

## **Expectations Formation and the 1990s ERM Crisis**

**Beeby M, Hall S<sup>1</sup>, Henry S and A Marcet<sup>2</sup>.**

December 2001.

Revised January 2002

This version August 2002.

*Abstract; The ERM crisis of 1992 appears to be due to a series of negative output shocks followed by devaluation. This stylised fact does not seem to conform to the “second-generation “ crisis model based on the REH. We show that by adopting bounded rational learning, the model yields a trade- off between the size and persistence of shocks leading to devaluation, a finding which makes the model more clearly applicable to these crisis episodes.*

JEL Classification.

Acknowledgement; Beeby, Henry and Marcet wish to thank the ERSC for financial support for this research, under the RoPA award [ ] .

---

<sup>1</sup> Imperial College, London.

<sup>2</sup> Barcelona

Contents;

1 Introduction

2. Currency crisis models and the 1992 and 1993 ERM episodes.

3. A simple Model and the implications of learning.

4. Model Simulations

5. Conclusions

## **1. Introduction**

The consensus view about the causes of the ERM crises in 1992 and 1993 is that these can be explained by the so-called “second generation” model of currency crises. (Eichengreen and Wyplosz (1993), and (1995)). Indeed, these models were developed with the express purpose of accounting for the ERM crises, which the first-generation models were thought to be incapable of doing. According to the second-generation model, currency crises are not driven by fundamental factors such as the depletion of a nation’s currency reserves, as emphasised in “first generation “ models, but are essentially self-fulfilling, expectations driven, processes. Second- generation, like first-generation models, place heavy emphasis on the Rational Expectations Hypothesis (REH), but the characteristic of the second-generation model is that it introduces explicit government incentives concerning the costs and benefits of maintaining a currency peg, and of the alternative - that of a realignment. The analysis it advances is essentially an extension of the Barro-Gordon closed-economy model of monetary policy setting to the open economy, with two regimes; pegging and floating. Taking the special case of a devaluation only, which is strictly the only case we are concerned with here, the RE solution for currency changes (equal to inflation in these simple models), is a weighted average of exchange rate behaviour in the two regimes, the weights being the probability that output shocks exceed or fall short of a critical “ threshold” value. This equation is quadratic in the output shock; so two equilibria are possible, depending on the size of the shock. (Obstfeld (1994)). With this feature of multiple equilibria, currency crises in such models are seen essentially as being produced by the exogenous output shock which shift the economy from one equilibrium or regime, to another. In the high expected depreciation case, there is a competitiveness and unemployment problem so the regime may only continue if the output shock is “small”. (The canonical version of the second-generation model is due to Obstfeld, see Obstfeld (1994) (1996) and Obstfeld and Rogoff (1996)).

There are basic features of the crises which do not fit this second generation model however. We will argue that the situation in the early 1990s was one where countries were experiencing a recession that had gone on for some time with attendant high unemployment, and competitiveness problems, both of which were long lasting. The UK provides a particularly clear example. Economic conditions in the UK had deteriorated substantially for some time before the September 1992 crisis. Thus, growth had slowed sharply in 1990 falling from 2.1 to 0.6 percent (Q1 on Q1 annualised rates for GDP. Source; LBS/OEF Economic Outlook (2000)). In 1991 GDP growth was substantially negative (-1.5) and only just about achieved a positive rate in 1992 (0.1). Unemployment rose sharply although lagging behind these changes in output, and by 1993 had risen above 10 percent both on the claimant count and on the international (ILO) measure, as compared with a rate of just over 6 percent (claimant count) and 7.3 percent (on the ILO definition) in 1989. Again, a similar judgement may be made about the other key real indicator- competitiveness- which many argued was seriously affected by a too high entry rate in 1989 (Wren-Lewis et al ( ) calculated that the rate was some 20% above its optimal value ). This worsened further as UK inflation increased sharply in 1990 when it reached almost 10 percent compared with just under 5 percent in 1988, although it needs to be noted inflation fell sharply thereafter. What these events underscore is that the deterioration in the aggregate UK economy was not just the result of a one-off shock, but a progressive worsening which persisted over 3-4 years. The economies of ERM countries coming under attack at this time experienced similar long lasting deterioration. Sweden's unemployment rate for example, was over 5 percent in 1993, having averaged under half that rate over the period 1982-91. And the deterioration in its government finances were equally profound, moving from an average surplus of 2.5 percent in 1987-1991, to a deficit of over 7 percent in 1992 (Obstfeld (1994)).

