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Is the Public Sector too Large in a Democracy?[0].

Abstract

The public sector supplies a club good financed by either a head tax or proportional taxation on exogenous incomes in a democracy. For a class of utility functions and club quality functions, the optimal club quality is independent of the income distribution, and hence of the identity of the median voter. With "uniform and universal" public provision , the median voter chooses the head tax or proportional tax rate. This can result in lower levels of club goods in either financing regime than would occur in the first best. However, provision in all the latter three regimes can be lower than would occur via market supply by a "not for profit" organisation.

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1 Introduction

Is the public sector too large in a democracy? This paper studies the democratic determination of the size of a public sector supplying a congestible and excludable shared good - a Buchanan (1965) club good - like healthcare or education. Such goods seem more prevalent than Samuelson pure public goods. Our simple model of direct democracy has consumer-voters differentiated only by their exogenous incomes. They get utility from a club good and a numeraire private good. With public sector provision, the club is financed by a head tax or a proportional income tax. In a democracy, these tax parameters are set by the median voter (MV). We show that, with "uniform and universal" public provision [cf. Besley and Coate (1991)] in a democracy, then, relative to the first best (FB), there can be underprovision of the club good in either tax regime. I.e., the public sector can be too small in a democracy. All of these three regimes can lead to less provision of the club good than occurs via a market supplied by break-even "not for profit" organisations.

Additionally, our model explains two empirical puzzles. First, why is municipal spending not always related to the median income in a polity? Second, why are there often no scale effects in providing shared goods - suggesting that such goods might be essentially private?

A classic source of the claim that the public sector is too large in a democracy is Meltzer and Richard (1981). They sought to explain how an expansion in suffrage and a reduction in median income relative to the mean might result in an expansion of the state. But, they focused on a government which engaged solely in the redistribution of (endogenous) incomes[1] . Earlier, Bowen (1943) had studied the provision of a pure public good financed by uniform taxation of exogenous incomes. He argued that the MV determination of its level might be inefficient relative to a "second best" (SB). The SB satisfies the well known "Samuelson rule". There is inefficiency if the MV's marginal valuation of the public good differs from the mean marginal valuation in the

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[1] Peltzman (1980) also focused on a government exclusively concerned with redistribution.

population and/or his tax price differs from the mean tax price.

Many later contributors also noted that a sizeable part of the public budget goes to providing shared goods[2] . Then [cf., e.g., Mueller (2003, 516)], tax-induced direct transfers of incomes alone à la Meltzer-Richard are unnecessary for redistribution. Redistribution is achievable by supplying shared goods with progressivity in the tax system. Several authors have refined this insight under a variety of electoral arrangements [Lizzeri and Persico (2001) being among the latest]. But attention has been confined to public goods. Arguably, club goods like garbage collection and the fire service are of at least equal economic significance to the few examples of pure public goods (e.g., clean air) which one can find. This partly explains our concentration on them.

Our other reason for focusing on club goods is twofold. First, excludability enables them to be provided in either the public or private sector. So, it is of interest to compare public and private provision, as the private sector might ameliorate any inefficiency in public provision[3] . Second, due to congestibility, there are quantity and quality dimensions to clubs. In the best of circumstances, this makes their analysis more involved than for a pure public good, for which quality and quantity are synonymous. For a start, it is not obvious which of two situations has a larger public sector if one has a larger quantity of the club good, but a greater intensity of use results in a lower quality of it as compared with the other. E.g., the public sector might provide more hospitals in one situation than another. But, if a larger throughput of patients in the first situation results in them receiving poorer care, on average, than in the second, it is unclear that the first situation has a larger public sector. A quality-adjusted notion of size might be appropriate. With democratic provision, there is also the difficulty that the Median Voter Theorem (MVT) does not apply to multidimensional choice problems, except in special cases[4] .

[2] See, e.g., Mueller and Murrell (1985) and Mueller's survey (2003, ch 21).

[3] But, unlike some analyses of public provision of *private* goods (e.g., cf. Besley and Coate, 1991), we treat private sector and public sector provison of shared goods as mutually exclusive and exhaustive. I.e., we do not allow some consumers to either augment public provision or substitute a higher quality private sector alternative. If the public sector exists, then the private sector does not.

[4] Perhaps the most up-to-date survey of the difficulties of obtaining consistency in voting in multidimensional choice contexts is Mueller (2003, especially chapter 5).

