

**Identifying The Wage Bargaining Process  
In Germany**

**By**

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# 1. Introduction

Standard models of wage determination have followed the general bargaining approach for many years (see for example Cartter(1951) or Leontief(1946)) and since the seminal paper of McDonald and Solow(1981) this approach has been dominant in the empirical literature. Spawning the many papers, which have followed in the Nickel/Layard tradition of empirical models. However, a major criticism of this approach was made by Manning(1993) who pointed that within the conventional model being applied to the labour market bargain, wage determination itself was not formally identified. Thus the key relationship in the bargain could not be identified or properly understood within this framework. While wage models have continued to be used (e.g. Bean1994)) this basic problem has remained at the heart of the approach.

In this paper we build on some recent proposals by Chamberline, Hall, Henry and Satchi(2000) which point out that within the new modelling approached based around systems of cointegrating equations it is now possible to identify the long run components of the bargaining model properly (a brief survey of these new techniques may be found in Hall, Mizon and Welfe(2000)). The intuition behind this insight is relatively straightforward. The essence of the Manning problem is that in a bargain both sides of the market affect the bargain and so all variables enter the wage equation. If no variables are excluded from this equation then the equation cannot be identified by the conventional rank or order conditions. Hence the bargaining process is unidentified. The argument put forward by Chamberline, Hall, Henry and Satchi(2000) is that the formal conditions for identifying the long run parameters of a cointegrated system (again the rank and order conditions, but now for the long run parameters only) are not a function of restrictions on individual equations but on individual cointegrating vectors. Hence an individual vectors may be structurally identified even though every vector appears in every equation, so no variables are excluded from any equations. Thus even though all sides of the bargain may be included in the wage equation the structural long run parameters are still identified.

This paper then applies these ideas to the German wage bargaining process in an attempt to estimate a structural model of the German bargaining process. Section 2 of the paper then outlines the new developments in formal identification, which make this approach possible. Section 3 then revisits some of the theory of wage bargaining in the light of section 2 to see if the required theoretical restrictions exist in the theory. Section 4 then outlines the empirical model, the German data to be used and follows an estimation strategy proposed by Greenslade Hall and Henry(2000) to implement the empirical model. Section 5 then concludes.

## 2. Identification in Non-Stationary Systems

The basic problem of identification in wage-price systems which has confounded economists since the seminal paper of Manning(1993) may be stated quite simply. In a bargaining model, all sides of the bargain affect wages and hence no variables may be excluded from the wage equation. Without any exclusion restrictions the conventional rank and order conditions for identification cannot be met and hence the wage equation is generically unidentified. However in this section we will argue that this is not the case for non-stationary cointegrated systems. Here identification is not carried out with respect to individual equations but with respect to individual cointegrating vectors. All vectors may be part of a particular equation (e.g. the wage equation) without losing identification. To make this crucial point clear, in this section we will outline recent developments in the identification of cointegrated systems.

Time series econometrics has been revolutionized by the developments in multivariate cointegration over the last fifteen years. Much of this advance has been achieved by the development of the statistical theory relevant for the analysis of cointegrated systems (see e.g., Johansen, 1995b). The issue of identification and economic interpretation of the parameters in such systems, though important, was not central to the early development of this theory. However, the Granger Representation Theorem (see Engle and Granger, 1987) by establishing that the vector autoregressive model (VAR) and the vector equilibrium correction model (VEqCM) are observationally equivalent, helped to bridge the gap between the statistical model and an economic interpretation of it. Subsequently much more attention has been paid to the development of models that are economically interpretable simplifications of the VAR. One approach is that which adopts the ‘structural’ VAR (SVAR) using economically interpretable restrictions to achieve identification, with further restrictions being testable as over-identifying hypotheses (see e.g., Davidson and Hall 1991 and for recent reviews Canova, 1995 and Pesaran and Smith, 1998). An alternative approach, developed in Hendry and Mizon 1993, exploits the fact that a statistically well specified full rank VAR is identified, and provides a valid basis against which to test simplification hypotheses such as those that arise from economic theory considerations and from the empirical evidence

In the SVAR approach to modeling the parameters of interest  $\mathbf{f}$  are a function of  $(\mathbf{A}_0, \mathbf{A}_1, \dots, \mathbf{A}_p, \mathbf{c}, \Phi)$  in:

$$\mathbf{A}_0 \mathbf{z}_t = \sum_{j=1}^p \mathbf{A}_j \mathbf{z}_{t-j} + \mathbf{c} + \mathbf{u}_t \quad \text{with} \quad \mathbf{u}_t \sim \text{IN}_N(\mathbf{0}, \Phi). \quad (1)$$

