the dashed one, the asymmetric information case, at $\theta = \bar{\theta}$ (efficiency at the top), and with the solid one, the private market case, at $\theta = \frac{y(R)}{\Delta - a_{\min}}$, because $z_1(0) = z_{\frac{\lambda}{1+\lambda}}(0)$.

Notice the information externality among universities in different local education markets: whether some students attend university or not depends on the cost conditions in the rest of the university sector, even though the cost of providing university education is fully determined at the local level.

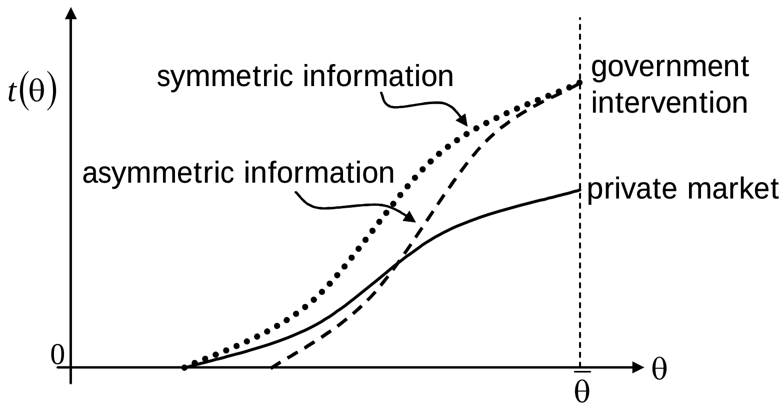


Figure 6: The number of students with private and government provision.

Recall that Figure 6 is drawn for the special case when Assumption 2 holds and the total amount of research R is the same in the three regimes. If this is not the case, the horizontal curve in Figure 5 would have a different position in each regime, and the comparison would have to be made taking into account of the different position of this curve. In general, however, note that this horizontal curve shifts *down* when R is higher: this naturally reflects the trade-off between teaching and research.

7 Extensions.

We have studied a taxation and higher education general equilibrium model, with a continuum of different local markets, a continuum of different students in each market, and information, teaching and research interactions among all the local markets mediated by a global labour market. This is a rich set-up, and to keep the complexity at a manageable level, we have introduced a number of simplifying assumptions. In this section, we illustrate that our results are likely to survive unscathed in a richer set-up where some of the assumptions are relaxed.

7.1 Concave utility function.

In normative analysis, it is customary to assume concavity of the utility function, to reflect the government's preference for redistribution. In the paper, consumers' utility function is instead assumed linear in income.[26] Conversely, we have also assumed that the shadow cost of public funds is strictly positive and exogenously given. We show in this Section that relaxing both assumptions together would generate a richer, but possibly intractable, general equilibrium model, where the shadow cost of public funds is determined *endogenously* and is higher when the utility function is "more concave". This justifies our assumption of a linear utility function with exogenously given λ, which is thus shown to be a good proxy for the government preference for redistribution.

Let the utility function of a type a consumer be given by $U(y-a)$, with $U'(\cdot) > 0$, $U''(\cdot) < 0$. To maintain tractability, let there be a single value of θ (that is, the support of f is degenerate): in this case, clearly, $R = r$, and the government's problem becomes one of complete information, which can be written as:

$$\max_{p,t,g,r} \int_{a_{\min}}^{\Phi^{-1}(t)} U(y(r) - p - a - g)\,\phi(a)\,\mathrm{d}a + (1-t)\,U(y(r) - g), \tag{37}$$

$$\text{s.t.: } g + pt - \frac{y(r)}{\theta}(r+t) = 0, \tag{38}$$

$$\Delta - p - \Phi^{-1}(t) = 0. \tag{39}$$

We have dropped the argument (θ) of the functions p, t, g, and r chosen by the government, omitted the constraint $t \geqslant 0$, and used the government budget constraint $g = h$.

Proposition 6 *If $U''(\cdot) < 0$, at the solution of problem (37)-(39), the Lagrange multiplier of constraint (38) is given by*

$$\frac{U'\left(y(r) - p - \Phi^{-1}(t) - g\right)}{\frac{\int_{a_{\min}}^{\Phi^{-1}(t)} U'(y(r)-p-a-h)\phi(a)\mathrm{d}a}{t}} - 1 \tag{40}$$

where p, t, g, and r are the optimal tuition fee, number of students, lump-sum grant and research effort.

The denominator of the first term is the average marginal utility of consumption for all the individuals who become students, and the numerator is the marginal utility of consumption of the marginal student. Since the average student's income, net of utility effort cost, is higher for the average than for the marginal student, if $U''(\cdot) < 0$ for at least some of the students, then the

[26] This is not restrictive for the positive analysis of Section 3, since with a single good, any strictly increasing utility function can be monotonically transformed into a linear one.

term is greater than 1, implying a strictly positive lagrange multiplier for the government budget constraint, that is a strictly positive shadow cost of public funds, even in the absence of distortionary and administrative costs of taxation. Moreover, the greater the difference between the denominator and the numerator in (40) the greater the shadow cost of public funds.

7.2 Students mobility

We have so far assumed that students can only study at their local institution. Therefore a university's potential demand is given by all university age individuals at the university location. In reality, students are not perfectly immobile. While students' mobility is not the focus of this paper, it is worth hinting at how it can be introduced in a natural way.

Suppose that some students are prepared to leave their preferred[27] local education market to attend a university in different one. A reduced form, compact, manner of accounting for this is to let the distribution of potential students of a university depend on the tuition fee it charges, p, and on the average tuition fee in the sector, say $\bar{p}$. Let therefore

$$\Phi_c\left(a;p,\bar{p}\right),$$

with density $\phi_c\left(a;p,\bar{p}\right)=\Phi_c'\left(a;p,\bar{p}\right)$, and monotonic hazard rate $\frac{\partial}{\partial a}\left(\frac{\Phi_c(a;p,\bar{p})}{\phi_c(a;p,\bar{p})}\right)>0$, be the distribution of potential students of a type θ university, charging fee p, when the sector average tuition fee is $\bar{p}$. Students preference for lower fees is captured by the assumption that if $p_1>p_0$, then $\Phi_c\left(a;p_1,\bar{p}\right)$ first order stochastically dominates $\Phi_c\left(a;p_0,\bar{p}\right)$. Repeating for the present case the analysis carried out in Section 3, a university of type θ charging tuition fee $p\left(\theta\right)$ enrols a number of student $t\left(\theta\right)$ satisfying:

$$t\left(\theta\right)=\Phi_c\left(\Delta-p\left(\theta\right),p\left(\theta\right),\bar{p}\right).$$

