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ECONOMETRIC MODELLING OF THE AGGREGATE TIME-SERIES RELATIONSHIP BETWEEN CONSUMERS' EXPENDITURE AND INCOME IN THE UNITED KINGDOM*

I. INTRODUCTION

Although the relationship between Consumers' Expenditure and Disposable Income is one of the most thoroughly researched topics in quantitative economics, no consensus seems to have emerged in the United Kingdom about the short-run dynamic interactions between these two important variables. In support of this contention, we would cite the plethora of substantially different quarterly regression equations which have been reported by Byron (1970), Deaton (1972, 1977), Hendry (1974), Ball *et al.* (1975), Bispham (1975), Shepherd *et al.* (1975), Wall *et al.* (1975), Townend (1976) and Bean (1977). Moreover, this list of studies is representative, rather than exhaustive.

The diversity of the published estimates is really surprising since most of the investigators seem to have based their regression equations on similar economic theories and seem to have used approximately the same data series. Specifically, therefore, we wish to explain why their results manifest quite dissimilar short-run multipliers, lag reactions and long-run responses. This requires examining the extent to which the estimates are mutually incompatible as well as their inconsistency with the empirical evidence. More generally, we hope to be able to specify which aspects of the methodology used were primarily responsible for creating the differences in the published results.

Close inspection of the above list of studies reveals that despite their superficial similarities, they differ in many respects the importance of which is not obvious *a priori*. Initially, therefore, to highlight the issues involved we concentrated on three studies only (Hendry (1974), Ball *et al.* (1975) and Wall *et al.* (1975), denoted *H*, *B*, *W* respectively). Rather than use the elegant but very technical theory recently developed for testing "non-nested" models (see Pesaran and Deaton, 1978) we have chosen to "standardise" those aspects of the three studies which do not seem crucial to explaining the original differences between the results. This allows analogues of the contending models to be embedded in

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a common framework within which nested tests are feasible. By stressing the implications for each model of the results obtained by others it will be seen below that our approach assigns a major role to mis-specification analysis (see Hendry, 1977*a*, for a discussion of mis-specification theory in dynamic systems).

A proliferation of non-nested models is symptomatic of certain inappropriate aspects of present practice in econometrics. We would suggest that this problem can be mitigated to some extent by adopting the following principles. First, we consider it an essential (if minimal) requirement that any new model should be related to existing "explanations" in a constructive research strategy such that previous models are only supplanted if new proposals account (so far as possible) for previously understood results, and also explain some new phenomena. Second, to avoid directionless "research" and uninterpretable measurements, a theoretical framework is also essential. Unfortunately, much existing economic analysis relates to hypothetical constructs (for example, "permanent income") and/or is based on unclearly specified but stringent *ceteris paribus* assumptions, and leaves many important decisions in formulating an operational model to *ad hoc* considerations (e.g. functional form, dynamic specification, error structure, treatment of seasonality, etc.). Nevertheless, economic theory does furnish some helpful postulates about behaviour in steady-state environments and to guide an empirical analysis it seems sensible to incorporate such information as is available explicitly. Third, to be empirically acceptable, an econometric model obviously must account for the properties of the data (e.g. the autocorrelation function in a time-series study). It is not valid to "accomplish" this aim simply by not looking for counter-evidence (for example, by claiming the absence of autocorrelation in a dynamic equation on the basis of an insignificant value for a Durbin–Watson *d*-statistic).

The combination of not encompassing previous findings, introducing *ad hoc* auxiliary assumptions and not rigorously testing data compatibility leaves plenty of room for a diversity of outcomes from model building even in a common theoretical framework with a common data set. Indeed, one could characterise "econometric modelling" as an attempt to match the hypothetical data generation process postulated by economic theory with the main properties of the observed data. Any model which fails to account for the "gestalt" of results which are obtained from the data set cannot constitute the actual data generation process. Consequently, a further minimal requirement when modelling from a common data set is that the chosen model should explain both the *results* obtained by other researchers and *why* their research methods led to their published conclusions. The former usually can be achieved through the appropriate mis-specification analysis from a sufficiently general model which could be based on *a priori* theory (see, for example, Hendry and Anderson, 1977) or empirical considerations. Any theory gains some plausibility by an explanation of different empirical results, but a data-based construction always must be susceptible to a potential *post hoc ergo propter hoc* fallacy. However, given the research methods which any investigator claimed to use it is not trivial even from a data based general model to explain why they reached certain conclusions. That the general model is not obtained by every investigator seems to depend on the operation of (self-imposed) constraints limiting the range of specifications, estimators, diagnostic tests, etc., which are employed. Such arbitrary and unnecessary constraints can play a large role in determining the final equations selected and a further major objective of this paper is to illustrate the advantages of using a wide range of different techniques (including both "econometric" and "time-series" methods) when analysing aggregate economic data.

