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INTEREST RATE LINKAGES:
A KALMAN FILTER APPROACH
TO DETECTING STRUCTURAL CHANGE

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Abstract

This paper investigates changes in the causal structure linking the G-7 short-term rates by estimating time-varying speed of adjustment coefficients in error correction equations using a Kalman filter approach. This technique allows us to detect structural breaks in the causal linkages or even a reversal in the direction of causality. The hypotheses of interest are the US world-wide leadership, the disengagement of UK monetary policy from those pursued in the Eurozone after the collapse of the ERM, and the German leadership hypothesis (GLH) within the European Union (EU). While we do not find any example of reversal of causality, the evidence points to a break in the causal linkages between the UK and other EU countries after the third-fourth quarter of 1992. The empirical results are also consistent with a US world-wide leadership and a weak German leadership within the Eurozone.

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1.   Introduction

In broad terms one can identify two views on how interest rates may be linked. If they are treated as analogous to other asset prices, then their movements are naturally interpreted as being determined by financial flows in fluid, profit-seeking capital markets. This will normally give rise to a set of arbitrage conditions such as the uncovered interest rate parity condition. Alternatively, they can be viewed as policy instruments, so that their time paths may be determined by a policy objective such as an exchange rate target or an inflation target. These two approaches are not necessarily inconsistent, deviations from interest rate parity may cause the exchange rate to move towards its policy target. As long as deviations of the exchange rate from its target are stationary the deviations from interest rate parity will also be stationary. Interest rate linkages have therefore often been analysed in the context of a specific policy framework such as the Exchange Rate Mechanism (ERM). For instance, numerous studies have attempted to test the so-called "German Leadership Hypothesis" (GLH), according to which Germany acts as the dominant player within the system, and monetary authorities in other ERM countries are unable to deviate from the course of interest rates set by the Bundesbank (see, e.g., Fratianni and von Hagen, 1990, and Kirchgassner and Wolters, 1993). Taking this view, co-movements in interest rates arise because of policy convergence. But under pure arbitrage conditions we also expect interest rates to move together in the long run. So the question naturally arises, what is the effect of a policy regime on the system and how will the system change if the policy regime changes?

In the existing literature on linkages, standard causality tests are carried out in the context of non-stationary VARs, even though the resulting test statistics have non-standard distributions, so that statistical inference is invalid. To avoid this pitfall, Caporale and Williams (1998b) take a different approach, and use the methodology advocated by Toda and Yamamoto (1995). This has the advantage, compared to other testing strategies, of not requiring pre-testing for the cointegration properties of the system, and yields statistics which follow standard distributions, thus making inference legitimate (for a detailed discussion, see Caporale and Pittis, 1999, who also present an empirical example). [1] Using this technique, Caporale and Williams (1998b) find that in the G-7 there is a marked difference between linkages in long-term rates (10-year bond yields) and linkages in short-term rates (3-month Treasury bills). Whilst there is little evidence that the former have been linked to one another over the last two decades, for the latter the evidence of co-movements is more compelling. Furthermore, the causal structure is not consistent with the standard characterisation of the ERM as an asymmetric system in which Germany was the dominant player - it suggests instead that there was German accommodation of French monetary policy within the ERM. [2] This result could be interpreted in the context of the "size effects" identified in recent theoretical research, according to which larger, more stable countries can achieve policy objectives more successfully via accommodation than by

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[1] More recently, Bruneau and Jondeau (1999) have shown that the non-causality conditions can be expressed as the nullity of a function of long-run dynamic multipliers and the parameters of a VAR in levels, and can be tested by means of a standard Wald test. Using this method, they analyse long-run causal links between US, German and French long rates, and find bi-directional causality between US and German rates, as well as between German and French rates.

[2] See also Karfakis and Moschos (1990), and Katsimbris and Miller (1993), both studies essentially rejecting the GLH.

compulsion (see Martin, 1997). The system was actually more flexible than normally recognised, as there were various "escape clauses" built into it (for instance, the options of exchange rate realignments, wider fluctuation bands, and capital controls).

The structure, though, might change over time. For instance, Caporale et al (1996) reported convergence in European rates after 1986, and Artis and Zhang (1998), using rolling window cointegration techniques, found that there is widespread cointegration between both US and German short rates and those on other ERM currencies up to 1995, after which US influence on world-wide rates vanishes. [3] [4] In the context of the ERM, with its target zones, there might be regime shifts owing to the policies pursued by central banks. Specifically, the stochastic properties of interest rates (volatility, level and speed of adjustment) are likely to be different in periods when the currency has to be defended from speculative attacks, compared to periods when the exchange rate is credible. Because of the uncovered interest parity (UIP) relation, switches in the process governing exchange rates are translated into switches in the process followed by interest rates. Such regime shifts tend to be more frequent and not to be as long-lived as changes in monetary policy regimes in the US, say. Dahlquist and Gray (2000) show that a Markov-switching model well characterises the behaviour of a number of EMS short rates.

In general, one can think of changes in structure as changes either in the long-run relationships themselves (the cointegrating vectors) or in causality links (the loading factors). It would be problematic to specify the source of structural change in a model allowing for both types of changes as such a model would typically not be identified. In the case of interest rates, as almost any theory suggests long-run co-movement, it is reasonable to assume the cointegrating vectors are constant but the direction of causality changes. Hence we concentrate on the latter source of change, and estimate time-varying parameter models for the loading weights. This has the advantage that one does not have to impose a priori restrictions on when the breaks in the relationships might have occurred. Instead, the relationships are allowed to evolve freely, and the revealed timing of the structural breaks can be very informative about the effects of policy changes (see, e.g., Haldane and Hall, 1991, who analyse sterling's relationship with the US dollar and the Deutschemark). Kalman filtering techniques were used by Hall et al (1992), who found convergence in inflation and interest rates within the EMS. An example of state space modelling of interest differentials can be found in Cavaglia (1992), who uses one-month Euro deposit rates for five industrial countries. He finds that ex ante real interest differentials are relatively short-lived and mean reverting to zero, which would suggest that the extent to which national authorities can exercise influence over their domestic financial markets is limited. However, this type of analysis is not very informative about the sources of shocks of interest rates. Identifying the nature of the shocks, estimating the importance of common factors, and studying the transmission mechanisms all require analysing linkages between the levels of interest rates.

A novel way to test for long-run causality and structural change is suggested by Beeby, Funke and Hall (1998). Broadly speaking, policy-making can be thought of as an attempt to change the structure of an economy. Specifically, one could think of

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[3] Note that the results from this estimation method are highly sensitive to the selection of the window width and of the lag length.
[4] See also the growing literature on testing for structural breaks, e.g. Banerjee et al (1998).

structural change as changes in the causal structure of a system, which do not affect the underlying structural relationships. For instance, UK and German interest rates may have been set primarily in line with US rates prior to entering the ERM, but once full membership is achieved the UK rates may be driven by the German monetary stance which is still set with a view to the US. In both case the three interest rates will move in line and be cointegrated on a pairwise basis but the direction of causality changes between the two policy regimes.

To be more precise, consider a Vector Error Correction Model (VECM), where the long-run matrix has been decomposed into a vector of loading weights and one of cointegrating relationships. A change in the structure of the system could occur through changes either in the former or in the latter. In order to make the problem of identifying the sources of structural change more tractable, one can assume that only the vector of loading weights is subject to change, either because the weights vary over time, or because they change from a zero to a non-zero value. Alternatively one can assume that cointegrating vectors which were not significant before may now enter a particular equation in the system, or viceversa. In other words, "new" cointegrating vectors might emerge as a result of structural changes. This means that a different set of variables exhibit long-run linkages depending on what period is considered. In the former case, the weights can be estimated either by using a time-varying parameter version of the Kalman filter, or by adopting recursive techniques. These amount to multiplying each cointegrating vector by a dummy variable, which is switched on and off for different sub-samples. A t-test is then implemented to establish over what period(s) each cointegrating vector is significant. This will indicate whether the process of interest has been driven by different variables in different periods. However, a difficulty with the tests reported in Beeby et al (1998) is that standard distributions cannot be relied upon to test for the significance of the t-statistic.

