INTEREST RATE LINKAGES: A KALMAN FILTER APPROACH TO DETECTING STRUCTURAL CHANGE

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Abstract

This paper investigates changes in the causal structure linking the G-7 short-term rates by estimating time-varying speed of adjustment coefficients in error correction equations using a Kalman filter approach. This technique allows us to detect structural breaks in the causal linkages that generate the cointegrating relations between the series. The testable hypotheses are the US world-wide leadership, the disengagement of UK monetary policy from those pursued in the Eurozone after the collapse of the ERM, and the German leadership hypothesis (GLH) within the European Union (EU). The evidence points to a break in the causal linkages between the UK and other EU countries after the third-fourth quarter of 1992. The empirical results are also consistent with a US world-wide leadership and a weak German leadership within the Eurozone.

Keywords: *Interest Rate Linkages, Long-Run Causality, Weak Exogeneity, Structural Change, Kalman Filter*

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1. Introduction

In economic theory convergence between short-term interest rates can be explained taking two different approaches. If interest rates are treated as analogous to other asset prices, then their movements are naturally interpreted as being determined by financial flows in profit-seeking capital markets. This will normally give rise to a set of arbitrage conditions such as uncovered interest rate parity (UIP). Alternatively, they can be viewed as policy instruments, with their time paths being determined by a policy objective such as an exchange rate or an inflation target. These two approaches are not necessarily inconsistent, since deviations from interest rate parity may cause the exchange rate to move towards its policy target. As long as its deviations from the target are stationary, so will be those from interest rate parity. Almost all empirical studies have found that the G-7 exchange rates are at most I(1) series. If one then makes the reasonable assumption that any risk premium, which may exist, in the relationship is stationary, the implication of these theories is that interest rates should be cointegrated on a bilateral basis.

In previous empirical papers, interest rate linkages have often been analysed in the context of specific policy frameworks such as the Exchange Rate Mechanism (ERM). For instance, numerous studies have attempted to test the so-called "German Leadership Hypothesis" (GLH), according to which Germany acts as the dominant player within Europe, and monetary authorities in other ERM countries are unable to deviate from the interest rates path set by the Bundesbank (see Fratianni and von Hagen, 1990, and Kirchgassner and Wolters, 1993). Taking this view, co-movements in interest rates arise because of policy convergence. But under pure arbitrage conditions one also expects interest rates to move together in the long run. So the question naturally arises: how is the system affected by a policy regime, and how will it change if there is a regime shift?

In cointegrated models, one can think of changes in the long-run structure as changes either in the long-run relationships themselves (the cointegrating vectors) or in causality links (the loading factors). Specifically, consider a Vector Error Correction Model (VECM) such as,

$$(1.1) \quad \Delta z_t = \Pi z_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta z_{t-i} + \varepsilon_t$$

where the long-run reduced rank matrix Π has been decomposed as $\Pi = \alpha\beta'$, with α being the matrix of loading weights and β the matrix of cointegrating coefficients. A change in the long-run structure of the system could occur through changes either in former or in the latter (see Hendry, 2000 among others).

A simple diagnostic test for structural change is suggested by Hao and Inder (1996), who extend the CUSUM test to the case of non-stationary regressors considering the FM-OLS residuals and replacing the error variance with the long-run variance estimate. Hansen (1992) derives the asymptotic distribution of a LM test for parameter instability against several alternatives in the context of cointegrated regression models. Quintos and Phillips (1993) develop a test for the null of parameter constancy in cointegrated regressions against the alternative that the coefficients follow a random walk. Seo (1998) defines LM tests statistics for structural changes in both the cointegrating vector and the vector of adjustment parameters for the cases of both a known and an unknown breakpoint. Hansen and Johansen (1999) suggest graphical procedures to evaluate the constancy of the long-run parameters of cointegrated systems. Barassi et al (2001b) propose an OLS-based sequential approach to test for a single permanent break in causality in structural cointegrated VARs and compare its performance to a time-varying parameter version of the Kalman filter. The two techniques are found to have good power in terms of detecting the breakpoints and the magnitude of the shift in the parameters of interest, with the Kalman filter displaying a slightly better performance.

Previous empirical studies on interest rates, such as Caporale et al (1996), reported convergence in European rates after 1986. Artis and Zhang (1998), using rolling window cointegration techniques[1], found that there is widespread cointegration between both US and German short rates and those on other ERM currencies up to 1995, after which the US influence on world-wide rates vanishes. In the context of the ERM, with its target zones, there might be regime shifts owing to the policies pursued by central banks. Specifically, the stochastic properties of interest rates (volatility, level and speed of adjustment) are likely to be different in periods when the currency has to be defended from speculative attacks, compared to periods when the exchange rate is credible. Because of the UIP relation, switches in the process governing exchange rates are translated into switches in the process followed by interest rates. Such regime shifts tend to be more frequent and not to be as long-lived as changes in monetary policy regimes in the US, say. Dahlquist and Gray (2000) show that a Markov-switching model characterises adequately the behaviour of a number of EMS short rates.

As already mentioned, in general, one can think of changes in structure as changes either in the long-run relationships themselves (the cointegrating vectors) or in causality links (the loading factors). It would be problematic to specify the source of structural change in a model allowing for both types of changes as such a model would typically not be identified. In the case of interest rates, as almost any theory suggests long-run co-movement, it is reasonable to assume the cointegrating vectors are constant but the direction of causality changes. Hence we concentrate on the latter source of change, and estimate time-varying parameter models for the loading weights. This has the advantage that one does not have to impose a priori restrictions on when the breaks in the relationships might have occurred. Instead, the relationships are allowed to evolve freely, and the revealed timing of the structural breaks can be very informative about the effects of policy changes (see, e.g., Haldane and Hall, 1991, who analyse sterling's relationship with the US dollar and the Deutschemark). Kalman filtering techniques were used by Hall et al (1992), who found convergence in inflation and interest rates within the EMS.

In this paper, we use a Kalman Filter approach to detect changes in the causal structure of cointegrated models. In particular we apply this procedure to bivariate systems linking the G-7 short interest rates as irreducible cointegrating relations (IC) (Davidson 1998a) in order to investigate the possibility of breaks in the causal structure of these linkages or reversals in the direction of causality. The empirical findings of this research will have important policy implications, as they will provide

[1] Note that the results from this estimation method are highly sensitive to the selection of the window width and the magnitude of the break (see Barassi et al. 2001b).

evidence on whether countries can still conduct an independent monetary policy despite the increasing integration of international financial markets (see Caporale and Williams, 1998, 2002). It appears that even in a system like the ERM which aims to produce policy coordination it has been possible for monetary authorities to disengage their policy from developments elsewhere and pursue an independent policy agenda over long periods. Such an option should remain available for non-participating countries, like the UK, after the establishment of the Euro. Therefore the UK authorities will not necessarily find their freedom of action greatly constrained by what is happening in the Euro zone. Within the Euro zone the policies of the European Central Bank (ECB) will not necessarily be as stable or credible as those adopted so far by the German authorities, since smaller countries will also have an influence on monetary policy (see Begg at al, 1998).

The paper is organised as follows: The next section (section 2) introduces the relevant concepts of long-run (weak) causality and exogeneity within the framework of cointegrated systems, and highlights the importance of testing for changes in causality. The econometric methodology is outlined in section 3, while section 4 discusses the empirical results and their economic implications. A summary concludes.

2. Weak exogeneity and long-run causality in cointegrated systems

Building an econometric model typically involves focusing on a set of (endogenous) variables of primary interest, which are explained in terms of other (exogenous) variables. The advantage of such an approach is that it is easier to model the endogenous variables conditional on the exogenous variables if these show some kind of irregular behaviour, which would be difficult to model within a VAR framework. It is very tempting to draw inference from the conditional or partial model whilst modelling the exogenous variables less carefully or not at all. The idea underlying such an approach is that if one could just draw inference about the cointegrating rank in the partial system, estimating β and testing for hypotheses on it, one would work with smaller systems in terms of the parameters to be estimated with a gain in efficiency.

The problem, however, is that such an approach is valid if and only if the assumption of weak exogeneity is satisfied (Engle, Hendry and Richard, 1983). Failure to satisfy such a requirement will make it problematic to derive the asymptotic distribution theory for the estimate of β. Harbo et al. (1998) show that even if weak exogeneity is assumed, the presence of deterministic terms in partial systems makes it difficult to determine the rank without modelling the full system, because the asymptotic distribution of the test statistic will be different from the one of the full model. As a consequence, one needs to work with full structural systems in error correction form, the partial systems being more the result of our inference than a starting point. The reason is that, rather than simply imposing restrictions, one would want to test for their validity. Valid estimation of a partial system requires not just exogeneity of some variables with respect to the parameters of interest, but also a precise long-run causal structure of the model. Within the framework of cointegrated systems these two issues coincide (as far as the long-run properties of the model are concerned). One can in fact show that long-run non-causality is necessary as well as sufficient for long-run weak exogeneity of a variable with respect to the parameters of interest.

2a Weak-Exogeneity

The basis for this discussion is provided by the analysis of joint and conditional densities and sequential factorisation (see Hendry, 1995). Let

2.1 $D_z(y_t, x_t | Z_{t-1}, \theta)$

be the sequential density at time t of the random vector $z_t = (y_t : x_t)'$ conditional on $Z_{t-1}=(Z_0, z_1,\ldots,z_{t-1})$, where $\theta = (\theta_1,\ldots,\theta_n)' \in \Theta$ which is a compact subset of $\Re^n$. Generally speaking, x_t is endogenous in the framework of the joint density function, but if x_t is weakly exogenous it is possible to factorise the joint density such that knowledge of how the process x_t is determined is not necessary in order to investigate the properties of the process y_t.

Let us allow for the existence of many one-to-one transformations from the original n parameters $\theta = (\theta_1,\ldots,\theta_n)' \in \Theta$ to any new set of parameters $\phi \in \Phi$, and also let $\phi = (\phi_1', \phi_2')$. We can then factorise the joint density function as:

2.2 $D_z(y_t, x_t | Z_{t-1}, \theta) = D_{y|x}(y_t | x_t, Z_{t-1}, \phi_1)\ D_x(x_t | Z_{t-1}, \phi_2)$.