We build a simple model of the democratic choice of the quality and quantity of a club good that lets us compare different supply regimes. The key is that we focus on the families of preferences for which there is unanimity regarding the optimal quality of the club good. This achieves two things. First, it reduces the effective dimension of the MV's problem. Second, more importantly, the same quality level is optimal in all the different regimes.[5] The model then allows a ranking of provision of a club good by its facility size, the one dimension which then differs when it is supplied in, respectively: (a) the first best (FB); (b) a democracy with head tax financing; (c) a democracy with proportional taxation; (d) "not-for-profit" undertakings. If the MV has lower than mean income, as is reasonable empirically, a democracy might underprovide a club good relative to the FB. The latter provision will be, in turn, lower than that in either a constrained second best or, equivalently in our model, a regime with market provision by not-for-profit organisations.

Our analysis also throws up some other surprising findings: not only might the FB and MV-determined sizes of the public sector coincide, irrespective of the relative size of the median and mean incomes, but also the public sector size in the MV equilibrium might depend on mean, but not median, income. The next section outlines the model. Section 3 considers the "first best" and Section 4 the democratic provision of the club good. Section 5, turns to a comparison of the size of the public sector in the different provision regimes. Section 6 briefly examines empirical implications of our analysis while section 7 concludes. An Appendix contains a proof.

2 The Model

Consider a single-club economy for simplicity[6] . There are N consumers, all with an identical utility function, $U[.]$, defined over the quantity, x, of a numeraire private commodity, visits to or

[5] If everyone has the same preferences, as is often assumed in applied political economy models, Kramer's (1973) theorem shows that the MV will be decisive in multidimensional choice contexts. However, this will not be sufficient to enable us to make a comparison of the club good provision in a democracy with that in other regimes where both different qualities and club facility sizes will generally be chosen.

[6] Even the most sophisticated comparisons of pure public good provision across different regimes that allow for many private goods [e.g., Gaube's (2000) study of the FB and SB] consider only one public good. Whether with public goods or club goods, aggregation problems would arise once we have more than one such good.

use of a single club good, v, and its quality, q. The private good is a necessity; the club good is not and need not be demanded at low incomes. We assume:

(A.1) U is strictly concave increasing in x, concave increasing in v and non-decreasing in q.

(A.2) Consumers' exogenously-given incomes $m \in [\underline{M}, \overline{M}]$ have a continuous density, $dF(m)$ (we ignore integer problems). So, national income is $Y = \int mdF(m)$. The distribution $F(m)$ is known to the government or any other club supplier. In the second best, such a supplier cannot identify the income of any particular individual for tax or price discrimination purposes. In the first best, or the environment with proportional income taxation studied below, we assume that it can.

(A.3) A club's quality increases in its facility size, y, and decreases in its aggregate utilisation, V: $\partial q(y, V)/\partial y \equiv q_1(y, V) > 0$; $\partial q(y, V)/\partial V \equiv q_2(y, V) < 0$

(A.4) (a) $q(.)$ is homogeneous of degree zero in y and V: $q(y, V) \equiv q(y/V), q' > 0$; (b) $q'' < 0$

NB: (i) with exogenous income, there are no incentive effects to providing and financing the club good; (ii) facility size is measured by the expenditure on the club: a unit of money buys a unit of "facility"; (iii) If q is of the form (A.4), quality depends solely on the facility provision per use of the club. Only with this form will the FB "toll," if levied, make the club break even, whatever the population size [Kolm (1974); Mohring and Harwitz (1962)]. Here, the FB "toll" is not a price but, rather, club users' identical marginal willingness to pay for a marginal visit by foregoing private consumption and equals the value of the quality degradation the marginal visit imposes on club users; (iv) zero homogeneity of the quality function means that, e.g., the quality of a pool as perceived by swimmers depends just on the average amount of space each has to swim in, not on the number of swimmers or the size of the pool independently.

Finally, we restrict attention to the families of utility functions for which optimal quality provision in the club is independent of the income distribution if (A.4) holds. These families are identified in the following theorem, due to [Fraser (2000, 2002b)]:

Theorem 1. *If the club quality function satisfies (A.4), then the optimal quality provision of* *the club good is independent of the income distribution if and only if all consumers have a utility function, $U(x, v, q)$, which belongs to a family which satisfies the partial differential equation*

$$v\partial U/\partial v = g\left(q\right)\partial U/\partial q, \tag{1}$$

for some function $g(q)$.

