IRREDUCIBILITY AND STRUCTURAL COINTEGRATING RELATIONS: AN APPLICATION TO THE G-7 LONG-TERM INTEREST RATES

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Abstract

In this paper we examine the causal linkages between the G-7 long-term interest rates by using a new technique, which enables the researcher to analyse relations between a set of I(1) series without imposing any identification conditions based on economic theory. Specifically, we apply the so-called Extended Davidson's Methodology (EDM), which is based on the innovative concept of an irreducible cointegrating (IC) vector, defined as a subset of a cointegrating relation that does not have any cointegrated subsets. Ranking the irreducible vectors according to the criterion of minimum variance allows us to distinguish between structural and solved relations. The empirical results provide support for the hypothesis that larger, more stable economies can achieve policy objectives more successfully by accommodating rather than driving other countries' policies. It appears that the driving force is Canada, which is linked to the US, UK and France in three out of the four fundamental relations, and which is a reference point for the US. Italian and German rates, which are not cointegrated, seem to be determined by country-specific factors.

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1. Introduction

An important issue in the presence of increasingly integrated international financial market is the ability of national authorities to conduct an independent monetary policy with respect to long-run interest rates (see Caporale and Williams, 1998c). If the fundamental determinants of (long-term) interest rates are national rather than international, then the interest rate is not given even for a small open economy, and interest rate policy still lies mainly in the hands of domestic policy makers. In a previous paper (see Barassi, Caporale and Hall, 2000), we found empirical evidence of convergence between the G-7 short-term rates, which supports the uncovered interest parity or open arbitrage conditions, hence indicating that international factors are relatively more important in the case of short rates.

It is common to model the expected long-term rates as some weighted average of short-term rates (Expectation Hypothesis)[1] plus a country related risk premium. If we then make the assumption either that the risk premium is stationary, or that there exists a stationary relation between the G-7 risk premia, the implication of these theories is that all interest rates should be cointegrated on a bilateral basis. By itself, therefore, cointegration between interest rates is neither surprising nor particularly informative. However, if interest rates are cointegrated then there must exist a causal structure, which gives rise to cointegration and is of great policy interest. The purpose of this paper is to see how far we can get in determining what this causal structure is without imposing an arbitrary set of identification conditions on the data, which might invalidate the inference we draw.

Much of the empirical evidence on interest rate linkages is based on causality test statistics, even though interest rates are typically I(1), and hence the tests do not follow standard distributions. So the inference is often invalid (see Caporale and Pittis, 1999). Recent work using an appropriate testing procedure put forward by Toda and Yamamoto (1995) shows that in fact, in the case of long rates, interest rate

[1] For an exhaustive account of the Expectation Hypothesis see Cuthbertson (1996), "Quantitative Financial Economics" chapters 9 and 10, J.Wiley & Sons.

movements are determined mainly by domestic policy objectives or country-related factors (see Caporale and Williams, 1998a, 1998c).

This paper examines long-term interest rate linkages in the G-7 as structural relations, using a method put forward by Davidson (1998) and later extended by Barassi, Caporale and Hall (2000), which involves testing for irreducibility of cointegrating relations and their ranking according to the criterion of minimum variance. The interesting feature of this method is that, under certain circumstances, it allows us to learn about the structural relationships linking cointegrated series from the data only, without imposing any arbitrary identifying conditions. In the case of Europe, linkages might have been affected by institutional changes in the ERM, and further changes are likely to have been associated with the inception of EMU. Our analysis therefore will also include exogeneity tests on irreducible cointegrating (IC) relations to shed some light on the likely impact of the creation of an integrated capital market in Europe. By suppressing exchange rate risk within the area and by fostering harmonisation measures, EMU will have an impact on asset prices and monetary and fiscal policy, which in turn will affect investment, real activity, capital flows and hence global interest rate linkages (see Portes and Rey, 1998).