These features do not seem to fit the predictions of the second-generation model, where a single "large" output shock shifts the exchange rate from one regime (fixed) to the other (floating). Instead, models are needed which imply both persistent output falls and real exchange rate deterioration.

According to second-generation models output shocks which precipitate currency crises have i.i.d characteristics; being a single large shock which brings the government's willingness to adhere to the currency peg into question. As we have argued, the output shocks affecting all the countries under threat in the ERM were in fact highly persistent. Moreover, where the motive for abandoning the currency peg is one of worsening competitiveness, then the second-generation model suggests this worsening is also likely to occur over a short time. Again, the evidence shows that competitiveness worries were long lasting in the 1990s.

We argue that it is possible to explain these crises without recourse to the REH, using an alternative to rational expectations, that of boundedly rational learning. More significantly we suggest that its implications fit the stylised data just reviewed,

whereas the implications from the multiple- equilibrium/ REH models of the second-generation models do not.

This paper extends the approach used in Marcet and Nicolini (1997) in analysing the phenomenon of recurrent hyperinflation in South America. Assuming the authorities financed spending by seignorage, unless inflation exceeded a preset target when the government used an exchange rate regime, their paper showed the relative ease with which the feature of recurrence could be introduced in a model where boundedly rational expectations formation ( henceforth “learning”) was assumed rather than rational expectations. In implementing learning, Marcet and Nicolini assume that the updating equations governing agent’s learning were of two types; the first is least-squares learning where inflation is high, but the second occurs when big shocks hit the inflation process, and then agents resort to a tracking rule. Part of the model’s resulting non-linearity arises as switches occur between these two learning mechanisms. The application of learning to the ERM crisis we give below, extends on these Marcet and Nicolini insights.

## **2. Models of Currency Crisis**

### **(a) First and Second Generation Models.**

There are two strands to the currency crises literature. The first, begun by Krugman (1979) and known as the speculative attack model, emphasises the inconsistency between (assumed exogenous) government policies, usually taken to be too high a rate of monetary growth, with the current value of the fixed nominal exchange rate. Krugman’s motivation was to explain the wave of devaluations across Latin America in the 1970s as rational events, and not as signs of market failure, and in this model currency crises are runs on the central banks’ holdings of foreign reserves. Any rebalancing of portfolios by investors requires the selling of domestic assets and results in an increase in domestic credit, leading the central bank to intervene to maintain the currency peg. An expansive monetary/fiscal policy will not be consistent with this peg and will mechanically result in a speculative attack. Therefore, speculative attacks need to be seen as rational events - if there was no run on reserves then the speculators would be able to foresee the exact date of devaluation and make certain profits. Perhaps a weakness of the model is that government policy is modelled as being exogenous. Speculative attacks occur because the policy making authorities pursue policies that are, by assumption, inconsistent with the exchange rate. Bad fundamentals are then simply those policies that lead to a devaluation. It should be noted however that Krugman was not trying to explain *why* governments pursue incorrect policies; rather he was interested in studying the consequences of poor policies. And the model does seem to fit events in 1970/80 Latin America where countries like Argentina, Chile and Brazil were attempting to stabilise their high inflation and large public sector deficit economies.

With the ERM crisis of 1992 and 1993, the limitations of assuming an exogenous government policy became clear. It is true that expansionist monetary policy may have been an issue in the devaluations by Italy and Spain, but in countries such as France and the UK problems arose because of the contractionary nature of the monetary policy pursued. German reunification led to inflationary pressure in Germany that the Bundesbank countered with a tight monetary policy. This forced upon the other ERM member countries interest rates higher than they would have preferred given the state of their domestic economies which were experiencing increasing levels of unemployment. This in turn increased the temptation to devalue since the level of interest rates needed to defend the exchange rate was inconsistent with the domestic needs of monetary policy. The first-generation model provided only a limited framework for analysing these issues, particularly given its narrow definition of exchange rate credibility. Further, the ERM crisis showed that broadening the set of economic fundamentals in that model was not sufficient. Unemployment had been high and increasing in Germany years prior to the crisis, so why was it that the crisis occurred at that specific time? What appeared to be needed was a model that explained speculative attacks not simply as a response to a given level of the economic fundamentals, but also allowed a role for self-fulfilling speculation. This led to the development of the second-generation model, labelled the “escape clause” model.

