

**INTEREST RATE LINKAGES:  
IDENTIFYING STRUCTURAL RELATIONS**

**Marco R. Barassi**

**Department of Economics University of Birmingham,  
Imperial College Management School,  
and  
Centre for Monetary and Financial Economics  
South Bank University, London**

**Guglielmo Maria Caporale**

**Centre for Monetary and Financial Economics  
South Bank University, London**

**Stephen G. Hall**

**Imperial College Management School**

## **Abstract**

*This paper examines the causal linkages which may exist between the G-7 national interest rates. Its aim is to exploit some new techniques in cointegration analysis to see to what extent conclusions can be drawn purely from the data without imposing any arbitrary identification conditions. Causality is intimately linked with our structural view of the economy, and it has not been practical in a traditional setting to go very much beyond the standard Granger causality testing procedures. This paper examines linkages between  $I(1)$  series as structural relations, using a method put forward by Davidson (1998a) that involves the introduction of the new concept of an irreducible cointegrating vector. In order to distinguish between structural and solved irreducible cointegrating relations, we extend this methodology introducing the ranking of irreducible cointegrating vectors according to a minimum variance criterion. The results suggest that over our sample period the US has been the dominant player in setting world interest rates; they also allow us to reject the hypothesis of a German leadership in Europe in favour of a US world-wide leadership.*

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*Address for correspondence:* Professor Guglielmo Maria Caporale, Centre for Monetary and Financial Economics, South Bank University, 103 Borough Road, London SE1 0AA, UK. Tel: +44 (0)20 7815 7012. Fax: +44 (0)20 7815 8226. E-mail: [g.m.caporale@sbu.ac.uk](mailto:g.m.caporale@sbu.ac.uk)

## Introduction

The uncovered interest parity condition (or the open arbitrage condition) suggests that in general international interest rate differentials should equal the (expected) change in exchange rates. Almost all empirical studies have found that the G-7 exchange rates are at most I(1) series. If we then make the reasonable assumption that any risk premium, which may exist, in the relationship is stationary, the implication of these theories is that interest rates should be cointegrated on a bilateral basis. In itself therefore cointegration between interest rates is neither surprising nor particularly informative. However, if these interest rates are cointegrated then there must exist a causal structure, which gives rise to cointegration and is of great policy interest. The purpose of this paper is to see how far we can get in determining what this causal structure is without imposing an arbitrary set of identification conditions on the data that might invalidate the inference we draw.

Much of the empirical evidence on interest rate linkages is based on causality test statistics, even though interest rates are typically I(1) and hence the tests do not follow standard distributions. So the inference is often invalid (see Caporale and Pittis, 1999). Recent work using an appropriate testing procedure put forward by Toda and Yamamoto (1995) shows that in fact, at least in the case of long rates, interest rate movements are determined mainly by domestic policy objectives (see Caporale and Williams, 2002, 1998c). International linkages in short-term rates appear to be much stronger, although the direction of causality is not always consistent with common priors about the functioning of the international financial system and of the ERM (see Caporale and Williams, 1998b, 1998c).

This paper examines interest rate linkages in the G-7 as structural relations, using a method put forward by Davidson (1998a) that involves the introduction of the new concept of an *irreducible cointegrating vector*. The interesting feature of this method is that, under certain circumstances, it allows us to learn about the structural relationship that links cointegrated series from the data only, without imposing any arbitrary identifying conditions. In this paper we borrow Davidson's main idea and extend it to analyse the structural relationships between the G-7's short-term interest rates as well as the underlying causal structure that links them. It is plausible to assume that larger financial units exert stronger influence on smaller ones, which would explain why the US has been dominant in international financial markets.

However, the adoption of a single European currency, which covers a very large single economic area, might mean that financial conditions in Europe now have more powerful effects on interest rates worldwide; if the Euro gradually becomes an international currency, this influence could be accentuated. Similarly, linkages within Europe might have been affected by institutional changes in the ERM, and further changes are likely to have been associated with the inception of EMU. Our analysis therefore might also shed some light on the likely impact of the creation of an integrated capital market in Europe. By suppressing exchange rate risk within the area and by fostering harmonisation measures, EMU will have an impact on asset prices and monetary and fiscal policy, which in turn will affect investment, real activity, capital flows and hence global interest rate linkages (see Portes and Rey, 1998).

An important issue is the ability of national authorities to conduct an independent monetary policy with respect to long-run interest rates even in the presence of increasingly integrated international financial markets (see Caporale and Williams, 1998c). If the fundamental determinants of (long-term) interest rates are national rather than international, then the interest rate is not given even for a small open economy, and interest rate policy still lies mainly in the hands of domestic policy makers. It appears that even in a system like the ERM which aims to produce policy co-ordination it has been possible for monetary authorities to disengage their long-term interest rate policy from developments elsewhere and pursue an independent policy agenda over long periods. Such an option should remain available for non-participating countries, like the UK, after the establishment of the Euro. Therefore the UK authorities will not necessarily find their freedom of action greatly constrained by what is happening in the Euro zone. Within the Euro zone the policies of the European Central Bank (ECB) will not necessarily be as stable or credible as those adopted so far by the German authorities, since smaller countries will also have an influence on monetary policy (see Begg et al, 1998). If in fact Germany has not been able to impose its interest rate policy on the other ERM countries, and if this becomes true of fiscal policy as well (notwithstanding the Growth and Stability Pact), long-term rates might rise (rather than decline) in the EU after 1999.

## **1. Analysing Interest Rate Linkages**

In broad terms one can identify two views on how interest rates may be linked. If they are treated as analogous to other asset prices, then their movements are naturally interpreted as being determined by financial flows in fluid, profit-seeking capital markets. Alternatively, they can be viewed as policy instruments, so that their time paths may be determined by a policy objective such as an exchange rate parity or an inflation target. Interest rate linkages have therefore often been analysed in the context of a specific policy framework such as the Exchange Rate Mechanism (ERM). For instance, numerous studies have attempted to test the so called “German Leadership Hypothesis” (GLH), according to which Germany acts as the dominant player within the system, and monetary authorities in other ERM countries are unable to deviate from the course set by the Bundesbank (see, e.g., Fratianni and Von Hagen, 1990).

