

Evaluating Policy Feedback Rules using the Joint Density Function of a Stochastic Model

R. Barrell¹

S.G.Hall¹²³

And

I. Hurst¹

Abstract

This paper argues that the dominant practise of evaluating the properties of feedback rules in stochastic models using marginal distributions of the variables of interest implies the loss of considerable information. It argues that it is both practical and important to base decisions on the full joint density function. This argument is illustrated by comparing the properties of three rules applied to a large stochastic model under rational expectations.

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- 1. National Institute of Economic and Social Research**
- 2. Dept of Economics, University of Leicester.**
- 3. Corresponding Author, NIESR, 2 Dean Trench St. Smith Sq. London SW1P 3HE, UK**

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1. Introduction

There has been an enormous literature in recent years examining the properties of various policy feedback rules in a stochastic environment. Here we list only a few key references, Taylor 1993, 1999, Svenson 1997, Woodford 2003, Svenson and Woodford, 2002, Orphanides and Williams 2002, Fuhrer 1997, Giannoni 2002. One almost universal feature of this literature is that where an author considers a stochastic model decisions are based on the marginal distribution of the endogenous variables and the joint distribution is ignored. That is to say we might look at the variance of output and the variance of inflation but we would not consider the joint distribution of output and inflation.

In this paper we argue a conceptually very simple point. The marginal distribution conceals a great deal of interesting information which is contained in the full joint distribution. Very often in standard analysis we are forced to assign largely arbitrary weights to policy objectives so as to trade them off against each other. However when we understand the full joint distribution a policy maker is presented with a much richer information set about the possible outcomes and often this may make the choice of an appropriate policy rule much easier.

The next section then sets out a framework which contrasts the information in the marginal and joint distribution. Section 3 then shows how this information can be used in a practical investigation of the National Institute's NIGEM model under three alternative policy rules. Finally section 4 concludes.

2. A Formal Framework

When we consider the above referenced literature on monetary rules and stabilisation policy the essence of the problem is that we are attempting to evaluate a density subject to certain assumptions about either the structure or the parameterisation of a particular rule. Formally this amounts to analysing how probability forecasts for a set of variables might change as the rule changes. So assume that we are concerned with the m -variable vector, $z_t = (z_{1t}, z_{2t}, \dots, z_{mt})'$ and that the density is evaluated subject to a parametric model $M(\phi)$ where ϕ is the parameterisation of the model. We assume that this parameterisation is general enough so that any rules being considered can be viewed as specific parameterisations of the general model M . The possible different density functions produced by different values of ϕ can then be characterised by assuming that ϕ lies in a compact parameter space Φ . Then,

$$M(\phi) = \{f(z_1, z_2, \dots, z_T; \Phi), \phi \in \Phi\} \quad 1.$$

This may be factorised to consider various density function of interest (see Garrat, Lee, Pesaran and Shin(2003)), we might factorise it sequentially to give the conditional distribution of z_s conditional on all earlier z values, this would correspond to the density of a standard VAR analysis. We could decompose z into exogenous and endogenous variables (x and y) and derive the density function of the y variables jointly conditional on the x 's. What is conventionally done in the monetary policy literature however is to calculate the marginal distribution of individual z 's, that is if we define the marginal distribution of z_{it} to be $F(z_{it})$ then

$$F(z_{nt}) = \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} f(z_1, z_2, \dots, z_T; \phi) dz_{ij} \quad i = 1 \dots m, j = 1 \dots T, i, j \neq n, t \quad 2.$$

If the z's are independent then

$$f(z_1, z_2, \dots, z_T; \phi) = \prod_{i=1}^m \prod_{j=1}^T F(z_{ij}) \quad 3.$$

And the set of marginal distributions carries all the information which is of interest in the joint distribution. However if this independence does not hold then there is no simple relationship between the joint probability distribution and the set of marginal distributions. Indeed in the typical applications in the monetary rules literature it seems unlikely that outcomes for inflation and the output gap will be independent. Hence the concentration on marginal distributions may be significantly distorting the results.

It is however possible to define the probability of a particular joint set of events occurring. So suppose a joint event was defined as

$$\mathcal{G}(z_1, z_2, \dots, z_T; \phi) < a \quad 4.$$

Then we can define the probability of this joint event occurring by

$$\pi(a, \mathcal{G}) = \Pr[\mathcal{G}(z_1, z_2, \dots, z_T) < a \mid M(\Phi)] \quad 5.$$

While the marginal distributions are normally what is calculated in the Monte Carlo analysis it is actually very straightforward to also calculate this joint probability from a Monte Carlo experiment.