In this paper, we investigate changes in the causal structure that links the G7 short-term rates using a Kalman filter approach. The empirical findings of this research will have important policy implications, as they will provide evidence on whether countries can still conduct an independent monetary policy despite the increasing integration of international financial markets (see Caporale and Williams, 1998a,c). It appears that even in a system like the ERM which aims to produce policy coordination it has been possible for monetary authorities to disengage their policy from developments elsewhere and pursue an independent policy agenda over long periods. Such an option should remain available for non-participating countries, like the UK, after the establishment of the Euro. Therefore the UK authorities will not necessarily find their freedom of action greatly constrained by what is happening in the Euro zone. Within the Euro zone the policies of the European Central Bank (ECB) will not necessarily be as stable or credible as those adopted so far by the German authorities, since smaller countries will also have an influence on monetary policy (see Begg at al, 1998). If in fact Germany has not been able to impose its interest rate policy on the other ERM countries, and if this becomes true of fiscal policy as well (notwithstanding the Growth and Stability Pact), long-term rates might rise (rather than decline) in the EU after 1999.

The paper is organised as follows: The next section (section 2) introduces some relevant concepts such as Granger-causality and exogeneity within the framework of

cointegrated systems. The Kalman filter method is outlined in section 3, while section 4 discusses the empirical results and their economic implications. A summary concludes.

2.   Exogeneity and Causality in Cointegrated Systems

Building an econometric model typically involves focusing on a set of (endogenous) variables of primary interest, which are explained in terms of other (exogenous) variables. The advantage of such an approach is that it is easier to model the endogenous variables conditional on the exogenous variables if these show some kind of irregular behaviour, which would be difficult to model within a VAR framework. It is very tempting to draw inference from the conditional or partial model whilst modelling the exogenous variables less carefully or not at all. The idea underlying such an approach is that if we could just draw inference about the cointegrating rank in the partial system, estimating β and testing for hypotheses on it, we would work with smaller systems in terms of parameters to be estimated with a gain in efficiency.

The problem, however, is that such an approach is valid if and only if the assumption of weak exogeneity is satisfied (Engle, Hendry and Richards, 1983). Failure to satisfy such a requirement will make it problematic to derive the asymptotic distribution theory for the estimate of β. Harbo et al. (1998) show that even if weak exogeneity is assumed, the presence of deterministic terms in partial systems makes it difficult to determine the rank without modelling the full system, because the asymptotic distribution of the test statistic will be different from the one of the full model. As a consequence, one needs to work with full structural systems in error correction form, the partial systems being more a result of our inference than a starting point. The reason is that, rather than simply imposing restrictions, we would want to test for their validity. Estimating validly a partial system requires not just exogeneity of some variables with respect to the parameters of interest, but also a precise long-run causal structure of the model. Within the framework of cointegrated systems these two issues coincide (as far as the long-run properties of the model are concerned). We can in fact show that long-run non-causality is necessary as well as sufficient for long-run weak exogeneity of a variable with respect to the parameters of interest.

2a   Weak-Exogeneity

The basis for this discussion is provided by the analysis of joint and conditional densities and sequential factorisation (see Hendry 1995 for a detailed account).

Let

2.1 $D_x (y_t , z_t \mid X_{t-1} , \theta)$

be the sequential density at time t of the random vector $x_t = (y_t : z_t)'$ conditional on $X_{t-1} = (X_0, x_1,\ldots,x_{t-1})$, where $\theta = (\theta_1,\ldots,\theta_n)' \in \Theta$ which is a subset of $\mathfrak{R}^n$.

Generally speaking, z_t is endogenous in the framework of the joint density function, but if z_t is weakly exogenous it is possible to factorise the joint density such that knowledge of how the process z_t is determined is not necessary in order to investigate the properties of the process y_t.

Let us allow for the existence of many one-one transformations from the original n parameters $\theta = (\theta_1,...,\theta_n)' \in \Theta$ to any new set of parameters $\phi \in \Phi$, and also let $\phi = (\phi_1', \phi_2')$. We can then factorise the joint density function as:

2.2 $\quad D_x (y_t, z_t \mid X_{t-1}, \theta) = D_{y|z} (y_t \mid z_t, X_{t-1}, \phi_1) \ D_z (z_t \mid X_{t-1}, \phi_2)$.

Let the joint density under analysis involve a subset ψ of the parameters θ, where ψ is a vector of parameters of interest. The first requirement for a variable z_t to be regarded as weakly exogenous for a set of parameters of interest ψ is that the marginal process for z_t should add no useful information about ψ, that is we must be able to learn about ψ from ϕ_1 alone. The second condition we need to justify taking z_t as given is that ϕ_1 should not depend on ϕ_2. If this were the case we could learn indirectly about ψ from ϕ_2.

We can then say that z_t is weakly-exogenous for ψ if and only if

- ψ is function of ϕ_1 and does not depend on ϕ_2;
- ϕ_1 and ϕ_2 are variation free.

2b   Granger-causality

In a famous paper, Granger (1969) shows that given two multivariate processes {x} and {y}, and the information on them contained in their past behaviour X_t and Y_t, {y} causes {x} at time t if the past of {y} provides additional information for the forecast of x_t with respect to considering the past of {x} alone. From the definition above we see that there is a linkage between weak exogeneity and causality. Indeed stating that a variable y has no role in the prediction of another variable x is tantamount to saying that the lagged values of y do not enter the equation for x, i.e. there is no feedback from y to x.

This result is very similar to the first condition for x to be weakly exogenous with respect to the parameters of interest, in that it seems that x in this case is determined outside the system by its own past. The problem is that this fulfils only the first requirement for weak-exogeneity, and therefore implies that in standard regression analysis non-causality is necessary but not sufficient for weak-exogeneity. Things are substantially different when working with non-stationary series and within a cointegration framework. Let us explain this point formally.

Consider a simple p-variate vector autoregression[5]:

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[5] We are omitting deterministic terms to keep the example as simple as possible, but these could be included without complications.

2.3　　$\Pi(L)z_t = \begin{bmatrix} \Pi_{11}(L) & \Pi_{12}(L) \\ \Pi_{21}(L) & \Pi_{22}(L) \end{bmatrix} \begin{bmatrix} y_t \\ x_t \end{bmatrix} = \begin{bmatrix} e_{yt} \\ e_{xt} \end{bmatrix} = e_t$

where $\Pi(0) = I_p$, $E(\varepsilon_t)=0$, $E(\varepsilon_t \ \varepsilon_s')= \lambda_{ts}\Omega$, and the maximum lag in $\Pi(L)$ is k. We assume that some of the roots of $|\Pi(L)|$ are equal to 1 while the others lie outside the unit circle in the complex plane. Let also y_t and x_t be of dimension p_1 and p_2 respectively and $p_1+p_2=p$.

In order to check whether the variables in z_t are cointegrated and y_t does not cause x_t we have to test whether $\Pi(L)$ is upper block triangular and $\Pi=\Pi(1)$ is non-zero and has reduced rank. As a first step we reparameterise the model in an Error Correction Form as follows:

2.4　　$\Delta z_t = \Gamma_1 +...+ \Gamma_{k-1}\Delta z_{t-k+1} + \Pi z_{t-k} + e_t$

or more compactly as

2.5　　$\Delta Z = \Gamma\nabla Z + \Pi Z + E$,

where $\Gamma = (\Gamma_1,....,\Gamma_{k-1})$, $\Delta Z = \Delta Z_t$, $\nabla Z = (\Delta Z_{t-1}',...,\Delta Z_{t-k+1}')'$, $Z=(z_1,...,z_{t-k})$.

In this framework (following Mosconi and Giannini 1992), y does not Granger-cause x if the hypothesis

2.6　　H_0: U'ΓV=0,　　U'ΠU$\perp$=0.

holds, where:

2.7　　$U = \begin{bmatrix} 0 \\ Ip_2 \end{bmatrix}$　$U\perp = \begin{bmatrix} Ip_1 \\ 0 \end{bmatrix}$　$V = I_{k-1} \otimes U\perp$,

and U is (pxp_2), U$\perp$ is (pxp_1), V is (p(k-1)xp_1$(k-1)).

