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COMPETITION AND GROWTH IN NEO-SCHUMPETERIAN MODELS

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Abstract

We study the effect of product market competition on the incentives to innovate and the economy's rate of growth in an endogenous growth model. We extend previous works in industrial organization by assuming that innovation is sequential and cumulative, and early endogenous growth models by accounting for the possibility that in each period many asymmetric firms (i.e., an endogenously determined number of successive innovators) are simultaneously active.

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We identify the price effect, the front loading of profits, and the productive efficiency effect associated with an increase in competitive pressure. The price effect reduces the incentives to innovate, but both the front loading of profits and the productive efficiency effect raise the incentives to innovate. We demonstrate circumstances in which the productive efficiency effect dominates the price effect. In these circumstances, the front loading of profits and the fact that the productive efficiency effect dominates the price effect compound to make the equilibrium rate of growth increase with the intensity of competition.

1. INTRODUCTION

It has often been claimed that competition is good for innovation and growth. Indeed, what empirical evidence is available suggests an increasing, or inverted U-shaped, relationship between competition and growth.[1] However, there is no straightforward theoretical explanation for such a positive link. Quite to the contrary, early models of endogenous growth tend to conclude that tougher competition erodes the innovator's prospective monopoly rents and is therefore detrimental to growth.

This paper aims to reconcile the Schumpeterian view that the search for monopoly rents is the primary engine of growth and empirical evidence that competition is good for growth. We argue that the conclusion drawn by early endogenous-growth models crucially depends upon the simplifying assumption that at every point in time the technological leader is the only active firm in each industry. In more highly structured models, which allow for two or more firms to be simultaneously active in the same industry, two qualitatively new effects arise – the front loading of profits and the productive efficiency effect – that can generate a positive relationship between product market competition, innovation and growth.

Any definition of competition involves the idea that more intense competition reduces the equilibrium price, thus exerting downward pressure on the innovator's prospective rents (we call this effect the *price effect*). However, in more competitive markets, a larger fraction of these rents accrues in the early stages of the innovative firm's life cycle (this we call the *front loading of profits* following Segal and Whinston (2003)) and low-cost firms have a larger portion of the market, which reduces total industry costs (*productive efficiency effect*). We find circumstances in which the productive efficiency effect dominates the price effect, namely, when the size of innovations is large and/or competition is strong. In these circumstances, the front loading of profits and the fact that the productive efficiency effect dominates the price effect compound to make the equilibrium rate of growth increase with the intensity of competition.

As a modeling strategy, we depart from standard quality ladder models of endogenous growth only to the extent that is necessary to allow for several firms to be

[1] See Nickell (1996), Blundell, Griffiths and van Reenen (1995) and Aghion et al. (2002).

simultaneously active in each industry. We therefore stick to the standard assumption that innovative technological knowledge is proprietary; this implies that firms are asymmetric in that they have access to different technologies. In early quality ladder models, the fact that only the technological leader is active in the product market rests on the assumption that either innovations are drastic (Aghion and Howitt (1992)),[2] or else firms compete *a là* Bertrand (Grossman and Helpman (1991)). To allow for different market structures, we focus on the case of non drastic innovations, contrasting Bertrand with Cournot competition. With asymmetric firms, the number of active firms and their respective market shares will depend on the mode of competition, Bertrand or Cournot, and the size of innovations. (In fact we use a more general reduced-form model which encompasses the Bertrand and Cournot equilibria as special cases and yields a continuous index of the intensity of competition.)

Our model possesses a steady state in which $m+1$ firms are simultaneously active, i.e., the latest innovator and m past innovators, where m is endogenously determined (with Bertrand competition, $m=0$). An innovator, that does not conduct any further research, will nonetheless remain active, and reap positive profits, for $m+1$ periods (a period is the random time interval between two innovations, as in Aghion and Howitt (1992)). As new innovations arrive, the original innovator's market share shrinks but he will exit the market only after $m+1$ successive innovations have occurred. Consequently, the value of an innovation, and hence the incentive to innovate, is a weighted average of all active firms' profits, where the weights reflect the expected length of time periods, discounting, and growth. In a stationary environment with no discounting each innovator would get total industry profits over time periods, irrespective of the mode of competition.

We show that a rise in competitive pressure makes profits accrue sooner to the innovator: for example, with Cournot competition each innovator collects its rents over various time periods, whereas with Bertrand competition all of the rents are obtained in the one period starting when the innovation is achieved. In a stationary environment with no discounting, such a front loading of profits would have no effect on the incentive to innovate. In our model, however, delayed profits increase over

[2] An innovation is drastic if the innovator is unconstrained by outside competition and can therefore engage in monopoly pricing.

time periods as the economy grows, but firms discount future rents. The transversality condition implies that discounting prevails over growth, and so the front loading of profits raises the incentive to innovate implying that competition tends to be positively associated with growth.

The intensity of competition affects the incentive to innovate also *via* its effect on total industry profits. We decompose the effect of product market competition on industry profits into a price effect and a productive efficiency effect. The price effect is the change in industry profits that would obtain if all active firms shared the same technology. This effect is negative, i.e. more intense competition would lead to lower industry profits if firms were symmetric. With asymmetric firms, however, a rise in the intensity of competition raises the market shares of low-cost firms, and lowers those of high-cost firms. For example, under Bertrand competition all of the output is produced by the most efficient firm – the latest innovator; whereas under Cournot competition high-cost firms produce a positive share of total output. Therefore, a rise in competitive pressure improves the productive efficiency of the industry which is good for industry profits.

We identify two circumstances in which the productive efficiency effect outweighs the price effect. First, when innovations are almost drastic, the equilibrium price is close to the monopoly price irrespective of the mode of competition. In this case, the price effect is second order. However, with Cournot competition the high-cost firm holds a positive market share (when innovations are almost drastic, only two firms are active in each period); the productive efficiency effect is therefore first order. Thus, with large innovations industry profits are greater under Bertrand competition than under Cournot competition. (In fact, with large innovations industry profits are monotonically increasing in the intensity of competition). Second, we show that in the vicinity of the Bertrand equilibrium the productive efficiency effect is remarkably large: indeed, a unit decrease in the equilibrium price lowers the industry average cost by as much as one! Therefore, independently of the size of innovations, when competition is strong a further increase in the intensity of competition must increase industry profits.

The rest of the paper is organized as follows. In Section 2, we discuss the related literature. In Section 3, we analyze the value of an innovation when innovation is sequential but innovators are not immediately displaced by the occurrence of the next innovation. We show that the incentive to innovate depends both on industry profits, and the distribution of profits across active firms. Section 4 studies how the intensity of product market competition impacts on the incentive to innovate. In Section 5, the insights obtained in Sections 3 and 4 are embedded in a simple general equilibrium endogenous growth model. Finally, Section 6 offers some concluding remarks.

2. RELATED LITERATURE

Our paper is related to two different literatures: the industrial organization literature that examines the effect of product market competition on the incentives to innovate, and the recent endogenous growth literature that tries to reconcile theory and evidence on the relationship between competition and growth.

The industrial organization literature.—

The debate on the effect of competition on the incentive to innovate goes back to Schumpeter (1942) and Arrow (1962). Schumpeter (1942) claims that there exists a positive correlation between innovation and market power. He argues that for a variety of reasons a monopoly may likely develop and employ a more advanced technology than a competitive industry. This claim has been countered by Arrow (1962), who argues that the incentive to innovate is higher in competitive industries, because a monopolist's post-innovation profits replace his pre-innovation profits, whereas this replacement effect vanishes under competition. Moving to the case of oligopoly, Delbono and Denicolò (1990) find that Bertrand duopolists have greater incentives to innovate than Cournot duopolists when the product is homogenous. However, Bester and Petrakis (1993) and Bonanno and Haworth (1998) show that this result can be reversed with horizontal and vertical product differentiation, respectively, and Symeonidis (2003) shows that the same is true when the products are both horizontally and vertically differentiated. Qiu (1997) develops a model in which the incentive to innovate is greater with Cournot competition even if the product is homogeneous. Boone (2000, 2001) generalizes these findings and shows that the relation between competition and incentives to innovate is generally non monotone. In short, the industrial organization literature on the effect of product market competition on the value of an innovation is largely inconclusive.[3] In part, these conflicting results are due to different assumptions on the nature of technical progress (tournament or non-tournament) and on who conducts the research (incumbents or outside firms). The remaining ambiguity rests on the fact that in more highly competitive industries the technological leader has a larger market share, and this market share effect may or may not outweigh the negative effect of more intense competition on the equilibrium price.

All of these papers focus on a single innovation framework and therefore identify the incentive to innovate with the (increase in the) profits of the technological leader. We depart from this literature by modeling an infinite sequence of innovations. In our framework the incentive to innovate cannot be equated to the leader's profits, but is a weighted average of all active firms' profits. As such, the positive effect of more intense competition on the leader's market share does not translate mechanically into higher incentives to innovate, but operates only *via* the productive efficiency effect and the front loading of profits. Our contribution is to show that these indirect effects may nevertheless be substantial.

Segal and Whinston (2003) independently study a model of successive innovations in which each innovator stays active for two periods. Our analysis has many elements in common with theirs, including the front loading of profits. However, their model differs from ours in various respects; for example, they posit a rectangular demand function, a fixed timing of innovations, and an exogenously given number of firms ($m = 1$). Moreover, they do not compare Bertrand and Cournot competition, but focus on various business practices that may or may not be anti-competitive. Notwithstanding these differences, our conclusions and theirs complement and reinforce each other.