It is important to notice that even in a system like the ERM which aims to produce policy co-ordination it has been possible for monetary authorities to disengage their long-term interest rate policy from developments elsewhere and pursue an independent policy agenda over long periods (possibly implying lack of cointegration between some of the series). Such an option should remain available for non-participating countries, like the UK, after the establishment of the Euro. Therefore the UK authorities will not necessarily find their freedom of action greatly constrained by what is happening in the Euro zone. Within the Euro zone the policies of the European Central Bank (ECB) will not necessarily be as stable or credible as those adopted so far by the German authorities, since smaller countries will also have an influence on monetary policy (see Begg et al, 1998). If in fact Germany has not been able to impose its interest rate policy on the other ERM countries, and if this becomes true of fiscal policy as well (notwithstanding the Growth and Stability Pact), long-term rates might rise (rather than decline) in the EU following its inception.

The layout of the paper is as follows. Section 2 puts forward a new method for identifying structural relations, ideally suited for analysing interest rate linkages, the details of which are discussed in Section 3. The empirical results are presented in Section 4. Some concluding remarks are offered in Section 5.

2. Analysing Interest Rate Linkages

In broad terms one can identify two views on how interest rates may be linked. If they are treated as analogous to other asset prices, then their movements are naturally interpreted as being determined by financial flows in fluid, profit-seeking capital markets. Alternatively, they can be viewed as policy instruments, so that their time paths may be determined by a policy objective such as an exchange rate parity or an inflation target. Interest rate linkages have therefore often been analysed in the context of a specific policy framework such as the Exchange Rate Mechanism (ERM). For instance, numerous studies have attempted to test the so called "German Leadership Hypothesis" (GLH), according to which Germany acts as the dominant player within the system, and monetary authorities in other ERM countries are unable to deviate from the course set by the Bundesbank (see, e.g., Fratianni and Von Hagen, 1990). Taking this view, co-movement in interest rates arises because of policy convergence.

Early studies had concluded that there is no cointegration between German rates and other EMS rates (see Karfakis and Moschos, 1990), and that there is stronger evidence of cointegration between US rates and EMS rates (see Katsimbris and Miller, 1993). Subsequent papers reported convergence in European rates after 1986 (see Caporale et al, 1996). Similar conclusions were reached by Hall et al (1992) using time-varying techniques. In a global context, Caporale and Williams (1998b) found a marked difference between linkages in long-term rates (10-year bond yields) and in short-term rates (3-month Treasury bills) in the G-7 economies, with more compelling evidence of co-movements for the latter.

The existence of strong linkages between short rates was confirmed by our earlier study (see Barassi, Caporale and Hall, 2000) in which we first applied what we will call the Extended Davidson's Methodology (EDM). Furthermore, we found that the causal structure is not consistent with the standard characterisation of the ERM as an asymmetric system in which Germany was the dominant player, it rather suggesting US worldwide leadership. In this paper we apply the EDM technique to long-term interest rates for the G-7 countries. Our objective is to identify the fundamental relationships (and their causal structure) linking long-term interest rates among the G-7, in order to test whether the predicted convergence in policy objectives holds.

To summarise the steps of our procedure - we first perform cointegration tests on the complete G-7 group of long-term rates to obtain its cointegrating rank. In order to identify the structural relationships, one would then usually orthonormalise the matrix of long-run coefficients, and test for identification of its columns. The problem with this approach is that it involves dealing with potentially redundant variables that interact with the other cointegrated series driving the cointegrating regression coefficients towards some other element of the cointegrating space. To eliminate the redundant or non-cointegrated series, we perform cointegration tests on pairs of rates, so that we can drop the series that are not cointegrated. This leaves us with a collection of cointegrating relations that are irreducible (IC), for they do not have any cointegrated subset of variables.

Not all these irreducible cointegrating vectors are structural, though. Some of them are solved vectors, namely linear combinations of structural relations. Therefore we need some device to distinguish structural irreducible cointegrating relations from solved ones. This consists of ranking the irreducible cointegrating vectors according to the criterion of lowest variance. The argument put forward here is that (asymptotically) if we have N variables and R structural IC vectors, where R is at most N-1, then there may exist up to K irreducible vectors which are simply combinations of the R structural ones, where K is at most $((R-1)^2+(R-1))/2$. So there are a total of R+K possible IC vectors. Then the R structural ones will be grouped amongst the lower group of vectors when we order them by the lowest variance of the long-run residuals of the cointegrating relationship as discussed later. This point is illustrated in greater detail in the following section, which also includes a brief account of Davidson's (1998) method.

