A CONSISTENT APPROACH TO MODELLING DYNAMIC FACTOR DEMANDS

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ABSTRACT

This paper presents a consistent approach to modelling dynamic factor demands. Typically firms are assumed to face adjustment costs when changing factor inputs. This has lead to a class of models that capture the dynamics of adjustment in a system of dynamic factor shares. We argue here that such models are based on a misspecification of the firm's objective function that assumes firms face costs when adjusting factor shares. In fact firms often minimise their costs by allowing shares to vary. We argue that costs are incurred in changing the factor volumes themselves and specify an objective function on this basis. This enables us to estimate a consistent set of factor demands equations for the UK economy. We show that by correctly modelling the dynamics we are able to impose all of the restrictions implied by economy theory while maintaining the empirical validity of the model.

Keywords: Dynamic Factor Demands, Translog cost functions, adjustment costs.

JEL Classification: E21, E22, E23

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CONTENTS

1 INTRODUCTION

The behaviour of capital investment and employment patterns over time has been an active area of research in economics for a number of years, undoubtedly because of their importance in understanding business cycles and tackling the unemployment that goes with them. However, static models of factor demands provide little insight into the behaviour of these macroeconomic aggregates over time. It is for example clear that firms cannot adjust there factor inputs instantaneously, since that would imply that discrete changes in the economic environment (such as a change in interest rates) would lead to infinite rates of factor accumulation or scrapping. As a consequence it is now widely recognised that firms face significant costs in adjusting to their desired level of factor inputs.

Typically, adjustment costs are assumed to be most significant for capital which leads to the modelling of the restricted production set, or in the two factor case, the inverting of the production function to give the demand for the flexible factor (see for example the NAIRU model of Layard, Nickell and Jackman (1991). This insures that the firm is always on its production function. However, in more general cases, that arguably have more empirical relevance, this requires cross equation restrictions on the dynamics of factor demands[1]. Relatively little work has been attempted using the dual; that is a dynamic conditional demand systems based around a consistent cost function.[2] Recently however, Allen and Urga (1995 and 1999) have proposed a dynamic cost function consistent with dynamic adjustment in factor shares.

This papers estimates an aggregate model of factor demands in the UK that has both a high degree of theoretical consistency and empirical relevance. The joint estimation of the cost function and consistent dynamic factor demands results in far greater efficiency of estimation and leading to more precise estimates of such parameters as the returns to scale and the rate of technical progress. As well as being intrinsically of value in its own right, accurate modelling of the production set is important to recover a measure of the Solow residual free of mis-specification. An appropriate model of the dynamics of adjustment is therefore crucial to recover unbiased estimated of the long run parameters of the production set. This paper investigates the dynamic cost function of Allen and Urga (op. cit.) and finds that the results from this approach appear to be significantly mis-specified with adjustment dynamics that are to be empirically unrealistic. . We argue that this is because their dynamic cost function is implicitly minimising adjustment costs in factor share. We believe that this is unrealistic and propose an alternative approach based in minimising adjustment costs in the factor volumes themselves. We show that this lead to improved dynamic adjustment and enables us to impose all the restrictions implied by economic theory on the long run solution. A further innovation is that we are able to model the economy at the aggregate level rather than just focussing on the non-energy business sectors as in Allen and Urga (1999). The enables us to test whether we can restrict aggregate technical progress to be Harrod neutral so that out model of factor demands has a stationary steady state consistent with in observations of growth theory.

2. AN AGGREGATE PRODUCTION STRUCTURE FOR THE UK ECONOMY

We model the supply side of the UK economy in terms of a representative, imperfectly competitive firm, operating in a small open economy with five aggregate commodities; goods, capital, labour, fuels and non-fuels. We believe that this level of disaggregation is the minimum required to achieve a good empirical description of the supply side. Fuels and non-fuels are assumed to be limitless raw materials whose price is set exogenously but may be imported. We assume there is a market for labour and capital which determines their respective prices, although in so far as the cost of capital is influenced by interest rates this too is exogenous, set by an inflation targeting authority. The imperfectly competitive firm then decides its required input volume, taking factor prices as given, to produce an expected level of output, given the current state of technology. It then sets price on the basis of a markup over marginal costs, which in turn determines the real value of factor incomes. This then determines actual demand, through the demand side of the economy.

Suppressing the time subscript for clarity, aggregating across firms and imposing symmetry, we consider the imperfectly completive firm's optimisation problem as

$$\min\ TCOST = C\ (P_K\ ,\ P_L\ ,\ P_E\ ,\ P_M\ ,\ Y,\ t) \tag{1}$$

where there are four inputs to production; capital (K), labour (L), energy (E) and imported materials (M) and where P_i is the corresponding factor price, and Y is expected demand. Additionally, we assume disembodied exogenous technical progress (t). By virtue of Sheppard's Lemma, differentiating the cost function with respect to each of the factor prices gives the conditional factor demands

$$x_i = \frac{\partial c}{\partial p_i} = c^{/}_{p_i}\ (\ p_i,\ y,\ t) \tag{2}$$

Turning to our empirical specification, we assume that the cost function can be approximated by second order translog cost function. This is a flexible functional form which can be interpreted as a second-order approximation to any arbitrary cost function (see Denny and Fuss, 1977). It has enough parameters to allow us to estimate an unrestricted set of elasticities of substitution. We therefore are not constrained to restrict all of the elasticities of substitution to be unity a priori as is implicit in a Cobb-Douglas specification. The equilibrium translog cost function takes the form:

$$\begin{aligned}\ln C = {} & \alpha_0 \\ & + \alpha_y \ln y \\ & + \alpha_t t \\ & + \sum_{i=1}^{n} \alpha_i \ln p_i \\ & + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_{ij} \ln p_i \, p_j \\ & + \sum_{i=1}^{n} \alpha_{iy} \ln y \ln p_i \\ & + \sum_{i=1}^{n} \alpha_{it} \, t \ln p_i \\ & + \alpha_{yt} \, t \ln y \\ & + \frac{1}{2} \alpha_{yy} (\ln y)^2 \\ & + \frac{1}{2} \alpha_{tt} \, t^2 \end{aligned} \qquad \mathbf{(3)}$$

where p_i is the ith input price, C is the equilibrium total cost, y is output, t is a time trend. By Sheppard's lemma, differentiating the long run cost function with respect to each of the factor input prices generates the firm's long run cost minimising factor demands. If we differentiate $\ln C$ with respect to $\ln p_i$ we obtain the following system of input share equations:

$$S^{*i} = \alpha_i + \sum_{j=1}^{n} \alpha_{ij} \ln p_j + \alpha_{iy} \ln y + \alpha_{it} \, t \qquad \mathbf{(4)}$$

$$\text{where} \qquad S_i^* = \frac{p_i \ X_i}{\sum_{j=1}^{n} p_j \ X_j} \tag{5}$$

and X_i is the quantity demanded of input i.