When  $\mathbf{A}_0$  is an  $N \times N$  matrix (which is often assumed to be non-singular although singularity is a possibility as pointed out by Davidson and Hall(1991)),  $\mathbf{A}_0 \mathbf{z}_t$  represents  $N$  linear combinations of the  $N$  variables in  $\mathbf{z}_t$  that characterise their determination in the context of some economic theory, especially those in which the simultaneous determination of  $\mathbf{z}_t$  is a feature. It is well known that  $(\mathbf{A}_0, \mathbf{A}_1, \dots, \mathbf{A}_p, \mathbf{c}, \Phi)$  are not identified.

Consider the associated reduced form closed VAR assuming  $p$  lag on a vector of  $N$  variables  $\mathbf{z}_t$ :

$$z_t = \sum_{j=1}^p D_j z_{t-j} + \mathbf{d} + \mathbf{e}_t \text{ with } \mathbf{e}_t \sim \text{IN}_N(\mathbf{0}, \Sigma), \quad (2)$$

Where  $\mathbf{D}_j$  is an  $N \times N$  matrix of autoregressive coefficients, and  $\mathbf{e}_t$  is a vector of  $N$  unobserved errors, which have a zero mean and constant covariance matrix  $\Sigma$ . Independently of whether the variables  $z_t$  are  $I(0)$  or  $I(1)$  the VAR (2)) can be re-parameterised as a VEqCM (see Johansen 1988, 1992c, and Hendry 1995a):

$$\Delta z_t = \sum_{j=1}^{p-1} \Gamma_j \Delta z_{t-j} + \Pi z_{t-1} + \mathbf{d} + \mathbf{e}_t, \quad (3)$$

Where  $\Delta$  is the first difference operator,  $\Gamma_j = -\sum_{i=j+1}^p D_i$  ( $j=1,2,\dots,p-1$ ) are the short run adjustment coefficient matrices and  $\Pi = -\left(\mathbf{I}_N - \sum_{i=1}^p \mathbf{D}_i\right)$  is the long run coefficient matrix.

We note that although the model has been defined with an  $N \times 1$  vector of intercepts  $\mathbf{d}$ , often there will be other deterministic variables included. For example, the deterministic variables might be  $\mathbf{d} + \mathbf{a}t + \mathbf{K}d_t$ , when  $\mathbf{d}$  is an intercept;  $\mathbf{a}t$  a linear trend restricted to the cointegration space (trend is so restricted since few variables exhibit quadratic trend), and  $\mathbf{d}_t$  some event specific dummy variables.

When  $\Pi$  has full rank  $N$  the variables  $z_t$  are  $I(0)$  and the parameters  $(\Gamma_1, \dots, \Gamma_{p-1}, \Pi, \mathbf{d}, \Sigma)$ , or equivalently  $(\mathbf{D}_1, \dots, \mathbf{D}_p, \mathbf{d}, \Sigma)$ , are all identified in that the maximum likelihood estimator of these parameters is unique. Since (2) and (3) are re-parameterisations of each other they are observationally equivalent, and the choice between them can be made on the basis of their interpretation. Indeed, an attraction of the parameterisation in (3) is the interpretation of its static long run solution,  $E(\Pi z_t + \mathbf{d}) = \mathbf{0}$  as the equilibria of the system, with  $(\Pi z_t + \mathbf{d})$  being the disequilibria at time  $t$ . Economic theory is often informative about such equilibria. The short run adjustment parameters  $\Gamma_j$  are also the subject of economic theory considerations concerning the time form of responses and speed of adjustment, though these are typically less precise than the hypotheses concerning equilibria. However, the parameters of interest ( $\mathbf{f}$ ) will not generally be those of (1) or (2), and so the identification and estimation of  $\mathbf{f}$  has to be considered separately.