3. The Methodology

Consider a cointegrated VAR(p), as analysed by Johansen (1988):

1) $$A(L)x_t = \alpha\beta' x_t + A^* (L)\Delta x_t = \varepsilon_t \quad (p \times 1),$$

where $x_t \sim I(1)$, L is the lag operator, $A(L) = \alpha\beta' + A^*(L)(1-L)$ such that $A(1) = \alpha\beta'$, and α and β are $p \times k$ matrices the loading weights matrix and the matrix of cointegrating vectors respectively.[2] When $k<p$ it can be shown that the system incorporates a set of long run relationships of the form $\beta' x_t = s_t$, where

2) $$s_t = (\alpha'\alpha)^{-1} \alpha' (\varepsilon_t - A^* (L)\Delta x_t) \sim I(0).$$

In this model there are *k* linearly independent cointegrating vectors, the columns of β. Note that without restrictions on β we can always *scale* the matrix of the cointegrating relations by post-multiplying it by any non-singular *k x k* matrix M, to get $M\beta' x_t = Ms_t$ that is observationally equivalent to $\beta' x_t = s_t$ with loading matrix αM^{-1}. The identification problem within the Johansen procedure is tackled by estimating a collection of orthonormalised vectors spanning the same space as β that are identified by the usual rank condition. Here we propose a method that allows the researcher to identify the structural relations in the case of over-identified systems extending it to the case of just-identified ones. Ours is an extension of the method put forward by Davidson (1998), of which we need to recall the main points:

Theorem 1 (Davidson, 1994). *If a column of β (say β_1) is identified by the rank condition, the OLS regression which includes just the variables having unrestricted non-zero coefficients in β_1 is consistent for β_1.*

The issue raised by this theorem is that within a non-stationary world if another variable is added to a cointegrating regression, its coefficient might not necessarily

[2] We have assumed for simplicity the absence of any deterministic terms in this representation of the system under analysis. The modifications necessary to relax these assumptions are straightforward and would not alter the substance of the results obtained using a simpler model.

converge to zero as we would expect in the case of an irrelevant variable within a regression involving stationary variables. In the case of cointegration the regression coefficients would generally converge to some other element of the cointegrating space. The main result of this is that, if a collection of I(1) variables is found to be cointegrated, it does not necessary follow that the estimated vectors can be interpreted as structural. In this framework it is useful to recall the definition of irreducible cointegrating vector given by Davidson (1998), that is,

Definition I. A set of I(1) variables will be called irreducibly cointegrated (IC) if they are cointegrated, but dropping any of the variables leaves a set that is not cointegrated.

Having formally defined the features of an IC it is worth mentioning the following important property of these vectors.

Theorem 2. An IC vector is unique, up to the choice of normalisation.

This theorem is proved by contradiction using the following argument. Let us assume that there exists for the IC variables a set of cointegrating vectors of rank at least two. We have already seen that any linear combination of these vectors would lie in an observationally equivalent cointegrating space. If this is true, we can always generate a combination having a zero element by choosing the weights appropriately. This would allow us to drop the variable in question without losing cointegration, but this contradicts the definition of IC itself.

Theorem 3 (Davidson, 1994). *If and only if a structural cointegrating relation is identified by the rank condition, it is irreducible.*

This tells us that at least some IC vectors are structural. When the cointegrating rank of the system is k, an IC relation can contain at most p - k + 1 variables. There are between k and (p - k + 1) of these vectors in total, the actual number depending on the degrees of over-identification of the relations of the system. This is to say that in addition to up to k identified structural relations, which, by theorem 3, are among the

IC vectors, there might also be a number of solved vectors that can be defined as follows:

Definition 2 (Davidson 1998). *A solved vector is a linear combination of structural vectors from which one or more common variables are eliminated by choice of offsetting weights such that the included variables are not a superset of any of the component relations.*

A solved vectors lies in the cointegrating space by construction. It may also be irreducible provided that it is a function of identified structural vectors. It is worth highlighting that the solved relations are comparable to the reduced-form equations of the conventional simultaneous equations models as they are solved from the structure[3].