The relationship between t and p is now given by (denote, with slight abuse of notation, by $\Phi_c^{-1}\left(t;p,\bar{p}\right)$ the inverse of $\Phi_c\left(a;p,\bar{p}\right)$ for given p and $\bar{p}$):

$$\frac{dp}{dt}=-\frac{1}{\phi\left(\Phi_c^{-1}\left(t;p,\bar{p}\right)\right)-\frac{\partial\Phi_c(a;p,\bar{p})}{\partial p}}<0,$$

since $\frac{\partial\Phi_c(a;p,\bar{p})}{\partial p}<0$. The analytical tractability of being able to define the function z_k independently of p is now lost, as the first order condition for the

[27] Their preferred location may be their parents' place of residence, or perhaps a distant one, if they *prefer* to leave home to go to university.

choice of t now becomes:

$$\left(-\frac{t}{\phi\left(\Phi_c^{-1}\left(t;p,\bar{p}\right)\right)-\frac{\partial\Phi_c(a;p,\bar{p})}{\partial p}}+\Delta-\Phi_c^{-1}\left(t;p,\bar{p}\right)\right)\frac{\theta}{y\left(R\right)}=1.$$

Since the qualitative features of the function z_k are unchanged in this more general set-up, we would expect to obtain similar results as before. An interesting by-product of this extension is likely to be the result that universities that have more students also have abler students: students' mobility breaks the rigid link between the number of students and the ability level of the lowest ability student in a given university.

7.3 Quality dependent willingness to pay.

In Section 7.2 students move to a different location in response to price differences. In practice, of course, students choose a given university on the basis of other factors, principally quality. In the model developed so far, there is no scope for this, as the *type* of the university attended by a student does not affect directly her utility. This is unrealistic, and has the unrealistic implication that more productive universities charge lower tuition fees.

A simple way to make the university type directly affect a student's utility is to define the cost of attending university as $c\left(a,\theta\right)$, a function of both the student's type, with $c_a\left(a,\theta\right)>0$, and the university's type, with $c_\theta\left(a,\theta\right)<0$: studying at a better university is easier, and so has a lower effort cost, or there is more personal satisfaction and pride, which increase utility. The function $z_k\left(t\right)$ given in (8) is now replaced by

$$z_k\left(t,\theta\right)=c\left(\Phi^{-1}\left(t\right),\theta\right)+\frac{ktc_a\left(\Phi^{-1}\left(t\right),\theta\right)}{\phi\left(\Phi^{-1}\left(t\right)\right)}.$$

Lemma 1 continues to hold, *mutatis mutandis* for the new notation. The university's first order condition for the choice of r, (A2) in the Appendix, is not changed conceptually, and so the relation between t and θ remains positive: more productive universities enrol more students. On the other hand, the relationship between productivity and prices is given by the sign of $\frac{dp}{d\theta}$. To calculate it, expand $p\left(t\left(\theta\right)\right)=\Delta-c\left(\Phi^{-1}\left(t\left(\theta\right)\right),\theta\right)$:

$$\frac{dp}{d\theta}=-\frac{c_a\left(\Phi^{-1}\left(t\left(\theta\right)\right),\theta\right)}{\phi\left(\Phi^{-1}\left(t\left(\theta\right)\right)\right)}\frac{dt}{d\theta}-c_\theta\left(\Phi^{-1}\left(t\left(\theta\right)\right),\theta\right).$$

The first term is negative, which gives Corollary 1 when $c\left(a,\theta\right)=a$ as in the rest of the paper. In the case considered here, $c_\theta\left(a,\theta\right)<0$, and, if it is sufficiently large in absolute value, $\frac{dp}{d\theta}$ can become positive in a range of values of θ, yielding the realistic conclusion that universities with higher productivity not only carry out more research and teach more students, but also charge higher fees.

7.4 Endogenous determination of θ.

The methodology of our paper is inspired by the regulation and procurement literature (Baron and Myerson 1982, Laffont and Tirole 1993), where the parameter distinguishing suppliers is, like here, their exogenously given productivity parameter. It may be argued that in the case of universities productivity depends in large measure on human capital, and so the assumption that θ is exogenously fixed is less justifiable than for the firms studied in the regulation and procurement literature, where it is instead largely determined by the existing technology and the environmental conditions. It is therefore worth hinting at how the model could be enriched by determining the productivity parameter as part of the equilibrium rather than exogenously imposing it from outside.

A good starting point to model formally the relationship between θ and a university's human capital is the observation that the parameter characterising a worker is in practice highly correlated with success in carrying out research: those for whom studying at university is relatively less costly are also likely to have a more productive academic career, and universities that appoint staff with lower a will, *ceteris paribus*, be more productive. In this subsection we capture in a highly stylised way the idea that differences in the productivity of universities are linked to differences in the distribution of a of their staff.

Time is divided in periods. Universities are long lived. Workers live two periods: they receive education during their first period of life, are hired by employers after completing education at the beginning of their second period of life, and cannot change job until they retire. In each period a university trains workers in their first period of life, and employs skilled workers in their second period of life. Within each period, the production and education processes take place after the labour markets have cleared.

The markets for skilled labour open at the beginning of each period, and close before research and teaching begin. Whilst the labour market is open, each university receives a stream of job applicants who are randomly drawn from $[a_{\min}, a_{\max}]$ according to the distribution of skilled workers determined by the education process of the previous period. Assume that a university cannot turn away individual workers, but must hire all those who come along, until it decides to stop hiring altogether. Assume also, somewhat artificially, that before commencing to teach and carry out research, a university can dismiss the last hired of its workers. Some of these hypotheses are clearly *ad hoc*, and we stress that the aim of this subsection is only to hint at what the building blocks of a model of endogenous determination of θ could be.