We believe that considerable insight can be achieved by trying to explain the interrelationships between the consumption function studies of Hendry (1974), Ball *et al.* (1975) and Wall *et al.* (1975). Our analysis proceeds by noting seven potential explanations for the main differences between these three studies, namely the choice of (i) data series, (ii) methods of seasonal adjustment, (iii) other data transformations, (iv) functional forms, (v) lag structures, (vi) diagnostic statistics and (vii) estimation methods. It proves possible to "standardise" the models on a common basis for (i)–(iv) such that the major differences between the studies persist. This allows us to nest the standardised contending theories as special cases of a general hypothesis and test to see which (if any) are acceptable on statistical grounds. Such an approach leads to the selection on *statistical criteria* of the equation which we consider to be the least reasonable of the three on the basis of *economic theory* considerations. To account for this outcome we investigate the role of measurement errors in the data, but draw a blank. Next, we develop an econometric relationship (which was originally obtained as an empirical description of the data series) and show that it satisfies our desired theory criteria, fits as well as the previously best fitting equation and includes the rejected models as special cases. Moreover, this relationship is such that if it were the true model, then it is reasonably easy to see in retrospect why the alternative research methods led to their various conclusions. Finally, we conduct a variety of tests on a modified version of our chosen model and show that it adequately accounts for the atypical consumption behaviour observed over the period 1971–5.

The data and the three econometric studies are described in Sections II and III respectively. Sections IV and V investigate the standardisation aspects and multicollinearity respectively and in Section VI we consider the selection of the equation which performs "best" on statistical criteria. Section VII discusses the effects of certain of the data transformations on measurement errors. In Section VIII we propose a possible explanation for all the previous results through a serious, but hard to detect, dynamic mis-specification, and conditional on this interpretation, re-evaluate the role of (v)–(vii) above. Inflation effects are considered in Section IX and Section X concludes the study.

It should be noted that throughout the paper we are only concerned with expenditure excluding durables. Also, we must stress that most of the modelling described below was carried out during 1974/5 using data series in 1963 prices and estimating up to the end of 1970 only. Re-estimation using an extended data set in 1970 prices was undertaken in early 1977 without re-specifying any of the earlier equations and still terminating the estimation period in 1970. The data to the end of 1975 was used for testing and the additional equations based on Deaton (1977) were included at this stage.

II. THE DATA

Let Y_t denote personal disposable income, Cd_t consumers' expenditure on durable goods, S_t personal saving and C_t consumers' expenditure on all other goods and services, all variables being in constant prices. The main series used in this study are taken from *Economic Trends* (1976 Annual Supplement) and are quarterly, seasonally unadjusted in £ million at 1970 prices. Although C_t and Cd_t are separately deflated, the series are such that $Y_t = C_t + Cd_t + S_t$. Fig. 1 shows the time series of Y_t and C_t for the period 1958 (i) to 1976 (ii) (the data for 1957 were used to create variables like $C_t - C_{t-4}$).

