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On the Identification of Cointegrated Systems in Small Samples: Practical Procedures with an Application to UK Wages and Prices

Jennifer V. Greenslade
(Bank of England)

Stephen G. Hall
(Imperial College Management School

And

S. G. Brian Henry
(Centre for International Macroeconomics, Oxford University)

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ABSTRACT

This paper discusses the practical application of identification in cointegrated systems. It will argue that in a common realistic modelling situation of a limited data set and the theory requirements of a fairly rich model, the techniques proposed in the existing literature are almost impossible to implement successfully. There are crucial decisions to be made over the order in which various restrictions are imposed in the move from a general unrestricted VECM to the fully (over) identified VECM. We will argue that imposing exogeneity restrictions at the earliest possible stage of the model reduction process and then restricting the dynamic adjustment of the model hugely increases the power of tests of overidentifying restrictions on the long-run cointegrating vectors. In practice this means that a thorough use of economic theory at an early stage, rather than treating a model as a pure statistical artefact, can yield enormous benefits.

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1. Introduction

Considerable work has been undertaken in recent years on the identification of cointegrated systems. Beginning with the contribution of Davidson and Hall (1991) it has become increasingly apparent that the structural identification of cointegrated systems is a crucial step in making economic sense of any statistical system, which includes more than one cointegrating vector. In his original contribution, Johansen (1988,1991) used purely statistical criteria to achieve identification in the general case of multiple cointegrating vectors, with the assumption of orthogonality between the vectors. Phillips (1991) presented a more structural approach in that the set of variables was partitioned into an exogenous and endogenous subset of variables with a recursive structure and this provided sufficient restrictions to give formal identification. Johansen (1992) considers the imposition of restrictions on the cointegrating vectors directly and proposes an algorithm for estimating some cointegrating vectors conditional on restrictions placed on others. Pesaran and Shin (1994) and Johansen (1995) have developed a full theory of identification for a general unrestricted model along with some suggestions for an estimation and testing strategy.

In this paper we wish to discuss the practical application of these developments. In particular we will argue that in a common realistic modelling situation of a limited data set and the theory requirements of a fairly rich model, the techniques proposed above are almost impossible to implement successfully in an objective way. There are crucial decisions to be made over the order in which various restrictions are imposed in the move from a general unrestricted VECM to the fully (over) identified restricted VECM. We will argue that imposing exogeneity restrictions at the earliest possible stage of the model reduction process and then restricting the dynamic adjustment of the model hugely increases the power of tests of overidentifying restrictions on the long-run cointegrating vectors. In practice this means that a thorough use of economic theory at an early stage, rather than treating a model as a pure statistical artefact, can yield enormous benefits.

The plan of the paper is as follows, in section 2 we will discuss the general identification problem and the choices which must be made in the order in which testing takes place. Section 3 will present some Monte Carlo evidence to illustrate the importance of the ordering issues discussed in section 3. Section 4 will then present an application to UK wages and prices of the suggested modelling procedure. Section 5 will draw some overall conclusions.

2. Identifying dynamic structural models

We begin by setting out the general structural form of the Vector Autoregressive system (VAR) which forms the basis of our analysis. The starting point is the complete, or closed form, VAR

$$D(L)Z_t = V_t \qquad \textbf{(1)}$$

Where Z is an N dimensioned vector which may be partitioned in general to give $Z_t = (Y_t, X_t)$ where Y is an Mx1 vector of endogenous variables and X is a Qx1 vector of weakly exogenous variables (N=M+Q) and D(.) a suitably dimensioned matrix in the lag operator. Following standard lines, we reparameterise the VAR as a structural VECM (Vector Error Correction model), i.e.

$$A_0 \Delta Z_t = \sum_{i=1}^{p-1} A_i \Delta Z_{t-i} + A^* Z_{t-p} + u_t \qquad \textbf{(2)}$$

Where there are r cointegrating relations in Z, and r < N which implies that A^* has rank r. This rank may be imposed in the usual way by defining $A^* = a^* b^{*\prime}$ where both a^* and b^* are Nxr matrices. However it is important to stress that a^* and b^* are the structurally identified loading weights and the cointegrating vectors, as defined by Davidson and Hall (1991) as the target relationships, not the unidentified ones which are produced in unrestricted estimation.

The Structural VECM (2), will normally be estimated as an unrestricted version of the reduced form given as

$$\Delta Z_t = \sum_{i=1}^{p-1} \Gamma_i \Delta Z_{t-i} + \Pi Z_{t-p} + v_t \qquad \textbf{(3)}$$

Where $A_0^{-1} A_i = \Gamma_i$, $A_0^{-1} u_t = v_t$ and $\Pi = A_0^{-1} A^*$. Identification in the presence of cointegrating vectors is different from that traditionally used for stationary VARs (i.e. The Sims or Blanchard-Quah identification criteria), this is discussed in detail in Robertson and Wickens (1994). In particular there are now two parts to the identification problem. Given that we impose the cointegrating rank of the system r by the standard decomposition of the long-run matrix $\Pi = ab'$ where both a and b are Nxr matrices, we need to consider both the identification of the contemporaneous coefficient matrix A_0 and the identification of the long-run coefficients b. Restrictions on the long-run coefficient matrix can in general tell us nothing about the identification of A_0 as this can only come from the dynamic part of the model using information either from Γ_i or a. In a similar fashion the dynamic part of the model can not help us in general to identify the long-run structure, b. This may be seen easily as $\Pi = ab' = A_0^{-1} a^* b^{*\prime}$, so even if we knew A_0 this would not allow us identify b^* without additional restrictions on b. For this reason Pesaran and Shin (1994) have set out a formal theory of the identification of the long-run structure in isolation.

In general the complete exact (or over) identification of the system will involve a combination of four types of restriction.

a) Restrictions on the cointegrating rank of Π, r<N
b) Restrictions on the dynamic path of adjustment (the Γ_i)
c) Restrictions on the cointegrating vectors , b where $\Pi = ab'$
d) Restrictions on the exogeneity or long-run causality of the system, which will imply restrictions on α.

The conventional VAR conditions (see Robertson and Wickens (1994)) for identification apply to the dynamic identification of the system and as long as a combination of restrictions across the Γ_i and a matrices meet the standard conditions then the model is identified with respect to the dynamics. These restrictions can come from a number of sources, some models have theoretical restrictions on the adjustment process, which may be used to simplify the Γ_i matrix, e.g. the well-known quadratic adjustment cost model is one such. The alternative practise in the absence of theoretical restrictions is to base the restriction process on a data based set of simplifications of the dynamics. In either case some further restrictions may be necessary to identify A_0.