A number of specific restrictions can then be tested for or imposed on this general model. Restrictions 1 and 2 described below are imposed on the model to ensure some degree of coherence; the remain restrictions 3, 4 and 5 however are tested

1. If the shares are to **sum to one**, the following parameter restrictions must hold. This also ensures **linearly homogeneity in factor prices**.

$$\begin{aligned} \sum_{i=1}^{n} \alpha_i &= 1 \\ \sum_{j=1}^{n} \alpha_{ij} &= 0 \\ \sum_{i=1}^{n} \alpha_{iy} &= 0 \end{aligned} \tag{6}$$

2. Equally we require **symmetry** for the translog to be viewed as a quadratic approximation to an arbitrary cost function[3] (Denny and Fuss, 1997). This requires:

$$\alpha_{ij} = \alpha_{ji}, \qquad i \neq j \tag{7}$$

3. Additionally, the cost function will be **linearly homogeneous in output** if:

$$\begin{aligned} \alpha_y &= 1 \\ \alpha_{yy} &= 0 \\ \alpha_{yt} &= 0 \end{aligned} \tag{8}$$

4. The homogeneous translog cost function will be **homothetic** if

$$\alpha_{iy} = 0 \qquad \textit{for all } i, \tag{9}$$

5. Finally, **labour augmenting technical progress** requires the following restrictions between the coefficients on the price of labour and the coefficients on the time trend:

$$\begin{aligned} \alpha_t &= \alpha_l \; v \\ \alpha_{it} &= \alpha_{li} \; v \\ \alpha_{yt} &= \alpha_{ly} \; v \\ \alpha_{tt} &= \alpha_l \; v \end{aligned} \qquad \textbf{(10)}$$

3. DYNAMIC FACTOR DEMANDS IN THE PRESENCE OF ADJUSTMENT COSTS

We assume that firms are unable to adjust their factor volumes instantaneously because of the presence of adjustment costs, where the cost incurred may be direct costs incurred such as construction or training costs, or else profit foregone incurred by producing a less that optimal scale. These adjustments costs themselves are likely to vary between factors, with capital the most costly to adjust. This in turn is likely to be reflected in different speeds of adjustment. In this section we therefore derive a dynamic cost function following the suggestion of Allen and Urga (1995). They take as their starting point the flexible dynamic demand system that Anderson and Blundell (1982) employed for analysing the demand for good in the consumption, as opposed to production decision. This is an obvious place to start from, when considering a system of interrelated demand equations. However, the demand system of Anderson and Blundell only consists of a the set of demand functions and hence runs into an identification problem; in that they can only identify the relative speeds of adjustment. Allen and Urga (1995) however derive a dynamic cost function which is compatible with the integratability conditions of the flexible dynamic demand system. The derivation of this underlying cost function allows us to considerably increase the efficiency of the estimation of the factor demand equations and fully identify the parameters of the underlying process. In addition, the direct derivation of the factor demand functions from the underlying objective function means that the factor demands are in fact optimal, conditionally on the technology, even in the short run (as in Norworthy and Harper, 1981). This is not the case in the standard Anderson and Blundell.

Allen and Urga (1995) derive a general cost function that contains both equilibrium and disequilibrium terms, consistent with the dynamic equations for the factor shares. They is done for the simplest possible formulation, involving a single lag in the adjustment process which turns out to be sufficient to describe our data set. The starting point is the dynamic share equation of Anderson and Blundell (1983). If we let the optimal level of the factor share be ˋ , then a dynamic VARDL (1,1) process for S_t can be expressed as follows:

$$S_t = A.S_t^* + B.S_{t-1}^* + C.S_{t-1} \qquad \textbf{(11)}$$

where **A** + **B** + **C** = **I**, an identity matrix. Anderson and Blundell (1983) show this can be reparametrised as an equivalent error correction model in factor shares:

$$\Delta S_t = G.\Delta S_t^* + K.(S_{t-1}^* - S_{t-1}) \quad \textbf{(12)}$$

where **G** = **A**, **K** = **I** - **C** = **A** + **B**. Hence, the disequilibria in n-1 factor shares at time t-1, influence the current-period adjustment and any particular share.

Since the sum of the changes in shares must be zero, ie.

$$\sum_{i}^{n} (S_i - S_i^*)_{t-1} = 0 \quad \textbf{(13)}$$

the columns of the K matrix must therefore also sum to zero. This implies that there are only n-1 independent dis-equilibrium shares; the K matrix is therefore of reduced rank (n-1) x (n-1). This means that the individual adjustment coefficients are not identified, rather we are only able to obtain the ratios k_{11}/k_{12} for example. Allen and Urga (1995) however suggest that the joint estimation with a suitably consistent dynamic cost function overcomes this problem. This is because the dynamic cost function utilises an extra constraint; namely that the sum of the factor prices times the factor volumes has to equal total costs each period. They propose a particular form of dynamic cost function that can be estimated jointly with share equations of the form of (13) by imposing the mapping that the G matrix is a scalar m. This gives the following dynamic cost function that contains both equilibrium and disequilibrium terms and satisfies the integratability conditions between the cost function and the factor shares.

$$\begin{aligned} \ln C_t &= m \ln C_t^* + (1-m) \ln C_{t-1} \\ &+ (1-m) \left(\sum_{i=n}^{n} S_{it-1} \ln p_{it} - \sum_{i=n}^{n} S_{it-1}^* \ln p_{it-1} \right) \\ &+ \sum_{i=n}^{n} \sum_{j=n}^{n} \beta_{ij} (S_{jt-1}^* - S_{jt-1}) \ln p_{it} \end{aligned} \quad \textbf{(14)}$$

where b_{ij} are elements of the matrix **B**.

