

# The existence of non-elite private schools

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## Abstract

We provide an explanation to the puzzle of the existence of paid-for private schools that offer lower quality education than some tuition-free public alternatives. We consider a model of a city composed of two communities: the urban area and the suburbs. The suburban public school provides higher quality education at an implicit price: the higher tax burden plus a housing rent premium. If that price is high enough and the urban public school has a sufficiently low quality, intermediate income households live in the urban area and use a private school. Intermediate quality private schools, then, exist to serve these households' demand. Lower and higher income households use different quality public schools. Therefore, perfect income stratification across public and private education does not characterize this equilibrium.

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## 1. Introduction

Private schools do not always provide higher quality education than public schools. Both casual observation and empirical evidence (e.g. Figlio and Stone, 1999) confirm it. This stylized fact is, however, puzzling as it clashes with conventional wisdom, basic price theory and most of the previous theoretical literature on competition between public and private schools (e.g. Epple and Romano, 1998). The puzzle is: why would any household pay for private schooling when higher quality tuition-free public education is available?

The literature on the economics of education offers some possible explanations. Figlio and Stone (1999) informally argue that private schools seem to offer some non-academic advantages over public schools (e.g. more discipline and security, more opportunities for extracurricular

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activities or, for some parents, a religious education). Along these lines, Brunello and Rocco (2005) demonstrate theoretically that paid-for private schools of lower academic quality than the public alternatives could successfully enter the market by setting a lower educational standard and thereby offering their students more leisure.

In this paper we provide an alternative explanation which does not resort to non-academic differences between public and private schools. Instead, it is based on the interactions between local public schooling, taxation and housing markets. We build a general equilibrium model with two communities — which can, for the sake of fixing ideas, be called the urban area and the suburbs — in which local public schools coexist with competitive private schools. We use a computational version of the model to construct examples where multiple equilibria may exist. These equilibria will be of one of two types, which we label *urban-trap* and *urban-mixing* equilibrium.

In an *urban-trap* equilibrium, low income households live in the urban area and use the local public school, while high income households live in the suburbs and send their children to the higher quality suburban public school. There also exists a set of intermediate income households who live in the urban area and opt out of public education. Intermediate quality (non-elite) private schools exist to serve their demand for private schooling. Finally, households from the top income classes may acquire elite private schooling or use the suburban public school instead. An economy may also end up in an *urban-mixing* equilibrium, in which, by contrast, only households with income above a certain threshold acquire (elite) private schooling. Those with income below such threshold perfectly stratify by income across the urban area and the suburbs and send their children to their respective local public school.

The intuition behind these results is the following. Local public schools follow residence-based admission policies. This turns housing and public education into complement goods, giving rise to a rent premium where public education is better — the suburbs say. As a consequence, an implicit price for attending the tuition-free public school in the suburbs emerges as the sum of the rent premium and the higher level of taxation necessary for funding higher quality public schooling. In an *urban-trap* equilibrium, the public school in the urban area offers education of too low a quality. At the same time, the very wealthy live in the suburbs pushing the quality of the local public school and its implicit price up. Intermediate income households, thus, prefer to live in the cheaper urban area, but because they value education, they acquire intermediate quality private schooling rather than sending their children in the low quality public school.

The cases where an *urban-trap* equilibrium arise constitute counterexamples to a rather robust result in the previous literature. The models in Stiglitz (1974), Epple and Romano (1996) and De Fraja (2001) predict that, if households differ along a single dimension, either income or the ability of the student, they should perfectly stratify across the public and private educational sectors along the relevant dimension, with the better-off using higher quality private schools. This property does not hold in *urban-trap* equilibria: while intermediate income households opt for private schools, lower and higher income households prefer to use a public alternative.

The stratification patterns in this type of equilibrium characterize situations that are common to many US metropolitan areas. While the central city provides low quality public schooling, the suburbs are very expensive places in which to live. In that context, it is easy to find relatively low income households that live in the inner city and use a non-elite private school. That is, as we argue in the paper, the empirical evidence in Neal (1997), Figlio and Stone (1999) and Altonji et al. (2005) accords well with the predictions of *urban-trap* equilibria.

The rest of the paper is organized as follows: Section 2 presents a two-community model which draws primarily on Epple and Romano (1996, 2002) and Nechyba (1999). Section 3 defines *urban-*

trap and urban-mixing equilibria. In Section 4 we provide the computational examples of equilibrium and a discussion of the results. Some concluding remarks are in Section 5.

## 2. The model

We study a static, general equilibrium model of community and school choice. The model represents a city made up of two communities  $C_j, j=u, s$ , which for exposition purposes we label the urban area and the suburbs, respectively. The city is inhabited by a continuum of households with unit measure. All households consist of a head and a school-aged child; they have identical preferences defined over the quality of education the child receives ( $x$ ) and consumption numeraire ( $b$ ). With an eye towards our computational examples of equilibrium, we adopt the strictly concave utility function, used by [Bearse et al. \(2001\)](#):

$$u(b, x) = \frac{1}{1-\sigma} [b^{1-\sigma} + \delta x^{1-\sigma}]; \quad \sigma, \delta > 0. \quad (1)$$

Each household belongs to one of  $I > 2$  income classes indexed by  $i=1, \dots, I$ , with  $y_{i-1} < y_i, i=2, \dots, I$ . The fraction of the population which belongs to income class  $i$  is denoted  $\lambda_i$ .

