Measuring the Capital Stock in Russia;
An Unobserved Component model

By

Stephen G Hall
The Management School
Imperial College of Science technology and medicine

And

Olivier Basdevant
Sorbonne University
And
Pretoria University

This work was undertaken as part of the European Commission TACIS programme under the European Expertise Service under project RF33.

Introduction

This paper is part of a wider project to model the Russian economy during its current process of transition. The key aspect of the transition process being examined here is the behaviour of the supply side of the economy and in particular the very difficult task of measuring the capital stock in Russia. In essence the problem being examined is quite simple; the official measures of the capital stock in Russia show no decline over the transition period. In particular despite the major fall in GDP and production generally the official decline in factors of production (labour and capital) is virtually insignificant. Anecdotally the explanation for this seems to be clear. A large part of the capital stock that was in use under the soviet system was simply not profitable at world prices. As the Russian economy has been opened up to world prices and competition from international imports this situation has become apparent and large sections of the capital stock are not being used. One way to view this is by defining a concept of an effective capital stock as opposed to the official measured capital stock. Anecdotal evidence would suggest that this effective capital stock has declined considerably in recent years.

In order to build a model that effectively captures the current supply constraints of the Russian economy we need to derive a measure of the effective capital stock. Our belief is that many of the main developments in Russia have been driven by this supply side constraint, e.g. the rapid decline in investment levels, the fall in exports and the rise in imports. The purpose of this note is to propose a technique based on the Kalman Filter, which will estimate a data series for the effective capital stock. This will be treating the capital stock as an unobserved component and using the Kalman filter as a way of deriving a measure of this unobserved variable.

The plan of the paper is as follows; Section 2 outlines a standard state space formulation and gives the Kalman Filter equations, which provide estimates of the unobserved state component. Section 3 gives a specific state space representation of our problem. Section 4 gives the results of the estimation exercise and section 5 draws some conclusions.

2. The Kalman Filter and the state space form.

In this section a standard state space formulation is presented along with the appropriate Kalman filter equations for the univariate case, following Harvey (1987) or Cuthbertson Hall and Taylor(1992).

Let

$$Y_t = \delta' z_t + \beta x_t + \varepsilon_t \qquad 1.$$

be the measurement equation, where y_t is a measured variable, z_t is the state vector of unobserved variables, δ and β are vectors of parameters, x_t is a vector of weakly exogenous conditioning variables and $\varepsilon_t \sim NID(0,\Gamma)$. The state equation is then given as:

$$z_t = \Psi z_{t-1} + \gamma x_t + \psi_t \qquad 2.$$

Where Ψ and γ are parameters and $\psi_\tau \sim NID(0,Q)$, Q is sometimes referred to as the hyperparameters

The appropriate Kalman filter prediction equations are then given by defining z^*_t as the best estimate of z_t based on information up to t, and P_t as the covariance matrix of the estimate z^*_t, and stating:

$$z^*_{t|t-1} = \Psi z^*_{t-1} + \gamma x_t \qquad 3.$$

And

$$P_{t|t-1} = \Psi P_{t-1} \Psi' + Q \qquad 4.$$

Once the current observation on y_t becomes available, we can update these estimates using the following equations:

$$z^*_t = z^*_{t|t-1} + P_{t|t-1}\delta(Y_t - \delta' z^*_{t|t-1} - \beta x_t)/(\delta' P_{t|t-1}\delta + \Gamma) \; 5.$$

And

$$P_t = P_{t|t-1} - P_{t|t-1}\delta\delta' P_{t|t-1} / (\delta' P_{t|t-1}\delta + \Gamma) \qquad 6.$$

Equations (4)-(6) then represents jointly the Kalman filter equations.

If we then define the one-step-ahead prediction errors as,

$$v_t = Y_t - \delta' z^*_{t|t-1} - \beta x_t \qquad 7.$$

Then the concentrated log likelihood function can be shown to be proportional to (see Crowder(1976) and Schweppe(1965))

$$\log(l) = \sum_{t=k}^{T} \log(f_t) + N\log(\sum_{t=k}^{T} v_t^2 / N f_t) \qquad 8.$$

Where $f_t = \delta' P_{t|t-1}\delta + \Gamma$ and N=T-k, where k is the number of periods needed to derive estimates of the state vector; that is, the likelihood function can be expressed as a function of the one-step-ahead prediction errors, suitably weighted.