Equation (1) identifies numerous families of utility functions. Examples include $U(x, v, q) = \Psi\left(x, vq^{\eta}\right)$, when $g\left(q\right) = q/\eta, \eta > 0$ being a scalar, and $U(x, v, q) = \Omega\left[x, \exp\left(q/k\right)\right]$, when $g\left(q\right) = k$, for some scalar $k > 0$. Such utility functions are precisely ones for which we can talk of the club good in terms of quality-adjusted or "efficiency" units. Fraser (2000) provides a detailed intuitive justification for Theorem 1. He shows that the utility functions which it generates produce very simple rules for optimal quality provision, such as "choose that level of investment in quality which maximises the quality per unit of investment" (i.e., "maximise the bangs per buck" with respect to quality). Clearly, the Theorem imposes a stronger restriction than weak separability of (v, q) from x upon $U(x, v, q)$.

We will only study explicitly perhaps the simplest utility satisfying (1), namely

$$U(x, v, q) \ = u\left(x, vq\right) \tag{2}$$

(where $\eta = 1$ above)[7] . However, identical considerations to those identified below will apply to the analysis of the other cases satisfying (1).

3 The First Best

In the FB, the government has full information about consumers' incomes. It can pool resources to get any allocation of goods, hence welfare, it thinks fit, subject to the economy's overall endowment. It can be shown that, as everyone has the same U and incomes are exogenous, to maximise

[7] Fraser (2000) shows that the "independence" referred to in Theorem (which he termed a "partial separation of efficiency from distribution") does not require everyone to have the same utility function. The utility functions just all need to belong to the same family. E.g., if the population comprises two types with the different utility functions $U_1\left(x, vq\right)$ and $U_2\left(x, vq\right)$, then the optimal q would be independent of the income distribution.

aggregate utilitarian welfare the government equalises utilities at the (x, v) bundle that maximises a single person's utility with all treated equally.

Let facility provision per use of the club be p - i.e., $p \equiv y/V$. By (A.4), $q = q(p)$. Let $\overline{m} \equiv Y/N$ denote mean income The FB problem is then:

$$\underset{p,v}{Max}.u\left[\overline{m} - pv, vq(p)\right] \tag{3}$$

Using (*) to indicate the FB, the two first-order conditions (FOC) characterising it are the complementary slack conditions:

$$\left\{-u_1\left[\overline{m} - p^*v^*, v^*q(p^*)\right]p^* + u_2\left[\overline{m} - p^*v^*, v^*q(p^*)\right]q\left(p^*\right) \leq 0; v^* \geq 0\right\} \tag{4}$$

$$\left\{-u_1\left[\overline{m} - p^*v^*, v^*q(p^*)\right] + u_2\left[\overline{m} - p^*v^*, v^*q(p^*)\right]q'\left(p^*\right) \leq 0; p^* \geq 0\right\} \tag{5}$$

At an interior solution, the FOCs reduce to

$$p^*q'(p^*) = q\left(p^*\right) \tag{6}$$

Note that any p^* which solves (6) depends only on the quality function, $q\left(p\right)$, and not on $\overline{m}$. This is just a reiteration of the fact that, with the utility function (2), p^* is independent of the distribution of income. If (6) has a unique solution, as we will assume henceforth, it identifies the unique p^*. This will also be the unique p^* for which the quality provision per unit of expenditure is maximised with the given utility function [Fraser (2000)]. Note additionally from (4) that, if $v^* = 0$,

$$-u_1(\overline{m}, 0)p^* + u_2(\overline{m}, 0)q(p^*) \leq 0 \tag{7}$$

If the club good is normal, when (7) holds with equality it identifies a unique mean income, m^* say, below which $v^* = 0$ and above which $v^* > 0$. In any case, the FB level of facility provision is then

$$y^* = Np^*v^* \tag{8}$$

4 Democratic provision of the club good

We will adopt the standard static approach which assumes that the level of provision of the shared good in a democracy is that which maximises the MV's utility. We will consider two alternative break-even tax-cum-provision regimes in turn. In the first, with proportional income taxation, the MV chooses the proportional tax rate and the level of club good provision, to be supplied uniformly to all, which maximises its utility. The income tax revenue just finances the aggregate provision of the club good. In the second regime, the MV again chooses the uniform club good provision level and the associated expenditure on the facility, now financed by a head tax.. Despite the multidimensional nature of the MV's problems in these two environments, we will see that the population's unanimity regarding the most desirable level of facility provision per unit use of the club effectively converts the problem to a unidimensional one[8] .

Our analysis applies to, e.g., the democratic determination of the characteristics of a club good like the non-tertiary (i.e., pre-university) education system. The quality of that education can be proxied by the number of pupils per teacher (the "class size"). The quantity of that education per child is the number of days schooling in the school career. The quantity and quality of education combine with the number of children to determine the size of the education system. Consequently, they determine the overall expenditure on it, given the average salary of teachers (taken to be exogenous). Ignoring minor sources of randomness like illness, class sizes and the number of days of schooling are completely within the control of the MV in a polity.