Skilled workers have a reservation wage, $y(R)$. Workers, universities and employers are all assumed to predict correctly the value that R will take at the end of the process. In equilibrium, a university's salary offer will therefore be exactly $y(R)$. The set of workers hired at any moment of the hiring process obey a distribution F^A, which is a monotonically increasing differentiable function from $[a_{\min}, a_{\max}]$ into $\mathbb{R}_+$, such that $F^A(a_{\min}) = 0$. Let $\mathcal{A}$ be the space of all such functions, and let Θ be a function:

$$\Theta : \mathcal{A} \longrightarrow \left[0, \bar{\theta}\right], \qquad \Theta : F^A \longmapsto \theta. \tag{41}$$

In words, if a university's distribution of staff abilities in period t is $F^A(a)$, then its productivity parameter in that period is $\Theta\left(F^A\right)$. To fix ideas, Θ may be thought of as mapping a distribution into its mean, but one can easily think of reasons why size, variance, skewness, minimum and maximum, and so on all matter, and so a generic functional form gives more flexibility at no cost. A natural, though not strictly necessary, requirement is that if $F^{A_1}(a_{\max}) = F^{A_2}(a_{\max})$ and F^{A_1} first order stochastically dominates F^{A_2}, then $\Theta\left(F^{A_1}\right) > \Theta\left(F^{A_2}\right)$.

Each university knows the slope of the relationship between n, the number of staff hired, and the productivity parameter θ which will prevail in equilibrium, $n(\theta)$: given the large number of universities, this relationship is deterministic, both in a private market and with government intervention. At each moment during the hiring process, the pair (θ, n) describing the number of staff hired, n, and the value of θ of the university given the distribution of its staff, has followed a path in the $\left[0, \bar{\theta}\right] \times \mathbb{R}_+$ space. This path associates the size of the university, n, to the value of the productivity the university had when the distribution of its academics was given by F^A, with $F^A(a_{\max}) = n$ and $\Theta\left(F^A\right) = \theta$. The current (n, θ) pair also determines the possible pairs (n, θ) which, with positive probability, may characterise the university in the future. Let $\mathcal{P}(n, \theta)$ denote the set of all such pairs.

A utility maximising university stops hiring when the intersection between $n(\theta)$ and the interior of the set $\mathcal{P}(n, \theta)$ is empty, and subsequently reduce its workforce to its preferred point on the intersection set of its past path and $n(\theta)$. Figure 7.4 illustrates the idea. It depicts the (θ, n) Cartesian plane. A university's first employee determines its initial quality, the point where the

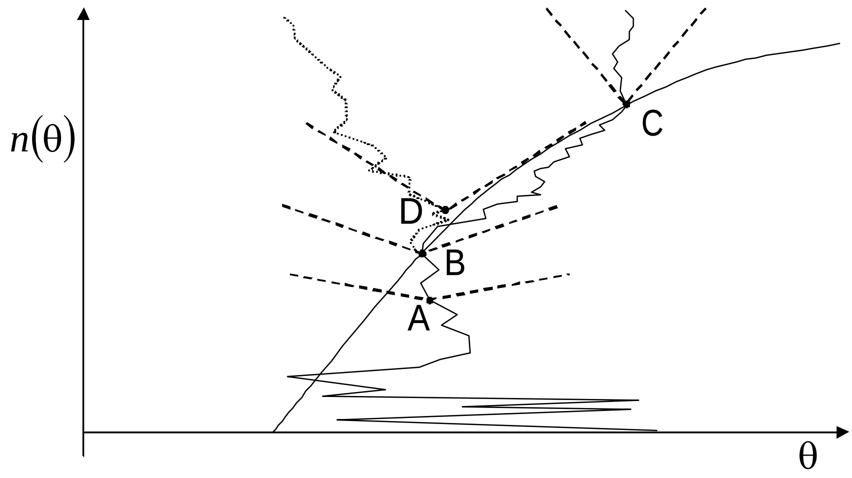


Figure 7: The endogenous determination of θ.

squiggly line meets the horizontal axis. Subsequent hires determine different values of θ, depicted by the path. A university that has reached a point like A will rationally continue to hire staff, because the (θ, n) combination represented by point A will not be "allowed" by the government or the private market. Consider next a university that has reached a point like B (say $\left(\theta^B, n^B\right)$). This combination will be possible (in the specific sense that universities of type θ^B will be given the incentive to choose to employ $n\left(\theta^B\right) = n^B$ academics). However, a university that has reached point B will not stop hiring. The reason is that it would be possible, if the university had a streak of high quality hires, to reach a better point, which will again be feasible, such as point C, along the solid path: the set $\mathcal{P}(n, \theta)$ (represented by the "cone" centred in B of the points which could theoretically be reached) does contain preferred point on the $n(\theta)$ curve. At point C, on the other hand, this is not the case, and a university that has reached point C will stop hiring. Finally, if the stream of recruits from point B had generated the dotted path, then at a D, then the university would need to stop hiring and to dismiss the most recent hires, to return to point B, the best among the feasible points reached during the hiring process.

This model could be further enriched to include, plausibly, some history dependence. A fully dynamic model may link the value of θ in one period to its past values, even though in each period hires are random. This could be due to better laboratories or structures inherited from the past. A shortcut to capture this formally is to replace the function (41) with

$$\Theta : \left[0, \bar{\theta}\right] \times \mathcal{A} \longrightarrow \left[0, \bar{\theta}\right], \qquad \Theta : \left(\theta_{t-1}, F^A\right) \longmapsto \theta_t.$$

In addition, skilled workers with a high a may prefer to work at a university which had a high θ in the previous period. In this case, the distribution of graduates seeking employment at a university could itself be affected by θ, and would be a differentiable function from $\left[0, \bar{\theta}\right] \times [a_{\min}, a_{\max}]$ into $\mathbb{R}_+$ such that $F^A(\theta, a_{\min}) = 0$ for every $\theta \in \left[0, \bar{\theta}\right]$ and monotonically increasing in the second argument..

The assumption which must be maintained to retain the qualitative structure of the model studied in Sections 2-6 is that a university cannot affect the quality of its staff by altering the salary it offers, or by selectively rejecting applicants with low a.[28]

8 Concluding remarks

This paper studies how a utilitarian government should intervene in the university sector. Intervention is beneficial because of an externality in research, which implies that, even though the total amount of research which is carried out in an unfettered private might be the optimal level, its *distribution* across universities is not. Specifically, we show that the private market spreads research too thinly: if the government could freely determine who does how much research, it would concentrate it in the most productive universities, and allow less productive universities as "teaching only" institutions, whose students subsidise research carried out elsewhere. The information disadvantage of the government vis-à-vis the universities implies that this is not possible, and the government must allow all teaching universities to do at least some research. This is inefficient, and it increases the overall cost of university provision. In response to this increase, the government reduces the number of universities relative to private provision. The overall effect on the number of students is ambiguous, because while there are fewer universities, the more productive ones are given stronger incentives to admit more students.