The formal identification of the long-run is the main subject of Pesaran and Shin (1994). Where it is demonstrated that the identification of b^* requires knowledge of r and then there is a necessary condition equivalent to the order condition which states that exact identification of the long-run coefficients requires $k=r^2$ restrictions. So the number of restrictions necessary to identify the long-run is a direct function of the number of cointegrating vectors. Pesaran and Shin (1994) also give a necessary and sufficient rank condition for exact identification, which is also a function of r^2. In general if the number of available restrictions $k<r^2$ the system is under identified, if $k=r^2$ then the system is exactly identified and when $k>r^2$ the system is over identified, according to the order condition and these over identifying restrictions may be tested. Based on asymptotic results from Phillips (1991) and Johansen (1991), Pesaran and Shin also demonstrate that the standard likelihood ratio test of the over identifying restrictions follows a $c^2(k-r^2)$ distribution.

This suggests that the long-run may be estimated and identified and the over identifying restrictions tested from the unrestricted VECM without identifying the model's dynamic structure. Asymptotically this is undoubtedly correct, but for the sample sizes available in most practical situations we argue that the interaction of dynamic identification and long-run identification can have an enormous effects on the size and power of the testing procedures conventionally used. There is then a very important choice to be made as to the order in which restrictions should be imposed and tested.

Figure 1 shows the main routes which we consider when moving from the unrestricted VECM to the exactly or over identified VECM. Note that we do not consider a route, which would involve restricting the dynamics of the VECM as a first step. This may well prove to be a useful approach. However, at present it has the serious disadvantage that the tests of r, the cointegrating rank of the system, have an unknown distribution when the dynamics are restricted in this way. We currently have an asymptotic distribution for tests of r when the rest of the VECM is unrestricted (the standard Johansen case) and we also have tests when some of the variables are exogenous (restrictions on a due to Pesaran, Shin and Smith (1997)). We will therefore confine our discussion in this paper to these two sets of routes.

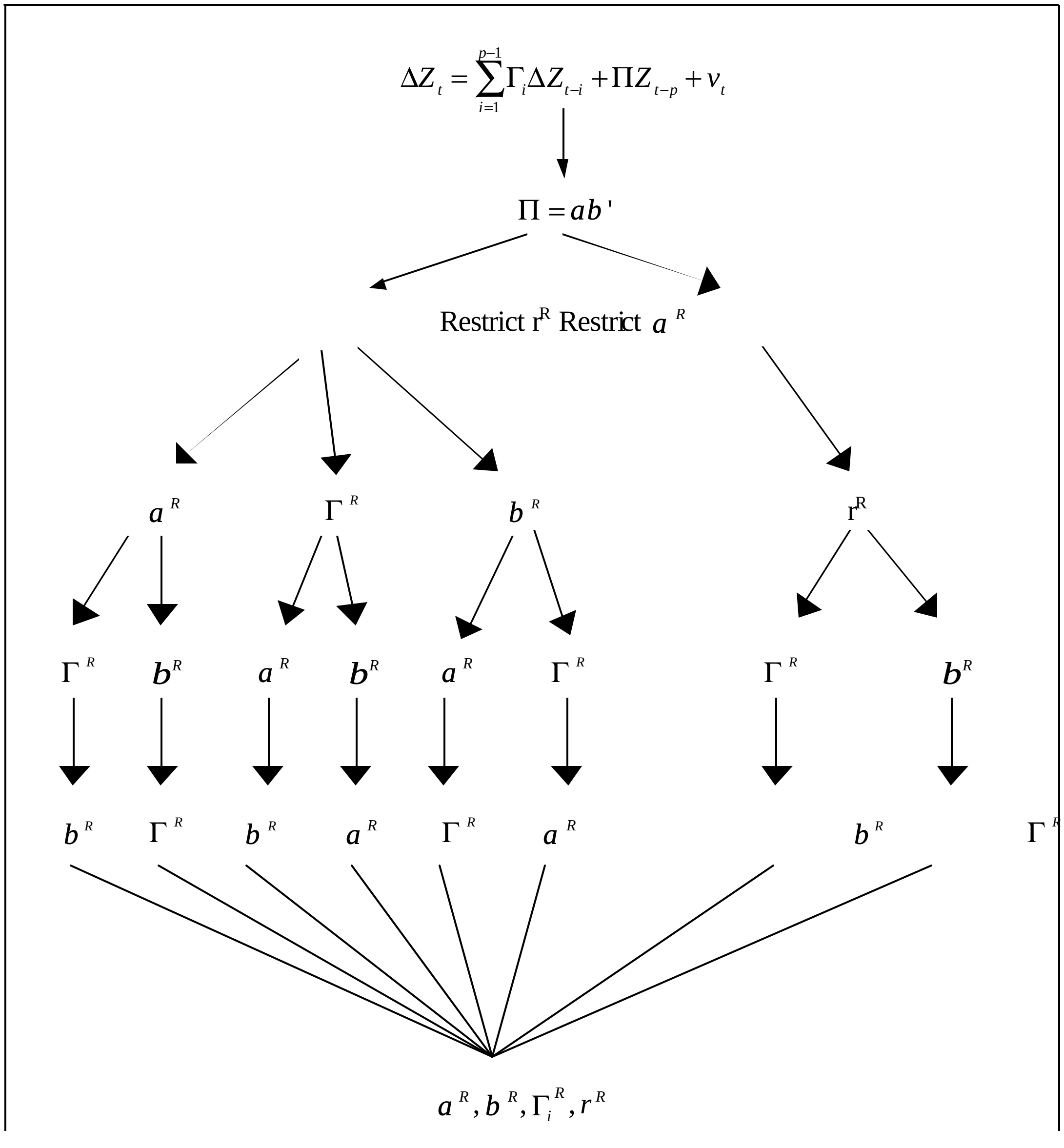

Figure 1. Alternative orders for applying the identifying restrictions

However, even within this restricted range there are still clearly many options, which the researcher has to choose from when constructing a model specification search. (Indeed it is even possible to mix the order of restriction, so that for example we might restrict some as then restrict Γ and then return to restricting more $a's$). Further it is not at all obvious that all the tests on one aspect of the model should be conducted together. It may well be sensible to begin by testing some aspects of the loading matrix, then simplify the dynamics then return to testing the loading matrix. This is not new of course as it has always been recognised in the dynamic modelling tradition that there is not a unique way of moving from the general to the most parsimonious model. This is simply the equivalent issue for the system as a whole.

