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PROFITABILITY OF HORIZONTAL MERGERS IN TRIGGER STRATEGY GAME

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Abstract

It is shown that, in a dynamic competition, an exogenous horizontal merger is profitable even if a small share of active firms merge. However, each firm has incentive to remain outside the merger because it would benefit more (Insiders' dilemma). We show that in an infinite repeated game in which the firms use trigger strategies an exogenous bilateral merger can be profitable and the Insiders' dilemma is mitigated.

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1 Introduction

The model by Salant, Switzer and Rynolds (1983)(henceforth, SSR) can be considered a seminal paper on horizontal mergers profitability. They construct a symmetric linear Cournot model with fixed number of firms and homogenous product where number of merger candidates is exogenously given. In this model the motives for the merger are to reduce competition and increase market power by abstracting from cost saving. Their main results are the following: a merger is profitable if and only if at least the 80% of the active firms merge[1], and each firm has an incentive to stay out even if the merger will be profitable (Insider's dilemma). These results are clearly in contrast with the observation that, in the real word, relative low size mergers (bilateral mergers) are observed in all industries, even when the merging firms face similar costs.

We are the first to show that if the number of merging firms is exogenous, then a bilateral merger among symmetric competitors is always profitable and the incentive to remain outside is mitigated.

The reason of the results in SSR is the following: when an exogenous number of firms merge, the new single entity (Insiders group) produces less than the sum of the quantities produced by each merging firm in absence of the merger[2]. In a Cournot framework, where the best replies of the players are strategic substitutes, the Outsiders (the remaining firms) react to the merger by increasing production but by less than the restriction of merged entity. This reaction implies that the post-merger industry output shrinks but that the resulting increase in price is not enough to make the merger profitable for the Insiders. Furthermore, once the merger has occurred, the Outsiders sell more at higher price and gain more profit than the merging firms (Stiger 1950).

We show that the problem of unprofitability can strongly depend on the nature of the competition. In particular, we construct an infinitely repeated trigger strategy game in which the two Insiders (entity merged) threaten to dissolve if the Outsider does not limit its post-merger production. The use of the trigger strategy allows enforcing the constraint on the Outsider's production over an infinite horizon. This constraint does not let Outsider strongly increase its production as replies to the merger, therefore the aggregate price remains high enough to make the merger profitable. Furthermore, this constrained production decreases the positive externality experienced by the not merging firm.

Some papers try to refine SSR remaining in a static contest. Fauli-Oller (1997) develops a Cournot model with homogenous product and show that if the market demand is sufficiently convex, then the threshold market share of merging firms required for a merger to be profitable moves from 80% to 50%.

Perry and Porter (1985) focus on the cost structures and show that in a Cournot model with linear demand and homogenous product a bilateral merger is profitable if there are sufficient cost savings. They assume that each merging firm has access to an asset of the other Insiders. The exploitation of this new

[1] A merger is profitable if each firm involved in the merger (Insider) gains a profit at least equal to profit gained in absence of merger.

[2] Farrel and Shapiro (1990) formalize this process.

asset allows to increase the production at a given average cost.

Some recent papers have tackled this problem in a dynamic contest. They find that with symmetric competitors the profitability for exogenous merger does not depend on the number of the Insiders.

Dockner and Gaunersdorfer (2001) and Benchekroun (2003) analyze the profitability of horizontal mergers in the case of dynamic competition with price stickiness and show that mergers among symmetric firms are profitable even if the Insiders represent a pre merger market share arbitrary small. Kabiraj and Mukherjee (2003) develop an exogenous specified symmetric Stackelberg game and find that a bilateral merger between a leader and a follower is profitable when the new merged entity behaves like a leader.

However, all the models assuming an exogenous number of the Insiders do not eliminate the incentive of each firm to remain Outsider.

The first formalization of Insider's dilemma in a model where the merger size is endogenously decided by the firms is due to Kamien and Zang (1990). They construct a model where buying and selling firms simultaneously offer bids and ask a price to conclude the merger. Nevertheless, they do not solve the Insider's dilemma because according to their results the merger may not occur despite the fact that monopoly is profitable.

Kamien and Zang (1993) consider a repeated static model and show that the incentive to remain out of the merger can be mitigated. Lindqvist (2003) dissolves the Insider's dilemma by constructing a three-stage model where a firm can buy a share in a competitive firm before an acquisition of another firm occurs (*outsider-toeholds*). He shows that the possibility of buying an outsider-toehold makes each acquiring firm able to increase its profitability by stealing part of the Outsider's profit.

The paper is organized as follows. In section 2 we reformulate the model of SSR (1983) as a two-stage static game. In section 3 we use this two-stage game as constituent game at an infinitely repeated dynamic game. In Appendix B we show that could exist an equilibrium in which the incentive to remain Outsider is completely solved.

2 The two-stage static game

The game described in this section if formally equivalent to the one under analysis in SSR. The purpose of this re-formulation is to construct a two-stage static game we will use as constituent game of the infinitely repeated dynamic game in section 3.

2.1 The game

We consider a market in which n identical firms compete with the aim of maximizing individual profit. They each supply the same homogeneous product. Consumers' inverse demand for the good is given by:

$$p\left(q_1, ..., q_n\right) = a - \sum_{i=1}^{n} q_i, \tag{1}$$

where p is the price and q_i is the output produced by the $i-th$ firm, $a > 0$. The cost function of each firm is assumed to be linear and is given by:

$$C\left(q_i\right) = cq_i, \qquad\qquad i = 1, ..., n \tag{2}$$

where $c > 0$, and $a > c$.

The choice of such cost function it motivated by the necessity to focus the analysis only on the anticompetitive aspect of the horizontal mergers; the absence of fixed cost and the fact that the marginal costs are equal means that there is no efficiency gains deriving from the merger. Hence, the only objective of the merging firm in this paper is simply to increase its market power.