Let the joint density under analysis involve a subset ψ of the parameters θ, where ψ is a vector of parameters of interest. The first requirement for a variable x_t to be regarded as weakly exogenous for a set of parameters of interest ψ is that the marginal process for x_t should add no useful information about ψ, that is one must be able to learn about ψ from ϕ_1 alone. The second condition one needs to justify taking x_t as given is that ϕ_1 should not depend on ϕ_2. If this were the case one could learn indirectly about ψ from ϕ_2.
One can then say that x_t is weakly-exogenous for ψ if and only if

- ψ is function of ϕ_1 and does not depend on ϕ_2;
- ϕ_1 and ϕ_2 are variation-free.

2b Long-run causality in cointegrated systems

In a famous paper, Granger (1969) shows that given two multivariate processes $\{x\}$ and $\{y\}$, and the information on them contained in their past behaviour X_t and Y_t, $\{y\}$ causes $\{x\}$ at time t if the past of $\{y\}$ provides additional information for the forecast of x_t with respect to considering the past of $\{x\}$ alone. From this definition one can see that there is a linkage between weak exogeneity and causality. Indeed, stating that a variable y has no role in the prediction of another variable x is tantamount to saying that the lagged values of y do not enter the equation for x, i.e. that there is no feedback from y to x.

This result is similar to the first condition for x to be weakly exogenous with respect to the parameters of interest, in that it seems that x in this case is determined outside the system by its own past. The problem is that this fulfils only the first requirement for weak exogeneity, and therefore implies that in standard regression analysis non-causality is necessary but not sufficient for weak exogeneity. Things are substantially different when working with non-stationary series and within a cointegration framework. Let us explain this point formally.

Consider a simple p-variate vector autoregression[2]:

$$2.3 \qquad \Pi(L)z_t = \begin{bmatrix} \Pi_{11}(L) & \Pi_{12}(L) \\ \Pi_{21}(L) & \Pi_{22}(L) \end{bmatrix} \begin{bmatrix} y_t \\ x_t \end{bmatrix} = \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{xt} \end{bmatrix} = \varepsilon_t$$

where $\Pi(0) = I_p$, $E(\varepsilon_t)=0$, $E(\varepsilon_t \ \varepsilon_s{}')= \lambda_{ts}\Omega$, and the maximum lag in $\Pi(L)$ is k. We assume that some of the roots of $|\Pi(L)|$ are equal to 1 while the others lie outside the unit circle in the complex plane. For simplicity we rule out the existence of seasonal unit roots and the presence of I(2) variables. Let also y_t and x_t be of dimension p_1 and p_2 respectively. Let $p= p_1+p_2$.

In order to check whether the variables in z_t are cointegrated and y_t does not cause x_t one has to test whether $\Pi(L)$ is upper block triangular and $\Pi=\Pi(1)$ is non-zero and has reduced rank. As a first step we reparameterise the model in an Error Correction Form as follows:

$$2.4 \qquad \Delta z_t = \Gamma_1 + \ldots + \Gamma_{k-1}\Delta z_{t-k+1} + \Pi z_{t-k} + \varepsilon_t$$

or more compactly as

$$2.5 \qquad \Delta Z = \Gamma \nabla Z + \Pi Z + \mathrm{E},$$

where $\Gamma= (\Gamma_1,\ldots,\Gamma_{k-1})$, $\Delta Z = \Delta Z_t$, $\nabla Z= (\Delta Z_{t-1}^{'},\ldots,\Delta Z_{t-k+1}^{'})'$, $Z=(z_1,\ldots,z_{t-k})$.

In this framework (following Mosconi and Giannini 1992), y does not Granger-cause x if the hypothesis

2.6 $\quad H_0$: $U'\Gamma V=0$, and $U'\Pi U\perp=0$.

holds, where:

$$2.7 \qquad U = \begin{bmatrix} 0 \\ Ip_2 \end{bmatrix} \quad U\perp = \begin{bmatrix} Ip_1 \\ 0 \end{bmatrix} \quad V = I_{k-1} \otimes U\perp,$$

and U is (pxp_2), $U\perp$ is (pxp_1), V is $(p(k-1)\text{x}\, p_1(k-1))$.

It is important to highlight that in cointegrated systems and VECMs, one can distinguish between two different types of causality, the first part of H_0 concerning short-run causality, while the hypothesis $U'\Pi U\perp=0$ is about long-run causality or weak causality as in Davidson and Hall (1991). Another way of formulating the

[2] We are omitting deterministic terms to keep the example as simple as possible, but these could be included without complications.

hypothesis $U'\Pi U\perp=0$, with reference to our initial system, is to test whether $\Pi_{21}=0$. This is equivalent to testing which rows of α are zero.

Testing for long-run non-causality also matters in the context of testing for weak exogeneity. We will see that, under the hypothesis of cointegration, long-run (weak) non-causality is necessary and sufficient for weak exogeneity. Let us rewrite our system in VECM form as

2.8 $$\Delta z_t = \Gamma_1 \Delta z_{t-1} + \ldots + \Gamma_{k-1} \Delta z_{t-k+1} + \Pi z_{t-k} + \varepsilon_t$$

where the parameters are defined as before. Assume for simplicity the absence of deterministic terms. The matrix $\Pi=\alpha\beta'$ contains information on the long-run relationships among the series in the model, with β containing the cointegrating relations and α representing the speed of adjustment to equilibrium. Also, we know that if there are $r \leq (p-1)$ cointegrating vectors in β, this implies that the last n-r columns of α are zero. To test how many $r \leq (p-1)$ cointegrating vectors exist in β is equivalent to testing how many columns of α are zero.

Focusing our attention on the non-zero columns of α, let the process z_t be decomposed into

2.9 $$z_t = (y_t', x_t')' \; and \; \alpha = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}, \Gamma_j = \begin{bmatrix} \Gamma_{1j} \\ \Gamma_{2j} \end{bmatrix}.$$

We can now rewrite the equations of the model as

2.10 $$\begin{aligned} \Delta y_t &= \alpha_{1t} \beta' z_{t-1} + \sum_{i=1}^{k-1} \Gamma_{1i} \Delta z_{t-i} + \varepsilon_{1t} \\ \Delta x_t &= \alpha_{2t} \beta' z_{t-1} + \sum_{i=1}^{k-1} \Gamma_{2i} \Delta z_{t-i} + \varepsilon_{2t} \end{aligned}$$

where ε_{ij} are iid N(0,Ω), and $\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{bmatrix}$.

Let us now consider the conditional model for Δy given the past and Δx, i.e.

2.11 $$\Delta y_t = \omega \Delta x_t + (\alpha_1 - \omega \alpha_2) \beta' z_{t-1} + \sum_{i=1}^{k-1} \widetilde{\Gamma}_{1i} \Delta z_{t-i} + \widetilde{\varepsilon}_{1t}$$

where $\omega = \Omega_{12}\Omega_{22}^{-1}, \widetilde{\Gamma}_{1i} = \Gamma_{1i} - \omega \Gamma_{2i}, and \; \widetilde{\varepsilon}_{1t} = \varepsilon_{1t} + \omega \varepsilon_{2t}$ with variance equal to $\Omega_{11.2} = \Omega_{11} - \Omega_{12}\Omega_{22}^{-1}\Omega_{21}$. We will now make the following statement:

The presence of all zeros in the i-th row of the matrix α_{ij}, $j = 1,\ldots,r$ indicates that the cointegrating vectors in β do not enter the equation determining Δz_{it}. This implies that there is no loss of information from not modelling the determinants of Δz_{it}, which can *therefore enter only the right-hand side of the system since there is no feedback from the other variables in the system.*

This implies long-run non-causality as well as weak exogeneity, as mentioned before. We can formalise it as follows, in the context of a system in error correction form:

$$\begin{aligned} \Delta y_t &= \omega \Delta x_t + (\alpha_1 + \omega\alpha_2)\beta' z_{t-1} + \sum_{i=1}^{k-1} \tilde{\Gamma}_{1i}\Delta z_{t-i} + \tilde{\varepsilon}_{1t} \\ \Delta x_t &= \alpha_{2t}\beta' z_{t-1} + \sum_{i=1}^{k-1} \Gamma_{2i}\Delta z_{t-i} + \varepsilon_{2t} \end{aligned} \quad 2.12$$

If α_2=0, then y_t is not causing x_t, and x_t is weakly exogenous for the parameters of interest (β, α_1). Therefore, the maximum likelihood estimator of β and α_1 can be inferred from the conditional model alone. This can be seen by rewriting the system under the hypothesis α_2=0, that is:

$$\begin{aligned} \Delta y_t &= \omega \Delta x_t + (\alpha_1 + \omega\alpha_2)\beta' z_{t-1} + \sum_{i=1}^{k-1} \tilde{\Gamma}_{1i}\Delta z_{t-i} + \tilde{\varepsilon}_{1t} \\ \Delta x_t &= \sum_{i=1}^{k-1} \Gamma_{2i}\Delta z_{t-i} + \varepsilon_{2t} \end{aligned} \quad 2.13$$

In fact, we can see that there is no trace of α_1 and β in the marginal model and therefore there is no trace of y_t, which is therefore not causing x_t, namely, there is no feedback from the former to the latter. Also, the condition that requires the parameters of the marginal to be unrelated to the parameters of the conditional model is fulfilled as a property of multivariate Gaussian distributions that do not have joint restrictions.[3]

3 The methodology

We first analyse the structural linkages between the G-7 short-term interest rates by means of the Extended Davidson Methodology (EDM) (see appendix A for a brief account) that relies on the concept of an irreducible cointegrating (IC) vectors (Davidson 1998a), i.e. cointegrating sets of variables that do not have any cointegrating subsets.

