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## DEPARTMENT OF ECONOMICS

# OPTIMAL DISTRIBUTION OF POWERS IN A FEDERATION: A SIMPLE, UNIFIED FRAMEWORK

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# Optimal Distribution Of Powers In A Federation: A Simple, Unified Framework[*]

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### Abstract

In a federation with $n \geq 2$ regions the relative optimality of six regimes- autarky, centralization, unregulated devolution, regulated devolution, direct democracy, and revenue maximising leviathan, is examined. Public policy consists of redistribution and regional public good provision. Regional incomes are uncertain and correlated while estimates of the usefulness of regional public goods are uncertain; the federal government's estimates are noisier relative to those of regional governments. The optimality of each regime is influenced by four margins- regional insurance, coarseness of federal information, internalisation of spillovers and 'raiding the commons'. Regulated devolution is the only regime that is capable of producing the constrained first best level of public goods. Federal insurance under the two regimes of direct democracy and a federal leviathan, can be inadequate relative to that under a utilitarian federal government. An increase in the number of regions has important implications for insurance and raiding the commons. The median region's choice of redistribution under direct democracy is influenced in important ways by the distribution of regional uncertainties. The paper synthesises a significant proportion of the existing literature in a single model and also provides several new results.

*Keywords:* Insurance, Spillovers, Information, Raiding the commons, Uncertainty.

*JEL Classification:* H77 (Intergovernmental Relations; Federalism), D78 (Positive Analysis of Policy-Making and Implementation), H41 (Public Goods).

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# 1. Introduction

## 1.1. The traditional view and the modern critique

In the traditional theory of fiscal federalism, the federal government is responsibile for undertaking macroeconomic stabilisation and redistribution across individuals. The primary function of local governments is to provide local public goods. If inter-regional spillovers exist then the federal government's responsibility is extended to include matching regional grants to the regions so that they internalise the spillovers. While the federal government levies federal taxes at some uniform rate, such as the income tax, the local governments rely on benefit taxes, such as property taxes. Indeed, when households are mobile, the Tiebout model demonstrates an efficient outcome, achieved by benefit taxes. In particular, the "Decentralisation Theorem" asserted that such an outcome is Pareto superior relative to federal provision of local public goods at a uniform rate; see for instance Oates (1972).

Several aspects of the traditional theory have been the subject of much recent investigation. Of relevance to this paper are the following.

First, regional governments might possess asymmetric information on the demand/ cost conditions while the federal governments are less informed[1]. Second, the recent literature on federal redistribution has stressed the ability of federal redistributive policy to internalize correlated shocks to income across regions[2]. Hence, even in the absence of mobility across regions, a federal redistributive policy might dominate a regional policy[3]; for instance, Persson and Tabellini (1996) and Alesina and Perotti (1998)[4]. Third, actual policy might have to be crafted in the face of substantial uncertainty on several counts. In particular, the demand/ cost conditions for the individual regions might be uncertain. Furthermore, regional incomes might not only be uncertain, but correlated with each other. The modern literature has only recently starting addressing these issues; for instance Lockwood (1999). Fourth, in the "Decentralisation Theorem", the alternative to local provision is uniform federal provision. This has been questioned by Besley and Coate (2003) who allow the federal government to undertake regional public good provision at a non-uniform rate. Fifth, the traditional literature overlooks the "hard budget constraint"

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[1]See for example Boadway et al. (1998), Bordignon et al. (1996), Bucovetsky et al. (1998), and Lockwood (1999). The federal government can then elicit the regional government's private information by conditioning federal grants on regional observables.

[2]Sala-I-Martin and Sachs (1992) calculate, using US data, that a $1 decrease in a region's per capita income triggers, on average, a decrease in federal taxes of 34 cents and an increase in transfers of about 6 cents. Asdrubali et al. (1996) estimate that 13 percent of shocks to GDP are smoothed by the federal government.

[3]Costs of bargaining and issues of asymmetric information might limit the practical applicability of regional contracts to share risks; see Inman and Rubinfeld (1997).

[4]Private markets typically do not enable regions or individuals to fully share all risks. Empirical evidence is supportive of this assumption; for instance, Campbell and Viceira (2002).

or "raiding the commons" problem. This refers to the tendency of regional governments to ignore the full costs of their actions when faced with the possibility of being bailed out by the federal government in times of financial distress; for example, Kornai, Maskin and Roland (2003). Hence, regional public good provision, as well as regional actions involving self-insurance, might be distorted.

## 1.2. Objectives of the paper and differences from related literature

The objective of this paper is to provide a unified model that simultaneously incorporates the modern critique of the traditional theory of federalism[5]. Most of the recent literature considers, exclusively, the tradeoff between only two of the following four margins alluded to in the modern critique above, viz. (i) information, (ii) insurance, (iii) raiding the commons, and (iv) internalisation of spillovers[6]. Furthermore, the paper differs in the following respects from the recent literature.

First, with the exception of Lockwood (1999), the issues of redistribution and public good provision in a federation are not simultaneously addressed. The relative optimality of alternative regimes might hinge on the tradeoff between redistribution and public good provision. Second, all sources of uncertainty are not considered simultaneously. It is conceivable that the balance of various uncertainties might determine the optimality of alternative regimes. Third, the literature on regional redistribution restricts attention to the special case of perfect negative correlation among regional incomes, while the degree of correlation might affect the relative optimality of regimes. Fourth, the implications of an increase in the number of regions in the federation are not explored. This raises important issues of insurance when risks are pooled and the commons are raided. Fifth, the political economy extensions have not fully considered the implications for regional redistribution as the distribution of regional incomes changes[7]. In particular, how is the redistributive tax rate chosen by a median region affected by uncertainty in own income relative to uncertainty elsewhere in the federation?

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[5]The appropriate framework to incorporate issues of information asymmetry is a mechanism design approach. However, as illustrated in Lockwood (1999) such an approach is not tractable when insurance considerations combine with incentive issues unless the problem is simplified. Hence, this paper models information asymmetry but not the information elicitation problem.

[6]The traditional literature, for instance, considered the tradeoff between information (interpreted as uniform provision) and internalisation of spillovers; for example Oates (1972). Lockwood (1999) considers the tradeoff between internalisation of spillovers and information. Besley and Coate (2003) consider the tradeoff between raiding the commons and internalisation of spillovers.

[7]See, however, Persson and Tabellini (1996). For $n = 2$ regions, among other issues, they make the point that federation-wide voting can lead to a suboptimal inter-regional insurance.

### 1.3. Brief description of the model

A federation comprises of $n \geq 2$ regions, each populated with a representative agent. The federation has an overarching federal government and, in addition, each region has its own regional government. There is no mobility of labour or capital between the regions. Public policy comprises of (i) non-distortionary redistribution across regions, and (ii) provision of a public good in each region, that causes inter-regional spillovers; there are no national public goods. Public policy can be exercised by either the federal or the regional governments.

At the time of public policy formulation, there are the following, simultaneous, sources of uncertainty. First, incomes in each region are uncertain and correlated. Second, estimates of the usefulness of public good provision in each region are uncertain[8]. Third, asymmetries exist in the estimates made by the federal and the regional governments of the usefulness of regional public good; the federal government's estimates are relatively noisier (coarser information).

The appropriate authorities first choose the tax rate and the provision of regional public goods. This is followed by the realisation of various sources of uncertainty. In the final stage, for each state of the world, state contingent tax revenues are collected, and private, public good consumption, takes place. Public goods are committed to, ex-ante, prior to the collection of the state dependent tax revenues[9].

Distribution of powers between the federal and regional governments in the exercise of public policy manifests in the form of the following six possible regimes; Autarky, Centralisation, Devolution, Regulated Devolution, Direct Democracy and a Revenue Maximising Leviathan who is risk averse. In all regimes other than autarky, the redistributive policy is undertaken at the federal level. Local public goods are provided by each of the regions in all regimes except for the regime of centralisation where they are provided by the federal government. Federal grants for purely redistributive purposes are provided in all regimes except under autarky, however, federal grants contingent on local public good provision are provided only under the regime of regulated devolution. The optimality of each regime is judged on the basis of four margins, namely, information, insurance, raiding the commons and spillovers.

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[8]For instance, before additional local police is introduced, the magnitude of reduction in crime is uncertain. Similar qualitative results are obtained under uncertainty about cost of public goods.