As in SSR, we allow an exogenous number of firms m to merge and take their quantity decision to maximize their joint profit. These firms will be named Insiders. We assume that $n > m \geq 2$. For notational simplicity, the potentially merging firms are those indexed from 1 to m; the generic merging firm is denoted by l, where $l = 1, ..., m$. The not merging firms are referred to as Outsiders, and are indexed from $m + 1$ to n ; the generic firm not merging is denoted by j, $j = m + 1, ..., n$. Since production cost is linear, any merged entity will be indifferent with respect to how to split its total output among the Insiders, hence every coalition of firms (or merged entity) behaves as a single firm.

We analyze a simple two-stage game, whose structure is illustrated in the Fig. 1.

At stage 1 of the game m firms/players can choose whether to enter a merging agreement. We stress the important assumption that the number and the identity of the m potentially merging firms is exogenously determined.

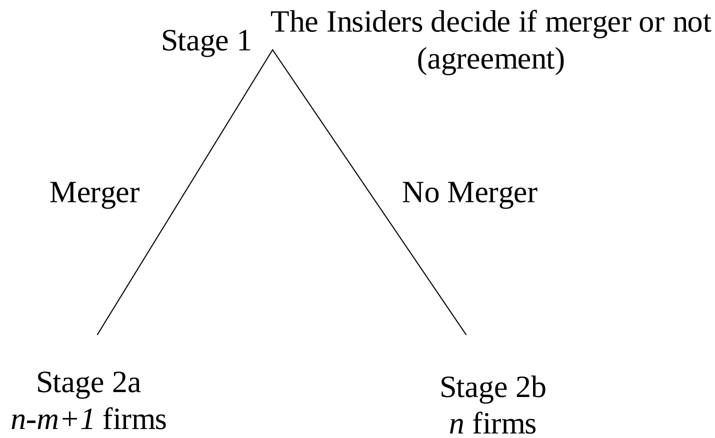

FIG. 1 The game tree

When entering the merging agreement, the firm agrees that, in the following stage, it will offer $\frac{1}{m}$ of the quantity maximizing the joint profit of the m firms or, otherwise, it will have to pay an infinitely high penalty payment[3].

The merger occurs when the potential Insiders forecast they can obtain a profit at least equal than in the absence of the agreement. In the stage 2 of the game, once the decision about merging is taken, a Nash-Cournot game takes place with, possibly, the merged firms choosing their quantity facing the additional constraint deriving from the merger agreement.

In Fig. 1 the second stage is composed by two sub-games starting from two different nodes ($2a$ and $2b$) and all the players find themselves at the two nodes, depending on the choice of the potential Insiders on merge. In both sub-games all players choose simultaneously their quantity as in standard Cournot game.

We assume that the information is perfect in the sense that players perfectly know the past history of the game at each stage in which they are taking an action. Because the structure of the game described above, the equilibrium concept adopted is that of sub-game perfect equilibrium (Selten 1975). The profit for the Insider is given by:

$$\pi_l\left(q_1, ..., q_l, ...q_m, q_{m+1}, ..., q_n\right) = \left(a - q_l - \sum_{\substack{k \neq l \\ k=1}}^{m} q_k - \sum_{j=m+1}^{n} q_j\right) q_l - cq_l. \qquad (3)$$

[3] Instead of such merging agreement it would be possible to assume that after the merger the Insiders give the managment of the merged entity to a single manager and he certantly will not cheat himself.

with $l = 1, ..., m$

while the Outsider's profit is given by

$$\pi_j (q_1, ...q_m, q_{m+1}, ..., q_j, ...q_n) = \left(a - \sum_{l=1}^{m} q_l - q_j - \sum_{\substack{h \neq j \\ h=m+1}}^{n} q_h \right) q_j - cq_j. \quad (4)$$

with $j = m + 1, ..., n$

For future ease of notation, it is useful to define here as Π the aggregate profits of the merging entity; in the case of merger, these are given by

$$\Pi (Q, q_{m+1}, ..., q_n) = \left(a - Q - \sum_{j=m+1}^{n} q_j \right) Q - cQ, \quad (5)$$

where clearly $Q = \sum_{l=1}^{m} q_l$ is the aggregate output produced by the merging entity.

Hence, the only difference between the two sub-games is the problem faced by the firms entering the agreement. Formally, in stage $2a$ the Insider's maximization problem is:

$$\max_{q_l} \pi_l (q_1, ..., q_l, ...q_m, q_{m+1}, ..., q_n), \quad (6)$$

$$s.t.\ q_l = \frac{1}{m} \arg\max_{Q} \Pi (Q, q_{m+1}, ..., q_n). \quad (7)$$

We recall that the static game presented in this section will be used as constituent game of the infinitely repeated dynamic game in section 3.

2.2 The equilibrium of the game

This section characterizes the equilibrium of the game under analysis.

We firstly characterize the equilibrium of the two sub-games ($2a$ and $2b$) forming the second stage.

Consider first the sub-game in the case in which the merger has not occurred in stage 1. In stage 2 the game is at the node $2b$ and all firms simultaneously choose their best replay to the quantity set by the rivals. It simply is an application of linear Nash-Cournot game so each firm i chooses $q_i \in R^+$ to maximize its profit given indifferently by (3) or (4), with $i = 1, ..., n$. The quantity chosen by the firms is given by the standard Nash-Cournot equilibrium quantities with n players and are the following

$$q_l^C = q_j^C = \left(\frac{a-c}{n+1}\right). \tag{8}$$

Let $\pi_j^C(.)$and $\pi_l^C(.)$ be the equilibrium Cournot profits for each firm; hence,

$$\pi_j^C\left(q_j^C, q_l^C\right) = \pi_l^C\left(q_l^C, q_j^C\right) = \left(\frac{a-c}{n+1}\right)^2, \tag{9}$$

