Irreducibility and Structural Cointegrating Relations: An Application to the G-7 Long Term Interest Rates

Marco R. Barassi
University of East London

Guglielmo Maria Caporale
University of East London

Stephen G. Hall
Imperial College Management School

March 2000

Abstract

JEL classification: C32, C51

Keywords: *Irreducible cointegrating vector, identification, structural, minimum variance, interest rates*

Financial assistance from ESRC grant no. R 00222955 is gratefully acknowledged

1. Introduction

An interesting issue in the presence of increasingly integrated international financial market is the ability of national authorities to conduct an independent monetary policy with respect to long run interest rates (see Caporale and Williams, 1998c). If the fundamental determinants of (long-term) interest rates are national rather than international, then the interest rate is not given even for a small open economy, and interest rate policy still lies mainly in the hands of domestic policy makers. In a previous paper (see Barassi, Caporale and Hall 2000), we found empirical evidence of convergence of the G-7 short-term rates to support the uncovered interest parity or open arbitrage conditions.

It is common to model the expected long-term rates as some weighted average of short-term rates (Expectation Hypothesis) plus a country related risk premium. If we then make the assumption that this risk premium, which may exist, in the relationship is either stationary, or that exist a stationary relation between the G-7 risk premia, the implication of these theories is that all interest rates should be cointegrated on a bilateral basis. In itself therefore cointegration between interest rates is neither surprising nor particularly informative. However if these interest rates are cointegrated then there must exist a causal structure, which gives rise to cointegration and is of great policy interest. The purpose of this paper is to see how far we can get in determining what this causal structure is without imposing an arbitrary set of identification conditions on the data, which might invalidate the inference we draw.

Much of the empirical evidence on interest rate linkages is based on causality test statistics, even though interest rates are typically [I(1)], and hence the tests do not follow standard distributions. So the inference is often invalid (see Caporale and Pittis, 1999). Recent work using an appropriate testing procedure put forward by Toda and Yamamoto (1995) shows that in fact, in the case of long rates, interest rate movements are determined mainly by domestic policy objectives or country related factors (see Caporale and Williams, 1998a, 1998c).

This paper examines long-term interest rate linkages in the G-7 as structural relations, using a method put forward by Davidson (1998a) and later extended by Barassi, Caporale and Hall (2000), which involves testing for irreducibility of cointegrating relations and their ranking according to the criterion of minimum variance. The interesting feature of this method is that, under certain circumstances, it allows us to learn only about the structural relationship that links cointegrated series from the data without imposing any arbitrary identifying conditions. Similarly, linkages within Europe might have been affected by institutional changes in the ERM, and further changes are likely to have been associated with the inception of EMU. Our analysis therefore will also include exogeneity tests on IC relations to shed some light on the likely impact of the creation of an integrated capital market in Europe. By suppressing exchange rate risk within the area and by fostering harmonisation measures, EMU will have an impact on asset prices and monetary and fiscal policy, which in turn will affect investment, real activity, capital flows and hence global interest rate linkages (see Portes and Rey, 1998).

Having said that, it is worth considering that even in a system like the ERM which aims to produce policy co-ordination it has been possible for monetary authorities to disengage their long term interest rate policy from developments elsewhere and pursue an independent policy agenda over long periods (possibly implying lack of cointegration between some of the series). Such an option should remain available for non-participating countries, like the UK, after the establishment of the Euro. Therefore the UK authorities will not necessarily find their freedom of action greatly constrained by what is happening in the Euro zone. Within the Euro zone the policies of the European Central Bank (ECB) will not necessarily be as stable or credible as those adopted so far by the German authorities, since smaller countries will also have an influence on monetary policy (see Begg at al, 1998). If in fact Germany has not been able to impose its interest rate policy on the other ERM countries, and if this becomes true of fiscal policy as well (notwithstanding the Growth and Stability Pact), long-term rates might rise (rather than decline) in the EU after 1999.