In general then the Kalman Filter will provide estimates of the unobserved variable z_t, while maximising the likelihood function can provide estimates of any other desired parameters. The Kalman Filter is an estimation technique which allows us to either estimate time varying parameter models by interpreting z as a vector of parameters to be estimated, or it allows us to think of z as a set of unobserved variables. It is this second interpretation which allows us to use the Kalman Filter and its associated smoothing algorithm as a means of deriving optimal estimates of an unobserved series such as the Russian capital stock. Further details of the Kalman Filter and the smoothing algorithms may be found in Cuthbertson, Hall and Taylor(1992) or Harvey(1992).

3. Formulating the Capital Stock Problem in State Space.

In this section we will show how the problem of estimating the capital stock may be formulated as a state space problem amenable to solution by the Kalman Filter. Define total production in the economy to be Y_t , employment to be L_t, the effective capital stock to be K_t and assume that the underlying technology is Cobb-Douglas. Output is a measured variable and it may be assumed to be generated by the following measurement equation.

$$Y_t = \alpha_0 + \alpha_1 \log(K_t) + \alpha_2 \log(L_t) + \alpha_3 t + e_t \qquad 9.$$

Where t is a trend representing neutral technical progress. We view Y, L, and t as variables which are directly observed and K and the depreciation rate of capital (δ_t) as unobserved state variables. The effective capital stock is generated by the following state equation.

$$\log(K_t) = \log(K_{t-1}) + \log(1 + I_t / K_0) - \delta_{t-1} + v_{1t} \qquad 10.$$

This is a standard state equation in the log (K_t), $1+I_t/K_0$ is investment expressed as a proportion of the initial capital stock[1], δ_{t-1} is the unobserved rate of depreciation and v is an error term. The unobserved depreciation rate is then assumed to be generated by a second state equation.

$$\delta_t = \delta_{t-1} + v_{2t} \qquad 11.$$

So the depreciation rate follows a random walk which allows it to freely increase to allow for the higher rate of scrapping during the transformation period in Russia. Together these three equations form a simple state space model and the Kalman Filter may be used to estimate the unobserved effective capital stock and the rate of depreciation. The associated smoothing algorithm to the Kalman filter then provides optimal estimates of these components based on the complete data sample.

4. Results[2]

[1] The initial capital stock comes from official data sources, we recognise that there are serious measurement error problems in the Russian capital stock and an additional problem is that no data seems to be available after 1995. For our purposes here however we would argue that the broad profile of the derived capital stock is more important than its precise level.

[2] The estimation work was carried out in the general Kalman Filter option of Stephen Halls' regression package REG-X; this package is freely available from HTTP://www.economics.ox.ac.uk/research/cim in the form of a zip file.

In implementing the above model we must begin by calibrating some of the parameters. This is due to the fact that it is not possible to estimate a co-efficient on an unobserved state variable. We must therefore fix the parameter α_1, in principle we could estimate the coefficient on labour (α_2) as this is directly observed. In order to produce a stable long run model we would require constant returns to scale and hence $\alpha_2 = 1 - \alpha_1$. It is therefore possible in principle to fully estimate the model subject to the assumption of constant returns to scale. However, in the case of Russia we are facing a number of problems which are rather more extreme than usual: first the available data sample is much shorter than we would like, essentially spanning only four years from mid 1994 to late 1998. Second the data itself is of poor quality, both the output and employment data are seriously miss-measured in a way which almost certainly gets worse during our sample. The employment data especially is strongly affected due to extreme under use of labour and labour hoarding by firms. Thirdly, even over this short sample period there are serious structural changes in Russia. For all these three reasons we found that free estimation of α_2 subject to constant returns to scale simply did not produce a reasonable set of parameter estimates. We therefore decided to calibrate α_1 on the basis of the approximate share of total income going to capital and labour in the Russian economy and hence $\alpha_1 = 0.4$ *and* $\alpha_2 = 0.6$. While we recognise the undesirability of calibration relative to proper estimation in any situation we would point out here that our main objective is to derive a measure of the capital stock consistent with a reasonable production function. We therefore experimented with a range of values for α_1 *and* α_2 subject to constant returns to scale. Although the exact size of the change in the capital stock was affected by a change in these parameters, the overall profile of the derived state variable did not change dramatically and the implications of our analysis proved not to be sensitive.