Taking this view, co-movement in interest rates arises because of policy convergence. Early studies had concluded that there is no cointegration between German rates and other EMS rates (see Karfakis and Moschos, 1990), and that there is stronger evidence of cointegration between US rates and EMS rates (see Katsimbris and Miller, 1993). Subsequent papers reported convergence in European rates after 1986 (see Caporale et al, 1996). Similar conclusions were reached by Hall et al (1992) using time-varying techniques. In a global context, Caporale and Williams (1998b) found a marked difference between linkages in long-term rates (10-year bond yields) and in short-term rates (3-month Treasury bills) in the G-7 economies. Whilst there is little evidence that the former have been linked to one another over the last two decades, for the latter the evidence of co-movement is more compelling. Furthermore, the causality structure is not consistent with the standard characterisation of the ERM as an asymmetric system in which Germany was the dominant player - it suggests instead that there was German accommodation of French monetary policy within the ERM. This result could be interpreted in the context of the “size effects” identified in recent theoretical research, according to which larger, more stable countries can achieve policy objectives more successfully via accommodation than by compulsion (see Martin, 1997). The system was actually more flexible than normally recognised, as there were various “escape clauses” built into it (for instance, the options of exchange rate realignments, wider fluctuation bands, and capital controls). In this

study our objective is to identify the fundamental relationships linking interest rates among the G-7, and to analyse their causal structure in order to test for hypotheses such as the GLH.

Our analysis is based on Davidson (1998a), who introduces the concept of an *irreducible cointegrating* (IC) relation, one from which no variable can be omitted without loss of the cointegration property. The focus is on the identification of long-run structural relations, where the word structure is used to refer to relations that are consistent with theory-based restrictions and therefore have a clear economic interpretation. This is in contrast to other definitions of structure as requiring invariance to regime shifts (see Hendry, 1995). Whilst the issue of identification is usually addressed in the context of a vector error correction model (VECM – see, e.g., Pesaran and Shin, 1994, Johansen, 1995), Davidson (1998a) provides a structural interpretation of single cointegrating regressions *a la* Engle and Granger (1987). The advantage of the procedure he suggests is that, under certain circumstances, when the model is overidentified, it enables the researcher to obtain information about the underlying structure directly from the data.

He crucially shows that an IC vector is unique (up to the choice of normalisation), and that if and only if a structural cointegrating relation is identified by the rank condition, it is irreducible (see Davidson, 1994). This means that, for the purpose of identifying the structure, cointegrating vectors with redundant variables are not useful. Not all the IC vectors, though, are structural. Some of them are *solved vectors*, namely linear combinations of structural vectors. Therefore one should first perform cointegration tests in order to eliminate all non-cointegrated sets and cointegrated supersets, and then concentrate on the cointegrated sets, which yield IC relations. Davidson (1998a) develops such an elimination procedure based on a GAUSS algorithm (MINIMAL).<sup>1</sup> Essentially, one is analysing all possible cointegrated relations and testing exclusion restrictions by means of suitably constructed Wald tests, which can be shown to follow standard distributions (see Davidson, 1998b). Furthermore, one can rank the cointegrating vectors according to the value of the Wald statistic for the vector itself, so as to establish which IC relations are most supported by the data.

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<sup>1</sup> Note that only in the case of maximum over-identification, i.e. when there is no overlap of the cointegrated subsets, it is possible to identify the structure in its entirety.

In this paper we extend Davidson's (1998a) method introducing the ranking of the irreducible cointegrating vectors according to the criterion of lowest variance. The argument put forward here is that (asymptotically) if we have  $N$  variables and  $R$  structural IC vectors where  $R$  is at most  $N-1$ , then there may also exist up to  $K$  irreducible vectors which are simply combinations of the  $R$  structural ones where  $K$  is at most  $((R-1)^2+(R-1))/2$ . So there are a total of  $R+K$  possible IC vectors. Then the  $R$  structural ones will be grouped amongst the lower group of vectors when we order them by the lowest variance of the long-run residuals of the cointegrating relationship as discussed later. We will apply these ideas to the G-7 short-term interest rates.

To summarise our procedure, we first perform cointegration tests on the complete G-7 group of series to obtain its cointegrating rank. After this the next logical step is to identify the structural relationships. It is standard practice to orthonormalise the matrix of long-run coefficients first, and then test for identification of its columns. The problem with this approach is that it involves dealing with the presence of potentially redundant variables that interact with the other cointegrated series, which drive the cointegrating regression coefficients towards some other element of the cointegrating space. To eliminate the redundant or non-cointegrated series, we perform cointegration tests on each pair of series, so that we obtain a certain number of cointegrating vectors, which, as we shall see, are by definition irreducible. Our next task is distinguishing structural irreducible cointegrating relations from solved ones. This is achieved by calculating the descriptive statistics of each of the irreducible cointegrating relations, and ranking these vectors on the basis of the magnitude of their standard deviation. Our idea is that the cointegrating relationships that display the lower variability should be the structural ones, the ones that have a high standard deviation being just solved cointegrating relations. This point is illustrated in greater detail in the following section, which also includes a more extensive discussion of Davidson's (1998a) methodology.