Testing for irreducibility is important in order to achieve a correct identification of the structural relations between the series involved in a system. It is common practice to build a presumed cointegrating regression in the light of some economic theory, the theory being considered to receive support if the hypothesis of non-cointegration is rejected. However, economic theory might suggest including some variable which does not in fact belong to that cointegrating relation but which, interacting with the other variables, might display a coefficient which does not converge to zero. This could well provide us with a stationary relation that could indeed be a wrong one, for as implied by *theorem 3*, a cointegrating relation that contains redundant elements is not of any interest. The theory could be wrong, in which case this is just an arbitrary element of the cointegrating space. If the theory is correct, the relation is revealed to be underidentified. The estimate is inconsistent and it represents a hybrid of different structural equations.

Irreducibility is an important diagnostic property of a cointegrating regression and testing for it allows us to determine what are the redundant variables in the system and to remove any unwanted effects. Once an IC relation is found, interest focuses on

[3] Note that in standard systems of simultaneous equations the reduced forms are defined with respect to a particular normalisation which is based on the distinction between endogenous and exogenous variables, which is not relevant in the cointegrating framework.

the problem of distinguishing between structural and solved forms. Of course, the theoretical model might answer this question for us, but this would then simply be using the theory to identify the model and so in the absence of overidentifying restrictions we could learn nothing about the validity of the theory itself. Generally speaking, the fewer variables an IC relation contains, and the fewer it shares with other IC relations, the better the chance that it is structural and not a solved form. In the extreme cases, we can actually draw definite conclusions, as the following pair of results show.

Theorem 4. If an IC relation contains strictly fewer variables than all those others having variables in common with it then, it is an overidentified structural relation.[4]

Theorem 5. If an IC relation contains a variable, which appears in no other IC relation, it is structural.

Thus, it is possible, in the context of simultaneous cointegrating relations, to discover structural economic relationships directly from a data analysis, without the use of any theory. To understand this consider a system that consists of four I(1) variables, x, y, z and w. Suppose we had tested for the cointegrating rank and had found a rank of two. Now assume we have tested for irreducibility and found the pairs (x, y) and (z, w) are found to be directly cointegrated (but not the pairs (x, z) or (y, w)). These two cointegrating relations, necessarily irreducible of course, are also necessarily structural. Neither can have arisen from solving out some more fundamental relationships. This is a case of maximal over-identification and is the framework within which Davidson's (1998) methodology performs at its best. Let us now assume that the system is made up of four I(1) variables, x, y, z and w. Now assume that having tested for irreducibility we found that the series x, y, z and w are directly cointegrated with each other as pairs. The cointegrating rank of this system is three, and we have a total of six IC relations. Three relations necessarily exist by being solved from two of the other three irreducible vectors.

[4] Note that this theorem is subject to the condition that a solved IC relation contains at least as many variables as each of the structural relations from which it is derived (see Lemma 1 in Davidson, 1998).

The problem is that we cannot know which, without a prior theory. In general, in the case of bivariate cointegration between each pair of variables in a set of N variables there will be R structural IC vectors where R is N-1, and there will exist K irreducible vectors which are simply combinations of the R structural ones where K is $((R-1)^2+(R-1))/2$. Now if we designate the first R cointegrating residuals as the structural ones, so that $e_1 ... e_R \sim NI(0, \sigma^2{}_1 ... \sigma^2{}_R)$, then clearly the solved cointegrating residuals will be combinations of these. However, the set of K solved residuals need not all be greater than all of the R structural residuals. For example, the first solved residual may be distributed as $N(0, \sigma^2{}_1 + \sigma^2{}_2)$ and there is no reason why this should be larger than any of the other structural cointegrating residuals (say $\sigma^2{}_3$). However, we can still use the variances to detect the structural residuals by carrying out a more complex comparison based on the following idea. The structural residuals will have a lower variance than any solved residual coming from an IC vector containing the same variable. The way this works and can be displayed is made obvious in the following table. Suppose we are considering a four variable case, (w, x, y, z), where the structural bivariate relationships are between w and x, y and z, then the following variances should be found (see Table 1).