The second-generation model was formalised by Obstfeld (see especially, Obstfeld (1994)). In this model, countries exited the fixed exchange rate regime not because government policies were inappropriate, but because governments are viewed as trading-off the costs and benefits of adhering to the currency peg on the one hand and realigning on the other. Under RE the private sector realise this and given a sufficiently adverse shock, will anticipate correctly that the government will abandon the peg.

Thus, in this model, governments commit to a fixed exchange rate regime only conditionally in the expectation of gaining anti-inflation credibility, but any commitment must be limited. If economic fundamentals deteriorate sufficiently then the government has an escape clause allowing it to change its exchange rate (e.g. float or devalue), which the private sector knows. Governments (and consequently speculators) are then in a continual process of evaluating the costs and benefits of maintaining a currency peg, in the light of exogenous shocks.

There are two major contributions of this model. First, the notion of economic fundamentals used is a broad one, it can be any variable that the policymaker decides is relevant for the decision on whether to exercise its escape clause. Second, and perhaps more importantly, it provides a new theory of self-fulfilling speculation and multiple equilibria. Causality can flow not only from narrow economic variables to market expectations but also in the opposite direction. Multiple equilibria can then arise from such circularity. For instance, in these models, any increase in the belief of private agents that the government may devalue can lead to an increase in interest

rates to strengthen the currency, which in turn raises the probability that devaluation actually occurs. Whether and when exchange rate crises result is now dependent upon the self-fulfilling mood of the participants in the market.

Incorporating multiple equilibria into exchange rate models is appealing for a number of reasons. First, many economists (and market participants such as George Soros) believe that speculation is motivated by more than just narrow economic fundamentals. Second, the spread of economic crises (i.e. contagion) and their timing can be more easily explained in models with this feature. Third, multiple equilibria is a compromise between those who believe that crises are a result of market failure and those that believe they are a response to bad government policies. To suggest that crises are purely a result of speculation appears to absolve governments of all the blame; before a currency becomes a potential target by speculators there has to be some economic fragility.

### **(b). The Escape Clause Model**

Obstfeld's escape clause model (1991, 1994 and 1996) is an open economy version of Barro and Gordon (1983), and applies to a government with an exchange rate objective. It yields the possibility of multiple equilibria in expected inflation rates in the model. The extension is based on there being fixed costs to currency changes, which the authorities will only countenance if there is a sufficiently large shock to output. When there is such a shock, the authorities devalue and use monetary policy to stabilise output. The superficial resemblance of this sequence to the ERM crisis seems clear.

The version of the model used here is Obstfeld (1994) as this focusses on the devaluation case, and links the exchange rate to output innovations in a way which is convenient for our present purpose. Let output be given by (variables in logs)

$$y_t = \alpha(e_t - w_t) + u_t \quad (1)$$

where  $y$  is the log of output,  $w$  the log of nominal wages and  $e$  is the log of the nominal exchange rate, which is equal to the domestic price level under PPP, and where the foreign price is normalised at unity. Hence, (1) gives an inverse relation between output and the real wage, which is shifted by an output shock  $u$  (assumed to be i.i.d). Wages in period  $t$  are set in period  $(t-1)$  based on information at that date, so does not include  $u_t$ . The government can respond to demand shocks in period  $t$  via changes in the current exchange rate. Under floating the government minimises

$$L_t = \sum_s^{\infty} \beta^{s-1} l_s \quad (2)$$

where  $\beta < 1$ , and where

$$l_s = (\theta/2)(e_s - e_{s-1})^2 + (1/2)(y_s - y^*)^2 \quad (3)$$

where  $y^*$  is the target output level, assumed to be non-zero which creates a dynamic inconsistency problem. It also suggests why a government might choose to tie its hands; the government sets the exchange rate  $e$  after observing  $u$  (unlike private agents), so any devaluation leads to costs. (see below)

Substituting (3) and (1) into (2) and minimising w.r.t  $e_t$ , assuming that  $w_t$  is given, gives the government's reaction function,

$$e_t - e_{t-1} = \lambda(u_t / \alpha) + \lambda(w_t - e_{t-1}) + \lambda(y^* / \alpha) \quad (4)$$

where  $\lambda = \alpha^2 / (\theta + \alpha^2)$ .

