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ALICE THROUGH THE LOOKING GLASS: STRATEGIC MONETARY AND FISCAL POLICY INTERACTION IN A LIQUIDITY TRAP

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Abstract

The recent experience with low inflation has reopened interest in the liquidity trap; which occurs when the nominal interest rate reaches its zero lower bound. To reduce the real interest rate, and to stimulate the economy, the modern literature highlights the role of high inflationary expectations. Using the Dixit-Lambertini (2003) framework of strategic policy interaction, we find that the optimal institutional response to the possibility of a liquidity trap has two main components. First, an optimal inflation target given to the Central Bank. Second, the Treasury, who retains control over fiscal policy and acts as leader, is given optimal output and inflation targets. This keeps inflationary expectations sufficiently high and achieves the optimal rational expectations pre-commitment solution. Simulations show that this arrangement is (1) optimal even when the Treasury has no inflation target but follow's the optimal output target and (2) 'near optimal' even when the Treasury follows its own agenda through a suboptimal output target but is willing to follow an optimal inflation target. Finally, if monetary policy is delegated to an independent central bank with an optimal inflation target, but the Treasury retains discretion over fiscal policy, then the outcome can be a very poor one.

Keywords: liquidity trap, strategic monetary-fiscal interaction, optimal Taylor rules.

JEL Classification: E63, E52, E58, E61.

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1. Introduction

In its classical form, the *liquidity trap*, a term coined by Keynes (1936), is a situation where an economy is caught up in a deflation and the nominal interest rate has been driven down to zero (the so called 'zero bound'). The source of a liquidity trap, in most circumstances, is a sharp fall in aggregate demand; see Keynes (1936), Bernanke (2002). Interest in the liquidity trap has revived in recent years due, in no small measure, to the experience of Japan since 1990. Woodford (2005, p29) discusses the near miss of the US economy from a liquidity trap in the summer of 2003. The era of successful delegation of monetary policy to independent central banks with low inflation targets[1] opens up the possibility that sufficiently large negative demand shocks might push an economy into a liquidity trap with huge associated welfare consequences[2]. For example, Blanchard (2003) raises the possibility of the following scenario: high USA deficits force a devaluation of the dollar and, hence, an appreciation of the Euro. With already low interest rates in the Euro area, and with fiscal policy circumscribed by the stability and growth pact, the Euro area may well be forced into a liquidity trap.

Traditional monetary policy loses its effectiveness because nominal interest rates can be reduced no further in order to boost the interest sensitive components of aggregate demand. Hence, reliance must be placed on other, possibly more expensive, policies. Keynes (1936), in the first policy prescription for a liquidity trap, suggested the use of fiscal policy, which works through the multiplier effect to boost output and employment.

However, the recent literature has largely focussed on monetary policy and the role of expectations. Krugman (1998, 1999) reformulated the liquidity trap as a situation where an economy requires a negative real interest rate. With nominal interest rates bound below by zero, the only way in which a negative real interest rate can be achieved is to have an expectation of positive inflation[3]. This, in turn, creates a need for a credible commitment to the future level of actual inflation because after the economy has escaped from the liquidity trap it is in the interest of all parties to reduce inflation. A forward looking private sector will anticipate this and expect low future inflation. But then the real interest rate remains positive, keeping the economy in a liquidity trap.

The subsequent literature on the liquidity trap has also considered exchange rate poli-

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[1] Average inflation rates in successive decades from the 1950's on to the current decade show a declining trend; see Table 1 in Svensson (2003).

[2] High unemployment is an obvious fallout of a liquidity trap. An increase in the real value of private debt has further adverse consequences particularly for the financial sector. An increase in the real public debt creates a difficult problem for the government to increase taxes to balance its books on the one hand but risk getting mired deeper into a recession on the other.

[3] The real interest rate is given by $r = i - \pi^e$ where i is the nominal interest rate and π^e is expected inflation. In a liquidity trap, $i = 0$ and typically $\pi^e < 0$, hence $r > 0$. To expand economic activity, the government needs to lower r; one possible solution is to generate positive inflationary expectations.

cies such as currency depreciation, integral stabilization, a carry tax on currency, open market operations in long term bonds, price level targets, and money growth rate pegs. The surveys in Svensson (2003) and Blinder (2000) consider these policies in detail, however, these policies have important limitations[4,5].

Eggertsson (2006a, 2006b) recommends abandonment of an independent central bank and a return to discretionary policy by a unitary monetary-fiscal authority. A debt financed fiscal expansion during a liquidity trap results, via the government budget constraint, in higher expectations of future inflation. Eggertsson shows that this solution is superior to either monetary policy alone or uncoordinated monetary and fiscal policy. However, as Eggertsson shows, even optimal discretion is inferior to the fully optimal rational expectations solution with commitment. Moreover, abandoning delegation of monetary policy to an independent central bank with a narrow mandate, in favor of a return to discretion, appears to be a retrograde step.[6]

In this paper, we find that the optimal institutional response to the possibility of a liquidity trap has two main components. First, an optimal inflation target given to the operationally independent Central Bank. Second, the Treasury, who retains control over fiscal policy and acts as leader, is given optimal output and inflation targets. This keeps inflationary expectations sufficiently high and achieves the optimal rational expectations pre-commitment solution. Simulations show that this arrangement is (1) optimal even when the Treasury has no inflation target but follows the optimal output target and (2) 'near optimal' even when the Treasury follows its own agenda through a suboptimal output target but is willing to follow an optimal inflation target. Finally, if monetary policy is delegated to an independent central bank with an optimal inflation target, but the Treasury

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[4]Variants of the devaluation approach can be found in McCallum (2000) and Svensson (2003). There are several potential problems with the devaluation option. First, calibrated models show that the magnitude of the devaluation required to get out of the liquidity trap might be too high. Second, using the uncovered interest rate parity condition when the domestic interest rate is zero, the expected appreciation of the home currency is fully locked-in by the foreign interest rate. Third, current devaluation will generate expectations of future appreciation of currency when the economy moves out of the liquidity trap, generating counter flows that frustrate attempts to devalue. Fourth, devaluations may bring about competitive devaluations or retaliations in the form of other barriers to trade.

[5]In a liquidity trap, zero nominal interest rates make bonds and money perfect substitutes. Hence, it might be difficult to engineer a price level increase. Furthermore, increases in money supply, suggested, for instance, in Clouse et al. (2003) and in Orphanides and Wieland (2000), for a long enough period that exceeds the duration of the liquidity trap, creates problems of credibility. While short term interest rates might be zero, long term interest rates might be strictly positive (this has been true of Japan during its deflationary experience). Hence, several authors such as Bernanke (2002) and Auerbach and Obstfeld (2005) have suggested open market operations in long term bonds. However, moving the long run yield curve on securities is confounded by the presence of the risk premium term whose behavior in a liquidity trap is not well known. A carry tax on money, suggested by Buiter and Panigirtzoglu (2003), works in theory but substantial practical problems of implementation are likely.

[6]Central bank independence has other benefits. For example, it shields monetary policy from political interference and allows the delegation of policy to the most competent experts etc.

retains discretion over fiscal policy, then the outcome can be a very poor one.

1.1. The Japanese experience: fiscal policy

The Japanese experience with the liquidity trap since the 1990's is now well documented; for instance, Posen (1998). Here we emphasize three points[7].

J1 *Potency of fiscal policy in a liquidity trap*: The large budget deficits in Japan over the 1990's, with debt reaching a peak of about 140 percent of GDP, have sometimes formed the basis for the conclusion that Japanese fiscal policy was not effective in the liquidity trap. However, this view is at variance with the empirical evidence; for instance Posen (1998), Kuttner and Posen (2001), Iwamura et al. (2005) and Ball (2005). Kuttner and Posen show that tax revenues fell through the deflation of the 1990's. Worried by the special demographic problems faced by Japan, the budget deficits largely funded existing expenditure commitments. It follows that the stabilization component of Japanese fiscal policy in the 1990's was quite weak. Kuttner and Posen show that when the fiscal stimulus was strong, such as in the fiscal package of 1995, it worked in stimulating GDP. On the whole, however, expansionary fiscal policies were largely offset by other contractionary components of fiscal policy such as an increase in the national consumption tax from 3 percent to 5 percent, increase in the contribution rates to social security and the repeal of temporary tax cuts. It is useful to cite more fully from Posen (1998). He writes "The reality of Japanese fiscal policy in the 1990's is less mysterious and ultimately, more disappointing. The actual amount injected into the economy by the Japanese government- through either public spending or tax reductions- was about a third of the total amount announced. This limited quantity of total fiscal stimulus was disbursed in inefficiently sized and inefficiently administered doses with the exception of the 1995 stimulus package. The package did result in solid growth in 1996, demonstrating that fiscal policy does work when it is tried....On net, the Japanese fiscal stance in the 1990's was barely expansionary." The empirical results of Iwamura et. al (2005) and Ball (2005) lend strong support to the finding of Kuttner and Posen. Eggertsson (2006b) calculates a deficit spending multiplier of 3.76, which is much higher than previously thought.

J2 *Lack of appropriate institutions and incentives for policy makers*: The inability of the Japanese Treasury to follow through with an appropriate fiscal stimulus suggests the possibility of inadequate institutional foundations to deal with the liquidity trap. For instance, the Japanese fiscal and monetary authorities did not have any explicit

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[7]There are clearly other relevant issues in the Japanese experience such as the ineffectiveness of monetary policy that we do not touch on here; see Blinder (2000).

output/ inflation targets prior to the onset of the liquidity trap that (1) might have created incentives for an appropriate response, and (2) altered expectations, particularly inflationary expectations, that could have dampened the liquidity trap.

J3 *Lack of coordination between the fiscal and monetary authorities*: Competing policy authorities might disagree on the appropriate response to a liquidity trap, possibly worsening the situation. For instance, the empirical results of Iwamura et. al (2005) indicate lack of coordination between the monetary and fiscal policy authorities. They write "It also suggests that policy coordination between the government and the Bank of Japan did not work well during this period, in the sense that the government deviated from the Ricardian rule towards fiscal tightening while the BOJ (Bank of Japan) adopted a zero interest rate policy and quantitative easing." Eggertsson (2006b) calculates a deficit spending multiplier of exactly zero, for this scenario.

1.2. About our paper

To motivate our paper we ask the following three questions.

Q1 *Is there strategic policy interaction between the various policy makers?*
Models of strategic monetary and fiscal policy interaction have recently been given a new impetus by the work of Dixit and Lambertini (2003) and Lambertini and Rovelli (2003) (which, however, do not consider a liquidity trap). Issues of strategic interaction between policy makers assume even greater significance during times of extreme recessions as the Japanese experience (J3 above) indicates. However, issues of strategic policy interaction between monetary and fiscal authorities are completely ignored by the theoretical work on the liquidity trap. Typically the only policy considered is monetary policy and so issues of strategic interaction do not arise[8]. On the other hand, when multiple policies are considered, their strategic interaction is not considered[9].

Q2 *Can liquidity traps occur in equilibrium?*
One strand of the literature considers policies that could mitigate the effects of liquidity traps. The other strand prescribes policies that would prevent the economy

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[8]Examples are Krugman (1998), Eggerston and Woodford (2003), Nishiyama (2003), Clouse et al. (2003), Buiter- Panigirtzoglou (2003), and Auerbach and Obstfeld (2005). Ball (2005) considers fiscal policy alone.

[9]Examples include (1) monetary and fiscal policy in Benhabib, Schmitt-Grohe and Uribe (2002), Iwamura et al. (2005) and (2) monetary and exchange rate policy in Orphanides and Wieland (2000), McCallum (2000) and Svensson (2003). Bernanke (2002) considers both monetary and fiscal policy but there is no theoretical analysis.

from ever falling into a liquidity trap[10]. In general, the optimal policy for our model allows the economy to fall into a liquidity trap with some probability. Thus our model is in the economics tradition that stresses limiting economic bads (e.g. externalities) to their 'optimal level', rather than complete elimination[11].

Q3 *Is the perspective ex-ante or ex-post?*

The literature typically asks either one of the following two questions. (1) What is the optimal institutional design (assignments of targets and instruments to the various policy makers) when there is the possibility of a liquidity trap in the future? (2) Given that the economy is in a liquidity trap, what actions can be taken to eliminate the liquidity trap[12]. There is considerable disagreement on both questions, particularly the latter. An ex-ante perspective allows one to plan optimally for a problem before it arises, while an ex-post approach is mainly concerned with damage control. Furthermore, the announcements of policy makers during a liquidity trap (an ex-post perspective) might carry little credibility for the public. Hence, ideally one would like to look at the appropriate institutional design prior to the onset of a liquidity trap (an ex-ante perspective).

We describe our paper as follows. We would answer yes to the first two questions and 'ex-ante perspective' to the third. We consider strategic interaction between monetary and fiscal authorities in a simple aggregate supply - aggregate demand model similar to the one in Dixit and Lambertini (2003) and Lambertini and Rovelli (2003) but extended to allow for a liquidity trap and the effect of inflationary expectations in the aggregate supply curve. There is some possibility that the economy will fall into a liquidity trap in some state of the world in the future. Our central concern is to identify optimal institutional arrangements[13] from an ex-ante perspective. Figure 1.1 summarizes our paper in relation to the existing literature.

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[10]In the first group are Krugman (1998), Eggertsson and Woodford (2003), Orphanides and Wieland (2000), McCallum (2000), and Svensson (2003). In the second group are Benhabib, Schmitt-Grohe and Uribe (2002), Nishiyama (2003), Clouse et al. (2003), Buiter- Panigirtzoglou (2003), and Auerbach and Obstfeld (2005).

[11]A dental analogy might be appropriate here. Tooth decay can be prevented by extracting all the child's teeth. But, normally, the optimal policy is not to extract; tooth decay then occurs with some probability.