It is important to highlight that in cointegrated systems and VECMs, one can distinguish between two different types of causality, the first part of H_0 concerning short-run causality, while the hypothesis U'ΠU$\perp$=0 is about long-run causality or weak causality as in Davidson and Hall (1991). Another way of formulating the hypothesis U'ΠU$\perp$=0, with reference to our initial system, is to test whether Π_{21}=0. We can show that this is equivalent to testing which rows of α are zero.

This also matters in the context of testing for weak exogeneity. We will show that long-run (weak) non-causality is necessary and sufficient for weak exogeneity under the hypothesis of cointegration. Let us rewrite our system in VECM form as

2.8　　$\Delta X_t = \Gamma_1\Delta X_{t-1} +...+ \Gamma_{k-1}\Delta X_{t-k+1} + \Pi X_{t-k} + e_t$

where the parameters are defined as before. Assume for simplicity the absence of deterministic terms. The matrix $\Pi = \alpha\beta'$ contains information on the long-run relationships among the series in the model, with β containing the cointegrating relations and α representing the speed of adjustment to equilibrium. Also, we know that if there are $r \leq (n-1)$ cointegrating vectors in β, this implies that the last n-r columns of α are zero. To test how many $r \leq (n-1)$ cointegrating vectors exist in β is equivalent to testing how many columns of α are zero.

Focusing our attention on the non-zero columns of α, let the process X_t be decomposed into

$$2.9 \qquad X_t = (X_{1t}', X_{2t}')' \, and \, \boldsymbol{a} = \begin{bmatrix} \boldsymbol{a}_1 \\ \boldsymbol{a}_2 \end{bmatrix}, \Gamma_j = \begin{bmatrix} \Gamma_{1j} \\ \Gamma_{2j} \end{bmatrix}.$$

We can now rewrite the equations of the model as

$$2.10 \qquad \Delta X_{1t} = \boldsymbol{a}_{1t} \boldsymbol{b}' X_{t-1} + \sum_{i=1}^{k-1} \Gamma_{1i} \Delta X_{t-i} + \boldsymbol{e}_{1t}$$

$$\Delta X_{2t} = \boldsymbol{a}_{2t} \boldsymbol{b}' X_{t-1} + \sum_{i=1}^{k-1} \Gamma_{2i} \Delta X_{t-i} + \boldsymbol{e}_{2t}$$

where ε_{ij} are iid $N(0, \Omega)$, and $\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{bmatrix}$.

Let us now consider the conditional model for ΔX_1 given the past and ΔX_2, i.e.

$$2.11 \qquad \Delta X_{1t} = \boldsymbol{w} \Delta X_{2t} + (\boldsymbol{a}_1 - \boldsymbol{w}\boldsymbol{a}_2) \boldsymbol{b}' X_{t-1} + \sum_{i=1}^{k-1} \tilde{\Gamma}_{1i} \Delta X_{t-i} + \tilde{\boldsymbol{e}}_{1t}$$

where $\boldsymbol{w} = \Omega_{12} \Omega_{22}^{-1}, \tilde{\Gamma}_{1i} = \Gamma_{1i} - \boldsymbol{w}\Gamma_{2i}, and \, \tilde{\boldsymbol{e}}_{1t} = \boldsymbol{e}_{1t} + \boldsymbol{w}\boldsymbol{e}_{2t}$ with variance equal to $\Omega_{11.2} = \Omega_{11} - \Omega_{12} \Omega_{22}^{-1} \Omega_{21}$. We will now make the following statement:

The presence of all zeros in the i-th row of the matrix $\boldsymbol{a}_{ij}$, j = 1,...,r indicates that the cointegrating vectors in $\boldsymbol{b}$ do not enter the equation determining $\boldsymbol{D}X_{it}$. This implies that there is no loss of information from not modelling the determinants of $\boldsymbol{D}X_{it}$, which can therefore enter only the right-hand side of the system since there is no feedback from the other variables in the system.

This implies long-run non-causality as well as weak exogeneity as mentioned before. We can formalise it as follows, in the context of a system in error correction form:

$$2.12 \qquad \Delta X_{1t} = \boldsymbol{w} \Delta X_{2t} + (\boldsymbol{a}_1 + \boldsymbol{w}\boldsymbol{a}_2) \boldsymbol{b}' X_{t-1} + \sum_{i=1}^{k-1} \tilde{\Gamma}_{1i} \Delta X_{t-i} + \tilde{\boldsymbol{e}}_{1t}$$

$$\Delta X_{2t} = \boldsymbol{a}_{2t} \boldsymbol{b}' X_{t-1} + \sum_{i=1}^{k-1} \Gamma_{2i} \Delta X_{t-i} + \boldsymbol{e}_{2t}$$

If $\alpha_2=0$ then X_{1t} is not causing X_{2t}, and X_{2t} is weakly exogenous for the parameters of interest (β,α_1) and the maximum likelihood estimator of β and α_1 can be inferred from the conditional model alone. This can be seen rewriting the system under the hypothesis $\alpha_2=0$, that is:

$$\Delta X_{1t} = \mathbf{w}\Delta X_{2t} + (\mathbf{a}_1 + \mathbf{w}\mathbf{a}_2)\,\mathbf{b}'X_{t-1} + \sum_{i=1}^{k-1}\widetilde{\Gamma}_{1i}\Delta X_{t-i} + \tilde{\mathbf{e}}_{1t}$$

2.13

$$\Delta X_{2t} = \sum_{i=1}^{k-1}\Gamma_{2i}\Delta X_{t-i} + \mathbf{e}_{2t}$$

In fact, we can see that there is no trace of α_1 and β in the marginal model and therefore there is no trace of X_{1t}, which is therefore not causing X_{2t}, namely, there is no feedback from the former to the latter. Also, the condition that requires the parameters of the marginal to be unrelated to the parameters of the conditional model is fulfilled as a property of multivariate Gaussian distributions that do not have joint restrictions. Note that if $\alpha_2=0$ then the sp$((0,I)')$ is contained in sp$(\alpha\perp)$, which means that $\sum_{i=1}^{T}\mathbf{e}_{2t}$ is a common trend in the sense that the errors in the equations for X_{2t} cumulate in the system giving rise to non-stationarity. X_2 will still be cointegrated with X_1 of course, as implied by the first equation in 2.13. The key point to note here is that as long as Π_{11} is of full rank then $\sum_{i=1}^{T}\mathbf{e}_{1t}$ will not be a common stochastic trend of the system and hence there will be no long-run link from X_1 to X_2. A related proof was presented in Hall and Wickens (1993).

3   Kalman filtering

The Kalman (1960, 1963) filter technique is adopted to estimate linear regression models with time-varying coefficients. [6] This class of models consist of two equations: the transition equation, describing the evolution of the state variables, and the measurement equation, describing how the observed data are generated from the state variables. This approach is extremely useful for investigating the issue of parameter constancy, because it is an updating method producing estimates for each time period based on the observations available up to the current period current period. It is important to realise that recursive OLS estimation (or moving window OLS estimation) is not a suitable technique to use here. Recursive estimation is essentially a test of structural stability. We can set up a null hypothesis that the parameters are constant and see if that can be rejected through recursive estimation. But as the underlying assumption of OLS is always that the parameters are constant, recursive estimation does not provide a consistent estimate of a time-varying parameter.