In the class of linear dynamic systems the VAR in (2), or equivalently the VEqCM in (3), characterise the distribution  $\mathbf{z}_t | \mathbf{Z}_{t-1}$  and thus provide the reduced form of (1). This implies that:

$$\mathbf{D}_j = \mathbf{A}_0^{-1} \mathbf{A}_j \quad (j=1,2,\dots,p), \quad \mathbf{d} = \mathbf{A}_0^{-1} \mathbf{c}, \quad \text{and} \quad \Sigma = \mathbf{A}_0^{-1} \Phi (\mathbf{A}_0^{-1})', \quad (4)$$

And leads to the conventional discussion of identification in simultaneous equations models in which restrictions on  $(\mathbf{A}_0, \mathbf{A}_1, \dots, \mathbf{A}_p, \mathbf{c}, \Phi)$  are required for (4) to have a unique solution for  $(\mathbf{A}_0, \mathbf{A}_1, \dots, \mathbf{A}_p, \mathbf{c}, \Phi)$  in terms of  $(\mathbf{D}_1, \dots, \mathbf{D}_p, \mathbf{d}, \Sigma)$  or  $(\Gamma_1, \dots, \Gamma_{p-1}, \Pi, \mathbf{d}, \Sigma)$  (see e.g., Johnston, 1972, Greene, 1991). This is the conventional approach to identification which was first set

out by the Cowles commission and which lies at the heart of the Manning identification problem.

However the nature of identification changes in a crucial way when the variables being modeled are  $I(1)$ , but satisfy  $r < N$  cointegrating relationships  $\mathbf{b}'\mathbf{z}_t$  that are  $I(0)$ . This is often the case for macroeconomic time series, and modeling of wage behavior has clearly fallen within this class of models since the work of Hall(1986). In this case the rank of  $\Pi$  is  $r$  a feature that can be incorporated into the model by defining  $\Pi = \mathbf{a}\mathbf{b}'$  with  $\mathbf{a}$  and  $\mathbf{b}$  being  $N \times r$  matrices of rank  $r$  thus leading to the reduced rank or cointegrated VEqCM:

$$\Delta\mathbf{z}_t = \sum_{j=1}^{p-1} \Gamma_j \Delta\mathbf{z}_{t-j} + \mathbf{a}\mathbf{b}'\mathbf{z}_{t-1} + \mathbf{d} + \mathbf{e}_t \quad (5)$$

Note though that  $\mathbf{a}$  and  $\mathbf{b}$  are not identified since  $\mathbf{a}\mathbf{b}' = \mathbf{a}^+ + \mathbf{b}^{+'} = \mathbf{a}\mathbf{P}\mathbf{P}^{-1}\mathbf{b}'$  for any non-singular  $r \times r$  matrix  $\mathbf{P}$  (rotation). Hence in the reduced rank case, with the reduced rank imposed, neither the VAR in (2) nor the VEqCM in (3) is identified. In particular, it is necessary to determine  $r$ , and identify  $\mathbf{a}$  and  $\mathbf{b}$ , and there are many routes in which this might be achieved in practice - see Figure 1 in Greensale, Hall and Henry (1998) for a diagrammatic representation of the possibilities.

Since  $r$  is not known *a priori* its value has to be determined empirically, and this provides one possible starting point. A commonly adopted procedure is the maximum likelihood one developed by Johansen (1988), which employs likelihood ratio criteria for determining  $r$ , and for a given choice of  $r$  yields a unique estimate of  $\Pi$  of rank  $r$ . Since the short run adjustment coefficients  $\Gamma_j (j = 1, 2, \dots, p-1)$  and the error covariance matrix  $\Sigma$  are identified, unique unrestricted maximum likelihood estimates of these parameters are available for a given value of  $r$ . The Johansen procedure also produces unique estimates of  $\mathbf{a}$  and  $\mathbf{b}$  satisfying  $\Pi = \mathbf{a}\mathbf{b}'$  as a result of imposing the restriction that the resulting  $\mathbf{b}$  be orthogonal. This restriction amounts to a sufficient set of restrictions to exactly identify  $\mathbf{b}$  although of course these restrictions can have no economic interpretation and are just one arbitrary set of restrictions amongst an infinite set of restrictions which would achieve exact identification. This is the meaning of the statement that this estimator estimates the space spanned by the  $\mathbf{b}$  matrix.