The growth literature.—

A small endogenous growth literature tries to reconcile theory and empirical evidence on the relationship between competition and growth. One strand of this literature introduces agency issues into the picture (Aghion, Dewatripont and Rey (1999)).

[3]Equally inconclusive is the related literature on the effects of product market competition on managerial incentives: see Raith (2003).

In these models, non-profit maximizing managers delay the adoption of new technologies until profits fall below a threshold level. The effect of tougher competition is to reduce profits thereby speeding up the adoption process.

In the non-tournament models of van de Klundert and Smulders (1997) and Peretto (1999), tougher competition reduces the equilibrium number of varieties and increases the size of active firms, which raises their incentive to innovate. These papers posit a positive, deterministic relationship between the level of R&D investment and the size of the innovation. In a related contribution, d'Aspremont, Dos Santos Ferreira and Gerard-Varet (2002) consider the case in which R&D investment affects the probability of success rather than the size of innovations, but still many firms can innovate simultaneously. Thus, in each period there are some firms which have successfully innovated, and others that have access only to the prior art, which is in the public domain. They compare the Cournot and Bertrand equilibria, and also analyze an intermediate case in which all successful innovators co-operatively engage in limit pricing. They show that growth is fastest in this intermediate case, and conclude that the relationship between competition and growth is inverted U-shaped.

Aghion et al. (2001) develop a general equilibrium model of step-by-step technical progress in which two firms produce horizontally differentiated products, and show that more competition (as measured by an increase in the degree of product substitutability)[4] may be beneficial to growth. In step-by-step models, firms' incentive to innovate is greatest when they are neck-and-neck (which can never occur in leapfrogging models). In such a state, the incentive to gain a technological lead is greater when competition is intense; however, with fierce competition the fraction of industries in which firms are neck-and-neck tends to be lower. The interaction of these effects can generate an increasing, or inverted U-shaped, relationship between competition and growth. Encaoua and Ulph (2000) argue that introducing into this model the possibility of leapfrogging strengthens the positive effect of competition on growth.

The main difference between these papers and ours is that we do not make any special assumption: we use the standard leapfrogging model with profit-maximizing

[4]In a simplified version of the model, Aghion, Harris and Vickers (1997) parametrize the intensity of competition also as a switch from Cournot to Bertrand competition.

firms and tournament technical progress. The novelty of our analysis lies in that we allow for several firms to be simultaneously active – which requires that innovations are non-drastic and competition is Cournot rather than Bertrand.

3. THE INCENTIVE TO INNOVATE WITH SEQUENTIAL INNOVATIONS

In this section we analyze the key determinants of the incentives to innovate in a model of repeated innovations. We extend previous works in industrial organization by assuming that innovation is sequential and cumulative, and earlier endogenous growth models by accounting for the possibility that in each period many firms are simultaneously active.

Throughout, the following assumptions will be maintained. Innovative activity happens at a rate determined by R&D efforts. In each period k, where $k-1$ is the number of past innovations, there is a patent race for innovation k. (Time is continuous but can be divided into periods, where a period is the random time interval between two innovations.) Patent races come in a sequence in the sense that only after one innovation is achieved can the race for the next one begin. The size of innovations is exogenous but the timing of innovations is a probabilistic function of the amount invested in R&D by research firms; specifically, the R&D effort determines the expected time of successful completion of the R&D project according to a Poisson discovery process with a hazard rate z_k. We assume that incumbents do no research; research is conducted by outsiders, and so in each period the current leader is systematically replaced.[5]

To fix ideas, suppose that there is perfect, infinitely-lived patent protection, so that nobody can imitate the innovation without infringing the patent.[6] Because innovative technological knowledge is proprietary, in period k only the $(k-i)$th innovator, who

[5] The possibility that incumbents invest in R&D so that technological leadership may persist over time periods will be discussed in the concluding section.

[6] In practice, there are various means of protecting innovative technological knowledge, including patents, copyrights, secrecy, lead time etc. Typically the protection an innovator enjoys declines over time, but for simplicity we abstract from this additional source of dynamics.

holds a patent on the $(k-i)$th innovation, can practice it.[7] Under the assumption that all innovations are obtained by outsiders, nobody holds multiple patents. In period k, innovator $k-1$ is the technological leader, but we allow for m past innovators to remain active. Let $\pi_{i,k}$ be the flow of profit earned by innovator $k-1-i$ in period k. Thus, $\pi_{0,k}$ is the technological leader's profit; $\pi_{1,k}$ is the profit of the second most efficient firm (i.e., innovator $k-2$); and so on. Later m and z_k will be determined endogenously (for example, $m=0$ with Bertrand competition), but for the moment we take them as given.

To determine the expected value of innovation k, $E(V_k)$, one must take into account that the kth innovator's rents will not be terminated by the occurrence of the $(k+1)$th innovation: although competition from the $(k+1)$th innovator will reduce all past innovators' market shares and profits, only the least efficient among active firms will be driven out of the market when a new innovation occurs. Thus, $E(V_k)$ is determined by the following asset condition:

$$rE(V_k) = \pi_{0,k+1} - z_{k+1}\left[E(V_k) - E(V_k^1)\right]$$

where r is the interest rate. This equation says that securities issued by the leader pay the flow profit $\pi_{0,k+1}$ in period $k+1$, less the expected capital loss $z_{k+1}\left[E(V_k) - E(V_k^1)\right]$ that will be incurred when the next innovation is achieved. Such a capital loss is the difference between the value of being leader and that of being the second most efficient firm in the market, i.e. $E(V_k) - E(V_k^1)$, where $E(V_k^h)$ is the value of innovation k after h periods, i.e. in period $k+h$. The value of being the second most efficient firm in the market, $E(V_k^1)$, is in turn determined by the asset condition

$$rE(V_k^1) = \pi_{1,k+2} - z_{k+2}\left[E(V_k^1) - E(V_k^2)\right],$$

and so on. Eventually, after $m+1$ innovations, the kth innovator will exit the market, so that $E(V_k^{m+1}) = 0$. Consequently, we have

$$rE(V_k^m) = \pi_{m,k+m+1} - z_{k+m+1}E(V_k^m).$$

[7] We follow the vast majority of endogenous growth models in ruling out patent licensing. The standard justification for this assumption is that licensing agreements between successive innovators would have anti-competitive effects and thus would be prohibited by antitrust authorities. When licensing improves productive efficiency, however, such a justification loses some of its strength. After developing our results, we discuss licensing agreements more fully in the concluding section.

These $m+1$ equations can be solved recursively yielding

$$\begin{aligned} E(V_k) &= \frac{\pi_{0,k+1}}{r+z_{k+1}} + \frac{z_{k+1}}{(r+z_{k+1})}\frac{\pi_{1,k+2}}{(r+z_{k+2})} + ... \\ &\quad + \left[\prod_{i=1}^{m}\frac{z_{k+i}}{(r+z_{k+i})}\right]\frac{\pi_{m,k+m+1}}{(r+z_{k+m+1})} \\ &= \sum_{i=0}^{m}\left[\frac{\pi_{i,k+i+1}}{(r+z_{k+i+1})}\prod_{h=1}^{i}\frac{z_{k+h}}{(r+z_{k+h})}\right] \end{aligned} \tag{1}$$

When $m=0$, this expression reduces to the standard formula

$$E(V_k) = \frac{\pi_{0,k+1}}{(r+z_{k+1})}$$

that is, the value of the kth innovation is the discounted value of the innovator's profits, where the interest rate is augmented by the factor z_{k+1} that captures the expected duration of the innovator's leadership. More generally, equation (1) says that the value of the kth innovation is the expected present value of all future profits that the innovator will get in the $m+1$ periods for which he will be active in the product market. In each period, the discount factor is augmented to keep into account the probability that the current flow of profits is terminated by the occurrence of the next innovation. Moreover, because innovation is cumulative future profits are weighted by the factors $\prod_{h=1}^{i}\frac{z_{k+h}}{(r+z_{k+h})}$, which can be interpreted as the "discounting-adjusted probabilities" that future innovations are achieved: with a Poisson discovery process, each future innovation eventually occurs with probability one, but since there is discounting, a delayed success counts less than instant success. Thus, $\prod_{h=1}^{i}\frac{z_{k+h}}{(r+z_{k+h})}$ is the "discounting-adjusted probability" that innovation $k+i$ occurs and period $k+i+1$ profits start accruing to the kth innovator.

More intuition on the determinants of the incentive to innovate can be gained by focusing on the case of a stationary environment in which z_k and $\pi_{i,k}$ are constant across periods. In the limiting case in which z tends to zero, the value of the innovation will then depend only on the technological leader's profit, π_0. This limiting case effectively corresponds to a single innovation framework, like that envisaged in the early industrial organization literature. In the polar case in which r tends to 0, the value of the innovation would depend only on the sum total of firms' profits,

$\Pi = \sum_{i=0}^{m} \pi_i$. In general, both industry profits and the profit distribution across firms matter.

4. INTENSITY OF COMPETITION AND INCENTIVE TO INNOVATE

In this section we analyze the effect of an increase in the intensity of competition on the incentives to innovate. Building on the result of the previous section, we focus on how competition affects industry profits and their distribution across firms. We identify the front loading of profits, the price effect, and the productive efficiency effect associated with a change in competitive pressure. We also demonstrate circumstances in which the productive efficiency effect dominates the price effect. To underscore that our results are independent of the details of the particular growth model that we develop below, the analysis is cast in a partial equilibrium framework.

