

Bargaining models and identifying the wage equation.

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Abstract; It is commonly asserted that the standard wage equation derived from bargaining theory cannot be identified. Here, it is argued that the case for this alleged failure rests on an outmoded definition of identification. Newer concepts based on non-stationarities, cointegration and reduced rank are appropriate. An empirical example applying these concepts shows that the standard model can be derived and that far from being underidentified, it is actually overidentified.

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1 Introduction

It has become widely accepted that the wage equation in the standard model of the wage-employment process is not identified. Manning (1993), Bean (1994), and Layard, Nickell and Jackman(1991), each attest to this property, and while some suggestions have been made for overcoming the problem, it is probably fair to say that they are not wholly successful (see the suggestion in Manning (1993) for estimating the Euler equation for example).

The problem is not difficult to see, and is best represented in a direct quote from Manning (op.cit). The structural model of wages and the labour demand equation (written with unemployment as the dependant variable) is

$$\begin{aligned}(w - p)_t &= -\gamma_1 u_t + \beta_{11} x_{1t} + \beta_{12} x_{2t} + \eta_{1t} \\ u_t &= \gamma_2 (w - p)_t + \beta_1 x_{1t} + \eta_{2t}\end{aligned}\tag{1}$$

The first equation is for the real wage the second is for employment (although here, following Manning, is written as an equation for unemployment). The vector x_1 includes tax wedges, the labour force and all variables affecting “productivity” (the capital stock, technical progress etc), and the vector x_2 includes variables affecting wage pressure (union power, the replacement ratio, etc).

As Manning points out, the problem with the model is then evident. The wage equation is not identified, because it must include all the variables in the second, labour demand, equation, and so it fails the familiar rank and order conditions for simultaneous equations.(i.e. Models of the Cowles Commission type). Moreover, the problem appears generic – that is, all wage models which are based on a wage bargain being struck conditional on the “productivity” variables contained in x_1 above, will fail to be identified.

In this paper we argue that this proposition is incorrect. Fundamentally, our argument is that it applies an outmoded identification criteria, one which is only suitable for identifying simultaneous equations of the “Cowles Commission” form. Such criteria ignore the fact that equation (1) is part of a complete set of dynamic equations determining the movements of *all* the variables in the model, so in specifying (1) we are actually using a large set of restrictions on this underlying dynamic system of equations which involve all these variables. This underlying model represents the co-movements over time of all the variables – not only (w-p), u, but also the two vectors

of variables x_1 and x_2 . To get to (1) from this underlying model a potentially large set of restrictions need to be applied. But these restrictions are not used when applying traditional – “Cowles-Commission” identification criteria as Manning does. Hence, our argument is that to decide whether the wage–employment model in (1) is potentially identifiable or not, the full model of all the variables in (1) needs to be set out, and the complete identification conditions appropriate for that full model applied. This is a general point and in making it we are simply calling on the very large literature on identifying dynamic systems which has developed since the seminal paper by Sims (1980), which was the first to draw attention to the need to recognise the dynamic interdependencies between all the variables in a model.

As is well known, the dynamic model proposed by Sims was a Vector Auto Regression (VAR), and his general conditions for identifying the VAR used a recursive ordering of the structural matrix and a diagonal variance-covariance matrix. These restrictions are necessary and sufficient for exact identification in a VAR (Sims (1980)). Under these conditions, structural responses to reduced form innovations can be uniquely recovered. As such, this feature was essentially the same concept of identification as appears in the Cowles Commission application; identification is equated with uniqueness in the correspondence between reduced form and structural parameters.

Since the seminal contribution by Sims, the literature which has developed on the identification of multi-equation dynamic models is too voluminous to cite in full, but for our purposes, the key contribution most relevant to our application is the extension of identification criteria to the case of non-stationary variables. These extensions to non-stationarity systems of equations, Vector Error Correction Models (VECMs), include contributions by Johansen (1992) and (1995a), and Pesaran and Shin(1994) among others. As we show in detail in Section 2 the overarching concept in both the VAR and VECM definitions of identification criteria is again that of uniqueness; but whereas in the VAR literature the meaning of identification is the same as the Cowles Commission notion (but with the recognition that it is dealing with a system of course), identification in the VECM concerns uniqueness of the long-run part of the dynamic model only. Building on this, we show in sections 2 and 3 below that by applying identification conditions and subsequently testing and accepting over-identifying restrictions on the long-run part of a dynamic VECM involving *all* the variables in the wage bargain, we are able to recover the “standard” model in (1).

What VARs and VECMs have in common, is that the model is treated as a complete system incorporating all the variables in the model. In the non-stationary case where the dynamic model is a VECM, attention is directed at establishing the conditions under which the long-run part of the model (as represented by the set of cointegrating vectors) are identified. Where there are r cointegrating vectors, then Pesaran and Shin (op.cit) show that r^2 conditions are needed to identify the long run part of the model. It from this point that our application in section 4 starts, and this shows that by applying economic-theoretic over identifying conditions, we can derive a data admissible model of the form given by equation (1).

Hence we argue that the appropriate identification criteria to apply to the wage equation are those which have been developed in the VECM case. This is a general point about the need to appropriate identification criteria when dealing with a system

of non-stationary variables. This does not itself mean that, where these are applied, an economically sensible wage and employment system results of course. The relevant question then is somewhat different. Rather than asking whether the wage equation is identified -since we know it almost certainly is- the question is whether, by using the appropriate identification criteria, is it possible to derive a wage, employment and price model of the form given by (1) above. The empirical example we give later shows that the standard model can be obtained.