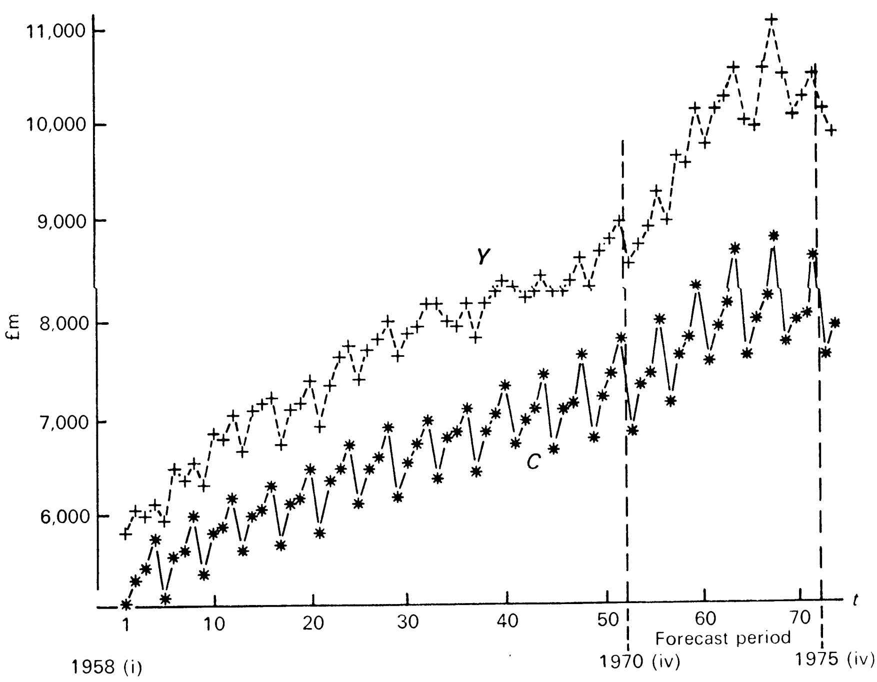

Fig. 1. Time paths of personal disposable income (Y) and consumers' expenditure (C).

The salient features of the data are the strong trends in both C_t and Y_t, the magnitude and stability of the seasonal pattern in C_t compared to that of Y_t (although the seasonal shape has tended to become increasingly "elongated" over time), the regularity of the "output" series C_t compared to the "input" series Y_t, and the marked change in the behaviour of the Y_t series after 1972. Detailed scrutiny reveals the presence of "business cycles" which are more clearly seen in the transformed series $\Delta_4 Y_t = Y_t - Y_{t-4}$ and $\Delta_4 C_t$ graphed in Fig. 2 (Δ_4 is referred to below as the four period or annual difference as compared with the fourth difference Δ_1^4). Fig. 2 also confirms the greater variance of the income series, and casual inspection suggests that using annual differences has removed most of the seasonality in both series. As shown in Fig. 3, the average propensity to consume (C_t/Y_t denoted APC) has fallen steadily over the sample

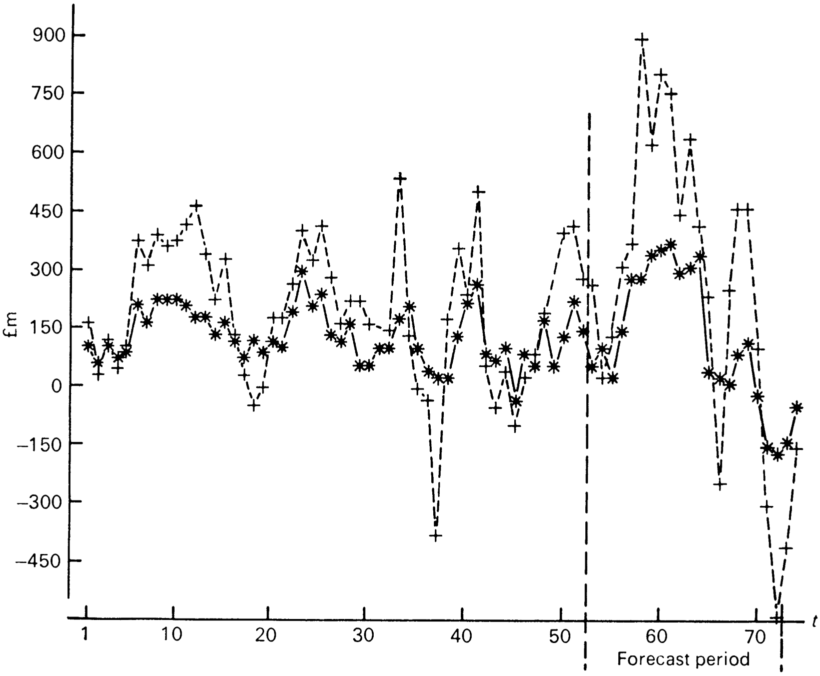

Fig. 2. Four period changes in data series. ∗——∗, $\Delta_4 C$; +——+, $\Delta_4 Y$.