Let the Kalman Filter measurement equation be:

3.1 $y_t = x'\beta + \varepsilon_t \qquad\qquad \varepsilon_t \sim N(0, H_t)$

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[6]   For a simple exposition of Kalman filtering, see Cuthbertson *et al* (1992).

and the transition equation be:

$$3.2 \quad \beta_t = T\beta_{t-1} + \eta_t \qquad \eta \sim N(0, Q_t)$$

with the initial conditions given by:

$$3.3 \quad \beta_0 \sim N(\beta_0, \sigma^2 P_0)$$

When T=I and Q_t=0, the model is reduced to the standard normal OLS regression model. The matrices T, H and Q are assumed to be known, and the problem is obtaining estimates of β_t using information I_t available up to time t. The process of evaluating the conditional expectation of β_t given I_t is known as filtering. The evaluation of β_t given I_s, with s>t, is instead referred to as smoothing, whereas the estimation of β_t with s<t is called prediction. Kalman (1960) derived the basic results to obtain filtered and smoothed estimates of β_t recursively. The prediction equation is given by:

$$3.4 \quad \hat{\beta}_{t/t-1} = T\hat{\beta}_{t-1}$$

and the covariance matrix is defined as:

$$3.5 \quad P_{t/t-1} = TP_{t-1}T' + Q_t$$

Finally, the updating formulae are given by:

$$3.6 \quad \hat{\beta}_t = \hat{\beta}_{t/t-1} + P_{t/t-1}x(y_t - x'\hat{\beta}_{t/t-1})(x'P_{t/t-1}x + H_t)$$

and

$$3.7 \quad P_t = P_{t/t-1} - P_{t/t-1}x'xP'_{t/t-1} / (x'P_{t/t-1}x + H_t)$$

As the estimates are updated recursively each period, Kalman filtering can be viewed as belonging to the class of Bayesian estimators. Before starting the estimation process, one has to specify the vector of prior coefficients β_t and the matrix Q. By estimating the long-run relationship in this way one obtains a vector containing the evolving state coefficients which show whether the relative importance of the factors driving the dependent variable has changed over time.

In our case we start from a model in error correction form such as:

$$3.8 \quad \Delta X_t = \Gamma_1 \Delta X_{t-1} + ... + \Gamma_{k-1}\Delta X_{t-k+1} + \boldsymbol{ab}'X_{t-k} + \boldsymbol{e}_t,$$

where ε_t are assumed to be iid N $(0,\Sigma)$, and taking the Γ_I and $\boldsymbol{b}$ as non time-varying we estimate the matrix of adjustment coefficients $\boldsymbol{a}_t$ with the Kalman filter, under the assumption that this matrix follows a random walk process such that

3.9 $\quad \boldsymbol{a}_t = \boldsymbol{a}_{t-1} + \boldsymbol{n}_t, \quad \boldsymbol{n}_t$ is $N(0, \boldsymbol{s}^2 I)$

In particular we apply this procedure to the bivariate systems linking the G-7 short interest rates as irreducible relations[7] (see Barassi, Caporale and Hall 2000) in order to investigate the possibility of breaks in the causal structure of these linkages or reversals in the direction of causality. What we expect, given the results presented in Barassi, Caporale and Hall (2000), is to find exogeneity of US interest rates with respect to all other rates, implying convergence to zero of the adjustment parameter in the error correction equations. The same should happen to the UK rate in the equations describing long-run linkages with the other European rates, at least after the collapse of the ERM in third/fourth quarter of 1992. Other hypotheses of interest are German leadership within the Eurozone and the stability of the causal structure within the G-7 system as a whole.

4.   Empirical Results

For the empirical analysis we use IMF quarterly data on the three-month Treasury bill rates covering the period between 1980:Q2 and 1998:Q3. Notice that for efficiency reasons we have estimated single equations in error correction form rather than bivariate vector error correction models. This is legitimate as in an earlier paper (see Barassi, Caporale and Hall, 2000) we found that there is bivariate cointegration between all the rates with unit cointegrating vectors. Once the cointegrating vectors are determined then single equation estimation becomes FIML.

More specifically, in Barassi, Caporale and Hall (2000) we analysed causal linkages between the G-7 short-term interest rates by applying a methodology due to Davidson (1998) based on the concept of irreducible cointegrating relations (IC). Evidence was found that cointegration is a property of the G-7 short rates, and that it is important to test for irreducibility as a diagnostic. We also extended Davidson's (1998) methodology introducing the ranking of the IC relations according to the criterion of minimum variance. This allowed us to distinguish between structural and solved IC vectors without any prior theoretical assumptions. Furthermore, we performed exogeneity tests on all IC relations in order to gather information on the causal structure that links interest rates.

Briefly, we found that the system of the G-7 interest rates has a rank of six. The immediate implication of this is the existence of six structural irreducible cointegrating regressions which we were able to isolate, ranking the IC relations according to the criterion of the minimum variance. The two most significant relations involve the US and Canada, and Italy and France. The other four relations involve Germany and Japan, and Japan with USA, France and UK. The causal structure obtained from testing restrictions on the matrix of loading weights in the bivariate irreducible systems suggested a US worldwide leadership and rejected the hypothesis of a German leadership in Europe, therefore confirming the findings of Caporale and

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[7] The concept of irreducible cointegrating vectors is due to Davidson (1998) and refers to cointegrating subsets of variables, which do not have any cointegrating subsets. The Davidson methodology (see Davidson 1998) decomposes hybrid cointegrating relations in irreducible ones allowing (in the case of over-identified systems) the identification of the structural relations without imposing theory based restrictions on the cointegrating vectors.

Williams (1998b) and of other authors (see, e.g., Katsimbris and Miller, 1993). Essentially, the US and Canada appear to constitute the fundamental block, UK rates respond more to non-European than to other European rates, Italy is following France, and France and Germany respond to world rates rather than to each other. Lastly, Japan acts as the link between US and European rates.

These results however are all predicated on the assumption that the causal structure is constant over the sample period. A casual consideration of the structural changes, which have occurred in the monetary policy structure of the world over the last 20 years, makes this an unlikely assumption. And so we will now apply the techniques outlined above to investigate the possible changes in exogeneity and weak causality structure, which may have occurred.

Below we discuss the time paths of the adjustment coefficients in the single-equation error-correction models corresponding to the irreducible cointegrating relations. It is worth mentioning that most of the first differences of the G-7 interest rates seem to follow an autoregressive process of order one, AR(1), apart from the US rates that can be modelled as an AR(3) or AR(4) process. We estimate the single equations by OLS, and having imposed the OLS coefficients as the fixed parameters of the observation equation we then re-estimate the same equations with the Kalman filter, assuming that the coefficient of the error correction term follows a random walk.

Six models were estimated for each country where in each equation we allow for the possibility that that particular country adjusts to one of the other G7 countries. We are therefore allowing for the possibility that each country is being influenced by any of the other six at any point in time. So if we found that for country A all six adjustment parameters were zero for the whole period, it would tell us that this country was not influenced by any of the other countries over this period. If we found that the adjustment coefficient involving country B became significant half way through the period, then this would indicate a shift in policy regime such that country A started to follow country B from that period onwards. We will not report all the short-run dynamic parameters of the model for the sake of brevity.

4a   USA

The empirical results seem to support the existence of US leadership. The speed of adjustment coefficients converge towards zero in almost all cases. Exogeneity of US interest rates with respect to the Italian and the German ones is clearly observable. The linkage with the Canadian rate seems to be the most significant one, although even this coefficient is below 0.1 and is falling. The same   (but with even smaller coefficients) can be said about the speed of adjustment to disequilibrium in the linkage with the French and Japanese rates. The situation is slightly different in the error correction equation containing the cointegrating relation that links US to UK rates. It seems that there was some (decreasing) feedback from UK to US rates until the collapse of the ERM in the last quarter of 1992. Afterwards, US rates appear to be exogenous and causality runs from US to UK rates only. Overall we can conclude that the evidence from the time-varying estimation supports the idea that the US economy is a constant point of reference for all the countries of the G-7 group.