We then show that wage equations of the type given in (1) are actually over identified. Our argument is based on two elements. The first is that bargaining models of the wage give rise to at least two co-integrating relationships; one which, for shorthand, we describe as a labour demand and the other the labour supply equation (or wage setting equation). Secondly, in VECMs, zero, one, or more cointegrating equations may enter a particular dynamic equation. Hence it is possible and indeed likely that these two separately identified cointegrating vectors will each enter the wage inflation equation. In this case, with the application of economic theoretic restrictions as well, the wage equation is over identified. Or, more precisely, we show that such an over identified long run part of the wage inflation equation, will be of the same form as given in (1).

The plan of the paper is as follows, section 2 reviews general identification procedures for the non-stationary case. Section 3 reconsiders the underlying bargaining theory of wage determination to draw out the required theoretical restrictions to meet the identification requirements. Section 4 then illustrates the analysis by estimating an over identified wage bargaining model for the UK, and section 5 concludes.

2. Identification in Non-Stationary Systems

Since the Granger Representation Theorem-which established that the vector autoregressive model (VAR) and the vector equilibrium correction model (VEqCM) are observationally equivalent- attention has been focused on the development of models that are economically interpretable simplifications of the VAR. (see Engle and Granger, 1987)

In this literature, the traditional concept of identification is best seen through the 'structural' VAR (SVAR) which uses economically interpretable restrictions to achieve identification, with further restrictions being testable as over-identifying hypotheses (see e.g., Davidson and Hall 1991 and for recent reviews Canova, 1995 and Pesaran and Smith, 1998. see also the alternative approach of Hendry and Mizon()).

In the SVAR the parameters of interest ϕ are a function of $(\mathbf{A}_0, \mathbf{A}_1, \dots, \mathbf{A}_p, \mathbf{c}, \Phi)$ in:

$$\mathbf{A}_0 \mathbf{z}_t = \sum_{j=1}^p \mathbf{A}_j \mathbf{z}_{t-j} + \mathbf{c} + \mathbf{u}_t \quad \text{with} \quad \mathbf{u}_t \sim \text{IN}_N(\mathbf{0}, \Phi). \quad (2)$$

where \mathbf{A}_0 is an $N \times N$ matrix (which is often assumed to be non-singular although singularity is a possibility as pointed out by Davidson and Hall(1991)), $\mathbf{A}_0 \mathbf{z}_t$ represents N linear combinations of the N variables in \mathbf{z}_t that characterise their determination using economic theory, especially where the simultaneous determination of \mathbf{z}_t is a feature. In the absence of restrictions, the matrices $(\mathbf{A}_0, \mathbf{A}_1, \dots, \mathbf{A}_p, \mathbf{c}, \Phi)$ are not identified.

Consider the associated reduced form, closed, VAR assuming p lags on a vector of N variables z_t :

$$z_t = \sum_{j=1}^p D_j z_{t-j} + \delta + \varepsilon_t \quad \text{with} \quad \varepsilon_t \sim \text{IN}_N(\mathbf{0}, \Sigma), \quad (3)$$

where \mathbf{D}_j is an $N \times N$ matrix of autoregressive coefficients, and ε_t is a vector of N unobserved errors, which have a zero mean and constant covariance matrix Σ . Independently of whether the variables z_t are $I(0)$ or $I(1)$ the VAR (3) can be re-parameterised as a VEqCM (see Johansen 1988, 1992c, and Hendry 1995a):

$$\Delta z_t = \sum_{j=1}^{p-1} \Gamma_j \Delta z_{t-j} + \Pi z_{t-1} + \delta + \varepsilon_t, \quad (4)$$

Where Δ is the first difference operator, $\Gamma_j = -\sum_{i=j+1}^p D_i$ ($j=1,2,\dots,p-1$) are the short run adjustment coefficient matrices and $\Pi = -(\mathbf{I}_N - \sum_{i=1}^p \mathbf{D}_i)$ is the long run coefficient matrix, and $**[q=???.$ When Π has full rank N the variables z_t are $I(0)$ and the parameters $(\Gamma_1, \dots, \Gamma_{p-1}, \Pi, \delta, \Sigma)$, or equivalently $(\mathbf{D}_1, \dots, \mathbf{D}_p, \delta, \Sigma)$, are all identified in that the maximum likelihood estimator of these parameters is unique.]**

Since (3) and (4) are re-parameterisations of each other they are observationally equivalent, and the choice between them can be made on the basis of their interpretation. Indeed, an attraction of the parameterisation in (4) is the interpretation of its static long run solution, $E(\Pi z_t + \delta) = \mathbf{0}$ as the equilibrium of the system, with $(\Pi z_t + \delta)$ being the disequilibria at time t . Economic theory is often informative about such equilibria. The short run adjustment parameters Γ_j are also the subject of economic theory considerations concerning the time form of responses and speed of adjustment, though these are typically less precise than the hypotheses concerning equilibria. However, the parameters of interest (ϕ) will not generally be those of (2) or (3), and so the identification and estimation of ϕ has to be considered separately.