Further, since we are modelling in the logs of the variables, the scaling of the unobserved capital stock will be determined by α_0. This parameter may therefore be fixed at any arbitrary level and will simple change the units of measurement of the capital stock and so we assign a value of zero to this parameter.

The state variable must be initialised as the Kalman filter recursions begin from an initial estimate of z_0 and P_0. In this application a diffuse prior has been used on z_0. This means that an arbitrary value of $z_0=1$ is set and P_0 is set to a very large number (effectively infinity). Over the first k-1 periods of the recursion, where k is the number of state variables plus one, the filter calibrates the initial values of the state vector to produce a perfect fit. The recursion then begins from this point. This is the reason why the likelihood function in (8) is evaluated from period k onwards only.

Applying the Kalman filter to the specific problem outlined above using monthly data from May 1994 to October 1998 then yields an estimate of the effective Russian capital stock based on the full smoothing algorithm. The results of this procedure are presented in Figures 1 and 2. Figure 1 contrasts the effective capital stock[3] (the dotted line), with a scaled time series for the logarithm of GDP (the solid line). The effective capital stock shows a sharp decline especially in 1998. Unfortunately there are no official comparable capital stock figures for this period although the annual data which is available up to 1996 has, so far, recorded no decline in the capital stock at all. Figure 2 shows the monthly rate of depreciation of the capital stock, again produced by the smoothing algorithm. This implies a very rapid depreciation of almost 10% a year in the mid 1990s, falling steadily through the period until it reaches a minimum of 4.5% per annum in early 1998. The depreciation rate then rose somewhat during the collapse in output to just under 6% a year.

The main estimated parameters of the model are the elements of the variance matricies $\Gamma \, and \, Q$, we estimate these by forming a concentrated likelihood function (8) which means that one element of them is fixed (in this case Γ) and the other two are then estimated relative to this value. We can then calculate the maximum likelihood estimate of the concentrated parameter and re-scale all the variances relative to this one. Only the estimated hyperparameters will have associated standard errors and 't' statistics, as the concentrated parameter is not part of the maximisation. The following table then gives estimates of the three hyperparameters.

Table 1: The estimated Hyperparameters

Parameter	Estimate	't' Statistic
Γ	0.00000012	-
Q_{11}	0.000015	1.8
Q_{22}	0.0036	3.5

The fall in the capital stock at the end of the period is mainly explained by a rising depreciation rate. Actual investment levels were very low during the whole period and there was no significant additional fall in investment at the end of the period to

[3] It is possible to give the state variables standard error bands based on the smoothed covariance matrix $P_{t|T}$, however this estimate may considerably understate the true uncertainty in the capital stock as it is calculated on the assumption that the model and its parameters are correct. We prefer therefore not to include these standard errors in figure 1. For completeness they have an average value of 0.025 for the first state variable, which puts a 95% confidence interval around the capital stock of about 5% on either side and an average value of 0.07 for the depreciation rate so that the 95% confidence band would be + or -. 0.14

cause the fall in the capital stock. The error term in the first state equation also explains very little of the fall as evidenced by its much small estimated variance. The major downturn in the Russian economy in the mid 90s is also explained primarily by a very high depreciation rate which well outstripped the prevailing investment rates. This confirms the anecdotal view of widespread scrapping taking place as Russia was opened up to world prices.

5.Conclusions

This note has proposed a new technique for estimating the stock of capital in an economy, which is undergoing transition and where the official data is either unavailable or believed to be highly inaccurate. This technique has been applied to the case of Russia using monthly data over the period 1994-1998. It suggests that over this period there has been a substantial reduction in the stock of effective capital of around 10%.