The corresponding cost share equations will therefore be of the form

$$\Delta S_{it} = m\Delta S^*_{it} + \sum_{j=n}^{n} k_{ij} \left(S^*_{jt-1} - S_{jt-1}\right) \qquad \textbf{(15)}$$

where k_{ij} are elements of **K** and where **K** = m**I** + **B**.

The consistent dynamic cost function described by Allen and Urga is derived by integrating error correction equations in factor shares. Christofides (1976) shows that this dynamic demand system can also be derived by considering the optimisation of a suitable objective function. For example, if we assume firms face costs C_i when adjusting factor shares and by being away from their optimal factor combinations, consistent with S^*; then a the firm's objective function would be:

$$L^* = (S_t - S^*_t)' \; C_1 \; (S_t - S^*_t) \; + \; \Delta S'_t \; C_2 \; \Delta S_t \; - \; \Delta S_t \; c_3 \; \Delta S^*_t \qquad \textbf{(6)}$$

where C_i (I = 1,2,3) are conformable adjustment matrices. Minimising L^* with respect to S_t we obtain generalised error feedback equations of the form as in Andersen and Blundell (1983):

$$\Delta S_t = G.\Delta S^*_t + K.(S^*_{t-1} - S_{t-1}) \qquad \textbf{(7)}$$

where the dis-equilibria in n-1 factor shares at time t-1 influence the current-period adjustment and any particular share and G is in general a matrix.

This suggests that the integration of dynamic equations in factor share proposed by Allen and Urga (1995) is implicitly the same as minimising adjustment in factor shares. Previous attempts to their apply this technique to production theory have proved unsatisfactory. In chapter 4 we found that the dynamic response from such a share system is implausibly fast. Moreover, the limited model they are able to estimate (by making the assumption that G is a scalar) is not an acceptable empirical restriction of the general dynamic specification. We believe that creating a dynamic model from costly adjustment in shares is simply implausible. It is not costly to adjust shares, in fact we often incur zero adjustment costs by allowing the shares to change rapidly in response to a shock. Rather firms face costs of adjusting the actual level of factor input.

We can therefore respecify the firm's objective function in terms of changes in factor volumes. If we assume firms face costs (C_2) when adjusting the volumes of factor inputs (x) in addition to an opportunity cost (C_1) for not producing at optimal factor shares, S* (ie. for producing with factor proportions that are not consistent with their long run optimal cost function). The firm's objective function would then be:

$$L^{*} = (Z_{i,t} - S^{*}_{i,t-1})^{/} \; C_1 \; (Z_{i,t} - S^{*}_{i,t-1}) \; + \; \Delta \ln(x)^{/}_{i,t} \; C_2 \; \Delta \ln(x)_{i,t} \qquad \textbf{(8)}$$

where Z is the actual factor proportion relative to optimal costs (ie. $\dfrac{P_{i,\,t}\; X_{i,}}{C^{*}_{t-1}}$) . The first order

conditions will then give rise to general factor demand functions which have the following general form (where for simplicity we assume C is diagonal):

$$\Delta\ln(x)_{i,\,t} \; = \; \gamma_{i,\,t}.\Delta\ln(x)_{i,\,t-1} \; + \; \beta_i \left(\frac{p_{i,\,t-1}.x_{i,\,t-1}}{C(p_{i,\,t-1},\; Y_{i,\,t-1})} \; - \; S^{*}_{i,\,t-1} \right) \qquad \textbf{(10)}$$

Where in general the dynamic factor demands are estimated as an unnormalised, non-linear system. We can obviously extend the model to allow for higher order adjustment costs to give rise to more lags or intertemporal optimisation to give rise to rational expectations effects.

4. ESTIMATING THE CONSISTENT DEMAND SYSTEM

We now attempt to estimate the dynamic cost function and system of dynamic factor shares discussed in the previous. Since the system is non-linear in factor prices and because we want to estimate a very specific adjustment mechanism we are not able to employ standard the Johansen technique (see Johansen 1988, 1991). Instead we estimate the system jointly using full information maximum likelihood (FIML). The approach we take is also an example of the ideas discussed in Greenslade et. al. (1999) in that we impose a high degree of theoretical structure on the data and estimate a very particular conditional system. We do this because in a small sample we are not confident of correctly being able to identify the cointegrating vectors that correspond to the factor demands we are attempting to estimate. We do however consider the cointegrating properties of our system by estimating the long run equations separately and testing for cointegration in a rather heuristic fashion using standard Augmented Dickey Fuller tests. This can be thought of as the first stage of the Engle-Granger procedure but generalised to a full system. We do not report tests for the order of integration of the data or problems associated with cointegration in non-linear systems as these are extensively discussed in (Allen 1997). But, in summary we treat all the variables of interest, prices, costs and output, as I(1). The shares themselves for example, can also be shown to be I(1) after an appropriate logit transformation. Allen (1997) also considers the implications of the non-linear nature of the system for cointegration and we do not repeat his discussion here. We apply our restrictions to the model in three stages, homogeneity, homotheticity and then Harrod neutrality, testing for continued cointegration at each stage. Having gauged the cointegrating properties of the system we then jointly estimate the full set of dynamic equations including the coefficients on levels. Ideally, we would like to test the validity of our restrictions via conventional likelihood ratio tests in the dynamic model.