Let $\Pi^C(.)$ be the equilibrium profit for the whole group composed by the m firms in the case of no merger: it clearly is:

$$\Pi^C\left(q_l^C, q_j^C\right) = m\left(\frac{a-c}{n+1}\right)^2. \tag{10}$$

Consider now the sub-game in the case in which the merger has occurred in stage 1. This sub-game starts from node $2a$. However, in order not to pay the infinitely high penalty in case of deviation from the merging agreement, each Insider chooses a quantity

$$q_l^M\left(q_{m+1}, ..., q_n\right) = \frac{1}{m} Q^M\left(q_{m+1}, ..., q_n\right), \tag{11}$$

where:

$$Q^M\left(q_{m+1}, ..., q_n\right) \equiv \arg\max \Pi\left(Q, q_{m+1}, ..., q_n\right), \tag{12}$$

while each Outsider maximizes (4), and chooses a quantity:

$$q_j^M\left(Q, q_{m+1}, .., q_{j-1}, q_{j+1}, .., q_n\right) \equiv \arg\max_{q_j}(a - q_j - \sum_{\substack{h=m+1 \\ h \neq j}} q_h - Q)q_j - cq_j \tag{13}$$

with $j = m+1, ..., n$.

Because each player simultaneously chooses its quantity and all of them have the same marginal cost, the equilibrium quantity of the sub-game $2a$ are given by

$$q_l^M = \frac{1}{m}\frac{a-c}{n-m+2}, \tag{14}$$

$$q_j^M = \frac{a-c}{n-m+2}. \tag{15}$$

Now let $\Pi^M(.)$and $\pi_j^M(.)$ be the profit of the merged entity and the profit for each Outsider respectively after that merger has occurred. Denote also by π_l^M the profit for each Insider. Hence, the equilibrium profits for this sub-game are given by:

$$\pi_j^M\left(Q^M, q_{m+1}^M, ..., q_n^M\right) = \left(\frac{a-c}{n-m+2}\right)^2 \quad \textit{with } j = m+1, ..., n, \tag{16}$$

and

$$\pi_l^M\left(Q^M, q_{m+1}^M, ..., q_n^M\right) \equiv \frac{1}{m}\Pi^M\left(Q^M, q_{m+1}^M, ..., q_n^M\right) = \frac{1}{m}\left(\frac{a-c}{n-m+2}\right)^2, \tag{17}$$

with $l = 1, .., m$

Having characterized the players' equilibrium profits for both sub-games of stage 2 , we can now turn to characterizing the SPE of the whole game.

This is done in the following Proposition.

Proposition 1 *The equilibrium of the two-stage static game is as follows:*

-if $\frac{m}{n} < 0.8$

stage 1: *Insiders do not sign merging agreement,*

stage 2: *each potential merging firm and Outsider respectively produce the Nash-Cournot equilibrium quantity. That is*

$$q_l = q_j = q_l^C = q_j^C \quad \textit{with } l = 1, ..., m \textit{ and } j = m+1, ..., n \tag{18}$$

-if $\frac{m}{n} > 0.8$

stage 1: *Insiders merge,*

stage 2: *each Insider and Outsider respectively produces:*

$$q_l = q_l^M \quad and \quad q_j = q_j^M \tag{19}$$

In the stage 1 the Insiders anticipate this equilibrium profit and decide to merge and enter the agreement if they forecast gaining a joint profit at least equal to the sum of the profits gained by each potential Insider in status quo, that is if

$$\Pi^C\left(q_l^C, q_j^C\right) = m\left(\frac{a-c}{n+1}\right)^2 \leq \left(\frac{a-c}{n-m+2}\right)^2 = \Pi^M\left(Q^M, q_{m+1}^M, ..., q_n^M\right). \tag{20}$$

Note that as SSR predicted, the Insider decides to merge only if the level of the pre-merger market share $\left(\frac{m}{n}\right)$ is higher than 0.80%.

Let's come back to the analysis of the SPE. The Proposition shows that the potential Insiders prefer to merger if and only if their pre-merger market share is sufficiently high.

The reason is the following. In stage 1 the potential Insiders forecast that their joint profit will depend on their pre-merger market share. They know that if the ratio $\frac{m}{n}$ is sufficiently low (in particular $\frac{m}{n} < 0.8$) their joint profit will be less than the sum of each pre-merger profit (status quo). Therefore, the optimal choice for them will be no merging so that in stage 2 a standard Nash-Cournot game with n players and n firms takes place. The equilibrium profits for each Outsider and potential merging firm are given by equation (9).

Consider now the case in which merging firms represent sufficiently high pre-merger market share($\frac{m}{n} > 0$). In this case at stage 1 the potential Insiders anticipate that their joint profit will be at least equal to the sum of each pre merger profit so from their point of view the merger will be profitable and the optimal choice will be merging. In stage 2 a standard Nash-Cournot game with n players and $n-m+1$ firms takes place and the equilibrium profits respectively for each Insider and outsider are given by equation (16) and (17).

So I can state the following result:

Proposition 2 *: the profitability of the merger for the Insiders increases in* $\frac{m}{n}$.

Proof. See Appendix A. ■

The above proposition illustrates that the incentive to participate in a merger strongly depends on the relative size of the merged entity. The higher the size more profitable will be the merger for the Insiders.

Consider the case in which the relative size of the merger is sufficiently low ($\frac{m}{n} < 0.8$). In such a market the number of firms not entering the merger is relatively high, therefore the negative effect on the aggregate price due to the increase in the Outsiders' production is more stronger.

However, the above results confirm that each firm has the incentive to become Outsider because it would benefit more (Insider's dilemma). The reason for this is the following. The formation of the merged entity creates positive externality on firms remaining outside the merger because the aggregate price increases but the Outsiders do not need to decrease their production.

Let's assume Δ_S^π is the difference between the profit of Outsider and Insider after the merger has occurred. Its value is defined as follows:

$$\Delta_S^\pi = \pi_j^M \left(Q^M, q_{m+1}^M, ..., q_n^M\right) - \pi_l^M \left(Q^M, q_{m+1}^M, ..., q_n^M\right). \qquad (21)$$

The subscript S stresses the fact that this difference is computed in static framework.