Davidson shows that an IC vector is unique (up to the choice of normalisation), and that if and only if a structural cointegrating relation is identified by the rank condition, it is irreducible (see Davidson, 1994). This means that, for the purpose of identifying the structure, cointegrating vectors with redundant variables are not useful. Not all the IC vectors, though, are structural. Some of them are *solved vectors*, namely linear combinations of structural vectors. Therefore one should first perform cointegration tests in order to eliminate all non-cointegrated sets and cointegrated supersets, and

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[3] Note that if α_2=0 then the sp((0,I)') is contained in sp($\alpha\perp$), which means that $\sum_{i=1}^{T} \varepsilon_{2t}$ is a common trend in the sense that the errors in the equations for x_t cumulate in the system giving rise to non-stationarity. x_t will still be cointegrated with y_t, of course, as implied by the first equation in 2.13. The key point to note here is that, as long as Π_{11} is of full rank, then $\sum_{i=1}^{T} \varepsilon_{1t}$ will not be a common stochastic trend of the system and hence there will be no long-run link from y_t to x_t. A related proof was presented in Hall and Wickens (1993).

then concentrate on the cointegrated sets, which yield IC relations. Davidson (1998a) develops such an elimination procedure based on a GAUSS algorithm (Minimal).[4] This algorithm basically analyses all possible cointegrated relations, testing exclusion restrictions by means of suitably constructed Wald tests, which can be shown to follow standard distributions (see Davidson, 1998b). When it terminates it will provide a set of cointegrating vectors that do not have any subsets, and are therefore irreducible. Furthermore, the irreducible cointegrating vectors can be ranked according to the value of the Wald statistic for the vector itself, so as to establish which IC relations are most supported by the data. Our extension to the Davidson's (1998a) method consists in performing a rank test in each step of the elimination process rather than testing for exclusion restrictions and the introduction of the ranking of the irreducible cointegrating vectors according to the criterion of lowest variance. Our idea is that the cointegrating relationships that display the lowest variability should be the structural ones, the ones with a high variance being just solved cointegrating relations. This point is illustrated in greater detail in appendix A.

The second part of our analysis consist in the detection of structural changes in the adjustment coefficients using a time-varying parameter version of the Kalman filter. The Kalman (1960, 1963) filter technique (for a brief account see appendix B) is adopted to estimate linear regression models with time-varying coefficients[5]. This class of models consists of two equations: the transition equation, describing the evolution of the state variables, and the measurement equation, describing how the observed data are generated from the state variables. This approach is extremely useful for investigating the issue of parameter constancy, because it is an updating method producing estimates for each time period based on the observations available up to the current period current period. It is important to realise that recursive OLS estimation (or moving window OLS estimation) is not a suitable technique to use here. Recursive estimation is essentially a test of structural stability. We can set up a null hypothesis that the parameters are constant and see if that can be rejected through recursive estimation. But as the underlying assumption of OLS is always that the parameters are constant, recursive estimation does not provide a consistent estimate of a time-varying parameter.

In our case we start from a model in error correction form such that the Kalman filter measurement equation will be:

3.1 $$\Delta z_t = \Gamma_1 \Delta z_{t-1} + \ldots + \Gamma_{k-1} \Delta z_{t-k+1} + \alpha\beta' z_{t-k} + \varepsilon_t ,$$

where ε_t are assumed to be *iid N (0,H),* and taking the Γ_i and β as non-time-varying we estimate the matrix of adjustment coefficients α_t so that the transition equation will be:

3.2 $$\alpha_t = T\alpha_{t-1} + \nu_t , \qquad \nu_t \text{ is } \mathrm{N}(0,Q)$$

with the initial conditions given by:

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[4] Note that only in the case of maximum over-identification, i.e. when there is no overlap of the cointegrated subsets, it is possible to identify the structure in its entirety.

[5] For an exhaustive exposition of Kalman filtering, see Harvey (1991) or Cuthbertson *et al* (1992)

3.3 $$\alpha_0 \sim N(\alpha_0, \sigma^2 P_0)$$

Here we assume that the coefficients in α follow a random walk process such that T=I. In particular we apply this procedure to the bivariate systems linking the G-7 short interest rates as irreducible relations in order to investigate the occurrence of breaks in the causal structure of these linkages.

4 Empirical results

For our analysis we used IMF quarterly data on the three-month Treasury bill rates covering the period between 1980:Q1 and 1998:Q3. We started by testing for cointegration on the bivariate systems involving in turn all the G7 short-term interest rates in order to rule out non-cointegrated series. Briefly, we found that cointegration holds between the G-7 interest rates on a pairwise basis (table 1), and with homogeneous coefficients implying that the G7 system has indeed a rank of six, and that there exist 21 IC vectors linking the 7 series. The immediate implication of this is the existence of six structural irreducible cointegrating regressions that we have isolated, ranking the IC relations according to the criterion of the minimum variance. The two most significant relations involve the US and Canada, and Italy and France. The other four relations involve Germany and Japan, and Japan with US, France and UK (table 2).

Next we estimated the error correction equations for each country and, bearing in mind the structural nature of some IC vectors, we inferred the causal structure that links the G-7 interest rates and tested for its stability. Specifically, six models were estimated for each country; in each equation we allowed for the possibility that a particular country adjusts to one of the other G-7. Notice that for efficiency reasons we have estimated single equations in error correction form rather than bivariate vector error correction models. This is legitimate as we found that there is bivariate cointegration between all the rates with unit cointegrating vectors, and once the cointegrating vectors are determined then single equation estimation becomes FIML.

The first observation to make is that most of the first differences of the G-7 interest rates seem to follow simple autoregressive processes of order one (AR(1)), apart from the US rates that require including some dummy variables to offset some heteroskedasticity in correspondence to the monetary base targeting pursued by the Fed during the early 1980s. The complete results for all the countries are presented on Tables 3-9, which report the coefficient estimates and both OLS and Newey-West corrected standard errors. Notice that for ease of interpretation the sign of the adjustment coefficients is always presented as negative (even in the cases in which this is positive due to the ordering of variables in the original bivariate VARs). At any rate, none of the series exhibit explosive behaviour.

The causal structure is obtained from the OLS estimation, and in particular from the third column of Tables 3-9 where the adjustment coefficients are reported. The first important result is the lack of feedback from all the other rates to the US one (Table 9). This is particularly evident in the error correction equations including the adjustment coefficients towards the structural long-run equilibrium relations with

Canadian and Japanese rates suggesting a US worldwide leadership. Interestingly Japan shares four out of the six IC relations with the US, France, Germany and the UK. It seems that Japanese rates (Table 7) represent the *trait d'union* between American and European rates being not-long-run caused by all other rates apart from the US and UK ones.

Another important result is the weak leadership of Germany within the European Monetary Union (EMU) highlighted by the fact that German rates do not adjust to disequilibrium in cointegrating relations with to Italian and French rates (Table 5). This is apparent, as German rates do not share any structural relations with the European ones and receive feedback from Japan (in a structural IC relation) and all the remaining rates included the UK. As for the UK rates, they seem to receive feedback from US and Canadian rates but not from the EMU countries. This may be due to the presence of a break in the causal relationships with the latter rates after the third/fourth quarter of 1992, and it represents one of the phenomena we want to test for in our exercise. Essentially, the US and Canada appear to constitute the fundamental block, UK rates respond to non-European rather than to other European rates, Italy is probably following France, and France and Germany respond to world rates rather than to each other. Lastly, Japan acts as the link between US and European rates.However, these results are all predicated on the assumption that the causal structure is constant over the sample period. A casual consideration of the structural changes, which have occurred in the monetary policy structure of the world over the last 20 years, suggests that this is an unlikely assumption. Therefore, we apply the Kalman filter to investigate the possible changes in long-run causality structure, (and in exogeneity) which may have occurred.

Below we discuss the time paths of the adjustment coefficients in the single-equation error-correction models corresponding to the irreducible cointegrating relations. We estimate the single equations by OLS, and having imposed the OLS coefficients as the fixed parameters of the observation equation we then re-estimate the same equations with the Kalman filter, assuming that the coefficient of the error correction term follows a random walk. Six models were estimated for each country where in each equation we allow for the possibility that that particular country adjusts to one of the other G7 countries. We are therefore allowing for the possibility that each country is being influenced by any of the other six at any point in time. So if we found that for country A all six adjustment parameters were zero for the whole period, it would tell us that this country was not influenced by any of the other countries over this period. If we found that the adjustment coefficient involving country B became different from zero half way through the period, then this would indicate a shift in policy regime such that country A started to follow country B from that period onwards.

4a US

The empirical results seem to support the existence of US leadership. The speed of adjustment coefficients converge towards zero in almost all cases. Exogeneity of US interest rates is clearly observable in the bivariate systems that link its rate to the Italian and the German ones (figure 7). The linkage with the Canadian rate seems to be the most significant one, although even this coefficient is below 0.1 and is falling. The same (but with even smaller coefficients) can be said about the speed of adjustment to disequilibrium in the linkage with the French and Japanese rates. The situation is slightly different in the error correction equation containing the cointegrating relation that links US to UK rates. It seems that there was some (decreasing) feedback from UK to US rates until the collapse of the ERM in the last quarter of 1992. Afterwards, US rates appear to be exogenous and causality runs from US to UK rates only. Overall we can conclude that the evidence from the time-varying estimation supports the idea that the US economy is a constant point of reference for all the countries of the G-7 group.

4b UK

The results for the UK are even more interesting. The main result is that the linkage between UK and other European rates has become weaker after the collapse of the ERM (figure 6). In particular, apart from the clear exogeneity of UK in the system that links it to the Italian one, it can easily be seen that in the 1990s the influence of the German and French rates on the British one has decreased overtime, even reaching zero in the case of the linkage with the French rate. It seems that the UK rate still responds to the German one (very weakly), but it is also clear that the influence on the UK of conditions in the non-European G-7 countries is still strong and even growing in the case of Canada. It is worth considering what happens in the causal relation with the US rate. Here we can observe a break in the causal linkage between 1989 and 1991. Notice that after this period the US influence on UK rates seems to grow again. Overall we can then say that UK rates seem to respond to non-European rates, and that, following the breakdown of the ERM, UK monetary authorities have pursued policies that are completely independent from those implemented in the economies of the Eurozone. This might help explain the differences in economic performance between UK and other European economies during the 1990s.