A number of attempts have been made to impose more meaningful restrictions to identify  $\mathbf{b}$ . Phillips (1991) presented an approach to identifying  $\mathbf{b}$  by partitioning the set of variables  $\mathbf{z}_t$  into  $\mathbf{z}_{1,t}$  and  $\mathbf{z}_{2,t}$ , and giving  $\mathbf{b}$  the form  $(-I_r, \mathbf{B})$  to yield a block recursive structure

$$\begin{aligned} \mathbf{z}_{1,t} &= \mathbf{B}\mathbf{z}_{2,t} + \mathbf{v}_{1,t} \\ \Delta\mathbf{z}_{2,t} &= \mathbf{v}_{2,t} \end{aligned} \quad (6)$$

In which  $\mathbf{z}_{1,t}$  is a vector of  $r$  variables,  $\mathbf{z}_{2,t}$  is an  $(N-r)$  vector, and  $\mathbf{v}_{1,t}$  and  $\mathbf{v}_{2,t}$  are independent  $I(0)$  processes which in general are temporally dependent. The assumptions underlying (6) are sufficient to exactly identify the system, but this is achieved by assuming that  $r$  is known, by imposing a very restricted block-recursive structure which may not often correspond to a relevant economic theory, and the outcome is not invariant to the ordering of

the variables within  $\mathbf{z}_t$ . Saikkonen (1993) discusses the complete identification of a VEqCM that has a similar partition of  $\mathbf{z}_t$  into  $\mathbf{z}_{1,t}$  and  $\mathbf{z}_{2,t}$  to that in the Phillips (1991) system

The question of identifying the parameters of a SVAR which has  $r < N$  cointegrating vectors has received increased attention, and there is now a reasonably complete understanding of the process of identifying a structural cointegrated system - see inter alia Johansen (1994), Johansen (1995a), and Robertson and Wickens (1994). In such cases (1), written in VEqCM format, becomes:

$$\mathbf{A}_0 \Delta \mathbf{z}_t = \sum_{j=1}^{p-1} \mathbf{C}_j \Delta \mathbf{z}_{t-j} + \mathbf{A}^* \mathbf{b}' \mathbf{z}_{t-1} + \mathbf{c} + \mathbf{u}_t \text{ with } \mathbf{u}_t \sim \text{IN}_N(\mathbf{0}, \Phi) \quad (7)$$

with  $\mathbf{C}_j = \mathbf{A}_0 \Gamma_j$  ( $j = 1, 2, \dots, p-1$ ) and  $\mathbf{A}^* = \mathbf{A}_0 \mathbf{a}$ . Conditional on having chosen the cointegrating rank  $r$  it is necessary to consider the identification of the contemporaneous coefficients  $\mathbf{A}_0$  and the long run coefficients  $\mathbf{b}$ , and these are essentially separate issues in that there are no mathematical links between restrictions on  $\mathbf{A}_0$  and those on  $\mathbf{b}$ . In particular, since a  $\Pi$  matrix of rank  $r$  is identified and satisfies  $\Pi = \mathbf{a} \mathbf{b}' = \mathbf{A}_0^{-1} \mathbf{A}^* \mathbf{b}'$ , it follows that restrictions are required to identify  $\mathbf{b}$  even if  $\mathbf{A}_0$  were known. Conversely, restrictions on  $\mathbf{b}$  have no mathematical implication for the restrictions on  $\mathbf{A}_0$ . It remains possible though that the economic interpretation of a restricted set of cointegrating vectors  $\mathbf{b}' \mathbf{z}_t$  may have implications for the nature of restrictions on  $\mathbf{A}_0$  that will be economically interesting, particularly when  $\mathbf{A}^*$  is restricted via  $\mathbf{a}$ . Mathematical, and possibly economic, linkages do exist between restrictions on the adjustment coefficients  $\mathbf{a}$  and those required to identify  $\mathbf{b}$  - see Doornik and Hendry (1997).

The formal identification of  $\mathbf{b}$  is the main subject of Johansen and Juselius (1992) and Pesaran and Shin (1997) where it was demonstrated that a necessary condition for exact identification is that there are  $k = r^2$  restrictions. Johansen (1995a) and Pesaran and Shin (1997) also give a necessary and sufficient rank condition for exact identification, which for example rules out dependence amongst the  $r^2$  restrictions. In general if the number of available restrictions  $k < r^2$  the system is under-identified, if  $k = r^2$  the system is exactly identified, and when  $k > r^2$  the system is over-identified, and subject to the rank condition being satisfied the over-identifying restrictions are testable.

The key point to be made here is that the structural identification of  $\mathbf{b}$  in (7), from (2) or (3) is a function of restrictions (possible although not necessarily exclusion ones) on the elements of each row in  $\mathbf{b}$ , that is on each cointegrating vector. However this does not imply that any cointegrating vectors are excluded from any individual equation in the system. Hence it is possible to imagine an identified long run system which has wages being affected by all the cointegrating vectors in the system and yet which is still fully identified. The key to identification is however that theory gives rise to sets of cointegrating vectors which can be identified uniquely. This requires us to use theory in a rather different way from the traditional approach, which has simply been to determine what variables might be excluded from an individual equation. Instead we must ask what the long run structural relationships may be which might represent cointegrating vectors.