Preliminaries.—

Consider an industry comprising $s+1$ asymmetric firms, indexed by $i = 0, 1, ..., s$, producing a homogeneous product. Let firms' marginal cost be constant at c_i per unit, and label firms so that $c_0 < c_1 < ... < c_s$. Thus, firm 0 is the technological leader (e.g. the latest innovator), firm 1 is the leader's most efficient competitor (e.g. the penultimate innovator) and so on. The number of firms that are active in equilibrium, $m+1$, is determined endogenously; firm m is the least efficient amongst active firms. Let demand be given by $X(p)$, where p is price, X is output, and $X(.)$ is a strictly decreasing and twice differentiable function on $[0, \bar{p}]$ and is zero on $[\bar{p}, \infty)$. It follows that inverse demand, $p(X)$, is decreasing and twice differentiable on $[0, X(0)]$. For simplicity, we assume that the following regularity condition holds: $2p'(X) + p''(X)X < 0$ on $[0, X(0)]$. This assumption of decreasing marginal revenue simplifies the exposition (in particular, it implies that the function $\Pi(X) = [p(X) - \psi]X$ is strictly concave for any constant $\psi < \bar{p}$) but is not needed for many of our results. The individual firm's profit function is $\pi_i = [p(X) - c_i]x_i$, where x_i is the individual firm's output. To keep the analysis interesting, assume that c_1 is lower than the monopoly price associated with c_0, $p^M(c_0) = \arg\max(p - c_0)X(p)$. If this assumption failed, firm 0 could engage in monopoly pricing without fear of being displaced by its competitors.

Bertrand and Cournot competition.—

Initially we parametrize the degree of competition by a switch from Cournot to Bertrand competition. With Bertrand competition, the outcome is a limit-pricing equilibrium in which price equals the marginal cost of the second most efficient firm and all of the output is produced by the low-cost firm: $p^B = c_1$, $m^B = 0$, and $x_0^B = X^B = X(c_1)$. In a Cournot equilibrium, the first-order conditions are[8]

$$p'(X^C)x_i^C + p^C = c_i \qquad i = 0, ..., m \tag{2}$$

where $X^C = \sum_{i=0}^{m} x_i^C$ and m^C is the greatest integer such that $p^C \geq c_m$ holds. For future reference, note that

$$\frac{x_i^C}{x_j^C} = \frac{p^C - c_i}{p^C - c_j} \qquad i, j = 0, ..., m$$

that is, in the Cournot equilibrium, *the ratio of any two active firms' market shares equals the ratio of their respective price-cost margins.* This relationship trivially holds also in the Bertrand equilibrium. However, in the Cournot equilibrium high-cost firms hold positive market shares, which means that the Cournot equilibrium exhibits productive inefficiency. This productive inefficiency is important to explain why industry profits can be larger under Bertrand competition, even if the Bertrand equilibrium price is lower than the Cournot price.

It is indeed well known that with Cournot competition both the equilibrium price and the number of active firms are higher than under Bertrand competition. Therefore, a switch from Cournot to Bertrand competition is associated with an increase in the intensity of competition. To facilitate the comparison we now present a general solution that encompasses the Bertrand and Cournot equilibria as special cases. This will also allow us to obtain a continuous index of the intensity of competition.

[8]With constant marginal costs, the assumption $2p'(X) + p''(X)X < 0$ ensures that the second order conditions are satisfied and the Cournot equilibrium is unique.

A reduced-form model.—

The intensity of competition can be measured in many different ways, but any definition of competition involves the idea that more intense competition reduces the equilibrium price of a homogeneous product. Accordingly, we use a reduced form specification in which the intensity of competition is simply identified with the (inverse of the) equilibrium price.[9] Thus, let p range from the Cournot equilibrium price p^C to the Bertrand equilibrium price $p^B = c_1$ as product market competition increases. Industry output is $X(p)$. To pin down the industry equilibrium, assume that the ratio of any two active firms' market shares equals the ratio of their respective price-cost margins without making any more specific assumption on the nature of product market competition:

$$\frac{x_i}{x_j} = \frac{p - c_i}{p - c_j} \qquad i, j = 0, ..., m. \tag{3}$$

The number of active firms, $m(p)$ is determined as a function of p as the largest integer such that $p > c_m$ (it is understood that $x_i = 0$ when $p < c_i$). The active firms' equilibrium outputs and profits are then uniquely determined by the adding up condition $\sum_{i=0}^{m(p)} x_i = X(p)$.

To show existence and uniqueness of the solution, note that for any given p, $m(p)$ is uniquely determined. Equations (3) provide $m(p)$ independent conditions. Together with the adding up condition, they comprise a system of $m(p)+1$ linearly independent equations in the $m(p) + 1$ unknowns $x_0, ..., x_m$, the solution of which exists and is unique. It is easy to show that individual outputs are

$$x_i = \frac{p - c_i}{p - \hat{c}} \frac{X(p)}{[m(p) + 1]} \tag{4}$$

where $\hat{c} = \sum_{i=0}^{m(p)} \frac{c_i}{[m(p) + 1]}$ is the unweighted average of the marginal costs of active

[9] Previous work on competition and growth has often measured the intensity of competition by the inverse of the elasticity of demand, which determines the size of the innovator's mark-up. Our reduced-form model allows us to obtain a continuous measure of the intensity of competition that disentangles the effects of a change in the degree of competition from those associated with changes in structural (taste and/or technology) parameters that ultimately determine the elasticity of demand.

firms.

Clearly, because (3) holds both at the Bertrand and Cournot equilibria, these equilibria are reproduced for $p = p^B$ and $p = p^C$, respectively. For intermediate values of the price, our solution can be interpreted as a reduced form of a more highly structured model where firms can collude partially (Cabral (1995)), or can choose both capacities and prices (Maggi (1996)).[10] Alternatively, and perhaps more prudently, the solution can be thought of as an analytical tool that helps compare the Bertrand and Cournot equilibria.

The productive efficiency effect.—

Consider now an increase in the intensity of competition, i.e. a fall in the equilibrium price. If the number of active firms and their respective market shares stayed constant, the fall in the equilibrium price would unambiguously reduce industry profits $\Pi = \sum_{i=0}^{s} \pi_i$. This is the *price effect.* The reason why the price effect is negative is that industry profits are $\Pi = [p(X) - \bar{c}]X$, where $\bar{c} = \sum_{i=0}^{m} \frac{x_i}{X} c_i$ is the *industry average cost*, i.e. weighted average of firms' marginal costs. With constant market shares, $\bar{c}$ is constant; since $\Pi(X)$ is quasi-concave, any fall in price must then reduce industry profits if the price is lower than the monopoly price. However, the number of active firms and their market shares change with the equilibrium price. As a consequence, $\bar{c}$ changes with the intensity of competition, and the associated change in industry costs and profits is the *productive efficiency effect.*

Formally, the change in industry profits associated with a change in the intensity

[10] If fact, our solution coincides with the conjectural variations equilibrium under the assumption that the conjectural variations parameter is the same for all firms. To see this, note that in a conjectural variations equilibrium, the first-order conditions (2) become

$$(1 + \phi_i)p'(X)x_i + p = c_i \qquad i = 0, ..., m$$

where ϕ_i is the conjectural variations parameter of firm i. Provided that ϕ_i is the same for all firms, it is immediate that (3) holds at equilibrium.

of competition is[11]

$$\frac{d\Pi}{dp} = \underbrace{X + (p - \bar{c})\frac{dX}{dp}}_{\text{price effect}} \qquad \underbrace{-\frac{d\bar{c}}{dp}X}_{\text{productive efficiency effect}} \tag{5}$$

We now show that an increase in the intensity of competition improves the productive efficiency of the industry; a fall in the equilibrium price, that is to say, lowers the industry average cost.

Lemma 1 *The productive efficiency effect is positive.*

Proof. Simple algebra (the details are in the Appendix) shows that

$$\frac{d\bar{c}}{dp}X = \sum_{i=0}^{m(p)}(c_i - \bar{c})\frac{dx_i}{dp} = \frac{X(p)\sigma_c^2}{[m(p)+1]\,(p - \hat{c})^2} > 0 \tag{6}$$

where σ_c^2 is the variance of active firms' marginal costs. ■

The intuition behind Lemma 1 is that a rise in competitive pressure raises the market shares of low-cost firms and lowers the market shares of high-cost firms. This reduces the total cost at which any given industry output is produced. An immediate corollary of Lemma 1 is that a switch from Cournot to Bertrand competition improves the productive efficiency of the industry. This is obvious, because with Bertrand competition all of the output is produced by the low-cost firm, whereas under Cournot competition high-cost firms have positive market shares.

Before proceeding, we pause here to show that an increase in price is positively associated with an (inverse) index of the intensity of competition that is commonly used in empirical work, namely, the industry average price-cost margin $(p - \bar{c})$. [12]

[11] Because $m(p)$ jumps up at certain critical points $c_1, c_2, ..., c_m$ as p increases, variables like outputs, profits etc. are piecewise differentiable. Consequently, caution should be used in differentiating those variables with respect to p. Any such derivative calculated at c_j, where c_j is the critical value of p at which m jumps up from $j-1$ to j, must be interpreted as the right derivative under the conventional assumption $x_j(c_j) = 0$.