In a system context the greatest gains are likely to come from the imposition of weak exogeneity. This is simply because the identification of each weakly exogenous variable allows us to reduce the number of equations in the system being estimated by one. So to illustrate in a realistically dimensioned problem, if we have a marginalisation of 8 variables and we believe that we need four lags in the unrestricted VAR, then we need to estimate a total of 256 parameters. But, if we are able to treat 5 of the 8 variables as weakly exogenous then the number of parameters to be estimated in the VAR will be 96.

The test for weak exogeneity is given by testing the appropriate row of the a matrix, see Ericsson, Hendry and Mizon(1998). This can be seen as follows. The dynamic system is given in full to show the weakly exogenous variables by separating out the equations for Y and X. Thus, the dynamic model in (3) becomes,

$$\Delta Y_t = \Gamma_{11}(L)\Delta Y_{t-1} + \Gamma_{12}(L)\Delta X_t + (\Pi_{11} \quad \Pi_{12})(Y_{t-p} \quad X_{t-p})' + v_{1t} \qquad \textbf{(4)}$$

And the marginal model for X is

$$\Delta X_t = \Gamma_{21}(L)\Delta Y_{t-1} + \Gamma_{22}(L)\Delta X_{t-1} + \Pi_{22} X_{t-p} + v_{2t} \qquad \textbf{(5)}$$

Equation (4) is now the conditional, (5) the marginal model. If X_t are indeed weakly exogenous, then the parameters of interest in the conditional model can be estimated without modelling (5). Tests of weak exogeneity are then necessary to decide whether it is possible to model only (4), or (4) and (5) together (Banerjee et al. (1993)). The basic test is based on the restriction in the above model that $\Pi_{21} = 0$ which is a restriction on the a matrix. Hall and Wickens demonstrated a notion of long-run causality when we add the assumption that Π_{11} is of full rank. Granger non-causality is defined by the condition that $\Pi_{21} = 0$ and $\Gamma_{21} = 0$.

The argument we advance for estimating the model as we do relies on its likely small sample properties compared with those of alternative estimation procedures. It is widely accepted that the correct specification of the structural VECM relies crucially on determining the correct number of cointegrating vectors. Once this is determined, the just-identifying restrictions on the cointegrating vectors can be imposed and over-identifying restrictions may be tested. However the small sample properties of the now familiar cointegration tests can be very poor in many practical situations. For example, in a very simple wage system we might easily have 8-10 variables. Estimating an unrestricted VAR using these 10 variables will very quickly use up most of the degrees of freedom in a typical data set of 100-120 quarterly observations. We believe that in this situation the task of determining the correct number of cointegrating vectors without the use of extra identifying assumptions (especially about the exogeneity of the system) is extremely difficult.

The method proposed here in many ways parallels the traditional approach to single equation dynamic modelling. We begin by estimating the unrestricted VAR and then search for a sequence of simplifying restrictions. Each of these restrictions should be tested and be congruent with the data.

But given each simplification we make on the model the power and size of subsequent tests will be changed and, we argue, improved. The main gain will probably come from first making (and where possible testing) a set of assumptions about the exogeneity of the model. This can considerably reduce the size of the modelling problem and arguably the performance of the tests for the number of cointegrating vectors can improve considerably. Having then determined the number of cointegrating vectors in the system we can then follow Pesaran and Shin in imposing a set of just-identifying restrictions on the cointegrating vectors. These cointegrating vectors can then be entered in the VECM i.e. - the conditional model (4) above - in an unrestricted way, so that each equation for the endogenous variables will have all the cointegrating vectors included in it. Then the complete dynamic model may be estimated and the dynamics can be simplified at the same time as the over- identifying restrictions on the cointegrating vectors are tested. At this stage, the causality structure of the model can be established by eliminating the "inappropriate" cointegrating vectors from each equation using likelihood ratio tests.

These alternatives illustrate the close interconnections between the classification of variables into endogenous and weakly exogenous and practical identification procedures.

In the next section we turn to a Monte Carlo evaluation of the effectiveness of different tests at varying points in the nesting down procedure.

3. A Monte Carlo study

To illustrate these points about the small sample properties of alternative testing procedures, we have conducted a series of Monte Carlo experiments. The idea of these experiments differs markedly from the bulk of Monte Carlo work in that we design a data generation process, which is deliberately quite complex. In contrast most Monte Carlo work examines quite small unrealistic data generation processes. Our exercise proposes that from a total of 8 variables, the system has three cointegrating vectors and in structural form each vector enters only one equation. There are therefore 5 weakly exogenous variables and 3 endogenous ones. The dynamic structure is also quite rich with a maximum lag up to 6. The system is deliberately chosen to be similar to the type of system commonly estimated for wages, prices and import prices in the literature (e.g. Greenslade, Henry and Jackman (1998)), with a typical sample size of 112 observations.

We begin by investigating the first choice, which needs to be made from Figure 1. This is, should we test the cointegrating rank using the unrestricted system first or should we test and impose assumptions about the weak exogeneity on the system and then test the cointegrating rank r?

Table 1 summarises the results from a large number of simulation experiments where we apply the standard test for the cointegrating rank of the system in a range of contexts using a number of alternative sets of critical values (the results for each simulation are from 10 000 replications). The variants we consider are divided up in the following way.

a) Simulations 1-4 perform the tests on the basis of all 8 variables being treated as endogenous while simulations 5-8 impose the correct exogeneity split on the model and treat 5 of the variables as exogenous.

b) Simulations 1, 2, 5 and 6 generate data for both the endogenous and exogenous variables at each replication, thus treating the exogenous variables as stochastic while simulations 3, 4, 7 and 8 generate data for the endogenous variables only at each replication thereby treating the exogenous variables as fixed.

c) Simulations 1, 3, 5 and 7 use the standard asymptotic maximal eigenvalue and trace tests while simulations 2, 4, 6 and 8 use a small sample correction to these two tests.

d) Finally simulations 5-8 are performed using both the standard critical values from Osterwald-Lenum and the critical values from Pesaran Shin and Smith based on the presence of exogenous variables

e) For each of these choices we report the results of the test based on a VAR of order 2, 4, 6 and 8, where the true maximum lag in the system is 6.