2. Analysing Interest Rate Linkages

In broad terms one can identify two views on how interest rates may be linked. If they are treated as analogous to other asset prices, then their movements are naturally interpreted as being determined by financial flows in fluid, profit-seeking capital markets. Alternatively, they can be viewed as policy instruments, so that their time paths may be determined by a policy objective such as an exchange rate parity or an inflation target. Interest rate linkages have therefore often been analysed in the context of a specific policy framework such as the Exchange Rate Mechanism (ERM). For instance, numerous studies have attempted to test the so called “German Leadership Hypothesis” (GLH), according to which Germany acts as the dominant player within the system, and monetary authorities in other ERM countries are unable to deviate from the course set by the Bundesbank (see, e.g., Fratianni and Von Hagen, 1990). Taking this view, co-movement in interest rates arises because of policy convergence. Early studies had concluded that there is no cointegration between German rates and other EMS rates (see Karfakis and Moschos, 1990), and that there is stronger evidence of cointegration between US rates and EMS rates (see Katsimbris and Miller, 1993). Subsequent papers reported convergence in European rates after 1986 (see Caporale et al, 1996). Similar conclusions were reached by Hall et al (1992) using time-varying techniques. In a global context, Caporale and Williams (1998b) found a marked difference between linkages in long-term rates (10-year bond yields) and in short-term rates (3-month Treasury bills) in the G-7 economies showing more compelling evidence of co-movements for the latter ones. Evidence of strong linkages between short rates has been confirmed in our previous work (Barassi, Caporale and Hall, 2000) in which we first applied what we will thereafter call Extended Davidson's Methodology (EDM). Furthermore, we found that the causality structure is not consistent with the standard characterisation of the ERM as an asymmetric system in which Germany was the dominant player -it rather suggesting a US worldwide leadership. In this paper we apply the EDM technique to long term interest rates for the G-7 countries. Our objective is to identify the fundamental relationships (and their causal structure) linking long-term interest rates among the G-7, so to test whether the predicted convergence in policy objectives holds.

To summarise the steps of our procedure, we first perform cointegration tests on the complete G-7 group of long-term rates so to obtain its cointegrating rank. After this, in order to identify the structural relationships, we should orthonormalise the matrix of long-run coefficients first, and then test for identification of its columns. The problem with this approach is dealing with the presence of potentially redundant variables that interact with the other cointegrated series driving the cointegrating regression coefficients towards some other element of the cointegrating space. To eliminate the redundant or non-cointegrated series, we perform cointegration tests on pairs of rates, so that we can rule out those series which are not cointegrated. This would leave us with a collection of cointegrating relations that are irreducible (IC) for that they do not have any cointegrated subset of variables. Not all these irreducible cointegrating vectors are structural though. Some of them are solved vectors, namely linear combinations of structural relations. This implies that we need some device to distinguish structural irreducible cointegrating relations from solved ones. The device consists of ranking the irreducible cointegrating vectors according to the criterion of lowest variance. The argument put forward here is that (asymptotically) if we have N variables and R structural IC vectors where R is at most N-1, then there may also exist up to K irreducible vectors which are simply combinations of the R structural ones where K is at most $((R-1)^2+(R-1))/2$. So there are a total of R+K possible IC vectors. Then the R structural ones will be grouped amongst the lower group of vectors when we order them by the lowest variance of the long run residuals of the cointegrating relationship as discussed later. This point is illustrated in a more detailed fashion in the following section, which also includes a brief account of Davidson's (1998a) methodology as well as the formal definition of some fundamental concepts involved.

3. The methodology

Consider a cointegrated VAR(p), as analysed by Johansen (1988):

$A(L)x_t = \alpha\beta'x_t + A^*(L)\Delta x_t = \varepsilon_t \quad (p \times 1)$,

where $x_t \sim I(1)$, L is the lag operator, $A(L) = \alpha\beta' + A^*(L)(1-L)$ such that $A(1) = \alpha\beta'$, and α and β are $p \times k$ matrices the loading weights matrix and the matrix of cointegrating vectors respectively.[1] When $k<p$ it can be shown that the system incorporates a set of long run relationships of the form $\beta'x_t = s_t$, where

$s_t = (\alpha'\alpha)^{-1}\alpha'(\varepsilon_t - A^*(L)\Delta x_t) \sim I(0)$.

In this model there are k linearly independent cointegrating vectors, the columns of β. Note that without restrictions on β we can always *scale* the matrix of the cointegrating relations by post-multiplying it by any non-singular $k \times k$ matrix M, to get $M\beta'x_t = Ms_t$ that is observationally equivalent to $\beta'x_t = s_t$ with loading matrix αM^{-1}. The identification problem within the Johansen procedure is tackled by estimating a collection of orthonormalised vectors spanning the same space as β that are identified by the usual rank condition. Here we propose to follow a method that allows the researcher to identify the structural relations in the case of over-identified systems extending it to the case of just-identified ones. Our methodology is an extension of a method put forward by Davidson (1998a) of which we need to recall the main points that follow.