[12] To measure the intensity of competition, Nickell (1996) and others also use indices of market concentration. In our model, however, an increase in price is negatively associated with market con-

Lemma 2 *A rise in price p (weakly) raises the industry average price-cost margin $(p - \bar{c})$.*

Proof. See the Appendix. ■

With these preliminary results in place, we are now ready to state the main results of this section.

Competition and industry profits.—

We start focusing on the effect of product market competition on industry profits. In particular, we look for circumstances in which the productive efficiency effect dominates the price effect so that more intense competition raises industry profits.

As is clear from equation (6), if firms were symmetric ($\sigma_c^2 = 0$) the productive efficiency effect would vanish. However, with asymmetric firms, $\sigma_c^2 > 0$, the productive efficiency effect is first order. When the price effect is second order, it must therefore be dominated by the productive efficiency effect. But the price effect will, indeed, be second order when the price is close to the monopoly price. This observation leads to the following result.

Proposition 1 *When the marginal cost of the second most efficient firm c_1 is close to the monopoly price $p^M(c_0)$, industry profits are greater under Bertrand competition than under Cournot competition.*

Proof. The proof is in the Appendix. Here we sketch the proof of a more general claim, i.e. that when c_1 is close to $p^M(c_0)$, industry profits are monotonically increasing in the intensity of competition. To prove this claim, note that when c_1 is just below $p^M(c_0)$, p must be close to $p^M(c_0)$;[13] moreover, x_1 must be close to zero and so

centration. More precisely, let $L_{\mathbf{X}}(\frac{h}{s+1}) = \sum_{i=s-h+1}^{s} \frac{x_i}{X}$ be the Lorenz curve of the output distribution, showing the proportion of industry output produced by any given percentage of the population of firms, starting from the smallest firm. Proceeding as in the proof of Proposition 3 below, it can be shown that a rise in price shifts the Lorenz curve $L_{\mathbf{X}}$ down. This implies that as the price increases, market concentration falls according to any of the most commonly used measures of market concentration, like C_n, the sum of the market shares of the largest n firms, or the Herfindhal index.

[13] Remember that p ranges from $p^B = c_1$ to p^C, which cannot exceed $p^M(c_0)$.

$\bar{c}$ must be close to c_0. Using (6), equation (5) therefore reduces to

$$\begin{aligned}\frac{d\Pi}{dp} &= \underbrace{X[p^M(c_0)] + [p^M(c_0) - c_0]\frac{dX}{dp}}_{=0} - X[p^M(c_0)]\frac{\sigma_c^2}{2[p^M(c_0) - \hat{c}]^2} \\ &= -X[p^M(c_0)]\frac{\sigma_c^2}{2[p^M(c_0) - \hat{c}]^2} < 0\end{aligned}$$

Thus, industry profits monotonically decrease with price.■

The intuition is that at $c_1 = p^M(c_0)$ both Bertrand and Cournot competition yield the monopoly solution. Starting from $c_1 = p^M(c_0)$, consider now the effect of decreasing c_1. With Bertrand competition, the presence of firm 1 now constrains the low-cost firm (i.e., firm 0) that must price at $p = c_1$, but when c_1 is close to the monopoly price the effect of competition on the low-cost firm's profit is second order – the profit function is flat at $p = p^M(c_0)$. With Cournot competition a fall in c_1 reduces the equilibrium price less than under Bertrand competition, but now it also increases the high-cost firm's market share. Since $c_1 > c_0$, with Cournot competition the negative effect on industry profits of a fall in c_1 is first order, whence the result follows.

Proposition 1 might suggest that the productive efficiency effect is small and can prevail over the price effect only if the latter is negligible. Quite to the contrary, the productive efficiency effect can be surprisingly large: a unit increase in the equilibrium price can raise the average industry cost by as much as one! (Lemma 2 ensures that $\bar{c}$ cannot increase by more than one.) This is indeed what happens in the vicinity of the Bertrand equilibrium – the equilibrium which most quality ladder models of endogenous growth with non-drastic innovations focus on. Consequently, starting at Bertrand competition, a small decrease in the intensity of competition will unambiguously lead to a fall in industry profits.

Proposition 2 *Starting at the Bertrand equilibrium price, a small increase in price lowers industry profits.*

Proof. From (5) and (6) we get

$$\frac{d\Pi}{dp} = X + (p - \bar{c})\frac{dX}{dp} - X\frac{\sigma_c^2}{2(p - \hat{c})^2}$$

When p is slightly increased starting at $p = p^B = c_1$, only two firms, 0 and 1, will be active. Consequently, $\sigma_c^2 = (c_1 - \hat{c})^2 + (c_0 - \hat{c})^2 = 2(c_1 - \hat{c})^2 = 2(p - \hat{c})^2$, whence we get

$$\frac{d\Pi}{dp} |_{p=p^B} = (p - \bar{c})\frac{dX}{dp} < 0. \blacksquare$$

Proposition 2 effectively shows that starting at the Bertrand equilibrium price, a small increase in price leaves the industry average price-cost margin $(p-\bar{c})$ unchanged. The intuitive reason is that when $p = c_1$, a unit increase in price raises the market share of firm 1 (the high-cost firm) from zero to $\frac{1}{c_1 - c_0}$, and this causes a unit increase in $\bar{c}$. Because the industry average price-cost margin $(p - \bar{c})$ is unchanged, a rise in price must necessarily lower industry profit.

[Figure 1 here]

Figures 1 and 2 illustrate Propositions 1 and 2, respectively, in the linear demand case $p = 1 - X$ with $c_0 = 0$. Figure 1 plots industry profits under Bertrand and Cournot competition as c_1 ranges from 0 (the symmetric case) to 0.50 (the monopoly price). Industry profits are greater with Bertrand competition for $c_1 > 0.28$. Figure 2 displays the regions in which industry profits increase or decrease with the intensity of competition as c_1 ranges from 0 to $\frac{1}{2}$ and p ranges from $p^B = c_1$ to $p^C = \frac{1+c_1}{3}$. Although our qualitative results are local, Figures 1 and 2 show that the productive efficiency effect prevails over the price effect in a sizeable region of parameter values.

[Figure 2 here]

Competition and the distribution of profits.—

Product market competition affects not only the sum total of all firms' profits, but also another key determinant of the incentive to innovate, namely, the distribution of profits across active firms. Accordingly, we now focus on how a rise in the intensity of competition affects the profit distribution, for any given level of industry profits.

Let the profit distribution be the $s+1$-dimensional vector $\mathbf{\Pi} = (\pi_0, \pi_1, ..., \pi_s)$. Because $c_0 < c_1 < ... < c_s$, we have $\pi_0 \geq \pi_1 \geq ... \geq \pi_s$, with strict inequalities whenever profits are strictly positive. The Lorenz curve of the profit distribution is $L_{\mathbf{\Pi}}(\frac{h}{s+1}) = \sum_{i=s-h+1}^{s} \frac{\pi_i}{\Pi}$, where Π is industry profits. It shows the proportion of industry profits earned by any given percentage of the population of firms, starting from the least profitable firm.

Our next result is that the profit distribution becomes more unequal according to the Lorenz dominance criterion as competition becomes more intense. That is to say, the Lorenz curve of the profit distribution shifts down as the intensity of competition increases. Lorenz dominance implies that profit inequality would increase as the intensity of competition increases according to a wide set of inequality measures.

Proposition 3 *If there are at least two active firms, an increase in the intensity of competition makes the profit distribution more unequal according to the Lorenz dominance criterion.*

Proof. See the Appendix. ■

Proposition 3 follows from the simple fact that low-cost firms gain, and high-cost firms lose in relative terms when the market becomes more competitive. The reason is twofold: first, the market shares of low-cost firms tend to increase with the intensity of competition, and second, when the equilibrium price falls, the percentage decrease in the price-cost margin is larger for high-cost firms.

In the dynamic model of successive innovations to be developed presently, each innovator will be active, and reap positive profits, for $m + 1$ periods: in the first period after his innovation is achieved, he is the technological leader, in the second period he is the second most efficient firm, in the third period he is the third most efficient amongst active firms, and so on. Over time periods, the innovator reaps total industry profits irrespective of the intensity of competition. However, the Lorenz dominance result means that as the intensity of competition increases the innovator will get a larger proportion of his prospective rents in the first i periods for which he is active, for all $i = 1, ..., m$ (over $m + 1$ periods he always gets 100 per cent of industry profits). This is the front loading of profits associated with more intense competition. Proposition 4 below shows that the front loading of profits tends to increase the incentives to innovate, for any given level of industry profits.

5. A GROWTH MODEL

We now embed the insights from the previous sections in a simple growth model. To eschew distracting assumptions, we use a one-sector version of the text-book model of Barro and Sala-i-Martin (1995, ch. 7), but the main results are more general and can be reproduced in many other models with quality improvements.[14]

Preferences and technology.—

The economy is populated by identical individuals whose mass is normalized to 1. Each individual has linear intertemporal preferences:

$$u(c) = \int_0^\infty c(t)e^{-rt}dt$$

so that the rate of time preference r coincides with the equilibrium rate of interest. Each individual inelastically offers one unit of labor.