Equation (4) is crucial, as it captures the main ingredients of the problem. Firstly, the equation shows that the government uses the exchange rate to offset output shocks, and secondly that it can attempt "surprise" depreciations if wage inflation affects competitiveness. Lastly, according to the last term in the equation, the government may attempt to push output above its natural rate.

As is standard in these escape-clause models, the problem of maintaining a currency peg is seen as one of the credibility of the commitment to the peg. This entails adding a cost of realignment to the objective function (3) above, i.e.

$$l_t^I = l_t + cZ_t \quad (5)$$

where  $Z$  is an indicator function with value=1, if  $\Delta e_t \neq 0$ , and is 0 otherwise. Using (5) it can be shown that the loss under a fixed exchange rate regime is

$$l_t^F = (1/2)(\alpha\pi_t + u_t + y^*)^2 \quad (6)$$

as  $\Delta e_t = 0$ , and  $\pi_t = w_t - e_{t-1}$ , assuming, as we do, that wages are indexed to prices (equal in this model to the exchange rate). Under realignment, from (4) the loss is

$$l_t^R = (1 - \lambda)l_t^F + c, \quad (7)$$

from which it follows that a realignment will occur when

$$(\lambda/2)(\alpha\pi_t + u_t + y^*)^2 > c. \quad (8)$$

Treating (8) as an equality, it is already evident that there are two solutions to this for  $u$ ;  $u > \bar{u}$ , where the government devalues, and  $u < \bar{u}$  where it revalues.

The trouble with this derivation so far is that these trigger points for upper and lower values of  $u$  depend on the market expectations of depreciation ((in this model given by  $\pi_t$ ), while at the same time these expectations depend on where the market thinks these trigger points are. To proceed to analyse this further, we limit the model to the devaluation option as anticipated earlier. Then,

(a) The market expectation of depreciation, given  $\bar{u}$ , is

$$\pi_t = \Delta e_t = P\{u_t \leq \bar{u}\}.0 + P\{u_t > \bar{u}\}.E\{\Delta e_t \mid u_t > \bar{u}\} \quad (9)$$

where  $P(\cdot)$  refers to Probability. Assuming that  $u$  has a uniform distribution between  $\{-\mu, \mu\}$ , then

$$P\{u_t > \bar{u}\} = (\mu - \bar{u}) / 2\mu, \quad \text{and}$$

$$E\{u_t \mid u_t > \bar{u}\} = (\mu + \bar{u}) / 2.$$

Using these in (4) we get,

$$E\{\Delta e_t \mid u_t > \bar{u}\} = \lambda((\mu + \bar{u}) / (2\alpha)) + \pi\lambda + \lambda(y^* / \alpha) \quad (10)$$

which is the market's (rational) expectation of devaluation given the threshold  $\bar{u}$ .



From (9) and (10) it follows that

$$\begin{aligned}\pi &= \lambda((\mu - \bar{u}) / 2\mu) [\{(\mu + \bar{u}) / 2\alpha\} + (y^* / \alpha)] / [1 - \lambda((\mu - \bar{u}) / 2\mu)] \\ &= \delta(\bar{u})\end{aligned}\tag{11}$$

(b) Calculating the Threshold

The government takes the expectation in (11) as given when minimising its loss function. Putting this into (8), the value of the *largest* shock consistent with maintaining the peg is

$$(\lambda / 2)[\alpha\delta(\bar{u}) + \tilde{u} + y^*]^2 = c.\tag{12}$$

Since  $\bar{u} = \tilde{u}$  in equilibrium, (12) is a quadratic in  $\bar{u}$  with generally two solutions.

For calibrated values of the parameters  $\{\alpha, \theta, \mu, \dots\}$  and  $y^*$ , Obstfeld (1994) shows there are two values for the threshold, and associated with these an expected depreciation (or inflation) rate. The second, higher, expected depreciation rate will normally trigger an actual devaluation, unless output shocks are very favourable.