Table 1 therefore reports cointegration tests as we successively impose the three groups of restrictions on the system estimated jointly but purely in levels terms. We find that in order to achieve cointegration we are required to extend our theoretical structure to include two additional variables. Perhaps most importantly, to apply linear homogeneity with respect to output requires that we take account of changes in capacity utilisation. This finding mirrors some the arguments made in the real business cycle literature about the need to measure capital services accurately (see for example Burnside, Eichenbaum and Rebelo, 1995). Clearly, it is utilised factors that go into the production function so this result is hardly surprising. In principle it would be possible to adjust the data for factor volumes employed (most easily labour could be multiplied by hours, for example). However, hours data is only available for manufacturing and in any case this series has been recently discontinued. Instead we take the expedient of include capacity utilisation in our system as an extra regressor. This implies the addition of six extra terms to the cost function; *cu*, $p_i.cu$, for I=1 to 4, and *cu.t*, with the appropriate cross equation restrictions between them.

In a similar vein, our measure of fuel input shows a marked drop at the start of the 1980s. This seems to reflect a fundamental asymmetry of response, possibly associated with irreversibility of investment or permanent technical change. Thus the long fall in real fuel prices over the 1980s has not resulted in a return to the same level of fuel use for a given level of output. Rather the price hike of the 1970s appears to have produced a permanent increase in fuel efficiency. To capture this effect, we additionally include a cumulated real fuel price as well as the share of manufacturing in GDP, as a two further regressors in our system. This again implies the addition of a further six terms and two more restrictions for each variable.

The results from the ADF tests on the residuals from each equation on the system, indicate that we can restrict our model to be consistent with economic theory and still maintain cointegration. Thus imposing linear homogeneity, homotheticity and Harrod neutrality improve the cointegration properties of the system without increasing the standard error of the regressions markedly.

Table 1a: Cointegration tests on Unrestricted Levels System

	ADF(n)	(n)	SSR	SE	LogL
TCOST	4.61 [.028]	(0)	.01064	.00912	2110
SL	6.44 [.000]	(3)	.00218	.00413	
SK	4.93 [.011]	(3)	.00345	.00519	
SF	5.01 [.008]	(0)	.00065	.00225	

Table 1b: Cointegration tests when linear homogeneity with respect to output is imposed

	ADF(n)	(n)	SSR	SE	LogL
TCOST	4.71 [.021]	(0)	.01608	.0112	2065
SL	5.81 [.000]	(1)	.00231	.00425	
SK	4.73 [.019]	(1)	.00467	.00604	
SF	5.57 [.001]	(0)	.00064	.00224	

Table 1c: Cointegration tests when homotheticity is imposed

	ADF(n)	(n)	SSR	SE	LogL
TCOST	5.52 [.001]	(0)	.0329	.0160	2033
SL	6.43 [.000]	(3)	.00214	.00409	
SK	5.12 [.005]	(3)	.00338	.00514	
SF	4.35 [.057]	(0)	.00071	.00236	

Table 1d: Cointegration tests when Harrod Neutral Technical Progress is imposed

	ADF(n)	(n)	SSR	SE	LogL
TCOST	4.41 [.049]	(0)	.05090	.0214	1940
SL	4.96 [.009]	(0)	.00327	.00505	
SK	5.03 [.007]	(3)	.00340	.00515	
SF	5.17 [.004]	(0)	.00071	.00236	

Notes: ADF tests of order n are reported for residuals on each equation, where n is the minium lag required to remove serial correlation from the ADF regression. Cointegration probability values are for 6 regressors and are for guidance only.

The second extension stems from the observation that our measure of fuel input shows a marked drop at the start of the 1980s. This seems to reflect a fundamental asymmetry of response, possibly associated with irreversibility of investment or permanent technical change. Thus the long fall in real fuel prices over the 1980s has not resulted in a return to the same level of fuel use for a given level of output. Rather the price hike of the 1970s appears to have produced a permanent increase in fuel efficiency. To capture this effect, we additionally include a cumulated real fuel price as well as the share of manufacturing in GDP, as a two further dummies in our system. This again implies the addition of a further six terms and two more restrictions for each dummy.

The results from the ADF tests on the residuals from each equation on the system, indicate that we can restrict our model to be consistent with economic theory and still maintain cointegration. Thus imposing linear homogeneity, homotheticity and Harrod neutrality improve the cointegration properties of the system without increasing the standard error of the regressions markedly. A closer inspection of the residuals of the fully restricted cost function suggests that there is clearly a systematic pattern to the residuals (see figure 4). Thus in contrast to Darby and Wren-Lewis (1992) we do not appear to be able to explain costs solely on the basis of factor inputs, substitution between factors, capacity utilisation and a deterministic trend. Given the pattern of residuals; low in the 1970s, high in the 1980s, we take this to be evidence of time varying technical progress (or possibly endogenous scrapping). Later work in our research programme will return to this.

Turning to the dynamic model shown in table 2, we are able to estimate the full non-linear system with error correction in factor shares. All of the error correction terms are significant and the system is well specified, passing diagnostic tests for autocorrelation and hetroskedelasticity. In terms of individual coefficients, we find that the necessary concavity conditions are global and that the estimated Allen elasticities are consistent with previous studies. The elasticity of substitution between capital and labour is 0.42 which is broadly in line with a wide average of studies (see Rowthorn, 1996 for a survey). Finally, our estimate of technical progress is for growth of around 2.3% per year.