### 3. Bargaining models and identifying the wage equation.

In this section we wish to re-visit some of the basic theory on wage bargaining in the light of the formal discussion above with a view to seeing how far we may be able to identify the structural relationships, which underlie the bargaining approach to wage formation. We base this analysis firmly on the McDonald –Solow(1981) paper, which has the advantage of both being theoretically very clear and of developing a range of models of alternative bargaining structures. We wish to argue here, not simply that one specific bargaining model gives rise to the restrictions we need for identification but that the bargaining process will generally give rise to the required identification conditions. The underlying thrust of our argument is that the bargain over wages takes place between two sides; the firm and the labor suppliers (sometimes unions) and each of these sides will generally give rise to a cointegrating vector. Both of these vectors will be in the wage equation but both are identifiable.

#### Case 1

We begin by considering the McDonald Solow simple Monopoly Union case (there section 1.). This has two ingredients, the firm's objective and the union's, These objectives stay constant throughout the later cases, all that changes is the nature of the bargaining process. The Objectives are

$$\text{Firm; Maximise Profits } R(L)-wL \quad (8)$$

$$\text{Union Maximise Utility } L(U(w) - \bar{U}) \quad (9)$$

Where  $R(L)$  is the firm's revenue function,  $w$  is wages,  $L$  is employment and the unions want high wages but to avoid unemployment,  $\bar{U}$  depends on the disutility of work and the utility of the alternative wage.

The first model is one of a monopoly union, which unilaterally sets the wage and then allows the firm to set the employment level (the so called right to manage model). In this case the firm maximises its profits given wages and thus it will always satisfy

$$R'(L)-w=0 \quad (10)$$

This means that the union will maximise (9) subject to (10) with respect to wages to yield the following first order condition.

$$(U(w)-\bar{U})/wU'(w)+LR''(L)/R'(L)=0 \quad (11)$$

Clearly both equations (10) and (11) in this model represent equilibrium conditions which we would expect to hold in the long run and so both would be cointegrating vectors. Only 11 would enter the wage equation in this model as (10) only affects employment. However, the parameters in  $R''$  and  $R'$  are the same as in (10),  $R'$  is identical to 10 and  $R''$  is just its derivative so the theory would allow (10) to be identified by exclusion restrictions and (11) to be identified by cross equation restrictions. This model is therefore identified.

## CASE 2 The Efficient Wage Model

The second case in McDonald Solow allows for the wage and employment level to be simultaneously determined by a true bargaining process. The difficulty with case one is that it is not a paraeto efficient outcome and hence both the firm and the union could improve their situation simultaneously. The solution to this model is given in terms of a contract curve which defines the set of points of tangency which might arise, this contract curve is defined by the following equation

$$(U(w)-U(\bar{w}))/U'(w) = w- R'(L) \quad (12)$$

And of course this can be expressed as

$$(U(w)-U(\bar{w}))/U'(w) = B \quad (13)$$

$$w- R'(L) = B \quad (14)$$

Of course if the economic environment, including the bargaining strengths of the two participants were to remain stable then B would be a constant and this would imply that both (13) and (14) will be separate cointegrating vectors.

We do however have to ask what happens to this equilibrium as factors affecting the bargain, or the general economic environment change. Here McDonald Solow demonstrate two important features. First the contract curve is bounded in employment to lie within a limited range. Second changing economic circumstances can shift the whole contract curve but of course when this happens (13) and (14) still hold. What this means of course is that B can change over time but only to a limited extent. This is hardly a surprising conclusion to any theorist, (14) is effectively saying that the monopoly power in the economy can drive a wedge between the firms marginal revenue product and the real wage. But it is inconceivable that this wedge could grow in an unbounded way. We may be pushed away from the perfectly competitive equilibrium but the departure from the competitive equilibrium must be by a bounded, stationary amount.

So if we accept that at most B is a stationary stochastic process then clearly (13) and (14) both represent cointegrating vectors and they are clearly identified, as there is essentially nothing in common between them. Of course the wage equation and potentially the employment equation in this model would both contain both cointegrating vectors but this is irrelevant to their identification.