The table reports the most likely number of cointegrating vectors which we would detect using each procedure, that is the mode of the distribution of the number of vectors. We have looked at the shape of the distribution of the number of cointegrating vectors and for the case of all 8 variables treated as endogenous the distribution was remarkable flat, indicating that we have a very similar chance of finding any number of cointegrating vectors. Under the assumption of exogeneity in 5 of the variables the distribution is very heavily weighted towards the correct answer.

In this table we can see that if we treat all variables as endogenous in the test then the asymptotic tests will generally overestimate the true number of cointegrating vectors for an adequate VAR length, typically finding anything from 4 to 7 vectors. While the small sample correction, for a large system such as this, is clearly making much too large a correction and almost always finds no cointegration at all.

If we correctly impose the exogeneity status of the variables (in the lower half of the table) then the situation changes dramatically. Using the Osterwald-Lenum critical values we always detect the true number of cointegrating vectors while using Pesaran, Shin and Smith's critical values we generally do quite well for a reasonable VAR length of 6-8. We suspect that the Osterwald-Lenum results are not actually so positive and that they are detecting the maximum number of vectors possible and the size of the test would not be good.[1]

The distinction between the exogenous variables being fixed or stochastic turns out to have very little consequence and the results are very similar whichever way the experiment is conducted.

So in the absence of restrictions on the a matrix there is very little prospect of successfully detecting the true number of cointegrating vectors, which underlie a system of this type with this size data set. This emphasises the need to know about the exogenous variables before we test r. However there is

1 In this case we find the maximum number of cointegrating vectors in almost all replications, hence we do not find the expected 5% of incorrect rejections.

also the question of the effectiveness of the testing procedure for weak exogeneity and we turn to this next.

Table 1: Testing the Cointegrating Rank

All 8 variables endogenous

	Maximal Eigenvalue				Trace			
Sim. \ VAR	2	4	6	8	2	4	6	8
1	1	2	4	7	2	3	6	7
2	0	0	0	0	0	0	0	0
3	2	2	4	7	3	5	6	7
4	1	0	0	0	2	0	0	0

3 variables endogenous: Osterwald-Lenum Critical Values

	Maximal Eigenvalue				Trace			
Sim. \ VAR	2	4	6	8	2	4	6	8
5	3	3	3	3	3	3	3	3
6	3	3	3	3	3	3	3	3
7	3	3	3	3	3	3	3	3
8	3	3	3	3	3	3	3	3

3 variables endogenous: Pesaran, Shin and Smith Critical Values

	Maximal Eigenvalue				Trace			
Sim. \ VAR	2	4	6	8	2	4	6	8
5	1	1	2	3	1	2	2	3
6	1	1	1	2	1	1	2	2
7	1	1	2	3	1	1	2	3
8	1	1	1	2	1	1	1	2

Simulation 1: stochastic exogenous case, 8 endogenous variables. Simulation 2: as 1, but with small sample correction. Simulation 3: fixed exogenous variable case, 8 endogenous variables. Simulation 4: as 3, but with small sample correction. Simulation 5: stochastic exogenous case, 3 endogenous variables. Simulation 6: as 5, but with small sample correction. Simulation 7: fixed exogenous variable case, 8 endogenous variables. Simulation 8: as 7, but with small sample correction.

In Table 2 we present the results of testing the weak exogeneity in the system. That is, the likelihood ratio test that a complete row of the a matrix is zero, the test is done under the assumption that the cointegrating rank has been determined and for a particular lag length in the VAR. In Table 2 we report the percentage of times we found the endogenous variables to be correctly identified as endogenous and the percentage of times we incorrectly found the exogenous variables to be endogenous (again based upon 10 000 replications per case). We do this under a range of assumptions regarding the lag length and the assumed cointegrating rank of the system. The true rank is given by r=3 and the VAR length is 6. We also report the results for the case of fixed and stochastic exogenous variables. The results of this exercise show that the test of weak exogeneity again has very poor small sample performance. If we correctly specify the cointegrating rank and the order of the VAR then the test has reasonable power; it rejects the assumption that $a_i = 0$ in 66% of cases when in fact this assumption is false. But the size of the test is far too large in that we also reject this assumption approximately 60% of the time when it is true. Interestingly the performance of the test does not change very much if we assume either that the cointegrating rank is larger than the true value or the VAR length is longer. However the performance of the test is severely distorted by underestimating the cointegrating rank. This is a useful result as it suggests that the test for weak exogeneity may be conducted under the assumption of full rank without affecting its performance very markedly.

Table 2: Testing the weak exogeneity of the system

Stochastic exogenous variable case

r. \ VAR	2	4	6	8
1	38/23	34/27	39/33	45/36
2	52/33	50/40	52/50	64/51
3	62/42	60/49	66/61	76/61
4	67/47	67/54	70/68	78/65
5	68//50	67/56	72/70	79/66
6	69/51	68/58	74/71	78//67
7	73/57	72/63	78/75	77//70

Fixed exogenous variable case

r. \ VAR	2	4	6	8
1	36/26	38/22	39/33	45/40
2	50/39	54/31	56/48	65/58
3	60/45	61/41	66/58	76/69
4	66/55	65/47	71/63	80/75
5	65/57	67/55	74/65	82/77
6	65/51	68/60	76/68	82/78
7	66/58	74/67	79/73	82/80

r is the assumed cointegrating rank, VAR is the lag length of the VAR used in the test, in each cell a/b, a is the percentage of times the endogenous variables are found to be endogenous and b is the percentage of times the exogenous variables are found to be endogenous.

These two sets of results lead us to the conclusion that it is often better in practise to use theory to define some of the variables as weakly exogenous, and to test this assumption. Then test the cointegrating rank of the system, rather than determining the cointegrating rank and then testing for weak exogeneity. There are two arguments for this. First, Table 1 demonstrates that the tests of r have reasonable power once the weak exogeneity assumption has been made. While Table 2 shows that restricting the cointegrating rank has little impact on the weak exogeneity test, at least as long as we do not restrict it to be less than the true rank. Second, we would argue that the marginalisation of the model often gives us a very good idea as to what the weak exogeneity of the system should be. The reason for this is that we must remember that weak exogeneity is not invariant to the marginalisation of the model, it is not an absolute property of a variable rather it is a property of a particular model.