Theorem 1 (Davidson, 1994). *If a column of* $\boldsymbol{b}$ *(say* $\boldsymbol{b}_1$*) is identified by the rank condition, the OLS regression which includes just the variables having unrestricted non-zero coefficients in* $\boldsymbol{b}_1$ *is consistent for* $\boldsymbol{b}_1$*.*

The issue raised by this theorem is that within a non-stationary world if another variable is added to a cointegrating regression, its coefficient might not necessarily converge to zero as we would expect in the case of an irrelevant variable within a regression involving stationary variables. In the case of cointegration the regression coefficients would generally converge to some other element of the cointegrating space. The main result of this is that, if a collection of I(1) variables is found to be cointegrated, it does not necessary follow that the estimated vectors can be interpreted as structural. In this framework it is useful to recall the definition of irreducible cointegrating vector in the way it is given in Davidson's (1998a) paper, that is,

Definition I. A set of I(1) variables will be called irreducibly cointegrated (IC) if they are cointegrated, but dropping any of the variables leaves a set that is not cointegrated.

Having formally defined the features of an IC it is worth mentioning the following important property of these vectors.

Theorem 2. An IC vector is unique, up to the choice of normalisation.

This theorem is proved by contradiction using the following argument. Let us assume that there exists for the IC variables a set of cointegrating vectors of rank at least two. We have already seen that any linear combination of these vectors would lie in an observationally equivalent cointegrating space. If this is true, we can always generate a combination having a zero element by choosing the weights appropriately. This would allow us to drop the variable in question without losing cointegration, but this contradicts the definition of IC itself.

Theorem 3 (Davidson, 1994). *If and only if a structural cointegrating relation is identified by the rank condition, it is irreducible.*

---

[1] We have assumed for simplicity the absence of any deterministic terms in this representation of the system under analysis. The modifications necessary to relax these assumptions are straightforward and would not alter the substance of the results obtained using a simpler model.

This tells us that at least some IC vectors are structural. When the cointegrating rank of the system is k, an IC relation can contain at most p - k + 1 variables. There are between k and (p - k + 1) of these vectors in total, the actual number depending on the degrees of over-identification of the relations of the system. This is to say that in addition to up to k identified structural relations, which, by theorem 3, are among the IC vectors, there might also be a number of solved vectors that can be defined as follows:

Definition 2 (Davidson 1998a). *A solved vector is a linear combination of structural vectors from which one or more common variables are eliminated by choice of offsetting weights such that the included variables are not a superset of any of the component relations.*

A solved vectors lies in the cointegrating space by construction. It may also be irreducible provided that it is a function of identified structural vectors. It is worth highlighting that solved relations are comparable to the reduced form equations of the conventional simultaneous equations models as they are solved from the structure[2].

Testing for irreducibility is an important diagnostic in order to achieve a correct identification of the structural relations between the series involved in a system. It is common practice to build a presumed cointegrating regression in the light of some economic theory, the theory being considered to receive support if the hypothesis of non-cointegration is rejected. However, economic theory might suggest including some variable which is in reality not really involved in that cointegrating relation but which, interacting with the other variables, might display a coefficient which does not converge to zero. This could well provide us with a stationary relation that could indeed be a wrong one, for that, as gathered from *theorem 3*, a cointegrating relation that contains redundant elements cannot be of any interest to us. The theory could be wrong, in which case this is just an arbitrary element of the cointegrating space. If the theory is correct, the relation is revealed to be underidentified. The estimate is inconsistent and it represents a hybrid of different structural equations. Irreducibility is an important diagnostic property of a cointegrating regression and testing for it allows us to determine what are the redundant variables in the system removing any unwanted effects. Once an IC relation is found, interest focuses on the problem of distinguishing between structural and solved forms. Of course, the theoretical model might answer this question for us, but this would then simply be using the theory to identify the model and so in the absence of overidentifying restrictions we could learn nothing about the validity of the theory itself. Generally speaking, the fewer variables an IC relation contains, and the fewer it shares with other IC relations, the better the chance that it is structural and not a solved form. In the extreme cases, we can actually draw definite conclusions, as the following pair of results show.

Theorem 4. If an IC relation contains strictly fewer variables than all those others having variables in common with it then, subject to the condition of Lemma l, it is an overidentified structural relation.

Theorem 5. If an IC relation contains a variable, which appears in no other IC relation, it is structural.