### 3. Boundedly Rational Learning

#### 3.1. The Solution under RE.

The rational expectations version of the crisis model set forward by Obstfeld (op.cit) is an extension of the Barro-Gordon set up; namely that wage bargainers know the government reaction function given by (4) above, and set wages according to it. Assume that wage bargainers (firms and unions) agree to wages which ensure a constant real wage, then

$$w_t = E_{t-1}(e_t) \quad (13)$$

so the nominal wage is set conditional only on information dated at t-1, and does not include the realisation of  $u_t$ , so the ex-post real wage in period t will be affected by this. However, the RE bargain ensures that the wage is set in the full knowledge of how the government will react to realisations of  $u_t$  as given by (4). Thus in the RE equilibrium, from (13) and (4) the wage is

$$w_t = e_{t-1} + (\lambda/(1-\lambda))[E_{t-1}(u_t/\alpha) + (y^*/\alpha)] \quad (14)$$

which, as the expectation of u is zero, is

$$w_t = e_{t-1} + (\lambda/(1-\lambda))(y^*/\alpha) \quad (15)$$

This, together with the government reaction function (4) implies the exchange rate is given by (16).

$$\Delta e_t = \lambda(u_t/\alpha) + (\lambda/(1-\lambda))(y^*/\alpha) \quad (16)$$

Equations (14) and (16) constitute the system of dynamic equations we will use.

### 3.2. Learning Mechanisms

To implement the alternative Boundedly Rational solution, we operate with the same basic model of output, wage setting, and reaction function as we have above. In contrast to the REH case we replace the assumption that expected exchange rates and wages are formed rationally with the following two assumptions. Under learning, the expected exchange rate is taken to be equal to the lagged rate with a constant and slope parameter that are time varying and which are updated using a Kalman Filter. Thus,

$$e_t^e = \varphi_{0t} + \varphi_{1t}e_{t-1} \quad (17)$$

while wage expectations are formed by the rule

$$\Delta w_t^e = \varphi_{3t} + \varphi_{4t}(y_{t-1} - y_{t-1}^*) \quad (18)$$

Equations (17) is a simple boundedly rational learning rule which says that expectations of the exchange rate are a function of past actual values, This rule has

been shown to work remarkably well in Beeby, Hall and Henry(2002). Equation (18) is of course a boundedly rational version of a standard Phillips curve relationship which simply states that the expected change in wage inflation is a function of the observed output gap.

#### 4. Model simulations.

In the simulation the government is assumed to minimise the loss function given by (2) and (3) above, and the two parameters  $(\beta, \theta)$  are set equal to (0.95, 0.15) following Obstfeld (1994). Then, as in the account given above, the government considers the advantages of maintaining the parity versus a depreciation, where the depreciation produces competitiveness gains, but incurs a fixed (political) cost  $C$  which we set at 0.001 in the simulations.

As in the theoretical model earlier, optimising (3) above w.r.t. the exchange rate  $e_t$ , the loss under the fixed regime is

$$L^F = 0.5 * (y_t - y_t^*)^2 \quad (19)$$

and under the floating regime,

$$L^R = 0.5 * (y_t^S - y_t^*)^2 + C + \theta / 2 (e_t^S - e_{t-1})^2 \quad (20)$$

The terms  $(y^S, e^S)$  are the level of output and the exchange rate which would come about if the government chose to devalue, defined below. We then obtain a dynamic equation for the exchange rate in terms of the disturbance  $(\mu_t)$  as follows,

$$\Delta e_t = (\text{diff} / (L^F - L^R)) (\beta (e_t^e - e_t) + (\gamma / \theta) (y_t^* - y_t)) \quad (21)$$

where

$$\text{diff} = \text{Max}(0, L^F - L^R)$$

The simulation is generated by defining values for output and the exchange rate conditioned upon the floating rate, i.e. what happens if the government chooses to go for the depreciation option. Thus,

$$y_t^S = \gamma (e_t^S - p_t^S) + \mu_t, \text{ and} \quad (22)$$

$$e_t^S = e_{t-1} + \beta (e_t^e - e_t^S) + (\gamma / \theta) (y_t^* - y_t^S) \quad (23)$$

Simulations take artificial data for a period of 40 observation to create a base solution. Then the disturbance is shocked by a range of values. Under Rational expectations the result should be fairly straight forward. A shock below the key threshold should leave the exchange rate unchanged and so no devaluation occurs. Because expectations are

fully rational, the past instantly becomes a bygone, and as further shocks of a similar magnitude occur they also have no effect. So the simple result is that either a shock is larger than the threshold, in which case devaluation occurs, or it is not and devaluation remains equally unlikely in the future.