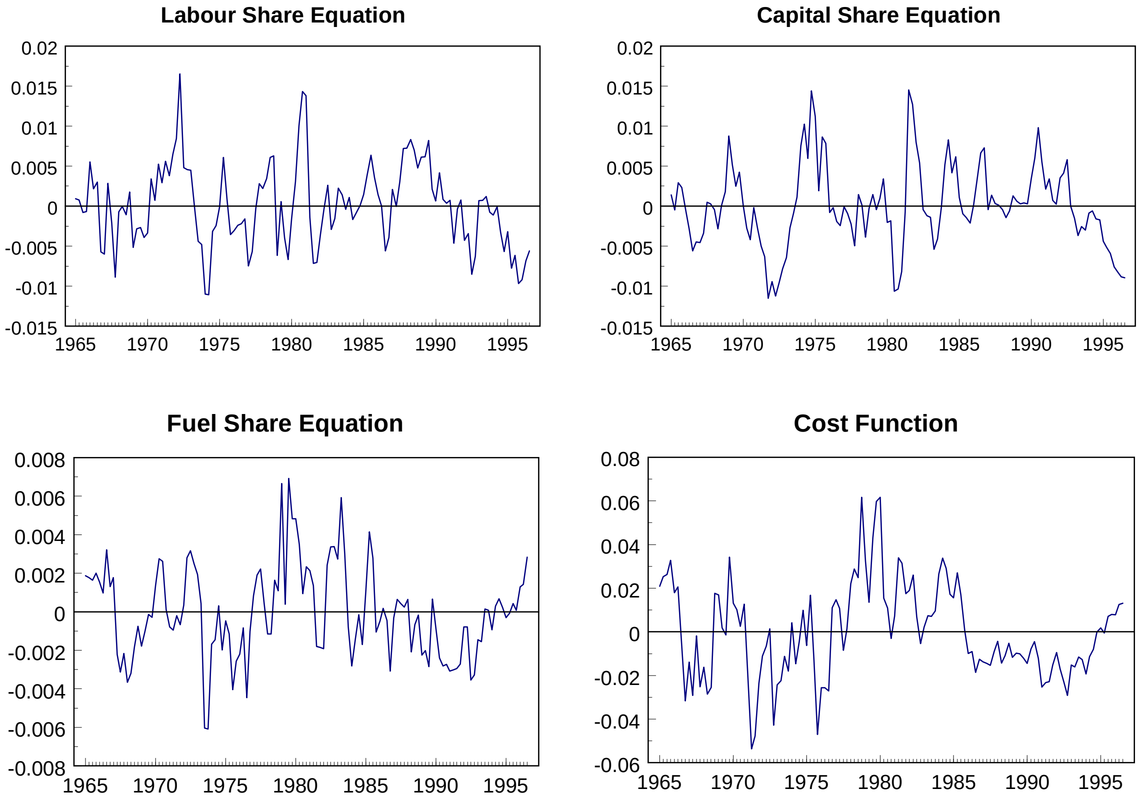

Figure 4: Cointegrating residuals from the fully restricted long run system

Table 2: Log likelihood Ratio tests of the restrictions on the dynamic System

	Log likelihood	Chi Sq	(n)	p (value)
Unrestricted	1762			
Homogeneity	1760	4.34	(3)	0.226
Homotheticity	1754	10.67	(4)	0.031
Harrod Neutrality	1746	15.44	(9)	.0799
All restrictions		30.45	(16)	.0159

Table 3: Levels and Dynamic Estimates of Main Coefficients

66q2 - 96q4	Static Model		Dynamic Model	
FIML:	Estimate	t-statistic	Estimate	t-statistic
A0	-2.604	-57.74	-0.46543	-.091175
A1	.4393	3.167	-.212794	-.987472
A2	.6959	65.71	1.24868	30.9058
A3	-.0289	-4.533	-.063788	-3.94799
A11	.1673	32.39	.163842	4.18784
A12	-.1075	-46.19	-.145369	-5.22424
A13	-.0462	-23.77	-.845225E-03	-.258361
A22	.1364	54.52	.165077	5.31508
A23	.0092	-10.47	-.010561	-3.98151
A33	.0436	40.66	.018169	2.89945
V	-.00435	-62.90	-.005578E-02	18.5859
B0			.538740E-02	1.85901
B1			-.221817	-2.93514
B2			-.491001E-02	2.38615
B3			-4.39729	2.62236
B4			-3.14291	2.21445
ΔN_{-1}			.323493	4.03552
ΔN_{-2}			.165658	2.17369
ΔK_{-4}			.486817	7.0986
ΔF_{-1}			-.299425	-4.15931
ΔF_{-1}			-261300	-3.62754
ΔM_{-1}			-.217297	2.88183

Notes: where *Ai j* are the production coefficients, where 1=labour (N), 2=capital (K), 3=fuels (F), and 4=non-fuels (M).

Table 4: Individual Equation Diagnostics

	R^2	S.E.	Q (1)	Q (2)	Q (3)	Q (4)	Arch (1)
COSTS	.1251	.0234	.154	.190	.688	8.51	.154
EMP	.5225	.00338	.708	1.67	2.66	2.86	4.61
KP	.0943	.01333	.885	1.10	1.25	6.41	.107
VFUEL	.3275	.0250	.004	.063	4.72	5.48	1.75
VNF	.1328	.0347	.194	.198	.283	3.67	.801

Notes Q(n) is Box-Pierce portmanteau test, distributed $\chi^2(n)$
Arch (1) is Q(1) performed on the squares of the residuals

Table 5a: Long Run Allen Elasticities of Substitution

	Labour	Capital	Fuel	Non-Fuel
Labour	-0.133	0.461	-0.479	0.366
Capital		-0.832	1.902	-2.511
Fuel			-6.876	3.5146
Non Fuel				-0.332

Table 5b: Long run Price Elasticities

	Labour	Capital	Fuel	Non-Fuel
Labour	-0.089	0.075	-0.027	0.041
Capital	0.215	-0.136	0.112	-0.284
Fuel	-0.183	0.278	-0.396	0.397
Non Fuel	0.379	-0.413	0.173	-0.038

Notes: Elasticities are evaluated at sample means.