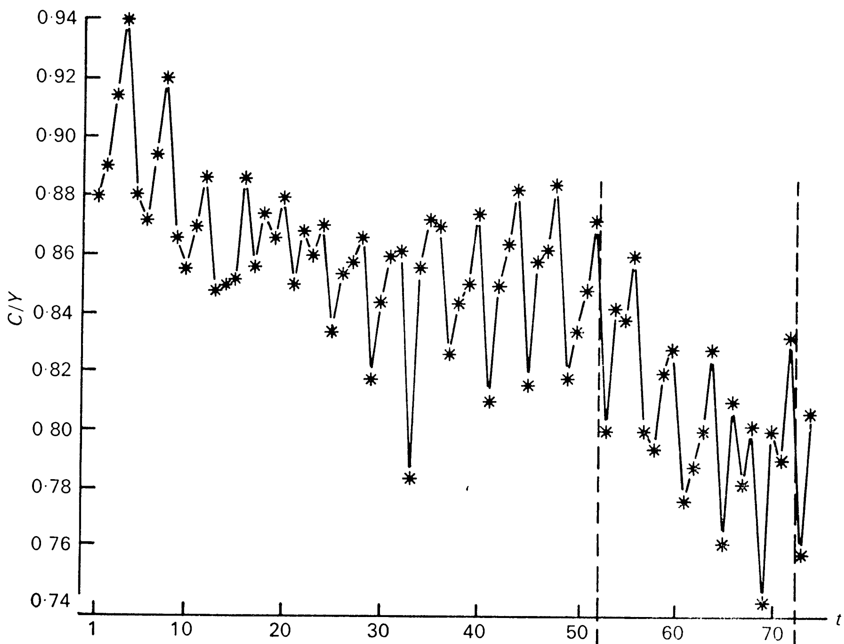

Fig. 3. The average propensity to consume.

period from around 0·9 to under 0·8, although as explained below, this evidence is still consonant with a long-run income elasticity of expenditure close to unity. If C_t is plotted against Y_t as in Fig. 4, marked differences in the average propensities to consume in the various quarters are clear. The upper and lower lines show the patterns of observations for the fourth and first quarters respectively. Finally, plotting $\Delta_4 C$ against $\Delta_4 Y$ yields a scatter diagram (see Fig. 5) in which the slope (MPC) of the "short-run" consumption function is much smaller than that of the relationship portrayed in Fig. 4, and a wide range of values of $\Delta_4 C$

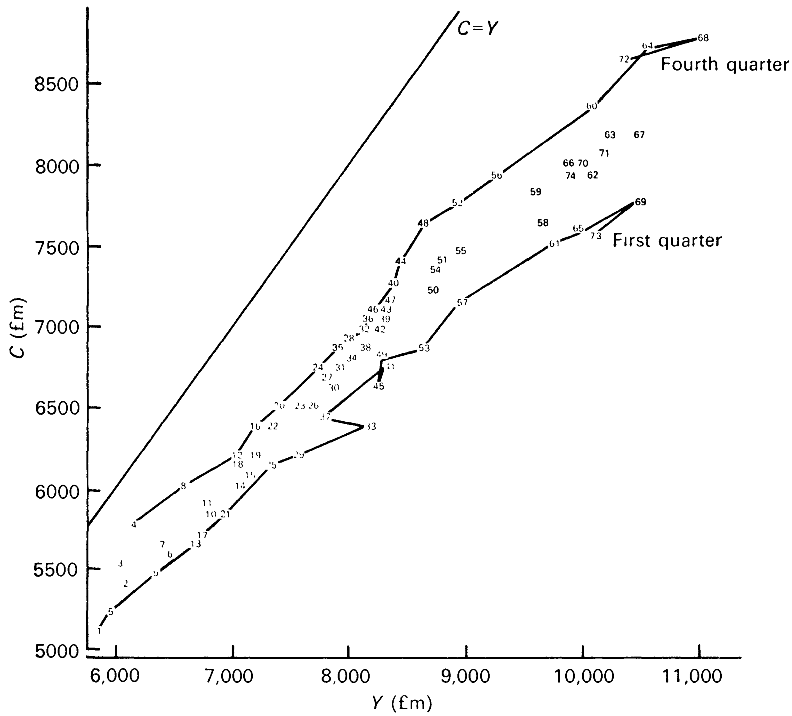

Fig. 4. Scatter diagram of personal disposable income and consumers' expenditure.