[12]In the first group are Krugman (1998), Eggertsson and Woodford (2003), Benhabib, Schmitt-Grohe and Uribe (2002), Shin-Ichi (2003), Clouse et al. (2003), Buiter- Panigirtzoglou (2003). In the second group are papers by Ball (2005), Auerbach and Obstfeld (2005). Finally there are papers that touch on both ex-ante and ex-post issues, for instance, Orphanides and Wieland (2000), McCallum (2000), Bernanke (2002), Svensson (2003).

[13]By optimality or near optimality we mean regimes that help us to attain or get very close to the optimal rational expectations (or pre-commitment) solution.

Paper	Timing	Strategic interaction between Monetary and Fiscal authorities	Policy mix	Can a liquidity trap occur in equilibrium?	Suggested Policy	Does the suggested institutional solution achieve the Precommitment solution?
Benhabib et. al (2002)	Ex-ante	No	Monetary and Fiscal	No	Inflation sensitive budget deficits, switch from interest rate rule to money growth rate peg in a liquidity trap.	No
Auerbach and Obstfeld (2005)	Ex-post	No	Monetary	N.A.	Open market operations in long term government bonds.	No
Buiter-Panigirtzoglou (2003)	Ex-Ante	No	Monetary	No	Carry tax on currency.	No
Clouse et al. (2003)	Ex-post	No	Monetary	N.A.	Open market purchases of Treasury bills.	No
Nishiyama (2003)	Ex-ante	No	Monetary	No	Inflation target for the central bank.	No
Eggertsson and Woodford (2003)	Ex-ante	No	Monetary	Yes	Commitment to adjust nominal interest rates to achieve a time varying price level target.	No
Krugman (1998)	Ex-ante	No	Monetary	Yes	Inflation target	No
Bernanke (2002)	Ex-ante and ex-post	No	Primarily monetary but also fiscal.	Yes	Buffer zone for the inflation rate, financial stability, ceilings on yields of longer maturity Treasury debt, tax cuts.	No
Orphanides-Wieland (2000); McCallum (2000)	Ex-ante and ex-post	No	Monetary and exchange rate.	Yes	Expansion of monetary base, currency depreciation, moving exchange rate target.	No
Svensson (2003)	Ex-ante, ex-post	No	Monetary, exchange rate.	Yes	Price level target, currency depreciation and temporary peg, exit strategy	No
Eggertsson (2006a,b)	Ex-ante	No	Monetary and Fiscal	Yes	Discretion by a unitary monetary-fiscal authority	No
Dixit-Lambertini (2000, 2003)	Ex-ante	Yes	Monetary and Fiscal	N.A.	Fiscal authority is Stackelberg leader, among others.	Yes
Lambertini-Rovelli (2003)	Ex-ante	Yes	Monetary and Fiscal	N.A.	Fiscal authority is Stackelberg leader.	N.A.
Dhami-al-Nowaihi (2006)	Ex-ante	Yes	Monetary and Fiscal	Yes	Inflation target for the central bank and a kind of Taylor rule for the Treasury that offers it a menu of choices.	Yes

Figure 1.1: **Relation of our paper with the existing literature**

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1.3. Some results and intuition

As pointed out above, Krugman identified the solution to a liquidity trap as creating high enough inflationary expectations. However, under discretion, promises of high inflation will not be believed. This is because outside a liquidity trap the correct value for the real interest rate can be achieved more cheaply with zero inflation. Therefore, if the economy turns out not to be liquidity trapped, the Treasury has an incentive to renege on its promise of high inflation. A rational forward looking private sector will anticipate this. The result is low inflation expectations, keeping the real interest rate too high in a liquidity trap. Notice that unlike the standard analysis conducted in the absence of a liquidity trap the discretionary outcome can be suboptimal relative to the precommitment outcome because it creates *too little* inflation (Eggertsson (2006a,b) calls this the deflation bias).

We suggest an institutional solution, the *optimal delegation regime*, that achieves the optimal rational expectations precommitment solution for all parameter values in our model. The *optimal delegation regime* seems broadly in line with the successful arrangements introduced by the British Chancellor Gordon Brown in 1997 and seems entirely natural. This regime has three components. First, the Treasury acts as Stackelberg leader and the Central Bank as follower. Second, an inflation target is given to a Central Bank who has exclusive control over monetary policy. Outside a liquidity trap, where monetary policy is effective, the Treasury would rather not use the relatively more costly fiscal stabilization policy, leaving the Central Bank to perform the stabilization function. Because the Central Bank is operationally independent and its sole objective is achieving monetary stability, this type of delegation provides a commitment to the necessary inflation level when the economy is not in a liquidity trap. Our third component is to give the Treasury, who retains control of fiscal policy, something like a *Taylor rule*, which penalizes deviations of output from an output target and inflation from the inflation target. This gives the Treasury the correct incentive to undertake the appropriate (but costly) fiscal stimulus in a liquidity trap where monetary policy is ineffective. Consequently, inflation expectations are at the right level to produce the correct value for the real interest rate in a liquidity trap. For a variety of reasons, e.g. electoral concerns, the output target of the Treasury may differ from the optimal target. In this case, we find that even if the Treasury's output target is substantially different from the optimal output target, this *suboptimal delegation regime* achieves *close* to the optimal solution and is much better than discretion.

Regime	Treasury follows optimal Output target	Treasury follows optimal inflation target	Treasury cares about inflation	Treasury follows personal output agenda	Outcome
Optimal Delegation	Yes	Yes	Yes	No	Precommitment Solution
Suboptimal Delegation	No	Yes	Yes	Yes	Near optimal
Output nutter	Yes	No	No	No	Precommitment Solution
Reckless nutter	Yes	No	No	No	Precommitment Solution
Output nutter	No	No	Yes	Yes	Much worse than discretion
Reckless nutter	No	No	No	Yes	Much worse than discretion

Figure 1.2: **Outcomes under various regimes**

While it may appear reasonable to assign an inflation target to the Central Bank, it may be asked why should the Treasury have an inflation target, as well as an output target? To answer this question, we define two further regimes: the *output nutter regime*, where the Treasury has an output target but not an inflation target; and the *reckless output nutter regime* where the Treasury has an output target but does not have an inflation target and does not care about the cost of fiscal policy. It turns out that so long as the Treasury follows the optimal output target, then delegation achieves the optimal solution even in the regimes of the output nutter and the reckless output nutter. However, in the latter two cases, the delegation regime is not robust; in the sense that if the output target of the Treasury is different from the optimal target, then performance is poor and can be much worse than under discretion. Hence, giving the Treasury an inflation target (as well as an output target), while not essential for optimality, adds to the robustness of the policy. In particular the hybrid regime where monetary policy is delegated to an independent central bank with an optimal inflation target, while the Treasury retains discretion over fiscal policy, can perform badly and much worse than had the Treasury retained discretion over both monetary and fiscal policy. We summarize these results in Figure 1.2. In each regime the central bank follows its optimally assigned inflation target.

Furthermore, the *optimal delegation regime* achieves the optimal mix between monetary and fiscal policy as we now explain. Theoretically, society could give a sufficiently high inflation target to the Central Bank which in turn generates sufficiently high inflation expectations so that the nominal interest rate never hits its zero floor. While this policy

would always avoid the liquidity trap, it is not optimal because inflation is costly. Analogously it is not optimal to give the Treasury too high an output target because if a liquidity trap occurs, it would use the costly fiscal policy excessively. The optimal solution then is to have a mix of both i.e. some inflation outside a liquidity trap and some dependence on costly fiscal policy in a liquidity trap.

The first best is achieved if one could remove the distortions that cause the liquidity trap. The second best obtains with the optimal rational expectations commitment solution. The third best is achieved with various institutional design features introduced into policy making. The fourth best obtains under discretion. It is well known that, in the absence of a liquidity trap, 'optimal institution design', such as Walsh contracts, can achieve the second best. Our suggested institutional design achieves the second best in the presence of a liquidity trap.

1.4. Optimal Control versus Game Theory[14]

To simplify the dynamic *game-theoretic* analysis we follow the tradition, established in the time-inconsistency literature[15], of abstracting from *structural* dynamic issues, notably, capital formation, the term structure of interest rates, exchange rate policy and the financing of the stabilization component of fiscal policy. Concentrating on the aggregate demand consequences of investment expenditure, but abstracting from its contribution to growth, is standard in models of the business cycle, and is a feature of all the models of the liquidity trap (as far as we know).

Eggertsson and Woodford (2003), in a structurally dynamic model of monetary policy with a financial sector and a zero lower bound on interest rates, show that the short-run interest rate (which is the instrument of policy) determines all other interest rates and exchange rates. As they clearly explain, open market operations only work to the extent that they enhance the credibility of policy. Thus, and in common with many models, we take the short-run interest rate as directly affecting aggregate demand and we abstract from open economy aspects.

We offer two arguments that mitigate not explicitly modelling the government budget constraint. First, we assign a higher welfare loss to the use of fiscal policy relative to monetary policy. The cost of using fiscal policy could include deadweight losses, costs of servicing debt and a risk premium for default. Second, in all equilibria of our model, fiscal policy is not used for stabilization purposes outside a liquidity trap. In a calibrated model of Japan, Ball (2005) shows that the combination of higher output, higher tax revenues and higher inflation outside the liquidity trap is more than adequate to finance the extra

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[14]We take this title from Blanchard (2006, p581).
[15]See, for example, Romer (2006, chapter 10) and Walsh (2003, chapter 8).

fiscal spending during the liquidity trap.

Nevertheless, we incorporate an element of structural dynamics resulting from persistence in demand shocks (section 5). We believe that our model thus reproduces the essentials of the problems associated with a liquidity trap: persistence, credibility and monetary-fiscal coordination, in a clear and simple way.

1.5. Schematic outline

The model is formulated in Section 2. Section 3 derives the two benchmark solutions: the *optimal rational expectations precommitment solution* and the *discretionary solution*. Section 4 derives the *optimal delegation solution*. Section 5 demonstrates the robustness of the model by allowing for the full set of parameters, persistence of demand shocks and several alternative formulations of the Treasury's objectives. Section 6 discusses the relation of our paper to the literature. Section 7 concludes with a brief summary. Proofs are relegated to appendices.

2. Model

In this section we describe the most parsimonious version of the model. In Section 5 below, we demonstrate the robustness of the results of this model with respect to the full set of parameters, persistent demand shocks, a general probability distribution over the two states of nature, and further considerations about the Treasury's objectives.

2.1. Aggregate Demand and Aggregate Supply

We use an aggregate demand and supply framework that is similar to Ball (2005), Dixit and Lambertini (2003) and Lambertini and Rovelli (2003). The aggregate demand and supply equations are given by, respectively

$$AD : y = f - (i - \pi^e) + \epsilon \tag{2.1}$$

$$AS : y = \pi - \pi^e \tag{2.2}$$

where y is the deviation of output from the natural rate and f captures fiscal policy[16]. For example, $f > 0$ could denote a fiscal deficit (either debt financed or money financed[17])

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[16]To be more precise, f is the *stabilization* component of fiscal policy (which varies over the business cycle). *Total* fiscal policy is then $F = f_0 + f$, where f_0 is *fixed* and chosen so that $F^e = f_0 + f^e = 0$, so that the government budget constraint on average.

[17]In principal these alternative modes of finance need not be equivalent. However, in the context of a liquidity trap, Ball (2005) shows that there are no long run differences arising from these alternative modes of finance.

while $f < 0$, a fiscal surplus. But f could also denote a temporary balanced budget reallocation of taxes and subsidies that has a net expansionary effect; for instance Dixit and Lambertini (2000). $i \geq 0$ is the nominal interest rate, π is the rate of inflation, π^e is expected inflation[18] and ϵ is a demand shock[19]. The instruments of policy are i and f. The demand shock ϵ takes two values, $a, -a$, with equal probability, where $a > 0$, hence

$$E\left[\epsilon\right] = 0, \; Var\left[\epsilon\right] = a^2. \tag{2.3}$$

The aggregate demand equation reflects the fact that demand is increasing in the fiscal impulse, f, and decreasing in the real interest rate; it is also affected by demand shocks. The aggregate supply equation shows that deviations of output from the natural rate are caused by unexpected movements in the rate of inflation. Note the absence of parameters in (2.1),(2.2). This is because our conclusions do not qualitatively depend on the values of such parameters (see Section 5). So we have suppressed them to improve readability.

Equating aggregate demand and supply we get from (2.1) and (2.2), our reduced form equations for output and inflation.

$$y = f - i + \pi^e + \epsilon \tag{2.4}$$

$$\pi = f - i + 2\pi^e + \epsilon \tag{2.5}$$

Hence, fiscal policy, monetary policy and inflation expectations (in the spirit of New Keynesian models) have an affect on output (and so also on unemployment) and inflation.

2.2. Microfoundations

Our model is inspired by the microfounded dynamic model of monopolistic competition and staggered price setting in Dixit and Lambertini (2000, 2003). Our structural model in (2.1),(2.2) (or its variant with the full set of parameters given in (5.1), (5.2) below) is

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[18]The following formulation might appear even more plausible

$$AD : y_t = f_t - \left(i_t - \pi^e_{t+1}\right) + \epsilon_t$$

$$AS : y_t = \pi_t - \pi^e_t$$

where $\pi^e_t = E_{t-1}\pi_t$ and $\pi^e_{t+1} = E_t\pi_{t+1}$. However, in our model, the private sector has to make its decision before the realization of the demand shock ϵ_t. Hence, in the aggregate demand curve, it has to forecast π^e_{t+1} at time $t - 1$. But $E_{t-1}\pi^e_{t+1} = E_{t-1}\left(E_t\pi_{t+1}\right) = E_{t-1}\left(\pi_{t+1}\right) = E_{t-1}\left(\pi_t\right) = \pi^e_t$. While this is true in our model, it is not true more generally.