4c The Eurozone

As already mentioned, one of the hypotheses of interest is the purported German leadership (GLH) within the Eurozone. In order to test it, we should only consider the three countries in our sample belonging to EMU, therefore investigating the speed of adjustment of German rates (figure 3) towards the long-run equilibria shared with the French and Italian rates. On the basis of the results obtained from this “partial” analysis, we might be tempted to conclude that the empirical evidence supports the GLH. This is because German rates do not look to be long-run caused by the French and the Italian ones. However, a closer look to the complete G-7 system shows that such a hypothesis does not have empirical support, for three main reasons.

First, while the only causal link to Germany seems to be from the US there are also strong direct links from the US to France and Italy (in both cases larger in magnitude than the link to Germany). So while the three rates may be moving in line there does not seem to be a strong case for arguing for one of them as the leader. Second, one of the six structural IC vectors links Italy and France (table 2), which implies that the relations between these two latter countries and Germany belong to the class of irreducible solved relations rather than to the structural ones. Third, we argue (table 2) that Germany does not share any of the six structural irreducible relations with another European country.

As for French and Italian rates (figures 2 and 4 respectively), there appears to receive feedback from all the world rates and from each other. This provides us with enough evidence to conclude that European rates are actually driven more by US rates, which give feedback to all the Eurozone rates. It is worth noticing that the time path of the speed of adjustment coefficients of the error correction equations of European rates displays a kink in the fourth quarter of 1992, which coincides with the collapse of the ERM. Also, subsequently it becomes a lot smoother, implying more stability within the Eurozone.

4d Canada and Japan

We have chosen to discuss the results on Canadian and Japanese rates together as they seem to act as the *trait d'union* between US and European rates within the G-7. The main feature of the Japanese rate is its high non-variability. Specifically, it is characterised by step-changes, clearly indicating that Japanese rates are determined by policy decisions rather than market conditions. This may help to explain the low variability of the time-varying parameter estimates of the adjustment coefficients in its error correction equations as well as their low (but non-zero) values. We find some weak feedback from more or less all the other rates (with the exception of the German one) to Japanese rates. [6] As already stated, it appears that Japanese rates, together with the Canadian ones, act as a linkage between European and US rates. In fact the feedback from both Japanese and Canadian rates to US rates is substantially weaker than the one in the opposite direction. As for the Canadian rate, it is linked to the other rates by a two-ways feedback relation, apart from its clear exogeneity in the system with the French rate in another of the six irreducible structural cointegrating relations.

5. Conclusions

In this paper, we have investigated changes in the causal structure linking the G-7 short-term rates by estimating time-varying parameter models using a Kalman filter approach. In particular, we have applied the technique to bivariate error correction systems linking the G-7 short-term rates as irreducible relations in the sense of Davidson (1998a). The analysis was aimed at examining the possibility of structural breaks in the causal linkages between rates, which in some cases might make it possible for monetary authorities to disengage their policy from developments elsewhere. Other hypotheses of interest concerned the US world-wide leadership, the degree of autonomy of monetary policy in the UK policy after the collapse of the ERM in September 1992, and the GLH in the Eurozone.

[6] Notice that the relation between these rates constitutes one of the six irreducible structural linkages between the G-7 rates.

We have found some evidence of breaks in the causal linkages between the rates under investigation. One of the most interesting results concerns the progressive disengagement of UK policies from developments elsewhere in the EU, especially after the collapse of the ERM. In the following period, UK rates seem to be linked much more to world rates, as shown by the higher speed of adjustment parameter after 1992 in the equations linking UK and world rates. As for the other results, the evidence seems to support the leadership of the US, the corresponding speed of adjustment coefficients being very close to zero. Furthermore, we have found some evidence of a German leadership in the Eurozone, since the German rate has been found exogenous in the systems with the French and Italian ones. Nevertheless, as German rates are in turn driven by other world rates (mainly US, UK and Japanese rates - recall that German and Japanese rates are linked by an irreducible structural relation), the German leadership is not substantial. Finally, Japan and Canada act as a linkage between US and European rates.

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Appendix A
The Davidson (IC) Methodology
and the Extended Davidson Methodology (EDM)

To outline Davidson's (1998a) identification methodology, consider the following cointegrated VAR as analysed by Johansen (1988, 1991):

(A.1) $A(L)X_t = \alpha\beta'X_t + A^*(L)\Delta X_t = \varepsilon_t \ (px1),$

where $X_t \sim I(1)$, L is the lag operator, $A(L)= \alpha\beta' + A^*(L)(1–L)$ such that $A(1) = \alpha\beta'$, and α and β are pxk matrices, the loading weights matrix and the matrix of cointegrating vectors respectively[7]. When $k<p$ the system incorporates a set of long run relationships of the form $\beta'x_t = s_t$, where $s_t = (\alpha'\alpha)^{-1}\alpha'(\varepsilon_t - A^*(L)\Delta x_t) \sim I(0)$. In this model there are k linearly independent cointegrating vectors, the columns of β.[8] The identification problem is usually (Johansen 1991, Pesaran and Shin 1994) tackled by estimating a collection of linearly or non-linearly restricted vectors spanning the same space as β that are identified by the rank condition. Davidson (1998a) proposes to follow a method that allows the researcher to use the data to identify the structural relations in the case of over-identified systems. We recall its main points.

Theorem 1 (Davidson, 1994). If a column of β (say β_l) is identified by the rank condition, the OLS regression which includes just the variables having unrestricted non-zero coefficients in β_l is consistent for β_l.

In a non-stationary world if another variable is added to a cointegrating regression, its coefficient might not necessarily converge to zero as we would expect in the case of an irrelevant variable within a regression involving stationary variables. In the case of cointegration the regression coefficients would generally converge to some other element of the cointegrating space. The main result of this is that, if a collection of I(1) variables is found to be cointegrated, it does not necessary follow that the estimated vectors can be interpreted as structural. In this framework it is useful to recall the definition of irreducible cointegrating vector introduced by Davidson (1998), that is,

Definition 1. A set of I(1) variables will be called irreducibly cointegrated (IC) if they are cointegrated, but dropping any of the variables leaves a set that is not cointegrated.

IC relations have the following important properties summarised in the following theorems.

Theorem 2. An IC vector is unique, up to the choice of normalisation.

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[7] We have assumed for simplicity the absence of any deterministic terms in this representation of the system under analysis. The modifications necessary to relax these assumptions are straightforward and would not alter the substance of the results obtained using a simpler model.

[8] Recall that without restrictions on β we can always scale the matrix of the cointegrating relations by post-multiplying it by any non-singular kxk matrix C, to get $C\beta'x_t = Cs_t$ that is observationally equivalent to $\beta'x_t = s_t$ with loading matrix αC^{-1}.

Theorem 3 (Davidson, 1994). If and only if a structural cointegrating relation is identified by the rank condition, it is irreducible.

This tells us that at least some IC vectors are structural. When the cointegrating rank of the system is k, an IC relation can contain at most p-k+1 variables. There are between k and (p-k+1) of these vectors in total, the actual number depending on the degrees of over-identification of the relations of the system. This is to say that in addition to up to k identified structural relations, which, by theorem 3, are among the IC vectors, there might also be a number of solved vectors that are defined as follows:

Definition 2. A solved vector is a linear combination of structural vectors from which one or more common variables are eliminated by choice of offsetting weights such that the included variables are not a superset of any of the component relations.

The original Davidson's (1998a) methodology tries to avoid just identifying restrictions from economic theory by searching for subsets of cointegrated variables (due to zero restrictions) which cannot be further reduced without losing the cointegration property. This irreducible set is found in a bottom-up approach by adding (and dropping) variables until cointegration is satisfied, after a certain rank has been chosen using a Johansen (1988) rank test. In detail, one is testing exclusion restrictions by means of suitably constructed Wald tests, which follow standard distributions (see Davidson, 1998b). Furthermore, one can rank the cointegrating vectors according to the value of the Wald statistic for the vector itself, so as to establish which IC relations are most supported by the data and are therefore structural.

In a previous paper (Barassi et al. 2001a) we have modified Davidson's original methodology in two ways. The first difference consists in that rather than of testing for exclusion restrictions, we perform a cointegration test in each step of the bottom up strategy without necessarily starting from the cointegrating rank indicated by the Johansen (1988) test on the complete set of variables. The advantage of this is that it allows to extend the IC methodology to exactly identified systems. These might have been suggested by economic theory, and therefore our technique allows us to test for the validity of the theory itself. The second difference consists in the fact that we use the criterion of the minimum variance of the disequilibrium errors rather than the value of the Wald test statistic to rank the IC vectors and distinguish between structural and solved vectors.

To introduce our extension of Davidson's procedure, we will analyse the case of bivariate cointegration as this gives rise to the largest number of IC vectors and solved cointegrating relationships for any number of variables. In addition, this is also the most relevant case for our application to the G7 interest rates. Consider a p-dimensional cointegrating system as (A.1). In general, in the case of bivariate cointegration between each pair of variables in a set of p variables there will be k structural IC vectors where k is p-1, and there will exist r irreducible vectors, which are simply combinations of the k structural ones where r is $((k-1)^2+(k-1))/2$. Now, if we designate the first k cointegrating residuals as the structural ones, so that for $u_1 \dots u_k \sim NI(0, \sigma^2{}_1 \dots \sigma^2{}_k)$, then clearly the solved cointegrating residuals will be combinations of these[9]. Because for any IC vector the variance may vary with the normalisation, we need two starting assumptions for our ranking criterion to be operative.

This first assumption involves the possibility of normalising the IC vectors using the normalisation, which yields a minimum variance. What we mean is that, in formal terms, given two cointegrated series x_{1t} and x_{2t}, we choose to normalise their cointegrating relation as

(A.2) $x_{1t} = \delta x_{2t} + u_{1t}, \qquad u_{1t} \sim (\mu_1, \sigma_1^2)$

rather than as

(A.3) $x_{2t} = \rho x_{1t} + u_{2t} \qquad u_{2t} \sim (\mu_2, \sigma_2^2)$,

if $\sigma_1^2 \leq \sigma_2^2$.

A second assumption is that, the residuals from the structural cointegrating relations are normally independent distributed with full rank diagonal covariance matrix Ω[10] (Sims 1980 formulates a similar assumption on error terms of the unobservable structural VAR[11], see also Hendry 1995 pp.784 and 807). Given these two starting assumptions, we formulate the following statement.