The VAR in (3), or equivalently the VEqCM in (4), characterize the distribution $\mathbf{z}_t | \mathbf{Z}_{t-1}$ and thus give the reduced form of (2), where:

$$\mathbf{D}_j = \mathbf{A}_0^{-1} \mathbf{A}_j (j=1,2,\dots,p), \quad \delta = \mathbf{A}_0^{-1} \mathbf{c}, \quad \text{and} \quad \Sigma = \mathbf{A}_0^{-1} \Phi (\mathbf{A}_0^{-1})', \quad (5)$$

and leads to the conventional discussion of identification in simultaneous equations models in which restrictions on $(\mathbf{A}_0, \mathbf{A}_1, \dots, \mathbf{A}_p, \mathbf{c}, \Phi)$ are required for (5) to have a unique solution for $(\mathbf{A}_0, \mathbf{A}_1, \dots, \mathbf{A}_p, \mathbf{c}, \Phi)$ in terms of $(\mathbf{D}_1, \dots, \mathbf{D}_p, \delta, \Sigma)$ or $(\Gamma_1, \dots, \Gamma_{p-1}, \Pi, \delta, \Sigma)$ (see e.g., Johnston, 1972, Greene, 1991). This, of course, is just the conventional approach to identification which was first set out by the Cowles commission and which lies at the heart of the identification problem in wage bargaining models noted in the Introduction. Even here though, observe that the above definitions rely on the entire dynamic model, i.e all the variables in the vector \mathbf{z}_t , without any restrictions whatsoever (e.g. on weak exogeneity, within-or between equation- restrictions) being used at this stage.

So far the discussion has centred on stationary variables. However the nature of identification changes in a crucial way when the variables being modeled are $I(1)$, but satisfy $r < N$ cointegrating relationships $\beta' \mathbf{z}_t$ that are $I(0)$. This is often the case for macroeconomic time series, and modeling of wage behavior has clearly fallen within this class of models since the work of Hall (1986). In this case the rank of Π is r , which is a feature that can be incorporated into the model by defining $\Pi = \alpha \beta'$ with α and β being $N \times r$ matrices of rank r , thus leading to the reduced rank or cointegrated VEqCM:

$$\Delta \mathbf{z}_t = \sum_{j=1}^{p-1} \Gamma_j \Delta \mathbf{z}_{t-j} + \alpha \beta' \mathbf{z}_{t-1} + \delta + \varepsilon_t \quad (6)$$

Note though that α and β are not identified since $\alpha \beta' = \alpha^+ + \beta^{+'} = \alpha \mathbf{P} \mathbf{P}^{-1} \beta'$ for any non-singular $r \times r$ matrix \mathbf{P} (rotation). Hence in the reduced rank case, with the reduced rank imposed, neither the VAR in (3) nor the VEqCM in (4) is identified, and this is the identification issue in the $I(1)$ case.. In particular, it is now necessary to determine r , and identify α and β , and there are many routes in which this might be achieved in practice - see Figure 1 in Greenslade, Hall and Henry (1998) for a diagrammatic representation of the possibilities.

Since r is not known *a priori* its value has to be determined empirically, and this provides one possible starting point. A commonly adopted procedure is the maximum likelihood one developed by Johansen (1988), which employs likelihood ratio criteria for determining r , and for a given choice of r yields a unique estimate of Π , of rank r . Since the short run adjustment coefficients $\Gamma_j (j=1,2,\dots,p-1)$ and the error covariance matrix Σ are identified, unique unrestricted maximum likelihood estimates of these parameters are available for a given value of r . The Johansen

procedure also produces unique estimates of α and β satisfying $\Pi = \alpha\beta'$ as a result of imposing the restriction that the resulting β be orthogonal. ^{**[to what??]} This restriction amounts to a sufficient set of restrictions to exactly identify β although of course these restrictions can have no economic interpretation and are just one arbitrary set of restrictions amongst an infinite set of restrictions which would achieve exact identification. A number of attempts have been made to impose more meaningful restrictions to identify β (see Phillips (1991), and Saikkonen (1993)).

The question of identifying the parameters of a SVAR which has $r < N$ cointegrating vectors is discussed in, inter-alia, Johansen (1994), Johansen (1995a), and Robertson and Wickens (1994). In such cases (2), written in VEqCM format, becomes:

$$\mathbf{A}_0 \Delta \mathbf{z}_t = \sum_{j=1}^{p-1} \mathbf{C}_j \Delta \mathbf{z}_{t-j} + \mathbf{A}^* \beta' \mathbf{z}_{t-1} + \mathbf{c} + \mathbf{u}_t \text{ with } \mathbf{u}_t \sim \text{IN}_N(\mathbf{0}, \Phi) \quad (7)$$

with $\mathbf{C}_j = \mathbf{A}_0 \Gamma_j$ ($j = 1, 2, \dots, p-1$) and $\mathbf{A}^* = \mathbf{A}_0 \alpha$. Conditional on having chosen the cointegrating rank r , it is necessary to consider the identification of the contemporaneous coefficients \mathbf{A}_0 and the long run coefficients β , and these are essentially separate issues in that there are no mathematical links between restrictions on \mathbf{A}_0 and those on β . In particular, since a Π matrix of rank r is identified and satisfies $\Pi = \alpha\beta' = \mathbf{A}_0^{-1} \mathbf{A}^* \beta'$, it follows that restrictions are required to identify β even if \mathbf{A}_0 were known. Conversely, restrictions on β have no mathematical implication for the restrictions on \mathbf{A}_0 . It remains possible though that the economic interpretation of a restricted set of cointegrating vectors $\beta' \mathbf{z}_t$ may have implications for the nature of restrictions on \mathbf{A}_0 that will be economically interesting, particularly when \mathbf{A}^* is restricted via α . Mathematical, and possibly economic, linkages do exist between restrictions on the adjustment coefficients α and those required to identify β - see Doornik and Hendry (1997).