4.1 Proportional Taxation

Let τ denote a proportional rate of income taxation. With proportional taxation, the MV sets a τ and a club consumption level, v, knowing that the club allocation to everyone is the same. Hence, aggregate club use would be $V = Nv$ and $y = \tau Y$ is the aggregate level of the facility provision.[9]

[8] It is not unusual for multidimensional choice problems with voting to be reduced to single-dimensional ones. This was so in Meltzer and Richard's analysis, which used an explicit utility function. See Roberts (1977) also.

[9] In terms of our educational example, y is the education budget.

By (A.4), the quality of the club good will be given by $q\left(y/V\right) = q\left(\tau Y/V\right) = q\left(\tau Y/Nv\right)$. A voter with income m would choose v and τ to solve:

$$\underset{\tau,v}{Max}.u\left[m\left(1-\tau\right), vq(\tau Y/Nv)\right] \tag{9}$$

Let $p^\tau \equiv \tau Y/Nv = \tau\overline{m}/v$ denote the level of facility provision per unit use of the club in this case. Let $(*)$ again show an optimum. Assuming an interior solution again, the solution to (9) then satisfies

$$\begin{aligned} \tau &: -mu_1\left[m\left(1-\tau^*\right), vq(p^{\tau*})\right] + u_2\left[m\left(1-\tau^*\right), vq(p^{\tau*})\right] q'\left(p^{\tau*}\right)\left(\overline{m}\right) = 0 \quad \text{(a)} \\ v &: u_2\left[m\left(1-\tau^*\right), vq(p^{\tau*})\right]\left[q(p^{\tau*}) - p^{\tau*}q'(p^{\tau*})\right] = 0 \quad \text{(b)} \end{aligned} \tag{10}$$

Equation (10)(b) collapses to (6). This confirms that $p^{\tau*} = p^*$: the optimal quality of the club good that would be chosen by any voter is independent of income. It also suggests that this quality is independent of the mode of financing the club good. In terms of our education example, this means that everyone would agree on the optimal class size, although they might disagree on the right length for the school year or the school career.

If $p^{\tau*} = p^*$ would be chosen by everyone, irrespective of income, including the person with median income, it follows that everyone's choice of τ and v must satisfy the positive linear relationship

$$\tau = \left(p^*/\overline{m}\right)v \tag{11}$$

Hence, a voter's two-dimensional choice is reduced to a one-dimensional choice of either τ or v. Problem (9) can be rewritten as

$$\underset{\tau}{Max}.u\left[m\left(1-\tau\right), \tau\overline{m}q(p^*)/p^*\right] \tag{12}$$

Provided $u\left[m\left(1-\tau\right), \tau\overline{m}q(p^*)/p^*\right]$ is single-peaked in τ, which it will be by concavity, we can apply the MVT to the determination of τ (and hence v) by the MV in the political equilibrium. Let $\widehat{m}$ denote the median income. The proportional tax and uniform club provision level, v^τ, chosen by the MV will satisfy (11), with $\tau^* = \left(p^*/\overline{m}\right)v^\tau$, and hence, from (10)(a),

$$-\widehat{m}u_1\left[\widehat{m}\left(1-\tau^*\right),v^\tau q(p^*)\right]+u_2\left[\widehat{m}\left(1-\tau^*\right),v^\tau q(p^*)\right]q'\left(p^*\right)\left(\overline{m}\right)=0 \tag{13}$$

The resulting size of the club facility will be

$$y^\tau = p^* N v^\tau \tag{14}$$

4.2 A Head Tax

When provision of the club good is financed by a head tax, an arbitrary voter would seek to choose the uniform level of club provision and the level of investment in the club facility per use of the club, which determines its quality, in order to maximise utility. Let p^t be the club facility per unit use chosen in this case, with $q\left(p^t\right)$ the corresponding quality. At uniform club provision level v, this determines the head tax $p^t v$.

This arbitrary voter would now solve the problem

$$\underset{p^t,v}{Max}.u\left[m-p^t v, vq(p^t)\right] \tag{15}$$

By inspection, (15) is identical to the FB problem (3), except that the arbitrary income level m will generally differ from the mean income, $\overline{m}$. It will again generate the FB investment in quality per unit use of the club good, $p^t = p^*$, irrespective of the voter's income. This unanimity means that the MVT can again be applied. Letting v^t be the level of club good provision chosen by the MV in this case, this satisfies

$$-u_1\left[\widehat{m}-p^*v^t, v^t q(p^*)\right]p^* + u_2\left[\widehat{m}-p^*v^t, v^t q(p^*)\right]q\left(p^*\right)=0 \tag{16}$$

with the resulting level of expenditure on the club facility being given by

$$y^t = p^* N v^t \tag{17}$$

A comparison of y^* ,y^τ and y^t indicates that their relative size ranking, hence the relative size of the state in the three instances, is the same as that of v^* ,v^τ and v^t.