The minimum variance normalised structural residuals will have a strictly lower variance than any solved residual coming from an IC vector containing the same variable. This proposition provides us with the immediate rule for distinguish structural from solved irreducible vectors. We prove it as follows.

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[9] However, we will see that the set of r solved residuals need not all to display a variance that strictly greater than all of the k structural residuals.

[10] However, given that the solved vectors are linear combinations of the structural ones it is likely that their cointegrating errors will be correlated with one or both the residuals of the solving relations.

[11] Sims' 1980 argument can be summarised as: The structural model is not directly observable, however a VAR can be estimated as the reduced form of the underlying structural model

$$\begin{pmatrix} X_{1t} \\ X_{2t} \end{pmatrix} = A^{-1}C(L)\begin{pmatrix} X_{1t-1} \\ X_{2t-1} \end{pmatrix} + \begin{pmatrix} e_t^{X_1} \\ e_t^{X_2} \end{pmatrix},$$

where **e** denotes the VAR residual vector, normally independent distributed with full variance-covariance matrix Ω. The relation between the residuals in **e** and the structural disturbances in **u** is,

$A\begin{pmatrix} e_t^{X_1} \\ e_t^{X_2} \end{pmatrix} = B\begin{pmatrix} u_t^{X_1} \\ u_t^{X_2} \end{pmatrix}$, so that $e_t = A^{-1}Bu_t$, and $E(e_t e_t^{'}) = A^{-1}BE(u_t u_t^{'})B' A^{-1}$. Thus, Ω contains $(p^2+p)/2$ different elements, that indicate the maximum number of identifiable parameters in thematrices A and B. In his seminal paper, Sims proposed the following identification strategy based on the Choleski decomposition of matrices:

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ a_{21} & 1 & 0 & 0 \\ .... & .... & 1 & 0 \\ a_{n1} & .... & a_{nn-1} & 1 \end{bmatrix}, B = \begin{bmatrix} b_{11} & 0 & 0 & 0 \\ 0 & b_{22} & 0 & 0 \\ 0 & 0 & b_{33} & 0 \\ 0 & 0 & 0 & b_{nn} \end{bmatrix}.$$

Let

(A.4) $\beta' X_t = u_t$, β (*pxk*) and X_t (*px1*)

be the structural cointegrating relations with β normalised such that $u_t \sim iN(0,\Omega)$. Here we assume that β has been normalised on the j[th] element of its columns such that the cointegrating relations have variances $\sigma^2_{i.j} < \sigma^2_{i.k}$, where σ^2_i are the elements on the main diagonal of *Ω*, and $j \neq k$ indicate different normalisations. Any solved vector will be given by a combination of the structural $\beta' X_t = u_t$ of the form

(A.5) $A\beta' X_t = Au_t$,

where A is (1xr) normalised on the corresponding element j. It follows that $Au_t \sim N(0, A\Omega A')$ and more explicitly $A\Omega A' = \sum_i a_i^2 \sigma_i^2$. Now recall that as at least one of the terms of this summation is given by $a_j^2 \sigma_j^2$, and $a_j = 1$, we can state that

(A.6) $A\Omega A' = \sum_i a_i^2 \sigma_i^2 > \sigma^2_{i.j}$. QED.

Appendix B.
The Kalman Filter

Let the Kalman Filter measurement equation be:

B.1 $y_t = x_t^{'}\beta_t + \varepsilon_t \qquad \varepsilon_t \sim N(0, H_t)$

and the transition equation be:

B.2 $\beta_t = T\beta_{t-1} + \eta_t \qquad \eta \sim N(0, Q_t)$

with the initial conditions given by:

B.3 $\beta_0 \sim N(\beta_0, \sigma^2 P_0)$

When T=I and Q_t=0, the model is reduced to the standard normal OLS regression model. The matrices T, H and Q are assumed to be known, and the problem is obtaining estimates of β_t using information B_t available up to time t. The process of evaluating the conditional expectation of β_t given B_t is known as filtering. The evaluation of β_t given B_s, with s>t, is instead referred to as smoothing, whereas the estimation of β_t with s<t is called prediction. Kalman (1960) derived the basic results to obtain filtered and smoothed estimates of β_t recursively. The prediction equation is given by:

B.4 $\hat{\beta}_{t/t-1} = T\hat{\beta}_{t-1}$

and the covariance matrix is defined as:

B.5 $P_{t/t-1} = TP_{t-1}T' + Q_t$

Finally, the updating formulae are given by:

B.6 $\hat{\beta}_t = \hat{\beta}_{t/t-1} + P_{t/t-1}x(y_t - x'\hat{\beta}_{t/t-1})(x'P_{t/t-1}x + H_t)$

and

B.7 $P_t = P_{t/t-1} - P_{t/t-1}x'xP_{t/t-1}^{'} / (x'P_{t/t-1}x + H_t)$

As the estimates are updated recursively each period, Kalman filtering can be viewed as belonging to the class of Bayesian estimators. Before starting the estimation process, one has to specify the starting values of the vector of prior coefficients β_t and the matrix Q_t. By estimating the long-run relationship in this way one obtains a vector containing the evolving state coefficients which show whether the relative importance of the factors driving the dependent variable has changed over time.

Table 1 Pairwise cointegration tests for the G7 short-term interest rates						
	$-T\log(1-\hat{\lambda}_p)$	-(T-nm)°	Upper 5% cv	$-T\sum_{i=p+1}^{n}\log(1-\hat{\lambda}_i)$	-(T-nm)°	Upper 5% cv
US-Canada	14.23*	9.678	14.1	16.58*	11.27	15.4
US-Japan	22.46**	14.57*	14.1	23.68**	15.36	15.4
US-UK	14.71*	9.936	14.1	17.31*	11.7	15.4
US-France	14.48*	11.92	14.1	15.48*	12.74	15.4
US-Italy	16*	13.16	14.1	19*	15.63*	15.4
US-Germany	24.45**	20.17**	14.1	24.45**	20.17**	15.4
Canada-France	15.52*	11.8	14.1	17.94*	13.64	15.4
Canada-UK	14.72*	12.15	14.1	17.39*	14.35	15.4
Canada-Italy	21.09**	18.52**	14.1	21.34**	18.74*	15.4
Canada-Japan	24.71**	20.39**	14.1	24.76**	20.43**	15.4
Canada-Germany	17.47*	13.93	14.1	17.47*	13.93	15.4
UK-France	15.95*	12.55	14.1	19.46*	15.31	15.4
UK-Germany	30.99**	23.74**	14.1	31.24**	23.93**	15.4
UK-Italy	17.8*	14.48	14.1	19.71*	16.03	15.4
UK-Japan	19.9**	15.31*	14.1	24.62**	18.94*	15.4
Germany-France	16.31*	14.32*	14.1	18.43*	16.18*	15.4
Germany-Italy	22.21**	18.7*	14.1	25.75**	21.68*	15.4
Germany-Japan	14.8*	13	14.1	17.03*	14.95	15.4
Italy-Japan	27.96**	25.96**	14.1	29.79**	27.67**	15.4
Italy-France	24.09**	21.15*	14.1	30.62**	26.89**	15.4
Japan-France	27.46**	21.9**	14.1	28.91**	23.05**	15.4

* indicates rejection of the null of no cointegration at 5% significance level

** indicates rejection of the null of no cointegration at 1% significance level

° -(T-nm) is a small sample correction replacing (-T) in the λ-max and λ-trace statistics

Table 2 Cross tabulation of standard deviations for the G7 short-term interest rates.							
	US	Canada	Japan	Germany	France	Italy	UK
US	-	**1.78**	**2.1**	2.5	2.4	2.6	2.3
Canada	1.78	-	2.5	2.7	2.1	2.5	2.1
Japan	2.1	2.5	-	**1.57**	**1.96**	2.5	**1.88**
Germany	2.5	2.7	1.57	-	2.2	2.8	2.4
France	2.4	2.1	1.96	2.2	-	**1.17**	2.6
Italy	2.6	2.5	2.5	2.8	1.17	-	3.2
UK	2.3	2.1	1.88	2.4	2.6	3.2	-
Structural IC relations are in bold.							

Table 3. Canada: OLS-based estimates for the ECMs		
Dependent Variable: DCanada(t)		Error correction term
Constant	DCanada(t-1)	Canada-France(t-1)
-0.105 (0.148) [0.151]	0.212 (0.127) [0.108]	-0.057 (0.077) [0.059]
Constant	DCanada(t-1)	Canada-Germany(t-1)
0.094 (0.232) [0.169]	0.220 (0.118) [0.097]	-0.046 (0.053) [0.046]
Constant	DCanada(t-1)	Canada-Italy(t-1)
-0.212 (0.257) [0.218]	0.247 (0.122) [0.104]	-0.051 (0.062) [0.049]
Constant	DCanada(t-1)	Canada-Japan(t-1)
0.662 (0.357) [0.286]	0.276 (0.114) [0.106]	-0.130 (0.059) [0.051]
Constant	DCanada(t-1)	Canada-UK(t-1)
-0.164 (0.142) [0.152]	0.277 (0.112) [0.970]	-0.172 (0.068) [0.062]
Constant	DCanada(t-1)	US-Canada(t-1)
0.362 (0.212) [0.249]	0.174 (0.110) [0.097]	-0.230 (0.078) [0.098]
OLS Standard errors in parenthesis		
Newey-West standard errors in brackets		

Table 4. France: OLS-based estimates for the ECMs		
Dependent Variable: DFrance(t)		Error correction term
Constant	DFrance(t-1)	Canada-France(t-1)
-0.114 (0.097) [0.102]	0.132 (0.110) [0.119]	-0.165 (0.047) [0.078]
Constant	DFrance(t-1)	Germany-France(t-1)
0.282 (0.195) [0.166]	0.292 (0.108) [0.110]	-0.093 (0.049) [0.032]
Constant	DFrance(t-1)	France-Italy(t-1)
-0.906 (0.323) [0.481]	0.263 (0.112) [0.104]	-0.232 (0.089) [0.141]
Constant	DFrance(t-1)	France-Japan(t-1)
0.564 (0.311) [0.243]	0.236 (0.112) [0.110]	-0.119 (0.052) [0.042]
Constant	DFrance(t-1)	France-UK(t-1)
-0.150 (0.103) [0.111]	0.174 (0.112) [0.128]	-0.087 (0.037) [0.036]
Constant	DFrance(t-1)	US-France(t-1)
0.260 (0.130) [0.174]	0.228 (0.103) [0.105]	-0.146 (0.041) [0.062]
OLS Standard errors in parenthesis		
Newey-West standard errors in brackets		