Thus, it is possible, in the context of simultaneous cointegrating relations, to discover structural economic relationships directly from a data analysis, without the use of any theory. To understand this assume a system that consists of four I(1) variables, x, y, z and w. Suppose we had tested for cointegrating rank and had found a rank of two. Now assume we have tested for irreducibility and found the pairs (x, y) and (z, w) are found to be directly cointegrated (but not the pairs (x, z) or (y, w)) these two cointegrating relations, necessarily irreducible of course, are also necessarily structural. Neither can have arisen as a result of solving out some more fundamental relationships. This is a case of maximal over-identification and is the framework within which Davidson methodology performs at its best. Let us now assume that our system made up of four I(1) variables x, y, z and w. Now assume that having tested for irreducibility we found that the series x, y, z and w are directly

[2] Note that in standard systems of simultaneous equations the reduced forms are defined with respect to a particular normalisation which is based on the distinction between endogenous and exogenous

cointegrated with each other as pairs . The cointegrating rank of this system is three, and we have a total of six IC relations. Three relations necessarily exist by being solved from two of the other three irreducible vectors. The problem is that we cannot know which, without a prior theory. In general in the case of bivariate cointegration between each pair of variables in a set of N variables there will be R structural IC vectors where R is N-1, there will exist K irreducible vectors which are simply combinations of the R structural ones where K is $((R-1)^2+(R-1))/2$. Now if we designate the first R cointegrating residuals as the structural ones, so that $e_1...e_R \sim NI(0, s^2{}_1...s^2{}_R)$. Then clearly the solved cointegrating residuals will be combinations of these. However the set of K solved residuals need not all be greater than all of the R structural residuals. For example, the first solved residual may be distributed as $N(0, s^2{}_1 + s^2{}_2)$ and there is no reason why this should be larger than any of the other structural cointegrating residuals (say $s^2{}_3$). However we can still use the variances to detect the structural residuals by carrying out a more complex comparison based on the following idea. The structural residuals will have a lower variance than any solved residual coming from an IC vector containing the same variable. The way this works and can be displayed is made obvious in the following table. Suppose we are considering a four variable case, (w, x, y, z), where the structural bivariate relationships are between w and x, y and z, then the following variances should be found.

In table 1 we can see that the structural relationships between w and the other variables always have the smallest variance in the column for any one variable. This is obvious,

Table 1. The relationship of cointegrating errors between structural and solved vectors

	W	X	Y	Z
w	0	$s^2{}_1$	$s^2{}_2$	$s^2{}_3$
x	0	0	$s^2{}_1 + s^2{}_2$	$s^2{}_1 + s^2{}_3$
y	0	0	0	$s^2{}_2 + s^2{}_3$
z	0	0	0	0

variables, which is not relevant in the cointegrating framework.

for example the whole column for z contains s^2_3 but only the structural model contains only this term so all other terms must be greater then the structural one.

A final complication which may arise in some circumstances, (although not in the case studied here), is that for any IC vector the variance may vary with the normalisation. In this case we suggest always using the normalisation which yields a minimum variance.

4. Empirical Analysis

a) The data-set

The sample under investigation covers the period between 1977:1-1998:3. The source for the data is International Financial Statistics of IMF.

We begin this study by pre-testing for the order of integration of the series using standard Augmented Dickey-Fuller (ADF) test. The number of lagged differences included in the test is decided on the basis of a criterion advised by Hendry and Doornik (1997) so to insure non-autocorrelated residuals on the auxiliary regressions. In each case the tests deliver the expected result that the series are all integrated of order one [I(1)], so that they follow stochastic trends. The results are shown in table 1.

Table 1. G-7 long-term interest rates unit root tests

ADF(canlong) = -0.9903	Critical values used in ADF test: 5%=-2.895 1%=-3.508
ADF(frlong) = -0.9384	Critical values used in ADF test: 5%=-2.895 1%=-3.508
ADF(itlong) = -1.01	Critical values used in ADF test: 5%=-2.896 1%=-3.51
ADF(usalong) = -1.338	Critical values used in ADF test: 5%=-2.895 1%=-3.508
ADF(uklong) = -0.4426	Critical values used in ADF test: 5%=-2.898 1%=-3.513
ADF(gerlong) = -2.073	Critical values used in ADF test: 5%=-2.899 1%=-3.516
ADF(jplong) = -0.7201	Critical values used in ADF test: 5%=-2.897 1%=-3.511