[28] Work is in progress (Carrillo-Tudela and De Fraja 2008) to incorporate the ideas sketched above into a rigorous search model, where graduates have preferences over the quality of the institution they join, and the latter is determined by the employees' ability.

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Appendix

Proof of Lemma 1. (i) Differentiate (8), writing $(\cdot)$ for $\left(\Phi^{-1}(t)\right)$:

$$z_k'(t) = \frac{dz_k}{dt} = \frac{1}{\phi(\cdot)} - \frac{\frac{\phi'(\cdot)}{\phi(\cdot)}kt}{\phi(\cdot)^2} + \frac{k}{\phi(\cdot)} \; .$$

This is positive if

$$\frac{kt}{\phi(\cdot)^2}\left(\frac{\frac{1+k}{k}\phi(\cdot)}{t} - \frac{\phi'(\cdot)}{\phi(\cdot)}\right) > 0 \; . \tag{A1}$$

Now notice that, given the assumption of a monotonic hazard rate for $\Phi(\cdot)$, we have

$$\frac{d}{dx}\left(\frac{\Phi(x)}{\phi(x)}\right) = 1 - \frac{\Phi(x)\,\phi'(x)}{\phi(x)^2} = \frac{\Phi(x)}{\phi(x)}\left(\frac{\phi(x)}{\Phi(x)} - \frac{\phi'(x)}{\phi(x)}\right) > 0 \; .$$

Evaluate the hazard rate at $x = \Phi^{-1}(t)$, and substitute it into (A1):

$$\frac{kt}{\phi(\cdot)^2}\left(\frac{\phi(\cdot)}{kt} + \frac{\phi(\cdot)}{t} - \frac{\phi'(\cdot)}{\phi(\cdot)}\right) \; ,$$

and (i) is established. (ii) follows immediately from (8). Consider (iii) next. $z_{k_1}(t) - z_{k_0}(t) = \frac{(k_1-k_0)t}{\phi(\Phi^{-1}(t))} > 0$, and

$$\frac{dz_k'(t)}{dk} = \frac{1}{\phi(\cdot)}\frac{d}{dx}\left(\frac{\Phi(\cdot)}{\phi(\cdot)}\right) > 0 \; .$$

This end the proof of the Lemma. ∎

Proof of Proposition 1. Substituting (2) into (3), we can write:

$$\frac{y(R)}{\theta}r - g = \left(p - \frac{y(R)}{\theta}\right)t = \left(p - \frac{y(R)}{\theta}\right)\Phi(\Delta - p)$$

A university of type θ chooses p to maximise the RHS of the above. If an internal solution exists, it satisfies the first order condition

$$\Phi(\Delta - p) - \phi(\Delta - p)\left(p - \frac{y(R)}{\theta}\right) = 0$$

or

$$\Delta - p + \frac{\Phi(\Delta - p)}{\phi(\Delta - p)} = \Delta - \frac{y(R)}{\theta} \tag{A2}$$

since $t = \Phi(\Delta - p)$, (10) follows and (9) from it. The derivative of the LHS of (A2) with respect to p is negative, implying that the second order condition is satisfied at the optimum, and the proof of the Proposition is complete. ∎

Proof of Corollary 1. The first assertion follows from (10), noting that z_k^{-1} is increasing. The second from the first and $\frac{dp}{dt} = -\frac{1}{\phi(\Phi^{-1}(t))} > 0$. For the third, develop $\frac{dr}{d\theta}$:

$$\frac{dr}{d\theta} = \left((\Delta - z_1(t)) \frac{\theta}{y(R)} - 1 \right) \frac{dt}{d\theta} + \frac{p(t(\theta))\, t(\theta) + g}{y(R)} > 0 \tag{A3}$$

since the first term vanishes by the first order condition on the choice of r, (A2). Finally,

$$\frac{dn}{d\theta} = \frac{\frac{dt}{d\theta} + \frac{dr}{d\theta}}{\theta} - \frac{t(\theta) + r(\theta)}{\theta^2} \ ,$$

using (A3) and $t(\theta) + r(\theta) = \frac{p(t(\theta))t(\theta)+g}{y(R)}\theta$, this simplifies to $\frac{1}{\theta}\frac{dt}{d\theta} > 0$. ■

Proof of Corollary 2. The preferred value of t is given by the intersection of the increasing function $z_1(t)$ with the horizontal line $\Delta - \frac{y(R)}{\theta}$ (see (10)). This intersection occurs for a positive value of t if $z_1(0) = \Phi^{-1}(0) = a_{\min} < \Delta - \frac{y(R)}{\theta}$. This establishes the Corollary. ■

Proof of Proposition 2. Let the government policy be $\{t(\theta), p(\theta), g(\theta)\}$. By choosing to report type $\hat{\theta} \in \left[0, \overline{\theta}\right]$, university of type θ is allowed to set a price for tuition $p\left(\hat{\theta}\right)$, receives a grant $g\left(\hat{\theta}\right)$, and is required to teach $t\left(\hat{\theta}\right)$ students. Given the market salary for its staff, $y(R)$, it employs:

$$\frac{p\left(\hat{\theta}\right) t\left(\hat{\theta}\right) + g\left(\hat{\theta}\right)}{y(R)} \tag{A4}$$

academics, which will enable it to carry out an amount of research x such that:

$$\frac{p\left(\hat{\theta}\right) t\left(\hat{\theta}\right) + g\left(\hat{\theta}\right)}{y(R)} = \frac{x + t\left(\hat{\theta}\right)}{\theta} \ .$$

Hence the utility of a university of type θ for reporting $\hat{\theta}$ is

$$\xi\left(\theta, \hat{\theta}\right) = \frac{p\left(\hat{\theta}\right) t\left(\hat{\theta}\right) + g\left(\hat{\theta}\right)}{y(R)}\theta - t\left(\hat{\theta}\right) \ .$$

The revelation principle requires that the above is maximised at $\hat{\theta} = \theta$. The first order condition for the choice of $\hat{\theta}$ is:

$$\left.\frac{\partial \xi\left(\theta, \hat{\theta}\right)}{\partial \hat{\theta}}\right|_{\hat{\theta}=\theta} = \left.\frac{\partial \left(\frac{p(\hat{\theta})t(\hat{\theta})+g(\hat{\theta})}{y(R)}\theta - t\left(\hat{\theta}\right) \right)}{\partial \hat{\theta}}\right|_{\hat{\theta}=\theta} = 0 \ , \tag{A5}$$

which gives:

$$\left(p\left(\theta\right)\dot{t}\left(\theta\right)+\dot{p}\left(\theta\right)t\left(\theta\right)+\dot{g}\left(\theta\right)\right)\frac{\theta}{y\left(R\right)}-\dot{t}\left(\theta\right)=0\ . \tag{A6}$$