[9]Public goods often have a gestation lag and require some sunk costs in the beginning for their realisation. Formal modeling of these issues is beyond the scope of this paper. Notice that the level of public goods only needs to be committed to, not actually provided, before the realisation of various sources of uncertainty. Actual consumption of public goods only takes place in the last stage.

### 1.4. The results

The relative optimality of a regime is an outcome of the following four margins: information, internalisation of spillovers, insurance and raiding the commons, as explained below:

1. *Autarky*: Each region undertakes its own redistribution and public good provision. Regions are unable to provide any insurance for themselves and ignore spillovers arising from their public good provision. Autarky has two advantages: better estimates about the usefulness of the regional public good, and no "raiding the commons" problem because regional public goods are financed by the region's own taxes.

2. *Centralization*: The federal government undertakes regional redistribution and regional public good provision. Centralisation corrects for the defects under autarky but suffers on account of the federal government's 'coarser' information.

3. *Unregulated Devolution*: The federal government undertakes regional redistribution. Each of the regions provides its own public good, financed from the pool of federal tax revenues. Hence, the drawbacks are that the regional governments raid the commons and ignore public good spillovers. The advantages of this regime are: availability of federal insurance and better regional information.

4. *Regulated Devolution*: This regime is similar to unregulated devolution but corrects for its two shortcomings by using (i) externality correcting federal grants, which force regional governments to internalise public good spillovers, and (ii) regional benefit taxes that eliminate the raiding of the common federal tax pool. These result in a constrained first best level of public good provision.

5. *Direct Democracy*: Under appropriate conditions, a decisive median voter chooses the federal redistributive tax rate while regions provide their own public goods. The analysis clarifies the asymmetric effects on federal redistribution arising from uncertainty in the median region and uncertainty in non-median regions and shows that relative to a utilitarian federal government, regional insurance might be inadequate.

6. *Revenue Maximising Leviathan*: Under uncertainty, the revenue maximising leviathan, exemplified in the public choice tradition, is modeled as caring about expected tax revenues and the variance of the uncertain tax revenues; so the leviathan is risk averse. The main contribution here is to show that the federally provided regional insurance might be inadequate.

Other results are as follows. The affect of an increase in the number of regions has important implications for the relative optimality of the regimes, a factor that has topical

significance in light of the current increase in the membership of the European Union. An increase in the number of regions has two main implications. First, it allows a better pooling of risks when incomes across regions are correlated. This increases the payoff under centralisation and various forms of devolution, relative to autarky. Second, the opportunity cost of 'raiding the commons' of a unit of pooled federal tax revenues for a region is $\frac{1}{n}$, which is the amount of federal transfers foregone by a region if it increases its regional public good by a unit. Hence, an increase in the number of regions, worsens the raiding the commons problem. Furthermore, it is shown that as the number of regions increases, regional covariances in incomes play a relatively more important role as compared to regional variances.

Direct democracy can have important distortionary effects on the federal tax rate. The decisive median voter increases the federal tax rate if its regional uncertainty in income increases; this is reminiscent of the social insurance role of taxation in Varian (1980). However, the median region lowers the federal tax rate following an increase in uncertainty in any other region; by doing so the region reduces its exposure to risk that comes from sharing the total tax revenues. A similar intuition accounts for an increase in the median region's choice of the federal tax rate when regional covariances in incomes in regions other than the median region increase.

Finally, a risk averse but revenue maximising leviathan can under-provide regional insurance through the tax system. In the absence of any tax distortions, complete insurance arises from pooling all federation-wide income by setting the federal tax rate equal to unity i.e. $t = 1$, and then distributing the tax proceeds equally among the regions. However, providing complete insurance might not be optimal for a leviathan because $t = 1$ might cause excessive variance in tax revenues that the risk-averse leviathan dislikes. Hence, the leviathan might settle for providing incomplete insurance.

### 1.5. Organisation of the paper

Section 2 outlines the structure of the model. Section 3 undertakes a normative analyses of redistribution and regional public goods provision under autarky, centralisation, devolution and regulated devolution. Section 4 addresses some political economy considerations under the regimes of direct democracy and a risk-averse but revenue maximising leviathan. Finally, Section 5 concludes. All proofs are contained in the appendix.

## 2. THE MODEL

There are $n \geq 2$ regions, indexed by $i = 1, 2..., n$. Each region is populated by a representative individual and the regions jointly comprise a federation. There is an overarching federal government and each region has its own regional government. Public policy, in the

model, comprises of (1) the provision of region-specific public goods $g_i$, $i = 1, 2..., n$ and, (2) a redistributive tax rate that, depending on the regime, can either be set at a uniform rate by the federal authorities or by the regional governments at region-specific rates.

## 2.1. Technology

Each region has an exogenously given income $Y_i = x_i (1 + \epsilon_i)$ where $x_i$ is the non-stochastic endowment of the region and $\epsilon_i$ is a random shock to income with mean 0, variance $\sigma_i^2$ and the covariance between $\epsilon_i$ and $\epsilon_j$, $i \neq j$, is given by $\sigma_{ij} \lesseqgtr 0$.

At the time of public good provision, the usefulness of the public good is a random variable. Denote by $\nu_i$, the per unit usefulness for the $i^{th}$ region of its regional public good, $g_i$[10]. The regional government estimates that $\nu_i$ is distributed with mean $\bar{\nu}_i \geq 1$ and variance $\theta_i^2$. The covariance between $\nu_i$ and $\nu_j$, $i \neq j$, is zero. The federal government, however, estimates the per unit usefulness of $g_i$, for region $i$, as $\nu_i + \varkappa_i$ where $\varkappa_i$ is distributed with zero mean and variance equal to $\xi_i^2$; the latter is a measure of the coarseness of federal information.

Each region is affected by spillovers or externalities (positive or negative) from public good provision in the other regions. The spillover into the $i^{th}$ region from public good provision in the $j^{th}$ region is $e_{ij}g_j$ where $e_{ij} \lesseqgtr 0$ is the per unit spillover. Hence, the aggregate spillover from public good provision in all regions for the $i^{th}$ region is $\sum_{j,j \neq i} e_{ij}g_j$.

Finally, one unit of tax revenues can produce a unit of any regional public good.

## 2.2. Preferences

In common with most of the literature, preferences of each region are additively separable between public good consumption and private consumption. Expected utility is as in Alesina and Perotti (1998) i.e. of the mean-variance form which depends positively on the expected value and negatively on the variance[11].

### 2.2.1. Preferences of the Regional Government

Expected utility of the $i^{th}$ region is given by

$$\Lambda_i = E[u(\nu_i g_i)] + \sum_{j,j \neq i} e_{ij}g_j + E[u(C_i)] \tag{2.1}$$

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[10]This can equivalently by understood as a shock to preferences or with minor modifications, as a shock to the cost of public good provision.

[11]The mean-variance form can be derived from more general utility functions under appropriate assumptions on the distribution of the random shocks. This is the case when (i) preferences are CARA and the random terms are normally distributed, or (ii) preferences are CRRA and the random terms are lognormally distributed. More details can be found in Campbell and Viceira (2002).

where $g_i$ and $C_i$ denote respectively public good and private good consumption in the $i^{th}$ region; a particular realisation of $C_i$ is denoted by $c_i$. When $\nu_i$, $C_i$ are random variables, mean-variance utility implies that (2.1) can be written as

$$\Lambda_i = \left( E\left[\nu_i g_i\right] - \frac{1}{2}Var\left[\nu_i g_i\right] \right) + \sum_{j,j \neq i} e_{ij}g_j + \left( E\left[C_i\right] - \frac{1}{2}Var\left[C_i\right] \right). \qquad (2.2)$$

### 2.2.2. Preferences of the Federal Government

The federal government maximizes the sum of the regional payoffs; its utility is

$$\Lambda = \sum_i \Lambda_i, \qquad (2.3)$$

where $\Lambda_i$ is defined in (2.2).
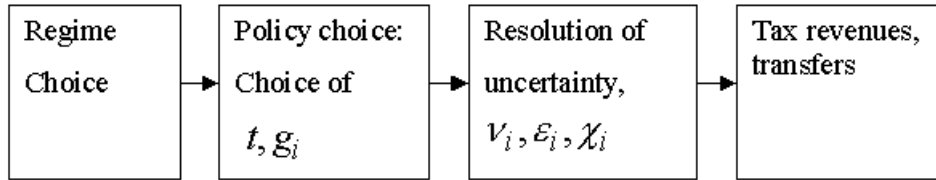
### 2.3. Redistributive Tax Policy

Redistributive policy is either exercised by the federal government or the regional governments, depending on the regime. It comprises of a non-distortionary linear tax rate, $t$, and a constant lumpsum transfer, $\alpha$, that is distributed equally among the regions in all states of the world.