Having obtained confirmation that all interest rates are integrated of order one we proceed by running cointegration tests for the complete G-7 series of short-term interest rates. For this and subsequent analysis we have used Johansen's (1988,1991) likelihood based cointegration tests. As suggested in Hall (1991) and Caporale, Hall, Urga and Williams (1997), in performing the rank tests we have specified the unrestricted VAR including as many lags of the variables as necessary to ensure non-autocorrelation in the residuals as well as one point dummies necessary to correct for non-normality or heteroscedasticity of the disturbances.

b) Empirical Results

We start by performing cointegration tests on the complete G-7 sample of interest rates obtaining the results displayed in table 2.

Table 2. G-7 long-term rates cointegration test

eigenvalue	loglik for rank	
	648.865	0
0.681239	695.169	1
0.469721	720.860	2
0.297578	735.166	3
0.200772	744.242	4
0.118348	749.344	5
0.0792845	752.689	6
0.0312077	753.973	7

Ho:rank=p	-Tlog(1-\mu)	using T-nm	95%	-T\Sum log(.)	using T-nm	95%
p == 0	92.61**	52.59**	45.3	210.2**	119.4	124.2
p <= 1	51.38**	29.18	39.4	117.6**	66.79	94.2
p <= 2	28.61	16.25	33.5	66.23	37.61	68.5
p <= 3	18.15	10.31	27.1	37.61	21.36	47.2
p <= 4	10.2	5.794	21.0	19.46	11.05	29.7
p <= 5	6.691	3.8	14.1	9.259	5.258	15.4
p <= 6	2.568	1.458	3.8	2.568	1.458	3.8

The results indicate the cointegrating rank of the system being two. Of course these tests only allow us to reject the hypothesis that there are less than two cointegrating vectors, they do not necessarily mean that there is not more. So, in order to learn something more about the structure of the linkages among these series, we perform cointegration tests on pairs of series of each of the two groups of rates to investigate whether direct cointegration holds among all of the series of the group. What we want to establish is the number of irreducible cointegrated relations and which are the series involved in them. Of course if we find that pairwise cointegration holds between each pair of rates this tells us that the rank of the whole seven variable system is in fact 6. The conflict between the two test procedures is then seen as simply one of the small sample power and size of the tests in different contexts. The results for pairwise cointegration tests are presented in table 3.

table 3: long-run rates pairwise cointegration tests

	Ho:rank=p	-Tlog(1-\mu)	using T-nm	95% CV	-T\Sum log(.)	using T-nm	95% CV
Usa-Canada	p == 0	20.39**	17.91*	14.1	23.08**	20.27**	15.4
Usa-Japan	p == 0	18.64**	16.82*	14.1	19.66*	17.74*	15.4
Usa-Uk	p == 0	14.43*	12.67	14.1	14.73	12.94	15.4
Usa-France	p == 0	24.07**	21.75**	14.1	24.34**	22**	15.4
Usa-Italy	p == 0	12.13	10.66	14.1	12.88	11.31	15.4
Usa-Germany	p == 0	5.524	4.985	14.1	6.001	5.415	15.4
Canada-France	p == 0	36.9**	31.43**	14.1	38.76**	33.01**	15.4
Canada-Uk	p == 0	16.55*	13.66	14.1	18.56*	15.31	15.4
Canada-Italy	p == 0	9.61	8.438	14.1	11.26	9.891	15.4
Canada-Japan	p == 0	16.55*	14.51*	14.1	16.86*	14.78	15.4
Canada-Germany	p == 0	8.095	7.305	14.1	8.731	7.88	15.4
Uk-France	p == 0	30.31**	26.32**	14.1	33.9**	29.44**	15.4
Uk-Germany	p == 0	11.59	7.781	14.1	14.1	11.9	15.4
Uk-Italy	p == 0	28.23**	22.96**	14.1	42.31**	34.41**	15.4
Uk-Italy	p <=1	14.08**	11.45**	3.8	14.08**	11.45**	3.8
Uk-Japan	p == 0	16.85*	15.21*	14.1	17.1*	15.43*	15.4
Germany-France	p == 0	10.71	9.401	14.1	14.93	13.99	15.4
Germany-Italy	p == 0	8.323	7.242	14.1	8.327	7.245	15.4
Germany-Japan	p == 0	13.96	12.24	14.1	14.7	12.89	15.4
Italy-Japan	p == 0	17.07*	15.84*	14.1	21.88*	20.3**	15.4
Italy-Japan	p <=1	4.805*	4.458*	3.8	4.805*	4.458*	3.8
Italy-France	p == 0	10.64	9.848	14.1	10.91	10.1	15.4
Japan-France	p == 0	20.05**	17.57*	14.1	20.1**	17.62*	15.4

* indicates rejection of the null of no cointegration at 95% level
** indicates rejection of the null of no cointegration at 99% level

The results that we can observe are basically three.