Next differentiate $r\left(\theta\right)$ given in (15),

$$\dot{r}\left(\theta\right)=\left[p\left(\theta\right)\dot{t}\left(\theta\right)+\dot{p}\left(\theta\right)t\left(\theta\right)+\dot{g}\left(\theta\right)\right]\frac{\theta}{y\left(R\right)}+\frac{p\left(\theta\right)t\left(\theta\right)+g\left(\theta\right)}{y\left(R\right)}-\dot{t}\left(\theta\right)\ ,$$

and substitute (A6) into it to obtain (16). Now (17): following Laffont and Tirole (1993, p 121), a sufficient condition for a policy to be incentive compatible is that:

$$\frac{\partial^2\xi\left(\theta,\hat{\theta}\right)}{\partial\theta\partial\hat{\theta}}\geqslant 0\ .$$

We have

$$\frac{\partial^2\xi\left(\theta,\hat{\theta}\right)}{\partial\theta\partial\hat{\theta}}=\frac{\partial\left(\frac{p(\hat{\theta})t(\hat{\theta})+g(\hat{\theta})}{y(R)}\right)}{\partial\hat{\theta}}=\frac{\partial\left(\frac{r(\hat{\theta})+t(\hat{\theta})}{\hat{\theta}}\right)}{\partial\hat{\theta}}\geqslant 0\ ;$$

$$\frac{1}{\hat{\theta}}\left(\dot{r}\left(\hat{\theta}\right)+\dot{t}\left(\hat{\theta}\right)-\frac{r\left(\hat{\theta}\right)+t\left(\hat{\theta}\right)}{\hat{\theta}}\right)\geqslant 0\ ,$$

substitute $\frac{r(\hat{\theta})+t(\hat{\theta})}{\hat{\theta}}=\frac{p(\hat{\theta})t(\hat{\theta})+g(\hat{\theta})}{y(R)}=\dot{r}\left(\hat{\theta}\right)$ from (16), to obtain (17). ■

Proof of Proposition 3. Consider local labour market θ. The total pre-tax utility of the potential students is:

$$\int_{a_{\min}}^{\Phi^{-1}(t(\theta))}\left(y\left(R\right)-p\left(\theta\right)-a-h\right)\phi\left(a\right)\mathrm{d}a+\left(1-t\left(\theta\right)\right)\left(y\left(R\right)-\Delta-h\right)\ ,$$

where the first term is the total utility of the individuals who go to university, and the second the total utility of those who work in the unskilled labour market. Rearrange to obtain (22). Integrating for $\theta>\underline{\theta}$, using the fact that $\left(1+\lambda\right)\int_{\underline{\theta}}^{\bar{\theta}}g\left(\theta\right)f\left(\theta\right)\mathrm{d}\theta=h$ (the total tax paid equals the total value of the subsidies given by the government to the university sector increased by the deadweight loss costs of taxation per unit of tax raised), adding the direct benefit of research, ωR, and rearranging gives (21). ■

Proof of Proposition 4. Begin by constructing the Lagrangean for (21):

$$\begin{aligned}\mathcal{L} = &\left((\Delta - p(\theta))\, t(\theta) - \int_{a_{\min}}^{\Phi^{-1}(t(\theta))} a\phi(a)\,\mathrm{d}a - (1+\lambda)\, g(\theta) \right) f(\theta) \\ &+ \mu(\theta)\, \delta \frac{p(\theta)\, t(\theta) + g(\theta)}{y(R)} + \beta(\theta) \left[\frac{y(R)}{\theta} (r(\theta) + t(\theta)) - g(\theta) - p(\theta)\, t(\theta) \right] \\ &+ \tau(\theta) \left[\Delta - \Phi^{-1}(t(\theta)) - p(\theta) \right] + \eta(\theta)\, t(\theta) + \rho r(\theta)\, f(\theta) \ , \end{aligned} \tag{A7}$$

where $\beta(\theta)$, $\tau(\theta)$, $\eta(\theta)$, are the Lagrange multipliers for constraints (18), (14), (19), respectively and $\mu(\theta)$ and ρ are the Pontagryin multipliers for the state variables in constraints (16) and (20), respectively. To simplify the analysis of the perfect information case, we have multiplied the incentive compatibility constraint (16) by an indicator $\delta \in \{0, 1\}$, with $\delta = 1$ for the imperfect information case, and $\delta = 0$ for the case in which the government can costlessly observe the type of the university and therefore is not subject to (16) and (17). The first order conditions are:

$$-\frac{\partial \mathcal{L}}{\partial r(\theta)} = \delta \dot{\mu}(\theta) = -\beta(\theta) \frac{y(R)}{\theta} - \rho f(\theta) \ ; \tag{A8}$$

$$\frac{\partial \mathcal{L}}{\partial g(\theta)} = -(1+\lambda) f(\theta) + \frac{\delta \mu(\theta)}{y(R)} - \beta(\theta) = 0 \ ; \tag{A9}$$

$$\frac{\partial \mathcal{L}}{\partial p(\theta)} = -t(\theta) f(\theta) + \frac{\delta \mu(\theta)\, t(\theta)}{y(R)} - \beta(\theta)\, t(\theta) - \tau(\theta) = 0 \ ; \tag{A10}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial t(\theta)} = &\left[\Delta - p(\theta) - \Phi^{-1}(t(\theta)) \right] f(\theta) + \frac{\delta \mu(\theta)\, p(\theta)}{y(R)} \\ &+ \beta(\theta) \left(\frac{y(R)}{\theta} - p(\theta) \right) - \frac{\tau(\theta)}{\phi(\Phi^{-1}(t(\theta)))} + \eta(\theta) = 0 \ , \end{aligned} \tag{A11}$$

and, for R and $\underline{\theta}$ (Leonard and van Long, 1992, Theorem 7.11.1, p 255):

$$\rho = y'(R) + \omega + \int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial \mathcal{L}(\theta)}{\partial R} \mathrm{d}\theta \ , \tag{A12}$$

$$\mathcal{L}(\underline{\theta}) = 0 \ . \tag{A13}$$

Derive $\beta(\theta)$ from (A9):

$$\beta(\theta) = \frac{\delta \mu(\theta)}{y(R)} - (1+\lambda) f(\theta) \ , \tag{A14}$$

and substitute it into (A8):

$$\delta \dot{\mu}(\theta) = -\frac{\delta \mu(\theta)}{\theta} + (1+\lambda)\, y(R) \frac{f(\theta)}{\theta} - \rho f(\theta) \ .$$