The final good y is produced in a perfectly competitive market using labor (which is in fixed supply) and an intermediate good the quality of which increases over time because of technical progress. We normalize at 1 the quality of the intermediate good at time 0, and we denote by $q > 1$ the size of each innovation. In period k, where $k - 1$ is the number of past innovations, the final good can be produced according to the following constant-returns-to-scale production function:

$$y_k = \widehat{X}_k^{\alpha} \text{ , } 0 < \alpha < 1, \tag{7}$$

where labor input is set equal to one, $(1 - \alpha)$ is the share of labor's income, and $\widehat{X}_k = \sum_{i=0}^{k} q^{i-1} x_i$ is the quality-adjusted index of a composite good which combines all past generations of intermediate goods. It is convenient to rewrite $\widehat{X}_k$ as $\widehat{X}_k = q^k X_k$, where $X_k = \sum_{i=0}^{k} q^{i-k-1} x_i$ measures the input of the composite intermediate good in efficiency units relative to the last vintage. From the production function (7) one obtains the demand for the intermediate good (measured in efficiency units)

$$X_k = \alpha^{\frac{1}{1-\alpha}} p_k^{-\frac{1}{1-\alpha}} q^{\frac{\alpha}{1-\alpha}k} \tag{8}$$

[14] In particular, the growth model we develop exhibits scale effects, but our results would continue to hold in a model with no scale effects, provided that greater incentives to innovate lead to faster growth (see for example Howitt (1999)).

where p_k is its price. The final good may be consumed, used to produce intermediate goods, or used in research. Independently of its quality, the intermediate good is produced using the final good with a constant marginal rate of transformation that is normalized to 1.

Technical progress.—

In each period there is a patent race. Incumbents do no research, and there is free entry by risk-neutral outsiders. In period k, each firm ℓ participating in the race decides its R&D effort $n_{\ell k}$ to obtain the kth innovation. The R&D effort determines the expected time of successful completion of the R&D project according to a Poisson discovery process with a hazard rate equal to $\lambda_k n_{\ell k}$, with $\lambda_k > 0$. The projects of different firms are independent, so that the aggregate instantaneous probability of success is simply the sum of the individual probabilities. Let $n_k = \sum_\ell n_{\ell k}$ denote aggregate R&D investment in period k. Then, the innovation occurs according to a Poisson process with a hazard rate $z_k = \lambda_k n_k$.[15]

If innovations were drastic, the technological leader would be unconstrained by outside competition and could engage in monopoly pricing, and so the model's equilibrium would be independent of the mode of competition in the product market. We therefore assume that innovations are non-drastic, which in the current setting means that

$$q \leq \frac{1}{\alpha}.$$

Steady state.—

In a steady-growth equilibrium the price of the (latest vintage of the) intermediate good, in terms of the consumption good, will be constant. This implies that $X_k$will grow at rate $q^{\frac{\alpha}{1-\alpha}}$, and from (7) it then follows immediately that y_k will also grow at rate $q^{\frac{\alpha}{1-\alpha}}$. This is the growth factor between periods, and we denote it by $g \equiv q^{\frac{\alpha}{1-\alpha}}$. In a steady state, output, consumption, the input of intermediate goods, profits, and R&D investment will all grow at rate g between periods.

In order to guarantee the existence of a steady state with positive growth, following

[15] Our results immediately extend to the case where $z_k = \lambda_k n_k^{\beta}$, with $0 < \beta \leq 1$. The case $\beta < 1$ may reflect the presence of external diseconomies in research.

Barro and Sala-i-Martin (1995, p. 250) we assume that $\lambda_k = \lambda g^{-k}$. Because in a steady state n_k grows at rate g across periods, under this assumption the hazard rate $z_k = \lambda_k n_k$ can be constant across periods. Finally, note that the following transversality condition must hold (see Barro and Sala-i-Martin, 1995, p. 248):

$$r > z(g-1).$$

If this condition is violated, consumers have an incentive to postpone consumption indefinitely.

Equilibrium in the product market.—

To proceed, remember that only the kth innovator, who holds a patent on his vintage of the good, can produce the intermediate good of vintage k. Independently of its quality, the intermediate good is produced using the final good on a one-to-one basis. However, in period k it takes q^{i-1} units of the intermediate good of vintage $k-i$ to make one unit of the intermediate good k *in efficiency units*. Innovator $k-i$'s unit cost of producing the intermediate good, measured in period k efficiency units, is therefore q^{i-1}. Thus we can proceed as if the intermediate good was homogeneous but firms had different production costs, i.e. 1 for the latest innovator, q for the penultimate innovator, q^2 for the third latest innovator and so on.

Given the demand function (8), the Cournot equilibrium price can then be easily calculated as

$$p^C = \frac{1+q+q^2+...+q^{m^C}}{m^C+\alpha} \tag{9}$$

where $m^C$is the largest integer such that[16]

$$\frac{1+q+q^2+...+q^m}{m+\alpha} \geq q^{m+1}.$$

[16]It can be shown that m^C is constant across periods. With m_k^C+1 active firms in period k, the Cournot equilibrium price is

$$p_k^C = \frac{1+q+q^2+...+q^{m_k^C}}{m_k^C+\alpha}$$

and so m_k^C is the largest integer such that

$$\frac{1+q+q^2+...+q^{m_k^C}}{m_k^C+\alpha} \geq q^{m_k^C+1}.$$

Because all the parameters in this inequality are constant, m_k^C must be constant across periods.

Individual outputs can be obtained by substituting (9) into (4). Clearly, in each period low-cost firms hold larger market shares than high-cost firms. However, when innovations are non-drastic different vintages of the intermediate good will be simultaneously produced, even if older vintages are less productive.

In contrast, the Bertrand equilibrium is a limit-pricing equilibrium where the leader prices at $p^B = q$ and drives its competitors out of the market. At this limit-pricing equilibrium, there is no productive inefficiency. The corresponding profits are $\pi_{i,k}^B = 0$ for $i \geq 1$, and:

$$\pi_{0,k}^B = \Pi_k^B = (q-1)\, q^{-\frac{1}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}} g^k.$$

Such a Bertrand equilibrium is standard in the endogenous growth literature. The next Lemma confirms that a switch from Cournot to Bertrand captures the notion of tougher competition.

Lemma 3 *The equilibrium price under Cournot competition is greater than under Bertrand competition.*

Proof. We have

$$\begin{aligned}
p^C &= \frac{1+q+q^2+...+q^m}{m+\alpha} \\
&> \frac{\alpha q+q+q^2+...+q^m}{m+\alpha} \text{ because innovations are non-drastic } (\alpha q<1) \\
&> \frac{\alpha q+mq}{m+\alpha} \quad \text{because } q>1 \\
&= q \\
&= p^B \qquad \blacksquare
\end{aligned}$$

Like in Section 4, we also use a more general reduced-form solution that encompasses the Bertrand and Cournot equilibria as special cases. In this solution, p ranges from $p^B = q$ to p^C, and the corresponding individual outputs are given by (4). The equilibrium number of active firms other than the latest innovator, $m(p)$, is the largest integer such that $m(p) \leq \frac{\log p}{\log q}$.

Equilibrium in the research industry.—

To complete the derivation of the model's equilibrium, we next focus on the research sector. The expected discounted profit of an outside firm that invests $n_{\ell k}$ units of the final good in period k to obtain innovation k, as of the start of the race and given that aggregate investment in R&D is n_k, is

$$\frac{\lambda_k n_{\ell k} E(V_k) - n_{\ell k}}{r + n_k \lambda_k}$$

where $E(V_k)$ is the expected value of the kth innovation, and is given by (1). At equilibrium, outsiders' expected net profit must be equal to zero:

$$\lambda_k E(V_k) = 1 \tag{10}$$

In a steady state, z is constant and profits grow at rate g between periods: $\pi_{i,k} = \pi_i g^k$, where $\pi_i = \pi_{i,0}$. Equation (1) then reduces to:

$$E(V_k) = \sum_{i=0}^{m} \frac{z^i g^{k+i+1} \pi_i}{(r+z)^{i+1}} \tag{11}$$

Equilibrium in the research industry is then determined by inserting (11) into the free-entry condition (10)

$$H(z) = \frac{1}{\lambda}. \tag{12}$$

where $H(z) \equiv \sum_{i=0}^{m} \frac{z^i g^{i+1} \pi_i}{(r+z)^{i+1}}$. Equation (12) determines the equilibrium hazard rate, z^*, and hence the economy's rate of growth. To see this, note that the growth factor between periods, g, is constant. This means that the equilibrium rate of growth is entirely determined by the expected length of each period, which in turn depends on the speed of technical progress: with an exponential distribution of the timing of success, the expected waiting time for each innovation is $\frac{1}{z}$. Consequently, an increase in z is associated with faster growth.

Lemma 4 *Assume that* $\frac{g\pi_0}{r} > \frac{1}{\lambda}$. *Then a unique, strictly positive equilibrium hazard rate* z^* *exists.*

Proof. See the Appendix. ■

Condition $\frac{g\pi_0}{r} > \frac{1}{\lambda}$ ensures that research is sufficiently profitable that some research is conducted at equilibrium. It is easy to show that the steady state level of research, z^*, is an increasing function of the productivity of R&D effort λ. For any other arbitrary parameter a that influences z^*, the sign of $\frac{\partial z^*}{\partial a}$ equals the sign of $\frac{\partial H}{\partial a}$. Using this fact, it is immediate to show that z^* is a decreasing function of the rate of time preference r and an increasing function the step size between innovations q. It is also clear that the economy's rate of growth increases with the incentives to innovate. Our next task is to analyze the impact of a switch from Cournot to Bertrand competition, or, more generally, a rise in competitive pressure, on the economy's rate of growth.