### 2.4. Sequence of Moves

At the beginning of the period, public policy is announced. This policy comprises of the following elements.

1. A commitment to a certain level of public good provision in each region.

2. Announcement of the redistributive tax rate $t$ and a commitment to redistribute tax revenues, net of the costs of public good provision, equally to each region, in each possible state of the world.

This is followed by the simultaneous realization of the random variables in the model i.e. shocks to income and usefulness of the public good, in each region. Following the shocks, the tax revenues are collected and distributedequally among the regions, net of the costs of public good provision. The sequence is shown below.

| Regime Choice | Policy choice: Choice of $t, g_i$ | Resolution of uncertainty, $v_i, \varepsilon_i, \chi_i$ | Tax revenues, transfers |
|---|---|---|---|

<div align="center">Generic Sequence Of Moves</div>

## 2.5. Types of Policy Regimes

The paper analyzes six main policy regimes; autarky, centralization, devolution, regulated devolution, direct democracy and a risk-averse but revenue maximising leviathan. The first four regimes are in the normative tradition, while the last two are in the political economy tradition. An explanation of the regimes, and consumption/ transfers in each is given below.

### 2.5.1. Regional Autarky

In this case, each of the regional governments specifies a choice of redistributive income tax $t_i$ and a commitment to supply $g_i$ units of the regional public good. Ex-post, i.e. after the realisation of uncertainty, tax revenues in any state of the world are $T_i = t_i y_i$, where $y_i$ is the realisation of $Y_i$, and so the regional government's budget constraint is

$$\alpha_i = t_i y_i - g_i. \tag{2.4}$$

Notice that the regional transfer, $\alpha_i$, is net of the regional public good, $g_i$, which is committed upfront. Regional consumption in any state of the world, $c_i$, is defined as

$$c_i = (1 - t_i) y_i + \alpha_i. \tag{2.5}$$

Substituting (2.4) in (2.5) it follows that consumption in any state is given by

$$c_i = y_i - g_i.$$

Since $c_i$ is offset one for one with additional public good provision in the region, the region fully internalises the cost of extra public good provision.

### 2.5.2. Centralization

In this case, prior to the resolution of any uncertainty, the constitution specifies that an overarching federal government will commit to (1) a uniform federal redistributive tax rate,

<div align="center">8</div>

$t$, and (2) the provision of regional public goods, $g_i$, $i = 1, ..., n$. Regional governments are redundant in this case. Denote total income in the federation by $Y = \sum_j Y_j$. The total tax revenues in any state of the world are given by $T = \sum_j t y_j$ where $y_j$ is a realisation of $Y_j$. Each region now receives an equal lumpsum transfer of income, $\alpha_i$, from the federal government, net of the total cost of public good provision. This is given by

$$\alpha_i = \frac{1}{n} \left( \sum_j t y_j - \sum_j g_j \right). \tag{2.6}$$

Consumption in the $i^{th}$ region is $c_i = (1 - t_i) y_i + \alpha_i$. On substituting (2.6), $c_i$ equals

$$c_i = (1 - t_i) y_i + \frac{1}{n} \left( \sum_j t y_j - \sum_j g_j \right). \tag{2.7}$$

### 2.5.3. Devolution

In a devolved regime, the constitution specifies the division of responsibilities for redistributive taxation and public good provision among, respectively, the federal and the regional governments. The federal tax rate '$t$' is uniform across the regions. The regional governments then simultaneously choose the levels of public good provision. The lumpsum transfers and private consumption of the $i^{th}$ region are given by (2.6) and (2.7) respectively, however, the level of public good provision in this case is different from the centralised regime. In particular, from (2.7), for each extra unit of public good that the $i^{th}$ region chooses, ceteris-paribus, it offsets its private consumption by only $\frac{1}{n}$, giving rise to the "raiding the commons" problem.

### 2.5.4. Regulated Devolution

Devolution can produce perverse incentives for public good provision. Regions attempt to raid the commons and ignore inter-regional spillovers arising from public good provision. Under this regime, by giving appropriate incentives to the regional government, in the form of matching grants, spillovers can be internalised. By adopting appropriate fiscal institutions such as requiring regions to raise benefit taxes in order to finance their public goods, the "raiding the commons" problem can be eliminated. Each region now gets two kinds of grants

1. A *matching-grant*, $m_i g_i$, where $m_i$ is the per-unit endogenous federal grant.

2. A *lumpsum-grant*, $\alpha_i$, which equals $\left( \frac{1}{n} \right)^{th}$ of tax revenues net of matching grants i.e.

$$\alpha_i = \frac{1}{n} \left( \sum_j t y_j - \sum_j m_j g_j \right). \tag{2.8}$$

Private consumption must now take full account of the benefit taxes imposed on the region to finance its local public good, hence

$$c_i = (1 - t_i)y_i - g_i + m_i g_i + \frac{1}{n}\left(\sum_j t y_j - \sum_j m_j g_j\right) \qquad (2.9)$$

Thus, each unit of $g_i$ now offsets one unit of $c_i$, forcing regions to internalise the true cost of public good provision.

### 2.5.5. Direct Democracy

Under direct democracy, the median region is decisive under appropriate conditions in directly choosing the federal redistributive tax rate. The lumpsum transfers and private consumption of the $i^{th}$ region are still given by (2.6) and (2.7). Since preferences are separable between public and private consumption, a variety of regimes for public good provision can be allowed, without altering the focus of this regime which is on the redistributive tax rate chosen by the median voter.

### 2.5.6. Risk-Averse But Revenue Maximising Leviathan

This regime follows from the public choice tradition of Brennan and Buchanan (1980) but adapts the definition of a revenue maximising leviathan to situations of uncertainty. The revenue maximising but risk-averse federal leviathan commits to a tax rate that maximises its expected utility, given by

$$W\left[T\right] = E\left[T\right] - \frac{1}{2}Var\left[T\right] \qquad (2.10)$$

where $E\left[T\right]$ and $Var\left[T\right]$ are respectively the mean and variance of tax revenues, $T$. Hence, the leviathan's optimisation problem is

$$t^* \in \arg\max W\left[T\right] \qquad (2.11)$$

The focus of this section is on the redistributive tax rate chosen by the federal leviathan. The tax revenues are used, as in the other regimes, for regional redistribution, net of public good provision. Under separability of regional preferences between public and private consumption, a variety of modes of public good provision, as, for instance, under devolution or regulated devolution, are possible.

## 3. NORMATIVE ANALYSIS

This section examines the properties of the four normative regimes - Autarky, Centralization, Devolution, and Regulated Devolution. The optimal values will be superscripted

with the first lowercase letter of the type of the regime. For pedagogical ease, the normative analysis restricts the non-stochastic endowment of each region to $x_i = 1$ for all $i$, hence, the exogenous income of each region is $Y_i = 1 + \epsilon_i$. The political economy analysis in Section 4 allows for a distribution of $x_i$, which is crucial in determining the median region.

## 3.1. REGIONAL AUTARKY

Using (2.5) check that $E[C_i] = 1 - g_i$ and $Var[C_i] = \sigma_i^2$. Recall that by the local government's estimation, $\nu_i$ is distributed with mean $\bar{\nu}_i$ and variance $\theta_i^2$. Substituting these values in (2.2), the optimization problem of the $i^{th}$ regional government is given by

$$g_i^a, t_i^a \in \arg\max \ \Lambda_i = \left[ \bar{\nu}_i g_i - \frac{1}{2}\theta_i^2 g_i^2 \right] + \sum_{j, j \neq i} e_{ij} g_j + \left[ 1 - g_i - \frac{1}{2}\sigma_i^2 \right]. \tag{3.1}$$

The first order condition with respect to $g_i$ is

$$\frac{\partial \Lambda_i}{\partial g_i} = (\bar{\nu}_i - 1) - \theta_i^2 g_i = 0. \tag{3.2}$$

From (3.2) the optimal value of public good provision under autarky, $g_i^a$, is

$$g_i^a = \frac{\bar{\nu}_i - 1}{\theta_i^2}, \tag{3.3}$$

and the following properties follow in a straightforward manner.