This second argument amounts to recognising that when we undertake practical modelling we are not equally interested in modelling the behaviour of all the variables in our system. We are typically interested in building a model of either a single variable or a small sub set of the variables. Many of the variables are there because we think they are relevant to the determination of the variable we want to model but we are not interested in explaining them. As an illustration of this in the next section we give a model of wage and price determination, the model includes unemployment because we believe that unemployment affects wages but it does not include the many variables, which we think might explain unemployment. It would therefore be surprising if a cointegrating vector existed which explained unemployment. Even though in the real world we have no doubt that wages do affect unemployment in our model unemployment can probably be treated as weakly exogenous.

So we have argued that making decisions about the weak exogeneity of the system and applying tests (although we should be aware of the problems of the power and size of these tests) of this assumption should be the first step in dealing with the identification of a large system. We can then hope to test the cointegrating rank of the system reasonably effectively. We must then decide if we should then test the dynamics of the model to derive a fully restricted dynamic system or if we should identify the long-run structure by imposing restrictions on b first. Table 3 reports on a series of Monte Carlo experiments to investigate this.

Table 3: Testing the over identifying restrictions

s. \ % level of rejection	10	5	1
1	3.2	3.8	5.7
2	54.6	58.5	64.5
3	74.9	77.0	83.2

s is the simulation number whereby in 1, there are 8 endogenous variables and unrestricted dynamics, in 2, there are 3 endogenous variables and unrestricted dynamics and in 3 there are 3 endogenous variables and restricted dynamics. In each cell the percentage of times the restrictions are accepted at the appropriate level of testing is reported.

In this table we report on the percentage of times we accept the true set of over-identifying restrictions, that is the size of the test. The first row shows the result for the case where all 8 variables are treated as endogenous and we have a full set of dynamics. This shows that in only about 4% of the cases do we accept the true over-identifying restrictions (in 96% of the cases we reject them). When we impose the set of exogeneity assumptions on the model and repeat this experiment we then find that we can accept the overidentifying restrictions in nearly 60% of the cases. Finally if we also simplify the dynamics first and then perform the test the acceptance rate rises to nearly 80%, which is fairly reasonable. This set of experiments illustrates the point that in this type of model and sample size the unrestricted model approach advocated by Pesaran and Shin will almost certainly reject the true set of over identifying restrictions.

4. Estimation Procedures

In this section we propose to illustrate the arguments made above by estimating a wage price system for the UK economy. Our primary interest is to estimate equations for wages and prices and given the important interaction with imported costs also for import prices. However inevitably the model will have to include a range of other variables which are important to these variables and so the unrestricted system will be quite large.

The discussion in section 3 leads to the following strategy as a way to estimate this wage-price model.

(i) Use economic theory to decide what the split between endogenous and weakly exogenous variables should be and to verify this by testing the a matrix.

(ii) Then determine the cointegrating rank of the conditional system.

(iii) Find a parsimonious representation of the dynamic terms in the system.

(iv) Then test the over identifying restrictions on the long-run coefficients b and test any further restrictions on the loading matrix a to arrive at the final, fully restricted model of the form given in (3).

4.1 The Model

We take a reasonably standard model of the wage-price system, building on previous studies, including Layard, Nickell and Jackman (LNJ) (1991), Gordon (1997), Manning(1993), Blanchard and Katz(1997) and Henry, Nixon and Williams (1997). [For a further discussion see, Greenslade, Henry and Jackman (1998)] There are three equations in the model, one for aggregate wages (W), the second is for the consumer price (P_c) and the third is for import prices (P_m). In schematic form, the long-run structural (static) form of the equations are (variables in logs, except for the unemployment variables).

$$W = a_0 + a_1 P_c + a_2 PROD + a_3 u + a_4 u^L + a_5 Z_w \qquad \textbf{(6)}$$

Equation (6) is a familiar wage equation, depending upon consumer price (P_c), productivity, (PROD), unemployment (u), and the ratio of long and medium duration unemployed to total unemployment (u^L). Consumer prices depend upon unit labour cost (ULC defined as W - PROD, variables in logs) and import costs, and import prices depend upon the nominal effective exchange rate (E) and world prices (PW). Each equation allows for additional factors, such as tax and import price wedges in wages (Z_w), demand effects in both equations, and possible currency of invoicing effects in the import price equation. In an earlier exercise, Greenslade, Henry and Jackman (1998), argue that (6)-(8) is an acceptable, parsimonious, form of the wage-price system.

$$P_c = b_0 + b_1 ULC + b_2 P_m + b_3 Z_{pc} \quad \textbf{(7)}$$

$$P_m = g_0 + g_1 E + g_2 PW + g_3 Z_{pm} \quad \textbf{(8)}$$

The theoretical basis of the equations is the standard wage bargaining model, where the wage is determined by the union and firm in a simple Nash framework and the firm then sets employment or, as here, prices. The model is explicitly extended to include overseas price effects on domestic wages and prices by equation (8). Note that individual equations (6) - (8) would be just identified if they each excluded two of the eight variables in the system (W, P_c, P_m, PROD, u u^L, E, PW) (given the normalisation in each equation). As written, each equation is overidentified; excluding three (equation 6), four (equation 7) and five (equation 8) variables respectively.

In more detail, the following restrictions are applied in the model shown by (6) - (8) for convenience, which show the variable excluded not the parameter.

P_m = E = PW = 0 in (6), i.e. 3 restrictions
u = u_L = E = PW = 0 in (7), i.e. 4 restrictions
P_c = W = A = u = u_L = 0 in (8) i.e. 5 restrictions

The model contained in (6) - (8) is static, so represents the long-run equilibrium of the system. Dynamic adjustment to these long-run equilibria is determined by lags in each equation, in wages these are due to non-synchronised wage contracts, and in prices it is assumed that changing prices can be costly. Details of these dynamics extensions are familiar, and are not repeated here.

So, moving directly to the dynamic model, this can be expressed in the same form as equation (4) earlier (i.e. as a dynamic Vector Autoregressive ECM (VECM)) for the conditional model, so

$$\Delta Y_t = A(L)\Delta Y_{t-1} + B(L)\Delta X_t + \Pi \; Z_{t-1} + V_t \quad \textbf{(9)}$$

Where $Y_t = (W, P_c, P_m), X_t = (PROD, u, u^L, E, PW), Z = (Y, X), A(L)$ and B (L) are matrices of polynomials in the lag operator (L) and Π is a matrix of dimension 3x3 (equal to ab').