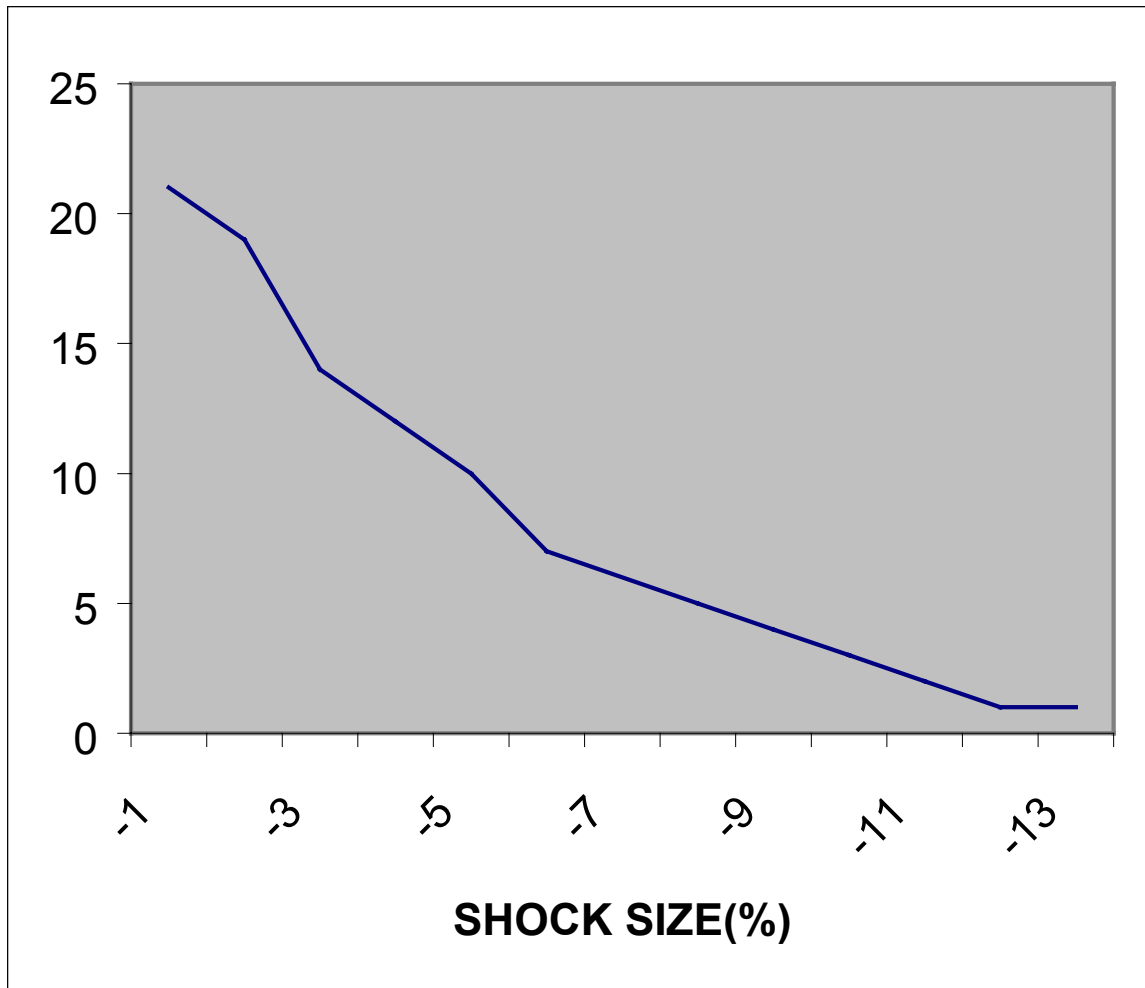
However, under learning the story becomes rather different and more complex. If a shock occurs which is below the key threshold to output, this begins to change wage expectations (as output will be different from the natural rate). This effect will then have an impact on future output levels. Of course if there were no further shocks this effect would almost certainly die away and, again, no devaluation would occur. But if a further similar shock should occur in the second period, then the combination of the two effects might trigger a devaluation even if each single shock does not reach the RE threshold. So what we would expect to find would be a combination of persistence and sizes of shocks, which trigger devaluation. One large shock would be enough, two smaller shocks might also be enough, even three small shocks in a row might be enough and so on. We are agnostic as to the distribution of the shocks. Even if we assume the shocks are actually i.i.d., then in any given simulation there is a chance that a series of similar sign shocks will occur, and these will trigger devaluation under learning. In the real world, as we argued in the introduction, it seems clear that shocks are often highly correlated.