As was noted in the previous section, the formal identification of β is the main subject of Johansen and Juselius (1992) and Pesaran and Shin (1997) where it is demonstrated that a necessary condition for exact identification is that there are $k = r^2$ restrictions. Johansen (1995a) and Pesaran and Shin (1997) also give a necessary and sufficient rank condition for exact identification, which for example rules out dependence amongst the r^2 restrictions. In general if the number of available restrictions $k < r^2$ the system is under-identified, if $k = r^2$ the system is exactly identified, and when $k > r^2$ the system is over-identified, and subject to the rank condition being satisfied the over-identifying restrictions are testable.

In the application we report below in section 4, we will use these identification criteria of Pesaran and Shin (op.cit) to derive over-identified wage, employment and price equations. In order to do so we also use the procedure advanced in Greenslade et.al () which, as they show, considerably increases the power of the tests for cointegration.

Finally, the key point to be made here is that the structural identification of β in (7), from (3) or (4) is a function of restrictions (possibly, although not necessarily, exclusion ones) on the elements of each row in β ; that is on each cointegrating vector. However this does not imply that any cointegrating vectors are excluded from any individual equation in the system. Hence it is possible to think of an identified long run system which has wages being affected by *all* the cointegrating vectors in the system and yet which is still fully identified. The key to identification in such non-stationary systems in our view is that economic theory gives rise to sets of cointegrating vectors which can be identified uniquely. This requires a use of economic theory which is rather different way from the traditional approach. Traditionally, this has simply been to determine what variables might be excluded from an individual equation. Instead we must ask what the long run structural relationships may be, and which might be representable by cointegrating vectors. This is our starting point for an application to the identification of the wage equation. Before moving to that in section 4, we first discuss why bargaining theory generally suggests the existence of two cointegrating vectors in the wage equation.

3. Bargaining models and cointegration

The argument made already is that, properly constituted identification criteria need to be applied when deciding if the wage equation is identified. This is hardly controversial. More of a problem is the next part of our claim which is that wage bargaining models generally give rise to two cointegrating vectors, each of which quite properly belongs in the wage equation. This section is devoted to a demonstration of this point and re-visits some wage bargaining models, specially that of the McDonald–Solow(1981) paper, which has the advantage of both being the basis of many subsequent applications including that of LNJ, and discusses a range of bargaining structures. This is important as the argument advanced here is not simply that a specific bargaining model gives rise to the restrictions needed for identification, but that models of the bargaining process will generally give rise to the required identification conditions. The underlying thrust of our argument is that the bargain over wages takes place between two sides; the firm and the labor suppliers (sometimes unions) and the behaviour of each of these sides of the bargain will give rise to a cointegrating vector. Both of these vectors will enter the wage equation but as both are identifiable, the wage equation is also identified.

Case 1: The Monopoly Union

We begin by considering the McDonald Solow simple Monopoly Union case. This has two ingredients, the firm's objective and the union's, which are the same in the later cases, so all that changes is the nature of the bargaining process. The objectives are

$$\text{Firm; Maximise Profits } R(L) - wL \quad (8)$$

$$\text{Union Maximise Utility } L(U(w) - \bar{U}) \quad (9)$$

where $R(L)$ is the firm's revenue function, w is wages, L is employment and the unions want high wages but to avoid unemployment, \bar{U} depends on the disutility of work and the utility of the alternative wage.

The model is of a monopoly union, which unilaterally sets the wage, where the firm then sets employment (the so called right to manage model). In this case the firm maximises its profits given wages. Thus

$$R'(L) - w = 0 \quad (10)$$

This means that the union will maximise (9) subject to (10) with respect to wages, which yields the following first order condition.

$$(U(w) - \bar{U})/wU'(w) + LR''(L)/R'(L) = 0 \quad (11)$$

Clearly both equations (10) and (11) in this model represent equilibrium conditions which we would expect to hold in the long run and so both would be cointegrating vectors. ****[Only (11) would enter the wage equation in this model as (10) only affects employment.]**** However, the parameters entering R'' and R' in (11) are the same as those in (10), so there are economic theory restrictions between the equations which generally would allow (10) to be identified by exclusion restrictions and (11) to be identified by cross equation restrictions. It appears therefore, that this model is identified.

CASE 2 The Efficient Wage Model

The second case allows for the wage and employment level to be simultaneously determined by a full bargaining process. The difficulty with the earlier case is that it is not a Pareto efficient outcome and hence both the firm and the union could improve their situation simultaneously. The solution to this present model is given by a contract curve which is defined by the following equation

$$(U(w) - U(\bar{w}))/U'(w) = w - R'(L) \quad (12)$$

and this can be expressed as the two equations

$$(U(w) - U(\bar{w}))/U'(w) = B \quad (13)$$

$$w - R'(L) = B \quad (14)$$

where B is variable capturing the relative bargaining strength of firms and unions, and the degree of product market imperfection (see below). In the case of constant market imperfection and where the bargaining strengths of the two participants is stable then B is a constant, and this implies that both (13) and (14) will each be a (separate) cointegrating vector.

We do however have to ask what happens to this equilibrium as factors affecting the bargain, or the general economic environment change (i.e. as B changes). Here it is possible to demonstrate two important things. First the contract curve in terms of employment is bounded. Second changing economic circumstances can shift the whole contract curve, but when this happens, (13) and (14) still hold. What this means is that B can change over time but this change is limited. This is hardly surprising; (14) is effectively saying that imperfections such as monopoly power can drive a wedge between the firm's marginal revenue product and the real wage. But it is unlikely that this wedge could grow in an unbounded way. Thus, in general, we may be pushed away from the perfectly competitive equilibrium but this departure must be by a bounded, stationary, amount.