5. COMPARING THE SYSTEM SIMULATION PROPERTIES

To begin with we note that the model will possess the following steady state properties:

- Our estimate of v gives an exogenous rate of technical progress of 2.3% p.a.
- If real wages grow at this amount, demand (hence output) also grows at this amount
- Harrod neutral technical progress ensures that employment is constant while other factors of production are utilised at rate v in order to produce more output. The capital output ratio is therefore a constant, as are the factor shares.
- Costs therefore grow at rate v plus some underlying rate of nominal (factor) price increase. Average costs and hence prices therefore grow at a constant rate. Inflation in the steady state is therefore a constant. This could be thought of as the rate of growth of the money supply or the authorities inflation target.[4]

Figures 5 and 6 compare the impulse response functions of the system estimated here with those estimated by Nixon and Urga (1997). The two figures below show deviations of each factor from base for 10 percent increases in fuel prices, with no feedbacks through other factor prices or wages. The two figures compare the dynamic response of the system estimated here with adjustment in factor volumes, with the system estimated in Nixon and Urga (1997) that has adjustment in factor shares. We clearly see that fuel and employment tend to be complements in our system, which substitute for capital and non fuels. In the dynamic share system (figure 5), the factor volumes make an initial jump and then just to a new steady state equilibrium, where it takes around five for complete adjustment. This is somewhat disappointing, in that even though the dynamic factors shares should in principal allow for different speeds of adjustment, adjustment in factor shares appears to impose a significant amount of common response. The speed of adjustment also appears quite rapid, especially in the context of the capital stock. The lower figures demonstrate the dynamic response of the system with the new approach to dynamic factor demands taken here. There is no initial jump in factor volumes in response to a price change, but instead the system adjusts to a new optimising position over a much more protracted horizon; significant adjustment is now spread over some eight to ten years.