Communities have a fixed stock of homogenous houses; each household rents out one unit of housing in the community in which they live at rent  $p_h^j$ .<sup>1</sup>

We model education as a (publicly provided) private good. Public and private schools use the same technology to produce educational services from the numeraire. Quality units are normalized such that one unit costs one unit of consumption per student.<sup>2</sup>

Each community imposes a proportional property tax on the value of housing and uses the proceeds to provide public education of quality  $E$ . The pair  $(E, t)$ , where  $t$  stands for the tax rate, is chosen through a political process, which we simplify to majority voting. There are no transfers from a central government: therefore, in community  $j$ , the local government's budget constraint is:

$$E_j = t_j p_h^j \frac{N_j}{n_j}; \quad j = u, s. \quad (2)$$

where  $N_j$  is the mass of households living in the community and  $n_j$  the mass of those using the local public school; the remaining households use private schools.

Public schools follow a residence-based admission policy: a household is required to live and pay taxes in a community in order to use its local public school. Households may opt out of the public sector and acquire private education of any level of quality in a competitive market. As in most models of education, we consider public and private alternatives as being mutually exclusive: a child cannot receive public and private education simultaneously.

Decisions are made in two stages. In the first stage, decision-makers simultaneously choose among communities and schools, taking into account their expectations over the equilibrium vector of public policies and housing prices  $e^* = (E_u, t_u, p_h^u, E_s, t_s, p_h^s)$ . In this stage, local housing markets clear. In the second one, with residence and schooling decisions committed, heads vote on their community education policy.<sup>3</sup>

<sup>1</sup> As is standard in the literature, we assume that all rents go to an absentee landlord to close the model.

<sup>2</sup> For simplicity sake, this technology assumes away the influence of peer group effects and other inputs such as student effort.

<sup>3</sup> This sequence of decisions is also found in [Nechyba \(1999\)](#). An alternative timing would allow households to choose school after voting ([Epple and Romano, 1996](#), [Bearse et al., 2001](#)). However, this may result in existence problems, as preferences are not single-peaked in that case ([Stiglitz, 1974](#)).

In this economy, an *equilibrium* is a partition of households across communities and schools, an allocation  $(b, x)$  across households and a vector of community policies and housing prices  $e^*$  satisfying: (i) *Rational choices*: every household maximizes utility; (ii) *housing market equilibrium*: housing demand equals housing supply in every community; (iii) *majority voting equilibrium*: for  $j = u, s$ , the pair  $(E_j, t_j)$  satisfies the local government budget constraint and it is majority-preferred by voters in community  $j$ , given the partition of households across schools and the price of housing in the community. A pair  $(E_j, t_j)$  is majority-preferred in community  $j$  if it is preferred by at least half the electorate of that community in a pair-wise contest with any other bundle satisfying the local government budget constraint.

The notions of stratification we adopt are the following: an equilibrium exhibits *perfect income stratification across public and private education* if every household using a private school has weakly higher income than all households sending their children to a public school; an equilibrium shows *perfect income stratification across communities* if every household living in one community has weakly higher income than all those living in the other community. We will also say that a community is *mixed* when some of its inhabitants use the local public school while others opt for a private alternative.

This kind of model has been extensively analyzed in previous literature. Several results are relevant for the analysis below and extend readily to our setting<sup>4</sup>: (i) *within a mixed community* any equilibrium exhibits perfect income stratification across public and private education (Epple and Romano, 1996); (ii) if communities differ in the quality of public schooling in equilibrium, the community with a better public school has a higher gross-of-tax price of housing<sup>5</sup>; (iii) households that use public schooling perfectly stratify by income across communities, with higher income households living in the community that provides higher quality public education<sup>6</sup>; (iv) households that acquire private education live in the community with lower gross-of-tax price of housing<sup>7</sup>; (v) a majority voting equilibrium exists in every community.<sup>8</sup>

### 3. Urban-trap vs. urban-mixing equilibrium

In the computational examples of equilibrium below both communities provide public education of different quality and private schools emerge. We assume the urban area offers lower quality public schooling, which implies that the gross-of-tax price of housing is lower there. In such case, an equilibrium will be of one of two types, which we label *urban-trap* and *urban-mixing* equilibria.

**Definition 1.** An *urban-trap equilibrium* is an equilibrium in which: (i) low income households live in the urban area and use the low quality local public school; (ii) a set of intermediate income households live in the urban area and acquire intermediate quality private education; (iii) (a set of)

<sup>4</sup> The Working Paper version of this paper (Martínez-Mora, 2003) formally proves that all these results hold in our model. There, we use a general strictly quasi-concave and non-separable utility function.

<sup>5</sup> This is due to the residential requirement of public schools, which makes housing and public schooling complement goods (e.g. Epple and Romano, 2002).