Justifying the use of the full distribution should hardly need further elaboration from one viewpoint. For example Woodford(2003, pg. 386, equation 1.3)) states a taylor series approximation to a general welfare criterion as;

$$\bar{U} + U_x E(\bar{x}) + \frac{1}{2} tr(U_{xx} \text{var}(x)) + tr(U_{x\xi} \text{cov}(\xi, \bar{x})) + \frac{1}{2} tr(U_{\xi\xi} \text{var}(\xi)) + O(\|\varphi, \xi\|^3) \quad 6.$$

Using his notation, where the crucial thing from our point of view is that var is the complete variance-covariance matrix. However a lot of the literature relies on a utility based welfare criterion which explicitly derives a welfare function involving only the variance of the output gap and the variance of inflation. The derivation of this criterion is surveyed in Woodford(2003, pages 392-463). In brief it proceeds from stating that the natural welfare function should be derived from a representative agent's utility function which is a function of consumption and the disutility of labour. A series of simplifying assumption are then made which eventually allow this to be represented approximately only by the two variances. Justifying the relevance of the joint density function can then be done by either questioning the initial assumption that welfare is only a function of the representative utility or questioning the specific assumptions made in the formal derivation. The first of these avenues would include arguing that governments often care about the degree of dispersion of individual income outcomes around the average as well as the average level of income. The second would lead us to question such assumptions as the idea that the natural level of output

is independent of the variance of output and inflation or that there is no hysteresis in the system. For instance, Byrne and Davis(2004), show that the equilibrium level of investment and capital stock and hence output in the US depends upon the volatility of inflation. Hence the level of welfare using a utility function based only on consumption will depend upon the variance of inflation.

Even if we accept the formal derivation of the simple welfare function policymakers may still be interested in knowing what to expect in terms of different combinations of output and inflation. The full density function may also be of use in model evaluation and matching stylised facts of the real world of course¹.

3. Results

In order to illustrate the relevance of the general points made above we now use the National Institutes world model NIGEM in some stochastic simulations² where we test 3 different rules for stabilising inflation and output in the European Monetary Union area. The first rule reflects the framework adopted by the European Central Bank where there is a nominal target (in their case M3, in ours nominal GDP, $P_t Y_t$) and an inflation target which may be described as a Wicksellian rule.

$$r_t = 0.5(\log(P_t Y_t) - \log(P_t^* Y_t^*)) + 0.75(\Delta \log P_{t+j} - \Delta \log P_{t+j}^*) \quad 7:$$

This rule is contrasted with two different Taylor rules that feedback on the output gap and inflation. The nominal target rule (7) differs from (8) in an important way as when output returns to its natural rate after a shock (7) ensures that the actual price level also returns to its previous level, ensuring that the target nominal aggregate for GDP is maintained. Rule 8 leaves the price level to be determined by the scale of the shock, the parameters of the feedback rule and the speed of response of the economy to the shock (see woodford(2003), pg. 92).

$$r_t = \gamma_1(\log(Y_t) - \log(Y_t^*)) + \gamma_2(\Delta \log P_{t+j} - \Delta \log P_{t+j}^*) \quad 8$$

The Taylor Rule coefficients start with the industry standard 0.5 on output and 1.5 on inflation (Rule 2). Our alternative has an increased weight on GDP at $\gamma_1 = 1.5$ (rule 3) with that on inflation at 1.5.

These rules are then compared using stochastic simulations on NiGEM, which is a large, estimated and calibrated New Keynesian global model. The stochastic simulations use bootstrapping to apply equation residuals recovered from 1991q1 to 1999q4 to the forecast baseline.

There are of course many ways to illustrate the joint density function, in a welfare context we could consider the normal variances and also the covariance of the joint distribution. However for illustrative purposes we feel that it emphasises the information coming from the joint distribution if we consider a set of target ranges. We therefore set up two targets which we wish to achieve, Inflation should be stabilised to within 0.3% of its target value and GDP

¹ We are grateful to an anonymous referee for this point.

² For more details of the model and a more extensive investigation of policy rules using this model see Barrell and Pina(2004))

should also be within 0.5% of its target value. Table 1 summarises information from the marginal and joint distributions.

Table 1. Summary Density Indicators

	Both fail	GDP pass	INF pass	Both Pass	INF only	GDP only
Combined Rule (rule 1)						
year 1	0.393	0.331	0.480	0.205	0.276	0.126
mean	0.496	0.247	0.356	0.100	0.256	0.147
Standard Taylor Rule (rule 2)						
year 1	0.393	0.333	0.450	0.176	0.274	0.157
mean	0.489	0.248	0.361	0.098	0.262	0.150
Output focused Taylor Rule (rule 3)						
year 1	0.404	0.393	0.403	0.201	0.202	0.193
mean	0.452	0.311	0.352	0.115	0.237	0.196

*The mean is calculated over 5 years
 cols 1+2+3-4 = 1, cols 1+4+5+6=1*

We are now able to analyse the decision problem faced by the policy maker much more fully. If we are interested in maximising the pass rate for inflation on average over the simulation we would pick rule 2 while if we are interested in GDP we would pick rule 3. The choice between the two rules is then far from obvious. However the joint probability of meeting both targets is maximised by rule 3 suggesting that this rule might be preferable. Similarly rule 3 minimises the probability of missing both targets. There is still a normative element in the choice but the extra information provided by the joint density is useful we would argue.

4. Conclusion

In this paper we have argued that the joint probability distribution of a number of policy objectives contains considerable information which is unavailable if we only consider the marginal distributions as is the case with almost all current work. We have shown that it is actually quite easy to calculate the joint distribution and that in a practical example this offers some important insights. In particular a rule chosen on the basis of the marginal distribution may not be chosen when the full density is investigated.

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