#### **4.1. The Solution under REH**

Given the chosen parameters the RE solution is exactly as expected and we find that a shock of approximately 12% of  $y$  is required to trigger devaluation.

#### **4.2. The Solution under Learning**

In this case, to illustrate the properties of the model, we perform a range of simulations where we apply a shock for a number of periods (40 periods in all), we then record how many periods elapse before devaluation occurs. We find a trade-off between the size of shock and the number of periods which elapse before devaluation. This trade-off is illustrated in figure 1. It shows that for a very small shock of 1% applied in every period, devaluation will eventually be triggered after 21 periods while for a very large shock it will occur in the first period



**Figure 1: The trade of between the shock size and the number of periods before devaluation.**

## 5. Conclusions

We have argued that a very clear stylised fact of the ERM crises is that the economy was hit by a series of negative shocks before the crises occurred, but that there is no clear single shock which could have triggered the crisis at the time of the crisis itself. This stylised fact does not conform to the standard second generation currency crises model. However by relaxing the assumption of rational expectations we find that the model generates a continuum of combinations between the size of the shock and its persistence, any of which will trigger devaluation. Many show that a sequence of even relatively “small” shocks will trigger devaluation if there is a sufficiently long sequence of them. It is this feature- of a sequence of similar signed shocks preceding the crisis, rather than a single large one – which we argue conforms very closely with the UK experience during this period.

## Appendix . The General Case

The model starts from the government loss function which, in this case is,

$$L = (y - y^*)^2 + \beta \varepsilon^2 + C(\varepsilon) \quad (1)$$

where all lower-case letters denote natural logs, and  $y$  is output,  $y^*$  the target level of output, and  $\varepsilon \equiv e - e_{-1}$  is the change in the exchange rate, defined as the price of foreign currency. This is equal to the inflation rate under PPP. The last term in (1),  $C(\cdot)$  captures the feature that the government has adopted an exchange rate objective and hence there are costs in deviating from it, as we describe below.

Output is determined by an expectations augmented Phillips curve,

$$y = \bar{y} + \alpha(\varepsilon - \varepsilon^e) - u \quad (2)$$

where  $\bar{y}$  is the ‘natural’ output level,  $\varepsilon^e$  is the domestic price setters’ expectation of  $\varepsilon$  based on lagged information (assumed time invariant), and  $u$  is an i.i.d. mean-zero supply shock. The assumption that  $y^* > \bar{y}$  creates a dynamic inconsistency problem and is needed to obtain multiple equilibria. It also suggests why a government might choose to tie its hands. The government sets the exchange rate  $e$  after observing  $u$  (unlike private agents), where any devaluation leads to costs of  $C(\varepsilon) = \bar{c}$  in (2) and a revaluation leads to costs of  $C(\varepsilon) = \underline{c}$ .