5 The Size of the Public Sector in the Different Regimes

A comparison of (10)(a) and (17) now leads immediately to our first result.

Proposition 1 *If the club good is normal and $\overline{m} > \widehat{m}$, then $v^* > v^t$*

This result holds for the following reason. Both the FB and the head tax cases involve the decision-maker effectively "buying" the club good at a constant per unit price of p^*. By the same token, the FB corresponds formally to an MV equilibrium with a head tax in which aggregate income is redistributed so that everyone has mean, and thus median, income. If this mean-cum-median income is larger than the true median income in the population, and the club good is normal, then $v^* > v^t$ must follow. In the FB, the fact that a head tax is usually regressive does not matter, given that everyone has the same income. In the real head tax regime, it does matter. The lower is median income, the lower will be the head tax and, hence v^t.

The above argument suggests that we should expect the size of the state to be larger with proportional taxation than with the more regressive head tax. This is indeed our next result, proven in the Appendix, under the following additional assumption:

(A.5) $u_{12} \geq 0$ (Edgeworth-Pareto complementarity).

Proposition 2 *If (A.5) holds and $\overline{m} > \widehat{m}$, then $v^\tau > v^t$.*

Will the public sector determined by the MV under proportional taxation be larger than the FB one in our model? There seem to be two principal opposing forces at work. First, by assumption, the MV has lower income than someone with mean income. As we have seen, the latter is effectively the decision-maker in the FB. This means that, OTRE, the MV would choose to pay for a lower level of the club good than would be provided in the FB, provided that the club good is normal. Second, as $\tau^*\widehat{m}/v^\tau < \tau^*\overline{m}/v^\tau = p^*$, the effective unit "price" for the club good of quality $q(p^*)$ that is paid by the MV is less than the real price. However, this real price, p^*, does have to be paid by the decision maker in the FB. This difference in prices means that there is an income effect and a substitution effect which both work in the direction of making the

MV paying a proportional tax tend to demand more of the club good than someone of the same income would demand if faced with the price p^*[10].

Unsurprisingly, in view of the above argument, we have the following result.

Proposition 3 *$v^\tau \{>, =, <\} v^*$ are all possible.*

We can prove this result simply by constructing examples to show that $v^\tau \{>, =, <\} v^*$ are all possible. More importantly, these examples will highlight how critical magnitudes, such as the relationship between $\overline{m}$ and $\widehat{m}$ and the attitude to risk or inequality, interact in determining the direction of inequality between v^τ and v^*, hence between y^τ and y^*.

Example 1. $u(x, vq) = x^{1/2} + (vq + \gamma_1)^{1/2}$, $\gamma_1 \geq 0$ being a scalar. Then[11]

$$v^\tau = \frac{1}{p^* q(p^*)} \left(\frac{\overline{m}^2 q^2 - \widehat{m}\gamma_1 p^{*2}}{p^* \widehat{m} + \overline{m} q} \right) \tag{18}$$

and

$$v^* = \frac{\left(\overline{m} q^2 - \gamma_1 p^{*2}\right)}{p^* q(p^*)(p^* + q)} \tag{19}$$

From these expressions, we can calculate that $\frac{v^\tau}{v^*} \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} 1$ as $\frac{\overline{m}}{\widehat{m}} \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} 1$. But, given our hypothesis that $\overline{m} > \widehat{m}$, we conclude that $v^\tau > v^*$.

Example 2. $u(x, vq) = A + vq - e^{-x}$, A being some scalar, and $q(p) = p^{1/2} - 1$. Then: $p^* = 4$, $q(p^*) = 1$, $v^* = [\overline{m} - \ln(p^*/q(p^*))]/p^*$ and $v^\tau = [\overline{m} - (\overline{m}/\widehat{m})\{\ln(p^*/q(p^*)) + \ln(\widehat{m}/\overline{m})\}]/p^*$. Hence, $\frac{v^\tau}{v^*} \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} 1$ as $\left(\frac{p^* \widehat{m}}{q(p^*)\overline{m}}\right)^{\overline{m}/\widehat{m}} \left\{ \begin{array}{c} < \\ = \\ > \end{array} \right\} p^*/q(p^*)$. It is clear that there can be any direction of inequality between v^τ and v^*, depending on the specifics of the quality function $q(p)$ and the

[10] Stiglitz (2000, ch. 7) advances similar arguments in the context of pure public goods. He concludes that public goods will be oversupplied with proportional (or progressive) taxation in an MV equilibrium. His focus is on oversupply in the sense of the Samuelson *rule* - where the social marginal valuation of the public good is less than its social marginal cost. Our focus is on ranking *levels* of goods in different regimes. See Chang (2000) and Gaube (2000) on the distinction between rankings by rule and by level.