**Lemma 1** : *Regional public good provision under autarky is increasing in its mean usefulness, $\bar{\nu}_i$, and decreasing in the uncertainty about its usefulness, $\theta_i^2$.*

A drawback of public good provision under autarky is that it does not internalize the externality $\sum_{k, k \neq i} e_{ki} g_i^a$ caused by $g_i^a$ to all other regions; this is well known in the literature.

The objective function in (3.1) is independent of the tax rate because under autarky, there is no possibility of sharing correlated inter-regional incomes through the tax system. Substituting (3.3) into (2.4), the government budget constraint in any state of the world is given by

$$\alpha_i = t_i y_i - g_i^a. \tag{3.4}$$

Hence, all combinations of $\alpha_i, t_i$ that satisfy (3.4) are admissible, as they all give the same payoff to the region. For a single region under autarky, there are no opportunities for insurance, hence, one solution to the model is:

$$\alpha_i^a = 0; \quad t_i^a = \frac{g_i^a}{y_i} \tag{3.5}$$

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Feasibility requires $t_i^a \in [0,1]$ which implies the restriction that $g_i^a \leq y_i$[12].

Substituting (3.5) and (3.3) in (3.1) the indirect utility of the region under autarky is

$$\Lambda_i^a = \frac{(\bar{\nu}_i - 1)^2}{2\theta_i^2} + \sum_{j \neq i} e_{ij} \frac{(\bar{\nu}_j - 1)}{\theta_j^2} + 1 - \frac{\sigma_i^2}{2} \tag{3.6}$$

The comparative static properties of $\Lambda_i^a$ are stated, without proof, in Proposition 1.

**Proposition 1** : *The $i^{th}$ region's payoff under autarky, $\Lambda_i^a$, is increasing in $\bar{\nu}_i$ but decreasing in the two sources of uncertainty, $\sigma_i^2$ and $\theta_i^2$. Furthermore, $\Lambda_i^a$ is decreasing (increasing) in $\theta_j^2$ if $e_i$ is positive (negative).*

Uncertainty in the usefulness of "own public good" and in the regional income diminishes the welfare of the $i^{th}$ region. However, if the region suffers negative externalities from the $j^{th}$ region's public good provision, its welfare is increasing in the $j^{th}$ region's uncertainty in estimating the usefulness of its public goods, $\theta_j^2$, which lowers $g_j$ and, hence, reduces inter-regional spillovers. The converse intuition applies to the case of positive externalities.

## 3.2. CENTRALISATION

Using (2.7), the mean and variance of consumption of the $i^{th}$ region under centralisation are given by

$$E[C_i] = 1 - \frac{1}{n} \sum_j g_j \tag{3.7}$$

$$Var[C_i] = \left( (1-t)^2 + \frac{1}{n} 2t(1-t) \right) \sigma_i^2 + \left( \frac{t}{n} \right)^2 \sum_i \sum_j \sigma_{ij} + \frac{2}{n} t(1-t) \sum_{j, j \neq i} \sigma_{ij} \tag{3.8}$$

Under centralisation, the federal government, whose responsibility it is to provide public goods, estimates that $\nu_i$ is distributed with mean $\bar{\nu}_i$ and variance $\theta_i^2 + \xi_i^2$. Using this, and substituting (3.7), (3.8) in (2.2) the payoff function of the $i^{th}$ region under centralisation is given by

$$\Lambda_i = \left[ \bar{\nu}_i g_i - \frac{1}{2} \left( \theta_i^2 + \xi_i^2 \right) g_i^2 \right] + \sum_{j, j \neq i} e_{ij} g_j + 1 - \frac{1}{n} \sum_j g_j$$

$$- \frac{1}{2} \left( \left( (1-t)^2 + \frac{1}{n} 2t(1-t) \right) \sigma_i^2 + \left( \frac{t}{n} \right)^2 \sum_i \sum_j \sigma_{ij} + \frac{1}{n} 2t(1-t) \sum_{j, j \neq i} \sigma_{ij} \right). \tag{3.9}$$

---

[12]Similar feasibility conditions are required for each regime; these are straightforward, and omitted in the subsequent analysis.

The maximization problem of the central government now is:

$$g_1^c, ..., g_n^c, t^c \in \arg\max \ \Lambda = \sum_i \Lambda_i \tag{3.10}$$

where $\Lambda_i$ is defined in (3.9).

The first order condition with respect to the tax rate is given by

$$\frac{\partial \Lambda}{\partial t} = \frac{(1-t)}{n} \left( (n-2) \sum_j \sigma_j^2 + \sum_i \sum_j \sigma_{ij} \right) \leq 0; t \geq 0 \tag{3.11}$$

Since $n \geq 2$, the first term in the braces in (3.11) is positive. The second term is simply the variance of aggregate federation-wide income, $\sum_j Y_j$, which is also positive. Hence, the only interior solution to the federal redistributive tax rate is unity, i.e.

$$t^c = 1. \tag{3.12}$$

In the absence of any distortions in the tax system, the federal government provides complete regional insurance; so, in all states of the world, all regional incomes are pooled and equally distributed to the regions. In the presence of tax distortions there is an additional tradeoff to consider, namely, insurance provided by the tax system and additional tax distortions. This only lowers the optimal tax rate (incomplete insurance) but does not alter any qualitative results[13].

The first order condition with respect to public good provision in the $i^{th}$ region is

$$\frac{\partial \Lambda}{\partial g_i} = (\bar{\nu}_i - 1) - (\theta_i^2 + \xi_i^2) g_i + \sum_{j, j \neq i} e_{ji} = 0,$$

which can be solved for the optimal provision under centralisation, $g_i^c$,

$$g_i^c = \frac{\bar{\nu}_i - 1 + \sum_{j, j \neq i} e_{ji}}{\theta_i^2 + \xi_i^2}. \tag{3.13}$$

Public good provision under centralisation does not suffer from the "raiding the commons" problem because the federal government accounts for the correct opportunity cost of regional public good provision i.e. unity. The federal government also takes account of the federation-wide spillovers caused by public good provision in any region. Furthermore, public good provision is decreasing in the degree of coarseness, $\xi_i^2$, of federal estimates about the usefulness of regional public goods.

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[13]An earlier version of the paper verified this by assuming that a fraction of tax revenues, representing deadweight losses of taxation, are lost in the redistributive process. Results are available from the author.

### 3.2.1. Comparison of public good provision under Autarky and Centralisation

Using (3.13) and (3.3), the comparison of public good provision under autarky and centralization is given by

$$g_i^a \gtreqless g_i^c \text{ as } \frac{\xi_i^2}{\theta_i^2} \gtreqless \frac{\sum_{j,j \neq i} e_{ji}}{\bar{\nu}_i - 1}. \tag{3.14}$$

Inequality (3.14) implies the following straightforward inference, stated without proof.

**Proposition 2** : *If aggregate spillovers from the provision of $g_i$ are negative i.e. $\sum_{j,j \neq i} e_{ji} < 0$, then $g_i^a > g_i^c$. If aggregate spillovers are positive i.e. $\sum_{j,j \neq i} e_{ji} > 0$ and large relative to the federal government's information disadvantage, as captured by the term $\xi_i^2/\theta_i^2$ then it is possible that $g_i^c > g_i^a$ .*

The intuition behind Proposition 2 is simple. Relatively coarser information on the part of the federal government leads it to supply a lower amount of public goods relative to the autarkic solution. Negative spillovers complement this reduction in public good provision and so in this case $g_i^a > g_i^c$. Positive spillovers increases public good provision under centralisation but coarser federal information reduces it; the net effect, relative to the autarkic solution, can be positive or negative.

### 3.2.2. Constrained first best level of public good provision

The constrained first best level of public good provision, $g_i^*$, would (1) internalize spillovers, (2) use the correct opportunity cost of public good provision, and (3) use the best possible information. Therefore, $g_i^*$ can be written as

$$g_i^* = \frac{\bar{\nu}_i - 1 + \sum_{j,j \neq i} e_{ji}}{\theta_i^2} \tag{3.15}$$

When expressed in terms of the constrained first best, the level of public good provision under the autarkic and the centralized regimes can be written as

$$g_i^a = g_i^* - \frac{\sum_{j,j \neq i} e_{ji}}{\theta_i^2},$$

$$g_i^c = g_i^* \left\{ 1 + \frac{\xi_i^2}{\theta_i^2} \right\}^{-1}.$$

The following Proposition is now self evident.