In table 1 we can see that the structural relationships between w and the other variables always have the smallest variance in the column for any one variable. This is obvious,

Table 1. The relationship of cointegrating errors between structural and solved vectors

	W	X	Y	Z
W	-	$\sigma^2{}_1$	$\sigma^2{}_2$	$\sigma^2{}_3$
X	-	-	$\sigma^2{}_1 + \sigma^2{}_2$	$\sigma^2{}_1 + \sigma^2{}_3$
Y	-	-	-	$\sigma^2{}_2 + \sigma^2{}_3$
Z	-	-	-	-

for example the whole column for z contains $\sigma^2{}_3$ but only the structural model contains only this term, so all other terms must be greater than the structural one.

A final complication, which may arise in some circumstances (although not in the case studied here), is that for any IC vector the variance may vary with the normalisation. In this case we suggest always using the normalisation which yields a minimum variance.

4. Empirical Analysis

a) The dataset

The sample under investigation covers the period between 1977:1-1998:3. The source for the data is International Financial Statistics of the IMF. The interest rates used are the 10-year government bond rates for all the G-7.

We begin the analysis by pre-testing for the order of integration of the series using standard Augmented Dickey-Fuller (ADF) tests. The number of lagged differences included in the test is decided on the basis of a criterion advised by Doornik and Hendry (1997) so to ensure non-autocorrelated residuals on the auxiliary regressions. In each case the tests deliver the expected result that the series are all integrated of order one [I(1)], so that they follow stochastic trends. These results are shown below on table 2.

Table 2. G-7 long-term interest rates unit root tests

ADF(canlong) = -0.9903	*Critical values: 5%=*	*-2.895,*	*1%=*	*-3.508*
ADF(frlong) = -0.9384	*Critical values: 5%=*	*-2.895,*	*1%=*	*-3.508*
ADF(itlong) = -1.01	*Critical values: 5%=*	*-2.896,*	*1%=*	*-3.51*
ADF(usalong) = -1.338	*Critical values: 5%=*	*-2.895,*	*1%=*	*-3.508*
ADF(uklong) = -0.4426	*Critical values: 5%=*	*-2.898,*	*1%=*	*-3.513*
ADF(gerlong) = -2.073	*Critical values: 5%=*	*-2.899,*	*1%=*	*-3.516*
ADF(jplong) = -0.7201	*Critical values: 5%=*	*-2.897,*	*1%=*	*-3.511*

Having obtained confirmation that all interest rates are integrated of order one we proceed by running cointegration tests for the complete set of G-7 long-term interest rates. For this and subsequent analysis we have used Johansen's (1988, 1991) likelihood based cointegration tests. As suggested in Hall (1991) and Caporale, Hall, Urga and Williams (1997), in performing the rank tests we have specified the unrestricted VAR to include as many lags as necessary to ensure non-autocorrelation in the residuals, as well as one-point dummies to correct for non-normality or heteroscedasticity of the disturbances.

b) Empirical Results

We start by performing cointegration tests on the complete set of G-7 interest rates obtaining the results displayed in table 3.

Table 3. G-7 long-term rates cointegration test

Eigenvalue	loglik for rank	
	648.865	0
0.681239	695.169	1
0.469721	720.860	2
0.297578	735.166	3
0.200772	744.242	4
0.118348	749.344	5
0.0792845	752.689	6
0.0312077	753.973	7

Ho:rank=p	-Tlog(1-\mu)	using T-nm	95%	-T\Sum log(.)	using T-nm	95%
p == 0	92.61**	52.59**	45.3	210.2**	119.4	124.2
p <= 1	51.38**	29.18	39.4	117.6**	66.79	94.2
p <= 2	28.61	16.25	33.5	66.23	37.61	68.5
p <= 3	18.15	10.31	27.1	37.61	21.36	47.2
p <= 4	10.2	5.794	21.0	19.46	11.05	29.7
p <= 5	6.691	3.8	14.1	9.259	5.258	15.4
p <= 6	2.568	1.458	3.8	2.568	1.458	3.8