So given that B is a stationary stochastic process at most, then clearly (13) and (14) each represent a cointegrating vector, and they must be separately identified, since essentially they contain nothing in common. In principle, it is possible that the dynamic wage equation and the dynamic employment equation in this model could each contain both cointegrating vectors (13) and (14), but this feature is irrelevant to their identification.

One further possibility remains; B is essentially capturing the relative bargaining strength of the firms and unions, so it possible that this relative bargaining strength has changed over time in a non-stationary way. Union membership in the UK as a proportion of employment (unionization) has been falling steadily over the 1980s and 1990s, and legislative changes since the beginning of the 1980s have successively weakened union's bargaining strength. This may mean that B is itself a non-stationary process. Even in this case, identification of the system can still be achieved under the fairly mild assumption that B is a function of a set of variables, such as union legislation or union membership. In which case, define a vector of such variables, Z, and assume that $B=g(Z)$, and restate (13) and (14) as

$$(U(w)-U(\bar{w}))/U'(w) = g(Z) \quad (15)$$

$$w - R'(L) = g(Z) \quad (16)$$

Again identification of these two vectors follows in a straightforward way.

One concluding point concerns the range of possible bargaining structures the nature of which might affect the contract curve (whether we have a dominant union or a dominant firm, as well as type bargain which might be struck- e.g. Nash solutions or otherwise). The essential point of the analysis is that under almost all circumstances the outcome is driven by the relationships in the contract curve. A particular model may be interpreted as selecting a particular point on the contract curve or even to drive a wedge between the contract curve and the outcome. Nevertheless the final solution will still be defined with respect to the basic contract curve (12) and hence the two basic cointegrating vectors we have already noted will enter that solution. In much of what follows we refer to these two long run relations as a labour "demand" and a labour "supply" equation, using this fairly obvious oversimplification merely for convenience.

4 Empirical Results

4.1 The Model

In this section we outline the system of theoretical equations which we plan to use when identifying the wage-price-employment system. As with all modelling we have to make pragmatic decisions to limit the size of the system which is estimated. As our primary interest is the estimated wage equation we use the variables which have commonly figured in the UK empirical work on wages. As we detail below, this means initiating our estimation with an eight-equation dynamic model. (Originally, we started with ten, but two variables, unionization and real unemployment benefits regularly proved incorrectly signed. Hence, we simplified the system to an eight equation one). So the full model is an eight equation VECM where the common lag length selected for each equation is four. Our testing method is to apply restrictions sequentially to this full model. The procedure we follow is to apply restrictions to the loading matrix first (tests for weak exogeneity) and then apply within- and between-equation restrictions implied by the theoretical model to the long run relations of the model. Precise details are given in 4.2 below. As the restrictions we apply to the long-run part of the model are economic-theoretic ones they enable us to derive, and test for the existence of, a long run model having the principal features of the “standard model”. Before proceeding to the tests, we describe the main parts of this standard model, using LNJ (op.cit) as its source.

The wage equation is of the form outlined in section 3 above. More specifically, it is obtained from a union-firm (Nash) bargain, where the union is concerned with the bargained wage relative to alternative income when not employed, and the firm’s objective is to maximize profit. The aggregate wage equation² is

$$w - p = \gamma_0 - \gamma_1 u + b_2 \alpha (k - l) + Z_w \quad (17)$$

where u is unemployment, $(k-l)$ is productivity, and Z_w is a set of wage “push” variables including real benefits and measures of union strength (LNJ p368).

The price equation is based on the maximising profit condition,

$$\kappa P_i = (1/\alpha) W_i Y_i^{(1-\alpha)/\alpha} \quad (18)$$

for the i ’th firm. Substituting for the firm’s product demand function (dependant on relative prices and an index of overall demand y^d), and aggregating across all firms, gives the aggregate price mark-up equation (the “price” equation)

² This is for the closed economy case. Extensions to the open economy do not alter the argument we are making

$$p - w = b_0 + (b_2 - b_1)(y^d - y^*) - b_2\alpha(k - l) \quad (19)$$

(see LNJ (op.cit), p338-339, and p 362-368), which assumes that the mark-up is affected by demand fluctuations, and productivity.

To obtain the employment equation, it is simpler to derive it using the elementary production function

$$Y_i = N_i^\alpha, \quad (20)$$

and substitute into (19) above to give the marginal revenue product condition

$$\alpha\kappa P_i N_i^{-(1-\alpha)} = W_i \quad (21)$$

which is solved for employment (N) in terms of the real wage and the mark-up. Thus in the aggregate,

$$n = a + (1/(1-\alpha)) \log \alpha\kappa - (1/(1-\alpha))(w - p) \quad (22)$$

In the wage bargaining model, the wage bargain is struck conditional up this employment relation, hence the identification problem that Manning cites. Extending the employment model by allowing for the effect of the capital stock, embodied or disembodied technical progress does not alter this feature.

To summarize, the long run equations we are aiming for are based on the theory reviewed in the previous section, which implies that the wage equation will contain both the cointegrating vector for the marginal revenue product condition and the vector derived from maximizing the union utility function (equation (17)). The employment equation (22) is generally also considered to be a function of the marginal revenue condition. In turn, the price equation (19) is based on the profit maximizing condition for the imperfectly competitive firm. So there are three relations of interest in the model, determining, in turn, wages, employment and prices. In keeping with almost all applications of the standard model, in what follows we assume the production function is Cobb-Douglas with constant returns to scale and autonomous technical progress.