## 2. The methodology

Consider a cointegrated VAR ( $p$ ), as analysed by Johansen (1988):

$$(2.1) \quad A(L)x_t = \alpha\beta'x_t + A^*(L)\Delta x_t = \varepsilon_t \quad (p \times 1),$$

where  $x_t \sim I(1)$ ,  $L$  is the lag operator,  $A(L) = \alpha\beta' + A^*(L)(1-L)$  such that  $A(1) = \alpha\beta'$ , and  $\alpha$  and  $\beta$  are  $p \times k$  matrices, the loading weights matrix and the matrix of cointegrating vectors respectively.<sup>2</sup> When  $k < p$  it can be shown that the system incorporates a set of long run relationships of the form  $\beta'x_t = s_t$ , where

$$(2.2) \quad s_t = (\alpha'\alpha)^{-1} \alpha' (\varepsilon_t - A^*(L)\Delta x_t) \sim I(0).$$

In this model there are  $k$  linearly independent cointegrating vectors, the columns of  $\beta$ . Note that without restrictions on  $\beta$  we can always *scale* the matrix of the cointegrating relations by post-multiplying it by any non-singular  $k \times k$  matrix  $C$ , to get  $C\beta'x_t = Cs_t$  that is observationally equivalent to  $\beta'x_t = s_t$  with loading matrix  $\alpha C^{-1}$ . The identification problem within the Johansen procedure is tackled by estimating a collection of orthonormalised vectors spanning the same space as  $\beta$  that are identified by the usual rank condition. Here we propose to follow a method that allows the researcher to identify the structural relations in the case of over-identified systems extending it to the case of just-identified ones. Our methodology is an extension of a method put forward by Davidson (1998a) of which we need to recall the main points that are formalised in five theorems.

*Theorem 1* (Davidson, 1994). *If a column of  $\beta$  (say  $\beta_l$ ) is identified by the rank condition, the OLS regression which includes just the variables having unrestricted non-zero coefficients in  $\beta_l$  is consistent for  $\beta_l$ .*

The issue raised by this theorem is that within a non-stationary world if another variable is added to a cointegrating regression, its coefficient might not necessarily converge to zero as we would expect in the case of an irrelevant variable within regression involving stationary variables. In the case of cointegration the regression coefficients would generally converge to some other element of the cointegrating space. The main result of this is that, if a collection of  $I(1)$  variables is found to be cointegrated, it does not necessarily follow that the estimated vectors can be interpreted as structural. In this framework it is useful to recall the definition of irreducible cointegrating vector introduced by Davidson's (1998a), that is,

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<sup>2</sup> We have assumed for simplicity the absence of any deterministic terms in this representation of the system under analysis. The modifications necessary to relax these assumptions are straightforward and would not alter the substance of the results obtained using a simpler model.



*Definition 1. A set of  $I(1)$  variables will be called irreducibly cointegrated (IC) if they are cointegrated, but dropping any of the variables leaves a set that is not cointegrated.*

Having formally defined the features of an IC it is worth mentioning the following important property of these vectors.

*Theorem 2. An IC vector is unique, up to the choice of normalisation.*

This theorem is proved using the following argument. Let us assume that there exists for the IC variables a set of cointegrating vectors of rank at least two. We have already seen that any linear combination of these vectors would lie in an observationally equivalent cointegrating space. If this is true, we can always generate a combination having a zero element by choosing the weights appropriately. This would allow us to drop the variable in question without losing cointegration, but this contradicts the definition of IC itself.

*Theorem 3 (Davidson, 1994). If and only if a structural cointegrating relation is identified by the rank condition, it is irreducible.*

This tells us that at least some IC vectors are structural. When the cointegrating rank of the system is  $k$ , an IC relation can contain at most  $p - k + 1$  variables. There are between  $k$  and  $(p - k + 1)$  of these vectors in total, the actual number depending on the degrees of over-identification of the relations of the system. This is to say that in addition to up to  $k$  identified structural relations, which, by theorem 3, are among the IC vectors, there might also be a number of solved vectors that can be defined as follows:

*Definition 2. A solved vector is a linear combination of structural vectors from which one or more common variables are eliminated by choice of offsetting weights such that the included variables are not a superset of any of the component relations.*

A solved vectors lies in the cointegrating space by construction. It may also be irreducible provided that it is a function of identified structural vectors. It is worth highlighting that solved IC relations are comparable to the reduced form equations of the conventional simultaneous equation models as they are solved from the structure<sup>3</sup>.

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<sup>3</sup> Note that in standard systems of simultaneous equations the reduced forms are defined with respect to a particular normalisation which is based on the distinction between endogenous and exogenous variables, which is not relevant in the cointegrating framework.

At this point a reasonable question would be, why does one need to test for irreducible cointegrating relations? And, how does one distinguish between structural and solved cointegrating vectors?

The answers are contained in the following argument. It is common practice to build a presumed cointegrating regression in the light of some economic theory, the theory being considered to receive support if the hypothesis of non-cointegration is rejected. However, economic theory might suggest including some variable which is in fact not really involved in that cointegrating relation but which, interacting with the other variables, might display a coefficient which does not converge to zero. This could well provide us with a stationary relation that could indeed be a wrong one, for that, as gathered from *theorem 3*, a cointegrating relation that contains redundant elements is not interest. The theory could be wrong, in which case this is just an arbitrary element of the cointegrating space. If the theory is correct, the relation is revealed to be underidentified. The estimate is inconsistent and it represents a hybrid of different structural equations.

Irreducibility is an important diagnostic property of a cointegrating regression, and testing for it allows us to determine what are the redundant variables in the system removing any unwanted effects. Once an IC relation is found, interest focuses on the problem of distinguishing between structural and solved forms. Of course, the theoretical model might answer this question for us, but this would then simply be using the theory to identify the model, and so in the absence of overidentifying restrictions we could learn nothing about the validity of the theory itself. Davidson (1998a) argues that important clues may also be provided by the data alone and we extend the argument later. To show this he uses the example discussed below. Prior to this, it is necessary to mention some more of his results, which will also be useful to understand our extension. These are formalised in a lemma and two more theorems.

*Lemma I. Provided  $\beta$  is restricted only by zero and normalisation restrictions, a solved IC relation contains at least as many variables as each of the identified structural relations from which it derives.*

In general, therefore, the fewer variables an IC relation contains, and the fewer it shares with other IC relations, the better the chance that it is structural and not a solved form. In the extreme cases, we can actually draw definite conclusions, as the following pair of results show.