Table 5. Germany: OLS-based estimates for the ECMs		
Dependent Variable: DGermany(t)		Error correction term
constant	DGermany(t-1)	Canada-Germany(t-1)
-0.140 (0.101 [0.068])	0.393 (0.106) [0.088]	-0.049 (0.023) [0.025]
constant	DGermany(t-1)	Germany-France(t-1)
-0.001 (0.118) [0.071]	0.448 (0.103) [0.103]	-0.001 (0.029) [0.023]
constant	DGermany(t-1)	Germany-Italy(t-1)
-0.028 (0.178) [0.112]	0.406 (0.107) [0.102]	-0.001 (0.024) [0.021]
constant	DGermany(t-1)	Germany-Japan(t-1)
0.204 (0.095) [0.102]	0.449 (0.096) [0.095]	-0.100 (0.036) [0.037]
constant	DGermany(t-1)	Germany-UK(t-1)
-0.247 (0.116) [0.102]	0.340 (0.106) [0.090]	-0.052 (0.025) [0.022]
constant	DGermany(t-1)	US-Germany(t-1)
-0.095 (0.069) [0.065]	0.378 (0.101) [0.094]	-0.063 (0.023) [0.025]
OLS standard errors in parenthesis		
Newey-West standard errors in brackets		

Table 6. Italy: OLS-based estimates for the ECMs		
Dependent Variable: DItaly(t)		Error correction term
Constant	DItaly(t-1)	Canada-Italy(t-1)
0.374 (0.153) [0.151]	0.277 (0.100) [0.076]	-0.127 (0.036) [0.040]
Constant	DItaly(t-1)	France-Italy(t-1)
0.723 (0.302) [0.289]	0.376 (0.103) [0.083]	-0.233 (0.081) [0.091]
Constant	DItaly(t-1)	Germany-Italy(t-1)
0.312 (0.264) [0.220]	0.344 (0.105) [0.084]	-0.059 (0.035) [0.028]
Constant	DItaly(t-1)	Italy-Japan(t-1)
0.749 (0.352) [0.346]	0.369 (0.101) [0.080]	-0.090 (0.037) [0.039]
Constant	DItaly(t-1)	Italy-UK(t-1)
0.063 (0.135) [0.113]	0.293 (0.108) [0.090]	-0.057 (0.030) [0.031]
Constant	DItaly(t-1)	US-Italy(t-1)
0.478 (0.217) [0.277]	0.324 (0.103) [0.071]	-0.099 (0.035) [0.046]
OLS standard errors in parenthesis		
Newey-West standard errors in brackets		

Table 7. Japan: OLS-based estimates for the ECMs		
Dependent Variable: DJapan(t)		Error correction term
Constant	DJapan(t-1)	Canada-Japan(t-1)
-0.295 (0.145) [0.158]	0.381 (0.183) [0.110]	-0.049 (0.027) [0.031]
Constant	DJapan(t-1)	France-Japan(t-1)
-0.229 (0.167) [0.157]	0.360 (0.106) [0.091]	-0.031 (0.028) [0.028]
Constant	DJapan(t-1)	Germany-Japan(t-1)
-0.058 (0.088) [0.104]	0.423 (0.105) [0.105]	-0.018 (0.034) [0.036]
Constant	DJapan(t-1)	Italy-Japan(t-1)
-0.088 (0.213) [0.144]	0.406 (0.105) [0.110]	-0.006 (0.023) [0.013]
Constant	DJapan(t-1)	Japan-UK(t-1)
-0.567 (0.190) [0.186]	0.324 (0.102) [0.102]	-0.087 (0.029) [0.031]
Constant	DJapan(t-1)	US-Japan(t-1)
-0.022 (0.081) [0.080]	0.321 (0.109) [0.122]	-0.043 (0.020) [0.021]
OLS standard errors in parenthesis		
Newey-West standard errors in brackets		

Table 8. UK: OLS-based estimates for the ECMs		
Dependent Variable: DUK(t)		Error correction term
Constant	DUK(t-1)	Canada-UK(t-1)
0.032 (0.110) [0.121]	0.239 (0.106) [0.114]	-0.117 (0.051) [0.051]
Constant	DUK(t-1)	UK-France(t-1)
-0.086 (0.116) [0.118]	0.177 (0.108) [0.113]	-0.032 (0.049) [0.047]
Constant	DUK(t-1)	Germany-UK(t-1)
0.071 (0.209) [0.204]	0.184 (0.119) [0.114]	-0.045 (0.045) [0.047]
Constant	DUK(t-1)	Italy-UK(t-1)
-0.118 (0.155) [0.147]	0.156 (0.120) [0.110]	-0.004 (0.036) [0.038]
Constant	DUK(t-1)	UK-Japan(t-1)
0.447 (0.414) [0.354]	0.262 (0.122) [0.118]	-0.080 (0.065) [0.064]
Constant	DUK(t-1)	US-UK(t-1)
0.245 (0.160) [0.145]	0.169 (0.111) [0.117]	-0.132 (0.045) [0.041]
OLS Standard errors in parenthesis		
Newey-West standard errors in brackets		

Table 9. US: OLS-based estimates for the ECMs		
Dependent Variable: DUS(t)*		Error correction term
Constant	DUS(t-1)	US-Canada(t-1)
0.040 (0.098) [0.071]	0.167 (0.072) [0.086]	-0.048 (0.038) [0.028]
Constant	DUS(t-1)	US-France(t-1)
-0.019 (0.108) [0.078]	0.251 (0.091) [0.113]	-0.019 (0.036) [0.023]
Constant	DUS(t-1)	US-Germany(t-1)
-0.054 (0.092) [0.078]	0.273 (0.087) [0.098]	-0.002 (0.032) [0.031]
Constant	DUS(t-1)	US-Italy(t-1)
0.056 (0.201) [0.175]	0.248 (0.090) [0.101]	-0.020 (0.033) [0.026]
Constant	DUS(t-1)	US-Japan(t-1)
0.261 (0.195) [0.251]	0.227 (0.109) [0.099]	-0.090 (0.048) [0.058]
Constant	DUS(t-1)	US-UK(t-1)
-0.162 (0.153) [0.162]	0.203 (0.112) [0.084]	-0.043 (0.045) [0.050]
(*) dummies for 1980:3, 1981:1, 1982:1, 1982:4 are included to account for the period corresponding to the Monetary Base Targeting pursued by the Fed during the early 1980s		
OLS Standard errors in parenthesis		
Newey-West standard errors in brackets		

Figure 1. Time path of adjustment coefficients in error correction equations for Canada

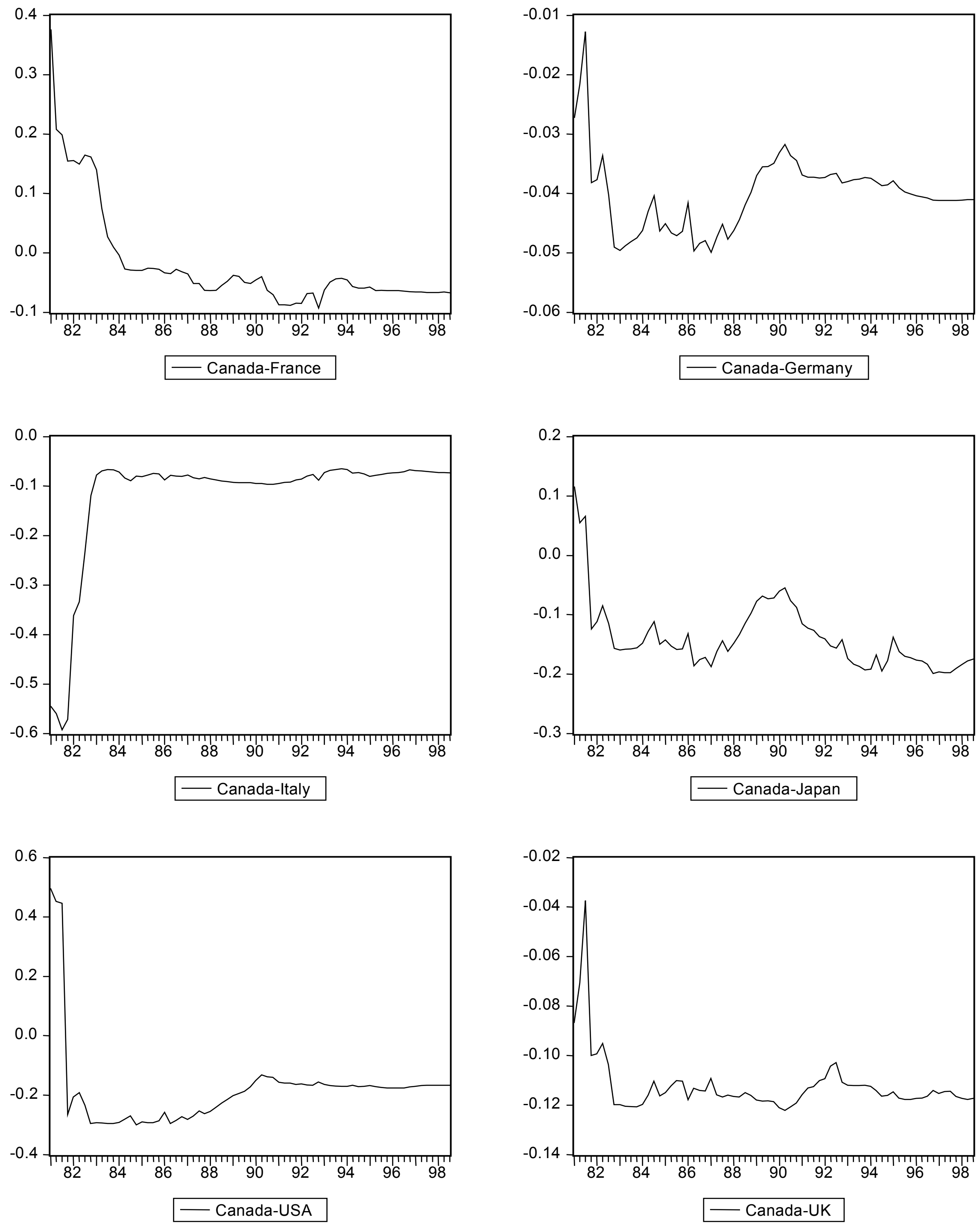

Figure 2. Time path of adjustment coefficients in error correction equations for France