Competition and growth.—

Proposition 4 *If industry profits are weakly increasing in the intensity of competition, an increase in the intensity of competition raises the equilibrium rate of growth.*

Proof. Let p and p' denote two price levels, with $p < p'$. The move from p' to p corresponds to an increase in the intensity of competition. By assumption, $\Pi(p) \geq \Pi(p')$. Proposition 3 then implies that

$$\sum_{i=0}^{h} \pi_i(p) \geq \sum_{i=0}^{h} \pi_i(p') \tag{13}$$

for all h, with a strict inequality for at least one h. We also know that $m(p) \leq m(p')$. We must show that $H(p) > H(p')$, i.e.

$$\sum_{i=0}^{m(p)} \pi_i(p) \left[\frac{gz}{(r+z)}\right]^i > \sum_{i=0}^{m(p')} \pi_i(p') \left[\frac{gz}{(r+z)}\right]^i .$$

This can be rewritten as

$$\sum_{h=0}^{m(p')} \left\{ \sum_{i=0}^{h} \pi_i(p) \left[\left(\frac{gz}{(r+z)}\right)^{h-1} - \left(\frac{gz}{(r+z)}\right)^{h} \right] \right\} >$$
$$> \sum_{i=0}^{m(p')} \left\{ \sum_{i=0}^{h} \pi_i(p) \left[\left(\frac{gz}{(r+z)}\right)^{h-1} - \left(\frac{gz}{(r+z)}\right)^{h} \right] \right\}$$

where the terms inside square brackets are positive by the transversality condition. By inequality (13), each term inside curly brackets on the left hand side is at least as large as the corresponding term on the right hand side, with at least one strict inequality. This completes the proof of the Proposition. ■

The intuition is as follows. We have shown in Section 3 that the value of an innovation is a weighted average of all active firms' profits, $\sum_{i=0}^{m} \omega_i \pi_i$, where the weights ω_i reflect the expected length of time periods, discounting, and growth. In a steady state, the expected length of time periods is constant. The transversality condition implies that discounting prevails over growth, and so the weights are decreasing in i : $\omega_0 \geq \omega_1 \geq ... \geq \omega_m$. The Lorenz dominance result (Proposition 3) shows that a rise in competitive pressure shifts profits from the least profitable firms to the most profitable ones. With declining weights, such a front loading of profits implies that the incentive to innovate $\sum_{i=0}^{m} \omega_i \pi_i$ increases with the intensity of competition, provided that industry profits do not fall. And in a neo-Schumpeterian model an increase in the incentive to innovate must cause an increase in the economy's rate of growth.

Proposition 4 leads to the following corollaries.

Corollary 1 *If innovations are sufficiently large (i.e., if the size of the innovations, q, is close to $\frac{1}{\alpha}$), then the rate of growth under Bertrand competition is higher than the rate of growth under Cournot competition.*

Proof. Follows from Propositions 1 and 4. ■

Numerical calculations show that as q falls, eventually $\Pi^B(q) < \Pi^C(q)$. By Proposition 4, this means that the rate of growth can (but need not) be greater with Cournot competition if the size of innovations is sufficiently small. Numerical calculations also show that the interval in which aggregate profits are greater under Bertrand competition, and thus more competition is surely associated with faster growth, can be quite large. Figure 3 illustrates.

[Figure 3 here]

Stokey (1995) notes that if innovations occur every few years, a reasonable range for q is 1.02 to 1.04; if innovations occur only a couple of times per century, then a reasonable range for q is 1.25 to 1.50. Barro and Sala-i-Martin (1995) note that reasonable values for α, the share of capital's income, range from 0.30 if capital is interpreted as physical capital to 0.70 if capital includes human capital. The shaded area in Figure 3 corresponds to the "reasonable" range $q \in [1.02, 1.50]$ and $\alpha \in [0.30, 0.70]$. Over this range, more intense competition may well be associated with faster growth.

Corollary 2 *If the intensity of competition is sufficiently high (i.e., p is close to the Bertrand equilibrium price q), a further increase in the intensity of competition raises the economy's rate of growth.*

Proof. Follows from Propositions 2 and 4. ■

In fact, the relationship between competition and growth is monotonically increasing when the size of innovations is large. For smaller innovations, numerical calculations show that industry profits first increase and then decrease as p increases. Consequently, unless the front loading of profits is sufficiently strong to outweigh the effect of tougher competition on total industry profits, the rate of growth first decreases and then increases as the intensity of competition increases.[17]

6. CONCLUDING REMARKS

In this paper, we have re-considered the relationship between competition and growth in a standard neo-Schumpeterian model with improvements in the quality of products. Focusing on the case of non-drastic innovations, we have modeled the notion of lower competition by a switch from Bertrand to Cournot competition, and more generally by a decrease in the equilibrium price.

We have shown that competition is good for growth either if the size of (non-drastic) innovations is large, or if the intensity of competition is high, or both. This result follows from two qualitatively new effects – the front loading of profits and the productive efficiency effect – that arise when innovators are not immediately displaced by the occurrence of the next innovation so that two or more asymmetric firms are simultaneously active in the same industry.

[17] These results hold provided that only two firms are active in the Cournot equilibrium. When innovations are still smaller, such that three firms are active under Cournot competition, the relationship between competition and growth can exhibit two local maxima.

We conclude with a brief discussion of repeated innovation by incumbents, patent licensing, welfare, and the inverted-U shaped relationship between competition and growth.

Persistent leadership.—

In our model, incumbents do no research and are systematically replaced by outsiders. However, there is ample evidence that incumbents account for much of the research done and the resulting pattern of persistent leadership is well documented in many industries. As discussed at greater length in Denicolò (2001), the literature that tries to explain this pattern of persistent leadership recognizes that in standard quality ladder models leapfrogging is indeed an equilibrium of a simultaneous moves R&D game if the size of innovations is not too small. In this case, the assumption that incumbents do no research is not restrictive. However, the standard model can be adapted to make room for the persistence of leadership in various ways. If, for example, it is assumed that incumbents have a first-mover advantage in the patent race starting after the latest innovation, the outcome is a pre-emption equilibrium in which all research is done by incumbents. However, the amount of research is still determined by the outsiders' zero-profit condition, and thus the incentive to innovate is driven by the same qualitative effects as in the leapfrogging equilibrium.

Patent licensing.—

We have followed the vast majority of endogenous growth models in ruling out patent licensing. The standard justification for this assumption is that licensing agreements between successive innovators would have anti-competitive effects and as such would be prohibited by antitrust authorities. In our model, however, patent licensing agreements could be arranged so as to improve productive efficiency with no anticompetitive effects, and so ruling them out is not as innocent an assumption as in earlier models.

If patent licensing was ubiquitous, all of the output would be produced with the most efficient technology and so the productive efficiency effect would vanish. However, a variety of transaction costs impede licensing agreements. As an example, royalty licensing is possible only if the output is verifiable; when individual output is not verifiable, and only fixed-fee licensing is feasible, licensing will occur at equilibrium only if the size of innovations is sufficiently small. As another example, incomplete information over the size of the innovation can lead parties to introduce inefficient terms in the licensing agreement. In addition, innovative technological knowledge can be difficult to codify and transmit to others. These transaction costs may likely result in an equilibrium outcome in which some active firms do not use the latest generation technology. To the extent that the product market equilibrium exhibits some productive inefficiency, our qualitative results are likely to continue to hold even if we allow for some licensing agreements.

Welfare.—

Although a detailed welfare analysis is outside the scope of this paper, a few remarks are in order. Our analysis shows that an increase in the intensity of competition has two effects on social welfare, a static effect and a dynamic effect. The static effect is unambiguously positive. Indeed, for any given state of the technology, the price of the intermediate good is lower and output is greater with tougher competition. Further, if competition is Bertrand, only the most efficient firm is active in the intermediate good industry and so only the highest quality good is produced in equilibrium, ensuring that productive efficiency is achieved. The dynamic effect, that operates via the incentive to innovate and the rate of growth, is more complex. As we have seen, competition can be growth-enhancing or growth-reducing. In addition, the equilibrium rate of growth can exceed the socially optimal rate, which means that faster growth is not necessarily socially beneficial. Therefore, the overall welfare effect of tougher competition is generally ambiguous.

An inverted-U shaped relationship.—

Nickell (1996) and Aghion et al. (2002) find evidence that the relationship between competition and growth is inverted-U shaped. Although this evidence is hardly conclusive,[18] it is tempting to speculate whether our model can generate an inverted-U shaped relationship. While we have let the equilibrium price range from p^C to $p^B = q$, it is possible to extend the analysis to the case $p < p^B$. Of course, in this interval only the technological leader would be active in the product market. The interpretation

[18] Aghion et al. (2002) recognize that the evidence is fragmentary, while Nickell (1996) calls the evidence "thin."

of the case $p < p^B$ is that because of imperfect patent protection and technological spillovers, competitive pressure exerted by the leader's most efficient rival sets a tighter constraint on the price that the leader can charge than in the standard Bertrand equilibrium.[19] When $p < p^B$, both the productive efficiency effect and the front loading of profits disappear, and so tougher competition unambiguously leads to lower profits and growth. Remember, however, that when the equilibrium price p is just above p^B, tougher product market competition leads to faster growth. This implies that growth is (locally) fastest with Bertrand competition, and in the vicinity of the Bertrand equilibrium the relationship between competition and growth is inverted U-shaped.