*Theorem 4. If an IC relation contains strictly fewer variables than all those others having variables in common with it then, subject to the condition of Lemma 1, it is an overidentified structural relation.*

*Theorem 5. If an IC relation contains a variable, which appears in no other IC relation, it is structural.*

Thus, it is possible, in the context of simultaneous cointegrating relations, to discover structural economic relationships directly from a data analysis, without the use of any theory. To understand this assume a system that consists of four I(1) variables,  $x$ ,  $y$ ,  $z$  and  $w$ . Suppose we had tested for cointegrating rank and had found a rank of two. The second step would involve testing for cointegration on pairs of the variables. If the pairs  $(x, y)$  and  $(z, w)$  are found to be cointegrated (but not the pairs  $(x, z)$  or  $(y, w)$ ), these two cointegrating relations, necessarily irreducible of course, are also necessarily structural. Neither can have arisen as a result of solving out some more fundamental relationships. This is a case of maximal over-identification and is the framework within which Davidson's (1998a) methodology performs at its best. This result is achieved by Davidson using an algorithm implemented in Gauss called MINIMAL.

The essence of it can be summarised as: start with a pair of variables, test these for cointegration, and then add to the set one variable at a time until a cointegrated subset is found. This procedure is then repeated in every possible way. When the routine has terminated, every subset of the variables has been tested for cointegration unless a subset of it has been previously found cointegrated. At the end of the procedure we should obtain a list of cointegrated subsets which do not have any cointegrated subsets. All the testing can be performed indifferently using the Engle-Granger OLS method or the Likelihood-based technique due to Johansen (1988).

To introduce our extension of Davidson's procedure, we will analyse the case of bivariate cointegration as this gives rise to the largest number of IC vectors and solved cointegrating relationships for any number of variables. In addition, this is also the most relevant case for our later application of the methodology to the G7 interest rates.

Consider an N-dimensional cointegrating system as analysed by Johansen (1988, 1991),

$$(2.3) \quad \Pi(L)X_t = \alpha\beta' X_t + \Pi^\circ(L)\Delta X_t = u_t,$$

where  $L$  is the lag operator,  $\Pi(L) = \alpha\beta' + \Pi^{\circ}(L)(1-L)$ , such that  $\Pi(1) = \alpha\beta'$ , where  $\alpha$  and  $\beta$  are of dimension  $N \times R$  and are respectively the loading matrix and the matrix of cointegrating vectors. When  $R < N$ , it can be shown that  $X_t \sim I(1)$  and the system incorporates a set of long run equilibrium relations of the form  $\beta'X_t = u_t$ .

In general, in the case of bivariate cointegration between each pair of variables in a set of  $N$  variables there will be  $R$  structural IC vectors where  $R$  is  $N-1$ , and there will exist  $K$  irreducible vectors, which are simply combinations of the  $R$  structural ones where  $K$  is  $((R-1)^2 + (R-1))/2$ . Now, if we designate the first  $R$  cointegrating residuals as the structural ones, so that for  $u_1 \dots u_R \sim NI(0, \sigma^2_1 \dots \sigma^2_R)$ , then clearly the solved cointegrating residuals will be combinations of these<sup>4</sup>. Because for any IC vector the variance may vary with the normalisation, we need two starting assumptions for our ranking criterion to be operative.

This first assumption involves the possibility of normalising the IC vectors using the normalisation which yields a minimum variance. Formally, given two cointegrated series  $x_{1t}$  and  $x_{2t}$  we choose to normalise their cointegrating relation as

$$(2.4) \quad x_{1t} = \delta x_{2t} + u_{1t}, \quad u_{1t} \sim (\mu_1, \sigma_1^2)$$

rather than as

$$(2.5) \quad x_{2t} = \rho x_{1t} + u_{2t} \quad u_{2t} \sim (\mu_2, \sigma_2^2),$$

if  $\sigma_1^2 \leq \sigma_2^2$ .

A second assumption is that, the residuals from the structural cointegrating relations are normally independent distributed with full rank diagonal covariance matrix  $\Sigma^5$  (Sims 1980 makes a similar assumption for the error terms of the unobservable structural VAR<sup>6</sup>; see also Hendry 1995 pp.784 and 807). Given these two starting assumptions, we make the following statement:

<sup>4</sup> However, we will see that the set of  $K$  solved residuals need not all to display a variance that is strictly greater than that all of the  $R$  structural residuals.

<sup>5</sup> However, given that the solved vectors are linear combinations of the structural ones it is likely that their cointegrating errors will be correlated with one or both the residuals of the solving relations.

<sup>6</sup> Sims' 1980 argument can be summarised as follows: The structural model is not directly observable, however a VAR can be estimated as the reduced form of the underlying structural model

$$\begin{pmatrix} X_{1t} \\ X_{2t} \end{pmatrix} = A^{-1}C(L) \begin{pmatrix} X_{1t-1} \\ X_{2t-1} \end{pmatrix} + \begin{pmatrix} e_t^{X_1} \\ e_t^{X_2} \end{pmatrix},$$

where  $e$  denotes the VAR residual vector, normally independent distributed with full variance-covariance matrix  $\Omega$ . The relation between the residuals in  $e$  and the structural disturbances in  $u$  is,

The minimum variance normalised structural residuals will have a strictly lower variance than any solved residual coming from an IC vector containing the same variable.

This proposition provides us with an immediate rule for distinguishing structural from solved irreducible vectors. We can prove it as follows.

Let

$$(2.6) \quad \beta' X_t = u_t, \beta (rxn) \text{ and } X_t (nxI)$$

be the structural cointegrating relations with  $\beta$  normalised such that  $u_t \sim iN(0, \Sigma)$ .

Here we assume that  $\beta$  has been normalised on the  $j^{\text{th}}$  element of its columns such that the cointegrating relations have variances  $\sigma_{i,j}^2 < \sigma_{i,k}^2$ , where  $\sigma_i^2$  are the elements on the main diagonal of  $\Sigma$ , and  $j \neq k$  indicate different normalisations. Any solved vector will be given by a combination of the structural  $\beta' X_t = u_t$  of the form

$$(2.7) \quad A\beta' X_t = Au_t,$$

where  $A$  is  $(Ixr)$  normalised on the corresponding element  $j$ .

It follows that  $Au_t \sim N(0, A\Sigma A')$  and more explicitly  $A\Sigma A' = \sum_i a_i^2 \sigma_i^2$ . Now recall

that as at least one of the terms of this summation is given by  $a_j^2 \sigma_j^2$ , and also  $a_j = 1$ .