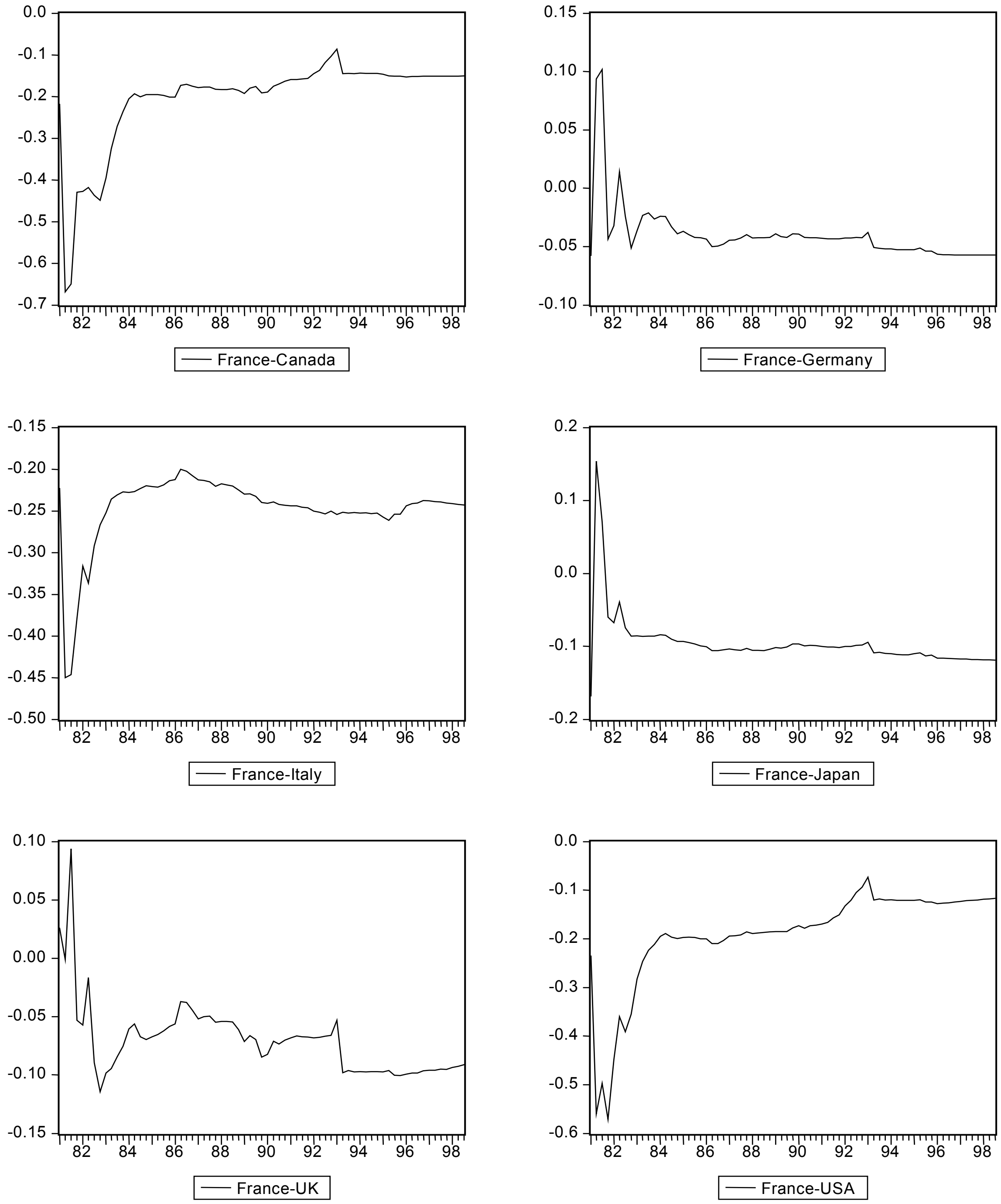

Figure 3. Time path of adjustment coefficients in error correction equations for Germany

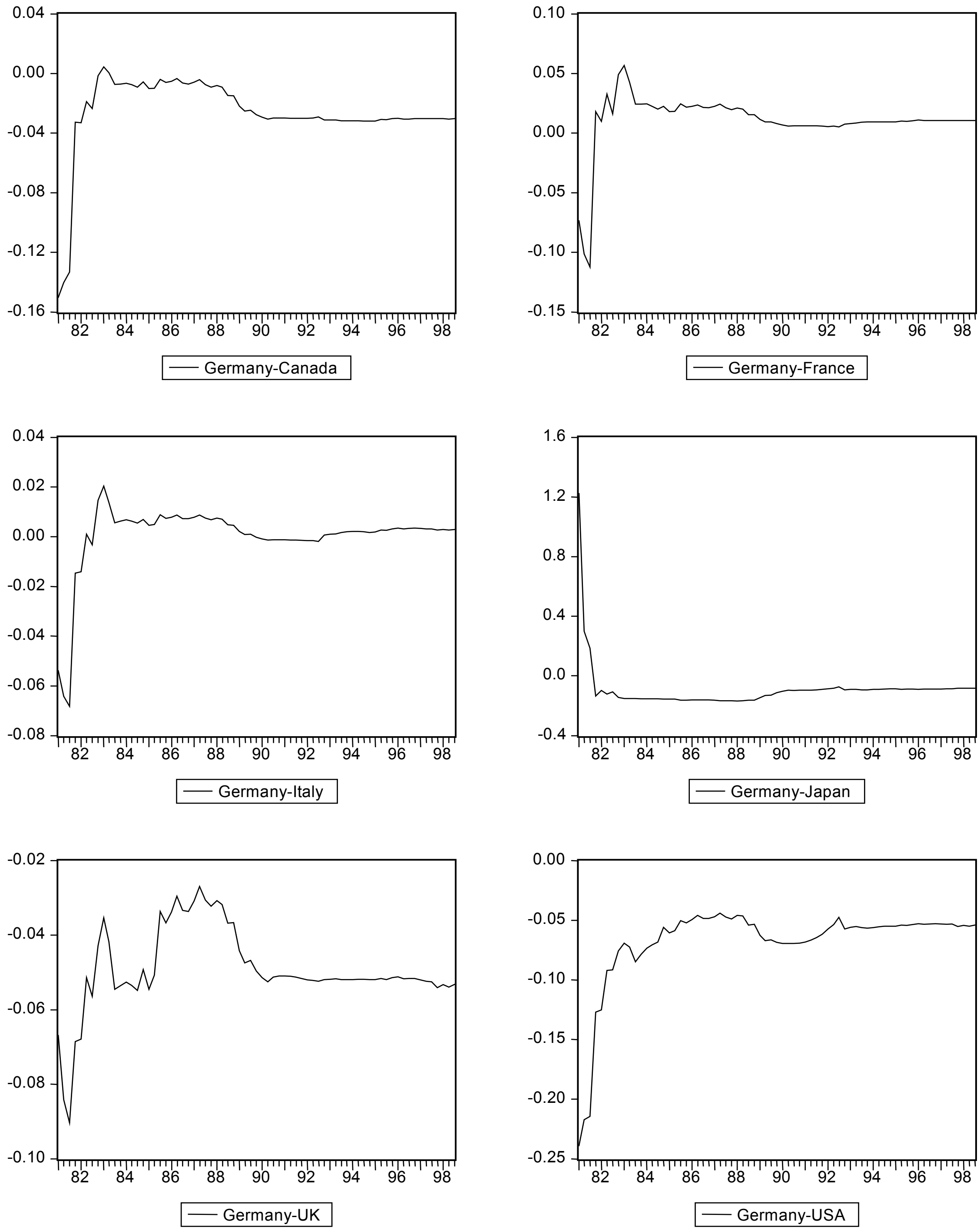

Figure 4. Time path of adjustment coefficients in error correction equations for Italy