<sup>6</sup> This result holds because the increase in the willingness to pay for a house in response to an increase in public education quality rises monotonically with income (e.g. Bénabou, 1996).

<sup>7</sup> Housing and schooling are not complements for these households, as there is no link between community and private school choices. Thus, they choose to live where their net income (income after paying for the taxes and the house) is maximum (Bears et al., 2001; Nechyba, 1999).

<sup>8</sup> Under the sequence of decisions in our model, preferences over the tax rate are single-peaked at the voting stage. The median voter theorem (Black, 1948), thus, applies. Note that the voting equilibrium in a mixed community will exhibit the ends against the middle property investigated in Epple and Romano (1996). Consequently, the median voter will have lower income than the median income household in the community.

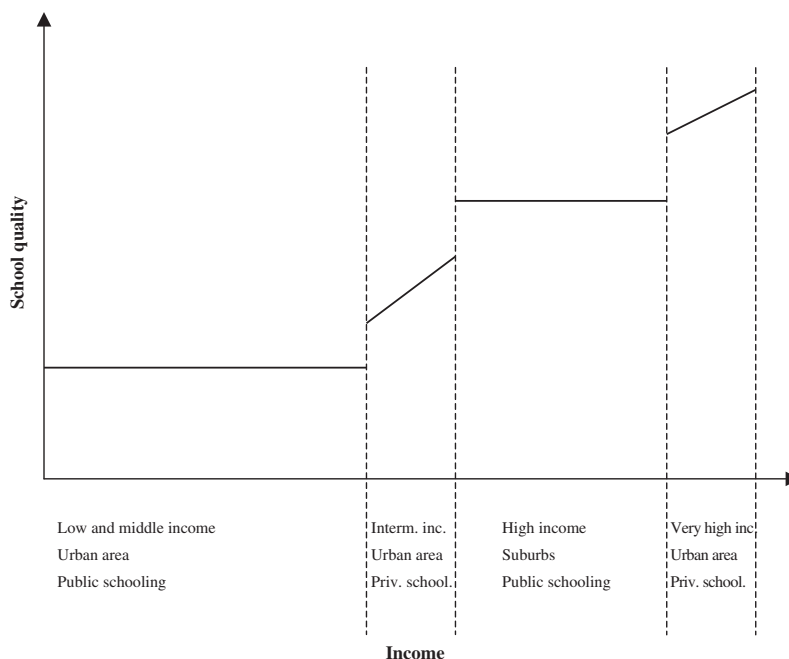


Fig. 1. *Urban-trap* equilibrium.

households with higher income live in the suburbs and send their children to the high quality local public school; and (iv) households from the top income classes may either live in the urban area and acquire elite private schooling or live in the suburbs and use the local public school.<sup>9</sup>

**Definition 2.** An *urban-mixing equilibrium* is an equilibrium in which: (i) low income households live in the urban area and send their children to the local public school; (ii) intermediate income households live in the suburbs and use the higher quality public school; (iii) rich households live in the urban area and acquire elite private schooling.

Figs. 1, 2 and 3 illustrate the allocation of households across communities and schools in both types of equilibrium. Taking community and school choices into account, they plot the quality of schooling received by the students against their households' income level.

*Urban-trap* equilibria contain the main contribution of the paper. Our examples of this type of equilibrium serve to prove two novel results on competition between public and private schools: in a two-community equilibrium, (i) *the allocation of households across public and private education does not necessarily exhibit perfect income stratification*; and (ii) *non-elite private schools* (i.e. *private*

<sup>9</sup> Equilibria that have some similarity have been found by Bénabou (1993), Rangazas (1995) and Epple and Romano (2002). Epple and Romano analyze a single-jurisdiction model with multiple neighbourhoods, each with one public school. Private schools are not allowed. Quality differentials among schools emerge due to differences in the quality of the peer group. Under an open-enrolment policy, if a positive transport cost exists, perfect income stratification arises across neighbourhoods. However, a set of intermediate income households that live in the poor neighbourhood are willing to pay the transport cost in order to receive higher quality education. Rangazas provides an example of a complete stratification equilibrium in which exogenous differences among communities lead to perfect income stratification across them. In this equilibrium, the better-off households in each community opt for private schooling. In Bénabou (1993), the poor community may not be able to supply education of enough quality. In that case, the whole population may choose not to exert any educational effort and join the informal economy.