The results indicate that the cointegrating rank of the system is two. Of course these tests only allow us to reject the hypothesis that there are less than two cointegrating vectors. They do not necessarily mean that there are not more. So, in order to investigate further the linkages among these series, we perform cointegration tests on each pair of series. What we want to establish is the number of irreducible cointegrated relations and which are the series involved in them. Of course if we find that pairwise cointegration holds between each pair of rates this tells us that the rank of the whole seven variable system is in fact 6. The conflict between the two test procedures is then seen as simply one of the small sample power and size of the tests in different contexts.

The results for pairwise cointegration tests are presented in table 4

.

Table 4: long-run rates pairwise cointegration tests

	Ho:rank=p	-Tlog(1-\mu)	using T-nm	95% CV	-T\Sum log(.)	using T-nm	95% CV
USA-Canada	*p == 0*	*20.39***	*17.91**	*14.1*	*23.08***	*20.27***	*15.4*
USA-Japan	*p == 0*	*18.64***	*16.82**	*14.1*	*19.66**	*17.74**	*15.4*
USA-UK	*p == 0*	*14.43**	*12.67*	*14.1*	*14.73*	*12.94*	*15.4*
USA-France	*p == 0*	*24.07***	*21.75***	*14.1*	*24.34***	*22***	*15.4*
USA-Italy	*p == 0*	*12.13*	*10.66*	*14.1*	*12.88*	*11.31*	*15.4*
USA-Germany	*p == 0*	*5.524*	*4.985*	*14.1*	*6.001*	*5.415*	*15.4*
Canada-France	*p == 0*	*36.9***	*31.43***	*14.1*	*38.76***	*33.01***	*15.4*
Canada-UK	*p == 0*	*16.55**	*13.66*	*14.1*	*18.56**	*15.31*	*15.4*
Canada-Italy	*p == 0*	*9.61*	*8.438*	*14.1*	*11.26*	*9.891*	*15.4*
Canada-Japan	*p == 0*	*16.55**	*14.51**	*14.1*	*16.86**	*14.78*	*15.4*
Canada-Germany	*p == 0*	*8.095*	*7.305*	*14.1*	*8.731*	*7.88*	*15.4*
UK-France	*p == 0*	*30.31***	*26.32***	*14.1*	*33.9***	*29.44***	*15.4*
UK-Germany	*p == 0*	*11.59*	*7.781*	*14.1*	*14.1*	*11.9*	*15.4*
UK-Italy	*p == 0*	*28.23***	*22.96***	*14.1*	*42.31***	*34.41***	*15.4*
UK-Italy	*p <=1*	*14.08***	*11.45***	*3.8*	*14.08***	*11.45***	*3.8*
UK-Japan	*p == 0*	*16.85**	*15.21**	*14.1*	*17.1**	*15.43**	*15.4*
Germany-France	*p == 0*	*10.71*	*9.401*	*14.1*	*14.93*	*13.99*	*15.4*
Germany-Italy	*p == 0*	*8.323*	*7.242*	*14.1*	*8.327*	*7.245*	*15.4*
Germany-Japan	*p == 0*	*13.96*	*12.24*	*14.1*	*14.7*	*12.89*	*15.4*
Italy-Japan	*p == 0*	*17.07**	*15.84**	*14.1*	*21.88**	*20.3***	*15.4*
Italy-Japan	*p <=1*	*4.805**	*4.458**	*3.8*	*4.805**	*4.458**	*3.8*
Italy-France	*p == 0*	*10.64*	*9.848*	*14.1*	*10.91*	*10.1*	*15.4*
Japan-France	*p == 0*	*20.05***	*17.57**	*14.1*	*20.1***	*17.62**	*15.4*

* indicates rejection of the null of no cointegration at 95% level
** indicates rejection of the null of no cointegration at 99% level

The results that we obtain are basically three:

- The true rank of the system is actually four.
- Direct cointegration holds among every pair of series (and with unit elasticity in all cases) but not between Italy and Germany. Therefore,
- We can rule out the possibility that Italy and Germany are involved in any of the structural relations.