Figure 1: GDP and the Capital Stock (indices 1994m7=1)

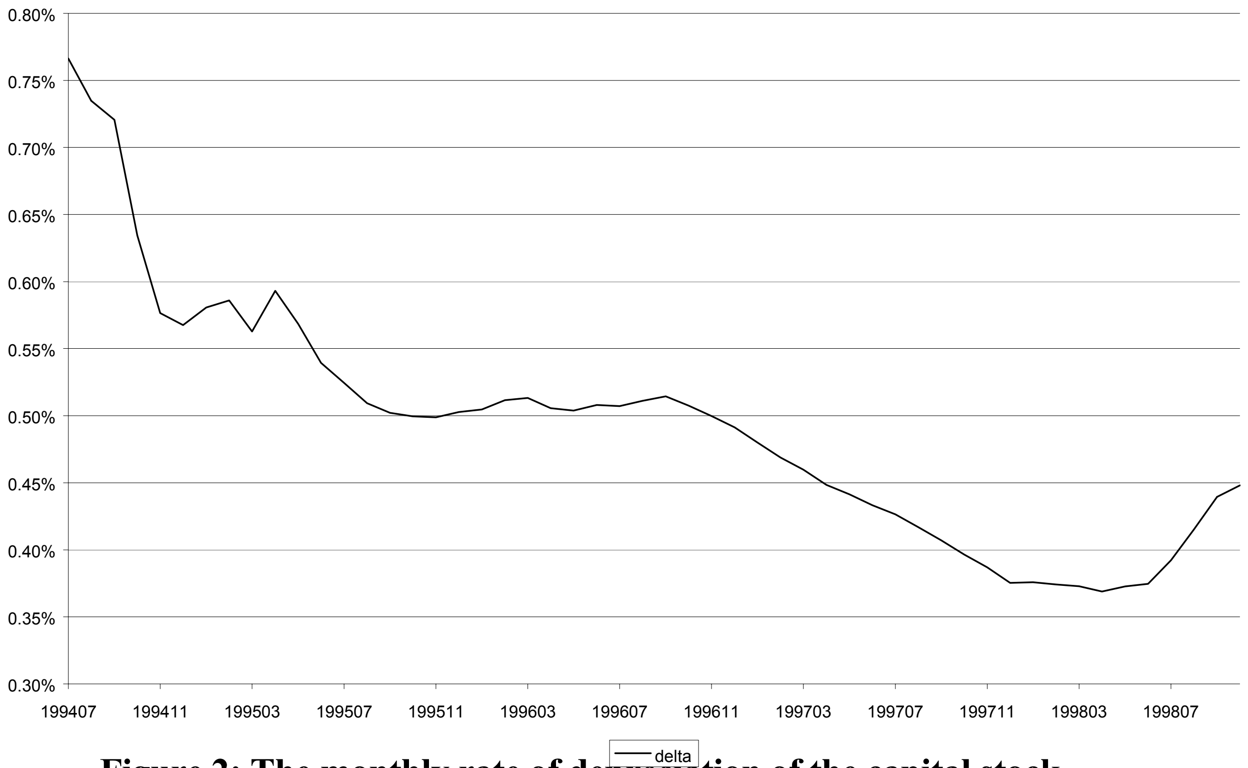

Figure 2: The monthly rate of depreciation of the capital stock

References

Crowder, M.J. (1976), `Maximum Likelihood Estimation for Dependent Observations', Journal of the Royal Statistical Society, Series B, vol.38, no.1, pp.45-53.

Cuthbertson K., Hall S.G. and Taylor M.P. (1992) 'Applied Econometric Techniques' University of Michigan Press, Ann Arbor.

Harvey, A.C. (1987), `Applications of the Kalman Filter in Econometrics', in T.F.Bewley (ed.) Advances in Econometrics: Fifth World Congress, vol. 1, Econometric Society Monograph No.13, Cambridge University Press.

Schweppe, F. (1965), `Evaluation of Likelihood Functions for Gaussian Signals', IEEE Transactions on Information Theory, vol.11, no.1, pp.61-70.