**Proposition 3** : *Centralized provision of public goods is always below the first best i.e. $g_i^c < g_i^*$. However, the autarkic provision can be greater or lower than the first best. In particular, $g_i^a < g_i^*$ when $\sum_{j,j \neq i} e_{ji} > 0$ and $g_i^a > g_i^*$ when $\sum_{j,j \neq i} e_{ji} < 0$.*

### 3.2.3. Indirect utility under Centralisation

Substituting (3.12) and (3.13) in (3.9) one obtains the indirect utility of the $i^{th}$ region under a centralized regime, $\Lambda_i^c$.

$$
\Lambda_i^c = \left( \bar{\nu}_i - \frac{1}{n} \right) \left( \frac{\bar{\nu}_i - 1 + \sum_{j,j\neq i} e_{ji}}{\theta_i^2 + \xi_i^2} \right) + 1 - \frac{1}{2} \frac{\left( \bar{\nu}_i - 1 + \sum_{j,j\neq i} e_{ji} \right)^2}{\theta_i^2 + \xi_i^2}
$$
$$
+ \sum_{j,j\neq i} \left( e_{ij} - \frac{1}{n} \right) \left( \frac{\bar{\nu}_j - 1 + \sum_{k,k\neq j} e_{kj}}{\theta_j^2 + \xi_j^2} \right) - \left( \frac{1}{2n^2} \right) \sum_i \sum_j \sigma_{ij} \quad (3.16)
$$

The effect of various parameters on $\Lambda_i^c$ is summarised, without proof, in Proposition 4.

**Proposition 4** : *The payoff of the $i^{th}$ region under centralisation, $\Lambda_i^c$, is decreasing in (1) federation-wide risk, $\sum_i \sum_j \sigma_{ij}$, and (2) federal uncertainty in the usefulness of public good provision in the $i^{th}$ region, $\theta_i^2 + \xi_i^2$. Furthermore, $\Lambda_i^c$ is increasing (decreasing) in federal uncertainty in the usefulness of public good provision in the $j^{th}$ region, $\theta_j^2 + \xi_j^2$, if $e_{ij} - \frac{1}{n}$ is negative (positive).*

The intuition behind the results stated in Proposition 4 is as follows.

1. Federal uncertainty about usefulness of public good provision in "own" region, $\theta_i^2 + \xi_i^2$, creates two reinforcing effects which reduce welfare. First, using (3.13), own uncertainty reduces the amount of own public good in equilibrium. Second, this creates greater uncertainty for the ex-post transfer received by the risk-averse region because transfers are net of the cost of public good provision.

2. A reduction in federal uncertainty about the usefulness of public good provision in the "neighboring" region, $\theta_j^2 + \xi_j^2$, increases public good provision in the neighbouring region, $g_j^c$. An increase in $g_j^c$ has two effects. First, it reduces the transfers received by the the $i^{th}$ region by an amount $\frac{1}{n}$. Second, it create a positive (negative) spillover for the $i^{th}$ region as $e_{ij}$ is positive (negative). When $e_{ij}$ is negative the two effects reinforce each other, whereas, if $e_{ij}$ is positive, the relative magnitudes of the two effects, $e_{ij} - \frac{1}{n}$, determines the overall outcome.

3. Federation-wide uncertainty in incomes, $\sum_i \sum_j \sigma_{ij}$, induces greater uncertainty in the consumption of the $i^{th}$ region because each region receives a transfer equal to $\frac{1}{n}$ of the total income of the federation.

### 3.2.4. Comparison of indirect utility under autarky and centralisation

Consider the case of symmetric uncertainty for public good production.

**Definition 1** : *Public goods are produced under "symmetric uncertainty" if for all $i, j$,
$\bar{\nu}_i = \bar{\nu}$, $\theta_i^2 = \theta^2$, $\xi_i^2 = \xi^2, e_{ij} = e$.*

If public goods are produced under symmetric uncertainty and $\bar{\nu} = 1$, then, the utility
of region $i$ under autarky, $\Lambda_i^a$, and centralisation, $\Lambda_i^c$, is respectively given by

$$\Lambda_i^a = 1 - \frac{\sigma_i^2}{2}$$

$$\Lambda_i^c = \frac{(n-1)^2 e^2}{2(\theta^2 + \xi^2)} + 1 - \left(\frac{1}{2n^2}\right) \sum_i \sum_j \sigma_{ij} \tag{3.17}$$

Check that in this case,

$$\Lambda_i^c \gtrless \Lambda_i^a \text{ as } \frac{(n-1)^2 e^2}{2\theta^2 \left(1 + \frac{\xi^2}{\theta^2}\right)} \gtrless \frac{\sigma_i^2}{2} \left(\frac{\sum_i \sum_j \sigma_{ij}}{\sigma_i^2} - 1\right) \tag{3.18}$$

The following conclusions are self evident from (3.18) and stated without proof in
Proposition 5

**Proposition 5** : *If public goods are produced under "symmetric uncertainty" and $\bar{\nu} = 1$,
then a region is likely to benefit from centralisation relative to autarky if (1) spillovers,
$(n-1) e$ are high, (2) federal government's information is not too coarse, i.e. $\frac{\xi^2}{\theta^2}$ is low,
(3) federation-wide risk is low relative to the risk in that region , i.e. $\frac{\sum_i \sum_j \sigma_{ij}}{\sigma_i^2}$ is low.*

Proposition 5 shows that an autarkic region is more likely to prefer a centralised regime
if the level of overall risk in the federation is relatively low, spillovers are important and
the federal government's information is not too coarse.

### 3.2.5. Number of regions in a federation

The results are best conveyed if public goods are produced under "symmetric uncertainty"
and there is "symmetric income uncertainty".

**Definition 2** : *A federation is characterised by "symmetric income uncertainty" if for all
$i, j$, $\sigma_i^2 = \sigma^2$ and $\sigma_{ij} = \bar{\sigma}$.*

**Definition 3** : *The "common pairwise correlation coefficient" between two regions in the
case of symmetric income uncertainty is defined as $r = \bar{\sigma}/\sigma^2$.*

If public goods are produced under symmetric uncertainty, $\bar{\nu} = 1$, and there is symmetric income uncertainty then (3.18) can be rewritten as

$$\Lambda_i^c \gtrless \Lambda_i^a \text{ as } \frac{e^2}{2\theta^2 \left(1 + \frac{\xi^2}{\theta^2}\right)} \gtrless \frac{\sigma^2}{2} \left( r \frac{n!}{(n-1)^2} + \frac{1}{(n-1)} \right) \qquad (3.19)$$

The following proposition draws the implications of this comparison.

**Proposition 6** : *Suppose that public goods are produced under "symmetric uncertainty", $\bar{\nu} = 1$, and there is "symmetric income uncertainty". If $n \to \infty$ and $r > 0$ then autarky unambiguously dominates centralisation. If $n \to \infty$ and $r < 0$ then centralisation unambiguously dominates autarky. Finally, as $n \to \infty$ the regional variances have a vanishing effect on the relatively optimality of the two regimes while regional covariances dominate in the relative payoffs under the two regimes.*

Proposition 6 shows that when $r < 0$, the gains from centralisation, in terms of the risk sharing benefits, are increasing in the number of regions, $n$, that form the federation, whereas the autarkic outcome is unaffected by $n$. Also, as $n$ increases, the covariance terms increase at a much faster rate than the variance terms, making them the main determinants of the choice between the two regimes.

If $\frac{\xi^2}{\theta^2}$ is high and so the federal government's information is very coarse, a much better outcome can be achieved if the regions produce their own public goods through the regime of "Devolution". Under devolution, the regions will ignore externalities arising through spillovers and not internalise the full costs of public good provision (because of raiding the commons problem), hence, a regime of "Regulated Devolution" might do even better. These issues are examined in the next two subsections.

### 3.3. UNREGULATED DEVOLUTION

The devolution problem is stated in subsection 2.5.3 above. The federal government and the regional governments take respective responsibility for redistribution and public good provision. The optimal magnitudes are superscripted with a '$d$'. The solution is by backward induction.

**Stage-II: The Regional Government's Problem**  Given any tax rate announced by the federal government, each regional government simultaneously chooses the level of its own public good. For any region $i$, the problem is to choose $g_i$ in order to maximise $\Lambda_i$, where $\Lambda_i$ is defined in (3.9). Solving out the first order condition, the optimal regional choice of public good provision, $g_i^d$, can be seen to equal

$$g_i^d = \frac{\bar{\nu}_i - \frac{1}{n}}{\theta_i^2}. \qquad (3.20)$$

Comparing (3.20) with the level of public good provision under autarky in (3.3) (3.13), as under autarky, regional governments do not take account of spillovers. However, unlike autarky, public good provision under devolution does not take account of the full cost of public good provision. It engages in "raiding the commons" and takes account of only $\left(\frac{1}{n}\right)^{th}$ of the total cost of providing for its own public good.