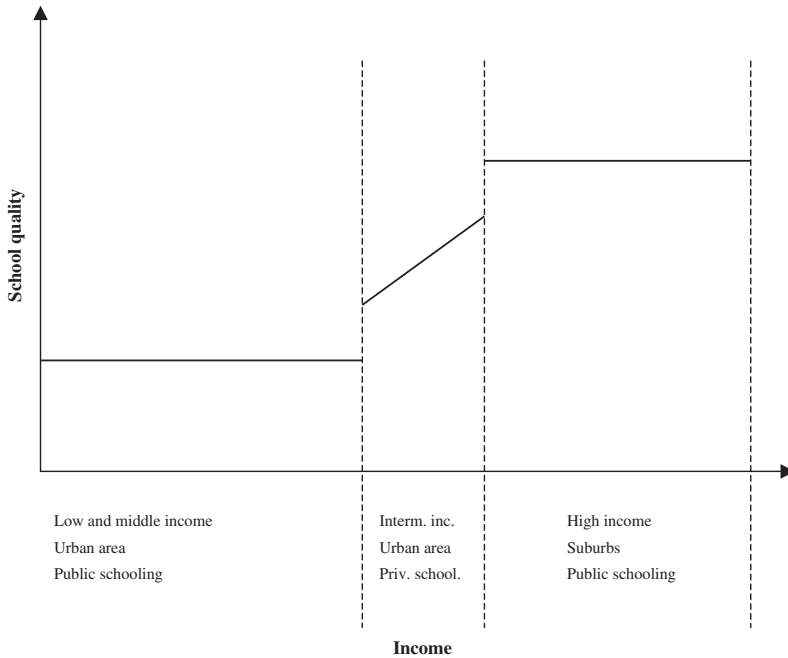


Fig. 2. *Urban-trap* equilibrium.

*schools that offer lower quality education than the high quality public alternative*) may exist (Figs. 1 and 2). Note also that *urban-trap* equilibria will exhibit perfect income stratification across communities if households from the top income classes live in the suburbs and use the local public school. In that case, private schooling exists but does not induce any income mixing within communities (Fig. 2).

*Urban-mixing* equilibria, in turn, exhibit perfect income stratification across public and private education. That is to say, only the rich opt for (elite) private schooling in this type of equilibrium. They all live in the urban area, where the gross-of-tax price of housing is lower. There they mix with low income households who send their children to the local public school instead (Fig. 3). The properties of what we call *urban-mixing* equilibria have already been analyzed in previous literature (Bears et al., 2001).

#### 4. Computational examples and discussion

We now present and discuss the computational examples of equilibrium.<sup>10</sup> To better highlight the intuitions and implications of the analysis, we choose two economies for which both types of equilibrium exist.<sup>11</sup> Let us describe what these economies look like: the first three rows in Table 1 contain the value of the parameters. Our income distributions are discretized versions of truncated lognormal distributions. They have twenty income classes ranging from  $y_{\min} = 5$  to  $y_{\max} = 100$  in constant intervals. The proportion of households in the  $i$ -th income class is given by<sup>12</sup>:  $\lambda_i = \Phi \cdot$

<sup>10</sup> Calculations that prove the examples are indeed equilibria of our model are available at [www.le.ac.uk/economics/fmm14.html](http://www.le.ac.uk/economics/fmm14.html).

<sup>11</sup> Note of course that multiple equilibria do not exist for every economy.

<sup>12</sup>  $F$  is the cumulative density function of the lognormal distribution,  $\Phi$  is a normalizing constant that assures  $\sum_i \lambda_i = 1$ .

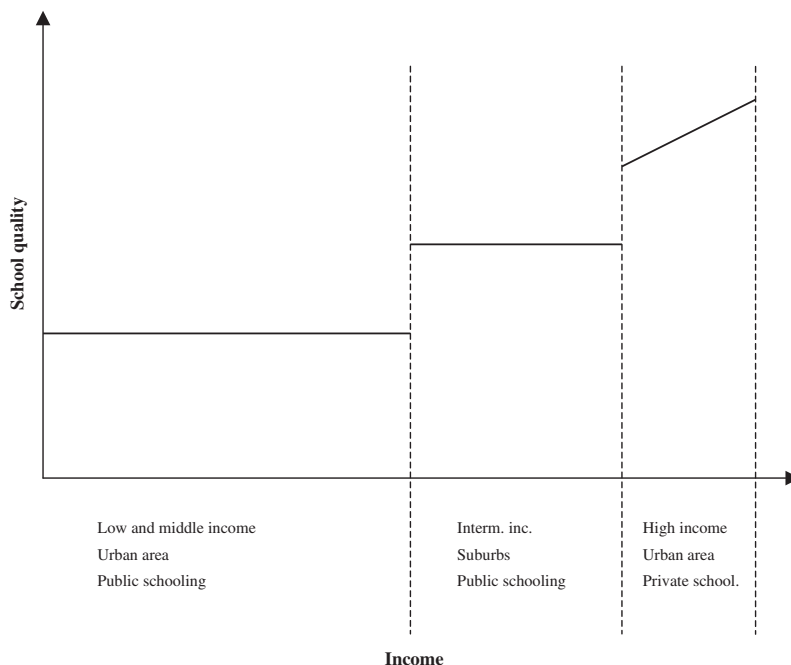


Fig. 3. Urban-mixing equilibrium.

$(F(y_i) - F(y_{i-1}))$ ,  $\forall i \neq 1$  with  $\lambda_1 = \Phi \cdot F(y_1)$ . The utility function is given by Eq. (1), with  $\sigma$  and  $\delta$  set at the values shown in the table. In our examples, the urban area is large relative to the suburbs:  $H_u$  is equal to 0.83 in example 1 and to 0.91 in example 2. Finally, the cost of construction of a house is set at  $c_h = 2.5$ . This assures that every household in the economy can at least afford to pay the gross-of-tax price of a house in  $C_u$ .