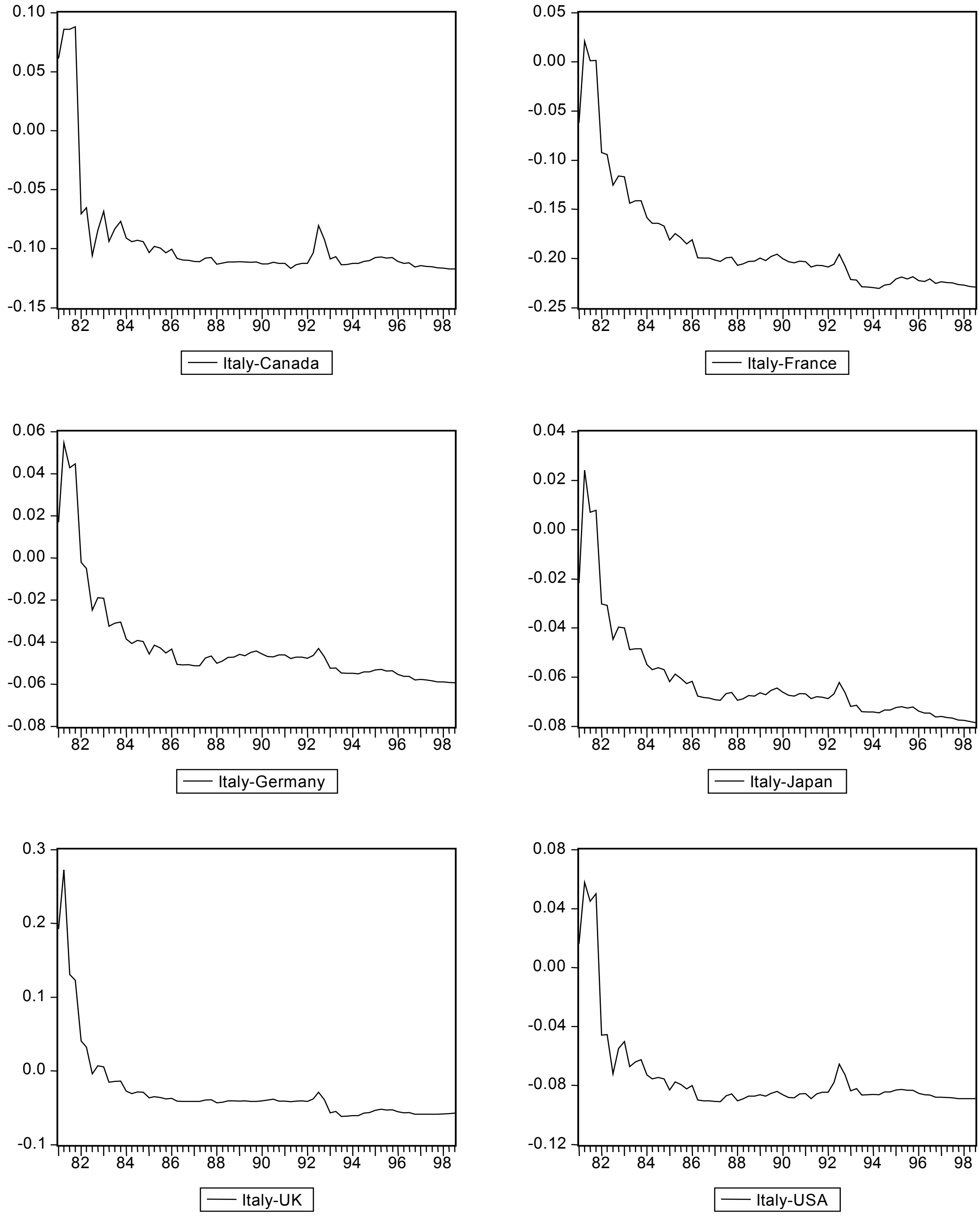

Figure 5. Time path of adjustment coefficients in error correction equations for Japan

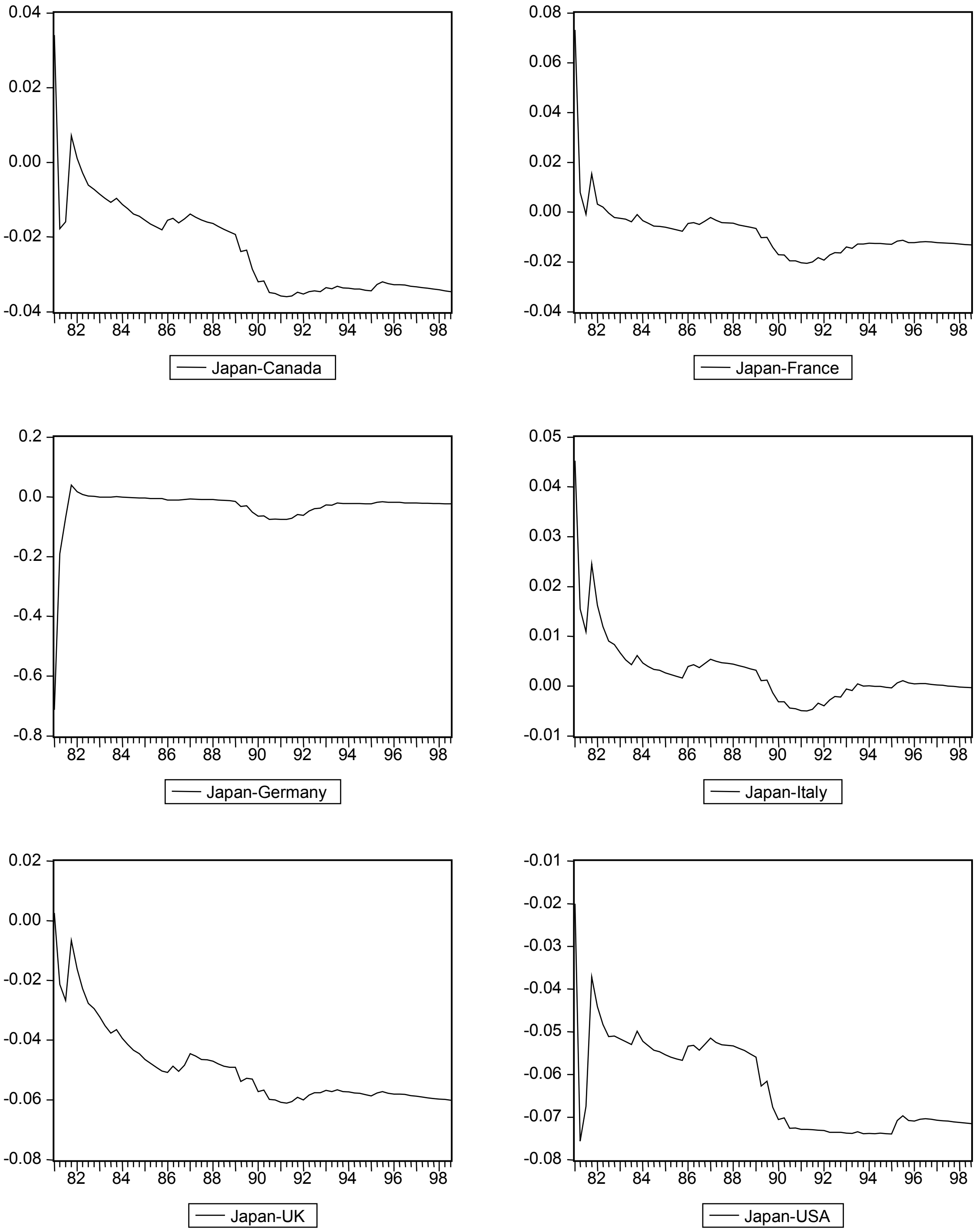

Figure 6. Time path of speed of adjustment coefficients in error correction equations for the UK

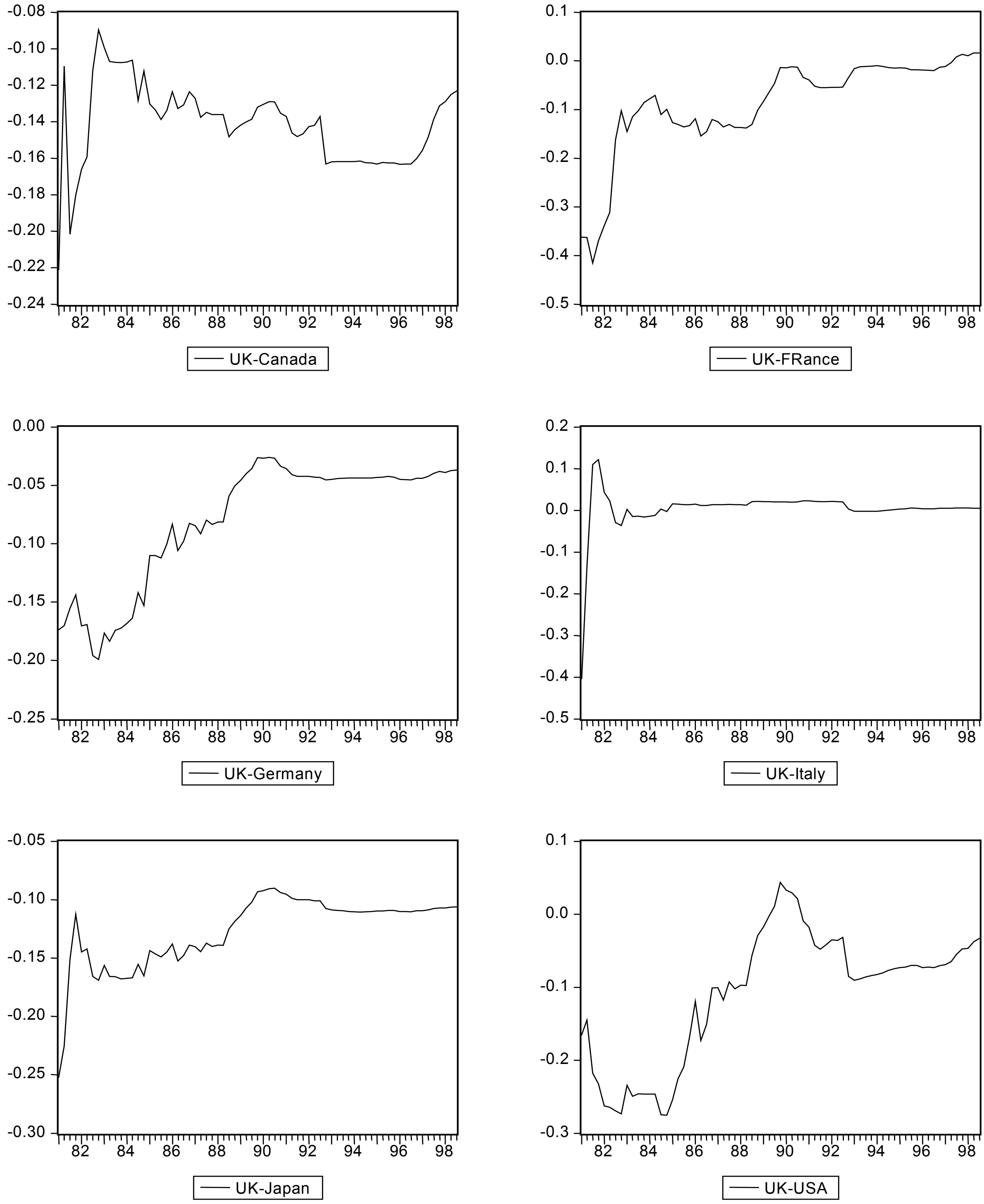

Figure 7. Time path of adjustment coefficients in error correction equations for the US