seems to be compatible with any given value for $\Delta_4 Y$. A closely similar picture emerges from the equivalent graphs of the logarithms of the data series, except that now the seasonal pattern for C_t does not appear to change over time (see Fig. 6). The correlograms for C_t, Y_t, $\Delta_4 C_t$ and $\Delta_4 Y_t$ over the period to 1970 (iv) are shown in Table 1.

From the slightly shorter data series, 1957 (i)–1967 (iv) in 1958 prices, Prothero and Wallis (1977) obtained a number of univariate time-series models for C_t and Y_t, no one of which was uniformly superior. Their most parsimonious descriptions were:

$$\Delta_1 \Delta_4 C_t = \underset{(0\cdot 14)}{(1 - 0\cdot 59 L^4)}\, \epsilon_t, \quad \hat{\sigma} = 32\cdot 0, \quad \chi^2_{15} = 6\cdot 8, \tag{1}$$

$$\Delta_1 \Delta_4 Y_t = \underset{(0\cdot 18)}{(1 - 0\cdot 58 L^4)}\, \epsilon_t, \quad \hat{\sigma} = 103\cdot 0, \quad \chi^2_{15} = 9\cdot 6. \tag{2}$$

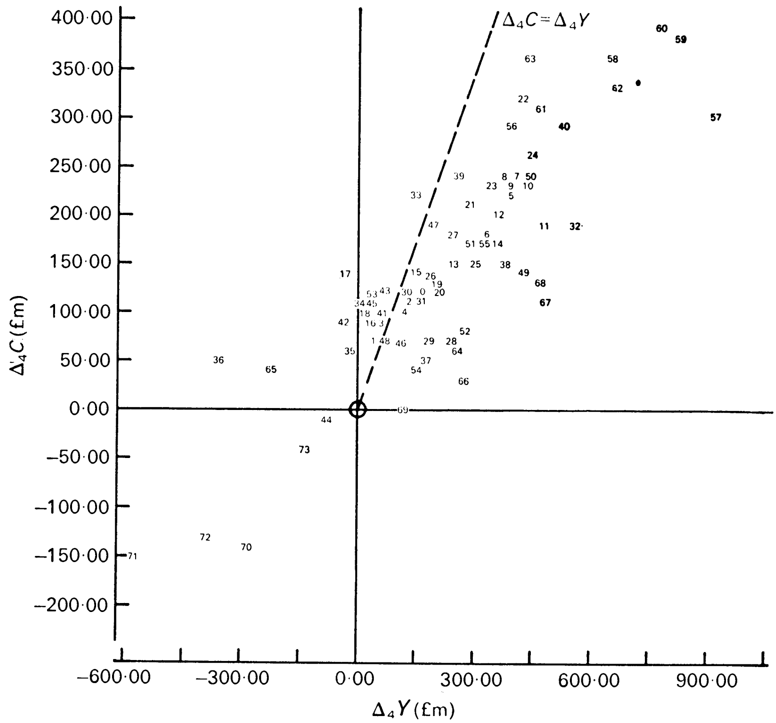

Fig. 5. Scatter diagram of four period changes.