One final possibility remains here, which is not discussed in the McDonald Solow paper. B is essentially capturing the relative bargaining strength of the firms and unions. It is certainly empirically plausible that this relative bargaining strength has changed in a non-stationary way over time. Union membership in the UK has been falling steadily over the 1980s and 1990s and legislative changes have successively weakened the union position. This may mean that B is itself a non-stationary process. Identification of the system can however still be maintained under the assumption that B is a function of a set of variables, such as legislation or union membership. So if we define a vector of such variables, Z, and assume that  $B=g(Z)$ . Then we may restate (13) and (14) as



$$(U(w)-U(\bar{w}))/U'(w) = g(Z) \quad (15)$$

$$w - R'(L) = g(Z) \quad (16)$$

And again identification of these two vectors is straightforward.

McDonald and Solow then discuss less formally in their paper a range of possibilities which might affect the contract curve including the possibility of a dominant union and a dominant firm and the form of various bargains which might be struck including a Nash solution and various more specific solutions. The essence of their analysis is that under almost all circumstances the outcome is driven by the relationships of the contract curve. The particular model may select a particular point on the contract curve or even drive a wedge between the contract curve and the outcome but the final solution will still be defined with respect to the basic contract curve (12) and hence the two basic cointegrating vectors will be present.

## 4.1 The Empirical Model

In this section we outline the system of equations, which will allow us to implement the full identification of the wage-price system. As with all modeling we have to make pragmatic decisions to limit the size of the system to be ultimately estimated. We do this by focusing on our primary interest of estimating a wage equation. To do this properly we need to include in our system of equations any equations, which may contain cointegrating vectors, which are common with the wage equation. Of course it is possible that some of these equations will contain further cointegrating vectors and then if these affect yet other variables these should also be included in the system. The data for Germany will be defined and discussed in detail in Appendix A.

With this in mind then the wage equation will contain the marginal revenue cointegrating vector and the union utility function vector. The price equation is generally also considered to be a function of the marginal revenue condition; this gives us the basic two equations. The marginal revenue condition of course contains parameters in common with the production function, so this relationship should also be included in the system. This gives us the basic three equation system, other possible equations which might need to be included are Import prices or possible exchange rates if these should prove to be not weakly exogenous. The identified, theoretical model will then be set up as a VEqCM with the following three basic equations being the core of the model and additional equations being added if weak exogeneity tests on the unrestricted model require them. If we assume that the production function is Cobb-Douglas with constant returns to scale and an autonomous technical progress effect then the following three equations represent our core system.

$$\Delta W = \nabla + \mathbf{a}_{11}(W - PPI - \mathbf{b}_1 T - \mathbf{b}_2 K + \mathbf{b}_2 L) + \mathbf{a}_{12}(W - CPI - \mathbf{b}_3 BEN - \mathbf{b}_4 U - \mathbf{b}_5 UNION)$$

$$\Delta P = \nabla + \mathbf{a}_{21}(W - PPI - \mathbf{b}_1 T - \mathbf{b}_2 K + \mathbf{b}_2 L) \quad (17)$$

$$\Delta Y = \nabla + \mathbf{a}_{31}(Y - \mathbf{b}_1 T - \mathbf{b}_2 K - (1 - \mathbf{b}_2)L)$$

Where all variables are measured in Logs except for T (time) and U (the percentage rate of unemployment). The other variables are, W, nominal wages, PPI, the producer price index, CPI, the consumer price index, K the capital stock, L, employment, BEN, unemployment

benefits, Union, union membership rates and Y is gross domestic production.  $\nabla$  represents all the dynamic terms in the system, in the unrestricted VAR approach this will imply a complete set of dynamics up to an adequate lag length, or we may interpret this as a restricted parsimonious set of dynamics following the arguments of Greenslade, Hall and Henry(2000).

Note that the wage equation has every variable in it, but as they are in two separate cointegrating vectors each vector is identified according to the criteria outlined in section 2. In fact the system is heavily over identified. It is also worth noting that as Y is not actually in the first two equations there would be no conventional weak exogeneity problem of dropping the production function from the estimation (as is usually done in the Nickel-Layard literature. However, as our objective is to produce the best estimates of the long run identified coefficients adding the production function to the system should considerably improve the efficiency of the system estimation.

It would also seem to be highly plausible that there should be a cointegrating vector linking the CPI with the PPI and possibly W as it would seem surprising if the relative prices in the system were trending in the long run.