[19]The modern literature on the liquidity trap stresses demand shocks as major contributory factors. We could also consider supply shocks. The main difference is as follows. A sufficiently *negative* demand shock will push the economy into a liquidity trap. On the other hand, a sufficiently *positive* supply shock will also create a liquidity trap. In either case, the real interest rate fails to drop sufficiently to match demand with supply. Hence our framework can be easily extended to incorporate supply, as well as demand, shocks.

similar to Dixit and Lambertini[20]. In the Dixit and Lambertini framework, unexpected movements in inflation have real effects because prices are staggered. Alternatively, a range of 'rational inattention' theories currently compete as potential explanations for the presence of the unexpected inflation term in (2.2). For instance, see Sims (2003).[21]

2.3. Notation

We shall write a variable with a subscript (sometimes a superscript) '+', for example, y_+, to denote the realization of that variable in the (good) state of the world, $\epsilon = a$. Analogously, to denote the realization of the same variable in the (bad) state of the world, $\epsilon = -a$, we use a subscript (sometimes a superscript) '−', for example, y_-.

2.4. Social Preferences

Society's preferences over output and inflation are given by the social welfare function,

$$U_S = -\frac{1}{2}(y - y_S)^2 - \frac{1}{2}\pi^2 - f^2. \tag{2.6}$$

The first term shows that departures of output from its desired level, y_S (note that y_S is the difference between desired output and the natural rate), are costly. We assume that

$$y_S \geq 0 \tag{2.7}$$

This captures the fact that the natural level of output is socially suboptimal (unless $y_S = 0$)[22].

The second term in (2.6) indicates that inflation reduces social welfare. The third term captures the fact that the exercise of fiscal policy is more costly than that of monetary

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[20]However, our model has the following differences from Dixit-Lambertini. (1) We normalize the natural rate of output to zero, hence, the additive shock ϵ (in (2.1) or in (2.4)) can also be interpreted as a shock to the natural rate of output. (2) Our model has the New Keynesian feature that expected inflation, π^e, also affects actual inflation, π. (3) Our stochastic structure allows persistence (see section 5 below). While there is no persistence in Dixit-Lambertini, they allow all parameters to be stochastic, hence, considering the possibility of non-additive shocks. (4) In our model a fiscal impulse acts on the demand side, creating greater output and inflation. However, in Dixit-Lambertini fiscal policy works on the supply side and takes the form of a subsidy to imperfectly competitive firms that increases output but reduces prices.

[21]Most dynamic structural models used in the analysis of a liquidity trap are forward looking New Keynesian models. Gertler (2003), Mankiw (2002) note dissatisfaction with this model in terms of its inability to explain persistence in the data. Recent work, for instance, Rudd and Whelan (2006), casts doubt even on the hybrid variant proposed by Gali and Gertler (1999). Of course, similar criticisms apply to the version of our model microfounded along the lines of Dixit and Lambertini (2003). Thus, all current macroeconomic models lack satisfactory microfoundations.

[22]The microfoundations for this in Dixit and Lambertini (2000, 2003) rest on the presence of monopolistic competition. Monopoly power in the product market reduces output below the efficient level, hence, giving policy makers an incentive to raise output. There are also a large number of other well known reasons for (2.7) but the ultimate cause, argue Alesina and Tabellini (1987), is the absence of non-distortionary taxes. For if they were available then other market failures could be corrected.

policy[23]. We model this as imposing a strictly positive cost of fiscal policy, f^2, but no cost of using the monetary policy[24]. The cost of using fiscal policy could include deadweight losses, as in Dixit and Lambertini (2003), costs of servicing debt and a risk premium for default.

From (2.6) we see that the first best obtains when $\pi = 0$, $f = 0$, and $y = y_S$. However, from (2.1) and (2.2), it follows that this cannot be an outcome of a rational expectations equilibrium (unless $y_S = 0$).

For expositional clarity we omit parameters in (2.6), but see Section 5. On the microfoundations of such a social welfare function, see Dixit and Lambertini (2000, 2003), Rotemberg and Woodford (1999).

2.4.1. Treasury and Social Preferences

We will assume for now that society can, if it desires, delegate policy to a "Treasury" that fully internalizes its objective function given in (2.6). So we will use society and Treasury interchangeably here. Other assumptions are considered in Section 5 below.

2.5. Sequence of Moves

At the first stage the economy designs its institutions, which assign powers of policy-making decisions to one or two independent policy makers. This is followed by the formation of inflationary expectations, π^e, and the signing of nominal wage contracts in anticipation of future inflation. Next, the demand shock, ϵ, is realized. In light of the actual realization of the shock, the relevant policy makers then decide on the optimal values of the policy variables, f and i. We shall also derive the optimal rational expectations solution (precommitment benchmark) in which the last stage is conducted up-front i.e. the (state contingent) policy variables f and i are announced to the economy prior to the resolution of demand uncertainty.

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[23]Fiscal policy is typically more cumbersome to alter, on account of the cost of changing it (balanced budget requirements, lobby groups etc.). Indeed the 'monetary policy committee' in the UK or the Fed in the USA meet on a regular basis to make decisions on the interest rate while changes to the tax rates are much less frequent.

[24]Strictly speaking, for our qualitative results to hold, we only require that fiscal policy be relatively more expensive than the (possibly strictly positive) cost of using monetary policy. Normalizing the cost of using monetary policy to zero, however, ensures greater tractability and transparency of the results.

3. The Precommitment and Discretionary Solutions

3.1. The Precommitment Regime (The optimal rational expectations solution)

In this section we calculate the globally optimal solution in the class of all rational expectations solutions[25]. The global optimality of the precommitment solution serves as a useful benchmark. The sequence of moves is described below.

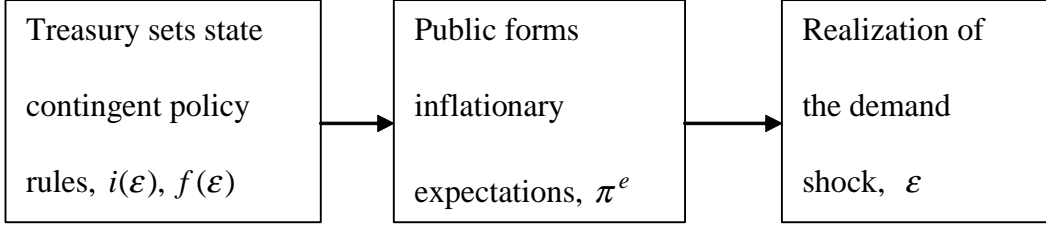| Treasury sets state contingent policy rules, $i(\varepsilon)$, $f(\varepsilon)$ | → | Public forms inflationary expectations, π^e | → | Realization of the demand shock, ε |

Figure 3.1: **Sequence of moves for the precommitment regime**

The solution method is to find state contingent rules for the policy variables, $i(\epsilon)$, $f(\epsilon)$, i.e., (i_-, f_-), (i_+, f_+), that maximize the expected value of the social welfare (2.6) under the constraints (2.4), (2.5) and the rational expectations condition $\pi^e = E[\pi]$, i.e.

$$\pi^e = \frac{1}{2}\pi_- + \frac{1}{2}\pi_+ \tag{3.1}$$

The results are summarized in Proposition 1. Superscript 'e' denotes expected value.

Proposition 1 : *The optimal state-contingent rational expectations precommitment solution is given by*

$\epsilon = -\mathbf{a} < \mathbf{0}$	$\epsilon = \mathbf{a} > \mathbf{0}$	$\epsilon^e = \mathbf{0}$
$i_- = 0$	$i_+ = \frac{6}{5}a$	$i^e = \frac{3}{5}a$
$f_- = \frac{2}{5}a$	$f_+ = 0$	$f^e = \frac{1}{5}a$
$y_- = -\frac{1}{5}a$	$y_+ = \frac{1}{5}a$	$y^e = 0$
$\pi_- = \frac{1}{5}a$	$\pi_+ = \frac{3}{5}a$	$\pi^e = \frac{2}{5}a$
$i_- - \pi^e = -\frac{2}{5}a$	$i_+ - \pi^e = \frac{4}{5}a$	$i^e - \pi^e = \frac{1}{5}a$

The expected utility in the precommitment regime is given by $E\left[U_S^{Opt}\right] = -\frac{1}{5}a^2 - \frac{1}{2}y_S^2$. Furthermore, $\left(\frac{\partial U_S}{\partial i}\right)_{Opt} < 0$ when $\epsilon = -a$ and $\left(\frac{\partial U_S}{\partial i}\right)_{Opt} = 0$ when $\epsilon = a$. ∎

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[25]Strictly speaking, this is a second best solution. The first best obtains if the imperfections responsible for the liquidity trap are removed. It is variously referred to as the 'precommitment solution', the 'optimal rational expectations solution', the 'second best solution' or simply the 'optimal solution'.

From Proposition 1 note that $\left(\frac{\partial U_S}{\partial i}\right)_{Opt} < 0$ when $\epsilon = -a$. Hence, the economy is always liquidity trapped when $\epsilon = -a$. In this case, monetary policy is not effective, $i_- = 0$. Hence, the government must commit to using expensive fiscal policy, $f_- = \frac{2}{5}a$, in order to 'lean against the wind'. By contrast, when $\epsilon = a$, monetary policy is effective, $i_+ = \frac{6}{5}a$, and the government has no need for the expensive fiscal instrument, $f_+ = 0$[26].

Also note that output is below the natural rate (which is normalized to zero) in the liquidity trap ($\epsilon = -a$) but above it otherwise ($\epsilon = a$). On average, it equals the natural rate (recall that y measures the deviation of output from the natural rate). Inflation is positive in both states of the world. The real interest rate is negative[27] in the liquidity trap but positive otherwise and on average.

Recalling that $Var[\epsilon] = a^2$, on average, ceteris paribus, inflation, interest rates and the fiscal instrument of the government will display greater variability in economies where demand shocks have a greater variance and precommitment is possible. Furthermore, the magnitude of policy instruments employed in the two states of the world, $f_- = \frac{2}{5}a$ and $i_+ = \frac{6}{5}a$, are increasing in the size of the shock. This is not surprising as each of these policies fulfills a stabilization role and a larger shock elicits a greater effort in "leaning against the wind".

The solution is independent of y_S, society's desired output relative to the natural rate. As in time consistency models in the absence of the liquidity trap, this occurs because, even if society has a high y_S, the precommitment technology allows it to counter expectations of ex-post surprise inflation (designed to push output towards the high target).

The magnitude of social welfare in this regime depends negatively on the variance of shocks hitting the economy, a^2, and also on the output target of society, y_S.

Finally, note that the values of i_+, i_-, f_+, f_- of the instruments are optimal *ex-ante*. However, after the realization of the shock, $\epsilon = -a$ or $\epsilon = a$, the *ex-post* optimal values of i, f will, in general, be different from these. Thus, for successful implementation, this optimal rational expectations solution needs a precommitment technology. We discuss this in Section 4 below. Next we turn to the second regime in the paper: Discretion.

3.2. Discretionary Regime

In this case, the monetary instrument, i, and the fiscal instrument, f, are both assigned to the Treasury. We calculate the time consistent discretionary policy. The sequence of moves is described below.

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[26]Recall that f refers only to the stabilization component of fiscal policy, hence, $f_+ = 0$ is consistent with a strictly positive level of government expenditure on other items such as redistribution etc.

[27]We conjecture that the combination of rigid wages-prices and a flexible nominal interest rate has the effect that the real interest rate, $i - \pi^e$, overshoots so as to equilibrate the economy.
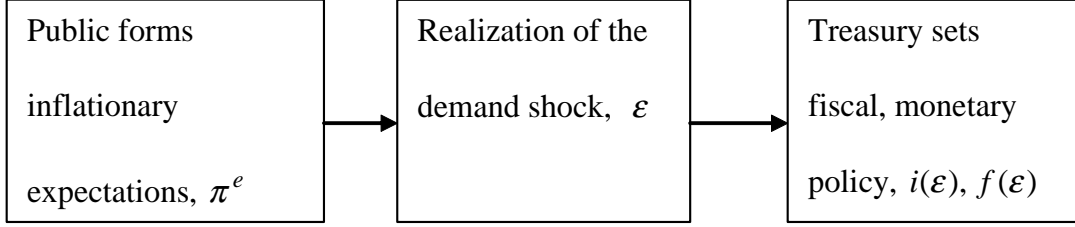
Figure 3.2: **Sequence of moves when Treasury controls i, f.**

To find the discretionary solution, first find state-contingent values of the policy variables $i_- (\pi^e)$, $f_- (\pi^e)$ and $i_+ (\pi^e)$, $f_+ (\pi^e)$ that maximize social welfare (2.6) under the constraints (2.1), (2.2) and conditional on given π^e, ϵ. This allows the computation of the state-contingent inflation rates $\pi_- (\pi^e)$ and $\pi_+ (\pi^e)$. Then one needs to find the fixed-point π^e by solving $\pi^e = E[\pi]$:

$$\pi^e = \frac{1}{2}\pi_- (\pi^e) + \frac{1}{2}\pi_+ (\pi^e) \tag{3.2}$$

Finally, substitute the value for π^e back into the state-contingent policy variables $i_- (\pi^e)$, $f_- (\pi^e)$ and $i_+ (\pi^e)$, $f_+ (\pi^e)$ to find the solution under discretion.

Depending on the parameter values, a liquidity trap may or may not arise. Proposition 2 below summarizes the results when a liquidity trap, which is the focus of this paper, arises[28].

Proposition 2 : For $\frac{1}{2}a \leq y_S < a$, the economy is liquidity trapped for $\epsilon = -a < 0$ but not liquidity trapped for $\epsilon = a > 0$. The solution under discretion is given by

$\epsilon = -a < 0$	$\epsilon = a > 0$	$\epsilon^e = 0$
$i_- = 0$	$i_+ = 4y_S - 2a$	$i^e = 2y_S - a$
$f_- = 2(a - y_S) > 0$	$f_+ = 0$	$f^e = (a - y_S) > 0$
$y_- = y_S - a < 0$	$y_+ = a - y_S > 0$	$y^e = 0$
$\pi_- = 4y_S - 3a$	$\pi_+ = 2y_S - a$	$\pi^e = 3y_S - 2a$
$i_- - \pi^e = 2a - 3y_S$	$i_+ - \pi^e = y_S > 0$	$i^e - \pi^e = a - y_S > 0$

and the expected social welfare is given by $E\left[U_S^{Disc}\right] = 12ay_S - 8y_S^2 - 5a^2$.