[11] Details of the calculations yielding the expressions in these examples are available on request.

relationship between $\widehat{m}$ and $\overline{m}$, among other things. With the special quality function chosen, $p^*/q(p^*) = 4$. Suppose $\widehat{m}/\overline{m} = 1/2$. Then, $\left(\frac{p^*\widehat{m}}{q(p^*)\overline{m}}\right)^{\overline{m}/\widehat{m}} = 4 = p^*/q(p^*)$ and $v^\tau = v^*$. If $\widehat{m}/\overline{m} = 1/3$, then $\left(\frac{p^*\widehat{m}}{q(p^*)\overline{m}}\right)^{\overline{m}/\widehat{m}} = 64/27 < 4 = p^*/q(p^*)$ and $v^\tau > v^*$. Conversely, if $\widehat{m}/\overline{m} = 7/8$, then $\left(\frac{p^*\widehat{m}}{q(p^*)\overline{m}}\right)^{\overline{m}/\widehat{m}} = 4.1859304(7d.p.) > 4 = p^*/q(p^*)$ and $v^\tau < v^*$. In this particular example, we validate the usual presumption that an increase in $\widehat{m}/\overline{m}$ makes $v^\tau < v^*$ more likely. This is because the price effects noted above are relatively less significant. In turn, this is because the closer in income is the median person to the mean one, the less the former benefits from the redistribution associated with the proportional tax financed uniform provision of the club good.

Example 3. $u(x, vq) = \frac{x^{1-\lambda}}{1-\lambda} + vq$, for some scalar $\lambda > 0$. Then $v^* = \left[\overline{m} - \left(\frac{p^*}{q(p^*)}\right)^{1/\lambda}\right]/p^*$ and $v^\tau = \left[\overline{m} - \frac{\overline{m}}{\widehat{m}}\left(\frac{p^*\widehat{m}}{q(p^*)\overline{m}}\right)^{1/\lambda}\right]/p^*$. Thus, now, $\frac{v^\tau}{v^*}\left\{\begin{array}{c} > \\ = \\ < \end{array}\right\} 1$ as $\frac{\overline{m}}{\widehat{m}}\left(\frac{p^*\widehat{m}}{q(p^*)\overline{m}}\right)^{1/\lambda}\left\{\begin{array}{c} < \\ = \\ > \end{array}\right\}\left(\frac{p^*}{q(p^*)}\right)^{1/\lambda}$.
Clearly, if $\lambda = 1$ (i.e., when $u(x, vq) = \ln x + vq$), then $v^\tau = v^*$. As $\widehat{m}/\overline{m} < 1$, by hypothesis, if $\lambda > 1$ then $(\widehat{m}/\overline{m})^{1/\lambda} > \widehat{m}/\overline{m}$, so $(\widehat{m}/\overline{m})^{1/\lambda}\,\overline{m}/\widehat{m} > 1$ and $v^* > v^\tau$, irrespective of the precise magnitude of $\widehat{m}/\overline{m}$. If $\lambda < 1$, the conclusion is reversed.

Example 4. $u(x, vq) = x^\alpha (vq + \gamma_1)^\beta$, for scalars $\alpha > 0$,$\beta > 0$. Then $v^* = (\beta\overline{m}q - \alpha p^*\gamma_1)/(\alpha + \beta)\,qp^* = v^\tau$, irrespective of the relationship between $\widehat{m}$ and $\overline{m}$, among other things.

Examples 1-3 illustrate some of the influences on v^* and v^τ, such as the elasticity of marginal utility w.r.t. private good consumption, which help to determine their relative magnitude in an intuitive way. Example 4 is perhaps most interesting. This is not only because it has $v^* = v^\tau$, irrespective of α and β, but also because, surprisingly, v^τ depends on $\overline{m}$, but not on $\widehat{m}$. Although this is obviously an artefact of the Cobb-Douglas-cum-Stone-Geary utility specification, we have no a priori basis for concluding that this specification is unreasonable and, hence, that we can rule this out empirically.