4b   UK

The results for the UK are even more interesting. The main result (see table 1) is that the linkage between UK and other European rates has become weaker after the collapse of the ERM. In particular, apart from the clear exogeneity of UK rate with respect to the Italian one, it can easily be seen that in the 1990s the influence of the German and French rates on the British one has decreased overtime, even reaching zero in the case of the linkage with the French rate. It seems that the UK rate still responds to the German one (very weakly), but it is also clear that the influence on the UK of conditions in the non-European G-7 countries is still strong and even growing in the case of Canada. It is worth considering what happens in the causal relation with the US rate. Here we can observe a break in the causal linkage between 1989 and 1991. Notice that after this period the US influence on UK rates seems to grow again. Overall we can then say that UK rates seem to respond to non-European rates, and that, following the breakdown of the ERM, UK monetary authorities have pursued policies that are completely independent from those implemented in the economies of the Eurozone. This might help explain the differences in economic performance between UK and other European economies during the 1990s.

4c   The Eurozone

As already mentioned, one of the hypotheses of interest is the purported German leadership (GLH) within the Eurozone. In order to test it, we should only consider the three countries in our sample belonging to EMU, therefore investigating the speed of adjustment of German rates towards the long-run equilibria shared with the French and Italian rates. On the basis of the results obtained from this "partial" analysis, we might be tempted to conclude that the empirical evidence supports the GLH. This is because of the exogeneity of German rates with respect to the French and the Italian ones. However, a closer look to the complete G-7 system shows that such a hypothesis does not have empirical support, for three main reasons.

First, while the only causal link to Germany seems to be from the US there are also strong direct links from the US to France and Italy (in both cases larger in magnitude than the link to Germany). So while the three rates may be moving in line there does not seem to be a strong case for arguing for one of them as the leader. Second, one of the six structural IC vectors links Italy and France, which implies that the relations between these two latter countries and Germany belong to the class of irreducible solved relations rather than to the structural ones. Third, Barassi, Caporale and Hall (2000) argue that Germany does not share any of the six structural irreducible relations with another European country.

As for Italian and French rates, there appears to be feedback from all the world rates and between themselves. This provides us with enough evidence to conclude that European rates are actually driven more by US rates, which are exogenous with respect to all the Eurozone rates. It is worth noticing that the time path of the speed of adjustment coefficients of the error correction equations of European rates displays a kink in the fourth quarter of 1992, which coincides with the collapse of the ERM.

Also, subsequently it becomes a lot smoother, implying more stability within the Eurozone.

4d   Canada and Japan

We have chosen to discuss the results on Canadian and Japanese rates together as they seem to act as the *trait d'union* between US and European rates within the G-7. The main feature of the Japanese rate is its high non-variability. Specifically, it is characterised by step-changes, clearly indicating that Japanese rates are determined by policy decisions rather than market conditions. This may help to explain the low variability of the time-varying parameter estimates of the adjustment coefficients in its error correction equations as well as their low (but non-zero) values. We find some weak feedback from more or less all the other rates (with the exception of the German one) to Japanese rates. [8] As already stated, it appears that Japanese rates, together with the Canadian ones, act as a linkage between European and US rates. In fact the feedback from both Japanese and Canadian rates to US rates is substantially weaker than the one in the opposite direction. As for the Canadian rate, it is linked to the other rates by a two-ways feedback relation, apart from its clear exogeneity with respect to the French rate in another of the six irreducible structural cointegrating relations.

5.   Conclusions

In this paper, we have investigated changes in the causal structure linking the G-7 short-term rates by estimating time-varying parameter models using a Kalman filter approach. In particular, we have applied the technique to bivariate error correction systems linking the G-7 short-term rates as irreducible relations in the sense of Davidson (1998). The analysis was aimed at examining the possibility of structural breaks in the causal linkages between rates, which in some cases might make it possible for monetary authorities to disengage their policy from developments elsewhere, or even for a reversal in the direction in causality between rates to occur. Other hypotheses of interest concerned the US world-wide leadership, the degree of autonomy of monetary policy in the UK policy after the collapse of the ERM in September 1992, and the GLH in the Eurozone.

While we have not found any examples of reversals of causality, we have found some evidence of breaks in the causal linkages between the rates under investigation. One of the most interesting results concerns the progressive disengagement of UK policies from developments elsewhere in the EU, especially after the collapse of the ERM. In the following period, UK rates seem to be linked much more to world rates, as shown by the higher speed of adjustment parameter after 1992 in the equations linking UK and world rates. As for the other results, the evidence seems to support the leadership of the US, the corresponding speed of adjustment coefficients being very close to zero. Furthermore, we have found some evidence of a German leadership in the Eurozone, since the German rate has been found exogenous with respect to the French

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[8] Notice that the relation between these rates constitutes one of the six irreducible structural linkages between the G-7 rates.

and Italian ones. Nevertheless, as German rates are in turn driven by other world rates (mainly US, UK and Japanese rates - recall that German and Japanese rates are linked by an irreducible structural relation), the German leadership is not substantial. Finally, Japan and Canada act as a linkage between US and European rates.

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Time path of adjustment coefficients in error correction equations for France
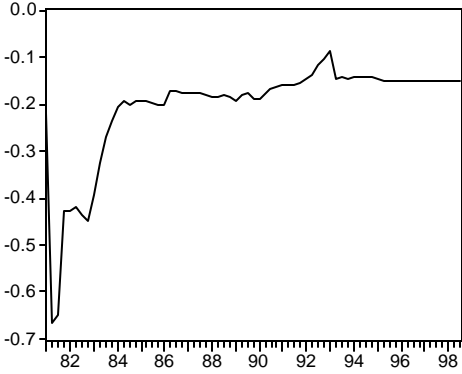
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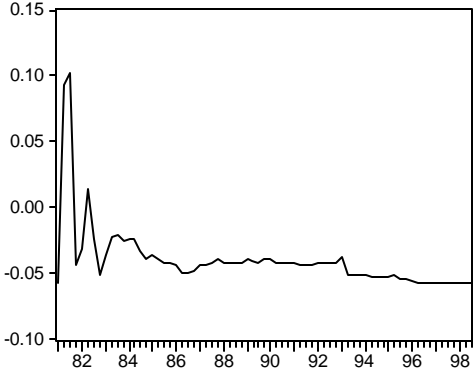
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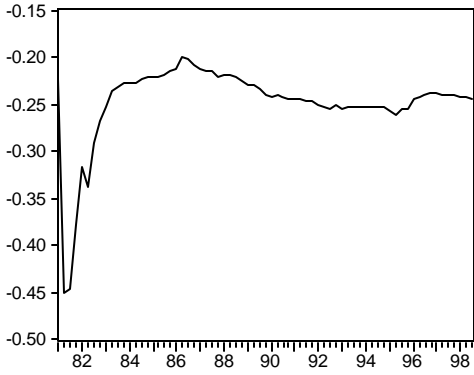
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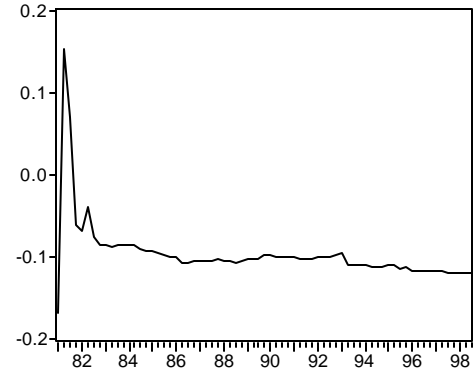
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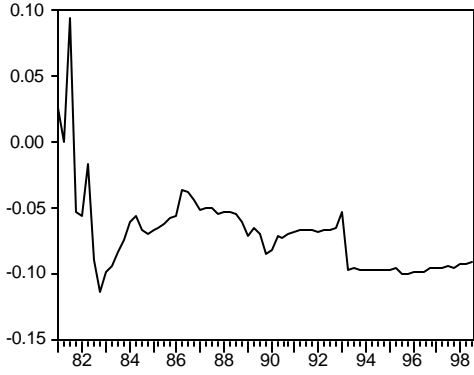
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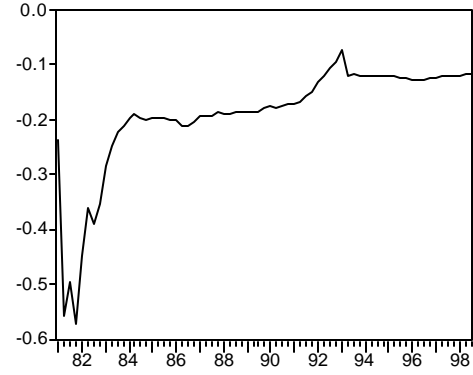
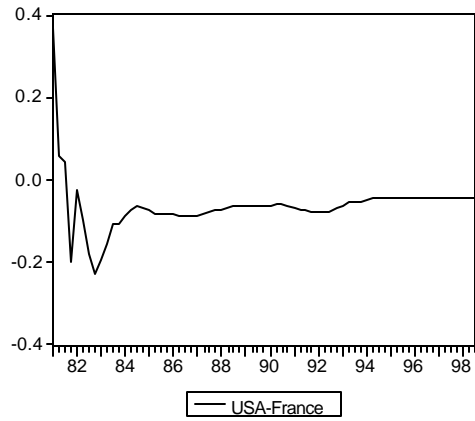France-USA