For stabilization purposes, the costly fiscal policy is used only in a liquidity trap when the monetary policy looses effectiveness. As in the precommitment solution, deviations of output from the natural rate are zero on average i.e. $y^e = 0$. The following corollary compares expected social welfare under *Precommitment* with that under *Discretion*.

---

[28]The full set of results under discretion is given in Appendix-B.

Corollary 1 : For $\frac{1}{2}a \leq y_S < a$, $E\left[U_S^{Opt}\right] - E\left[U_S^{Disc}\right] = \frac{3}{10}\left(5y_S - 4a\right)^2 \geq 0$.

As one would expect, the presence of a liquidity trap does not alter the ranking between the Precommitment and the Discretion regimes, from a social welfare point of view.

3.3. Alice through the looking glass

Krugman (1998) observed that 'applying conventional modelling to liquidity trap conditions produces unconventional conclusions and policy recommendations'. To which he added (1999) 'The whole subject of the liquidity trap has a sort of Alice-through-the-looking-glass quality'. And indeed, our model exhibits these features, as we will now see.

3.3.1. Precommitment can have higher inflation than Discretionary

In the traditional time inconsistency literature, in the absence of a liquidity trap, the optimal level of average inflation is zero (given the welfare function (2.6)) while under discretion it is positive (unless $y_S = 0$, in which case it is also zero); as is well known. The reason is that under discretion, agents perceive (correctly) that the government has an ex-post incentive to create surprise inflation, while under precommitment ex-post surprise inflation is institutionally ruled out.

When a liquidity trap occurs with a positive probability this changes dramatically. From Proposition 1 we see that the optimal level of average inflation under precommitment now is positive ($\pi^e = \frac{2a}{5}$), rather than zero. Under discretion π^e depends on y_S. For $y_S = \frac{1}{2}a$, Proposition 2 gives a negative average expected inflation rate ($\pi^e = -\frac{1}{2}a$), rather than a positive one. Eggertsson (2006a, 2006b) calls this the deflation bias.

The intuitive explanation is as follows. Under precommitment, it is optimal to have positive inflation on average ($\pi^e = \frac{2a}{5}$), despite its cost, to be able to deliver negative real interest rates ($i_- - \pi^e = -\frac{2a}{5}$) in the bad state of the world ($\epsilon = -a$). However, this optimal policy is time inconsistent. If ex-post, the economy is in the good state ($\epsilon = a$) then the optimal real interest rate is positive ($i_+ - \pi^e = \frac{4a}{5}$) which can be achieved more cheaply with zero inflation. Hence, the policy maker has the incentive to renege on its commitment to positive inflation. The rational private sector will perceive this and expect low future inflation. This destroys the credibility of the announcement of high inflation, unless a commitment technology is available.

3.3.2. Higher output targets are a good thing

In the standard textbook model in the absence of a liquidity trap, a higher value of desired output relative to the natural rate, $y_S > 0$, is bad because it leads to high inflation and no gain in output ($y^e = 0$). The reverse occurs with a liquidity trap, $y_S > 0$ is now good!

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The intuition is that a higher y_S increases inflationary expectations (see Proposition 2) which, by reducing the real interest rate in a liquidity trap, reduces the need for using the expensive fiscal instrument.

If society has a high enough output target (and the Treasury follows it) then, in the discretionary regime, ex-post, a liquidity trap will not arise. However, this outcome might require using the costly fiscal instrument excessively, which could be suboptimal. In section 4, below, we show this to be precisely the case.

4. Institutions and Delegation

In the delegation regime considered in this section, society gives the Central Bank the mandate of achieving an inflation target π_B. The monetary instrument, which is the nominal interest rate, i, is assigned to the Central Bank whose objective is to attain the inflation target π_B. We formalize this by assigning the following objective function to the Central Bank:

$$U_B = -\frac{1}{2}(\pi - \pi_B)^2 \tag{4.1}$$

The fiscal instrument, f, is controlled by the Treasury whose objective function is similar to that of society (2.6) but with, possibly, different inflation and output targets:

$$U_T = -\frac{1}{2}(y - y_T)^2 - \frac{1}{2}(\pi - \pi_T)^2 - f^2 \tag{4.2}$$

where y_T, π_T are the output and inflation targets respectively of the Treasury. It is important to bear in mind the difference between the socially desirable output level, y_S, and the Treasury's output target, y_T. The optimal value, y_T^*, of y_T, i.e., the value of y_T that maximizes expected social welfare, might be very different from y_S. In fact, our simulations show that y_T^* is well below y_S. Thus a fiscal authority should be 'conservative' in the sense that it should aim for a lower output target than that desired by society, as in Rogoff (1985). See, for example, Table 1, below.

4.1. The Optimal Delegation Regime

Under optimal delegation, the game has five stages, shown in Figure 4.1.
The Treasury acts as Stackelberg leader with an output target, y_T, and an inflation target π_T. The Central Bank is the follower with an inflation target π_B. In this subsection we consider the case $\pi_T = \pi_B$ (section 5, below, allows $\pi_T \neq \pi_B$). The Central Bank sets monetary policy taking the fiscal policy, set by the Treasury, as given. The Treasury sets fiscal policy, taking into account the anticipated response of the Central Bank. We solve the game backwards. First we obtain the Central Bank's reaction function $i = i(\pi_B, \pi^e, f, \epsilon)$ by maximizing U_B. Second, we find the Treasury's reaction function $f = f(y_T, \pi_B, \pi^e, \epsilon)$ by
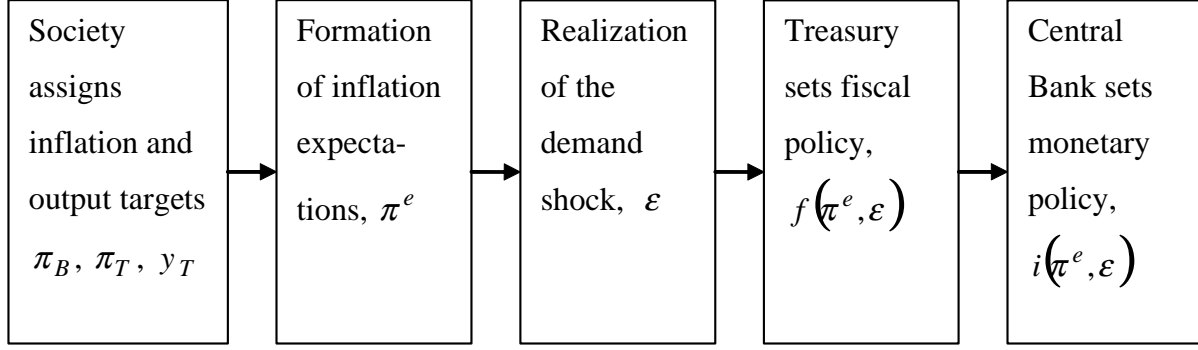
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Figure 4.1: **Sequence of moves in the optimal delegation regime**

maximizing U_T. This allows us to derive output and inflation as functions of $y_T, \pi_B, \pi^e, \epsilon$. Third, we determine π^e, assuming rational expectations on the part of the private sector. Fourth, we find the expected social welfare, EU_S, as a function of y_T, π_B, which we maximize to find the optimal values of y_T, π_B which are denoted by y_T^*, π_B^*. We assume that the Treasury and Central Bank adopt the optimal inflation target, π_B^*, and that the Treasury fully complies with the optimal output target, y_T^*. Section 5, below, explores the possibility that the Treasury might not care for inflation and/ or be unwilling to follow the optimal output target, y_T^*, because it has its own output target, y_T. For ease of reference, these concepts are summarized in the following definition.

Definition 1 : *y_S is the output level preferred by society (0 is the inflation level preferred by society, see (2.6)). y_T and π_T are output and inflation targets for the Treasury. π_B is the inflation target for the Central Bank. y_T^* and π_B^* are the values of y_T and π_B that maximize expected social welfare, EU_S, subject to the constraints of the model, where U_S is given by (2.6). In section 5, below, we allow the Treasury to adopt an output target, y_T, different from y_T^*, consistent with its own agenda.*

Proposition 3 : *Assume that monetary policy is delegated to an independent central bank with inflation target $\pi_B^* = \frac{3}{5}a$. Fiscal policy is retained by the Treasury with output target $y_T^* = \frac{1}{5}a$ and acts as Stackelberg leader. Then the optimal rational expectations (precommitment) solution (see Proposition 1) is achieved. Society's expected utility in the optimal delegation regime is given by $E\left[U_S^{OD}\right] = -\frac{1}{5}a^2 - \frac{1}{2}y_S^2$. The economy is liquidity trapped only under adverse demand shocks. Inflation and output targets are achieved in the good state but not in the bad state.*[29]

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[29]As stressed by Eggertsson and Woodford (2003), failure to meet the inflation target in the liquidity trap does not signify failure of policy. A similar remark can be made with respect to the output target.

So why does the optimal delegation regime perform so well? The inflation target given to the Central Bank provides a commitment to the necessary inflation level when the economy is not in a liquidity trap. This affects the (ex-ante) inflation expectations which also apply to the (ex-post) liquidity trap ensuring the correct value for the real interest rate in a liquidity trap. Furthermore, inflationary expectations are also influenced correctly by the output and inflation targets given to the Treasury that provide it with the incentive to use the appropriate level of fiscal policy in a liquidity trap. Such an institutional regime achieves the optimal balance between fiscal and monetary policy by neither having to rely too much on costly inflation outside the liquidity trap nor relying too much on costly fiscal policy in a liquidity trap.

5. The general model

How are our results altered when we introduce the full set of parameters in the model and allow for persistence in the demand shocks with a general probability distribution? What if the Treasury has its own agenda, perhaps on account of electoral concerns or other political economy considerations such as lobbying or interest groups? These issues are considered in this Section. We demonstrate that the results of our model are robust to the following five extensions.

E1. Introduction of the full set of parameters.

E2. Persistent demand shocks.

E3. General probability distribution over the two states of the world.

E4. The Treasury might follow an output target, y_T, different from the optimal output target, y_T^*. Recall that y_T^* will, in general, be different from the output level, y_S, most desired by society.

E5. The Treasury and the Central Bank can have distinct inflation targets i.e. $\pi_T \neq \pi_B$.

5.1. A note on output and inflation targets

5.1.1. Inflation Targets

There are two main cases. The inflation targets of the Treasury and Central Bank either coincide (i.e. $\pi_T = \pi_B$), or differ (i.e. $\pi_T \neq \pi_B$). In Section 4 we restricted attention to the case $\pi_T = \pi_B$. However, in Subsection 5.5, both cases i.e. $\pi_T = \pi_B$ and $\pi_T \neq \pi_B$ are considered. We show that the optimal delegation regime works equally well in each of these two cases and achieves the optimal rational expectations precommitment solution. Whilst

this does not have implications for the optimality of our suggested delegation regime we find the case $\pi_T = \pi_B$ more natural and easier to interpret. Furthermore, we show in Subsection 5.8 that the optimal rational expectations solution can also be achieved if the central bank *alone* has an inflation target while the Treasury simply follows the optimal output target given to it by society.

5.1.2. Output Targets

The Treasury is an arm of the government. If the natural rate of output is socially suboptimal, say on account of monopolistic competition, then the government may have an incentive to use fiscal instruments to increase output beyond its natural rate, at least temporarily and a rational private sector will foresee this. The problem of assigning output targets is compounded by the difficulty of measuring deviation of output from its natural rate (compared with the lesser difficulty of measuring deviation of inflation from its target value) and by the fact that output stability is only one (though important) consideration for government and voters (by contrast, monetary stability can be made the sole objective of the central bank). Hence, it is important to consider the case where the Treasury pursues its own agenda and sticks to its preferred value of the output target, y_T, rather than follow the optimal output target, y_T^*, that society assigns to it. Although in section 4 we restricted attention to the case $y_T = y_T^*$, Section 5.5 below considers both cases i.e. $y_T = y_T^*$ and $y_T \neq y_T^*$.

We proceed as follows. First, we derive the optimal rational expectations precommitment solution in this more general setting (Proposition 4). In general, this solution is time-inconsistent and, therefore, requires a commitment technology. We then consider the *optimal delegation regime* (first considered in Section 4.1, above). If the Treasury follows the optimal output target (i.e. $y_T = y_T^*$), then the optimal delegation regime achieves the precommitment solution for all values of the parameters (Proposition 5). If, however, the Treasury cannot be given the optimal output target, and has its own agenda (i.e. $y_T \neq y_T^*$), then Section 5.7, below, shows that a 'near optimal' solution can still be achieved. What if the Treasury is not given an inflation target or does not care about inflation at all, but is willing to adopt the socially optimal output target? Section 5.8, below, shows that the optimal precommitment solution can still be achieved.