Therefore, we can see that

$$(2.8) \quad A\Sigma A' = \sum_i a_i^2 \sigma_i^2 > \sigma_{i,j}^2. \quad \text{QED.}$$

As an illustration, consider the case when  $N=4$ , such that  $X_t = \{x_1, x_2, x_3, x_4\}$ . Assume that, in testing for irreducibility, we have found that cointegration holds between all the possible pairs of series. In this case the system has a rank of  $(N-1)=3$  and we have a collection of 6 irreducible cointegrating relations, of which 3 are structural and 3 are

$$A \begin{pmatrix} e_t^{X_1} \\ e_t^{X_2} \end{pmatrix} = B \begin{pmatrix} u_t^{X_1} \\ u_t^{X_2} \end{pmatrix}, \text{ so that } e_t = A^{-1} B u_t, \text{ and } E(e_t e_t') = A^{-1} B E(u_t u_t') B' A^{-1}. \text{ Thus, } \Omega$$

contains  $(n^2+n)/2$  different elements, that indicate the maximum number of identifiable parameters in the matrices  $A$  and  $B$ . In his seminal paper, Sims proposed the following identification strategy based on the Choleski decomposition of matrices:

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ a_{21} & 1 & 0 & 0 \\ \cdot & & 1 & 0 \\ a_{n1} & a_{m-1} & & 1 \end{bmatrix}, B = \begin{bmatrix} b_{11} & 0 & 0 & 0 \\ 0 & b_{22} & 0 & 0 \\ 0 & 0 & b_{33} & 0 \\ 0 & 0 & 0 & b_{mm} \end{bmatrix}.$$

just solved relations. Suppose that the minimum variance normalisation delivers us the following irreducible structural relations

$$(2.9) \quad x_{1t} = \delta x_{2t} + u_{1t}, \quad u_{1t} \sim N(0, \sigma_1^2)$$

$$(2.10) \quad x_{2t} = \gamma x_{3t} + u_{2t}, \quad u_{2t} \sim N(0, \sigma_2^2)$$

$$(2.11) \quad x_{3t} = \phi x_{4t} + u_{3t}, \quad u_{3t} \sim N(\mu_3, \sigma_3^2).$$

The three solved vectors will be linking respectively  $x_{1t}$  to  $x_{3t}$ ,  $x_{1t}$  to  $x_{4t}$ , and  $x_{2t}$  to  $x_{4t}$ . They will look like

$$(2.12) \quad x_{1t} = \theta x_{3t} + u_{4t} \quad \text{with } \theta = \delta\gamma, \text{ and } u_{4t} = u_{1t} + \delta u_{2t},$$

$$(2.13) \quad x_{1t} = \lambda x_{4t} + u_{5t}, \quad \text{with } \lambda = \delta\gamma\phi, \text{ and } u_{5t} = u_{1t} + \delta u_{2t} + \delta\gamma u_{3t}, \text{ and}$$

$$(2.14) \quad x_{2t} = \psi x_{4t} + u_{6t}, \quad \text{with } \psi = \gamma\phi, \text{ and } u_{6t} = u_{2t} + \gamma u_{3t}.$$

Basic statistics tells us that  $u_{4t}$  will be distributed as

$$(2.15) \quad u_{4t} \sim N(0, [\sigma_1^2 + \delta^2 \sigma_2^2 + 2\delta \text{cov}(u_{1t}, u_{2t})]).$$

And assuming that  $u_{1t}$  and  $u_{2t}$  are uncorrelated

$$(2.16) \quad u_{4t} \sim N(0, \sigma_1^2 + \delta^2 \sigma_2^2)$$

In the same way

$$(2.17) \quad u_{5t} \sim N(0, \sigma_1^2 + \delta^2 \sigma_2^2 + \delta^2 \gamma^2 \sigma_3^2), \text{ and}$$

$$(2.18) \quad u_{6t} \sim N(0, \sigma_2^2 + \gamma^2 \sigma_3^2) \text{ respectively.}$$

We can see that the magnitude of the variances of the solved vector depends on the values of the cointegrating parameters  $\delta, \gamma$ , and  $\phi$ , and it seems somewhat difficult to distinguish whether for example  $\sigma_1^2 < (\sigma_2^2 + \gamma^2 \sigma_3^2)$  without any prior knowledge about  $\gamma$ . This does not mean that we cannot use the criterion of the minimum variances to detect the structural residuals. This is achieved by carrying out a more complex comparison, which involves, rather than comparing the variances of the irreducible cointegrating relations in absolute terms (simply with each other), comparing the variances of vectors relative to the same variable.

Table 1 shows a four variable case, as in the example presented above, where the structural bivariate relationships are between  $x_1$  and  $x_2$ ,  $x_2$  and  $x_3$  and  $x_3$  and  $x_4$ . The variances reported are in fact the ones of  $u_1, \dots, u_6$ . There we can see that the structural relationship between  $x_1$  and  $x_2$  always has the smallest variance in the row corresponding to vectors normalised for  $x_1$ . In fact we can observe that  $\sigma_1^2$  must be

strictly smaller than both  $\sigma_{1t}^2 + \delta^2 \sigma_{2t}^2$ , and  $\sigma_{1t}^2 + \delta^2 \sigma_{2t}^2 + \delta^2 \gamma^2 \sigma_{3t}^2$  regardless of the values or the signs of the cointegrating parameters  $\delta$  and  $\gamma$ . The same applies to the structural relationship between  $x_2$  and  $x_3$  as compared to the one linking  $x_2$  and  $x_4$ , and so on.

It is interesting to notice that irreducibility might also be useful to simplify the analysis of causal linkages between variables. This is because by ruling out non cointegrated series we would be working with smaller systems with a gain in efficiency. Therefore, in order to shed some light on the causal structure of the cointegrating system we perform likelihood ratio tests on the adjustment coefficients towards the irreducibly cointegrated relations. Below we apply the proposed methodology to the G-7 short-run interest rates with interesting results.