The remaining of Table 1 presents the value of key endogenous variables. For each equilibrium, it reports the quality of public and private schools, the median voter income, the equilibrium tax rates, the implicit price of the suburban public school ( $\tau$ ) and the percentage of students attending private schools. Moreover,  $y_1$ ,  $y_2$  and  $y_3$  characterize the allocation of households across communities and schools: households with income  $y \in [y_{\min}, y_1]$  live in the urban area and use the urban public school; households with income  $y \in (y_1, y_2]$  live in the urban area but send their children to a private school; households with income  $y \in [y_2, y_3]$  live in the suburbs and use the suburban public school; finally, households with income  $y \in (y_3, y_{\max}]$  live in the urban area and use an elite private school.<sup>13</sup>

The possibility that intermediate income households choose non-elite private schooling is due to the interactions among local public schooling, taxation and housing markets and to the convexity of preferences. Local provision of public education gives rise to differences across communities with respect to the level of taxation and expenditure in education, and thus in public school quality. Moreover, the residence-based admission policy of local public schools makes

<sup>13</sup> Note that in urban-mixing equilibrium,  $y_1 = y_2$  and no intermediate income household opts for private schooling. Note also that if  $y_3 = y_{\max}$  in urban-trap equilibrium, households from the top income class do not opt for private schooling and the equilibrium is characterized by perfect income stratification across communities.

Table 1  
Computational examples of equilibrium

Parameters	Example 1		Example 2	
Income distribution	$y_{\text{mean}}=40.91; y_{\text{median}}=35$ $y_{\text{min}}=5; y_{\text{max}}=100$		$y_{\text{mean}}=41.72; y_{\text{median}}=35$ $y_{\text{min}}=5; y_{\text{max}}=100$	
Utility function	$\sigma=1.83; \delta=0.0028$		$\sigma=1.71; \delta=0.0026$	
Housing market	$c_h=2.50; H_u=0.83; H_s=0.17$		$c_h=2.50; H_u=0.91; H_s=0.09$	
	Urban-trap (1a)	Urban-mixing (1b)	Urban-trap (2a)	Urban-mixing (2b)
<i>Public policies</i>				
$E_u$	0.95	1.11	0.75	0.88
$E_s$	2.97	2.58	2.59	1.99
$t_u$	0.33	0.41	0.25	0.31
$t_s$	0.93	0.76	0.84	0.61
Median voter income, urban area	25	30	25	30
Median voter income, suburbs	80	70	90	70
<i>Private education</i>				
Minimum private school quality	2.00	3.35	1.70	2.29
Maximum private school quality	3.74	3.73	2.30	2.89
Students, non-elite priv. schools	10.09	–	15.23	–
Students, private schools	11.67	5.34	15.23	9.96
<i>Housing prices</i>				
$p_h^u$	2.50	2.50	2.50	2.50
$p_h^s$	3.21	3.37	3.09	3.27
$p_u = p_h^u (1 + t_u)$	3.32	3.53	3.12	3.28
$p_s = p_h^s (1 + t_s)$	6.18	5.95	5.69	5.26
$\tau = p_s - p_u$	2.86	2.42	2.57	1.98
<i>'Border' incomes</i>				
$y_1$	50	60	55	65
$y_2$	65	60	80	65
$y_3$	95	85	100	75

Definition of *border incomes*:  $y_1$ , highest income of households living in the urban area and using the local public school;  $y_2$ , lowest income of household living in the suburbs and using the local public school;  $y_3$ , highest of households living in the suburbs and using the local public school.

housing and public education to be complement goods, which in equilibrium results in a rent premium where public school quality is higher.<sup>14</sup> Therefore, the higher quality (and tuition-free) suburban public school has an implicit price equal to the sum of the rent premium and the higher tax levied on that community's residents:  $\tau = p_h^s (1 + t_s) - p_h^u (1 + t_u)$ .

Because preferences are convex, that implicit price may make some intermediate income households find the school quality-consumption numeraire bundles associated with the public alternatives unsatisfactory. This is shown in Fig. 4, which illustrates the optimal choice of households with income  $y=45, 55, 65$  in the *urban-trap* equilibrium of example 1. The alternatives

<sup>14</sup> For this to happen housing supply cannot be perfectly elastic. We assume a fixed housing supply for simplicity. However, such rent premium also arises in equilibrium in multi-community models in which the supply of housing is elastic (e.g. Epplé et al., 1984). Note also that the existence of housing markets is not a *conditio sine qua non* for *urban-trap* equilibria to exist. It is easy to find this kind of equilibrium in a version of the model with head taxation and no housing markets.