When $\delta = 1$, the two differential equations:

$$\dot{\mu}(\theta) = -\frac{\mu(\theta)}{\theta} + (1+\lambda)\, y(R)\, \frac{f(\theta)}{\theta} - \rho f(\theta) \qquad \mu(\underline{\theta}) \text{ free} \quad \mu(\bar{\theta}) = 0 \; ;$$
$$\dot{r}(\theta) = \frac{p(\theta)\, t(\theta) + g(\theta)}{y(R)} \qquad r(\bar{\theta}) \text{ free} \quad r(\underline{\theta}) = 0,$$

determine the state variable $r(\theta)$ and the multiplier $\mu(\theta)$:

$$\mu(\theta) = -\frac{\rho \int_{\theta}^{\bar{\theta}} \tilde{\theta} f\left(\tilde{\theta}\right) \mathrm{d}\tilde{\theta} - (1 - F(\theta))\,(1+\lambda)\, y(R)}{\theta} \; . \tag{A15}$$

Next substitute (A14) into (A10), to obtain:

$$\tau(\theta) = \lambda t(\theta)\, f(\theta) \; . \tag{A16}$$

(A16) implies that $\tau(\theta) > 0$ if $\lambda > 0$ and $t(\theta) > 0$, and so (14) holds as an equality: $p(\theta) = \Delta - \Phi^{-1}(t(\theta))$. Substitute this, $\beta(\theta)$ from (A14), $\tau(\theta)$ from (A16) and $\eta(\theta) = 0$ (because $t(\theta) > 0$) into (A11) and re-arrange:

$$\frac{\partial \mathcal{L}}{\partial t(\theta)} = \frac{\delta\mu(\theta)\, p(\theta)}{y(R)} + \left(\frac{\delta\mu(\theta)}{y(R)} - (1+\lambda)\, f(\theta)\right) \frac{y(R)}{\theta} -$$
$$\left(\frac{\delta\mu(\theta)}{y(R)} - (1+\lambda)\, f(\theta)\right) p(\theta) - \frac{\lambda t(\theta)\, f(\theta)}{\phi\left(\Phi^{-1}(t(\theta))\right)} = 0 \; ,$$

that is

$$\Phi^{-1}(t(\theta)) = \frac{\delta\mu(\theta)}{(1+\lambda)\, f(\theta)\, \theta} - \frac{y(R)}{\theta} + \Delta - \frac{\frac{\lambda}{1+\lambda} t(\theta)}{\phi\left(\Phi^{-1}(t(\theta))\right)} \; . \tag{A17}$$

Substitute $\mu(\theta)$ from (A15) to obtain:

$$z_{\frac{\lambda}{1+\lambda}}(t(\theta)) = \Delta - \frac{y(R)}{\theta} - \delta \frac{\frac{\rho \int_{\theta}^{\bar{\theta}} \tilde{\theta} f(\tilde{\theta}) \mathrm{d}\tilde{\theta} - (1-F(\theta))(1+\lambda) y(R)}{\theta}}{(1+\lambda)\, f(\theta)\, \theta} \; . \tag{A18}$$

To continue with the proof, we need to derive the value of the multiplier ρ. Expanding (A12) we have

$$\rho = y'(R) \left(1 - (1+\lambda) \int_{\underline{\theta}}^{\bar{\theta}} \frac{r(\theta) + t(\theta)}{\theta} f(\theta)\, \mathrm{d}\theta\right) + \omega \; ;$$

Let $N = \int_{\underline{\theta}}^{\bar{\theta}} \frac{r(\theta)+t(\theta)}{\theta} f(\theta)\, \mathrm{d}\theta$, – that is N is the total employment in universities –, and the above becomes:

$$\frac{\rho}{1+\lambda} = y'(R) \left(\frac{1}{1+\lambda} - N\right) + \frac{\omega}{1+\lambda} \; . \tag{A19}$$

Now substitute (A19) into (A18) and use (23) to obtain (27).

Lemma A1 *Let (30) hold, then* $-\frac{y(R)}{\theta}+\zeta(\theta)$ *is increasing in* θ*.*

Proof. Differentiate $-\frac{y(R)}{\theta}+\zeta(\theta)$ with respect to θ.

$$\begin{aligned}&\frac{y(R)}{\theta}\Bigg\{\left(1+\frac{f'(\theta)\theta+2f(\theta)}{f(\theta)^2\theta^2}\int_\theta^{\bar\theta}\tilde\theta f\left(\tilde\theta\right)\mathrm{d}\tilde\theta\right)\frac{\left(y'(R)\left(\frac{1}{1+\lambda}-N\right)-\frac{\omega}{1+\lambda}\right)}{y(R)}\\&-\frac{f'(\theta)\theta+2f(\theta)}{f(\theta)^2\theta^2}(1-F(\theta))\Bigg\}\ .\end{aligned} \tag{A20}$$

We want to divide the above by the term

$$\left(1+\frac{f'(\theta)\theta+2f(\theta)}{f(\theta)^2\theta^2}\int_\theta^{\bar\theta}\tilde\theta f\left(\tilde\theta\right)\mathrm{d}\tilde\theta\right)y(R)\ . \tag{A21}$$

(A21) is clearly positive if $f'(\theta)\theta+2f(\theta)\geqslant 0$. Consider therefore the case $f'(\theta)\theta+2f(\theta)<0$. Expand the derivative of the hazard rate:

$$\frac{d}{d\theta}\left(\frac{1-F(\theta)}{f(\theta)}\right)=-1-\frac{1-F(\theta)}{f(\theta)^2}f'(\theta)\,. \tag{A22}$$

Derive $f'(\theta)$ from (A22), and substitute it into (A21), to see that, if (1) holds, then (A21) is positive. Next divide (A22) by (A21) and re-arrange to obtain (A20) and establish the Lemma. ■

Since $z_k(t)$ is increasing, Lemma A1 and (27) imply that $t(\theta)$ also is and so (17) is satisfied. Since $t(\theta)$ is 0 for some $\theta\in\left[0,\bar\theta\right]$ (indeed for some $\theta\in\left[1,\bar\theta\right]$, there is a threshold value of θ, call it $\underline{\theta}$, such that $t(\theta)>0$ if and only if $\theta>\underline{\theta}$.