### **3. Empirical Analysis**

#### **a) The dataset**

The sample under investigation covers the period between 1977:1-1998:3. The short-term interest rates we employed are those that are most likely to be used as policy instruments, namely the three-month Treasury bills. The only exception is Japan, where only the discount rate was available for the whole period under investigation. The source for the data is the IMF's International Financial Statistics.

We begin the analysis by pre-testing for the order of integration of the series using standard Augmented Dickey-Fuller (ADF) tests. The number of lagged differences included in the test is decided on the basis of a criterion advised by Doornik and Hendry (1997), so as to ensure non-autocorrelated residuals on the auxiliary regressions. In each case the tests deliver the expected result that the series are all integrated of order one [I(1)], and hence follow stochastic trends. The results are shown in table 2.

Having obtained confirmation that all interest rates are integrated of order one, we proceed by running cointegration tests for the complete G-7 series of short-term interest rates. For this and subsequent analysis we have used Johansen's (1988, 1991) likelihood based cointegration tests. Our theory would suggest that amongst the seven series there should be six cointegrating vectors. As suggested in Hall (1991) and Caporale et al (1997), in performing the rank tests we have been particularly careful

about correctly specifying the unrestricted VAR, therefore including as many lags of the variables as necessary to ensure non-autocorrelation in the residuals, as well as one-point dummies to correct for non-normality or heteroscedasticity of the disturbances.

## **b) Empirical Results**

We start by performing cointegration tests on the complete G-7 set of interest rates obtaining the results displayed in table 3. The test statistics indicate that the cointegrating rank of the system is four. Of course these tests only allow us to reject the hypothesis that there are less than four cointegrating vectors - they do not necessarily mean that there is not more. So, in order to learn something more about the structure of the linkages among these series, we perform cointegration tests on each pair of series to investigate whether cointegration holds among all of the series of the group. The aim is to establish the number of irreducible cointegrated relations and what series are involved in them. Of course if we find that pairwise cointegration holds between each pair of rates this tells us that the rank of the whole seven variable system is in fact 6. The conflict between the two test procedures is then seen as simply one of the small sample power and size of the tests in different contexts.

The results for pairwise cointegration tests are presented in table 4. Looking at the results we get confirmation of what we suspected about the possible true rank of the system. The main result is that cointegration holds among every pair of series and with a unit elasticity in all cases but in two, namely the relationship Japan-Canada and Japan-France. The likely reason is the fact that in the case of Japan we used the discount rate, which appears to behave differently and to change much more slowly compared to other rates, therefore producing different results.

Cointegration is clearly a property of the series and this is confirmed by the existence of nineteen irreducible cointegrating relations with unit coefficients out of twenty-one cointegrating tests. Of course, given such widespread cointegration it is almost certain that in fact all pairwise interest rates cointegrate. For example Italy and Japan clearly cointegrate, as do Italy and Germany, but the test between Germany and Japan is less significant. However, as the first two cointegrate, Germany and Japan must also cointegrate.



The second part of our analysis involves the ranking of the cointegrating vectors according to the criterion of minimum variability and the discussion of exogeneity issues which will help to clarify the relationships among the series (table 5). The first two interesting results are that two of the minimum variance vectors link the US to Canada and Italy to France, both of which can be considered irreducible structural relationships. It then becomes rather hard to interpret the ranking when presented in the form of table 5. We therefore collate the information from table 5 in the form given in table 6.

This table now presents a set of results, which are relatively easy to interpret. We have highlighted in bold the relationships which seem to be structural on a column by column interpretation. Clearly, on a minimum standard deviation criteria the US and Canada is a structural relationship, as is Germany and Japan, France and Italy, UK and Japan, Germany and Japan, and US and Japan.

In the simple ranking of standard deviations given in table 6 these all appear in the lowest 7 places; we can exclude the relation between Canada and France, which is in position 6, from being structural as in the French column France and Japan have a lower standard deviation. The exogeneity tests reported in Table 5 add a little to this understanding. In the Japan Germany relationship, Japan is exogenous, and in the France-Italy relationship, France is exogenous. Perhaps most significantly, the US is found to be exogenous with respect to Japan. This suggests that Italy was largely following French rates and Germany was following Japanese rates, while Japan followed the US. All the other relationships seem to be bi-directional.

Overall we have a picture of the US and Canada being a clear block, then most of the other countries being linked primarily through Japanese rates (these are not of course acting in a dominant way as no exogeneity can be established except with respect to Germany). Italy was clearly following France. The UK responded more to non-European rates than to the other European countries, and, surprisingly, France and Germany seem to be responding more to world rates than to each other.

These results clearly confirm the previous analysis conducted by Caporale and Williams (1998b), which rejected the so-called German Leadership Hypothesis (GLH), since Germany does not appear to be driving the European financial markets. This is suggested by exogeneity of US and Japanese rates in the pairwise cointegration tests with the German one. Also, in the tests with other European

countries, Germany does not appear to be exogenous in any but one of the cases. It does not seem that a leader exists in Europe. This conclusion is justified by the fact that we find feedback effects in causality among most of the series involved and in the other cases we find a sort of “*circular*” causality among the series, indicating that there is not a country that represents the leader.

#### **4. Summary and Conclusions**

In this paper we have examined the causal linkages that exist between the G-7 short-term interest rates. We have done so applying a methodology due to Davidson (1998a) which is based on the innovative concept of an irreducible cointegrating (IC) vector which can be defined as a subset of a cointegrating relation that does not have any cointegrated subsets. Application of this method has provided us with the proof that cointegration is a property of the G-7 short rates showing the importance of testing for irreducibility as a diagnostic. We have also extended Davidson's (1998a) methodology introducing the ranking of the IC relations according to the criterion of minimum variance. This has allowed us to distinguish between structural and solved IC vectors without any prior theoretical assumptions. Furthermore, we have performed exogeneity tests on all IC relations in order to gather information on the causal structure that links the rates.