It is easily shown that:

$$\Delta_S^\pi = \left(\frac{a-c}{n-m+2}\right)^2 \left(1 - \frac{1}{m}\right) > 0 \text{ with } m \geq 2. \qquad (22)$$

We now turn to analyzing the quantity choice of the Outsider which grants the profitability of the merger in a situation in which the merger itself would be unprofitable if the Outsiders choose their best reply to the rivals. The interest in the analysis is two fold: first it further clarifies the nature of the externality described above, in the sense that it shows that, after the merger, Outsiders produce more at a price higher than the pre-merger price. This larger quantity chosen by the Outsiders implies that the post merger industry price does not increase enough to make the merger profitable also for Insiders. Second, the analysis provides some results that we will use in the rest of the paper when the 2-stage game under analysis is the constituent game of an infinitely repeated dynamic game.

To simplify the analysis we consider a market with three firms where two of these are allowed to merge. With three firms the study is restricted to the interesting case in which $\frac{m}{n} < 0.8$ and therefore exists only an unmerged equilibrium because the merger is unprofitable[4].

The Insider chooses q_l according the following maximization problem:

$$\max_{q_l} \pi_l \left(q_1, ..., q_l, q_3\right) \qquad (23)$$

[4] The case of three firms is also used in Kamien and Zang (1990) and Kabiraj and Arijit Mukherjee (2003).

$$s.t.\ q_l = \frac{1}{2}\arg\max_{Q}\Pi\left(Q, q_3\right) \tag{24}$$

with $l = 1, 2$

The solution of the maximization in 23-24 gives the Insider's best reply, that is:

$$\overline{q}_l\left(q_3\right) = \frac{1}{4}a - \frac{1}{4}q_3 - \frac{1}{4}c. \tag{25}$$

Since production cost is linear any merged entity will be indifferent with respect to how to split its total output among the Insiders, hence every coalition of firms (or merged entity) behaves as a single firm, that is $q_l = q_1 = q_2$.

Consider the maximization problem for firm 3 in case a merger has occurred and when a constraint on the profitability of the Insiders is imposed.

The Outsider solves the following problem:

$$\max_{q_3}\ \pi_3\left(q_l, q_l, q_3\right) = \left(a - 2q_l - q_3\right)q_3 - cq_3, \tag{26}$$

$$s.t.\ \pi_1\left(q_l, q_l, q_3\right) \geq \pi_1^C\left(q_1^C, q_2^C, q_3^C\right), \tag{27}$$

$$\pi_2\left(q_l, q_l, q_3\right) \geq \pi_2^C\left(q_2^C, q_1^C, q_3^C\right). \tag{28}$$

The constraints (27) and (28) mean that the Outsider chooses a quantity such that the profit for the new merged entity formed by firm 1 and 2 (Insiders) is (weakly) greater than its profit in case of no merger.

The economic reasoning behind constraints (27) and (28) is the following. To mitigate the positive externality gained by Outsider we constrain its quantity so that the aggregate market price in post-merger phase remains sufficiently high. Once the Outsider has constrained its production, the cause of the unprofitability for the Insiders is partially ruled out. Intuitively the solution of problem (26)-(28) will be a quantity higher than the quantity produced by each producer in standard Nash-Cournot game with 3 firms but less than the quantity each Outsider would produce without constraining its maximization.

The quantities q_l and q_3 are simultaneously chosen as in the standard Nash-Cournot game. Notice that despite the Outsider chooses a suboptimal quantity, it obtains nevertheless higher profits than in the absence of the merger.

The only difference with the previous standard Cournot game is that now Outsider's choices are constrained by (27) and (28).

The Outsider's best reply is derived by solving problem (26)-(28).

The maximum conditions for the Outsider's maximization problem are:

$$\frac{\partial L}{\partial q_3} = -2q_3 + a - 2q_l - c - 2\lambda q_l = 0,\ q_3 \frac{\partial L}{\partial q_3} = 0 \tag{29}$$

$$\frac{\partial L}{\partial \lambda} = -2\left(\frac{a-c}{4}\right)^2 + 2\left(\left(a - 2q_l - q_3\right) q_l - cq_l\right) \geq 0,\ \lambda \geq 0,\ \lambda \frac{\partial L}{\partial \lambda} \tag{30}$$

where λ is the Lagrangian multiplier.
The Outsider's best reply satisfying the conditions (29) and (30) is:

$$\overline{q}_3\left(q_l\right) = -\frac{1}{16} \frac{a^2 - 2ac + c^2 - 16q_l a + 32q_l^2 + 16cq_l}{q_l}. \tag{31}$$

Now, combining (25) and (31) we obtain the following optimal quantities:

$$\overline{q}_l^* = \frac{1}{8}\sqrt{2}\left(a-c\right), \tag{32}$$

$$\overline{q}_3^* = \left(a-c\right)\left(1 - \frac{\sqrt{2}}{2}\right), \tag{33}$$

The profits are:

$$\pi_3\left(\overline{q}_3^*, \overline{q}_l^*\right) = \frac{1}{4}\left(\sqrt{2}-1\right)\left(a-c\right)^2 \tag{34}$$

$$\pi_1\left(\overline{q}_l^*, \overline{q}_3^*\right) = \pi_2\left(\overline{q}_l^*, \overline{q}_3^*\right) = \frac{1}{16}\left(a-c\right)^2 = \frac{1}{2}\Pi\left(Q^M, q_3^M\right) \tag{35}$$

Profits in (34) and (35) show that there exists a market outcome (not an equilibrium) in which a bilateral merger is never unprofitable for Insiders. In such an outcome each Insider produces more than the quantity it would produce in the standard merger case $\left(q_l^M\right)$ but less than standard Cournot quantity $\left(q_l^C\right)$. The Outsider produces more than standard Cournot quantity $\left(q_3^C\right)$ but less than standard merger case $\left(q_j^M\right)$. Given the quantities $\overline{q}_l^*$ and $\overline{q}_3^*$, the positive externality experienced by the Outsider decreases and the profits for merged entity are equal to the sum of the two pre-merger standard Cournot profits. Despite in such bilateral merger the Outsider is forced to produce less than its preferred quantity, it gains a profit higher than the standard Cournot one.