## 4.2 The Data Analysis

The first and most crucial step is the determination of the cointegrating rank of the system of variables, It has been emphasised in Greenslade Hall and Henry(2000) that the conventional tests of the cointegrating rank can have rather poor power and size for large systems and small data sets, and the applied researcher need to bare this in mind at this crucial stage in the analysis. At this stage we have a theoretical expectation of at least three long run vectors with probably at least one more and we would give some weight to this expectation in our final determination. We begin by presenting the conventional Johansen likelihood ratio test of the cointegrating rank of the system along with a small sample correction.

**Table 1. Testing the Cointegrating Rank of the System**

Ho: $r=$	Asymptotic LR	Small Sample LR	95% Critical Value
0	505.5	383.4	192.9
1	245.7	186.3	156.0
2	182.8	138.6	124.2
3	134.3	101.8	94.1
4	90.1	68.3	68.5
5	53.2	40.4	47.2
6	28.1	21.3	29.7
7	16.0	12.1	15.4
8	5.0	3.5	3.8

Estimation carried out from 1961Q4-1999Q4, the model is a VAR(4) with unrestricted constants, there is also an additional deterministic trend in the VAR.. The SBC criteria suggested a VAR length of 3 while the AIC criteria suggested a much longer lag, based on the SBC criteria and in view of the fact that we were using quarterly data a VAR length of 4 seemed adequate.

On a strict interpretation of the asymptotic LR test we would conclude that there are 9 cointegrating vectors, which would imply that all the variables are stationary. This is not a tenable hypothesis and is in keeping with the Monte carol results of Greenslade, Hall and Henry(2000). The strict conclusion from the small sample corrected LR tests are that there are 4 cointegrating vectors linking this set of variables. This is a more tenable conclusion, it fits well with our prior expectations and so we propose to continue n the basis of this assumption.

The next stage is to see if any of the variables in the system may reasonably be treated as weakly exogenous. This would allow us to reduce the size of the estimated model by dropping these equations from the estimated system. The following table presents the Wald test for each variable in our system.

**Table 2. Testing Weak Exogeneity**

Variable	Wald test $\chi^2(4)$	Wald test $\chi^2(4)$	Wald test $\chi^2(4)$
K	447.0	364.8	348.7
Y	41.4	36.2	32.4
UNION	1.8	-	-
W	28.6	25.3	16.0
E	19.6	23.3	27.0
BEN	13.0	13.2	-
PPI	34.6	39.6	46.2
CPI	66.1	66.5	56.5
U	23.8	31.1	35.0

In this table we begin by calculating the weak exogeneity tests of the  $\mathbf{a}$  matrix conditional on the whole system (this is the first column of results in table 2). This shows that we may easily accept the hypothesis that Union membership is exogenous. We then repeat the weak exogeneity tests conditional on this assumption, in fact very little changes by reconditioning the system. The only variable, which is close to the borderline of accepting weak exogeneity, is the Benefit variable. It seems theoretically plausible that this variable should be weakly exogenous so it was decided to again recondition the system on this assumption, these results are then presented in the final column of table 2. The only dramatic change is that the test value for wages then falls somewhat, although not to below the 95% critical value. We decided, given these results to proceed on the assumption that union membership and benefits are weakly exogenous in this system but that everything else is to be treated as endogenous.

This gives us a 7-equation system to estimate with four long run relationships, which we hope to identify as the production function, the marginal product condition, the unions objective condition and a relationship relating the prices in the system. The fully identified long run system may then be set up in fully dynamic VecM form and estimated by maximum likelihood. Following the arguments in Greenslade, Hall and Henry(2000) we seek to form a set of dynamic terms which are as parsimonious as possible, each equation includes (at least initially before simplification) a dummy variable which is 1 after 1991 and a split time trend which starts in 1991 to help capture any structural instability which may have occurred following on from German unification. This procedure produced the following estimates for the long run structural relationships.

$$\text{ECM1} \quad Y = 0.002 T + 0.42 E + 0.58 K$$

$$(13.7) \quad (-) \quad (30.8)$$

$$\text{ECM2} \quad W = \text{PPI} + 0.002T - 0.58 E + 0.58 K$$

$$(0.002) \quad (30.8) \quad (30.8)$$

$$\text{ECM3} \quad W = \text{CPI} + (\text{BEN} - \text{CPI}) - 3.0 U - 0.03 \text{UNION} + 0.01T$$

$$(1.0) \quad (0.7) \quad (2.1)$$

$$\text{ECM4} \quad \text{CPI} = 0.67 \text{ PPI} + 0.18 \text{ W}$$

(21.6)      (11.5)