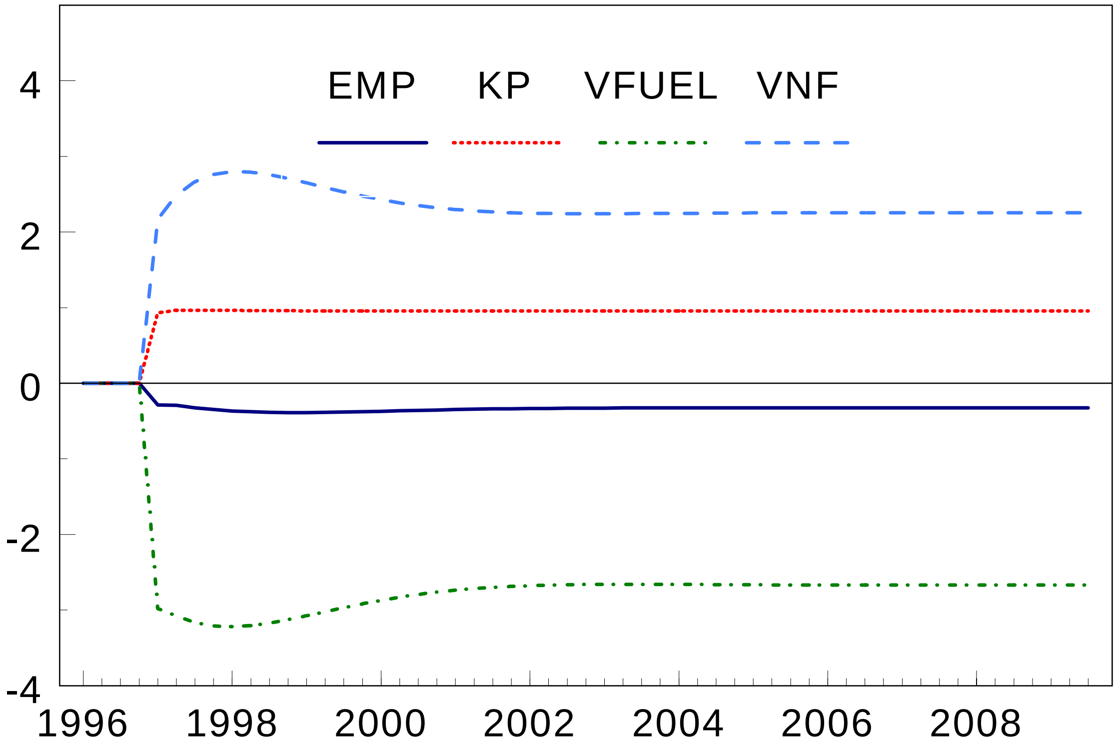

Figure 5: Factor volumes: % deviation from base after 10% fuel price increase

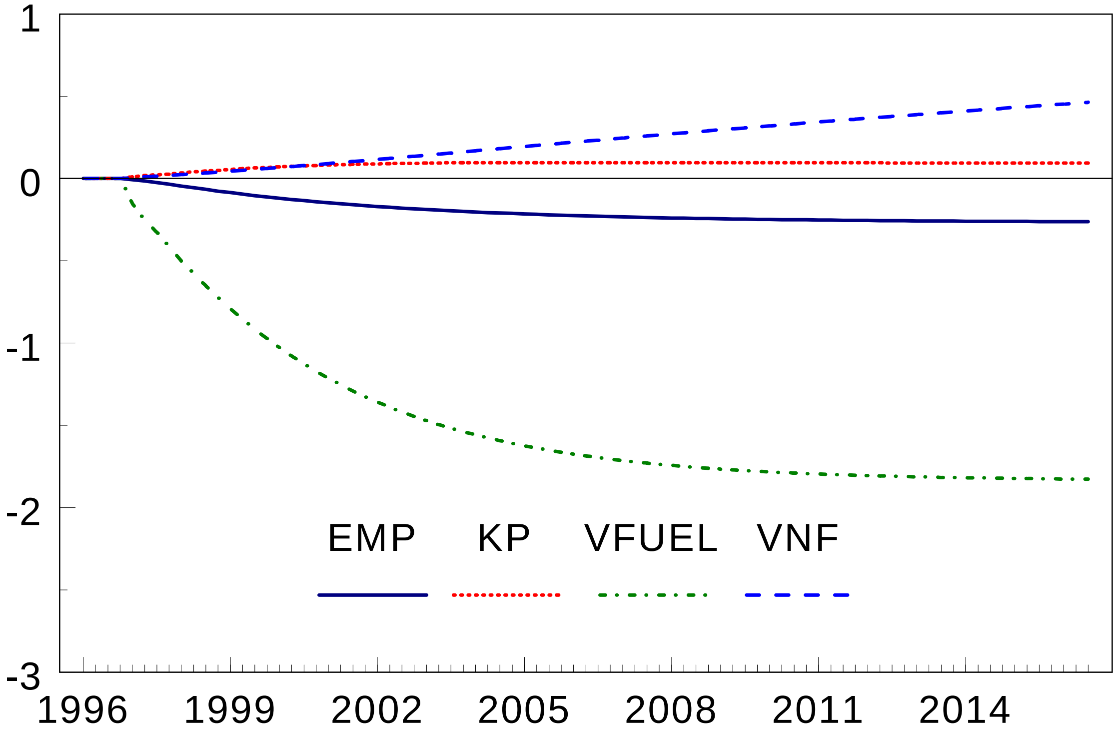

Figure 6: Factor Volumes: % deviation from base after a 10% fuel price increase

6. CONCLUSION

In this paper we have estimated a consistent aggregate production structure for the supply side of the UK economy incorporating a new approach to dynamic factor demands. We have paid particular attention to the restrictions required to ensure our system has a steady state solution and those restrictions required for a classical dichotomy between real and nominal values. We assume firms face adjustment costs in altering factor inputs, which leads us to estimate the system as a set of dynamic factor demands with error correction in factor shares.

The empirical results reported in this paper are encouraging;

- we are able to estimate a full dynamic model for the aggregate UK supply side that is theoretically consistent and has desirable economic properties. In particular the model has an identified steady state.

- The model has sensible non unit elasticities of substitution, where the concavity requirements (for negative own price elasticities of demand) are met and is dynamically stable. The model therefore converges to a steady state growth path.

- We find no strong evidence for increasing returns to scale once variations in factor utilisations are accounted for. We do however find evidence that technical progress cannot be represented by a deterministic trend and that the UK appears to have gone through something of a productivity cycle, with low productivity in the 1970s and higher productivity in the 1980s.

- By reformulating the firm's objective function as we suggest here, the dynamic behaviour of the model is much improved. Not only is this a desirable property in its own right but we show that the correct formulation of the models dynamics enables us to test and impose the long run restrictions implied by economic theory, while avoiding the misspeciation that was typical of earlier models.

FOOTNOTES

1. See for example Nadir and Rosen (1973) who consider a seven input production structure for the United States

2. Dynamic demand systems were first introduced by Anderson and Blundell (1982, 1983, 1984) in the context of consumption modelling but have been widely used in the empirical analysis of production. See for example Holly and Smith (1989) who employ this technique to model factor demands in UK manufacturing.This approach, typically leads to models in dynamic factor shares.

3. As an exact functional form, the translog cannot adequately represent a separable technology as a flexible second-order approximation. The set of constraints required for weak separability impose strong restrictions on either the micro aggregation functions or the macro function (see Diewert, 1976 for a general discussion of aggregation, while Blackorby et.al discuss the restrictions). In order to avoid these restrictions, the weaker notion of a second-order approximation at a point has been adopted. It is not clear that this loss is trivial since the behaviour of the approximation away from the point of approximation will depend on the data set. Typically, this is not an issue when one is estimating point estimates of the elasticities of substitution but is more problematic when the translog is pressed into time series analysis.

4. Clearly, if this were different from the world rate of inflation our steady state we describe here would only be consistent with a depreciation in the nominal exchange rate.

APPENDIX 1 : EXPRESSIONS FOR SHORT AND LONG RUN ELASTICITY

The most commonly used measures of price responsiveness are the Allen-Uzawa partial elasticity of substitution (σ_{ij}) and the price elasticity of demand (η_{ij}). It can be shown that for the translog function, the long run elasticities are given by

$$\sigma_{ii}^{*} = \frac{\alpha_{ii} + S_i^{*2} - S_j^{*}}{S_i^{*2}} \qquad \textbf{(11)}$$

$$\sigma_{ij}^{*} = \frac{\alpha_{ii} + S_i^{*} S_j^{*}}{S_i^{*} S_j^{*}} \qquad \textbf{(12)}$$

$$\eta_{ii}^{*} = S_i^{*}\sigma_{ii}^{*} \qquad \eta_{ij}^{*} = S_j^{*}\sigma_{ij}^{*} \qquad \textbf{(13)}$$

In the dynamic case, the short run elasticities are then given by

$$\eta_{ii} = \frac{m\alpha_{ii}}{S_i} + S_i - 1 \qquad \textbf{(14)}$$

$$\eta_{ii} = \frac{m\alpha_{ij}}{S_i} + S_j \qquad \textbf{(15)}$$

given that

$$\frac{\delta S_i}{\delta p_j} = \frac{m\ \alpha_{ij}}{S_i} \qquad and \qquad \frac{\delta lnC}{\delta lnP_j} = S_j \qquad \textbf{(16)}$$

APPENDIX 2 : DATA

The most difficult series to gather data for is the aggregate capital stock. There is a significant amount of evidence which points to an increase in capital scrapping during the recession of 1980-1 (see especially Mayes and Young, 1994). This increase in scrapping rates is not captured in the national accounts which suggests that official figures for the capital stock, as published in the blue book, might be a significant overestimate (by as much as 25%). However, in 1982 the ONS reduced the estimates of the average life of capital equipment that it utilises in its calculation of the UK capital stock. The current figures while still probably an overestimate do capture some of the reduction in the level of the capital stock the occurred in the 1980s. So while official estimates may be subject to significant bias they are better than simply cumulating investment flows with a constant depreciation rate (as in Lynde and Richmond, 1993). But clearly, this is of central importance if we are looking for a correlation between unemployment and the capital stock.

For example, in figure A2 the suitably deflated private non housing capital stock is plotted against the predicted values from a simple perpetual inventory model (PIM) where

$$K_t = (1 - \delta) K_{t-1} + I_t \qquad \textbf{(17)}$$

Over the whole sample period δ is estimated to be 0.0163, a depreciation rate of a little over 6%. As can be clearly seen, for the period up until 1980 this is an over estimate, there is then a period of capital scraping, before finally the estimated relationship comes back on line. Critically, these two series would have exactly opposite implications for unemployment, with the actual series growing more slowly at the same time of unemployment rose.