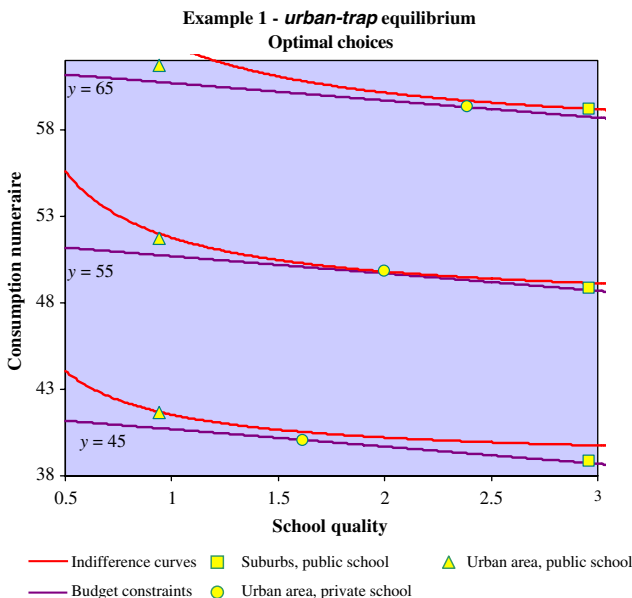


Fig. 4. Example 1 — urban-trap equilibrium. Optimal choices. — Indifference curves; — Budget constraints;  Suburbs, public school; ▲ Urban area, public school; ● Urban area, private school.

these households face are: (i) to live in the urban area and use the local public school, the triangular point, (ii) to live in the urban area and acquire their most desired level of private education, the circle, and (iii) to live in the suburbs and send the child to that local public school, the square.<sup>15</sup> As the figure shows, households with income  $y=55$  maximize utility by choosing the second alternative. This choice provides them with a preferable combination of school quality and consumption numeraire. Fig. 4 also shows the optimal choice of households with income  $y=45$ , who prefer to live in the urban area and use the local public school, and of households with income  $y=65$ , who in turn choose to live in the suburbs and send their children in the suburban public school.

It is remarkable that, as our examples reveal, both types of equilibrium may exist for the same economy. The emergence of one or the other hinges, on the one hand, on the level of political support for public schooling in the urban area, and on the other hand, on the result of the competition for living in the suburbs — which in turn depends on the result of the local political process and on how the willingness to pay for a house there varies with income. Let us further expand on this: let  $\hat{p}_h^s(y, E_u, t_u, p_u^u, E_s, t_s)$  be the maximum rent a household with income  $y$  is willing to pay for a house in the suburbs given  $E_u, t_u, p_u^u, E_s$  and  $t_s$ . This can be written as:

$$\begin{aligned}
 \hat{p}_h^s(\cdot) &= \frac{y - [(y - p_u)^{1-\sigma} + \delta E_u^{1-\sigma} - \delta E_s^{1-\sigma}]^{\frac{1}{1-\sigma}}}{1 + t_s} && \text{if } y \in [y_{\min}^s, y_u^r]; \\
 \hat{p}_h^s(\cdot) &= \frac{y - [(y - p_u - x_u(y, p_u))^{1-\sigma} + \delta x_u(y, p_u)^{1-\sigma} - \delta E_s^{1-\sigma}]^{\frac{1}{1-\sigma}}}{1 + t_s} && \text{if } y \in [y_u^r, y_{\max}^s].
 \end{aligned}
 \tag{3}$$

<sup>15</sup> Keep in mind that all households opting-out of the public sector strictly prefer to live in the urban area.

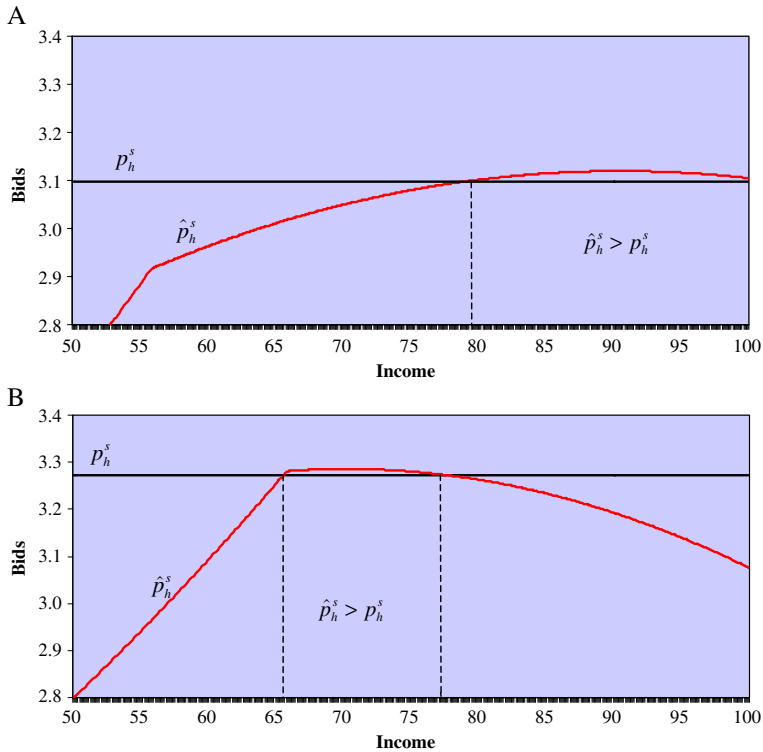


Fig. 5. a: Bid function. Example 2 — urban-trap equilibrium. b: Bid function Example 2 — urban-mixing equilibrium.

where  $p_j = p_h^j (1 + t_j)$  is the gross-of-tax price of housing in  $C_j$ ,  $x_j(y, p_j)$  is the private education demand function and  $y_u^r$  is the level of income that makes a household living in the urban area indifferent between using the local public school and sending the child to their most preferred private alternative.<sup>16</sup> Households with income  $y$  such that  $\hat{p}_h^s(y, \cdot) \geq p_h^s$  live in the suburbs in equilibrium. Next, let us define  $\hat{y}$  as the level of income at which a household demand for private education, were they living in  $C_u$ , would just equal  $E_s$ , i.e.  $x_u(\hat{y}, p_u) = E_s$ . Lemma 1 analyzes the behavior of  $\hat{p}_h^s$  as a function of income (Fig. 5).