If the term  $C(\varepsilon)$  in (1) is initially ignored, so the exchange rate can be freely altered, from (1) and (2), the government chooses

$$\varepsilon = \frac{\alpha(y^* - \bar{y} + u) + \alpha^2 \varepsilon^e}{\alpha^2 + \beta} \quad (3)$$

and substituting this into (2) gives,

$$y = \bar{y} + \frac{\alpha^2(y^* - \bar{y}) - \beta u - \alpha \beta \varepsilon^e}{\alpha^2 + \beta}$$

and a policy loss of

$$L^{flex} = \frac{\beta}{\alpha^2 + \beta} (y^* - \bar{y} + u + \alpha \varepsilon^e)^2$$

In the case where the government cannot change the exchange rate, then the loss is instead,

$$L^{fix} = (y^* - \bar{y} + u + \alpha \varepsilon^e)^2$$

Reintroducing the fixed costs term  $C(\varepsilon)$  means that the government only changes the exchange rate when the supply shock  $u$  is sufficiently large that  $L^{flex} + \bar{c} < L^{fix}$  or so small that  $L^{flex} + \underline{c} < L^{fix}$ . Devaluation then occurs for  $u > \bar{u}$ , and revaluation for  $u < \underline{u}$ , where

$$\bar{u} = \frac{1}{\alpha} \sqrt{\bar{c}(\alpha^2 + \beta)} - y^* + \bar{y} - \alpha \varepsilon^e, \underline{u} = -\frac{1}{\alpha} \sqrt{\underline{c}(\alpha^2 + \beta)} - y^* + \bar{y} - \alpha \varepsilon^e$$

So what happens is that the authorities defend the exchange rate against all but very large shocks, but where these occur, they devalue incurring the fixed cost of so doing, and once the fixed cost is paid, they use monetary policy to stabilise output.

Next, multiple equilibria for equilibrium expected inflation rates can arise because the rational expectation of next period's  $\varepsilon$ , given price setters' expectation  $\varepsilon^e$ , is

$$E(\varepsilon) = E(\varepsilon | u < \underline{u}) \Pr(u < \underline{u}) + E(\varepsilon | u < \bar{u}) \Pr(u < \bar{u}),$$

which shows that the relation between ex-post inflation and expected inflation is a complicated one, and it is this property which suggests the possibility of multiple equilibrium expected inflation rates. (i.e. where  $E(\varepsilon) = \varepsilon^e$ ).

Obstfeld (1994) provides a calibrated version for the case of a depreciation, and we use that here. Thus assuming that the supply shock ( $u$ ) is uniformly distributed on the interval  $[-\mu, \mu]$ , then

$$E(\varepsilon) = \frac{\alpha}{\alpha^2 + \beta} \left[ \left(1 - \frac{\bar{u} - u}{2\mu}\right) (y^* - \bar{y} + \alpha \varepsilon^e) - \frac{\bar{u}^2 - u^2}{4\mu} \right] \quad (4)$$

Assuming rational expectations by wage setters gives,

$$E(\varepsilon) = \varepsilon^e,$$

and usually this produces one solution. It is different in this case. To see this consider the slope of (4) shown below.

$$\frac{dE(\varepsilon)}{d\varepsilon^e} = \begin{cases} \frac{\alpha^2}{\alpha^2 + \beta} & \text{for } \underline{u} > -\mu \\ \frac{\alpha}{\alpha^2 + \beta} \left[ \frac{\alpha}{2} + \frac{\alpha}{2\mu} (y^* - \bar{y} + \alpha\varepsilon^e) \right] & \text{for } \underline{u} = -\mu \\ \frac{\alpha^2}{\alpha^2 + \beta} & \text{for } \bar{u} = -\mu \end{cases}$$

This implies that there are three equilibria possible in this model which are given by the three different devaluation probabilities and the consequent size of the exchange rate change. The three equilibrium expected depreciation rates are  $\varepsilon_1$ ,  $\varepsilon_2$ , and  $\varepsilon_3$  each corresponding to three different devaluation probabilities (see Obstfeld (1996) for details).

## References

Beeby M, Hall S and Henry S (2000).

Krugman. P. (1979) "A Model of Balance of Payments Crises", Journal of Money, Credit and Banking, 11 August, p311-25.

Marcet. A and Nicolini J. P. (1997) "Recurrent Hyperinflations and Learning" mimeo, Universitat Pomeu Fabra.

Obstfeld. M. (1994) "The Logic of Currency Crises" Banque De France- Cahiers Economiques et monetaires, No 43.

Obstfeld. M. and Rogoff K (1996) "Foundations of International Macroeconomics" The MIT Press.