5.2. Description of the general model

The model is described by the following basic equations:

$$\text{Aggregate Demand} : y = \varphi f - \lambda \left(i - \pi^e \right) + \epsilon \qquad (5.1)$$

$$\text{Aggregate Supply} : y = \mu \left(\pi - \pi^e \right) \qquad (5.2)$$

$$\text{Society's Objective}: \quad U_S = -\frac{1}{2}\alpha\pi^2 - \frac{1}{2}\beta(y - y_S)^2 - \frac{1}{2}\gamma f^2 \tag{5.3}$$

The parameters α, β, γ, φ, λ, μ are all strictly positive. φ and λ are a measure of the effectiveness of fiscal and monetary policy respectively in influencing aggregate demand and μ indicates the strength of inflation surprises in influencing aggregate supply. Finally, α, β, γ are the relative weights given to the various terms in the objective function. The state contingent values of the demand shock, ϵ, are:

$$\text{Bad State: } \epsilon_- = \rho x - (1-p)s \tag{5.4}$$

$$\text{Good State: } \epsilon_+ = \rho x + ps \tag{5.5}$$

where $0 < p < 1$, $s > 0$ and $0 \leq \rho < 1$. The variable x represents the previous period's shock and so ρ is a measure of the persistence in the shock. The second component in (5.4), (5.5) shows the innovation terms. With probability p the shock takes the value ϵ_- and with probability $1 - p$ it takes a value ϵ_+. Hence $E[\epsilon] = p\epsilon_- + (1-p)\epsilon_+ = \rho x$ and so in the absence of the persistence term, $E[\epsilon] = 0$ as in the model presented in Section 2.[30] Thus, if an economy is close to a liquidity trap, a negative shock can push the economy into it. Because of persistence, it may take the economy several periods to get out of the liquidity trap.

5.3. Sequence of moves

The sequence of moves under the regimes of precommitment, discretion and the optimal delegation are as in Figures 3.1, 3.2, 4.1 respectively, except that in any period, the realization of ϵ depends on the value of the of the shock in the previous period, x. The main effect of this is to cause the optimal inflation (and output) targets to be state dependent. This is in line with the results of Eggertsson and Woodford (2003) who, however, consider only monetary policy.

5.4. Optimal Solution

The optimal rational expectations precommitment solution, the analogue of Proposition 1, is described below in Proposition 4. The intuition behind the results is similar to that behind Proposition 1 except that the magnitude of demand shocks in the past influence the state of the economy in the current period and so one needs to distinguish between 3 cases. Our main focus is on Case (b) where the economy is liquidity trapped in the bad state. The proof is derived analogously to that of Proposition 1 and, so, is omitted.

---

[30]In more standard but less convenient notation, $x_t = \rho x_{t-1} + z_t$, where $z_t = -(1-p)s$ with probability p and $z_t = ps$ with probability $1 - p$.

Proposition 4 : **(a)** If $x < -ps\frac{(\alpha+\beta\mu^2)(\alpha\varphi^2+\gamma\lambda^2)}{\alpha\rho(\gamma\mu^2+\varphi^2(\alpha+\beta\mu^2))}$ then the economy is liquidity trapped in both states and the commitment solution is given by $i_- = i_+ = 0$,

$$f_- = \varphi\left(\frac{(\alpha+\beta\mu^2)s(1-p)}{\gamma\mu^2+\varphi^2(\alpha+\beta\mu^2)} - \frac{\alpha\rho x}{\alpha\varphi^2+\gamma\lambda^2}\right) > 0$$

$$f_+ = -\varphi\left(\frac{(\alpha+\beta\mu^2)sp}{\gamma\mu^2+\varphi^2(\alpha+\beta\mu^2)} + \frac{\alpha\rho x}{\alpha\varphi^2+\gamma\lambda^2}\right) > 0$$

(b) If $-ps\frac{(\alpha+\beta\mu^2)(\alpha\varphi^2+\gamma\lambda^2)}{\alpha\rho(\gamma\mu^2+\varphi^2(\alpha+\beta\mu^2))} \leq x < (1-p)\frac{s}{\rho}$ then the economy is liquidity trapped in the bad state only and the commitment solution is given by $i_- = f_+ = 0$,

$$f_- = \frac{\alpha\varphi(\alpha+\mu^2\beta)((1-p)s-\rho x)}{(\alpha+\beta\mu^2)(\alpha\varphi^2+\gamma\lambda^2 p)+\alpha\gamma\mu^2(1-p)} > 0$$

$$i_+ = \frac{(\gamma\lambda^2+\alpha\varphi^2)(\alpha+\beta\mu^2)sp+(\beta\varphi^2\mu^2+\gamma\mu^2+\alpha\varphi^2)\alpha\rho x}{\lambda((\alpha+\beta\mu^2)(\alpha\varphi^2+\gamma\lambda^2 p)+\alpha\gamma\mu^2(1-p))} \geq 0$$

(c) If $x \geq (1-p)\frac{s}{\rho}$ then the economy is liquidity trapped in neither state and the commitment solution is given by $f_- = f_+ = 0$,

$$i_- = \frac{\rho x - (1-p)s}{\lambda} \geq 0$$

$$i_+ = \frac{\rho x + ps}{\lambda} > i_- \geq 0$$

Proposition 4 illustrates the evolution of the economy over time. Suppose that the economy is liquidity trapped in period t. How does it get out of a liquidity trap? Proposition 4 (b), (c) gives the conditions required on how big the shocks must be in period t so that in period $t+1$ the economy is not liquidity trapped in at least in one state of the world[31].

5.5. The Optimal Delegation Regime

In this section we examine the possibility of achieving the optimal precommitment solution through appropriate institutional design. Here we extend the *optimal delegation* framework of Section 4.1 (details are suppressed to avoid repetition) to incorporate the five extensions E1 through E5 stated at the beginning of Section 5. The Treasury's objective function is given by

$$U_T = -\frac{1}{2}\alpha(\pi - \pi_T)^2 - \frac{1}{2}\beta(y-y_T)^2 - \frac{1}{2}\gamma f^2 \tag{5.6}$$

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[31] This might not be a bad descriptor of the actual occurrence of a liquidity trap given the deep reservations expressed about the efficacy of most macroeconomic policies; see Blinder (2000) for an excellent survey.

Note that the parameters α, β, γ are the same as in society's welfare function given in (5.3). Denote the optimal inflation target of the Central Bank by π_B^* and the optimal output and inflation targets of the Treasury by y_T^* and π_T^* respectively. Proposition 5, below, states the results under optimal delegation. As in Proposition 4, the magnitude of the demand shock in the previous period gives rise to three subcases, although we are primarily interested in Case (b). The proof is similar to that of Proposition 3, so it is omitted.

Proposition 5 : **(a)** *Under the condition of Proposition 4(a), give the Central Bank any inflation target, π_B^*, that satisfies $\pi_B^* > \gamma \left(\frac{\mu s p}{\beta \varphi^2 \mu^2 + \alpha \varphi^2 + \gamma \mu^2} + \frac{\lambda \rho(-x)}{\alpha \varphi^2 + \gamma \lambda^2} \right)$ and give the Treasury any output and inflation target pair (y_T, π_T) that satisfy*

$$y_T(\pi_T) = k - \frac{\alpha}{\beta \mu} \pi_T \tag{5.7}$$

where $k = \alpha \frac{(\lambda + \mu) \gamma \rho(-x)}{\beta \mu \left(\alpha \varphi^2 + \lambda^2 \gamma \right)}$. Then the solution under optimal delegation is the same as under precommitment, and given by Proposition 4(a).

(b) *Under the conditions of Proposition 4(b), give the Central Bank the inflation target*

$$\pi_B^* = \frac{\gamma \left(\beta \mu^2 \lambda + \alpha(\lambda + \mu) \right) \left(s(1-p) - \rho x \right) p}{\left(\alpha + \mu^2 \beta \right) \left(\alpha \varphi^2 + \gamma \lambda^2 p \right) + \gamma \mu^2 \alpha (1 - p)} > 0 \tag{5.8}$$

and give the Treasury any output and inflation target pair (y_T, π_T) that satisfies

$$y_T(\pi_T) = K - \frac{\alpha}{\beta \mu} \pi_T \tag{5.9}$$

where $K = \frac{\alpha \gamma p}{\mu \beta} \frac{(\lambda + \mu) \left(\alpha + \mu^2 \beta \right) \left(\epsilon(1-p) - \rho x \right)}{\left(\alpha + \beta \mu^2 \right) \left(\alpha \varphi^2 + \gamma \lambda^2 p \right) + \gamma \mu^2 \alpha (1-p)}$. Then the solution under optimal delegation is the same as under precommitment and is given by Proposition 4(b). Furthermore, $\pi_+ = \pi_B^*$.
(c) *Under the condition of Proposition 4(c), give the Central Bank the inflation target $\pi_B^* = 0$. Then, for any output and inflation target pair (y_T, π_T) for the Treasury, the solution under optimal delegation is the same as under commitment and is given by Proposition 4(c). Furthermore, $\pi_+ = \pi_- = \pi_B^* = 0$.*

The intuition behind the optimality of this delegation regime is as in Section 4.1 above. If the economy is not liquidity trapped in any state of the world we are in the standard textbook case where delegation to an independent Central Bank achieves the precommitment solution. Proposition 5(c) deals with this case. Our main case of interest, however, is when the economy is liquidity trapped in the bad state only; this is stated in Proposition 5(b). Here, the inflation target of the Central Bank is uniquely determined while the Treasury's target pair y_T, π_T can be chosen from a *menu of contracts* that satisfy (5.9).
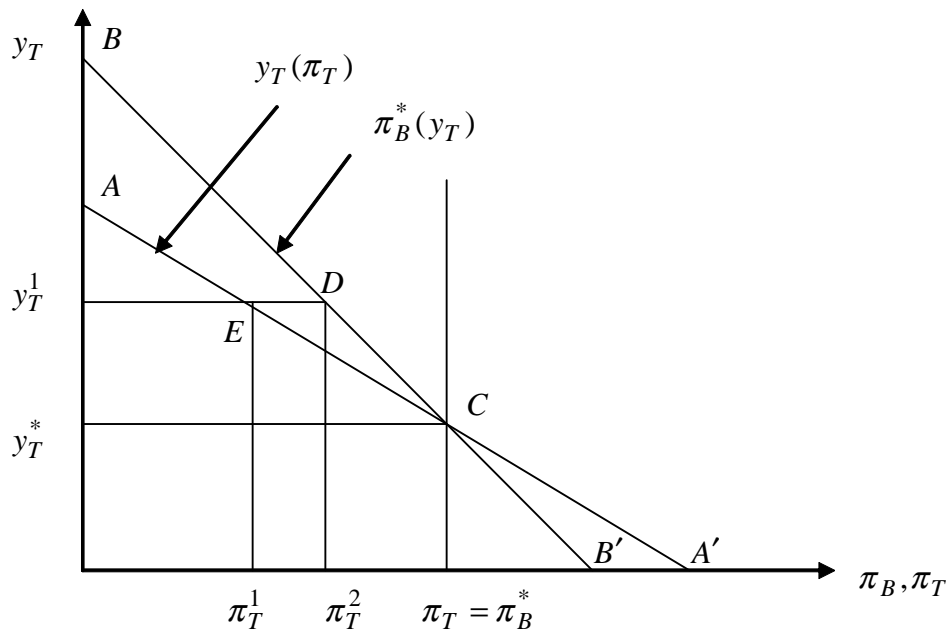
Figure 5.1: **Output and inflation targets under the optimal and suboptimal delegation regimes.**

To explain the indeterminacy of y_T and π_T, note that the Treasury has *two* targets, y_T and π_T, but just *one* instrument, f. Hence, the best it can hope for is hit just one of these targets or, more generally, a linear combination of them. Maximizing society's expected welfare yields the optimal linear combination of y_T and π_T. This is given by (5.7) in the case of Proposition 5(a) and (5.9) in the case of Proposition 5(b). The negative slope signifies that a high output bias is needed to compensate a low inflation target for the Treasury.

What if the inflation targets of the Treasury and the Central Bank are identical? Corollary 2 describes the results when the economy is liquidity trapped in the bad state.

Corollary 2 : *Under the conditions of Proposition 4(b), if $\pi_T = \pi_B^*$, then the optimal output target for the Treasury is*

$$y_T^* = \alpha\gamma p\mu^2 \frac{s\,(1-p) - \rho x}{(\alpha + \beta\mu^2)\,(\alpha\varphi^2 + \gamma\lambda^2 p) + \gamma\mu^2\alpha\,(1-p)} > 0. \qquad (5.10)$$

and the Treasury attains this target in the good state i.e. $y_+ = y_T^*$.

In Figure 5.1, the downward sloping line AA' is a graph of $y_T\,(\pi_T)$ defined in (5.7) or (5.9). The vertical line positioned at π_B^* reflects the inflation target for the central bank given in 5.8. Ignore the downward sloping line BB' for the moment.

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Proposition 5 then shows how the *optimal delegation regime* can achieve the *optimal precommitment solution* in the following two cases,

1. *The Treasury and the Central Bank can be given the same inflation target*
   Figure 5.1 shows that the optimal delegation solution is given by point C, where $\pi_B = \pi_T = \pi_B^*$ (given in (5.8)) and $y_T = y_T^*$ (given in (5.10)).

2. *The Treasury and the Central Bank are given distinct inflation targets*
   Figure 5.1 shows one possible solution. The Central Bank is given the uniquely determined inflation target i.e. $\pi_B = \pi_B^*$ (see (5.8)). The Treasury is given any output, inflation target along the line AA, for instance, corresponding to point E i.e. $(y_T, \pi_T) = (y_T^1, \pi_T^1)$.

5.6. Discretion

The results under discretion when we extend the basic model to extensions E1-E5 are similar to those stated in Proposition 2. The full set of results are given in Appendix-B; the method of proof is identical to that of Proposition 2, and is omitted. Denote by EU^{Disc}, the expected welfare level under discretion; we make use of it in Section 5.7 below.

5.7. Suboptimal Delegation: Treasury follows its own agenda ($y_T \neq y_T^*$)

We now consider the case where the Treasury does not adopt the optimal output target (see discussion in subsection 5.1.2 above); we call this regime '*suboptimal delegation*'. The output target y_T now represents the Treasury's own agenda and it refuses to accept the optimal output target, y_T^*. The objective function of the Treasury is given in (5.6). For pedagogical simplicity, we stick here to the more natural case where the inflation targets of the Treasury and the Central Bank are equal i.e. $\pi_B = \pi_T$.