- The true rank of the system is actually four.

- Direct cointegration holds among every pair of series (and with unit elasticity in all cases) but in the tests which involve Italy and Germany. Therefore,
- We can rule out the possibility that Italy and Germany are involved in any of the structural relations.

Indeed, in the light of the results obtained in our previous study concerning short term rates, we would not expect non convergence of Italian and German rates, therefore the fact that these rates do not cointegrate with each other and consequently with the others comes as a surprise. Consequently we investigate the issue further. It is useful to recall that in the introduction we had made the assumption that risk premia in the G-7 group were either stationary, or that there existed a stationary relation between them. Now recall that we had defined long term interest rates as,

$$LR = \sum_r r w_r + \lambda$$

where LR are long term rates, r refers to short term rates, w_r are weights attached to each of the rs and λ is the term that we called risk premium, which is made up of country specific factors. If we make the reasonable assumption that expectations on future inflation enter the risk premium term. This means that, having found that cointegration holds between the short rates of Italy and Germany, we can test for cointegration between the inflation rates of Italy and Germany to investigate the cause of non-convergence of the long-term rates. The results displayed in table 4 indicate full rank of the system and therefore stationarity of the two inflation rates. This result is clearly unacceptable because we had already performed ADF-tests on the two series obtaining results that indicate non-stationarity of the two series as displayed in table 5.

At this point what we can do is to test the stationarity of the two cointegrating vectors indicated by the rank test to check whether any of them is indeed stationary. The answer is definitely no. The ADF-tests (Table 6) cannot reject the hypothesis of a unit-root in both of the cointegrating vectors indicating that the inflation rates of Italy and Germany are not cointegrated and they can actually play a relevant part in explaining non convergence of Italian and German long term interest rates.

Table 4. Cointegration test between the inflation rates of Italy and Germany

eigenvalue	loglik for rank	
	130.258	0
0.210912	139.259	1
0.0854975	142.655	2

Ho:rank=p	-Tlog(1-\mu)	using T-nm	95%	-T\Sum log(.)	using T-nm	95%
p == 0	18*	14.69*	14.1	24.8**	20.23**	15.4
p <= 1	6.793*	5.541*	3.8	6.793**	5.541*	3.8

Table 5. ADF-test on inflation rates of Italy and Germany

ADF(ita-infl) = -0.9596	Critical values used in ADF test: 5%=-2.9 1%=-3.519
ADF(ger-infl) = -2.326	Critical values used in ADF test: 5%=-2.9 1%=-3.519

Table 6. ADF-test on Cointegrating vectors indicated from rank test in table 4.

ADF(CIvec1) = -1.872	Critical values used in ADF test: 5%=-2.903 1%=-3.525
ADF(CIvec2) = -2.584	Critical values used in ADF test: 5%=-2.903 1%=-3.525

Moving to the second part of our investigation we proceed with ranking the irreducible cointegrating vectors indicated from the tests before so to obtain table 7.

Table 7. Ranking of irreducible cointegrating vectors

IC vectors	**eigenvalue**	**standard deviation**	**exogeneity restrictions**
usa-canada	0.220188	0.706543	canada exogenous
canada-uk	0.186922	0.879522	uk exogenous
uk-japan	0.185781	0.947805	feedback
canada-france	0.365902	0.970788	canada exogenous
usa-france	0.251763	1.041766	feedback
canada-japan	0.184848	1.125332	japan exogenous
usa-uk	0.16136	1.210084	uk exogenous
germany-japan**	0.158347	1.282394	-
uk-germany**	0.139702	1.307491	-
uk-france	0.328857	1.403686	uk exogenous

usa-japan	0.203316	1.421847	japan exogenous
japan-france	0.219263	1.445948	japan exogenous
canada-germany**	0.0620838	1.499267	-
italy-france**	0.0506678	1.584742	-
usa-germany**	0.0645576	1.726671	-
germany-france**	0.10124	2.062937	-
canada-italy**	0.110588	2.179342	-
usa-italy**	0.137558	2.275801	-
italy-japan*	0.163975	2.467952	-
uk-italy*	0.260275	2.480295	-
germany-italy**	0.0911157	2.865093	-

* test indicates full rank
** no cointegration has been found between the series

Some explanations are necessary on the table. First, notice that cointegration with omogeneous coefficients has been imposed to the non cointegrated series. Therefore, the standard deviation has been calculated on these cointegrating vectors and they have been ranked accordingly. Second, the exogeneity restrictions on the vectors involving Italy and Germany have been omitted for they are not very meaningful because they are performed on non-cointegrated series under the hypothesis of cointegration.