In the next section we construct a dynamic framework to characterize such market outcome as an equilibrium.

3 Dynamic game

In this section we study an infinitely repeated game whose constituent game G is the 2-stage game of the previous section. The player are three: two Insiders and one Outsider (firm 3).

Now, let $G^{\infty}(\delta)$ be the supergame formed by infinitely repeating the two-stage game G. In each period t the players take their actions and maximize the discounted sum of profits, where $\delta_i \in (0,1)$ is discount factor for each of them with $i = l, 3$. The discount factor reflects time preference and continuation probability. Let $q^t \equiv (q_l^t, q_3^t)$ be the quantities chosen in period t respectively by each Insider and the Outsider, with $q \in \widehat{Q}$ (where $\widehat{Q}$ is the quantity space). We assume perfect monitoring in the sense that at the end of each stage each player observes the actions (or equivalently the output) taken by the other players up to that stage. At the end of period each profit is realized.

Assume that the game starts in period 0, with the null history h^0. For $t \geq 1$, let h^t be the vector of previous actions undertaken by the players in period up to $t-1$. Let also $h^{t(1)}$ be the vector of previous actions undertaken by the players up to the first stage of period t. Let $H^t = \widehat{Q}^{t-1}$ be the set of possible t-period histories with $H^1 = \{0\}$ and $H = \cup_{t=1}^{\infty} H^t$ the set of all possible histories.

The repeated game strategy for the Insiders and the Outsider are respectively indicated by $s_l(h^t)$ and $s_3(h^{t(1)})$.

Now, let us define the payoffs for each player.

The Insider's payoff function is defined as:

$$V_l = (1-\delta) \sum_{t=0}^{\infty} \delta_l^t \pi_l \left(s_l \left(h^t \right), s_3(h^{t(1)}) \right), \tag{36}$$

while the Outsider's payoff function is:

$$V_3 = (1-\delta) \sum_{t=0}^{\infty} \delta_3^t \pi_3 \left(s_3(h^{t(1)}), s_l \left(h^t \right) \right) ^{5} \tag{37}$$

From section 2 we know that for $m = 2$ and $n = 3$ the 2-stage game G has an unique Nash equilibrium in pure strategies. This equilibrium is characterized by the following strategies: at stage 1, the Insiders decide not to merge and at stage 2 the Insiders and the Outsiders play a standard Cournot game. The reason of this choice is that, at stage 1, the Insiders forecast gaining a joint profit less than the sum of the profits gained by each potential Insider in status quo.

[5] The repeated game payoffs are often normalized by multiplying by $(1-\delta)$. This normalization factor allows to measure the profit in the stage game and in the repeated game in the same units

The aim of this section is to show that the market outcome presented in 32-35 characterizes a sub game perfect equilibrium of supergame $G^{\infty}(\delta)$.

Firstly we state the following definition:

Definition 3 *: Let $\widehat{h}^{t}$ be an history at time t such that up to time $t-1$ the Insiders have always chosen to sign the merger agreement and to choose the joint-profit best reply $\overline{q}_{l}(q_{3})$, and the Outsider has always chosen the constrained best reply $\overline{q}_{3}(q_{l})$.*

Definition 4 *Let $\widehat{h}^{t(1)}$ be an history at time t such that up to the first stage of time t the Insiders have always chosen to sign the merger agreement and to choose the joint-profit best reply $\overline{q}_{l}(q_{3})$, and up to time $t-1$ the Outsider has always chosen the constrained best reply $\overline{q}_{3}(q_{l})$*

Now, let us define the following strategy for the Insiders(l) and Outsider(denoted by 3):

Definition 5 *Let the Insiders' strategy $\widetilde{s}_{l}(h^{t})$ be defined by the following sequence of actions:*

Stage 1

- *if $h^{t}=h^{0}$, enter the merging agreement (merge, henceforth);*
- *if $h^{t}=\widehat{h}^{t}$, merge;*
- *do not merge, otherwise.*

Stage 2

- *play $\overline{q}_{l}(q_{3})$ if played "merge" in the previous stage;*
- *play $q_{l}^{C}\left(q_{-l}^{C},q_{3}\right)$, otherwise.*

Definition 6 *Let the Outsider's strategy $\widetilde{s}_{3}(h^{t(1)})$ be defined by the following sequence of actions:*

Stage 1

- *do nothing ;*

Stage 2

- *play* $\overline{q}_3(q_l)$ *if* $h^{t(1)} = \widehat{h}^{t(1)}$*;*
- *play* $q_3^C(q_1, q_2)$*, otherwise.*

Definition 5 and 6 enable us to state the proposition representing the main result of the paper

Proposition 7 *Let* $\widetilde{\delta}_3 = 0.15$*. When* $\widetilde{\delta}_3 < \delta < 1$ *the strategy profile* $\{\widetilde{s}_l, \widetilde{s}_3\}$ *are a SNE of the game*

Proof. 1) First, prove $\widetilde{s}_l(h^t)$ and $\widetilde{s}_3\left(h^{t(1)}\right)$ are NE
1.A)
Assume the Insider plays $\widetilde{s}_l(.)$. Is $\widetilde{s}_3(.)$ BR[6] for the Outsider?
-if the Outsider plays $\overline{q}_3(q_l)$, then it obtains

$$\pi_3(\overline{q}_3^*, \overline{q}_l^*) = \frac{1}{4}\left(\sqrt{2}-1\right)(a-c)^2. \tag{38}$$

as in (34). See Appendix A for the computation.
Its infinite-horizon payoff is then given by

$$\overline{V}_3 = \overline{\pi}_3 + \delta_3 \overline{V}_3, \tag{39}$$

$$\overline{V}_3 = \frac{1}{1-\delta_3}\overline{\pi}_3, \tag{40}$$

-if the Outsider "deviates", it solves the unconstrained maximization problem and obtains:

$$\max_{q_3} \pi_3(\overline{q}_l, q_3) = \overline{\overline{\pi}}_3(\overline{q}_l, \overline{q}_l, q_3) = \frac{1}{8}(a-c)^2. \tag{41}$$