The results can be summarised as follows. First, the rank of the system appears to be 6 when Davidson's approach is taken, compared to 4, which is the result obtained performing a rank test *a la* Johansen (1988) on the full system. Second, we have been able to isolate six irreducible structural relations, of which the two most significant ones involve the US and Canada, and Italy and France. Third, exogeneity tests seem to indicate a US world-wide leadership and reject the hypothesis of a German leadership in Europe, therefore confirming the findings of Caporale and Williams (1998b) and of other authors (see, e.g., Katsimbris and Miller, 1993). In brief, US and Canada appear to constitute the fundamental block, UK rates respond more to non-European rates than to other European countries, Italy is clearly following France, and France and Germany respond to world rates rather than to each other. Last, it is important to mention the role of Japan that acts as the link between European rates and the US one.

Table 1. The relationship of variances of cointegrating errors between structural and solved vectors

	$x_1$	$x_2$	$x_3$	$x_4$
$x_1$	----	$\sigma^2_{1t}$	$\sigma^2_{1t} + \delta^2 \sigma^2_{2t}$	$\sigma^2_{1t} + \delta^2 \sigma^2_{2t} + \delta^2 \gamma^2 \sigma^2_{3t}$
$x_2$	----	----	$\sigma^2_{2t}$	$\sigma^2_{2t} + \gamma^2 \sigma^2_{3t}$
$x_3$	----	----	----	$\sigma^2_{3t}$
$x_4$	----	----	----	----

Table 2. Unit root test on G7 short-term rates

	ADF test value	Critical value 95%	Critical value 99%
Canada	-1.852	-2.89	-3.51
France	-1.383	-2.89	-3.51
Italy	-0.829	-2.89	-3.51
USA	-2.202	-2.89	-3.51
UK	-2.340	-2.89	-3.51
Germany	-2.749	-2.89	-3.51
Japan	-1.465	-2.89	-3.51

Table 3. Unit root test on G7 long-term rates

	ADF test value	Critical value 95%	Critical value 99%
Canada	-0.990	-2.89	-3.51
France	-0.938	-2.89	-3.51
Italy	-1.010	-2.89	-3.51
USA	-1.338	-2.89	-3.51
UK	-0.442	-2.89	-3.51
Germany	-2.073	-2.89	-3.51
Japan	-0.720	-2.89	-3.51

Table 4. G7 short term rates cointegration test						
Eigenvalue	Log-likelihood	rank				
	481.013	0				
0.7557	540.215	1				
0.5184	570.903	2				
0.3961	592.086	3				
0.2975	606.915	4				
0.1550	613.99	5				
0.0877	617.845	6				
0.0370	619.428	7				
Ho:rank=p	$-T \log(1 - \hat{\lambda}_p)$	(T-nm) <sup>o</sup>	95% cv	$-T \sum_{i=p+1}^n \log(1 - \hat{\lambda}_i)$	(T-nm) <sup>o</sup>	95%cv
P == 0	118.4**	88.8**	45.3	276.8**	207.6**	124.2
P <= 1	61.38**	46.03**	39.4	158.4**	118.8**	94.2
P <= 2	42.37**	31.77	33.5	97.05**	72.79*	68.5
P <= 3	29.66*	22.24	27.1	54.68**	41.01	47.2
P <= 4	14.15	10.61	21	25.03	18.77	29.7
P <= 5	7.71	5.783	14.1	10.87	8.156	15.4
P <= 6	3.164	2.373	3.8	3.164	2.373	3.8
*indicates rejection of the null at the 95% level						
** indicates rejection of the null at the 99% level						
<sup>o</sup> -(T-nm) is a small sample correction replacing (-T) in the $\lambda$ -max and $\lambda$ -trace statistics						

	$-T \log(1 - \hat{\lambda}_p)$	$-(T-nm)^\circ$	95% cv	$-T \sum_{i=p+1}^n \log(1 - \hat{\lambda}_i)$	$-(T-nm)^\circ$	95% cv
USA-Canada	14.23*	9.678	14.1	16.58*	11.27	15.4
USA-Japan	22.46**	14.57*	14.1	23.68**	15.36	15.4
USA-UK	14.71*	9.936	14.1	17.31*	11.7	15.4
USA-France	14.48*	11.92	14.1	15.48*	12.74	15.4
USA-Italy	16*	13.16	14.1	19*	15.63*	15.4
USA-Germany	24.45**	20.17**	14.1	24.45**	20.17**	15.4
Canada-France	15.52*	11.8	14.1	17.94*	13.64	15.4
Canada-UK	14.72*	12.15	14.1	17.39*	14.35	15.4
Canada-Italy	21.09**	18.52**	14.1	21.34**	18.74*	15.4
Canada-Japan	24.71**	20.39**	14.1	24.76**	20.43**	15.4
Canada-Germany	17.47*	13.93	14.1	17.47*	13.93	15.4
UK-France	15.95*	12.55	14.1	19.46*	15.31	15.4
UK-Germany	30.99**	23.74**	14.1	31.24**	23.93**	15.4
UK-Italy	17.8*	14.48	14.1	19.71*	16.03	15.4
UK-Japan	19.9**	15.31*	14.1	24.62**	18.94*	15.4
Germany-France	16.31*	14.32*	14.1	18.43*	16.18*	15.4
Germany-Italy	22.21**	18.7*	14.1	25.75**	21.68*	15.4
Germany-Japan	14.8*	13	14.1	17.03*	14.95	15.4
Italy-Japan	27.96**	25.96**	14.1	29.79**	27.67**	15.4
Italy-France	24.09**	21.15*	14.1	30.62**	26.89**	15.4
Japan-France	27.46**	21.9**	14.1	28.91**	23.05**	15.4

\* indicates rejection of the null of no cointegration at 95% level  
\*\* indicates rejection of the null of no cointegration at 99% level  
<sup>o</sup>  $-(T-nm)$  is a small sample correction replacing  $(-T)$  in the  $\lambda$ -max and  $\lambda$ -trace statistics