The structural restrictions, which concern us, are then the exclusion restrictions given above, and the requirement that the model is neutral in the long-run, i.e. both long-run levels and derivative homogeneity should hold. These latter conditions are discussed in Greenslade et al. (1998).

4.2 Estimation Results

As noted in Section 3, we proceed to estimate the model by first making assumptions about the weak exogeneity of the system and then testing for the cointegrating rank. The basic time series properties of the data are reported in Appendix 1, which illustrates that all the variables under consideration are almost certainly non-stationary with the possible exception of productivity. Given space constraints we will not discuss this aspect of the analysis further as it has little consequences for our analysis once we are working in a framework which correctly allows for non-stationarity.

4.2.1 Weak Exogeneity

In order to give reasonable power to the tests of the cointegrating rank we begin by trying to impose some congruent weak exogeneity assumptions on the model. Our theoretical model above suggests that 5 of the 8 variables should be weakly exogenous. For example, the exchange rate and productivity are almost certainly weakly exogenous. There are none of the interest rate variables or other exchange rate related variables such as oil prices, which we would expect to need to successfully model the long-run behaviour of exchange rates and similar arguments apply to productivity. There is a problem here that before we can test for exogeneity we must make a decision about the cointegrating rank of the system. We will attempt to deal with this by repeating the exercise first on the theoretically based assumption that there are three cointegrating vectors and then on the general unrestricted assumption that there are 7 vectors. Based on the assumption that there are 3 vectors the hypothesis that the exchange rate, productivity and unemployment are each weakly exogenous (that is, the relevant row of the alpha matrix is equal to zero) may not be rejected, as shown in Table 4. On the basis of this evidence, we then assume that these variables are weakly exogenous and set up a system with 5 endogenous variables and 3 exogenous variables in order to test the exogeneity of long-term unemployment and world prices. We obtain a test statistic of $c^2(3) = 10.45$ (probability 0.0151) for the hypothesis that long-term unemployment is weakly exogenous, which although not accepted at the 95% level of testing, is accepted at the 99% level. If we then assume that long-term unemployment is weakly exogenous and re-estimate the system so that there are now a total of four exogenous and four endogenous variables, we can test the exogeneity of world prices. We conclude that world prices may also be treated as weakly exogenous, with a test statistic of $c^2(3) = 2.03$ (probability 0.5660). We then test again that our system contains three endogenous variables, wages, consumer prices and import prices, together with five weakly exogenous variables, the exchange rate, productivity, unemployment, long-term unemployment and world prices. The hypothesis that wages are weakly exogenous is easily rejected at the 5% level, ($c^2(2) = 9.0$), as is consumer prices ($c^2(2) = 8.4$) and import prices ($c^2(2) = 61.8$). We then repeat this exercise on the assumption that there are seven cointegrating vectors. The formal test that each variable is weakly exogenous may be rejected in all cases except productivity, as shown in Table 4a. If we then assume that productivity is weakly exogenous and re-estimate the system with 7 endogenous variables, one exogenous variable and six cointegrating vectors, there is no evidence to suggest that any of the endogenous variables are weakly exogenous.

Table 4 Testing weak exogeneity: Three cointegrating vectors

Testing exogeneity of each variable		
	$c^2(3)$	Probability

W	5.88	0.1174
P_c	7.39	0.0604
P_m	26.72	0.0000
PROD	0.79	0.8524
u	5.46	0.1413
u^L	13.86	0.0031
E	5.58	0.1338
PW	14.24	0.0026

PROD, u and E weakly exogenous		
	$c^2(3)$	Probability
u^L	10.45	0.0151
PW	19.02	0.0003

PROD, u, E and u^L weakly exogenous		
	$c^2(3)$	Probability
PW	2.0308	0.5660

Table 4a Testing weak exogeneity: seven cointegrating vectors

Testing exogeneity of one variable		
	c (7)	Probability
W	31.66	0.0000
P_c	28.26	0.0000
P_m	46.92	0.0000
PROD	13.91	0.0528
u	16.70	0.0194
u^L	26.21	0.0005
E	20.19	0.0052
PW	25.11	0.0007

PROD, weakly exogenous		
	c (6)	Probability
W	18.19	0.0058
P_c	29.28	0.0001
P_m	49.54	0.0000
u	18.28	0.0056
u^L	26.74	0.0002
E	21.91	0.0013
PW	23.51	0.0006

Clearly these results are conflicting and illustrate the point made in the Monte Carlo simulations that the test results are very sensitive in such large systems. Given the Monte Carlo evidence of the tendency to overestimate the number of cointegrating vectors in this sample size we intend to impose our prior view that there are only 3 endogenous variables in this system and that all the remaining variables are weakly exogenous.

4.2.2 Testing the cointegrating rank

We now apply the standard Johansen tests for the number of cointegrating vectors in the system (W, P_c, P_m, u, u^L, PROD, E, PW) where we assume that the exchange rate, world prices, unemployment, long-run employment and productivity are weakly exogenous. The cointegrating rank appears to be at least three. The table below (Table 5) gives the Johansen eigenvalue and trace tests, which confirms this, with the 95% critical values is given in brackets. In Table 5a we show the tests for the cointegrating rank of the system when all eight variables are treated as endogenous. The maximal eigenvalue test statistic suggests that there are four cointegrating vectors whereas according to the trace statistic, there are six cointegrating vectors. These results confirm our earlier Monte Carlo tests, in the sense that we now seem to find an implausibly high number of vectors.