Since "deviation" will trigger a decision not to merger forever by all the Insiders and there is not lag in the collection of the information from one period to another, the Outsider's payoff is given by:

$$\overline{\overline{V}}_3 = \overline{\overline{\pi}}_3 + \frac{\delta_3}{1-\delta_3}\pi_3^C, \tag{42}$$

Let $\widetilde{\delta}_3$ be the critical value of δ such that

$$\frac{1}{1-\delta_3}\overline{\pi}_3 \geq \overline{\overline{\pi}}_3 + \frac{\delta_3}{1-\delta_3}\pi_3^C, \tag{43}$$

[6] BR means best reply

Then, for any $\delta > \widetilde{\delta}_3 = 0.15$[7], $\widetilde{s}_3\left(h^{t(1)}\right)$ is BR to $\widetilde{s}_l\left(h^t\right)$.

1.B)

Since there is an infinitely high penalty for Insider's unilateral deviation, no Insider would want (independently from the other Insider) to firstly agree to merge and then deviate. However, the problem of cooperation between Insiders and Outsider could remain. At this regard we proceed as in part 1.A.

Assume the Outsider plays $\widetilde{s}_3(.)$, then $\widetilde{s}_l(.)$ is BR for the Insiders?

-if the Insiders play $\overline{q}_l(q_3)$, then obtains:

$$\overline{\pi}_l\left(\overline{q}_l^*, \overline{q}_3^*\right) = \frac{1}{16}(a-c)^2 = \frac{1}{2}\Pi\left(Q^M, q_3^M\right), \tag{44}$$

as in (35).

Their infinite-horizon payoff is then given by:

$$\overline{V}_l = \overline{\pi}_l + \delta_l \overline{V}_l, \qquad l = 1, 2 \tag{45}$$

$$\overline{V}_l = \frac{1}{1-\delta_l}\overline{\pi}_l. \tag{46}$$

-if the Insider "deviates" in the second stage despite that in the first it played "merge", it will pay infinitely high cost. Therefore it never deviates in stage 2 if it has entered the agreement at stage 1.

However, since "deviation" will trigger a decision not to use constrained best reply forever by the Outsider and the "cooperative" outcome is weakly preferred by the Insider to the Cournot outcome, then the Insiders have no incentive to deviate either at stage 1.

Hence, the Insiders always want to merger in any stage of the game and $\widetilde{s}_l\left(h^t\right)$ is BR to $\widetilde{s}_3\left(h^{t(1)}\right)$ $\forall\ \delta_l \in (0,1)$.

2) Second, prove $\widetilde{s}_l\left(h^t\right)$ and $\widetilde{s}_3\left(h^{t(1)}\right)$ are SNE

Let me define two class of sub-games

Class A: history is $\widehat{h}^t$

Class B: history is not $\widehat{h}^t$

For Class A sub-games, previous analysis shows that players' strategy are SNE of each sub-games. This is because each sub-game is identical to the whole game for which we just proved strategies are NE.

Consider Class B games:

Since the Outsider will never play constrained quantity, the Insiders are better off adhering to the strategy $\widetilde{s}_l$ and not to merging. Since the Insider will never merge, the Outsider is weakly better off adhering to the strategy and removing the constraint from its maximization problem. So in the case of Class B the strategy for both the Insider and the Outsider is not merging forever, which is a NE for static game. ■

[7] The computation for the value of $\overline{\overline{\pi}}_3\left(\overline{q}_l, \overline{q}_l, q_3\right)$ and $\widetilde{\delta}_3$ are in Appendix A.

Proposition 7 illustrates that under infinite horizon interaction, if the Outsider is sufficiently patient, then in equilibrium the Insiders remain merged in every period and the Outsider reacts to this merger by limiting its quantity. In such a way the merger is never unprofitable for the potential Insiders and the Outsider gains higher profit than in absence of merger. This result clearly reverts the prediction in SSR which show that in a market with linear demand and constant marginal costs a bilateral horizontal merger is profitable if and only if at least the 80% of the active firms merge.

The same results on the profitability are also found in Dockner & Gaunersdorfer (2001) and Benchekron (2003). In particular, they apply the "sticky price model" introduced by Fershtman and Kamien (1987) and study the limit case in which the prices instantaneously adjust. They use Markovian strategy and show that the strict profitability for horizontal merger does not depend on the number of merging firms. However, this Markovian strategy does not allow them to affect the incentives for the firms to remain Outsiders.

In our paper, the use of the trigger strategy allows enforcing the constraint on the Outsider's production over an infinite horizon. This constraint does not let Outsider strongly increase its production as replies to the merger, therefore the aggregate price remains high enough to make the merger profitable.

Proposition 8 *Let be $n = 3$ and $m = 2$, the sub-game perfect equilibrium characterized by the strategy profiles $\widetilde{s}_l\left(h^t\right)$ and $\widetilde{s}_3\left(h^{t(1)}\right)$ guarantees a market outcome in which the incentive for the Insider to become Outsider is mitigated.*

Proof. Let Δ_D^π be the difference between the profits in 34 and 35. Is is defined as follows:

$$\Delta_D^\pi = \pi_3\left(\overline{q}_3^*, \overline{q}_l^*\right) - \pi_l\left(\overline{q}_3^*, \overline{q}_l^*\right) = \frac{1}{16}\left(4\sqrt{2}-5\right)(a-c)^2, \tag{47}$$

with $l = 1, 2$.