Let $\pi_B^*(y_T)$ maximize society's expected welfare, given the output target, y_T, of the Treasury. For the general case in Section 5 the expression for $y_T(\pi_B^*)$ is too unwieldy, but for the simple model presented in Section 2 it is given by

$$y_T(\pi_B^*) = \frac{7}{4}a - \frac{11}{2}\pi_B^*$$

In Figure 5.1, the line BB' is a sketch of (the inverse of) $\pi_B^*(y_T)$. Any point on the line BB' gives the optimal inflation target for the Central Bank, $\pi_B^*(y_T)$, conditional on the Treasury's private, but not necessarily optimal, output target, y_T. which is steeper than the schedule $y_T(\pi_T)$ plotted as line AA'.

Suppose that the Treasury's output target is given by $y_T = y_T^1$. Then, at one possible suboptimal equilibrium $\pi_B = \pi_T = \pi_T^2$ while $y_T = y_T^1$ i.e. the Treasury's equilibrium targets are shown by the point D. Because point D is off the line AA, which plots the

optimal menu of contracts for the Treasury, how well does the suboptimal delegation regime fare, relative to the optimal precommitment solution? Simulations, below, show that the performance of the *suboptimal delegation regime* is 'near optimal' and much better than *discretion.*

Denote the expected social welfare level under suboptimal delegation by EU_S^{SD}. The state contingent values of the policy variables in this case run into several pages, so we confine ourselves to reporting a representative sample of simulation results. Towards this end we define the following variables.

$q = EU_S^{Opt}/EU_S^{SD}$ is the expected welfare level under the optimal solution relative to the expected welfare under suboptimal delegation. Note that $0 < q \leq 1$ and $q = 1$ when $y_T = y_T^*$ (see Proposition 5).

$\omega = EU_S^{Disc}/EU_S^{SD}$ is the ratio of the expected welfare under discretion relative to that under suboptimal delegation. Note that $\omega > 0$ because the numerator and denominator are both negative.

$Q = \frac{EU_S^{SD} - EU_S^{Disc}}{EU_S^{Opt} - EU_S^{Disc}}$ is the ratio of the welfare loss under suboptimal delegation relative to that under the optimal solution when each is expressed as a difference from the expected welfare level under discretion. Hence, relative to the discretionary solution as a benchmark, this is the proportional loss to society in moving from the optimal solution to the suboptimal delegation solution. Note that $Q = 1$ for $y_T = y_T^*$ (see Proposition 5).

$o = y_S/y_T^*$ is the output target of society relative to the optimal output target given to the Treasury.

$t = y_T/y_T^*$ is the output target of the Treasury relative to the optimal output target given to it.

The feasible set of parameters belongs to a ten dimensional set. We give below simulations for a representative sample of parameters in Tables 1, 2 below. Tables 4 through 6 in Appendix-C give further simulation results to support our assertions. To simplify results, we focus on cases where the output targets of the Treasury and society coincide i.e. $y_T = y_S$ (and so $o = t$) and the inflation targets of the Treasury and the Central Bank also coincide i.e. $\pi_T = \pi_B$.

The main results of the simulations can be summarized as follows.

Proposition 6 : *Even if the private agenda of the Treasury, i.e. y_T, is substantially different from the optimal output target, y_T^*, the expected welfare level under the suboptimal delegation solution is very close to the optimal precommitment solution i.e. q is very close to 1. Suppose that we start with the minimal institutional framework of the discretionary regime. Then moving to the institutional regime of suboptimal delegation recovers, for all parameter values that we have investigated, a very large percentage of the benefit that might accrue if one could move to the optimal solution i.e. Q is typically very close to 1.*

α	φ	λ	β	γ	μ	q	ω	Q	$o = t$	$\pi_B^*(y_T)$
$\frac{10}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	0.9999	1.007	0.9844	404.4	$0.045s$
$\frac{10}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{10}{10}$	$\frac{10}{10}$	0.9936	1.039	0.8589	6.422	$0.146s$
$\frac{10}{10}$	$\frac{10}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	0.9999	1.270	0.9995	602.6	$0.178s$
$\frac{10}{10}$	$\frac{10}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	$\frac{10}{10}$	$\frac{10}{10}$	0.9921	1.451	0.9828	8.6	$0.216s$
$\frac{1}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	0.9989	1.067	0.9844	44.42	$0.406s$
$\frac{10}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	1.0000	3.931	1.0000	8.006	$2.497s$
$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{10}{10}$	$\frac{10}{10}$	0.9999	1.006	0.9851	2.048	$0.585\,s$
$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{10}{10}$	$\frac{10}{10}$	0.9873	1.039	0.7521	2.84	$0.371s$

In Table 1, the economy is liquidity trapped in the bad state only. Even if the output target of the Treasury is up to 602.2 times higher than the optimal output target (i.e. $o = t = 602.2$), q and Q are still very close to 1. Tables 4-6, in the appendix, confirm these results for other parameter values. In Table 2, below, constructed under the conditions of Proposition 5(a), the economy is liquidity trapped in both states and there is a very high level of persistence in the demand shocks.

Table 2: $p = \frac{1}{50}, y_T = y_S = ps, x = -(1-p)s, \rho = \frac{9}{10}$

p	α	φ	λ	β	γ	μ	ω	π_B^*
$\frac{1}{50}$	$\frac{10}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	1.0397	$0.17445s$
$\frac{1}{50}$	$\frac{10}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{10}{10}$	$\frac{10}{10}$	2.4680	$0.95859s$
$\frac{1}{50}$	$\frac{10}{10}$	$\frac{10}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	2.4215	$0.4849s$
$\frac{1}{50}$	$\frac{1}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	1.4805	$1.6016s$

From Table 2, the social loss in the discretionary regime is, in some cases, twice that under suboptimal delegation.

5.8. What happens if the Treasury does not have an inflation target?

Here we consider two alternative regimes. In both of these cases, the Central Bank is given an inflation target π_B, i.e., has the objective function given in (4.1) but the Treasury is not given an inflation target. We find that these regimes are able to achieve the precommitment solution.

The Treasury is an "output nutter"

If the Treasury is not given an inflation target, we call it an *output nutter*. Its objective function is then given by

$$U_T = -\frac{1}{2}\beta (y - y_T)^2 - \frac{1}{2}\gamma f^2$$

The Treasury is a "reckless output nutter"

If the Treasury cares neither about inflation nor the costs of fiscal policy we call it a *reckless output nutter*. Its objective function is then given by

$$U_T = -\frac{1}{2}\left(y - y_T\right)^2$$

We are interested in evaluating the performance of the alternative institutional regimes in which the Treasury does not care about inflation. Proposition 7, below, shows that the optimal precommitment solution can be achieved; the proof is identical to that of Proposition 3 and, so, is omitted.

Proposition 7 : *Unless the economy is liquidity trapped in both states of the world, if the Treasury can be assigned an optimal output target y_T^* and the Central Bank is assigned an optimal inflation target, π_B^*, then the outcome in the "output nutter" and the "reckless output nutter" cases is identical to the precommitment regime.*

However, and unlike the suboptimal delegation regime, if the Treasury does not adopt the optimal output target, y_T^*, then the outcome can be very poor, and much worse than the outcome under discretion. Table 3 gives a sample of results for the "output nutter" case.

Table 3: Treasury is an "output nutter" $\left(p = \frac{1}{50}, y_T = y_S = s \neq y_T^*, x = 0\right)$

p	φ	λ	β	γ	μ	q	Q
$\frac{1}{50}$	$\frac{10}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	$.08\,584\,9$	-33829
$\frac{1}{50}$	$\frac{10}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	$0.957\,78$	$0.382\,82$
$\frac{1}{50}$	$\frac{10}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	$0.202\,25$	-3821.0
$\frac{1}{50}$	$\frac{10}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	$\frac{10}{10}$	$\frac{10}{10}$	$0.959\,11$	$0.280\,7$

In this case, Q can take extreme negative values i.e. the output nutter regime turns out to be much worse than discretion; we summarize this result in the Proposition below.

Proposition 8 : *If the Treasury is not assigned the optimal output target, y_T^*, then the performance of the "output nutter" and the "reckless output nutter" regimes can be very adverse and, possibly, much worse than the discretionary regime. In particular, if monetary policy is delegated to an independent central bank, with an optimal inflation target, while the Treasury retains discretion over fiscal policy, then the outcome can be poor and much worse than had the Treasury retained discretion over both monetary and fiscal policy.*

Proposition 8 indicates the serious consequences that can arise if the Treasury/ government does not have the appropriate inflation or output targets even if it follows society's most preferred output target (note $y_T = y_S$ in Table 3). This has relevance for understanding the Japanese experience in which the fiscal authorities, as pointed out earlier, were not delegated with the optimally designed targets.

5.9. Summary

Proposition 5 and Corollary 2 establish that the *optimal delegation regime* (where the Bank has an optimal inflation target and the Treasury has optimal output and inflation targets) achieves the precommitment solution for all parameter values. Proposition 6 shows that performance of the *suboptimal delegation regime* (similar to the *optimal delegation regime*, except that the Treasury has its own output target) is near optimal, and much better than *discretion*, even when the Treasury deviates considerably from the optimal output target. Proposition 7 establishes that the *output nutter* and the *reckless output nutter* (in both cases the Bank and Treasury are given optimal inflation and output targets, respectively, but the Treasury is not given an inflation target) regimes also achieve the precommitment solution. However, Proposition 8 shows that the latter two regimes, unlike the *suboptimal delegation* regime, perform poorly, and can be much worse than *discretion*, if the Treasury deviates from the optimal output target. Thus, although giving the Treasury an inflation target as well as an output target is not necessary for optimality, it is necessary to achieve robustness. In particular, a hybrid system, where monetary policy is delegated to an independent central bank with an inflation target, but where the Treasury retains discretion over fiscal policy, can perform poorly and much worse than had the Treasury retained discretion over both monetary and fiscal policy.

6. Relation to the literature

The role of fiscal policy in theoretical models on the liquidity trap has not been adequately stressed despite this being Keynes's (1936) original solution to the problem. This is puzzling in light of the empirical evidence from Posen (1998), Kuttner and Posen (2001) which suggests that fiscal policy, when used in Japan, has been potent. The simulation exercises of Ball (2005) show that fiscal transfers equal to 6.6 percent of GDP could have ended Japan's output slump. There have been other suggestions in the literature, without a full theoretical model, that advocate fiscal policy in a liquidity trap. Bernanke (2002) recommends a broad based tax cut while Gertler (2003) recommends transitory fiscal policy. We consider fiscal policy explicitly in a Dixit and Lambertini (2003) framework when there is the possibility of a liquidity trap.

The theoretical literature has considered aspects of our *optimal delegation regime*, that achieves the precommitment solution. For instance, inflation targets have been suggested in Krugman (1998), Nishiyama (2003), and Iwamura et al. (2005). Other variants of monetary policy commitment have also been considered. Benhabib Schmitt-Grohe and Uribe (2002) consider a commitment to switch from an interest rate rule to a money growth rate peg in a liquidity trap. Eggertsson and Woodford (2003) propose a commitment to

adjust nominal interest rates to achieve a time varying price level target. Bernanke (2002) suggests a commitment to a buffer zone for the inflation rate. Svensson (2003) advocates a price level target (as part of a larger set of policies). However, none of these models allow for the possibility of strategic interaction between monetary and fiscal authorities nor jointly derive the optimal set of targets and instruments of the two policy making authorities.

Eggertsson (2006a, 2006b) studies the liquidity trap within a new Keynesian stochastic general equilibrium model with a government budget constraint and explicit microfoundations. Eggertsson recommends abandonment of an independent central bank and a return to discretionary policy by a unitary monetary-fiscal authority. A debt financed fiscal expansion during a liquidity trap results, via the government budget constraint, in higher expectations of future inflation. Eggertsson shows that this solution is superior to either monetary policy alone or uncoordinated monetary and fiscal policy. However, as Eggertsson shows, even optimal discretion is inferior to the fully optimal rational expectations solution with commitment. Moreover, abandoning delegation of monetary policy to an independent central bank with a narrow mandate, in favor of a return to discretion, appears to be a retrograde step.

Dixit and Lambertini (2003) and Lambertini and Rovelli (2003) consider strategic interaction between fiscal and monetary authorities, but in the absence of a liquidity trap. Lambertini and Rovelli (2003) show that the equilibrium with the fiscal authority acting as leader is superior to the Nash equilibrium. Dixit and Lambertini (2003) show that this regime can achieve the optimal precommitment rational expectations solution.

One of the important lessons of our paper (see Figure 1.2 and Section 5) is that an optimally derived target for one policy maker while ignoring the incentives and constraints facing the other policy maker can lead to extremely poor outcomes; witness the last row in Figure 1.2.

Furthermore, the *optimal delegation regime* achieves the optimal mix between monetary and fiscal policy as we now explain. Theoretically, society could give a sufficiently high inflation target to the Central Bank which in turn generates sufficiently high inflation expectations so that the nominal interest rate never hits its zero floor. While this policy would always avoid the liquidity trap, it is not optimal because inflation is costly. Analogously it is not optimal to give the Treasury too high an output target because if a liquidity trap occurs, it would use the costly fiscal policy excessively. The optimal solution then is to have a mix of both i.e. some inflation outside a liquidity trap and some dependence on costly fiscal policy in a liquidity trap.

The optimality of fiscal delegation to the Treasury in our paper is quite similar to the successful policy arrangements introduced in Britain by the chancellor Gordon Brown.

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The intuition is that if there were no liquidity trap, and the Treasury had its own agenda[32], then it would undermine the Central Bank's monetary commitment. However, appropriate delegation of policy to the Treasury, far from undermining monetary commitment, gives it an incentive to engage in an 'appropriate' fiscal stimulus in a liquidity trap, where the independent Central Bank is ineffective.