Comparing (3.20) and (3.13) it can be checked that the relative public good provision under Centralisation and Unregulated Devolution is given by

$$g_i^c \gtrless g_i^d \text{ as } \frac{\bar{\nu}_i - 1 + \sum_{j,j\neq i} e_{ji}}{\bar{\nu}_i - \frac{1}{n}} \gtrless \left(1 + \frac{\xi^2}{\theta^2}\right)$$

While an increase in spillovers, $\sum_{j,j\neq i} e_{ji}$, and a decrease in the relative coarseness of federal information, $\frac{\xi^2}{\theta^2}$, are conducive to greater public good provision under centralisation, an increase in the number of regions, $n$, by making it easier to 'raid the commons', is conducive to greater public good provision under devolution.

### 3.3.1. Stage-I The Federal Government's Problem

Given the subsequent provision of regional public goods given in (3.20), the federal government chooses the federal redistributive tax rate to maximise $\Lambda$ given in (3.10). Since the objective is separable in the tax rate and public good provision, the federal government's choice of the optimal tax rate is unaltered relative to centralisation, hence, the optimal tax rate $t = t^d$ is given by

$$t^d = 1 \tag{3.21}$$

The federal government carries responsibility for regional redistribution under centralisation and unregulated devolution. In each case, in the absence of any tax distortions, and separability of the tax problem from public good provision, it provides complete insurance.

### 3.3.2. Regional payoff under Unregulated Devolution

Substituting (3.20), (3.21) in (3.9), the payoff of the $i^{th}$ region under unregulated devolution, $\Lambda_i^d$, is given by

$$\Lambda_i^d = \frac{1}{2\theta_i^2}\left(\bar{\nu}_i - \frac{1}{n}\right)^2 + \sum_{j\neq i}\left(e_{ij} - \frac{1}{n}\right)\frac{1}{\theta_j^2}\left(\bar{\nu}_i - \frac{1}{n}\right) + 1 - \left(\frac{1}{2n^2}\right)\sum_i \sum_j \sigma_{ij} \tag{3.22}$$

The explanation of the payoff is similar to that under centralisation (see Proposition 4) with three differences. First, in the unregulated devolution case, there is a "raiding the commons" problem. Second, public good provision is not subject to the coarseness of the federal government's information. Third, the regions do not take account of spillovers arising from public good provision.

### 3.3.3. Relative payoffs under Centralisation and Unregulated Devolution

The relative payoffs are best illustrated when public good production takes place under "symmetric uncertainty" (see Definition 1) and $\bar{\nu} = 1$. Check that in this case, the utility of region $i$ under unregulated devolution is given by

$$\Lambda_i^d = \frac{e}{2\theta^2}(n-1)\left(1 - \frac{1}{n}\right) + 1 - \left(\frac{1}{2n^2}\right)\sum_i\sum_j\sigma_{ij} \tag{3.23}$$

Comparing (3.17) and (3.23) it follows that

$$\Lambda_i^d \gtreqless \Lambda_i^c \text{ as } \frac{1}{(n-1)e}\left(1 - \frac{1}{n}\right) \gtreqless \frac{1}{1 + \frac{\xi^2}{\theta^2}} \tag{3.24}$$

Using (3.24), the following inferences, stated without proof, can be made.

**Proposition 7** : *If public good production takes place under symmetric uncertainty and $\bar{\nu} = 1$ then the regional payoff under unregulated devolution is likely to dominate that under centralisation if (1) spillovers, $(n-1)e$, are small, (2) $n$ is large, and (3) federal information is too coarse i.e. $\frac{\xi^2}{\theta^2}$ is high.*

An advantage of centralisation is its ability to internalise spillovers, while a disadvantage is coarseness of federal information. If the former is not too important and the latter is a severe problem then devolution is likely to be more attractive than centralisation. Another attractiveness of devolution for a region, relative to centralisation, and that depends on the number of regions, is the possibility of raiding the commons.

### 3.4. REGULATED DEVOLUTION

Regional consumption under regulated devolution is defined in (2.9). The expected value of the (random) consumption is given by

$$E[C_i] = 1 - g_i + m_i g_i - \frac{1}{n}\sum_j m_j g_j, \tag{3.25}$$

while the variance of consumption is given by

$$Var[C_i] = \left((1-t)^2 + \frac{1}{n}2t(1-t)\right)\sigma_i^2 + \left(\frac{t}{n}\right)^2\sum_i\sum_j\sigma_{ij} + \frac{2}{n}t(1-t)\sum_{j,j\neq i}\sigma_{ij} \tag{3.26}$$

Using the distributional properties of $\nu_i$ and substituting (3.25), (3.26) in (2.2) the utility of the $i^{th}$ region under regulated devolution is given by

$$\Lambda_i = \left[\bar{\nu}_i g_i - \frac{1}{2}\theta_i^2 g_i^2\right] + \sum_{j,j\neq i} e_{ij}g_j + 1 - g_i + m_i g_i - \frac{1}{n}\sum_j m_j g_j$$

$$- \frac{1}{2}\left(\left((1-t)^2 + \frac{1}{n}2t(1-t)\right)\sigma_i^2 + \left(\frac{t}{n}\right)^2\sum_i\sum_j\sigma_{ij} + \frac{1}{n}2t(1-t)\sum_{j,j\neq i}\sigma_{ij}\right) \tag{3.27}$$

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### 3.4.1. The optimal choice of public good provision

Subsequent to the choice of the redistributive tax rate by the federal government, but prior to the resolution of uncertainty, each of the regional governments simultaneously chooses its public good provision. The problem faced by the $i^{th}$ region is to maximise $\Lambda_i$ given in (3.27) by a suitable choice of $g_i$, given that $g_i$ is financed from the region's own tax revenues. Solving out for the optimal $g_i = g_i^r$ one obtains

$$g_i^r = \frac{1}{\theta_i^2} \left( \bar{\nu}_i - 1 + m_i \left( 1 - \frac{1}{n} \right) \right) \tag{3.28}$$

Choosing the endogenous matching grants $m_i$, $i = 1, ..., n$ appropriately, it is obvious that one can produce the constrained first best level of public good for each region. This is shown in Proposition 8.

**Proposition 8** : *By setting $m_i = \frac{n}{n-1} \sum_{j, j \neq i} e_{ji}$, the constrained first best level of public good provision, $g_i^*$, can be decentralized under the regime of regulated devolution. In the case of symmetric spillovers i.e. $e_{ji} = e$ for all $i, j$ the matching grant rate that decentralises the constrained first best is $m_i = ne$.*

The regulated devolution regime corrects for the two drawbacks of devolution which distort public good provision. First, externalities are corrected by using matching grants. Second, benefit taxes raised in each region ensure that the propensity to raid commons is eliminated.

### 3.4.2. Federal government's choice of the redistributive tax rate

Given the subsequent choice of public good provision given in (3.28), the federal government chooses the tax rate $t$ in order to maximise $\sum_i \Lambda_i$ where $\Lambda_i$ is given in (3.27). Since each region's payoff is separable in public and private consumption, the first order condition is identical to (3.11) and, hence, the optimal tax, $t = t^r$, is

$$t^r = 1.$$

Thus, in all regimes where the federal government is responsible for the provision of regional insurance, in the absence of any tax distortions and separability of its objective function in redistribution and public good provision, it provides full insurance.

## 4. POLITICAL ECONOMY

The two regimes considered in this section are (i) direct democracy, and (ii) a risk averse federal leviathan. Each region has an exogenously given income $Y_i = x_i (1 + \epsilon_i)$ where

$x_i$, the non-stochastic endowment of the region is ordered across the $n$ regions such that $x_1 < x_2 < ... < x_n$.

## 4.1. DIRECT DEMOCRACY

Under direct democracy, the analysis makes the following assumptions.

A1. The distribution of $\epsilon_i$ is identical for each region i.e. the federation is characterised by "symmetric income uncertainty" (see Definition 2 above). Hence, the source of hetergeniety among the regions is the non-stochastic component of income, $x_i$.

A2. The number of regions, $n$, is odd.

A3. Each region supplies its own public good as under unregulated devolution[14].