To summarise, the VECM we base the exercise around is

$$\Delta \mathbf{z}_t = \sum_{j=1}^{p-1} \Gamma_j \Delta \mathbf{z}_{t-j} + \alpha \beta' \mathbf{z}_{t-1} \quad (23)$$

following (6) above, and ignoring intercepts and error terms. Here, the vector \mathbf{z} comprises the eight variables already discussed, and the lag length of the model (p) is set at four.

In what follows, we aim to reformulate (23) into a *conditional* model and a *marginal* one which captures the movement of the weakly exogenous variables, using tests for weak exogeneity. Hence, at that stage the model becomes

$$\Delta y_t = \Gamma_1(L)\Delta z_{t-1} + \alpha^* \beta z_{t-1} \quad (24)$$

$$\Delta x_t = \Gamma_2(L)\Delta z_{t-1} \quad (25)$$

where $y_t = (w, l, p)_t$ are endogenous, α^* is a suitably dimensioned sub-matrix of the unrestricted loading matrix α in (23) above, and x_t are the remaining variables in the system and are weakly exogenous. The final stage then applies within- and between-equation restrictions to the unrestricted conditional model (24), written more fully as (26) below

$$\Delta w_t = \Gamma_{11}(L)(\Delta y_{t-1}, \Delta x_{t-1}) + \alpha_1(L)(y_{t-1} - \beta' x_{t-1}) \quad (26)$$

$$\Delta l_t = \Gamma_{12}(L)(\Delta y_{t-1}, \Delta x_{t-1}) + \alpha_2(L)(y_{t-1} - \beta' x_{t-1})$$

$$\Delta p_t = \Gamma_{13}(L)(\Delta y_{t-1}, \Delta x_{t-1}) + \alpha_{3_2}(L)(y_{t-1} - \beta' x_{t-1})$$

The first equation in (26) is for wage inflation, the second is employment adjustment and the third the price inflation equation. As written, the long run relations are unrestricted. Thus the vector y is defined as (w, l, p) and the vector x represents all other variables in the model; so each cointegrating vector involving all the variables in the model enter each dynamic equation. Section 4 below describes how we move from this general, unrestricted, version to the restricted model which is in keeping with the standard model.

In the rest of the paper we describe how it is possible to estimate an over-identified wage-employment-price model which yields a wage equation which is of the same form as the standard model ((1) above). To show that this standard wage model is actually not just identified but actually over identified needs a number of distinct steps. Firstly, we show the model contains non-stationary variables. Secondly, it is shown that cointegration exists between the non-stationary variables, and there are separate cointegration equations for what we earlier described as the “demand” and the “supply” side of the wage bargain. Thirdly, and finally, we need to test that the conditions for over-identification when applied to the model are acceptable. This over-identified wage equation is the same general form to the standard model.

The sections which follow describe the application of each of these steps. First, we discuss the variables and their time-series properties to establish the existence of non-

stationarity. Next the exogeneity properties of the variables are assessed. Finally we estimate the full model, and apply and test the over-identifying conditions which we claim yield the standard model.

4.1 Time-Series Analysis

One aim is to explore the identification issue in the context of the standard model, so our choice of equations (26) above, and hence the variables that go into them, are influenced by the variables commonly used in that model. Thus we initially use the set of ten variables $z = (y, x)$, where the full set z comprises the wage rate (w), employment (l), real output (q), producer prices (pp), the consumer price (pc), the capital stock (k), unemployment (u), the tax and price wedge (wd), the replacement ratio (rr), and union density (ud). In other respects too, our choice represents in part the results of recent research. So although later versions of the standard model have progressively resorted to a larger set of “labour supply” variables such as alternative measures of union power, indicators of industrial turbulence, and skill mismatch (see, e.g. Nickell (1998)), it has been shown that these variables are not robust to sample changes, so are not used here (see, e.g. Henry and Nixon (2000)). The model is purposely simplified in other ways, not allowing open-economy influences on the equilibrium rate of unemployment for example, other than the effect of the import price wedge in the wage equation (For a discussion, see Nickell (1998) and Greenslade, Henry and Jackman (2000)).

Within the initial set of ten variables, we find strong evidence that two – union density and the replacement ratio – are wrongly signed. This appears a robust finding, and is confirmed in other studies (apart from those just cited, see also Greenslade, Henry and Jackman (2000)). Hence in what follows the analysis is restricted to the remaining eight variables only. Data for these is quarterly. And the sample is for 1973q1-1998q4. Full data definitions are given in the appendix.

The time-series properties of these variables is first addressed, and Table 1 shows the tests for stationarities.

Table 1: Stationarity Tests.
(Sample; 1973q1-1998q)

Variable	DF	DF(4)
Q	-1.65	-2.37
L	-0.86	-3.2
W	-2.76	-2.76
Pp	-7.3	-3.2
Pc	-5.56	-3.2
U	-0.13	-1.68
K	-2.7	-2.8
Wd	-3.0	-2.8
Rr	-1.03	-1.51
Ud	-6.4	-2.79

The 95% cut value for the DF test is -3.46 . All results refer to the test with a deterministic trend included.