7. Conclusions

In a liquidity trap, with nominal interest rates bound below by zero, an expectation of positive inflation is needed. This in turn needs a credible commitment to a future level of positive actual inflation. The credibility problem comes about because after the economy has escaped from the liquidity trap it is in the interest of all parties to renegotiate and reduce inflation. A forward looking private sector will anticipate this and expect low future inflation. With low expected future inflation, the real interest rate remains positive, keeping the economy in the liquidity trap; see for instance Krugman (1998).

The first best solution obtains when the rigidities that give rise to the liquidity trap are removed. But removal of these distortions is usually slow and difficult (witness the experience of Japan). In this case, macroeconomic policy can have an important role. Furthermore, the Japanese experience suggests that issues of strategic monetary fiscal policy interaction assume even greater importance in a liquidity trap.

In the solution considered here, society delegates monetary policy to an operationally independent Central Bank with an inflation target. Fiscal policy is delegated to the Treasury with inflation and output targets. Furthermore, the Treasury acts as a leader and the Central Bank is the follower. The required institutional arrangements are quite natural and are able to achieve the second best solution, namely, the best rational expectations precommitment solution. This institutional setting provides (1) the appropriate level of inflation and, hence, inflation expectations and (2) the optimal balance between monetary and fiscal policy. Even if the Treasury deviates considerably from the optimal output target, we find that the performance of this solution is still 'near optimal' and much better than the regime where the Treasury is given discretion over monetary and fiscal policy.

On the other hand, we find that the hybrid system where monetary policy is delegated to an independent central bank with an optimal inflation target, but where the Treasury retains discretion over fiscal policy, can perform badly and much worse than had the Treasury retained discretion over both fiscal and monetary policy. This is in line with the case when there is no liquidity trap considered by Dixit and Lambertini (2003, p1523, point 4): "Commitment achieves the second best only if it can be extended to both monetary

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[32]In Dixit and Lambertini (2003) the Treasury never has its own agenda and fully internalizes society's social welfare function.

and fiscal policy".

8. Appendix

Generic Equilibrium: To save space, we carry out some calculations that are relevant to both Proposition 1 (Precommitment) and Proposition 2 (Discretion).

Substituting (2.4) and (2.5) into (2.6),

$$U_S = -\frac{1}{2}\left(f - i + \pi^e + \epsilon - y_S\right)^2 - \frac{1}{2}\left(f - i + 2\pi^e + \epsilon\right)^2 - f^2 \tag{8.1}$$

Since $f \gtrless 0$ and $i \geq 0$ the first order conditions are as follows.

$$\frac{\partial U}{\partial f} = y_S - 2\epsilon - 4f - 3\pi^e + 2i = 0; \ f \gtrless 0 \tag{8.2}$$

$$\frac{\partial U_S}{\partial i} = 2f - 2i + 3\pi^e + 2\epsilon - y_S \leq 0; \ i \geq 0 \text{ and } i\frac{\partial U}{\partial i} = 0 \tag{8.3}$$

Since f is unrestricted, the optimal f satisfies $\frac{\partial U}{\partial f} = 0$, hence

$$f = \frac{1}{4}y_S - \frac{3}{4}\pi^e + \frac{1}{2}i - \frac{1}{2}\epsilon \tag{8.4}$$

Recall that values in the liquidity trap are distinguished by a '$-$' subscript and those in the complementary case by the '$+$' subscript. From (8.3), either $i \geq 0$ and $\frac{\partial U_S}{\partial i} = 0$ or $i = 0$ and $\frac{\partial U_F}{\partial i} < 0$, hence

$$i_+ = f_+ + \frac{3}{2}\pi^e - \frac{1}{2}y_S + a \text{ and } f_+ + \frac{3}{2}\pi^e - \frac{1}{2}y_S + a \geq 0 \tag{8.5}$$

$$i_- = 0 \text{ and } f_- + \frac{3}{2}\pi^e - \frac{1}{2}y_S - a < 0 \tag{8.6}$$

Substituting from (8.4), these two conditions can be restated as

$$i_+ = \frac{3}{2}\pi^e - \frac{1}{2}y_S + a \text{ and } 3\pi^e - y_S + 2a \geq 0 \tag{8.7}$$

$$i_- = 0 \text{ and } 3\pi^e - y_S - 2a < 0 \tag{8.8}$$

From (2.5), (8.4) (8.7), (8.8),

$$f_+ = 0 \tag{8.9}$$

$$f_- = \frac{1}{4}y_S - \frac{3}{4}\pi^e + \frac{1}{2}a \tag{8.10}$$

$$\pi_+ = \frac{1}{2}\left(y_S + \pi^e\right) \tag{8.11}$$

$$\pi_- = \frac{1}{4}y_S + \frac{5}{4}\pi^e - \frac{1}{2}a \text{ (liquidity trapped)} \tag{8.12}$$

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This completes the description of the generic equilibrium. ∎

Proof of Proposition 1 (Precommitment)

Since the two possible values of $\epsilon = -a$ and $\epsilon = a$ are equally probable, using (8.1) the expected social welfare is

$$E\left[U_S\right] = \frac{1}{2}\left(-\frac{1}{2}\left(f_+ - i_+ + \pi^e + a - y_S\right)^2 - \frac{1}{2}\left(f_+ - i_+ + 2\pi^e + a\right)^2 - f_+^2\right)$$
$$+ \frac{1}{2}\left(-\frac{1}{2}\left(f_- - i_- + \pi^e - a - y_S\right)^2 - \frac{1}{2}\left(f_- - i_- + 2\pi^e - a\right)^2 - f_-^2\right) \quad (8.13)$$

From (2.5), $\pi^e = f^e - i^e + 2\pi^e$, hence

$$\pi^e = \frac{1}{2}\left(i_+ + i_-\right) - \frac{1}{2}\left(f_+ + f_-\right) \quad (8.14)$$

Substituting (8.14) in (8.13) the expected social welfare is

$$E\left[U_S\right] = -\frac{1}{4}\left(\frac{1}{2}\left(f_+ - f_-\right) - \frac{1}{2}\left(i_+ - i_-\right) + a - y_S\right)^2 - \frac{1}{4}\left(i_- - f_- + a\right)^2 - \frac{1}{2}f_+^2$$
$$- \frac{1}{4}\left(\frac{1}{2}\left(i_+ - i_-\right) - \frac{1}{2}\left(f_+ - f_-\right) - a - y_S\right)^2 - \frac{1}{4}\left(i_+ - f_+ - a\right)^2 - \frac{1}{2}f_-^2$$

We maximize $E\left[U_S\right]$ subject to $i_+ \geq 0$ and $i_- \geq 0$. Formally

$$\underset{\{f_-, f_+, i_-, i_+\}}{Max} \ E\left[U_S\right]$$

subject to

$$i_+ \geq 0, \ i_- \geq 0$$

Solving the first order conditions simultaneously, using the condition of rational expectations (8.14) and the equations for output and inflation in (2.4) and (2.5), one obtains the solution for the policy variables and the macroeconomic magnitudes reported in Proposition 1. ∎

Proof of Proposition 2 (Discretion: Economy is liquidity trapped only under adverse demand conditions, $\epsilon = -a$)

Since $\epsilon = -a$ and $\epsilon = a$, each occur with probability $\frac{1}{2}$, the condition for rational expectations, using (8.11) and (8.12) gives:

$$\pi^e = \frac{1}{2}\left(-\frac{1}{2}a + \frac{1}{4}y_S + \frac{5}{4}\pi^e\right) + \frac{1}{2}\left(\frac{1}{2}\left(y_S + \pi^e\right)\right)$$

Hence the fixed point of π^e is readily found as

$$\pi^e = 3y_S - 2a \quad (8.15)$$

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(8.7), (8.8)and (8.15) give

$$\frac{1}{2}a \leq y_S < a \tag{8.16}$$

which is the necessary and sufficient condition for this case to arise.

Substituting (8.15) in (8.7)-(8.10) gives the magnitudes of the policy instruments:

$$i_- = 0 \tag{8.17}$$

$$f_- = 2(a - y_S) > 0 \tag{8.18}$$

$$i_+ = 4y_S - 2a \tag{8.19}$$

$$f_+ = 0 \tag{8.20}$$

The magnitudes of output and inflation can now be found from (2.4), (2.5), (8.15), and (8.17)-(8.20):

$$y_- = y_S - a < 0 \tag{8.21}$$

$$\pi_- = 4y_S - 3a \tag{8.22}$$

$$y_+ = a - y_S > 0 \tag{8.23}$$

$$\pi_+ = 2y_S - a \tag{8.24}$$

The expected values (where expectations are taken over the demand shock ϵ) of i, f and y are given by

$$i^e = 2y_S - a$$

$$f^e = a - y_S > 0$$

$$y^e = 0$$

Hence, on average macroeconomic policy ensures that there are no deviations of output from the natural level ($y^e = 0$). To find the state-contingent levels of social welfare, substitute (8.18), (8.20), (8.21)-(8.24) into (2.6) then take expectations over the demand shock to get the expected social welfare

$$E\left[U_S^{Disc}\right] = 12ay_S - 8y_S^2 - 5a^2 \tag{8.25}$$

This completes the proof of the proposition. ∎

Proof of Proposition 3 (Solution under the optimal delegation regime)
Monetary authority's reaction function

The monetary authority's reaction function can be found by maximizing U_B in (4.1). Since $i \geq 0$, the first order conditions for maximizing U_B are $\frac{\partial U_B}{\partial i} \leq 0$, $i \geq 0$, $i\frac{\partial U_B}{\partial i} = 0$. Using (2.5), this gives

$$i\left(f - i + 2\pi^e - \pi_B + \epsilon\right) \leq 0 \tag{8.26}$$

We start with the case where the economy is liquidity trapped in the the bad state ($\epsilon = -a$) only. The other cases will be considered at the end.

The economy is in a liquidity trap ($\epsilon = -a$)

In this case, at $\epsilon = -a$ the interest rate can go no lower than zero. Using (8.26), $f_- + 2\pi^e - \pi_B - a < 0$, and so

$$i_- = 0 \tag{8.27}$$

The economy is not in a liquidity trap ($\epsilon = a$)

In this case, $i \geq 0$, hence (8.26) holds with equality. Solving out for i at $\epsilon = a$, gives

$$i_+ = f_+ + 2\pi^e - \pi_B + a \tag{8.28}$$

The state contingent reaction function of the monetary authority is given by (8.27) and (8.28).

Fiscal authority's reaction function

The Treasury now chooses its state contingent fiscal policy f to maximize the objective function (4.2) after observing π^e and ϵ and knowing that the state contingent reaction function of the monetary authority is given by (8.27) and (8.28).

Case-I: Liquidity trap ($\epsilon = -a$)

In this case, the subsequent monetary policy is $i_- = 0$, hence, using (2.4), (2.5), (4.2) and $\pi_T = \pi_B$, the government maximizes:

$$U_T^- = -\frac{1}{2}\left(f_- + \pi^e - a - y_T\right)^2 - \frac{1}{2}\left(f_- + 2\pi^e - a - \pi_B\right)^2 - f_-^2 \tag{8.29}$$

Maximizing U_T^- with respect to f_- gives

$$f_- = \frac{1}{2}a + \frac{1}{4}y_T + \frac{1}{4}\pi_B - \frac{3}{4}\pi^e \tag{8.30}$$

Case-II: No liquidity trap ($\epsilon = a$)

The subsequent monetary policy is given by (8.28), hence, using (2.4), (2.5), (4.2) and $\pi_T = \pi_B$, the government maximizes

$$U_T^+ = -\frac{1}{2}\left(\pi_B - \pi^e - y_T\right)^2 - f_+^2 \tag{8.31}$$

Maximizing U_T^+ with respect to f_+ gives

$$f_+ = 0 \tag{8.32}$$

The state contingent reaction function of the fiscal authority is given by (8.30) and (8.32) respectively.

Substituting the state contingent monetary and fiscal policy reaction functions in (2.4) and (2.5) one obtains

$$y_- = -\frac{1}{2}a + \frac{1}{4}y_T + \frac{1}{4}\pi_B + \frac{1}{4}\pi^e \tag{8.33}$$

$$\pi_- = -\frac{1}{2}a + \frac{1}{4}y_T + \frac{1}{4}\pi_B + \frac{5}{4}\pi^e \tag{8.34}$$

$$y_+ = \pi_B - \pi^e \tag{8.35}$$

$$\pi_+ = \pi_B \tag{8.36}$$

Calculation of expected inflation

Since the two states of the world are equally probable, π^e is simply a weighted average of inflation in (8.34) and (8.36) respectively

$$\pi^e = \frac{1}{3}y_T - \frac{2}{3}a + \frac{5}{3}\pi_B \tag{8.37}$$

Substituting π^e in (8.28), (8.30), (8.33)-(8.35), one obtains

$$f_- = a - \pi_B \tag{8.38}$$

$$y_- = -\frac{2}{3}a + \frac{1}{3}y_T + \frac{2}{3}\pi_B \tag{8.39}$$

$$\pi_- = -\frac{4}{3}a + \frac{2}{3}y_T + \frac{7}{3}\pi_B \tag{8.40}$$

$$i_+ = \frac{2}{3}y_T - \frac{1}{3}a + \frac{7}{3}\pi_B \tag{8.41}$$

$$y_+ = \frac{2}{3}a - \frac{1}{3}y_T - \frac{2}{3}\pi_B \tag{8.42}$$

Calculation of the optimal inflation target

Substituting (8.32), (8.36), (8.38), (8.39) (8.40), (8.42) in (4.2) the expected social welfare can be simplified and written as:

$$E\left[U_S^{SD}\right] = 3a\pi_B + \frac{2}{3}ay_T - \frac{7}{3}\pi_B^2 - \pi_B y_T - \frac{7}{6}a^2 - \frac{1}{6}y_T^2 - \frac{1}{2}y_S^2 \tag{8.43}$$