Now, given $n = 3$ and $m = 2$ we rewrite 21 as follows

$$\Delta_S^\pi = \pi_j^M\left(Q^M, q_3^M\right) - \pi_l^M\left(Q^M, q_3^M\right) = \frac{1}{18}(a-c)^2. \tag{48}$$

Hence, we have:

$$\Delta_S^\pi > \Delta_D^\pi \tag{49}$$

■

Proposition 8 says that the difference between the profits gained by the Outsider and the Insider in static contest is higher than the difference between the profits they gain in each period of the dynamic game. The reason is because in the SNE, in each period the Outsider reacts to the merger by not increasing its production too much. This less "aggressive" response by the Outsider implies that, once the merger has occurred, the aggregate price remains sufficiently high to rule out the unprofitability for the Insiders. The constraint on the quantity reduces the positive externality that the Outsider exploits after the merger, therefore the incentive for the Insider to become Outsider decreases and the Insider's dilemma is mitigated.

The use of trigger strategy would also allow us to characterize a sub-game Nash equilibrium in which the merger is strictly profitable for both the Insider and the Outsider, and the incentive for remaining outside the merger completely disappears. To achieve such equilibrium we just need to construct a trigger strategy in which the Outsider is forced to produce less than the quantity of the Insider. In this outcome, despite that the quantity of the Outsider is strongly low, the aggregate price is so high that after the merger, the Outsider still gains higher profit than in the status quo. We characterize this equilibrium in Appendix B.

4 Conclusions

This paper shows that, in a dynamic framework, a bilateral horizontal merger between symmetric firms is never unprofitable and the incentive to remain Outsider is mitigated. We construct an infinitely repeated two-stage game where the number of the merger participants is exogenously given. At stage 1, the Insiders can choose whether to enter a merging agreement. The merging agreement entails that, when entering the agreement, the firm chooses to offer $\frac{1}{m}$ of the quantity maximizing the joint profit of the m firms or, otherwise, it will have to pay an infinitely high penalty payment. At stage 2, a Cournot game takes place.

Given the structure of the game, we find an equilibrium in trigger strategies such that the Insiders threaten to dissolve or not to merger if the Outsider does not limit its post-merger production. The limitation on the Outsider reaction determines the results of the paper in a fundamental way. Firstly, if the firms remaining out of the merger do not strongly increase their quantity, then the post-merger aggregate price remains sufficiently high and the unprofitability for the Insiders is ruled out. Secondly, this quantity constraint reduces the positive externality gained by the Outsider with the consequence that the incentive to remain out of the merger is mitigated.

Several papers study the profitability of horizontal mergers in the case of both endogenous and exogenous merging process. Salant and alter (1983) is considered the seminal paper belonging to the latter approach. They predict that, in a static framework where the number of merging firms is exogenously given, a bilateral horizontal merger between symmetric competitors is not profitable and each firm has an incentive to remain out of the merger because it would benefit more (Insider's dilemma).

The most recent studies have partially countered these results. Dockner & Gaunersdorfer (2001) and Benchekron (2003) develop a sticky price dynamic model in Markovian strategies where the merger results from an exogenous change in the number of active firms. They find that the profitability for the horizontal mergers does not depend on the number of merging firms. However the drawback of these papers is that the Insider dilemma, predicted by Stiger and formalized by SSR, still remains: each firm has incentive to remain Outsider, despite the merger is profitable.

Lindqvist (2003) is the first to solve the Insider dilemma in a model where the merger process is endogenously determined by the firms. He constructs a model where each firm can buy a share in a competitive firm before an acquisition of another competitor. The acquisition of this share allows the Insider to increase its profitability by stealing part of the Outsider's profit.

We are the first to show that, in a model where the number of merger participants is exogenous, a bilateral horizontal merger between symmetric firms is profitable and the Insider dilemma is eliminated. As in Dockner & Gaunersdorfer (2001) and Benchekron (2003), we use an infinitely repeated game, but the difference is that we find an equilibrium in trigger strategy. Differently from the Markovian strategy used in Dockner & Gaunersdorfer and Benchekron, the trigger strategy allows the merger participants to force the Outsider on not to strongly increase its production as replay to the merger. Hence, the positive externalities which the Outsider would gain in absence of this quantity limitation are decreasing and the aggregate price remains high enough to make the merger profitable for the Insiders.

We show that the solution of the Insider dilemma does not arise from the endogenezation of the merger process, but depends on the nature of the competition.

This paper provides a theoretical explanation for the several bilateral mergers observed in the industries where the merger size is exogenously decided.

APPENDIX A

We will sketch the proofs of proposition 2, 6.

Proof of Proposition 2
Given a fix value for n we simply have:

$$\frac{\partial \Pi^M \left(Q^M, q_{m+1}^M, ..., q_n^M\right)}{\partial \left(\frac{m}{n}\right)} = \frac{d\left(\left(\frac{a-c}{n-m+2}\right)^2\right)}{d\frac{m}{n}} = \frac{(a-c)^2}{(n-m+2)^2 m} n > 0, \tag{50}$$

since $n \geq m > 2.$■

Proof of Proposition 7 (computation)
This strategy profile s_3 is a Nash equilibrium for all δ satisfying the following inequality:

$$(1-\delta_3) \max_{q_3} \pi_3 \left(\overline{q}_l, q_3\right) + \delta_3 \pi_3^C < \overline{\pi}_3 \left(\overline{q}_3^*, \overline{q}_l^*\right). \tag{51}$$

We solve the Outsider's maximization problem in the period in which it has deviated. Since the other techniques used to solve (51) are only substitutions, then we omit them.

$$\max_{q_3} \pi_3 \left(\overline{q}_l, q_3\right) = \left(a - 2\overline{q}_l - q_3\right) q_3 - cq_3 \tag{52}$$