In the light of the findings reported in our earlier study on short-term rates, we would expect convergence of Italian and German rates, and therefore the fact that these rates do not cointegrate with each other and consequently with the others comes as a surprise. Therefore, we investigate the issue further. It is useful to recall that we had assumed earlier that either risk premia in the G-7 group were stationary, or there existed a stationary relation between them. More specifically we had defined long-term interest rates as (see Cuthbertson, 1996):

3) $$(1+LR)^T = \prod_{i=1}^{T}(1+rw_{t+i}+\lambda)$$

where *LR* are long-term rates, *r* refers to short-term rates, *w* are weights attached to each of the *r*s and λ is the term that we called risk premium, which is made up of country-specific factors. If we make the reasonable assumption that expectations on future inflation enter the risk premium term, having found that cointegration holds between the short rates of Italy and Germany, we can test for cointegration between the inflation rates of Italy and Germany to investigate the cause of non-convergence in long-term rates. The results displayed in table 5 indicate full rank of the system and therefore stationarity of the two inflation rates. This result is clearly unacceptable because we had already performed ADF-tests on the two series obtaining results that indicate non-stationarity of the two series as displayed in table 6.

We can then test the stationarity of the two cointegrating vectors indicated by the rank test to check whether either of them is indeed stationary. The answer is definitely no. The ADF-tests (see Table 7) cannot reject the hypothesis of a unit root in both cointegrating vectors indicating that the inflation rates of Italy and Germany are not cointegrated and can actually play a role in explaining non-convergence of Italian and German long-term interest rates.

Table 5. Cointegration test between the inflation rates of Italy and Germany

Eigenvalue	loglik for ratio					
	130.258 0					
0.210912	139.259 1					
0.085497	142.655 2					

Ho:rank=p	-Tlog(1-\mu)	using T-nm	95%	-T\Sum log(.)	using T-nm	95%
p == 0	18*	14.69*	14.1	24.8**	20.23**	15.4
p <= 1	6.793*	5.541*	3.8	6.793**	5.541*	3.8

Table 6. ADF-test on inflation rates of Italy and Germany

ADF(ita-infl) = -0.9596	Critical values: 5%=	-2.9,	1%=	-3.519
ADF(ger-infl) = -2.326	Critical values: 5%=	-2.9,	1%=	-3.519

Table 7. ADF-test on Cointegrating vectors indicated from rank test in table 4.
ADF(CIvec1) = -1.872 Critical values: 5%= -2.903, 1%= -3.525
ADF(CIvec2) = -2.584 Critical values: 5%= -2.903, 1%= -3.525

Moving on to the second part of our investigation, we next rank the irreducible cointegrating vectors indicated from the tests carried out before and obtain table 8.

Table 8. Ranking of irreducible cointegrating vectors

IC vectors	*Eigenvalue*	*standard deviation*	*exogeneity restrictions*
USA-Canada	*0.220188*	***0.706543***	*Canada exogenous*
Canada-UK	*0.186922*	***0.879522***	*UK exogenous*
UK-Japan	*0.185781*	***0.947805***	*feedback*
Canada-France	*0.365902*	***0.970788***	*Canada exogenous*
USA-France	*0.251763*	*1.041766*	*feedback*
Canada-Japan	*0.184848*	*1.125332*	*Japan exogenous*
USA-UK	*0.16136*	*1.210084*	*UK exogenous*
*Germany-Japan***	*0.158347*	*1.282394*	-
*UK-Germany***	*0.139702*	*1.307491*	-
UK-France	*0.328857*	*1.403686*	*UK exogenous*
USA-Japan	*0.203316*	*1.421847*	*Japan exogenous*
Japan-France	*0.219263*	*1.445948*	*Japan exogenous*
*Canada-Germany***	*0.0620838*	*1.499267*	-
*Italy-France***	*0.0506678*	*1.584742*	-
*USA-Germany***	*0.0645576*	*1.726671*	-
*Germany-France***	*0.10124*	*2.062937*	-
*Canada-Italy***	*0.110588*	*2.179342*	-
*USA-Italy***	*0.137558*	*2.275801*	-
*Italy-Japan**	*0.163975*	*2.467952*	-
*UK-Italy**	*0.260275*	*2.480295*	-
*Germany-Italy***	*0.0911157*	*2.865093*	-
* test indicates full rank			
** no cointegration has been found between the series			