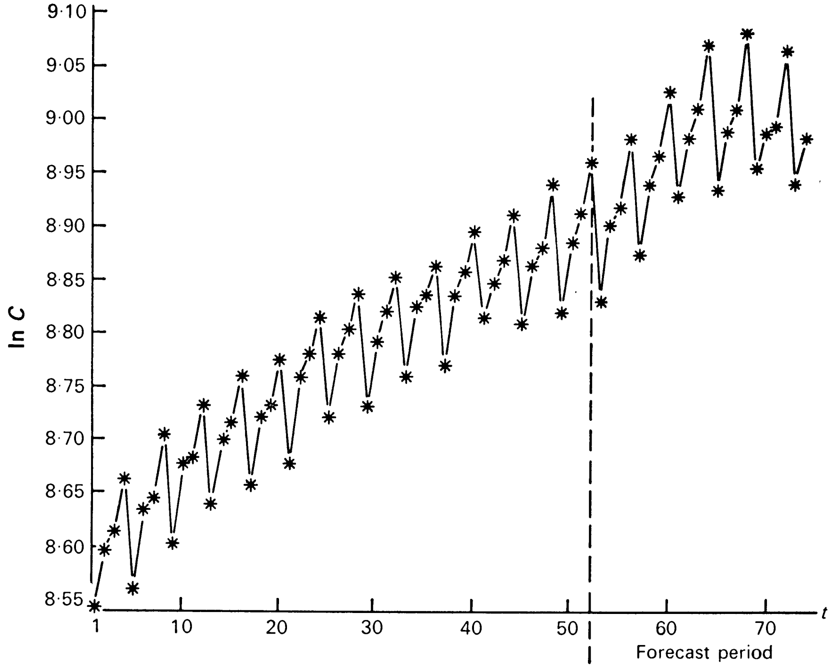

Fig. 6. Logarithms of consumers' expenditure.

In (1) and (2), L denotes the lag operator such that $L^j C_t = C_{t-j}$, ϵ_t represents a white-noise error process with estimated standard deviation $\hat{\sigma}$, and χ^2_{15} is the Box–Pierce (1970) test for a random residual correlogram. Such time-series descriptions show C_t and Y_t to obey similar equations but with the variance of the random component of Y_t nearly ten times as large as that of C_t.

Table 1

Correlograms for C_t, Y_t, $\Delta_4 C_t$ and $\Delta_4 Y_t$

Lag...	1	2	3	4	5	6	7	8
C	0·79	0·80	0·75	0·99	0·76	0·76	0·72	0·99
Y	0·95	0·95	0·93	0·97	0·93	0·93	0·91	0·96
$\Delta_4 C$	0·49	0·24	−0·02	−0·28	−0·23	−0·24	−0·02	−0·07
$\Delta_4 Y$	0·50	0·24	0·04	−0·33	−0·17	−0·16	−0·16	−0·02

Lag...	9	10	11	12	13	14	15	16
C	0·72	0·73	0·67	0·99	0·68	0·68	0·60	0·98
Y	0·92	0·92	0·89	0·96	0·91	0·90	0·86	0·95
$\Delta_4 C$	−0·05	0·09	0·02	0·04	0·02	0·06	0·20	0·35
$\Delta_4 Y$	−0·03	0·01	−0·02	−0·03	−0·04	−0·08	0·04	0·18

III. THREE ECONOMETRIC STUDIES AND THEIR RESEARCH METHODS

Since the main objective of this study is to explain why a large number of econometric descriptions of the data have been offered, there seems no need for a long section dealing with relevant economic theories. However, we do wish to stress that most theories of the consumption function were formulated to reconcile the low short-run *MPC* with the relative stability claimed for the *APC* over medium to long data periods (see *inter alia* Duesenberry, 1949; Brown, 1952; Friedman, 1957; Ando and Modigliani, 1963). Broadly speaking, all of these theories postulate lag mechanisms which mediate the response of C_t to changes in Y_t (e.g. previous highest C_t, C_{t-1}, "permanent income" and wealth respectively). Thus, the Permanent Income Hypothesis (*PIH*) assumes that:

$$C_t = \theta Y_{pt} + u_t, \tag{3}$$

where u_t is independent of Y_{pt} and has finite variance, and where Y_{pt} is "permanent income". Friedman (1957) approximated Y_{pt} using

$$(1 - \lambda L)\, Y_{pt} = (1 - \lambda)\, Y_t \tag{4}$$

to obtain

$$C_t = \theta(1 - \lambda)\,(1 - \lambda L)^{-1} Y_t + u_t, \tag{5}$$

while Sargent (1977) interprets this as a rational expectations formulation when Y_t is generated by