Maximizing $E\left[U_S^{SD}\right]$ in (8.43) with respect to π_B and y_T gives the following optimal inflation and output targets

$$\pi_B^* = \frac{3}{5}a \tag{8.44}$$

$$y_T^* = \frac{1}{5}a \tag{8.45}$$

Substituting (8.44) and (8.45) in (4.2) gives the final expression for expected social welfare in the Stackelberg delegation case

$$E\left[U_S^{SD}\right] = -\frac{1}{5}a^2 - \frac{1}{2}y_S^2 \tag{8.46}$$

Comparing with Proposition 1, we see that the inflation and output targets achieve the optimal solution, with the economy liquidity trapped in the bad state only. Hence, the two other cases, when the economy is never liquidity trapped and when the economy is liquidity trapped in both states, need not be considered; thus the proof is complete. ■

8.1. Appendix- B: The discretionary regime in the general case

Proposition 9 : **(a)** Let $\sigma = \text{signum}\left(\alpha\varphi^2 - \gamma\lambda\mu\right)$. If

$$\sigma x < -\frac{\sigma}{\alpha\rho}\left(\beta\lambda\mu y_S + \frac{\left(\alpha\varphi^2 - \mu\lambda\gamma\right)\left(\alpha + \beta\mu^2\right)sp}{\gamma\mu^2 + \varphi^2\left(\alpha + \beta\mu^2\right)}\right)$$

then the economy is liquidity trapped in both states and the solution under discretion is given by $i_- = i_+ = 0$

$$f_- = \varphi\left(\frac{\left(\alpha + \beta\mu^2\right)s\left(1 - p\right)}{\gamma\mu^2 + \varphi^2\left(\alpha + \beta\mu^2\right)} - \frac{\alpha\rho x + \beta\lambda\mu y_S}{\alpha\varphi^2 - \gamma\lambda\mu}\right) > 0$$

$$f_+ = \varphi\left(-\frac{\left(\alpha + \beta\mu^2\right)sp}{\gamma\mu^2 + \varphi^2\left(\alpha + \beta\mu^2\right)} - \frac{\alpha\rho x + \beta\lambda\mu y_S}{\alpha\varphi^2 - \mu\lambda\gamma}\right) > 0$$

(b) Let $\sigma = \text{signum}\left(\left(\gamma\lambda\mu p - \alpha\varphi^2\right)\left(\alpha + \beta\mu^2\right) - \alpha\gamma\mu^2\left(1 - p\right)\right)$. If

$$-\frac{\sigma}{\alpha\rho}\left(\alpha sp + \beta\lambda\mu y_S - \alpha s\right) < \sigma x \leq -\frac{\sigma}{\alpha\rho}\left(\beta\lambda\mu y_S + \frac{\left(\alpha\varphi^2 - \gamma\lambda\mu\right)\left(\alpha + \beta\mu^2\right)sp}{\gamma\mu^2 + \varphi^2\left(\alpha + \beta\mu^2\right)}\right)$$

then the economy is liquidity trapped in the bad state only and the solution under discretion is given by $i_- = f_+ = 0$

$$f_- = \varphi\frac{\left(\alpha + \beta\mu^2\right)\beta\lambda\mu y_S + \left(\alpha + \beta\mu^2\right)\alpha\rho x - \alpha s\left(\alpha + \mu^2\beta\right)\left(1 - p\right)}{\left(\alpha\mu + \lambda\left(\alpha + \beta\mu^2\right)\right)\gamma\mu p - \alpha\left(\gamma\mu^2 + \varphi^2\left(\alpha + \beta\mu^2\right)\right)} > 0$$

$$i_+ = \frac{-\left(\alpha\rho x + \beta\lambda\mu y_S\right)\left(\gamma\mu^2 + \varphi^2\left(\alpha + \beta\mu^2\right)\right) + \left(\alpha + \mu^2\beta\right)\left(\gamma\lambda\mu - \alpha\varphi^2\right)sp}{\lambda\left(\left(\gamma\lambda\mu p - \alpha\varphi^2\right)\left(\alpha + \beta\mu^2\right) - \gamma\mu^2\alpha\left(1 - p\right)\right)} \geq 0.$$

(c) If $x \geq \frac{\alpha s(1-p) - \lambda\beta\mu y_S}{\alpha\rho}$ then the economy is liquidity trapped in neither state and the solution under discretion is given by $f_- = f_+ = 0$,

$$i_- = \frac{\alpha\rho x + \alpha sp + \beta\lambda\mu y_S - \alpha s}{\alpha\lambda} \geq 0$$

$$i_+ = \frac{\alpha\rho x + \alpha sp + \beta\lambda\mu y_S}{\alpha\lambda} > i_- \geq 0$$

8.2. Appendix-C: Further simulation results

Tables 4, 5, 6 report the most interesting case: the economy is liquidity trapped in the bad state only.

Table 4 below confirms results similar to those in Table 1 when the probability of falling into the liquidity trap is very remote i.e. $p = \frac{1}{50}$.

Table 4: $p = \frac{1}{50}, y_T = y_S = s, x = 0$

α	φ	λ	β	γ	μ	q	ω	Q	$o = t$	$\pi_B^* \times 10^2$
$\frac{10}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	1.0000	1.000	0.9483	5158.2	$0.037s$
$\frac{10}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{10}{10}$	$\frac{10}{10}$	0.9992	1.070	0.9882	106.13	$0.095s$
$\frac{10}{10}$	$\frac{10}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	1.0000	1.001	0.9842	5259.3	$2.073s$
$\frac{10}{10}$	$\frac{10}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	$\frac{10}{10}$	$\frac{10}{10}$	0.9992	1.058	0.9858	107.24	$1.863s$
$\frac{1}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	0.9998	1.003	0.9576	566.34	$3.369s$
$\frac{10}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	1.0000	1.034	1.0000	102.09	$1.959s$
$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{10}{10}$	$\frac{10}{10}$	1.0000	1.887	1.0000	50.122	$2.392\,s$
$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{10}{10}$	$\frac{10}{10}$	0.9984	1.902	0.9983	60.224	$1.825\,s$

In Table 4, even if the output target of the Treasury, y_T, is 5158.2 times that of the optimal output target, y_T^*, results R1 and R2 above still hold. Tables 5, 6 below confirm the two main results, R1 and R2, for much smaller output targets of the Treasury $y_T = y_S = ps$ when the probability of falling into the liquidity trap takes a high and a low value respectively.

Table 5: $p = \frac{1}{2}, y_T = y_S = ps, x = 0$

α	φ	λ	β	γ	μ	q	ω	Q	$o = t$	π_B^*
$\frac{10}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	1.0000	1.01	0.9969	202.2	$0.01s$
$\frac{10}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{10}{10}$	$\frac{10}{10}$	0.9982	1.18	0.9904	3.211	$0.16s$
$\frac{10}{10}$	$\frac{10}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	0.9999	1.48	0.9999	301.3	$0.18\,s$
$\frac{10}{10}$	$\frac{10}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	$\frac{10}{10}$	$\frac{10}{10}$	0.9973	2.48	0.9982	4.3	$0.23\,s$
$\frac{1}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	0.9996	1.14	0.9971	22.21	$0.43s$
$\frac{10}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	1.0000	3.98	1.0000	4.003	$2.50s$
$\frac{1}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{10}{10}$	$\frac{10}{10}$	0.9985	1.17	0.9913	1.42	$0.41s$

Table 6: $p = \frac{1}{50}, y_T = y_S = ps, x = 0$

α	φ	λ	β	γ	μ	q	ω	Q	$o=t$	$\pi_B^* \times 10^2$
$\frac{10}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	1.0	1.001	1.0010	11.33	$0.3875\,s$
$\frac{10}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{10}{10}$	$\frac{10}{10}$	1.0	1.009	0.9999	2.123	$1.045s$
$\frac{10}{10}$	$\frac{10}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	1.0	1.024	1.0000	105.19	$2.093s$
$\frac{10}{10}$	$\frac{10}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	$\frac{10}{10}$	$\frac{10}{10}$	1.0	1.035	1.0000	2.145	$1.957\,s$
$\frac{1}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	1.0	1.006	0.9999	11.327	$3.546s$
$\frac{10}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	1.0	1.041	1.0000	2.042	$1.960s$
$\frac{1}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{10}{10}$	$\frac{10}{10}$	1.0	1.008	0.9999	1.205	$1.992\,s$

References

[1] Alesina, A. and Tabellini, G. (1987) "Rules and discretion with non-coordinated monetary and fiscal policies" Economic Inquiry : 619-630.

[2] Auerbach, A.J. and Obstfeld, Maurice (2005) "The case for open market operations in a liquidity trap" American Economic Review, 2005, 95: 110-137

[3] Ball, L. (2005) "Fiscal remedies for Japan's slump", NBER Working Paper 11374.

[4] Benhabib, J. Schmitt-Grohe, S. and Uribe, M. (2002) "Avoiding liquidity traps", Journal of Political Economy, 110:535-563.

[5] Bernanke, Ben S. (2002) "Deflation: Making sure that it does not happen here" speech on November 21, Federal Reserve Board.

[6] Blanchard, O. (2003) "Monetary Policy and Unemployment" presented at the conference "Monetary policy and the labour market. A conference in honor of James Tobin".

[7] Blanchard, O. (2006) "Macroeconomics" Pearson Education, Inc., 4th edition.

[8] Blinder, A. (2000) "Monetary policy at the zero lower bound: Balancing the risks" Journal of Money Credit and Banking, 32: 1093-99.

[9] Buiter, W. H. and Panigirtzoglou, N. (2003) Overcoming the zero bound on nominal interest rates with negative interest on currency: Gesell's solution" The Economic Journal, 113: 723–746.

[10] Clouse, J., Henderson, D., Orphanides, A., Small, D., and Tinsley, P. (2003) "Monetary policy when the nominal short term interest rate is zero," Topics in Macroeconomics, www.bepress.com.

[11] Dixit, A. and Lambertini, L. (2003) "Interactions of commitment and discretion in monetary and fiscal policies" American Economic Review, 93: 1522-1542.

[12] Dixit, A. and Lambertini, L. (2000) "Fiscal discretion destroys monetary commitment" mimeo., Princeton University.

[13] Eggertsson, G. (2006a) "The deflationary bias and committing to being irresponsible" Journal of Money, Credit and Banking, Vol. 38. No. 2.

[14] Eggertsson, G. (2006b) "Fiscal multipliers and policy coordination" Federal Reserve Bank of New York, Staff Report no. 241.

[15] Eggertsson, G. and Woodford, M. (2003) "The zero bound on interest rates and optimal monetary policy" Brookings Papers On Economic Activity, 1: 139-211.

[16] Gali, J. and Gertler, M. (1999) "Inflation dynamics: A structural econometric analysis," Journal of Monetary Economics, 44: 195-222.

[17] Gertler, M. (2003) "Comments on: The zero lower bound on interest rates and optimal monetary policy" Paper prepared for the Brookings Panel on Economic Activity, 1:219-227.

[18] Iwamura, M., Kudo, T. and Watanabe, T. (2005) "Monetary and fiscal policy in a liquidity trap: The Japanese experience 1999-2004", NBER Working Paper No. W11151. Available at SSRN: http://ssrn.com/abstract=669450.

[19] Keynes, M (1936) The general theory of employment, interest and money (Macmillan: London)

[20] Krugman, P. (1999) "Thinking about the liquidity trap", mimeo., http://web.mit.edu/Krugman/www/trioshrt.html.

[21] Krugman, P. (1998) "Its baaack! Japan's slump and the return of the liquidity trap" Brookings Papers on Economic Activity, 2: 137-187.

[22] Kuttner, K. N. and Posen, A. (2001). The Great Recession: Lessons for Macroeconomic Policy from Japan," Brookings Papers on Economic Activity, 2001: 93-185.

[23] Lambertini, L. and Rovelli, R. (2003) "Monetary and fiscal policy coordination and macroeconomic stabilization: A theoretical analysis" Available at SSRN: http://ssrn.com/abstract=380322 or DOI: 10.2139/ssrn.380322.

[24] Mankiw, N. G. (2001) "The Inexorable And Mysterious Trade-off Between Inflation And Unemployment," Economic Journal, 111:45-61.

[25] McCallum, Bennet T. (2000) "Theoretical analysis regarding a zero lower bound on nominal interest rates" Journal of Money Credit and Banking, 32: 870-904.

[26] Nishiyama, Shin-Ichi (2003) "Inflation target as a buffer against liquidity trap" Institute for Monetary and Economic Studies, Discussion Paper No. 2003-E-8.

[27] Orphanides, A. and Wieland, V. (2002) "Efficient monetary policy design near price stability" Journal of the Japanese and International Economies, 14: 327-365.

[28] Rogoff, K. (1985) "The optimal commitment to an intermediate monetary target" Quarterly Journal of Economics, 100(4), 1169-1189.

[29] Romer, D. (2006) "Advanced Macroeconomics" 3rd edition, McGraw-Hill/Irwin.

[30] Rotemberg and Woodford (1999) "Interest rate rules in an estimated sticky price model" in Monetary Policy Rules. in John B. Taylor, ed., Monetary Policy Rules, University of Chicago Press, 57-126.

[31] Ruud, J. and Whelan, K. (2006) "Can rational expectations sticky-price models explain inflation dynamics?" American Economic Review, 96: 303-320.

[32] Posen, A. S. (1998) Restoring Japan's Economic Growth, Institute for International Economics, Washington DC.

[33] Sims, C.A. (2003) "Implications of rational inattention" Journal of Monetary Economics, 50, 665-690.

[34] Svensson, Lars E. O. (2003) "Escaping from a liquidity trap and deflation: The foolproof way and others", Journal of Economic Perspectives, 17: 145-166.

[35] Walsh, C.E. (2003) "Monetary Theory and Policy' 2nd edition, The MIT Press.

[36] Woodford, M. (2005) "Central bank communication and policy effectiveness" Presented at the Federal Reserve Bank of Kansas City Symposium, "The Greenspan Era: Lessons for the Future" Jackson Hole, Wyoming, 25-27 August 2005.