Having said that it is worth to notice that in general, not surprisingly, non cointegrated series display a higher variability than the cointegrated ones with the exception of two cases which concern always Germany, with Japan and UK respectively. This is not something to worry about because the ranking in table 7 is made in terms of the absolute magnitude of standard deviations. To make clear our point and to interpret more easily the ranking on the IC vectors we need to use a different format for the results to be displayed as in table 8.

Table 8. Ranking of IC vectors per country.

	US	CA	JP	GER	FR	IT	UK
US	-	**0.70**	1.42	*1.72*	1.04	*2.27*	1.21
CA	0.70	-	**1.12**	*2.7*	**0.97**	*2.17*	**0.87**
JP	1.42	1.12	-	*1.28*	1.44	*2.46*	0.94
GER	*1.72*	*2.7*	*1.28*	-	*2.06*	*2.86*	*1.3*
FR	1.04	0.97	1.44	*2.06*	-	*1.58*	1.4

IT	*2.27*	*2.17*	*2.46*	*2.86*	*1.58*	-	*2.48*
UK	1.21	0.87	0.94	*1.3*	1.4	*2.48*	-

We have used bold for the vectors we consider irreducible and structural on a column by column interpretation and italic for the ones involving Italy and Germany.

Given a rank of four, on a minimum standard deviation criterion, USA and Canada is a structural relationship and so are UK and Canada and France and Canada. The forth involves Japan and Canada again. If we collate this information with the one on the "exogeneity restrictions" column in table 7, we get a complete picture of what seem to be happening. Overall, the conclusion that we can draw is a support for the hypothesis that larger and more stable economies can achieve policy objectives more successfully accomodating rather than driving other countries' policies. This would explain endogeneity of US long rates in the IC relations with all the other countries. The two driving forces are instead identifiable as UK and Japan, which result exogenous in all the systems and between which there is feedback. Notice that France and Canada follow UK and Japan as well.

5. Summary and Conclusions

In this paper we have examined the causal linkages that exist between the G-7 long term interest rates. We have done so applying what we named Extended Davidson's Methodology (EDM) which is based on the innovative concept of an irreducible cointegrating (IC) vector which can be defined as a subset of a cointegrating relation that does not have any cointegrated subsets. Application of this method has confirmed once again the importance of testing for irreducibility as a diagnostic. We have in fact obtained a rank of four for the system of long term rates compared to a rank of two as indicated from the rank test on the whole group of series. The ranking of the IC relations according to the criterion of minimum variance and exogeneity tests on all IC relations have provided us with a methodology to distinguish between structural and solved relations and clarify the causal structure that links the rates respectively.

We have been able to isolate four irreducible structural relations which involve USA and Canada and again Canada with UK, France and Japan respectively. We also found evidence of non-convergence of Italian and German rates. This is explained by the fact that, even if the short rates of the two countries are linked through a long run relationship, their risk premia are non-stationary and indeed do not cointegrate. Overall, the countries that seem to be the driving forces are the UK and Japan, that are linked through a relationship with causal feedback. They seem to be the point of reference for USA that seem to be behaving in a more accomodating fashion. It is worth recalling that long-term rates in Italy and Germany, being non-cointegrated, seem to be totally independent from every other country's rates.

References

Artis, M.J. and W. Zhang (1998), "The linkage of interest rates within the EMS", *Weltwirtschaftliches Archiv*, 134, 1, 117-132.