**Lemma 1.** For  $E_u < E_s$ ,  $\hat{p}_h^s(\cdot)$  is quasiconcave in  $y$  and the global maximum is at  $y = \hat{y}$  if  $y_u^r < \hat{y}$ , and at  $y = y_u^r$  if  $y_u^r \geq \hat{y}$ .

**Proof.** For given  $E_u, t_u, p_h^u, E_s$  and  $t_s$ , let us denote the first and second ‘pieces’ of  $\hat{p}_h^s(\cdot)$  as  $\rho_a(y)$  and  $\rho_b(y)$  respectively. Note first that for  $E_u < E_s$ , the maximum gross-of-tax rent a household with

<sup>16</sup> From Epple and Romano (1996), we know that all households with income above this threshold ( $y_u^r$ ) strictly prefer to opt out over using the public school in  $C_u$  and vice versa. Taking this into account, the bid function is easily deduced from the equality between  $u(y - p_u, E_u) = u(y - \hat{p}_h^s(\cdot)(1 + t_s), E_s)$  in the income interval in which  $y < y_u^r$  and from the equality among  $u(y - p_u - x_u(y, p_u), x_u(y, p_u)) = u(y - \hat{p}_h^s(\cdot)(1 + t_s), E_s)$  for  $y \geq y_u^r$ . It is continuous, as by definition  $u(y - p_u - x_u(y, p_u), x_u(y, p_u)) = u(y - p_u, E_u)$  at  $y = y_u^r$ , and differentiable except at  $y = y_u^r$ .

income below  $y_u^r$  is willing to pay for a house in  $C_s$ ,  $\rho_a(y)(1+t_s)$ , is always larger than  $p_u$ . This makes  $\rho_a$  to be increasing in income in all its support:

$$\rho'_a(y) = \frac{[y - \rho_a(y)(1+t_s)]^{-\sigma} - (y - p_u)^{-\sigma}}{[y - \rho_a(y)(1+t_s)]^{-\sigma}(1+t_s)} > 0 \tag{4}$$

The second ‘piece’ of  $\hat{p}_h^s(\cdot)$ ,  $\rho_b(y)$ , has a stationary point at  $\hat{y}$  such that  $\hat{y} - \rho_b(\hat{y})(1+t_s) = \hat{y} - p_u - x_u(\hat{y}, p_u)$ , as in that case:

$$\rho'_b(\hat{y}) = \frac{[\hat{y} - \rho_b(\hat{y})(1+t_s)]^{-\sigma} - [\hat{y} - p_u - x_u(\hat{y}, p_u)]^{-\sigma}}{[\hat{y} - \rho_b(\hat{y})(1+t_s)]^{-\sigma}(1+t_s)} = 0 \tag{5}$$

After some manipulation, the second derivative of  $\rho_b$  at  $\hat{y}$  can be written as:

$$\rho''(\hat{y}) = \frac{-\sigma \left[ \frac{\partial x_u(\hat{y}, p_u)}{\partial y} \right]}{[\hat{y} - \rho_b(\hat{y})(1+t_s)](1+t_s)} < 0 \tag{6}$$

which is always negative because the demand for education is normal. This implies the existence of a unique stationary point and that it is a global maximum. Now consider  $\hat{y}$ . By definition  $x_u(\hat{y}, p_u) = E_s$ , hence  $\rho_b(\hat{y})$  must be such that the level of numeraire consumption — and thus utility — of households with income equal to  $\hat{y}$  is equalized across the relevant alternatives, i.e. such that:  $\hat{y} - \rho_b(\hat{y})(1+t_s) = \hat{y} - p_u - x_u(\hat{y}, p_u)$ . This is the condition for  $\rho_b(y)$  to have a stationary point at  $\hat{y}$ . Therefore, as by definition  $\rho_a(y_u^r) = \rho_b(y_u^r)$ ,  $\hat{p}_h^s(\cdot)$  is quasiconcave in income and it has a single maximum at  $y = \hat{y}$  if  $y_u^r < \hat{y}$ , and at  $y = y_u^r$  if  $y_u^r \geq \hat{y}$ . □

By Lemma 1, the willingness to pay for a house in the suburbs increases with income in the range within which households prefer the urban public school over the private sector. Once income is above the threshold  $y_u^r$ , however, a household’s bid is higher the closer their demand for school quality is to the quality of the suburban public school. Therefore, it increases with income for all  $y$  such that  $x_u(y, p_u) < E_s$ , decreases with income for all  $y$  such that  $x_u(y, p_u) > E_s$ , and is maximum at  $\hat{y}$ .