Now we want to establish that the lowest θ determined in (24)-(26), $\underline{\theta}$, is also the value of θ such that $t(\theta)=0$ and $t(\theta)>0$ in a right neighbourhood. Expand (A13). At $\underline{\theta}$, the terms in the square brackets in (A7) and the term $\eta(\underline{\theta})t(\underline{\theta})$ are all 0 because of the slackness complementarity constraints. Also 0 is the term $\rho r(\underline{\theta})f(\underline{\theta})$, because $r(\underline{\theta})=0$, and so:

$$\begin{aligned}\mathcal{L}(\underline{\theta})=&\left((\Delta-p(\underline{\theta}))t(\underline{\theta})-\int_{a_{\min}}^{\Phi^{-1}(t(\underline{\theta}))}a\phi(a)\,\mathrm{d}a-(1+\lambda)g(\underline{\theta})\right)f(\underline{\theta})\\&+\delta\mu(\underline{\theta})\frac{p(\underline{\theta})t(\underline{\theta})+g(\underline{\theta})}{y(R)}t(\underline{\theta})f(\underline{\theta})=0\ .\end{aligned}$$

Since $r(\underline{\theta})=0$,

$$g(\underline{\theta})=\left(\frac{y(R)}{\underline{\theta}}-p(\underline{\theta})\right)t(\underline{\theta})\ ,$$

and so $\mathcal{L}(\underline{\theta}) = 0$ implies:

$$\mathcal{L}(\underline{\theta}) = \left\{ \left[\Delta - p(\underline{\theta}) - (1+\lambda)\left(\tfrac{y(R)}{\underline{\theta}} - p(\underline{\theta})\right) + \tfrac{\delta\mu(\underline{\theta})}{f(\underline{\theta})\underline{\theta}}\right] t(\underline{\theta}) - \int_{a_{\min}}^{\Phi^{-1}(t(\underline{\theta}))} a\phi(a)\,\mathrm{d}a \right\} f(\underline{\theta}) = 0 \;. \tag{A23}$$

Write (A17) (with $\eta(\theta) = 0$) as

$$\frac{\delta\mu(\theta)}{f(\theta)\,\theta} = \left(z_{\frac{\lambda}{1+\lambda}}(t(\theta)) + \frac{y(R)}{\theta} - \Delta\right)(1+\lambda) \;,$$

and so (A23) becomes:

$$\mathcal{L}(\underline{\theta}) = \left\{ \left[-\lambda\Phi^{-1}(t(\underline{\theta})) + z_{\frac{\lambda}{1+\lambda}}(t(\underline{\theta}))(1+\lambda)\right] t(\underline{\theta}) - \int_{a_{\min}}^{\Phi^{-1}(t(\underline{\theta}))} a\phi(a)\,\mathrm{d}a \right\} f(\underline{\theta}) = 0.$$

This is 0 at $t(\underline{\theta}) = 0$, moreover

$$\frac{\partial \mathcal{L}}{\partial t(\underline{\theta})} = f(\underline{\theta}) \left[\frac{-\frac{\lambda}{1+\lambda} t(\underline{\theta})}{\phi(\Phi^{-1}(t(\underline{\theta})))} + t(\underline{\theta})\, z'_{\frac{\lambda}{1+\lambda}}(t(\underline{\theta})) - \Phi^{-1}(t(\underline{\theta})) + z_{\frac{\lambda}{1+\lambda}}(t(\underline{\theta})) \right] =$$
$$= t(\underline{\theta})\, z'_{\frac{\lambda}{1+\lambda}}(t(\underline{\theta}))\, f(\underline{\theta}) \,.$$

This is strictly positive for $t(\underline{\theta}) > 0$. Therefore $\mathcal{L}(\underline{\theta}) = 0$ is increasing at $t(\underline{\theta}) = 0$, making 0 the only value of t where $\mathcal{L}(\underline{\theta}) = 0$, and so $t(\underline{\theta}) = 0$. What remains to be established are (28) and (29). The first follows from (14). To derive (29), start from the following equality:

$$r(\theta) = [p(\theta)\, t(\theta) + g(\theta)] \frac{\theta}{y(R)} - t(\theta) = \int_{\underline{\theta}}^{\theta} \frac{p\left(\tilde{\theta}\right) t\left(\tilde{\theta}\right) + g\left(\tilde{\theta}\right)}{y(R)} \mathrm{d}\tilde{\theta} \;,$$

and write it as:

$$g(\theta)\,\theta = \int_{\underline{\theta}}^{\theta} g\left(\tilde{\theta}\right) \mathrm{d}\tilde{\theta} + \int_{\underline{\theta}}^{\theta} p\left(\tilde{\theta}\right) t\left(\tilde{\theta}\right) \mathrm{d}\tilde{\theta} + t(\theta)\, y(R) - p(\theta)\, t(\theta)\, \theta \;.$$

Differentiate both sides with respect to θ, and divide by θ:

$$\dot{g}(\theta) = \frac{\dot{t}(\theta)}{\theta} y(R) - \frac{d\left(p(\theta)\, t(\theta)\right)}{d\theta} \;.$$

Integrate both sides in the above to get

$$g(\theta) = \left(\frac{t(\theta)}{\theta} + \int_{\underline{\theta}}^{\theta} \frac{t\left(\tilde{\theta}\right)}{\tilde{\theta}^2} \mathrm{d}\tilde{\theta} \right) y(R) - p(\theta)\, t(\theta) \;,$$

and therefore:

$$r(\theta) = \left[p(\theta)\,t(\theta) + \left(\frac{t(\theta)}{\theta} + \int_{\underline{\theta}}^{\theta} \frac{t\left(\tilde{\theta}\right)}{\tilde{\theta}^2}\mathrm{d}\tilde{\theta}\right) y(R) - p(\theta)\,t(\theta)\right] \frac{\theta}{y(R)} - t(\theta) \ ,$$
$$= \theta \int_{\underline{\theta}}^{\theta} \frac{t\left(\tilde{\theta}\right)}{\tilde{\theta}^2}\mathrm{d}\tilde{\theta} \ ,$$

from which (29) is derived. Integrate it to obtain (25). ■

Proof of Corollary 3. Take a given θ and $\varepsilon > 0$. We have $\dot{t}(\theta_t) = \frac{t(\theta+\varepsilon)-t(\theta)}{\varepsilon}$ for some $\theta_t \in [\theta, \theta+\varepsilon]$, from which we can write $\varepsilon = \frac{t(\theta+\varepsilon)-t(\theta)}{\dot{t}(\theta_t)}$ and $\dot{T}_F(\theta_T) = \frac{T_F(\theta+\varepsilon)-T_F(\theta)}{\varepsilon}$ for some $\theta_T \in [\theta, \theta+\varepsilon]$, that is

$$\dot{T}_F(\theta_T) = \frac{T_F(\theta+\varepsilon) - T_F(\theta)}{t(\theta+\varepsilon) - t(\theta)}\dot{t}(\theta_t) = \frac{y(R)}{\theta}\dot{t}(\theta_T) \ .$$