$$\Delta_1 Y_t = a + (1 - \lambda L)\,\epsilon_t, \tag{6}$$

which would add an intercept to (5). Since Y_t is assumed exogenous, (3) and (6) ensure that C_t and Y_t will have similar time-series properties, and as Sargent (1977) shows,

$$\begin{aligned}\Delta_1 C_t &= \theta(1 - \lambda)\,\epsilon_t + \theta a + u_t - u_{t-1} \\ &= \theta a + (1 - \lambda^* L)\,\epsilon_t^*, \end{aligned} \tag{7}$$

where ϵ_t^* is white noise and λ^* depends on θ, λ, σ_u^2 and σ_ϵ^2. Since $\sigma_{\epsilon*}^2 > \sigma_u^2$, it is of course more efficient to analyse (5) than (7).

At the aggregate level, a steady-state form of the Life Cycle Hypothesis (*LCH*) is exposited by Modigliani (1975) as:

$$C_t = \alpha Y_t + (\delta - r) A_t, \tag{8}$$

where A_t is end period private wealth and r (the rate of return on assets), α and δ are constant. Out of steady-state, α and δ (like θ in (3)) vary with a number of factors including the rate of interest and the expected growth of productivity. If capital gains and interest are included in income, then from (8) and the identity

$$A_t = A_{t-1} + Y_{t-1} - C_{t-1} \tag{9}$$

we can obtain

$$\Delta_1 C_t = \alpha \Delta_1 Y_t + (\delta - r)(Y_{t-1} - C_{t-1})$$

or

$$C_t = \alpha Y_t + (\delta - r - \alpha) Y_{t-1} + (1 - \delta + r) C_{t-1}, \tag{10}$$

which again produces a distributed lag model of C_t on Y_t.[1]

It is noticeable that neither the *PIH* nor the *LCH* is much concerned with seasonal patterns of expenditure and models based on such theories are often estimated from annual or from seasonally adjusted quarterly data series.

Against this background we can consider the three econometric studies.

(*H*) Hendry (1974) estimated several equations of the form

$$C_t = a_0 + a_1 Y_t + a_2 C_{t-1} + \sum_{j=1}^{3} b_j Q_{jt} + \sum_{j=1}^{4} d_j Q_{jt} t + \epsilon_t \quad (t = 1, \ldots, T), \tag{11}$$

where Q_{jt} denotes a dummy variable for the *j*th quarter. He imposed various restrictions on the parameters, used a number of different estimators and considered various autocorrelation structures for ϵ_t. For example, assuming a constant seasonal pattern ($d_j = 0$, $j = 1, \ldots, 4$), no autocorrelation and using raw data, 1957(i)–1967(iv) in 1958 prices, least-squares estimation yielded (see *H*, table 1):

$$\underset{(76)}{\hat{C}_t = 377} + \underset{(0{\cdot}06)}{0{\cdot}10 Y_t} + \underset{(0{\cdot}09)}{0{\cdot}84 C_{t-1}} + \hat{S}_t, \tag{12}$$

$$R^2 = 0{\cdot}994, \quad \hat{\sigma} = 30{\cdot}4, \quad \text{dw} = 2{\cdot}6.$$

In (12) the numbers in parentheses are standard errors,

$$\hat{S}_t = \sum_{j=1}^{3} \hat{b}_j Q_{jt},$$

R^2 is the squared coefficient of multiple correlation, $\hat{\sigma}$ is the standard deviation of the residuals and dw is the Durbin–Watson statistic. Then testing (12) for (i) fourth-order autoregressive residuals, (ii) omitted four period lagged values of C_t, Y_t and C_{t-1} and (iii) an evolving seasonal pattern, *H* found that each of these three factors was present if allowed for *separately*. When included in com-

[1] The derivation of (10) is less convincing if a white noise error is included in (8), since the error on (10) would be a first-order moving average with a root of minus unity, reflecting the inappropriateness of differencing (8).

binations, however, they appeared to act as substitutes since only the last remained significant in the three sets of pairwise comparisons.