[19] It could be debated whether the existence of incomplete patent protection and technological spillovers can be equated to a rise in competitive pressure. However, the two phenomena are difficult to distinguish observationally, as they both translate into lower price-cost margins.

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APPENDIX

Omitted details in the proof of Lemma 1.—

To prove the first equality in (6), i.e.

$$\frac{d\bar{c}}{dp}X = \sum_{i=0}^{m}(c_i - \bar{c})\frac{dx_i}{dp}$$

note that

$$\begin{aligned}
\frac{d\bar{c}}{dp} &= \frac{d}{dp}\sum_{i=0}^{m}\frac{x_i}{X}c_i \\
&= \frac{\sum_{i=0}^{m} c_i X \frac{dx_i}{dp} - \sum_{i=0}^{m} c_i x_i \frac{dX}{dp}}{X^2} \\
&= \frac{\sum_{i=0}^{m} c_i \frac{dx_i}{dp} - \frac{dX}{dp}\sum_{i=0}^{m} c_i \frac{x_i}{X}}{X} \\
&= \frac{\sum_{i=0}^{m} c_i \frac{dx_i}{dp} - \sum_{i=0}^{m} \frac{dx_i}{dp}\bar{c}}{X}
\end{aligned}$$

whence the result follows immediately. Next, note that

$$\sum_{i=0}^{m}(c_i - \bar{c})\frac{dx_i}{dp} = \sum_{i=0}^{m} c_i \frac{dx_i}{dp} - \bar{c}\frac{dX}{dp}$$

From (4) we get

$$\frac{dx_i}{dp} = \frac{x_i}{X}\frac{dX}{dp} + X(p)\frac{c_i - \hat{c}}{(p-\hat{c})^2}\frac{1}{[m(p)+1]}$$

Substituting into the above expression we get

$$\begin{aligned}
\sum_{i=0}^{m} c_i \frac{dx_i}{dp} - \bar{c}\frac{dX}{dp} &= \underbrace{\sum_{i=0}^{m} c_i \frac{x_i}{X}}_{=\bar{c}}\frac{dX}{dp} + \sum_{i=0}^{m} X(p) c_i \frac{c_i - \hat{c}}{(p-\hat{c})^2}\frac{1}{[m(p)+1]} - \bar{c}\frac{dX}{dp} \\
&= \frac{X(p)}{[m(p)+1]\,(p-\hat{c})^2}\sum_{i=0}^{m} c_i\,(c_i - \hat{c})
\end{aligned}$$

because the first and third term on the right hand side cancel out. But $\sum_{i=0}^{m} c_i\,(c_i - \hat{c})$ is the variance of active firms' marginal costs, σ_c^2, and so the second equality in (6) is obtained. ■

Proof of Lemma 2.—

From (6) we have

$$\begin{aligned}\frac{d}{dp}(p-\bar{c}) &= 1-\frac{\sum_{i=0}^{m(p)}(c_i-\hat{c})^2}{[m(p)+1]\,(p-\hat{c})^2}\\ &= \frac{\sum_{i=0}^{m(p)}\left[(p-\hat{c})^2-(c_i-\hat{c})^2\right]}{[m(p)+1]\,(p-\hat{c})^2}\geq 0\end{aligned}$$

where the inequality follows because $p \geq c_{m(p)}$. ■

Proof of Proposition 1.—

When $c_1 = p^M(c_0)$ we have $p^B = p^C = p^M(c_0)$ and $x_1^C = x_1^B = 0$; consequently, $\Pi^B = \Pi^C$. Starting from $c_1 = p^M(c_0)$, let us consider the effect of a small decrease in c_1, such that x_1^C becomes positive. Because $\Pi^B(c_1) = (c_1 - c_0)X(c_1)$, we have

$$\frac{d\Pi^B}{dc_1} = X(c_1) + (c_1 - c_0)X'(c_1)$$

and so $\frac{d\Pi^B}{dc_1} = 0$ at $c_1 = p^M(c_0)$. On the other hand, when c_1 is just below $p^M(c_0)$, exactly two firms will be active in the Cournot equilibrium. Thus, $\Pi^C(c_1) = (p^C - c_0)x_0^C + (p^C - c_1)x_1^C$. Differentiating we get

$$\frac{d\Pi^C}{dc_1} = X^C\frac{dp^C}{dc_1} - x_1^C + (p^C - c_0)\frac{dx_0^C}{dc_1} + (p^C - c_1)\frac{dx_1^C}{dc_1}$$

At $c_1 = p^M(c_0)$, the second and fourth term vanish, and so

$$\frac{d\Pi^C}{dc_1}\Big|_{c_1=p^M(c_0)} = X^C\frac{dp^C}{dc_1} + (p^C - c_0)\frac{dx_0^C}{dc_1}$$

From the first order conditions (3) one obtains

$$\frac{dx_0}{dc_1} = -\frac{p'(X) + p''(X)x_0}{[p'(X)]^2}\frac{dp^C}{dc_1}.$$

At $c_1 = p^M(c_0)$ we have $x_0 = X^C$ and thus

$$\frac{d\Pi^C}{dc_1}\Big|_{c_1=p^M(c_0)} = X^C\frac{dp^C}{dc_1} - (p^C - c_0)\frac{p'(X^C) + p''(X^C)X^C}{[p'(X^C)]^2}\frac{dp^C}{dc_1}.$$

But $(p^C - c_0) = -p'(X)x_0$ and so the derivative reduces to

$$\frac{d\Pi^C}{dc_1}\Big|_{c_1=p^M(c_0)} = \frac{2p'(X^C) + p''(X^C)X^C}{p'(X^C)}\frac{dp^C}{dc_1}.$$

The fraction $\frac{2p'(X^C)+p''(X^C)X^C}{p'(X^C)}$ is positive given the assumption of decreasing marginal revenue. Clearly, under our assumptions of constant marginal costs and decreasing marginal revenue we have $\frac{dp^C}{dc_1} > 0$.[20] It follows that

$$\frac{d\Pi^C}{dc_1}\Big|_{c_1=p^M(c_0)} > 0$$

This means that $\Pi^C(c_1)$ raises more steeply than $\Pi^B(c_1)$ in a left neighborhood of $p^M(c_0)$. By continuity, it follows that $\Pi^B(c_1) > \Pi^C(c_1)$ in a left neighborhood of $p^M(c_0)$. ■

Proof of Proposition 3.—

Let p and p' denote two price levels, with $p < p'$. The move from p' to p corresponds to an increase in the intensity of competition. We must show that $L_{\Pi(p)}(\frac{h}{s+1}) \leq L_{\Pi(p')}(\frac{h}{s+1})$ for all h, with at least one strict inequality. Note that (3) implies

$$\frac{\pi_i}{\pi_j} = \left(\frac{p - c_i}{p - c_j}\right)^2 \qquad i, j = 0, 1, ..., m$$

whenever firms i and j are active at equilibrium. Differentiating we get

$$\frac{d\frac{\pi_i}{\pi_j}}{dp} = 2\frac{(p - c_i)}{(p - c_j)^3}(c_i - c_j)$$

whence it immediately follows that

$$\frac{\pi_i(p)}{\pi_j(p)} > \frac{\pi_i(p')}{\pi_j(p')} \quad \text{for all } i, j = 0, 1, ..., m \text{ with } j > i. \tag{A1}$$

[20] As is well known, under Cournot competition and constant asymmetric marginal costs, the price is the same as if all firms shared the same cost $\hat{c}$. It is also well known that in a symmetric model with decreasing marginal revenue, the Cournot equilibrium price increases with the marginal cost. These two facts immediately imply that $\frac{dp^C}{dc_1} > 0$.

Inequalities (A1) imply
$$\frac{\pi_0(p)}{\Pi(p)} > \frac{\pi_0(p')}{\Pi(p')}$$
provided that there are at least two active firms at p', whence it follows that
$$\frac{\pi_s(p) + \pi_{s-1}(p) + ... + \pi_1(p)}{\Pi(p)} < \frac{\pi_s(p') + \pi_{s-1}(p') + ... + \pi_1(p')}{\Pi(p')}$$
i.e., $L_{\Pi(p)}(\frac{s}{s+1}) < L_{\Pi(p')}(\frac{s}{s+1})$.