Table 6. Short-term rates, ranking of IC vectors			
IC vectors	eigenvalue	standard deviation	exogeneity restrictions
Italy-France	0.254565	1.173084	France exogenous
Germany-Japan	0.165175	1.571049	Japan exogenous
USA-Canada	0.172846	1.782502	Feedback
UK-Japan	0.225159	1.883635	Feedback
Japan-France*	0.293614	1.967761	Feedback
Canada-France	0.186971	2.108287	Canada exogenous
USA-Japan	0.261784	2.126364	USA exogenous
Canada-UK	0.168093	2.138164	Feedback
Germany-France	0.18824	2.21158	Feedback
USA-UK	0.180218	2.359356	USA exogenous
USA-France	0.167506	2.405468	France exogenous
UK-Germany	0.307000	2.467542	Feedback
Canada-Japan*	0.265748	2.496532	Feedback
Canada-Italy	0.226752	2.497663	Feedback
Italy-Japan	0.283097	2.540980	Feedback
USA-Germany	0.263376	2.545527	USA exogenous
USA-Italy	0.183315	2.647584	Italy exogenous
UK-France	0.191583	2.674037	UK exogenous
Canada-Germany	0.198368	2.772337	Canada exogenous
Germany-Italy	0.253395	2.820897	Germany exogenous
UK-Italy	0.211309	3.190997	Feedback
* indicates cointegration with non-homogeneous coefficients			

Table 7 Cross tabulation of Standard deviations for the G7 short rates.							
	USA	Canada	Japan	Germany	France	Italy	UK
USA	-	<b>1.78</b>	<b>2.1</b>	2.5	2.4	2.6	2.3
Canada	1.78	-	2.5	2.7	2.1	2.5	2.1
Japan	2.1	2.5	-	<b>1.57</b>	<b>1.96</b>	2.5	<b>1.88</b>
Germany	2.5	2.7	1.57	-	2.2	2.8	2.4
France	2.4	2.1	1.96	2.2	-	<b>1.17</b>	2.6
Italy	2.6	2.5	2.5	2.8	1.17	-	3.2
UK	2.3	2.1	1.88	2.4	2.6	3.2	-
Structural IC relations are in highlighted in bold							

## References

- Begg, D., Giavazzi, F., De Grauwe, P., Uhlig, H., and C. Wyplosz (1998), *The ECB: Safe at Any Speed?*, CEPR Report on *Monitoring the European Central Bank*.
- Caporale, G.M., Kalyvitis, S. and N. Pittis (1996), "Interest rate convergence, capital controls, risk premia and foreign exchange market efficiency in the EMS", *Journal of Macroeconomics*, 18, 4, 693-714.
- Caporale, G.M. and N. Pittis (1999), "Efficient estimation of cointegrating vectors and testing for causality in vector autoregressions", *Journal of Economic Surveys*, 13, 1, 1-32.
- Caporale, G.M. and G. Williams (2002), "Long term nominal interest rates and domestic fundamentals", *Review of Financial Economics*, forthcoming.
- Caporale, G.M. and G. Williams (1998b), "International linkages in short- and long-term interest rates", *D.P. no.14-98*, Centre for Economic Forecasting, London Business School.
- Caporale, G.M. and G. Williams (1998c), "Can interest rate policy be independent?", *Economic Outlook*, 22, 4, 10-15.
- Caporale, G.M., Hall, S.G., Urga G. and G. Williams (1997), "Aggregate money demand functions in five industrial countries: are they cointegrated? ", *D.P. no.19-97*, Centre for Economic Forecasting, London Business School.
- Davidson, J. (1994), "Identifying cointegrating regressions by the rank condition", *Oxford Bulletin of Economics and Statistics*, 56, 1, 103-108.
- Davidson, J. (1998a), "Structural relations, cointegration and identification: some simple results and their application", *Journal of Econometrics*, 87, 87-113.



Davidson, J. (1998b), "A Wald test of restrictions on the cointegrating space based on Johansen's estimator", *Economics Letters*, 59, 2, 183-187.

Engle, R.F. and C.W.J. Granger (1987), "Cointegration and error correction: representation, estimation and testing", *Econometrica*, 55, 2, 251-276.

Fratianni, M. and J. von Hagen (1990), "German dominance in the EMS: evidence from interest rates", *Journal of International Money and Finance*, 9, 358-375.

Doornik, J.A. and D.F. Hendry (1997), *Modelling Dynamic Systems Using Pc-Fiml 9.0*, International Thomson Business Press, London

Hall, S.G. (1991), "The effect of varying length VAR models on the maximum likelihood estimates of cointegrating vectors", *Scottish Journal of Political Economy*, 38, 4, 317-323.

Hall, S.G., Robertson, D. and M.R. Wickens (1992), "Measuring convergence of the EC economies", *Manchester School*, 60, Supplement, 99-111.

Hendry, D.F. (1995), *Dynamic Econometrics*, Oxford University Press, Oxford.

Johansen, S. (1988), "Statistical analysis of cointegration vectors", *Journal of Economic Dynamics and Control*, 12, 231-254

Johansen, S. (1991), "Estimation and hypothesis testing of cointegration vectors in Gaussian vector autoregressive models", *Econometrica*, 59, 1551-1580.

Johansen, S. (1995), "Identifying restrictions of linear equations with applications to simultaneous equations and cointegration", *Journal of Econometrics*, 69, 111-132.

Karfakis, J.C. and D.M. Moschos (1990), "Interest rate linkages within the European Monetary System: a time series analysis", *Journal of Money, Credit and Banking*, 22, 3, 388-394.

Katsimbris, G.M. and S.M. Miller (1993), "Interest rate linkages within the European Monetary System: further evidence", *Journal of Money, Credit and Banking*, 25, 4, 771-779.

Martin, P. (1997), "The exchange rate policy of the Euro: A matter of size?", *CEPR D.P. no.1646*.

Pesaran, M.H. and Y. Shin (1994), "Long-run structural modelling", *Working Paper, Department of Applied Economics, University of Cambridge*.

Portes, R. and H. Rey (1998), "The emergence of the Euro as an international currency", *Economic Policy*, 26, April 1998.

Sims, C. A. (1980), "Macroeconomics and Reality". *Econometrica*, 48, 1-48

Toda, H.Y. and T. Yamamoto (1995), "Statistical inference in vector autoregressions with possibly integrated processes", *Journal of Econometrics*, 66, 225-250.