In most cases these tests show that there is clear evidence that the variables are non-stationary. The exceptions to this finding are the DF test for the two price series and for union density. In each of these cases though, the first order DF test shown in the first column, showed clear evidence of serial correlation. For this reason the augmented tests, shown in the second column, are to be preferred. Hence we proceed with the analysis of the model under the assumption that the relevant variables are indeed non-stationary.

4.2 Testing for Weak Exogeneity

Following Greenslade et al.(2000), we first analyse the exogeneity status of the variables since this is the decision which appears most to affect the power of the tests, including the tests on the long-run behaviour of the model, which follow. Taking the eight variables - w, pp, pc, l, k, u, y and wd – we first decided that wd, the tax and import price wedge, should be treated as weakly exogenous on a-priori grounds. As this variable is entirely affected by overseas and policy changes, this decision seems reasonable, and it also simplifies the tests which follow. In these tests, the evidence appears that there is at least 3 co-integrating vectors in the set of the eight variables, and so this hypothesis is maintained throughout.

Our approach to the tests of weak exogeneity is then a sequential one. In summary these first show that unemployment is weakly exogenous. Based on this it then seems that the capital stock is also. At this point the evidence is that the system is characterised as having five endogenous variables (w, pp, pc, l, y) and three weakly exogenous (k, u, wd). Table 2 gives details.

Table 2: Exogeneity tests

Variables	Weakly exogenous	Wald Test
W, pc, pp, u, l, y, k	U	7.2 (7.8)
W, pc, pp, l, y, k	K	1.39(7.8)

The table gives the set of variables which are used in the tests, and the second column then shows the variables in this set which appear weakly exogenous. All tests are done with the wage taken as weakly exogenous at the outset. The results indicate that first unemployment and then the capital stock can be taken as weakly exogenous. From the set of the five remaining variables- w, pc, pp, l, and y – all appear endogenous according to these tests.

To summarise so far, our initial tests show that we can reformulate the reduced-form VECM given by (23) into *Conditional* and *Marginal* sub models. The conditional model includes the endogenous variables which depend on the cointegrating vectors in the complete model. It is with this subset that we are concerned here, and within that, the wage, price and employment equation especially. The next section describes the results of the requisite tests on this part of the model.

4.3 Estimating the VECM

The previous tests show that we are dealing with a non-stationary system with three exogenous variables in a set of eight. Based on these results, we next consider the evidence on the co-integrating rank of the system, and the evidence for co-integration particularly between the variables on the demand side of the labour bargain and, separately, on the supply side.

4.3.1. Cointegrating Wage Equations

The analytical model set out in section 3 describes the bargaining model in terms of at least two cointegrating vectors, one for the demand side originating in a marginal productivity condition, the other from the supply side capturing the effects of employment probabilities, income out of work, and tax. We say “at least two” since, as our earlier discussion of identification made clear, there is no reason why a fully identified wage equation should not include all the cvs in the complete VECM system. But as our argument about the identification of the standard model in Section 3 makes clear, we claim that the two key relationships in the wage model – the demand and the supply equations- are each independently are cointegrating vectors. Table 3 sets out the results of tests of co-integration among sub-sets of the eight

variables. The first is of the underlying CD production function, the second the implied marginal productivity –real product wage relation, and the third the supply of labour relation between the real consumption wage, unemployment and the price –tax wedge.

Table 3; Co-integration Tests

Variables	LR (V)	LR(T)
q,l,k	26.4(25.4)	63.3(42.3)
w, pp ,k,l	24.1(25.4)	52.9(42.3)
w, pc, u, wd	35.9(31.8)	70.2(63.0)

Note; LR(V) is the Likelihood Ratio test based on the Maximal eigenvector, and LR(T) the trace test. The tests are each for $r=1$ against the null of $r=0$.

The tests support the assumption that the relevant subsets of variables co-integrate.

4.3.3.Results for the VECM

Having completed the necessary preliminaries, this section turns to estimating the model of interest. To estimate the model we use Full Information Maximum Likelihood (FIML), since although we are estimating a reduced form, non-linear restrictions including between-equation restrictions, figure in the restricted model, thus necessitating a full-information approach.

In keeping with the standard wage bargaining model we will be assuming that the production function is linear homogeneous CD. To ensure that it, and the accompanying marginal productivity condition co-integrate, it proves necessary to allow for observed changes in the rate of disembodied technical progress in the UK and cyclical variations in factor utilisation. Thus, where technology is cast in a CD framework, there is widespread evidence that the rate of overall productivity growth has varied significantly over the last three decades in the UK. Harvey, Henry and Wren-Lewis(1986) for example, give time-varying estimates of the trend for the manufacturing sector, which suggests that the apparent increase in productivity in the 1980s appeared to be a resumption of the pre-1970s trend with a shift down in the level of the trend in the 1970s. To capture these effects we use additional time trends. The first starts in 1981q4 to represent the apparent change in the productivity trend which emerged at the beginning of the 1980s. The second captures the slowdown in the trend starting at the end of 1990 (1990q4).

There is also the need to make allowance for variations in factor utilisation in both the production function and the marginal productivity relation. A full treatment of this needs an explicit inter temporal model of factor use based on fundamental distinctions

between fixed (including quasi-fixed) and variable factors. (Morrison (1986), Chamberlin, Hall, Henry and Satchi (2001)) Such an extension would take this study far outside the confines of the present study, and instead we proxy cyclical variations in factor use with “V” shaped intervention dummies to capture the main periods of under-utilisation. There are two of these, the first for the late 1970s early 1980s recession and recovery, and this runs from 1979q3 to 1984q3, and the second for the 1990s recession and recovery and runs from 1990q1 to 1994q4.