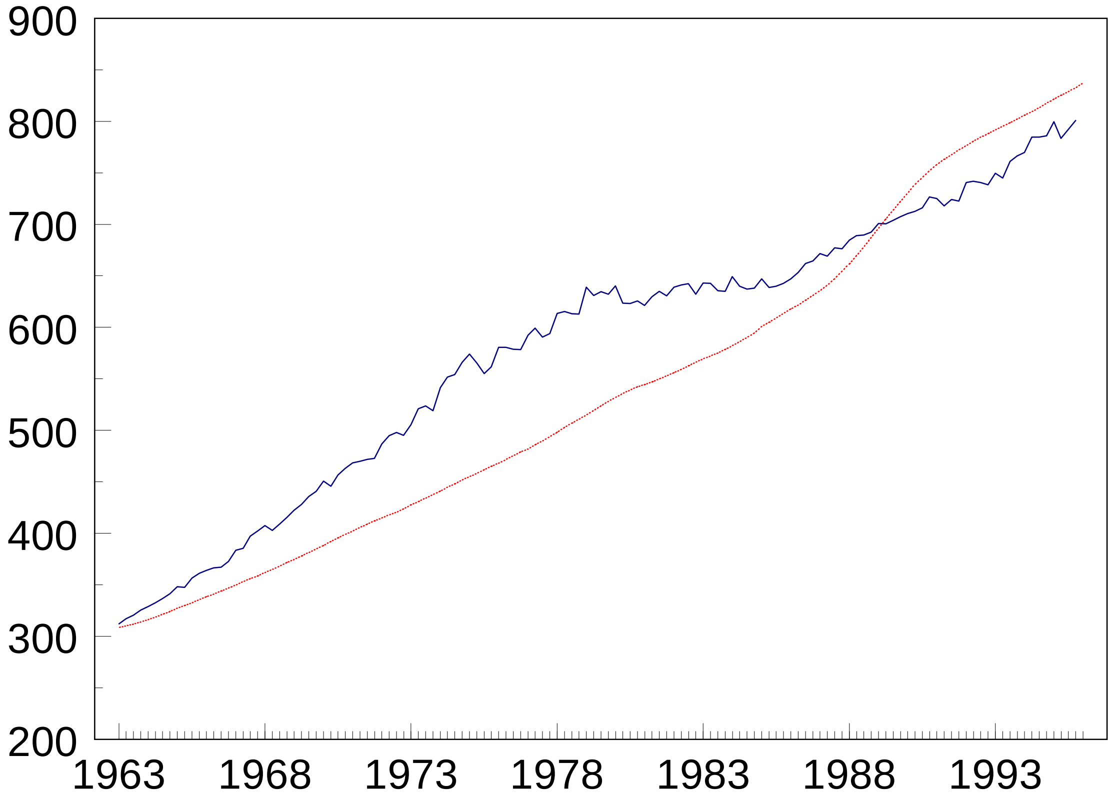

Figure A2: Actual capital stock and fitted cumulated investment.

It is also not clear whether the appropriate definition of capital should be gross or net of depreciation: in gross figures a unit of capital remains in the stock until it is retired. This seems to accord how we expect a particular unit of capital to be combined with labour: ie. a machine will continue to require n number of people until such time as it is actually retired. A net definition suggests that a production line for example would require a declining number of people as it gets older. However using a gross definition in a macro model would essentially lead one down the vintage production structure road, which is itself not with its problems (see Young, 1996). Rather a putty-putty depiction of the capital stock is suggestive of the idea that a micro level sufficient adjustments can be made to render the macro capital stock completely flexible. We therefore use the net capital stock figures reported in table 14.7 of the Blue Book (which also give the longest run of data back to 1947. These figures are at current replacement cost and therefore need to be deflated. The choice of appropriate deflator is itself not un-problematic but we use an implied deflator for the non housing investment.

There is also evidence that the public capital stock contributes to aggregate economic wide productivity (see Lynde and Richmond, 1993). Incorporating this into a macro model is likely to be problematic. Public investment cannot be said to arise from the firm's optimisation behaviour and is likely to possess a very different dynamic. Equally, lumping public investment in with private investment would require special attention when doing public expenditure simulations. We therefore take the extreme view that public investment is excluded from the economy production function - at least as a starting point. Since we also wish to exclude housing we therefore develop the investment and capital account on the basis of a non general government, non housing private sector. Public corporations are therefore included as part of the private sector.

2. Investment

Private non housing investment is private investment+investment by public corporations minus private investment in dwellings. The investment deflator is derived by dividing the nominal series by the real series (see table A8 in the UK Economic Accounts).

3. Other Factor Volumes

ET:	Total Employment
VFUEL:	Volume of Fuel Input (£million 1990 prices)
VNF:	Volume of Non-fuel input (£million 1990 prices)

4. Prices

AWB:	Price of Labour: ie. Average wage bill to the employer, defined as YEM/ET
PK	Price of Capital, defined as UCC/400*PIP
	where PIP is Private investment deflator such that PIP*KP=KP#
PFUEL	Price of Fuel Input (1990=100)
PNF	Price of Non Fuel Input (1990=100)

5. Values

YEM: Total Income from employment (£million)
ie. Wages and salaries + employer's contribution

VALK: Value of Capital Input (£ million), defined as (UCC/400)*KP#*1000
where UCC is User cost of capital and KP# is capital stock in current prices

VALFUEL: Value of fuel input (£million)

VALNF: Value of non fuel input (£million)

6. Total Costs

Total costs are given as the sum of the value of each of the inputs in £million

TCOST: YEM+VALK+VALFUEL+VALNF

This means that total costs are not given simply by $\Sigma P_i.X_i$ where P is price of input and X is volume of input. This is necessary because the price variables used as a mixture of indices and piece (wage) rates. Defining costs in the above manner means that factor shares are more representative of their true economic value. The is important because the estimated elasticities are a function of factor shares.

7. Output

VGO: Volume of Gross output (£million 1990 Prices)
Defined as VG0= C+I+G+S+X-MFMAN-MS-MRES-FCA-ADJ-RESO

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