Formally, the problem under direct democracy is similar to the devolution regime except that the redistributive federal tax is chosen by the median region, rather than the federal government. The payoff of any $i^{th}$ region is similar to (3.9)

$$\Lambda_i = \left[\bar{\nu}_i g_i - \frac{1}{2}\theta_i^2 g_i^2\right] + \sum_{j,j\neq i} e_{ij}g_j + x_i(1-t) + \frac{t}{n}\sum_j x_j - \frac{1}{n}\sum_j g_j$$
$$- \frac{1}{2}\left(\begin{array}{c}\left((1-t)^2 + \frac{1}{n}2t(1-t)\right)x_i^2\sigma^2 + \left(\frac{t}{n}\right)^2\left(\sigma^2\sum_j x_j^2 + 2\bar{\sigma}\sum_i\sum_{j,j\neq i}x_i x_j\right) \\ + \frac{1}{n}2t(1-t)\bar{\sigma}x_i\sum_{j,j\neq i}x_j\end{array}\right) \quad (4.1)$$

Given assumption A3, optimal public good provision, $g_i = g_i^m$, is

$$g_i^m = g_i^d = \frac{\bar{\nu}_i - \frac{1}{n}}{\theta_i^2}$$

Differentiating (4.1), an interior solution to the ideal tax rate preferred by the $i^{th}$ region is found by solving the following first order condition

$$\frac{\partial\Lambda_i}{\partial t_i} = \frac{1}{n}\sum_j x_j + \left(1 - \frac{1}{n}\right)x_i^2\sigma^2 - \frac{1}{n}\bar{\sigma}x_i\sum_{j,j\neq i}x_j$$
$$- t\left(\left(1 - \frac{2}{n}\right)x_i^2\sigma^2 + \left(\frac{1}{n}\right)^2\left(\sigma^2\sum_j x_j^2 + 2\bar{\sigma}\sum_i\sum_{j,j\neq i}x_i x_j\right) - \frac{2}{n}\bar{\sigma}x_i\sum_{j,j\neq i}x_j\right) = 0$$

---

[14]The focus of this section is on the redistributive tax rate chosen by the median region. The separability of the problem in public and private consumption ensures that the properties of the redistributive tax chosen do not depend on the institutional structure of public good provision.

The second order condition, which is sufficient for a unique interior solution is

$$\frac{\partial^2 \Lambda_i}{\partial t_i^2} = \frac{2}{n}\bar{\sigma}x_i \sum_{j,j\neq i} x_j - \left(1 - \frac{2}{n}\right) x_i^2 \sigma^2 - \left(\frac{1}{n}\right)^2 \left(\sigma^2 \sum_j x_j^2 + 2\bar{\sigma} \sum_i \sum_{j,j\neq i} x_i x_j\right) \leq 0 \quad (4.2)$$

The second order condition always holds if $\bar{\sigma} < 0$, where $\bar{\sigma}$ is the pairwise covariance between any two regions. $\bar{\sigma} < 0$ ensures the first term in (4.2) is negative, the second term is positive and the third term, which is the variance of aggregate income in the federation, is also positive. The literature typically focusses only on the case $\bar{\sigma} < -1$;for instance Persson and Tabellini (1996), in which case the preferences are always single peaked over the tax rate and so the median voter theorem applies. From (**??**), the ideal tax rate of the $m^{th}$ region, $t_m^*$, is

$$t_m^* = \frac{\frac{1}{n}\sum_j x_j + \left(1 - \frac{1}{n}\right) x_m^2 \sigma^2 - \frac{1}{n}\bar{\sigma}x_m \sum_{j,j\neq m} x_j}{\left(1 - \frac{2}{n}\right) x_m^2 \sigma^2 + \left(\frac{1}{n}\right)^2 \left(\sigma^2 \sum_j x_j^2 + 2\bar{\sigma} \sum_i \sum_{j,j\neq i} x_i x_j\right) - \frac{2}{n}\bar{\sigma}x_m \sum_{j,j\neq m} x_j} \quad (4.3)$$

Proposition 9 formalises the applicability of the median voter theorem.

**Proposition 9** : *If $\bar{\sigma} < 0$, then the region with non-stochastic income $x_m$ such that $x_m$ is the median of the strictly ordered sequence $x_1 < x_2 < ... < x_n$, is decisive in the choice of the federation-wide tax rate when the regions directly and sincerely vote on the federal tax rate. Furthermore, the ideal tax rate of any region is increasing in $x_i$.*

The properties of the median voter's choice, $t_m^*$, are given in Proposition 10.

**Proposition 10** : *The federal tax rate most preferred by the median voter, $t_m^*$, is increasing in (i) average non-stochastic endowment of the federation, $\frac{1}{n}\sum_j x_j$, and (ii) income uncertainty in the median region, $x_m^2\sigma^2$. Furthermore, $t_m^*$ is decreasing in (i) income uncertainty in any other region, $x_j^2\sigma^2$, $j \neq m$, and (ii) covariance in incomes between any two regions other than the $m^{th}$ region, $2\bar{\sigma}x_i x_j$.*

Under direct democracy, the redistributive tax rate is predicted to be higher if the average federal income, $\frac{1}{n}\sum_j x_j$, is higher. There is some evidence that richer democracies engage in greater redistribution; for instance, Persson and Tabellini (2000). The federal redistributive tax will also be high if income uncertainty in the decisive median region, $x_m^2\sigma^2$, is high. This is analogous to the social insurance role of taxation uncovered by Varian (1980). However, the converse occurs following an increase in uncertainty in any non-median region, $x_j^2\sigma^2$, $j \neq m$ which increases the variance in consumption of the median voter, arising from sharing the average tax proceeds of the federation-wide tax revenues; an increase in the federal tax rate mitigates this increase in variance. For the same reason, $t_m^*$ is decreasing in $2\bar{\sigma}x_i x_j$ : the covariance in income between any two non-median regions. An interesting insight of the analysis is that the median region reacts asymmetrically to an increase in "own uncertainty", relative to uncertainty in non-median regions.

## 4.2. RISK AVERSE FEDERAL LEVIATHAN

In the public choice approach to federalism, exemplified in Brennan and Buchanan (1980), the federal government is thought of as a revenue maximising leviathan, which is reminiscent of Niskanen's idea of budget maximising bureaucrats. In their model, Brennan and Buchanan argue that competition among regional governments, in the presence of mobile households, acts as a fiscal constraint on their power to tax. Since, such competition is not faced by a leviathan federal government, decentralisation can be welfare improving.

Subsection 2.5.6 sets up the problem of a risk averse federal leviathan. The analysis below identifies another weakness of a federal leviathan. It might not fully provide regional insurance. Since the utility functions of regions are separable in private and public good consumption, one can assume any fiscal mode for public good provision, and focus on the redistributive tax rate chosen by the leviathan. Tax revenues in any state of the world are given by $T = t \sum_j y_j$ and each region receives a federal transfer, $\alpha_i$ such that

$$\alpha_i = \frac{1}{n} \sum_{j=1}^{n} (ty_j - g_j)$$

where $g_j = g_j^d$ if, for instance, the public goods are provided by each region as in unregulated devolution and $g_j = g_j^r$ if public goods are provided as under regulated devolution.

The first order condition to (2.11) is given by

$$\sum_j E[Y_j] - tVar\left[\sum_j Y_j\right] = 0 \qquad (4.4)$$

where $E[Y_j] = x_j$ and $Var\left[\sum_j Y_j\right] = \sum_j x_j^2 \sigma_j^2 + 2\sum_i \sum_{j,j \neq i} x_i x_j \sigma_{ij}$. From (4.4) the optimal tax rate, $t^l$, of the federal leviathan can be solved out as

$$t^l = \frac{1}{\rho \sum_j x_j} \qquad (4.5)$$

where $\rho$ is the coefficient of variation of total federal income. A risk averse federal leviathan chooses a higher tax rate if average federal income is higher but a lower tax rate if variance of aggregate federal income is lower. There is no presumption that $\rho \sum_j x_j \leq 1$. Indeed, when $\rho \sum_j x_j > 1$, $t^l < 1$, and so, relative to a utilitarian federal government, the federal leviathan provides incomplete insurance. Decentralisation of taxing powers to regional governments, in the presence of mobility (which is not explored in this paper), would then carry even greater merit, especially the larger is $\rho \sum_j x_j$.

Each of the two regimes in a political economy setting have important implications for the magnitude of regional insurance, relative to the utilitarian federal government in the normative analysis. The implications of the other three margins: raiding the commons,

internalisation of spillovers and information, depend on the modes of public good provision that are chosen.