Turning to the part of the *conditional* model which concerns us - for the real wage, prices and employment, the general form is given by (26) above. The cointegration results for the complete set of eight variables earlier show that there are three cvs in the VECM. Our intention is to show that these can be restricted according to the requirements of the standard neoclassical demand for labour, the supply of labour, and the production function respectively, and that these restrictions will be data-acceptable.

Before applying and testing these tests on the long-run part of the model, we simplify the dynamics of the conditional model, and this data-based simplification shows that second-order model is sufficient.

Turning now to the tests on the long-run part of the model, we first restrict the unrestricted form given by equation (26) above, by using the cointegrating vectors given in Table 3, which are cointegrating subsets of the original eight variables, and contain just the variables which go into the production function, the labour demand and supply equations respectively. This involves five restrictions in the production function and four each in the remaining two equations, giving thirteen in total. Recalling the Pesaran and Shin show that to exactly identify this conditional model nine restrictions are needed (i.e. r^2 restrictions where r is the number of cvs in the model), it is evident that the model is already overidentified at this stage. Formal tests of these additional four restrictions (i.e. 13 minus the 9 of the just identified case) proceed as follows. The Johansen procedure gives an estimate of the just identified model using orthogonality conditions. It is known that any just identified model will yield the same likelihood value as the Johansen estimate. (Pesaran and Shin (op.cit)). Using this result we can test the additional thirty overidentifying restrictions by computing the Likelihood Ratio $2*(LL_{UR} - LL_R)$, where the first likelihood is the Johansen estimate of the just identified model and the second the likelihood when using the same model but with the cvs from Table 3. Applying this test gave an LR of 4.4, which when compared to the 95% critical value of 9.49($\chi^2(4)$) shows these restrictions are acceptable.

The next step is then to apply, and test, a final set of exclusion and within- and between – equation restrictions to the long run relations, and exclusion restrictions to the adjustment matrix, to obtain the standard model. Table 5 below shows the results of these tests. The first set of restrictions are to the adjustment matrix of the conditional model, and apply exclusion restrictions to the dynamic equations for wages, prices and employment. These restrictions exclude the production function from the wage and price inflation equation and the employment equation. In addition,

the long run labour supply equation from the price inflation equation and the employment adjustment equation. These give five restrictions in total. These are shown as the first entry of the Table, and these restriction are upheld. The second set of restrictions are the linear homogeneity of the production function, which are shown as the second entry. Again, these are satisfied.

Table 5; Tests of Overidentifying restrictions

	Restriction	Number of restrictions (k)	LR	$\chi(k)$
1	Restricting the adjustment matrix	5	9.5	11.07
2	Within equation restrictions	2	0.24	5.99

These restrictions appear to be clearly accepted, and so in the final version of the model shown below they are applied. As is clear from our earlier discussion of identification in VECMs, it is evident that the system as a whole is over identified, having substantially in excess restrictions than the nine which are required for exact identification. For a VECM with cointegrating rank of three, as we have here, the condition for exact identification is that nine restrictions are applied to the long-run part of the model. To further refine the model to produce the structure shown below, we have applied and accepted further overidentifying restrictions to the long run part of the model and restrictions on the matrix of loading factors in the conditional model (The matrix α^* in equation (24) above), totalling eleven in all.

When the restricted model is estimated as the fully restricted VECM, the results are as shown below. To estimate this model Full Information Maximum Likelihood (FIML) is used.

$$\Delta w = 0.25 * \Delta w(-1) \quad (2.4)$$

$$-0.208(w - pp - [0.78 * (y - l)])(-1) + 0.0036 * t \quad (27)$$

(2.4) (21.3) (62.3)

$$-0.32 * (w - pc + 0.40 * wd + 0.02 * u + 0.012 * t)(-1)$$

(3.45) (0.7) (3.87)

$R^2=0.36$, $DW=1.76$.

established for a number of cases, including where data is non-stationary as is the case here. These identification criteria are different from, and generalise, the Cowles-Commission rank and order conditions, but which imply exclusion restrictions and exogeneity assumptions, neither of which are tested in the traditional framework. Hence our thesis is that there is a need to reconsider the identification of the standard wage bargaining model in the modern framework. In applying these identification concepts we have framed the central question as whether the standard wage equation can be found to be data-admissible when full systems modelling based on the non-stationary unrestricted VECM is used as the starting point. Then the question is whether over identifying restrictions applied to this VECM give a wage equation of the same form as the standard model and whether these restrictions are statistically acceptable. We report estimates which show that the answer to both is in the affirmative. We draw attention to a common feature in VECMs that more than one cointegrating equation per dynamic equation is a perfectly reasonable and interpretable property. We have argued that wage bargaining theory suggests there is more than one target relation in the wage equation; one deriving from the firm's side, the other the union. It is most likely that each will be a cointegrating vector. In the dynamic wage equation we report we find evidence supports the inclusion of two cointegrating vectors having these firm (or demand) and union (or supply) interpretations, so it is a wage equation of the standard form. But as we have shown, when properly interpreted, the resulting model is far from being under identified and is actually over identified.

Data Appendix

All data are quarterly, seasonally unadjusted. The sample period is 1973q1-1998q4.

w Average earnings

l Total employment

pp Producer prices

pc Consumer prices

u Unemployment rate

y GDP

wd Tax and import price wedge

k the capital stock

rr the replacement ratio

ud union density

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