Examples 1-4 convey a more general lesson: although the subutility associated with the club good, vq, and the associated optimal club quality are the same in all four examples, they nevertheless generate very different optimal MV demands for the club good. This is simply because the underlying preferences over the club and private goods are very different in the 4 examples. In consequence, different jurisdictions with, e.g., different demographic characteristics and thus different preferences as between the club good and the private good, might demand different levels of the club good, even if they have the same median income and agree on the optimal quality of that good.

The Second Best

We can use the Fraser-Hollander model of constrained second best clubs [cf.: Fraser and Hollander, Cornes and Sandler (1996), Fraser (2000)] to provide a comparator for the other regimes. In this model, which builds on the approach of, e.g., Brito and Oakland (1980) and Fraser (1996) for excludable public goods, atomistic consumers confront a per visit price, facility size and conjectured quality for a club good. These latter magnitudes can be regarded as being determined by a benevolent government, as in the FB. However, unlike in the FB, it is assumed that the government does not have the information to redistribute incomes directly. Taking these price and quality for the club good as parametric, consumers self-select to club membership. In any resulting equilibrium, their simultaneous actions must validate the club congestion, hence quality, which they conjectured. In turn, the government or any entrepreneurial club good supplier can use the demand schedule generated by the consumers' joint actions, which it (correctly) anticipates, to fix the optimal price and level of facility provision that fulfill its objectives. One interpretation of this model is that it involves market provision of the club good by break-even, "not for profit" organisations. Alternatively, we can regard it as representing hypothecated tax-financed provision by a government with limited information on individual consumers' characteristics. Under the same assumptions on preferences and the club quality function as before, the SB will generally involve exclusion and a price for the club good equal to the p^* identified previously.

Using this model, al-Nowaihi and Fraser (2003) have shown that, with everyone having the same class of utility function as used in this paper, the level of club good provision in the SB will exceed that in the FB, provided that the demand function for the club good is convex. This will be the case, e.g., in Examples 1-4 above. If $y^* > y^\tau (> y^t)$, as we have argued is quite likely, then a democracy can lead to too small a public sector while private sector, not for profit, provision of the same shared goods would be socially excessive. Which leads to the greater welfare loss is unclear.

6 Empirical Implications

Our analysis enables us to address two empirically puzzling phenomena. The first is the common finding of either no, or a weak, relationship between median voter income (or median income relative to mean income) and the size of the public sector. (E.g., cf. Gouveia and Masai, 1998; Kristov, Lindert and McClelland, 1992, and Mueller's survey (2003, 243-6). Second is the apparent absence of a significant scale effect in the provision of shared goods [12] . Regarding the first, we have shown that there can be any relationship between the democratic and first best provision levels of a club good. Empirically, perhaps more interesting is the possibility that there can be any relationship or none (as in Example 4) between the size of the public sector and the median income in an MV equilibrium.

Regarding the explanation of the presence or absence of congestion effects, we have shown that congestion effects might apparently be absent if *both* of two circumstances are met: (i) congestion/quality functions are homogeneous of degree zero in the facility size and the aggregate club use, *and* (ii) agents have preferences which, given (i), result in a separation of efficiency from distribution in that the optimal quality of the club good is independent of the median voter's income. The same optimal amount will then be spent per unit of the club facility, irrespective of the facility size, its number of users and the income of the MV. Suppose, e.g., everyone had the same subutility from the club good in two different jurisdictions satisfying (i) and (ii), where agents within a jurisdiction had identical tastes, but tastes differed between the jurisdictions. We

[12] Mueller (2003, 246-7) surveys the empirics on this issue. Reiter and Wiechenrieder (1999) both discuss theoretical issues in the measurement of quality and comment on the empirical findings.

know from Theorem 1 that we would observe an identical quality provision of the club good in the two jurisdictions. Because of the different tastes, MV income and/or populations, we might observe different overall levels of club provision in the two jurisdictions, but at a common and constant cost per unit of size - as, e.g., in Examples 1-4. Note that neither of (i) or (ii) *alone* is sufficient for this outcome. It is a moot point whether the empirical observations on the absence of scale effects in the cost of providing shared goods in turn provides a justification for the utility and quality specifications we have employed in this paper.

7 Conclusion

We have shown that, when the public sector supplies club goods, there might be very little basis for the widespread belief that the public sector is too large in a democracy. The MV-determined public sector size might even be independent of median income. Club good provision has quality and quantity dimensions. So, to allow for comparability between different institutional settings, we focused on cases in which the optimal quality provision was independent of the distribution of income and, hence, of median income. Therefore, we needed only to rank the quantity of provision in the different environments. As we have argued, such cases are ones which also conform with another frequent emprical finding - namely, the absence of scale effects in public sector provision.