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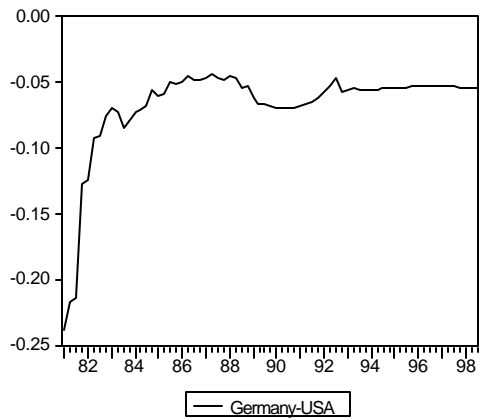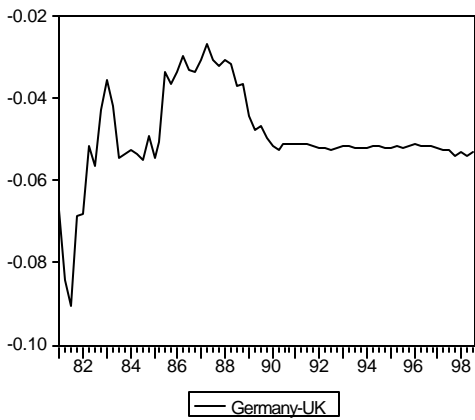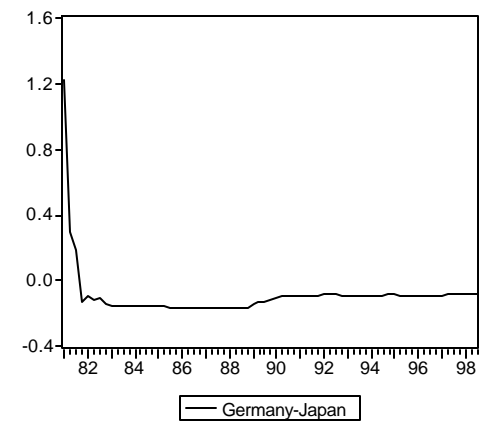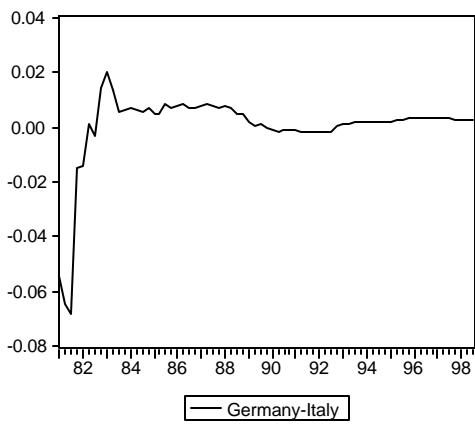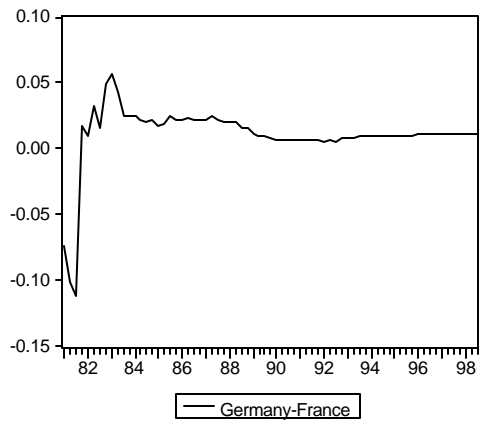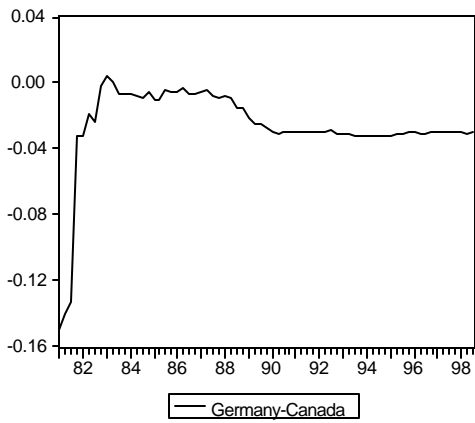
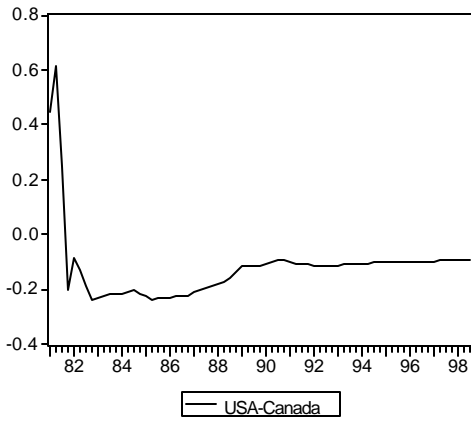Time path of speed of adjustment coefficients in error correction equations for Italy
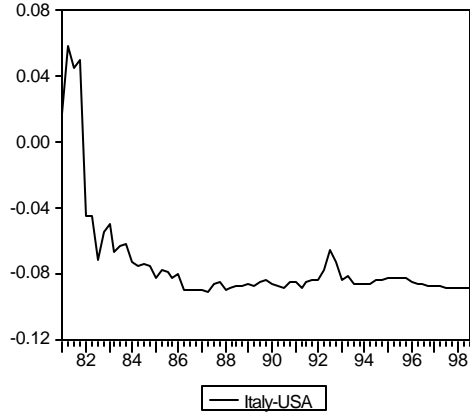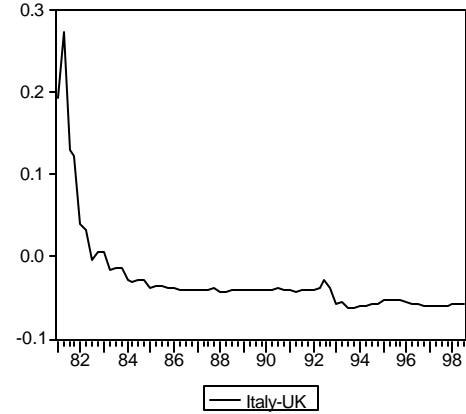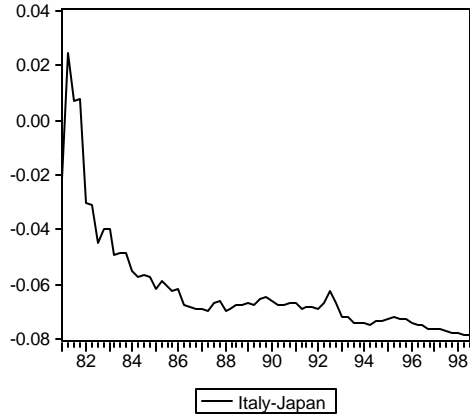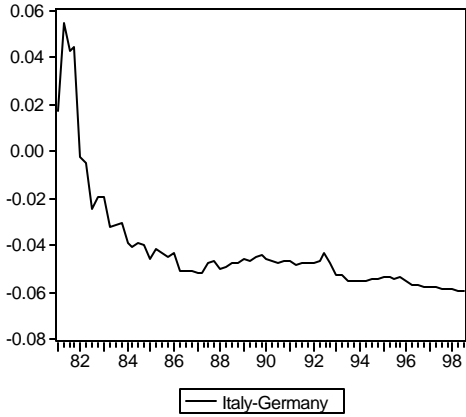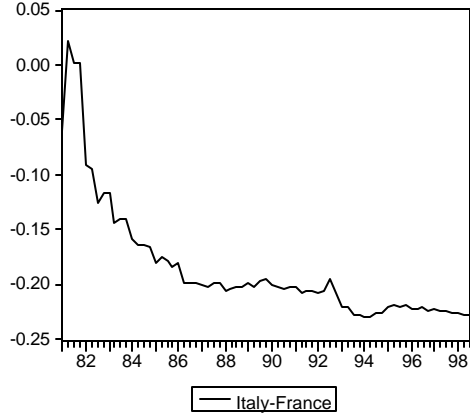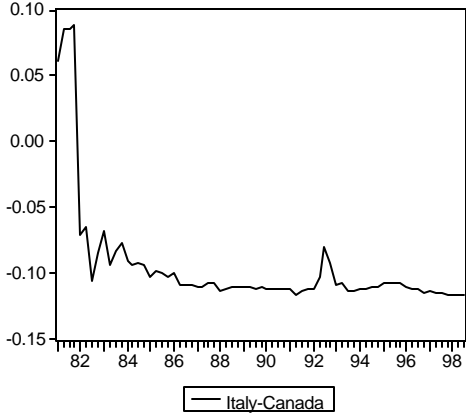
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Japan-France

