This short document briefly reviews some related concepts, which already appeared in the literature. This discussion will be incorporated into the paper in future revisions.

Al Najjar (1993) considers monotone and *strongly convex* binary relations.[1] A binary relation $\succcurlyeq$ is strongly convex if for all $x \in \mathbf{X}$, there exists a normalized vector $g(x)$ (i.e., $||g|| = 1$) such that (i) $y \succcurlyeq x$ and $y \neq x$ implies that $g(x) \cdot (y - x) > 0$ and (ii) for any normalized g' different from $g(x)$, there exists x' such that $x' \succ x$ and $g' \cdot (x' - x) < 0$. Intuitively, if preferences are smooth and strictly convex, there exists a unique hyperplane supporting $\{y : y \succcurlyeq x\}$ at x and $g(x)$ is the normalized normal to the hyperplane. The vector field g can thus be thought as a representation of the binary relation $\succcurlyeq$. Clearly, $g(x)$ is the unique normalized ordient of $\succcurlyeq$ at x. However, the converse is not true. For a counter-example, consider the lexicographic order.

Another related concept is to assume convex preferences and to consider the negative of the normal cone (in the sense of convex analysis) of the upper contour set $\{y : y \succcurlyeq x\}$ at x as a generalization of the concept of gradient (e.g., Guttiérez (1997) and Bonnisseau (2003)). But this is nothing else than the set of decreasing ordients of convex $\succcurlyeq$ at x and, thus, our concept also encapsulates this related notion.

Lastly, Shafer (1974) considers complete, continuous and strictly convex binary relations (transitivity is not assumed) and proves the existence of a continuous function $k : \mathbf{X} \times \mathbf{X}$ such that (i) $k(x, y) > 0$ if and only if $x \succ y$, (ii) $k(x, y) < 0$ if and only if $y \succ x$, and (iii) $k(x, y) = -k(x, y)$. The function k can be thought as a representation in that if x^* maximizes $\succcurlyeq$ on the convex set X, then $k(x^*, y) \geq 0$ for all $y \in X$. From Proposition 3, we have that $g \cdot (y - x) < 0$ implies $y \prec x$ and $y \succ x$ implies $g \cdot (y - x) > 0$. Hence, we must have that $g(x^*) \cdot (y - x^*) \leq 0$ for all $y \in X$. This complements Shafer's result.

References

[1] Nabil Al-Najjar, Non-transitive Smooth Preferences, Journal of Economic Theory, 1993, 60, pp. 14-41.

[2] Jean-Marc Bonnisseau, Regular economies with non-ordered preferences, Journal of Mathematical Economics, 2003, 39, pp. 153174.

[1] Neither transitivity nor completeness is assumed

[3] J.M Gutiérrez, A Generalization of the Quasiconvex Optimization Problem, Journal of Convex Analysis, 1997, 4, pp.281-287.

[4] Wayne J. Shafer, The Nontransitive Consumer, Econometrica, 1974, 42, pp. 913-919.