Some comments are in order. First, notice that cointegration with homogeneous coefficients has been imposed between the non-cointegrated series. Therefore, the standard deviation has been calculated for these cointegrating vectors and they have been ranked accordingly. Second, the exogeneity restrictions on the vectors involving Italy and Germany have been omitted for they are not very meaningful, as they are performed on non-cointegrated series under the hypothesis of cointegration.

We find that in general, not surprisingly, non-cointegrated series display a higher variability than cointegrated ones, with the exception of the relations between German rates, and Japanese and UK rates respectively. This is not something to worry about because the ranking in table 8 is made in terms of the absolute magnitude of the standard deviations. To make this point clear and to interpret more easily the ranking of the IC vectors it is convenient to use a different format for the results and to display them as in table 9.

Table 9. Ranking of IC vectors per country.

	US	CA	JP	GER	FR	IT	UK
US	-	**0.70**	1.42	*1.72*	1.04	*2.27*	1.21
CA	0.70	-	1.12	*2.7*	**0.97**	*2.17*	**0.87**
JP	1.42	1.12	-	*1.28*	1.44	*2.46*	0.94
GER	*1.72*	*2.7*	*1.28*	-	*2.06*	*2.86*	*1.3*
FR	1.04	0.97	1.44	*2.06*	-	*1.58*	1.4
IT	*2.27*	*2.17*	*2.46*	*2.86*	*1.58*	-	*2.48*
UK	1.21	0.87	**0.94**	*1.3*	1.4	*2.48*	-

We have used bold for the vectors we consider irreducible and structural on a column by column interpretation and italic for the ones involving Italy and Germany.

Given a rank of four, on a minimum standard deviation criterion, US and Canada is a structural relationship and so are UK and Canada and UK and Japan. The fourth involves France and Canada again. If we collate this information with the one on the "exogeneity restrictions" column in table 8, we obtain the overall picture. On the whole, we can conclude that the evidence supports the hypothesis that larger and more stable economies can achieve policy objectives more successfully by accommodating rather than driving other countries' policies (see Martin, 1997). This would explain the endogeneity of the US long rates in the IC relations with all the other countries. The Canadian rate can be identified as the driving force. This is a reasonable result given the strong linkage between Canadian and US markets. Notice that the French rate follows the Canadian one in another of the fundamental relationships. Interestingly, Canadian long rates accommodate the UK ones according to one of the IC relations, and Japan and UK are linked by a relation with a causal feedback.

5. Conclusions

In this paper we have examined the causal linkages that exist between the G-7 long term interest rates. Specifically, we have applied the so-called Extended Davidson's Methodology (EDM), which is based on the innovative concept of an irreducible cointegrating (IC) vector, defined as a subset of a cointegrating relation that does not have any cointegrated subsets. The application of this method has confirmed the importance of testing for irreducibility as a diagnostic. We have in fact obtained a rank of four for the system of long-term rates compared to a rank of two as indicated by the rank test on the whole group of series. The ranking of the IC relations according to the criterion of minimum variance and exogeneity tests on all IC relations have provided us with a methodology to distinguish between structural and solved relations and to clarify the causal structure that links the rates respectively.

We have been able to isolate four irreducible structural relations that link the US and Canada, and the latter with the UK, France and Japan respectively. It appears that the driving force is Canada, which is linked to the US, UK and France in three out of the four fundamental relations, and which seems to be a reference point for the US that behave in an accommodating fashion. It is worth recalling that long-term rates in Italy and Germany, being non-cointegrated, seem to be determined by factors that are specific to these two countries. To investigate this matter further, we have analysed their expected inflation rates that are likely to affect the respective risk premia. The results obtained provide evidence that non-convergence of Italian and German rates is justifiable on the basis of non-convergence of their risk premia, and therefore interest rates in these two countries are independent from every other country's rates.

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