Barassi, M. R., Caporale, G. M. and S. G. Hall (2000), "Interest rates linkages: identifying structural relations" *D.P. no.(--------) ,University of East London.*

Beeby, M., Funke, M. and S.G. Hall (1995), "Long-run causality and structural change: an application of P-star analysis to the UK's entry into the ERM", *D.P.no.19-95, Centre for Economic Forecasting, London Business School.*

Begg, D., Giavazzi, F., De Grauwe, P., Uhlig, H., and C. Wyplosz (1998), *The ECB: Safe at Any Speed?*, CEPR Report on *Monitoring the European Central Bank.*

Caporale, G.M., Kalyvitis, S. and N. Pittis (1996), "Interest rate convergence, capital controls, risk premia and foreign exchange market efficiency in the EMS", *Journal of Macroeconomics*, 18, 4, 693-714.

Caporale, G.M. and N. Pittis (1999), “Efficient estimation of cointegrating vectors and testing for causality in vector autoregressions”, *Journal of Economic Surveys*, 13, 1, 1-32.

Caporale, G.M. and G. Williams (1998a), “Bond markets and macroeconomic performance”, *D.P. no.13-98, Centre for Economic Forecasting, London Business School.*

Caporale, G.M. and G. Williams (1998b), “International linkages in short- and long-term interest rates”, *D.P. no.14-98, Centre for Economic Forecasting, London Business School.*

Caporale, G.M. and G. Williams (1998c), “Can interest rate policy be independent?”, *Economic Outlook*, 22, 4, 10-15.

Caporale, G.M., Hall, S.G., Urga G. and G. Williams (1997), "Aggregate money demand functions in five industrial countries: are they cointegrated? ", ”, *D.P. no.19-97, Centre for Economic Forecasting, London Business School.*

Davidson, J. (1994), “Identifying cointegrating regressions by the rank condition”, *Oxford Bulletin of Economics and Statistics*, 56, 1, 103-108.

Davidson, J. (1998a), “Structural relations, cointegration and identification: some simple results and their application”, *Journal of Econometrics,* 87, 87-113.

Davidson, J. (1998b), “A Wald test of restrictions on the cointegrating space based on Johansen’s estimator”, *Economics Letters*, 59, 2, 183-187.

Doornik, J.A. and D.F. Hendry (1997), *Modelling Dynamic Systems Using PcFiml 9.0*, International Thomson Business Press.

Engle, R.F. and C.W.J. Granger (1987), “Cointegration and error correction: representation, estimation and testing”, *Econometrica*, 55, 2, 251-276.

Fratianni, M. and J. von Hagen (1990), “German dominance in the EMS: evidence from interest rates”, *Journal of International Money and Finance,* 9, 358-375.

Goodfriend, M.," Interest rate policy and the inflation scare problem: 1979-1992," Federal Reserve Bank of Richmond, *Economic Quarterly,* vol. 79, winter 93, pp. 1-24

Hall, S.G. (1991), "The effect of varying length VAR models on the maximum likelihood estimates of cointegrating vectors", *Scottish Journal of Political Economy,* 38, 4, 317-323.

Hall, S.G., Robertson, D. and M.R. Wickens (1992), “Measuring convergence of the EC economies”, *Manchester School*, 60, Supplement, 99-111.

Hendry, D.F. (1995), *Dynamic Econometrics*, Oxford University Press, Oxford.

Johansen, S. (1988), " Statistical analysis of cointegration vectors", *Journal of Economic Dynamics and Control*, 12, 231-254

Johansen, S. (1995), “Identifying restrictions of linear equations with applications to simultaneous equations and cointegration”, *Journal of Econometrics*, 69, 111-132.

Karfakis, J.C. and D.M. Moschos (1990), “Interest rate linkages within the European Monetary System: a time series analysis”, *Journal of Money, Credit and Banking*, 22, 3, 388-394.

Katsimbris, G.M. and S.M. Miller (1993), “Interest rate linkages within the European Monetary System: further evidence”, *Journal of Money, Credit and Banking,* 25, 4, 771-779.

Martin, P. (1997), "The exchange rate policy of the euro: A matter of size?", *CEPR D.P. no.1646.*

Mehra, Y.P., (1996) "Monetary policy and long term interest rates", Federal Reserve Bank of Richmond, *Economic Quarterly,* vol. 82/3, summer 96, pp. 27-49

Pesaran, M.H. and Y. Shin (1994), "Long-run structural modelling", *Working Paper, Department of Applied Economics, University of Cambridge*.

Portes, R. and H. Rey (1998), "The emergence of the euro as an international currency", *Economic Policy,* 26, April 1998.

Toda, H.Y. and T. Yamamoto (1995), "Statistical inference in vector autoregressions with possibly integrated processes", *Journal of Econometrics,* 66, 225-250.