Since at stage 1, the Insiders have decide to merger and produce $\overline{q}_l$, then the optimal value $\overline{\overline{q}}_3$ maximizing (52) is defined by the following expression:

$$\arg\max_{q_3} \pi_3 \left(\overline{q}_l, q_3\right) = \overline{\overline{q}}_3 \tag{53}$$

where:

$$\overline{\overline{q}}_3 = \frac{1}{2}a - \frac{1}{2}c - q_l, \tag{54}$$

Now, to find the equilibrium quantities we solve the system between (54) and (25) and obtain:

$$\overline{q}_l = \frac{a-c}{6}, \tag{55}$$

$$\overline{\overline{q}}_3 = \frac{a-c}{3}, \tag{56}$$

by substituting (56) and (57) in the Outsider's profit function we obtain:

$$\overline{\overline{\pi}}_3\left(\overline{q}_l, \overline{q}_l, \overline{\overline{q}}_3\right) = \frac{(a-c)^2}{9}, \tag{57}$$

Finally, we substitute (57),(9) and (34) in (51) and obtain:

$$(1-\delta_3)\frac{(a-c)^2}{9} + \delta_3\left(\frac{a-c}{4}\right)^2 < \frac{1}{4}\left(\sqrt{2}-1\right)(a-c)^2, \tag{58}$$

that holds for every $\delta_3 > 0.15.\blacksquare$

APPENDIX -B

We show that there exists an equilibrium in which the incentive to remain Outsider is completely solved. To do that we proceed in two steps.

First step

We define the outcome in which the merger is strictly profitable for both Insider and Outsider and the Insider's dilemma is completely solved.

To do that we need to find the quantity the Outsider would have to produce to gain a profit at least equal to the standard Cournot triopoly one but lower than the profit of the Insider.

We assume that the Insiders do not change the production respect to the standard merger case in section 2, therefore their quantity remains:

$$q_l^M = \frac{1}{2}\frac{a-c}{3}. \tag{59}$$

Let Outsider produce the quantity q_3 such that the following identity is satisfied:

$$\pi_3\left(q_l^M, q_l^M, q_3\right) = \pi^C\left(q_l^C, q_3^C\right) + \gamma\left(\pi_3\left(Q^M, q_3^M\right) - \pi^C\left(q_l^C, q_3^C\right)\right), \tag{60}$$

where $\gamma \in [0, 1]$.

The LHS of (60) represents the Outsider's profit function when the two Insiders produce the quantity given in (59).

The first term in the RHS is the standard Cournot profit with three firms, the term in brackets is the difference between the post-standard merger profit defined in (17) and the Cournot one.

The Outsider's quantity q_3^γ solving the identity (60) is the following:

$$q_3^\gamma = (a-c)\left(\frac{4-\sqrt{7}\sqrt{(1-\gamma)}}{12}\right). \tag{61}$$

Given (61) and (59), the profits are:

$$\overline{\pi}_3\left(q_l^M, q_l^M, q_3^\gamma\right) = \frac{1}{144}(c-a)^2(7\gamma+9), \tag{62}$$

$$\overline{\pi}_l\left(q_l^M, q_l^M, q_3^\gamma\right) = \frac{1}{72}(a-c)^2\left(4+\sqrt{7}\sqrt{(1-\gamma)}\right), \tag{63}$$

where:

$$\overline{\pi}_l\left(q_l^M, q_l^M, q_3^\gamma\right) \geq \pi^C\left(q_l^C, q_3^C\right) \quad \text{for } \gamma \in \left[0, \frac{27}{28}\right], ^{8} \tag{64}$$

$$\overline{\pi}_3\left(q_l^M, q_l^M, q_3^\gamma\right) > \pi^C\left(q_l^C, q_3^C\right) \quad \text{for } \gamma \in (0,1)\,, ^{9} \tag{65}$$

$$q_3^\gamma < q_3^\gamma \text{ for } \gamma \in \left(0, \frac{6}{7}\right)^{10}. \tag{66}$$

$$\overline{\pi}_3\left(q_l^M, q_l^M, q_3^\gamma\right) \leq \overline{\pi}_l\left(q_l^M, q_l^M, q_3^\gamma\right) \text{ and } q_3^\gamma \leq q_l^M \text{ for } \gamma \in \left(0, \frac{3}{7}\right) \tag{67}$$

Now, let us write the measure of Insider's Dilemma as follows:

$$\Delta^\pi = \overline{\pi}_l\left(q_l^M, q_l^M, q_3^\gamma\right) - \overline{\pi}_3\left(q_l^M, q_l^M, q_3^\gamma\right), \tag{68}$$

given (62) and (63), we have:

[9] Where $\overline{\pi}_3\left(q_l^M, q_l^M, \overline{q}_3\right) = \pi^C\left(q_l^C, q_3^C\right)$ for $\gamma = 0$

$$\Delta^{\pi} = \left(-\frac{1}{144}\right)(a-c)^2\left(7\gamma - 2\sqrt{7}\sqrt{1-\gamma}+1\right) \geq 0 \Leftrightarrow \gamma \in \left(0, \frac{3}{7}\right] \qquad (69)$$

Then the Insider's Dilemma disappears when the Insiders and the Outsider respectively produce q_l^M and q_3^γ , and $\gamma \in \left(0, \frac{3}{7}\right]$.

Second step

We characterizing such market outcome as equilibrium. We rewrite the strategies given by Definition 5 and 6 by substituting $\overline{q}_3(q_l)$ and $\overline{q}_l(q_3)$ respectively with q_3^γ and q_l^M, and assuming $\gamma \in \left(0, \frac{3}{7}\right]$. Following the same proof of Proposition 7, we show that the new strategies defined on q_3^γ and q_l^M characterize a SNE for every discount factor $\delta_3 \geq \widetilde{\delta}_3 = 1-\gamma$. Where $\gamma \in \left(0, \frac{3}{7}\right]$ implies that $\widetilde{\delta}_3 \in \left[\frac{4}{7}, 1\right)$.

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