While our model was relatively simple, incorporating one club good and one private good, we would anticipate our results to be robust to at least two possible generalisations. The first is the incorporation of additional club goods, provided each is characterised by a zero homogeneous quality function, as in (A.4), and enters everyone's utility identically in the separable fashion identified by Theorem 1. In that event, the quality provision, hence "price", of each club good would be independent of distribution. We could therefore define a Hicksian "composite" club good and conduct the analysis as before. The second generalisation incorporates endogenous labour supply and incomes in a model where consumers differ only in exogenous skill. It can be shown that Theorem 1 carries over to this environment [cf. Fraser (2002a)]. Consequently, with unanimity w.r.t. the optimal quality for a single club good, we could in principle again rank the first-best and democratic club good provision simply according to the respective quantities provided in the two cases.

A further generalisation which might be worth exploring in subsequent work is consideration of a model of representative democracy in which the median voter is *not* decisive. An example would be Osborne and Slivinski's (1996) and Besley and Coate's (1997) citizen-candidate models. We will not speculate on the possible implications of this extension here.

8 Appendix

Proof. of Proposition 2. From (13), using $q'(p^*) = q(p^*)/p^*$ and $\overline{m} = p^* v^\tau / \tau^*$, the interior proportional tax equilibrium satisfies

$$u_1 \left[\widehat{m}\left(1-\tau^*\right), v^\tau q(p^*)\right] \tau^* \widehat{m}/v^\tau = u_2 \left[\widehat{m}\left(1-\tau^*\right), v^\tau q(p^*)\right] q(p^*) \tag{20}$$

while the head tax satisfies, from (16),

$$u_1 \left[\widehat{m} - p^* v^t, v^t q(p^*)\right] p^* = u_2 \left[\widehat{m} - p^* v^t, v^t q(p^*)\right] q\left(p^*\right) \tag{21}$$

Now, $\tau^* \widehat{m}/v^\tau < \tau^* \overline{m}/v^\tau = p^*$. I.e., the effective unit price for the club good of quality $q\left(p^*\right)$ that is paid by the MV is less than the real price. Moreover, it follows that $p^* v^\tau = \tau^* \overline{m} > \tau^* \widehat{m}$ as $\overline{m} > \widehat{m}$ by assumption. Suppose $v^\tau = v^t$. Then, as $p^* v^t = p^* v^\tau > \tau^* \widehat{m}$, it follows by concavity and $u_{12} \geq 0$ that $u_2 \left[\widehat{m} - \tau^* \widehat{m}, v^\tau q(p^*)\right] q(p^*) > u_2 \left[\widehat{m} - p^* v^t, v^t q(p^*)\right] q\left(p^*\right)$. So, we must have $u_1 \left[\widehat{m} - \tau^* \widehat{m}, v^\tau q(p^*)\right] \tau^* \widehat{m}/v^\tau > u_1 \left[\widehat{m} - p^* v^t, v^t q(p^*)\right] p^*$. But, with $v^\tau q(p^*) = v^t q(p^*)$, by hypothesis, and $p^* v^t > \tau^* \widehat{m}$, it follows by concavity that $u_1 \left[\widehat{m} - \tau^* \widehat{m}, v^\tau q(p^*)\right] \tau^* \widehat{m}/v^\tau < u_1 \left[\widehat{m} - p^* v^t, v^t q(p^*)\right] p^*$. This is a contradiction. So, we cannot have $v^\tau = v^t$. Suppose, next, $v^t > v^\tau$. Then $v^t q\left(p^*\right) > v^\tau q\left(p^*\right)$ and $u_2 \left[\widehat{m} - \tau^* \widehat{m}, v^\tau q(p^*)\right] q(p^*) > u_2 \left[\widehat{m} - p^* v^t, v^t q(p^*)\right] q\left(p^*\right)$ again, by concavity and $u_{12} \geq 0$. So, from the F.O.C.s, we must have $u_1 \left[\widehat{m} - \tau^* \widehat{m}, v^\tau q(p^*)\right] \tau^* \widehat{m}/v^\tau > u_1 \left[\widehat{m} - p^* v^t, v^t q(p^*)\right] p^*$. But, $u_1 \left[\widehat{m} - \tau^* \widehat{m}, v^\tau q(p^*)\right] \tau^* \widehat{m}/v^\tau < u_1 \left[\widehat{m} - p^* v^t, v^t q(p^*)\right] p^*$, again by concavity and $u_{12} \geq 0$. Hence there is again a contradiction. So, we must have $v^{\tau} > v^{t}$. ■

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