As we observe in Table 1, in an *urban-trap* equilibrium, the level of quality of the suburban public school is very high. This induces wealthy households, whose demand for school quality is close to that level, to outbid lower income households with a lower demand for school quality from the suburbs (Fig. 5a). At the same time, the urban public school offers education of too low a quality. Consequently, some middle income households who are ‘trapped’ in the urban area opt out of the public sector and acquire private education of intermediate quality. The resulting allocation of households across communities and educational sectors sustains the level of quality in both public schools as a voting equilibrium. In the urban area a large share of the population opts for a private school. Therefore, the political process results in a low level of expenditure per student and in low quality public schooling. The rich population of the suburbs, in turn, supports a high level of expenditure per student and thereby high quality public schooling.

In an *urban-mixing* equilibrium, in turn, the wealthy are not willing to pay as much as some lower income households for a house in the suburbs (Fig. 5b). This is because they demand higher quality education than that offered by the suburban public school. Thus, they live in the urban area and acquire higher quality private schooling. Intermediate income households, by contrast, do not opt out of public education. On the one hand, some of them are willing to pay the price for living in the suburbs in order to use the local public school there. On the other hand, the quality of the urban public school is not low enough to induce those of them living in the urban area to opt out.

Some further remarks are worth making. Firstly, the empirical evidence on the positive effects of (Catholic) private schools on urban minority students (e.g. Neal, 1997; Figlio and Stone, 1999; Altonji et al., 2005) is consistent with the stratification patterns that characterize *urban-trap* equilibria. This is not surprising. This kind of equilibrium sketches a picture that is common to many metropolitan areas in the US: some urban students attend and, especially those from minorities, benefit from Catholic private schools because (in Neal's words) "their local public school alternatives are poor (and) because their families can afford the modest tuition that are common in Catholic schools, but (...) can seldom afford housing in the exclusive neighborhoods with the best public schools". This was already observed by Friedman (1962).<sup>17</sup>

Secondly, *urban-trap* equilibria have clearly undesirable properties from an equity point of view. The quality of schooling students from low income households receive is lower. This is associated to a higher level of private school attendance in the economy, which in turn increases income sorting across individual schools. Furthermore, the argument that private schooling tends to reduce residential income stratification (e.g. Nechyba, 1999; Barse et al., 2001) is weaker when referred to this kind of equilibrium: as example 2 illustrates, perfect income stratification across communities may characterize an *urban-trap* equilibrium.

Thirdly, the possibility that the economy is in an *urban-trap* equilibrium should be considered in the analysis of voucher policy proposals. Nechyba (1999, 2000) and Luengo-Prado and Volij (2003) have investigated the consequences of different voucher policies in a multi-community setting. It seems important to extend this research to an economy which is in this type of equilibrium before the voucher policy is implemented. In this respect, Rangazas (1995) took a first step by providing an example of his *complete stratification* equilibrium in which private school vouchers lead to an increase in public school quality in both the rich and the poor communities.

Finally, our results shed some light on how private schools distribute along the quality space. In particular, *urban-trap* equilibria explain the stylized fact that some private schools are very good while others are not (Marks et al., 2001), and support the notion that private schools emerge to fill the gaps in the menu of public alternatives.

## 5. Concluding remarks

The market for education in a multi-community economy is complex. According to conventional wisdom one would expect an equilibrium with public and private schools to be such that the rich use private schools and the rest of the population use public schools. Moreover, all private schools should be of higher quality, as no one would pay tuition for private education when higher quality "free" education is available. We show that this is not necessarily the case in a multi-community context. An implicit price for high quality public education exists in the form of a higher tax bill and a premium in the price of housing. Consequently, some middle class households may not be able to afford to live in wealthy communities. If they are unsatisfied with the quality of public schooling in their community, they will acquire private education of intermediate quality instead.

<sup>17</sup> Another branch of empirical research focuses on the determinants of households' choices between public and private schools. One result of this literature also seems to support the empirical relevance of urban-trap equilibria. Buddin et al. (1998) obtain that Blacks and Hispanic whites are, ceteris paribus, more likely to attend a private school. A plausible explanation is that, ceteris paribus, a Black or Hispanic white student is more likely to live in segregated parts of cities where public schools are poor.

This equilibrium, which we label *urban-trap* equilibrium, reflects a situation that is common to many metropolitan areas in the US: the central city provides low quality education and the suburbs are very exclusive. This makes some relatively low income households, who cannot afford to live in the suburbs, acquire non-elite private schooling. A picture like this seems to be behind the evidence of Neal (1997), Figlio and Stone (1999) and Altonji et al. (2005). This branch of empirical research could therefore benefit from our results.

The possibility that an *urban-trap* equilibrium emerges gives rise to some equity concerns: central cities public schools — and not elite public schools — could be those most damaged by the cream-skimming effect of private schooling. Therefore, students left-behind in inner-city public schools after a voucher policy could face insurmountable obstacles to succeed at compulsory education. This should be taken into account when designing and analyzing vouchers proposals.

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