Japan-Germany

Japan-Italy
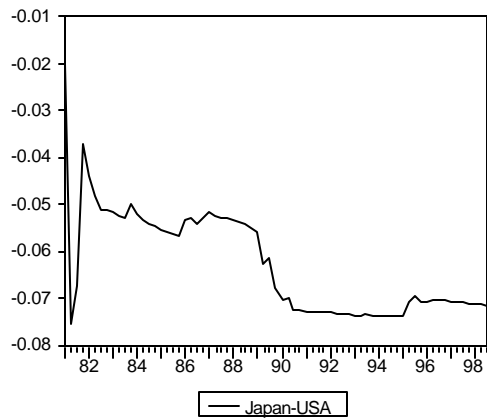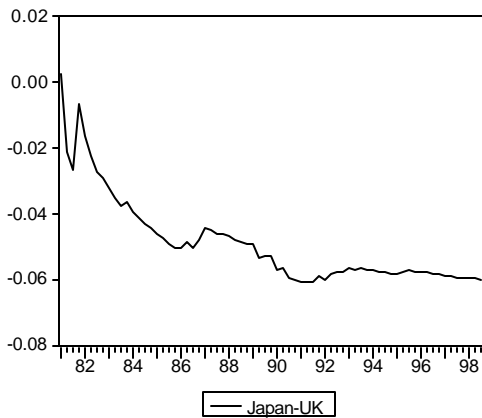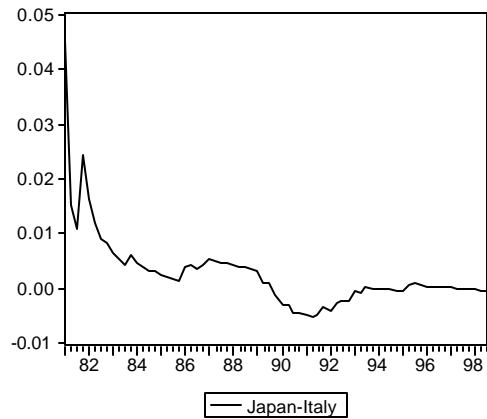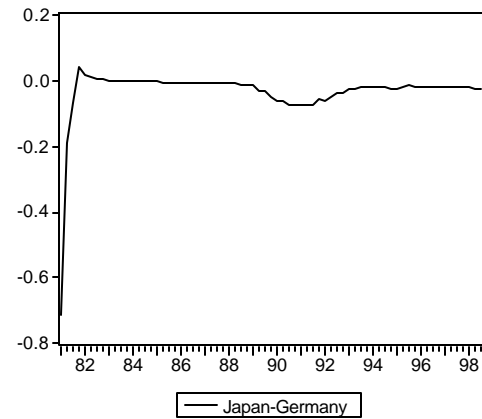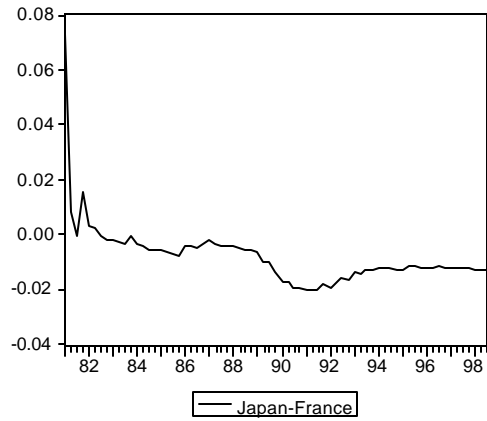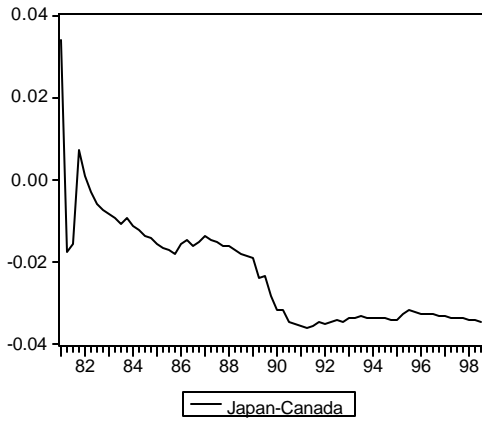
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Japan-USA

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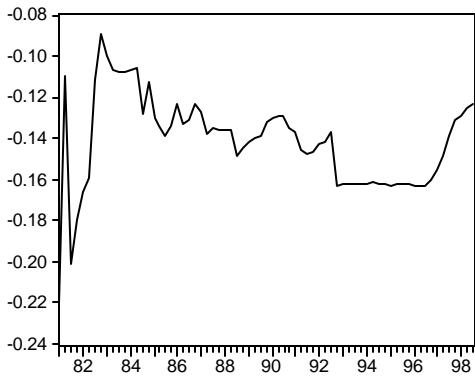
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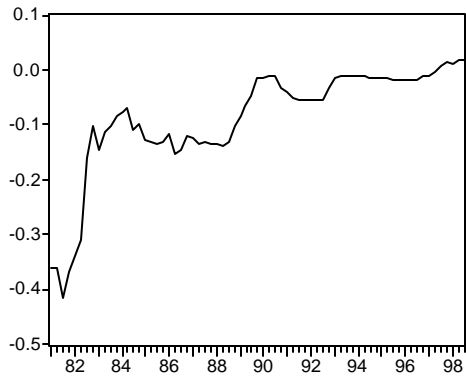
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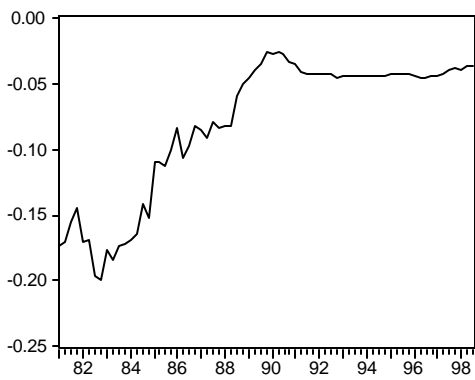
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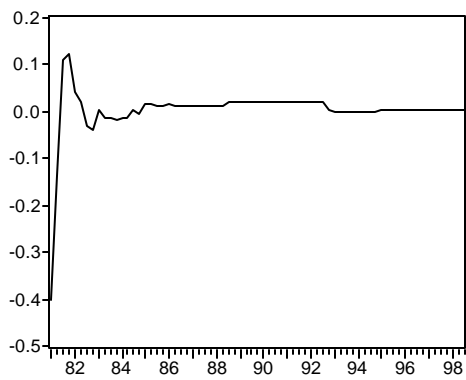
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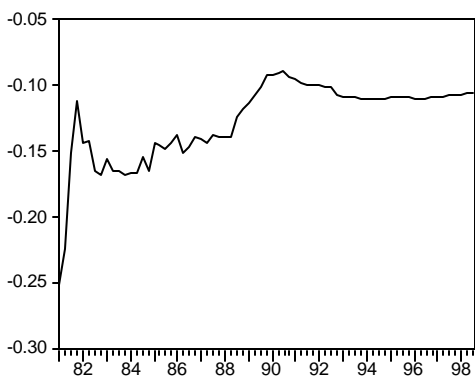
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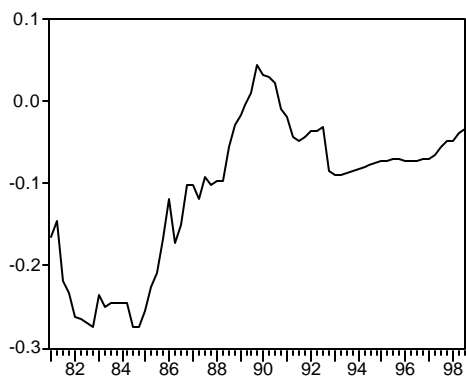
UK-USA