't' statistics are presented in parenthesis for each estimated parameter. ECM1 is a standard Cob-Douglas production function with constant returns to scale. ECM2 sets the real wage equal to the marginal product of labor with cross parameter restrictions from ECM1. ECM3 is the unions side of the wage bargain, the target real wage of the unions increases proportionally with real unemployment benefits, it responds negatively with respect to unemployment, as expected. The effect from union membership is very small and insignificant and of the wrong sign so it seems that membership rates have very little impact on the German unions bargaining position. This is not surprising as union membership rates have been very volatile since unification and this is not part of the bargaining process. Finally the unions target real wage is affected positively by a trend term, which may reflect rising aspirations and perhaps a steady reduction in the social consensus, which has held union power in, check in post war Germany. The final term, ECM4, Simply represents the relationship between the CPI and producer and wage costs.

We will not report all seven equations in the estimated system in detail but the wage equation will be reported as this is the primary focus of the analysis presented here. So the dynamic wage equation is

$$\Delta W = 0.01 + 0.6 \Delta W_{t-1} - 0.03 \text{ECM2}_{t-1} - 0.02 \text{ECM3}_{t-1} + 0.045 \text{DUM91} - 0.0003 \text{TDUM91}$$

(0.2) (9.9)      (2.3)      (2.7)      (1.2)      (1.0)

Both the firm side of the Bargain and the union side have significant and correctly signed adjustment coefficients, although they both suggest rather slow adjustment, which is perhaps not surprising for the case of Germany.

The main long term driving forces behind real wage growth in Germany are then the long-term development in the marginal product of labor (or labor productivity) and the growth in real unemployment benefits. More traditional labor supply factors such as union membership and unemployment rates seem to have rather weak effects, although there is an important role for a trend effect which we have been unable to associate with a specific variable. This may represent the steady change in the implicit social contract between firms and workers which anecdotal evidence would suggest is an important factor in Germany. The weights given to both sides of the bargain seems to be fairly even, in the sense that the adjustment coefficients for each ECM term are very similar in size but adjustment towards the equilibrium of the system is unusually slow by international standards. This may well reflect the many well-documented rigidities in German labor markets.

Unfortunately it has not proved possible to test the full set of overidentifying restrictions in this model as it has not proved possible to obtain convergence of the model with only exactly identified long run restrictions. We are however able to test individual restrictions of the model by relaxing restrictions in the final model, one at a time. The following table gives the likelihood ratio test of some relevant restrictions

Table 3. Some tests of Restrictions on the long run Model

restriction	LR test $\chi^2(1)$
Price Homogeneity in ECM2	0.2
Unit coefficient on real benefits in ECM3	6.0
Constant returns to scale	6.9
Price Homogeneity in ECM3	5.0

So Price Homogeneity in ECM2 is easily accepted the other restrictions are marginally over the standard critical value but not massively so for a system of such large size and in each case a wald test of the restriction easily accepts the restriction. The unrestricted constant return to scale parameter for example gives slight increasing returns to scale (1.1) but which is not significantly different from one on the wald test.

## 5. Conclusions

This paper has formulated a new model of the German wage formation process within the context of system cointegration estimation but by implementing some new suggestions by, Chamberline, Hall, Henry and Satchi(2000) on the process of identifying the structure of the wage bargain. This procedure overcomes the problems, which have been centered on the Manning Criticism of empirical wage models, that they are not identified in the conventional sense. By drawing on the new literature of identification in cointegrated systems and returning to the original theory of wage bargaining it is possible to identify the long run structure of the wage bargain formally. These ideas have then been implemented in a seven equation VeqCM system with four cointegrating vectors which has been estimated subject to the over identifying restrictions of the full theoretical model. The estimation procedure has yielded sensible parameter estimates for the four structural long run relationships and the wage equation finds that the interaction of the firm side and the union side in the wage bargain is both sensible and equally balanced.

## **Appendix A, The Data for Germany**

The raw data ,as defined below, for Germany is all standard official data from official German data sources supplied by the RWI research institute in Essen. Most data for Germany is now only published as whole Germany data, prior to 1991 only west German data is available. For most series an overlap exists of four or five years. The break in some series is considerable, eg, the capital stock, GDP or employment. This major data problem has been addressed in two ways: First all west German data was rescaled so that it smoothly linked on with the unified German data, the rescaled data moves exactly in line with west German data prior to the existence of true whole Germany data. Second all estimated equations included a step dummy and a split time trend beginning in 1991 to help to capture the possibility of any structural change after 1991.

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