Table 5: Johansen Cointegration Tests: Three Endogenous Variables

Endogenous variables: W, P_c & P_m. Exogenous variables: u, u^L, PROD, E & PW.				
			Eigenvalue	Trace
1	r = 0	r $\geq$ 1	72.96 (40.12)	148.90 (76.82)
2	r = 1	r $\geq$ 2	42.76 (33.26)	75.94 (49.52)
3	r = 2	r $\geq$ 3	33.18 (25.70)	33.18 (25.70)

Table 5a: Johansen Cointegration Tests: Eight Endogenous Variables

Endogenous variables: W, P_c, P_m, u, u^L, PROD, E & PW.				
			Eigenvalue	Trace
1	r = 0	r $\geq$ 1	81.01 (55.14)	318.90 (182.92)
2	r = 1	r $\geq$ 2	73.91 (49.32)	237.89 (147.27)
3	r = 2	r $\geq$ 3	49.74 (43.61)	163.97 (115.85)
4	r = 3	r $\geq$ 4	38.14 (37.86)	114.23 (87.17)
5	r = 4	r $\geq$ 5	30.75 (31.79)	76.09 (63.00)
6	r = 5	r $\geq$ 6	22.04 (25.42)	45.35 (42.34)
7	r = 6	r $\geq$ 7	15.52 (19.22)	23.31 (25.77)

4.2.3 The dynamic model

The general model may then be described in the following way

$$\begin{aligned}
\Delta W_t &= \sum_1 a_{li}\Delta W_{t-i} + \sum_1 a_{2j}\Delta Pc_{t-j} + \sum_1 a_{3i}\Delta Pm_{t-i} + \sum_1 a_{4i}\Delta X_{t-i} + a_{11}e_{1t-1} + \\
&\quad a_{12}e_{2t-1} + a_{13}e_{3t-1} \\
\Delta P_{ct} &= \sum_1 b_{lj}\Delta W_{t-j} + \sum_1 b_{2i}\Delta Pc_{t-i} + \sum_1 b_{3i}\Delta Pm_{t-i} + \sum_1 b_{4i}\Delta X_{t-i} + a_{21}e_{1t-1} + \\
&\quad a_{22}e_{2t-1} + a_{23}e_{3t-1} \\
\Delta Pm_t &= \sum_1 c_{1i}\Delta Pm_{t-i} + \sum_1 c_{2i}\Delta Pc_{t-i} + \sum_1 c_{3i}\Delta W_{t-i} + \sum_1 c_{ei}\Delta X_{t-i} + a_{31}e_{1t-1} \\
&\quad + a_{32}e_{2t-1} + a_{33}e_{3t-1}
\end{aligned} \tag{12}$$

In this reduced form of the model, all of the cointegrating vectors enter each equation, where e_i are the just identified form of the relevant cointegrating vector in each case [these use the Johansen just identified estimates: the position Pesaran-Shin start from].

Next, we proceed to test down from this model. Following the discussion in Sections 2 and 3, there is no unique way of reducing the model, when restricting the dynamics and the long-run relations (Γ and b in Figure 1). In the next section we show the results when undertaking data based simplifications of the dynamics and tests of derivative homogeneity, followed by tests of the over identifying restrictions on the long-run relationships (estimated using FIML).

4.2.4 Test of Dynamic Restrictions

We now proceed to a model with a more parsimonious dynamic structure before testing the overidentifying long-run restrictions as suggested by the Monte Carlo results in table 3. We seek a set of data based simplifications on the dynamics of the model, which does not lead to any undesirable properties in the models residuals (serial correlation, non-normality, heteroskedasticity etc.). This approach leads to excluding variables with individual t ratios less than 1.3. We are then able to test for the presence of long-run dynamic homogeneity in this parsimonious form of the model. This requires that the sum of coefficients on the lagged nominal dynamic terms sum to unity in the first two equations. While in the P_m equation, the requirement is that the long-run effect of a change in exchange rate inflation or a change in world price inflation be unity (See Greenslade et al. (1998) for details). Again, these restrictions can be tested one equation at a time, using standard Wald tests. In the wage equation, a Wald test of the hypothesis that the sum of the parameters on lagged wage and price inflation equals unity gave 0.30 (for $\chi^2(1)$), so the hypothesis of dynamic homogeneity is not rejected. In the consumer price equation, a Wald test statistic of 3.72 (for $\chi^2(1)$) was obtained for the hypothesis that the sum of the dynamic terms is equal to unity. For import prices, the Wald statistics are 3.58 and 0.81 (both are $\chi^2(1)$)[2]. Again these statistics do not violate the hypothesis of dynamic homogeneity. Table 6 presents the parsimonious dynamic model with dynamic homogeneity.

Table 6: The VECM With Restricted Dynamics

[2] The condition for dynamic homogeneity for import prices is $-(c_{41})=(c_{34}+c_{35}+c_{51})=1$, due to the assumption that we make about the exogeneity of the exchange rate.

	a_{11}	a_{21}	a_{22}	a_{41}	a_{42}	a_{53}	α_{11}	α_{12}	α_{13}	R^2
Δw	0.30 (3.27)	0.32 (2.86)	0.38 -	0.10 (4.44)	-0.09 (-3.54)	-0.23 (-2.57)	-0.006 (-2.10)	0.013 (3.11)	0.007 (1.37)	0.69
	b_{11}	b_{12}	b_{14}	b_{21}	b_{34}		α_{21}	α_{22}	α_{23}	
ΔP_c	0.18 (2.10)	0.24 (2.48)	0.15 (2.73)	0.37 (3.45)	0.06 -		0.002 (1.10)	-0.002 (-0.60)	0.013 (3.37)	0.73
	c_{34}	c_{35}	c_{41}	c_{51}			α_{31}	α_{32}	α_{33}	
ΔP_m	0.17 (2.56)	0.16 (1.71)	-0.32 (-2.32)	0.99 -			-0.011 (-2.27)	0.013 (2.38)	-0.032 (-2.51)	0.52
System Log Likelihood: 991:42										

a_{22}, b_{34} and c_{51} are imposed due to the restriction of dynamic homogeneity.

In these equations, a_{41} is the parameter on Δu^L_{t-1}, a_{42} is the parameter on Δu^L_{t-2}, a_{53} is the parameter on $\Delta PROD_{t-3}$, c_{41} is the parameter on ΔE_{t-1} and c_{51} is the parameter on ΔPW_{t-1}.

4.2.5 Tests of Long-run Restrictions

This section applies over identifying restrictions on each of the long-run equations, so that they may each be given a structural interpretation consistent with economic theory. In full, we apply the twelve exclusion restrictions noted earlier. (Excluding import prices, the exchange rate and world prices from the wage equation, unemployment, long-term unemployment, the exchange rate and world prices from the consumer price equation, and consumer prices, wages, productivity and the two unemployment terms from the import prices equation.) Next, we apply the restrictions required to ensure that the levels relationships are homogeneous. This involves one restriction in the wage equation, two in the consumer price equation and two in the import price equation[3]. We also apply the restriction that productivity is neutral in the long-run. Including the three normalisation restrictions this gives 21 total restrictions, which implies 11 over-identifying restrictions, which on the basis of a Likelihood Ratio test are not rejected (LR test statistic 6.70, χ^2 (11)).