## 5. Conclusions

The paper provides new results, insights and a synthesis of several important results in the literature on fiscal federalism under uncertainty. Four of the regimes, namely, autarky, centralisation, devolution, and regulated devolution, undertake a normative investigation while two regimes, namely, direct democracy and a risk-averse revenue maximising leviathan undertake a positive investigation. The regimes differ in the appropriate division of powers between hierarchical levels of government.

The paper demonstrates the mechanism by which the following factors, viz. information, internalisation of spillovers, insurance and problems of a soft budget constraint, interact with each other to influence the tradeoff between redistribution and public good provision. Furthermore, the optimality of the regimes is also influenced by the balance of several, simultaneous, sources of uncertainty arising from regional incomes and estimates about the usefulness of public goods.

Regulated devolution can provide the constrained first best level of public goods. In the absence of tax distortions, utilitarian federal governments provide complete regional insurance. However, federal insurance under (i) direct democracy and (ii) a risk-averse federal leviathan, can be inadequate. The median region's choice of redistribution under direct democracy is influenced asymmetrically by an increase in uncertainty about own income relative to that in non-median regions. This helps clarify the operation of different channels; social insurance in the first case and a dampening of uncertainty that feeds into the median region on account of sharing in the federal tax revenues, in the second case.

An increase in the number of regions has important implications that are relevant in light of, say, the recent expansion in the European Union. Such an increase helps provide for better federal insurance if regional incomes are negatively correlated. However, the downside is that it exacerbates the 'raiding the commons' problem, because, in providing for its public good each region takes account of only $\frac{1}{n}$ (where $n$ is the number of regions) of the cost in terms of foregone transfers.

The paper can potentially be extended in several directions. The most obvious being the incorporation of explicit information revelation mechanisms that allow federal governments to elicit the better estimates of the regional governments. While this will remove the distortion arising from coarser federal information, it will create an extra distortion that arises from providing information rents to regions. Hence, the federal government will not always want to elicit local information, especially if the cost of its own, coarser, information is not too high. Furthermore, such an approach can be fairly difficult to

apply in the presence of insurance considerations (both parties are risk averse) and its application will then involve a substantial simplication of the model in this paper; see for instance, Lockwood (1999). The second potential extension is to adapt the results in the paper about an increase in the number of regions to issues of European expansion, by introducing appropriate fiscal institutions and features such as the growth and stability pact. Finally, a fruitful area of research would be the design of institutions that eliminate or reduce the raiding the commons problem. Some research on this has already been done; for instance, Oates (2004).

# 6. Appendix

*Proof of Proposition 6*

The first part of the Proposition can be simply inferred from (3.19) by noting that $\lim_{n\to\infty} \frac{n-2!}{n-1} = \infty$. To see the second part of the proposition, rewrite (3.19) as

$$\frac{e^2}{2\theta^2 \left(1 + \frac{\xi^2}{\theta^2}\right)} \gtreqless \frac{1}{2}\left(\bar{\sigma}\frac{n!}{(n-1)^2} - \frac{\sigma^2}{(n-1)}\right)$$

The proof now follows by noting that $\lim_{n\to\infty} \frac{\sigma^2}{(n-1)} = 0$, $\lim_{n\to\infty} \frac{n!}{(n-1)^2} = \infty$ and noting that the coefficient of $\frac{n!}{(n-1)^2}$ is $\bar{\sigma}$.∎

*Proof of Proposition 8*

Comparing (3.15) with (3.28) it follows that $g_i^* = g_i^d$ when $m_i = \frac{n}{n-1}\sum_{j,j\neq i} e_{ji}$. For the case of symmetric spillovers i.e. $e_{ji} = e$, $\frac{n}{n-1}\sum_{j,j\neq i} e_{ji} = ne$, $m_i = ne$.∎

*Proof of Proposition 9*

Implicitly differentiating (4.3)

$$\frac{\partial t_i^*}{\partial x_i} = \left[-\frac{\partial^2 \Lambda_i}{\partial t_i^2}\right]^{-1}\left(2x_i\sigma^2(n-1)(n-2) - 3\bar{\sigma}\sum_{j,j\neq i} x_j\right) > 0$$

Because $\bar{\sigma} < 0$, the second order condition holds. Thus, each region prefers a unique federal tax rate and it is increasing in $x_i$. Since $n$ is odd and $x_1 < x_2 < ... < x_n$, hence, $\exists$ $x_m$, the median region, with some ideal tax rate $t_m^*$ such that half the regions prefer a tax rate lower than $t_m^*$ while the other half prefer a tax rate higher than $t_m^*$ when voting takes place sincerely on the federal tax rate. ∎

*Proof of Proposition 10*

Differentiating (4.3) with respect to the exogenous variables in the Proposition:

$$\frac{\partial t_m^*}{\partial \frac{1}{n}\sum_j x_j} = \left[-\frac{\partial^2 \Lambda_i}{\partial t_i^2}\right]^{-1} > 0 \tag{6.1}$$

The sign of (6.1) follows from the second order condition.

$$\frac{\partial t_m^*}{\partial x_m^2 \sigma^2} = \left[ -\frac{\partial^2 \Lambda_i}{\partial t_i^2} \right]^{-1} \left( \left( 1 - \frac{1}{n} \right) (1 - t_m^*) + \frac{t_m^*}{n} \left( 1 + \frac{1}{n} \right) \right) > 0 \qquad (6.2)$$

The sign of (6.2) follows from $n \geq 2$, $t_m^* \in [0, 1]$, and the second order condition.

$$\frac{\partial t_m^*}{\partial 2 \bar{\sigma} x_i x_j} = \frac{\partial t_m^*}{\partial x_j^2 \sigma^2} = \left[ \frac{\partial^2 \Lambda_i}{\partial t_i^2} \right]^{-1} \frac{t_m^*}{n^2} \leq 0 \qquad (6.3)$$

The sign of (6.3) follows from $t_m^* \in [0, 1]$, and the second order condition. ∎

# 7. References

# References

[1] Alesina, A. and Perotti, R. (1998) "Economic risk and political risk in fiscal unions" Economic Journal, 108: 989-1008.

[2] Asdrubali, P., Sorenson, B. E. and Yosha, O. (1996) Channels of interstate risk sharing: United States 1963-1990.

[3] Besley, T. and Coate, S. (2003) "Centralized versus decentralized provision of local public goods: A political economy approach," Journal of Public Economics, 87: 2611-37.

[4] Boadway, R., Horiba, I., Jha, R. (1999) "The provision of public services by government funded decentralized agencies" Public Choice, 100: 157-84

[5] Bordignon, M., Manasse, P., Tabellini, G. (1996) "Optimal regional redistribution under asymmetric information" Unpublished paper.

[6] Bucovetsky, S. Marchand, M., and Pestieau, P. (1998) "Tax competition and revelation of preferences for public expenditure" Journal of Urban Economics, 44: 367-390.

[7] Campbell, J. Y. and Viceira, L. M. (2002) Strategic Asset Allocation: Portfolio Choice for Long-Term Investors (Oxford University Press: Oxford).

[8] Inman, R. and Rubinfeld, L. (1997) "Rethinking Federalism", Journal of Economic Perspectives, 11:43-64.

[9] Kornai, J. Maskin, E. and Roland, G. "Understanding the soft budget constraint" Journal of Economic Literature, 41: 1095-1136.

[10] Lockwood, B. (1999) "Inter-regional insurance", Journal of Public Economics, 72: 1-37.

[11] Oates, W. (2004) "Towards a second generation theory of federalism", mimeo.

[12] Oates, W. (1972) Fiscal federalism (Harcourt Brace Jovanovich: New York).

[13] Persson, T. and Tabellini, G. (2000) Political institutions: Explaining economic policy (MIT Press: Cambridge, Mass.)

[14] Persson, T. and Tabellini, G. (1996) "Federal fiscal constitutions: risk sharing and redistribution", Journal of Political Economy, 104: 979-1009.

[15] Sala-I-Martin, X. and Sachs, J. (1992) "Fiscal federalism and optimum currency areas: evidence for Europe from the United States" in Canzoneri, M. B. et. al. (ed) Establishing a central bank: issues in Europe and lessons from the US (Cambridge University Press: Cambridge).

[16] Varian, H. R. (1980) "Redistributive taxation as social insurance" Journal of Public Economics, 14: 49-68