Time path of speed of adjustment coefficients in error correction equations for USA

USA-Canada

USA-France

USA-Germany

USA-Italy

USA-Japan

USA-UK
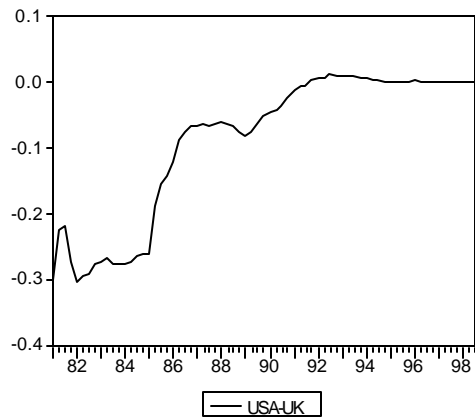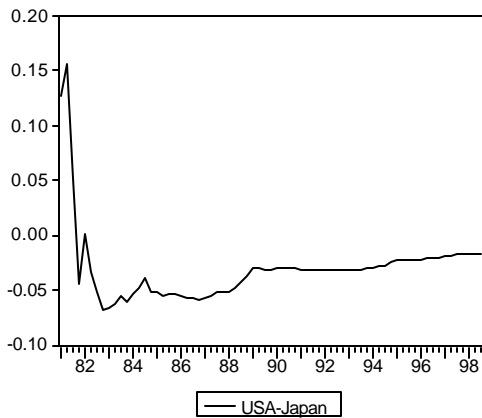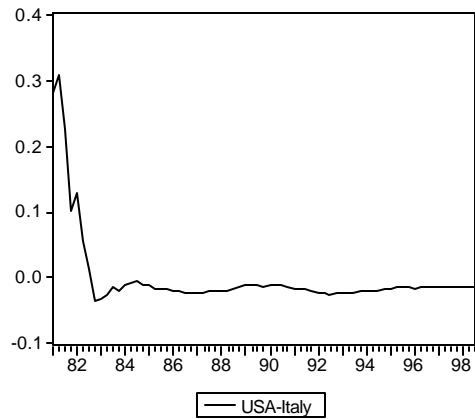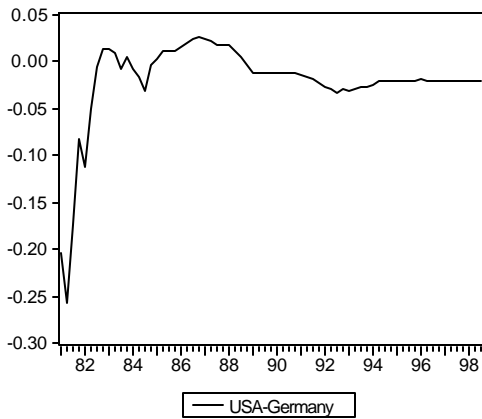
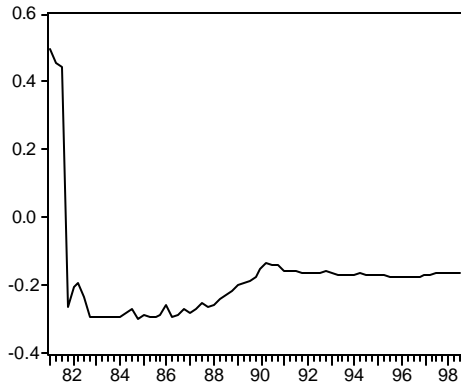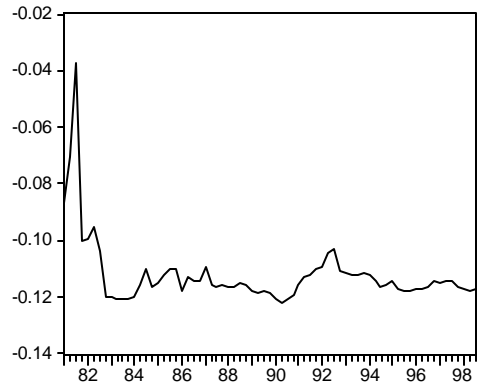Time path of speed of adjustment coefficients in error correction equations for Canada
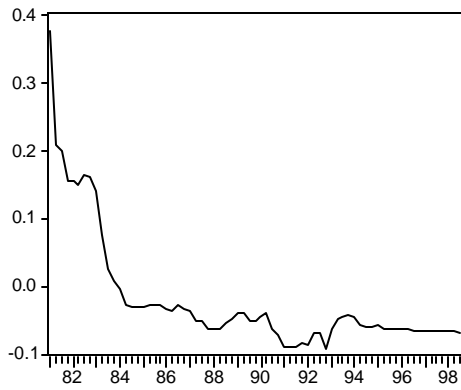
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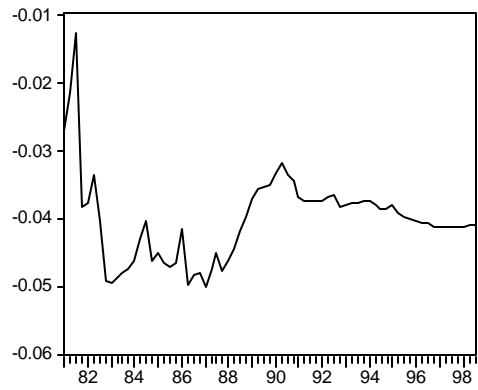
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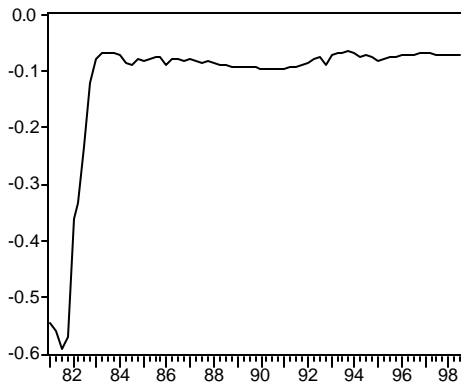
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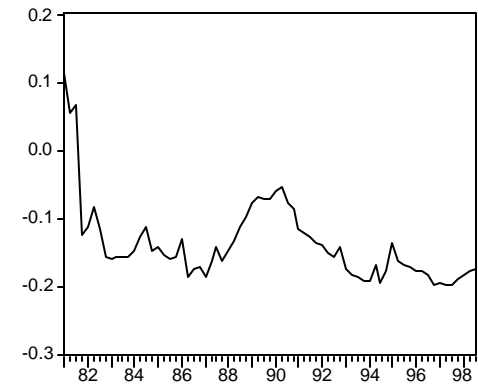
Canada-Japan

Canada-USA

Canada-UK