Clearly, (A1) also implies $L_{\Pi(p)}(\frac{1}{s+1}) \leq L_{\Pi(p')}(\frac{1}{s+1})$. Now suppose to the contrary that there exists i such that $L_{\Pi(p)}(\frac{i}{s+1}) > L_{\Pi(p')}(\frac{i}{s+1})$. Let h be the minimum value of i for which inequality $L_{\Pi(p)}(\frac{i}{s+1}) > L_{\Pi(p')}(\frac{i}{s+1})$ holds, so that
$$\frac{\pi_s(p) + \pi_{s-1}(p) + ... + \pi_{s-h}(p)}{\Pi(p)} > \frac{\pi_s(p') + \pi_{s-1}(p') + ... + \pi_{s-h}(p')}{\Pi(p')}.$$
and
$$\frac{\pi_s(p) + \pi_{s-1}(p) + ... + \pi_{s-h-1}(p)}{\Pi(p)} \leq \frac{\pi_s(p') + \pi_{s-1}(p') + ... + \pi_{s-h-1}(p')}{\Pi(p')}.$$
These inequalities imply
$$\frac{\pi_{s-h-1}(p)}{\Pi(p)} < \frac{\pi_{s-h-1}(p')}{\Pi(p')} \tag{A2}$$
and that there exists at least one $j > s - h - 1$ such that
$$\frac{\pi_j(p)}{\Pi(p)} > \frac{\pi_j(p')}{\Pi(p')} \tag{A3}$$
Combining (A2) and (A3) we get
$$\frac{\pi_{s-h-1}(p)}{\pi_j(p)} < \frac{\pi_{s-h-1}(p')}{\pi_j(p')}$$
but this violates (A1). This contradiction establishes the result. ■

Proof of Lemma 4.—

First of all, we show that $H(z)$ is monotonically decreasing in z. Differentiating $H(z)$ we get:
$$H'(z) = \frac{d}{dz}\sum_{i=0}^{m} \frac{z^i g^{i+1}\pi_i}{(r+z)^{i+1}} = -\sum_{i=0}^{m-1} \frac{(i+1)z^i g^i(\pi_i - g\pi_{i+1})}{(r+z)^{i+2}} - \frac{(m+1)z^m g^{m+1}\pi_m}{(r+z)^{m+2}}$$

A sufficient condition for $H'(z)$ to be negative is that $\pi_i \geq g\pi_{i+1}$. We know from the proof of Proposition 3 that the ratio $\frac{\pi_i(p)}{\pi_{i+1}(p)}$ decreases with p. Moreover, because

$$\frac{\pi_i}{\pi_{i+1}} = \left(\frac{p - q^i}{p - q^{i+1}}\right)^2 \tag{A4}$$

and q^i is a convex function of i, we have

$$\frac{\pi_i}{\pi_{i+1}} < \frac{\pi_{i+1}}{\pi_{i+2}}$$

for all i and for all p. Consequently, it suffices to show that $\pi_0(p) \geq g\pi_1(p)$ when p equals the monopoly price $\frac{1}{\alpha}$, which always exceeds the Cournot equilibrium price p^C. From (A4) we have

$$\frac{\pi_0(\frac{1}{\alpha})}{\pi_1(\frac{1}{\alpha})} = \left(\frac{\frac{1}{\alpha} - 1}{\frac{1}{\alpha} - q}\right)^2$$

Therefore, we must prove that

$$\left(\frac{1-\alpha}{1-\alpha q}\right)^2 \geq g = q^{\frac{\alpha}{1-\alpha}}$$

or

$$(1-\alpha)^2 \geq q^{\frac{\alpha}{1-\alpha}} (1-\alpha q)^2$$

At $q = 1$, the weak inequality is satisfied as an equality. To conclude the proof, it suffices to show that the derivative with respect to q of the right hand side of the above inequality is negative. Differentiating we get

$$\frac{d}{dq}\left[q^{\frac{\alpha}{1-\alpha}} (1-\alpha q)^2\right] = -\frac{\alpha}{1-\alpha}(1-\alpha q)\, q^{\frac{\alpha}{1-\alpha}-1}\left[(q-1) + q(1-\alpha)\right] < 0.$$

This completes the proof that $H'(z) > 0$, which implies that the equilibrium, if it exists, is unique. To show existence, note that $H(0) = \frac{g\pi_0}{r} > \frac{1}{\lambda}$ and $\lim_{z\to\infty} H(z) = 0$. Because $H(z)$ is continuous, an equilibrium exists. ■

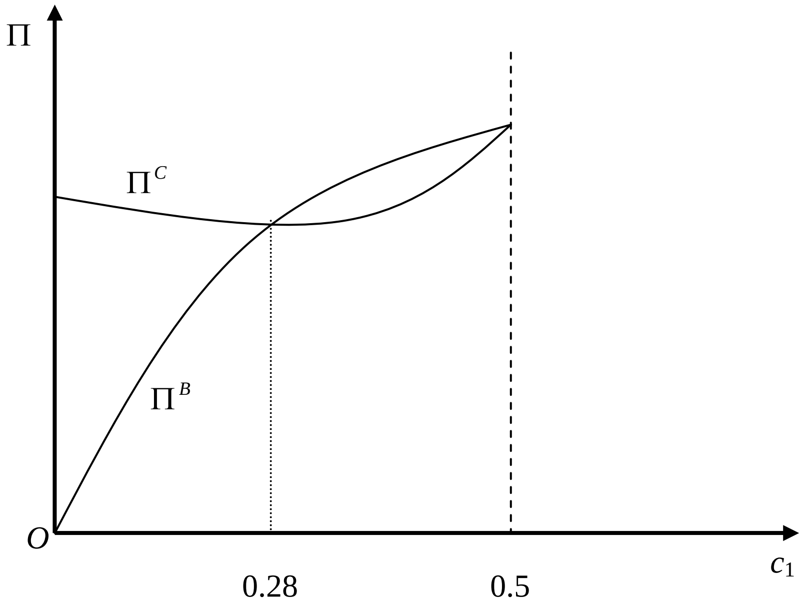

Figure 1
Industry profits under Bertrand and Cournot competition as a function of c_1 when $p = 1 - X$ and $c_0 = 0$

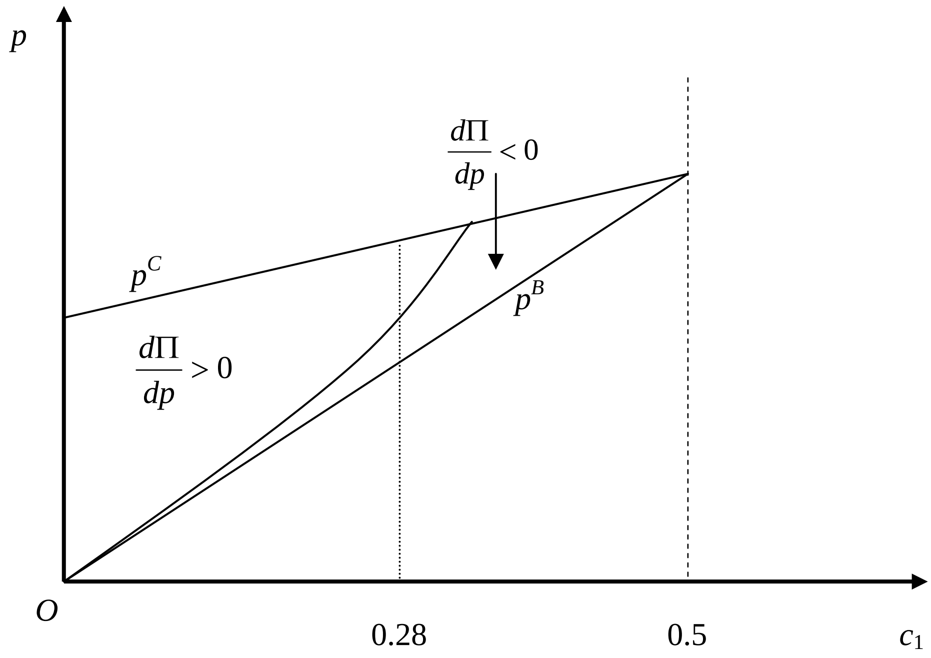

Figure 2
The effect of an increase in the equilibrium price p on industry profits as a function of c_1 and p when $X = 1 - p$ and $c_0 = 0$

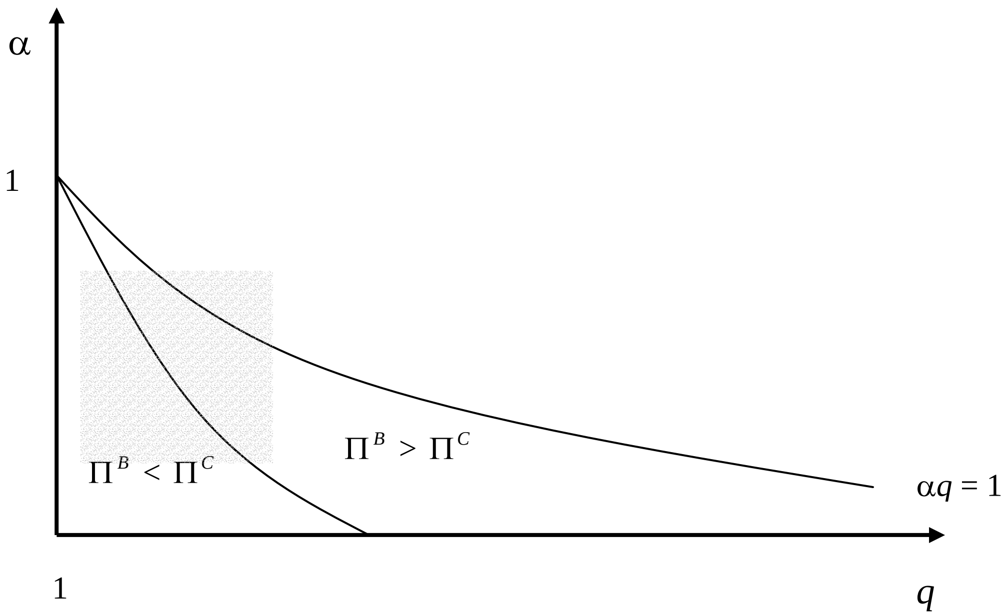

Figure 3
Industry profits under Bertrand and Cournot competition as a function of the elasticity of demand α and the size of innovations q