The second equality is (31). Take the limit $\varepsilon \to 0$, which implies $(\theta_t - \theta_T) \to 0$ and the above is

$$\frac{dT_F(\theta)}{dt(\theta)} = \frac{y(R)}{\theta} \ ,$$

with a slight abuse of notation. The LHS is decreasing in θ and therefore in t, which shows that the slope of the curve in the north-east quadrant is decreasing, establishing the Corollary. ■

Proof of Proposition 5. Impose $\delta = 0$ in the proof of Proposition 4. This eliminates the constraint given by the information disadvantage of the government. (A9) becomes:

$$-\beta(\theta) - (1+\lambda) f(\theta) = 0 \ . \tag{A24}$$

Because of the possibility that the optimum is a corner solution, (A8) must be replaced by

$$\begin{aligned} r = 0 \quad \text{if} \quad & \frac{\partial\mathcal{L}}{\partial r(\theta)} = (1+\lambda) f(\theta)\frac{y(R)}{\theta} - \rho f(\theta) < 0 \ ; \\ & \frac{\partial\mathcal{L}}{\partial r(\theta)} = (1+\lambda) f(\theta)\frac{y(R)}{\theta} - \rho f(\theta) = 0 \quad \text{for} \quad r \in (0, r_{\max}) \ ; \\ r = r_{\max} \quad \text{if} \quad & \frac{\partial\mathcal{L}}{\partial r(\theta)} = (1+\lambda) f(\theta)\frac{y(R)}{\theta} - \rho f(\theta) > 0 \ . \end{aligned}$$

Using (A24), this implies that

$$\begin{aligned} r &= 0 \text{ if } \theta < \frac{1+\lambda}{\rho} y(R) \ ; \\ r &= r_{\max} \text{ if } \theta > \frac{1+\lambda}{\rho} y(R) \ . \end{aligned}$$

That is, the solution is "bang-bang". (A10) becomes

$$-t(\theta) f(\theta) - \beta(\theta) t(\theta) - \tau(\theta) = 0 \ ,$$
$$\tau(\theta) = \lambda f(\theta) t(\theta) \ ,$$

as before. (A18) in turn becomes:

$$z_{\frac{\lambda}{1+\lambda}}(t(\theta)) = \Delta - \frac{y(R)}{\theta} \ ,$$

and the last university active is given by the solution in θ of

$$a_{\min} = \Delta - \frac{y(R)}{\theta} \ .$$

The multipliers ρ is still given by (A19), and substituting the value of ρ into the above, the Proposition is obtained. ■

Proof of Proposition 6. From the Lagrangean $\mathcal{L}$, with β and τ the multipliers for constraints (38) and (39):

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial t} =& \frac{1}{\phi(\Phi^{-1}(t))} U(y(r) - p - a - g) \phi\left(\Phi^{-1}(t)\right) - U(y(r) - \Delta - g) \\ &- \beta\left(\frac{y(r)}{\theta} - p\right) - \frac{\tau}{\phi(\Phi^{-1}(t))} \ ; \qquad (A25) \\ \frac{\partial \mathcal{L}}{\partial p} =& - \int_{a_{\min}}^{\Phi^{-1}(t)} U'(y(r) - p - a - g) \phi(a) \, \mathrm{d}a + \beta t - \tau \ ; \qquad (A26) \\ \frac{\partial \mathcal{L}}{\partial g} =& - \int_{a_{\min}}^{\Phi^{-1}(t)} U'(y(r) - p - a - g) \phi(a) \, \mathrm{d}a - (1-t) U'(y(r) - g) + \beta \ ; \qquad (A27) \\ \frac{\partial \mathcal{L}}{\partial r} =& \left(\int_{a_{\min}}^{\Phi^{-1}(t)} U'(y(r) - p - a - g) \phi(a) \, \mathrm{d}a + (1-t) U'(y(r) - g) \right. \\ &\left. + \frac{r+t}{\theta} \right) y'(r) - \beta \frac{y(r)}{\theta} \ . \qquad (A28)\end{aligned}$$

Note that, by the indifference condition (39), $U(y(r) - p - a - g) = U(y(r) - g)$, and so (A25) simplifies to

$$\frac{\partial \mathcal{L}}{\partial t} = -\beta\left(\frac{y(r)}{\theta} - p\right) + \frac{\tau}{\phi(\Phi^{-1}(t))} \ .$$

Define $u(t)$ to be the expression at the denominator of (40), so the integral in (A26)-(A28) is $u(t)\,t$. Setting the first order conditions to 0, and rearrange to get:

$$\beta = u(t)\,t + (1-t)\,U'(\cdot) \ , \tag{A29}$$
$$\tau = \beta t - u(t)\,t = t\,(1-t)\,\big(U'(\cdot) - u(t)\big) \ ,$$
$$\left(\Phi^{-1}(t) + \frac{(1-t)\,(U'(\cdot) - u(t))}{u(t)\,t + (1-t)\,U'(\cdot)} \frac{t}{\phi\,(\Phi^{-1}(t))}\right) = \Delta - \frac{y(r)}{\theta} \ .$$

This corresponds to (34): notice that the left hand side is the function z with

$$k = \frac{(1-t)\,(U'(\cdot) - u(t))}{u(t)\,t + (1-t)\,U'(\cdot)} \ .$$

Recall that if λ is the shadow cost of public funds, then the function z_k is $z_{\frac{\lambda}{1+\lambda}}$. Solve $\frac{(1-t)(U'(\cdot)-u(t))}{u(t)t+(1-t)U'(\cdot)} = \frac{\lambda}{1+\lambda}$ to obtain:

$$\lambda = \frac{U'(\cdot) - u(t)}{u(t)} \ .$$

This establishes the Proposition. ■