Table 7: Full Model with Restricted Dynamics and Long-run

	a_{11}	a_{21}	a_{22}	a_{41}	a_{42}	a_{53}	α_{11}	α_{12}	α_{13}	R^2
Δw	0.35 (3.36)	0.31 (2.72)	0.34 -	0.10 (4.16)	-0.10 (3.74)	-0.25 (2.96)	-0.06 (-1.23)	-0.03 (-0.31)	0.04 (1.74)	0.70
	b_{11}	b_{12}	b_{14}	b_{21}	b_{34}		α_{21}	α_{22}	α_{23}	
ΔP_c	0.19 (2.16)	0.22 (2.12)	0.13 (2.08)	0.40 (3.08)	0.06 -		-0.04 (-0.85)	-0.11 (-1.46)	-0.003 (-0.15)	0.71
	c_{34}	c_{35}	c_{41}	c_{51}			α_{31}	α_{32}	α_{33}	
ΔP_m	0.10 (1.58)	0.10 (1.38)	-0.21 (-1.63)	1.01 -			0.18 (0.97)	0.38 (1.57)	-0.14 (-2.70)	0.58
System Log Likelihood: 983:41										

a_{22}, b_{34} and c_{51} are imposed due to the restriction of dynamic homogeneity.

[3]The restrictions are as follows: wage equation, the coefficient on P_c = unity; consumer price equation, the coefficient of PROD = - coefficient on W (required for unit labour costs) and the sum of unit labour cost plus import prices is unity, and in the import price equation, the coefficients on both the exchange rate and world prices = unity.

It is now possible to conduct further tests: on the α matrix, for example, is there a simple direction of causation from each identified structural long-run relationship. In the above estimation, it is the case that several of the cointegrating vectors are clearly insignificant in some equations (with t-ratios of 0.32 or below) and it would seem reasonable to remove these. If we delete α_{12} and α_{23}, α_{21} remains clearly insignificant, and when deleted results in α_{32} having a t-ratio of under 0.6. If we then delete α_{32} , α_{13} is not significant at conventional levels of testing. The results of deleting these cointegrating vectors are given in Table 8.

Table 8: Restricted Levels Model

	a_{11}	a_{21}	a_{22}	a_{41}	A_{42}	a_{53}	a_{11}	a_{12}	a_{13}	R^2
Δw	0.42 (5.46)	0.29 (2.81)	0.29 -	0.10 (4.30)	-0.10 (3.90)	-0.30 (3.35)	-0.09 (-2.65)	-	-	0.68
	b_{11}	b_{12}	b_{14}	b_{21}	B_{34}		a_{21}	a_{22}	a_{23}	
ΔP_c	0.22 (2.72)	0.22 (2.27)	0.12 (2.09)	0.38 (3.67)	0.06 -		-	-0.01 (1.43)	-	0.69
	c_{34}	c_{35}	c_{41}	c_{51}			a_{31}	a_{32}	a_{33}	
ΔP_m	0.07 (0.98)	0.09 (1.13)	-0.20 (-1.46)	1.03 -			-	-	-0.15 (-3.15)	0.55

a_{22}, b_{34} and c_{51} are imposed due to the restriction of dynamic homogeneity.

This then produces the result that the identified wage cointegrating vector only enters the wage equation, the price one only enters the price equation and the import price one only enters the import price equation. This then yields our final model with parsimonious dynamics and full identified long-run relationships, which conform to our theoretical priors. The fact that the a matrix is diagonal is not of course a requirement for the final model, but it does indicate that, at this data frequency, the simultaneous inter-relationships in the structural model are weak (or zero) and that the primary long-run determinates in each equation is its own identified structural relationship. This may be appealing from a theoretical point of view. This model is both theory consistent and congruent with the data.

5. Conclusions

The joint questions of identification and exogeneity are hotly contested matters in time-series analysis at present. Progress on these methodological matters is essential, as they are prerequisites to any evaluation of structural behaviour, including establishing whether any structural change has taken place, a possibility, which has been frequently raised in the context of UK wage and price behaviour. In this paper, we propose a method of identifying wage and price structures in the presence of cointegration, which builds upon elements of previous work, including that of Bardsen and Fisher (1995) and Pesaran and Shin (1994). The advantages of our procedure are that it recognises the importance - perhaps the overriding importance - of the limits placed upon estimations and inference in cointegrating systems by data limitations. Our Monte Carlo exercises have shown that in small samples of the sort typically used by the applied researcher (about 100 quarterly observations say), substantially improved test performance (especially with respect to the size of the test) is obtained by restricting the model at the outset according to theory. The most important step is to first decide on the endogeneity and weak exogeneity status of the variables. We show that the crucial choice of the rank of the cointegration matrix (Π) is heavily dependent on this. Once these decisions have been made, the next steps require both restricting the model's dynamics and applying restrictions to its long-run (cointegrating) relationships. The latter, in particular, has been the focus of most of the previous literature in this area. The present contribution is to make the application of these overidentifying tests more [reliable] given the limited data sets commonly used. Following the outline of the steps we recommend in identifying a VECM, coupled with supporting Monte Carlo experimental evidence, we conclude with an empirical example. Our illustration confirms that a structural model of the labour market is identifiable using our procedures, with economically meaningful long-term behaviour.

Appendix

Time Series analysis

Time series properties of the individual series used later showed that most appeared difference stationary (I(1)). Table A1 shows the results.

Table A1
Time series properties of the data. (Sample 1966Q1 - 1998Q4)

		DF	**ADF (4)**
Levels	W	2.25	-0.74
	P_c	1.81	-1.18
	P_m	-0.96	-0.52
	u	-0.12	-1.88
	u^L	-1.92	-1.88
	PROD	-2.98	-2.90
	E	-1.86	-2.01
	PW	0.78	-1.02
Differences	)W	-6.15	-3.23
	)P_c	-4.71	-2.48
	)P_m	-12.16	-4.47
	)PROD	-12.44	-4.85
	)E	-9.14	-5.40
	)PW	-4.81	-4.14

All variables are logs except unemployment rates
95% critical value = 3.45

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