Consequently, H selected (11) (with ϵ_t assumed serially independent) as the best description of his data and obtained the following least squares estimates (instrumental variables estimates were similar):

$$\underset{}{\hat{C}_t} = \underset{(369)}{1994} + \underset{(0{\cdot}04)}{0{\cdot}19}Y_t + \underset{(0{\cdot}14)}{0{\cdot}22}C_{t-1} + \hat{S}_t + \sum_{j=1}^{4} \hat{d}_j Q_{jt} t, \tag{13}$$

$$R^2 = 0{\cdot}998, \quad \hat{\sigma} = 17{\cdot}6, \quad \text{dw} = 2{\cdot}2.$$

When selecting this outcome, all other potential mis-specifications were apparently deliberately ignored by H to highlight the problems of stochastic specification. Even granting this escape from sins of omission, there are several important drawbacks to econometric formulations like (11)–(13), and many of the following criticisms apply to other published regression equations. First, the assumed seasonal pattern is *ad hoc* and would yield meaningless results if extrapolated much beyond the sample period. Moreover, one of the more interesting aspects of the data (the regular seasonal pattern of C_t) is attributed to unexplained factors, where by contrast a model like

$$C_t = \sum_{i=1}^{4} \alpha_i Q_{it} Y_t$$

would at least correspond to the possible behavioural hypothesis of a different MPC in each quarter of the year. Secondly, since the derived mean lag and long-run (static equilibrium) marginal propensity to consume coefficients in (12) are given by $\hat{a}_2/(1-\hat{a}_2)$ and $\hat{a}_1/(1-\hat{a}_2)$ respectively, these can be altered considerably by minor changes in $\hat{a}_2$ when that coefficient is close to unity. In turn, $\hat{a}_2$ can vary markedly with different treatments of residual autocorrelation. Next, both the short-run and long-run MPC's are very small in (13) (yielding a long-run elasticity of about 0·2) and are radically different from the corresponding estimates in (12) although only the treatment of seasonality has changed. In part this is due to the inclusion of a trend in (13), but this is hardly an explanation and simply prompts the question as to why the trend is significant when one believes that the economic variables are actually determining the behaviour of C_t (most of the very close fit of (13) is due to the trend and seasonals). Finally, it is difficult to evaluate the plausibility of the results *as presented* in (12) and (13). For example, R^2 is unhelpful since the data are trending (see Granger and Newbold, 1974). Also, dw has both low power against high-order residual autocorrelation in static equations and an incorrect conventional significance level in dynamic equations[1] (see Durbin, 1970). No forecast or parameter stability tests are presented and the appropriateness of least squares is not obvious (although Hendry, 1974, did in fact publish forecast tests, other diagnostic checks, and used less restrictive estimators).

Overall, with or without evolving seasonals, (11) does not seem to be a useful

[1] dw is quoted below as a conventional statistic (from which, for example, Durbin's h test could be calculated if desired).

specification for studying consumption–income responses, however well it may happen to describe the data for a short time period.

(*B*) Ball *et al.* (1975) present an equation rather like (12), but based on seasonally adjusted data (abbreviated to *SA* below and denoted by a superscript *a*) for the period 1959 (ii)–1970 (iv), estimated by least squares:

$$\widehat{(C-G)}_t^a = \underset{(65)}{185} + \underset{(0.07)}{0.23}(Y-G)_t^a + \underset{(0.09)}{0.69}(C-G)_{t-1}^a + (\hat{\phi}_1 D_t + \hat{\phi}_2 D_{t-1}), \quad (14)$$

$$\bar{R}^2 = 0.99, \quad \hat{\sigma} = 29.3, \quad \text{dw} = 2.3,$$

D_t represents a dummy variable with zero values everywhere except for 1968 (ii) when it is unity (1968 (i) and (ii) are anomalous quarters owing to advance warning in the first quarter of 1968 of possible purchase tax increases in the second quarter; these duly materialised, and considerable switching of expenditure between these quarters resulted). G_t denotes direct transfer payments to individuals, and as specified in (14), G_t is immediately and completely spent. $\bar{R}^2$ is the adjusted value of R^2.

Many of the criticisms noted in *H* apply to (14) and in addition, the use of *SA* data must be considered. Seasonal adjustment methods can be interpreted as filters applied to time series to remove frequency components at or near seasonal frequencies, such filters often being many-period weighted moving averages (e.g. 24 periods for quarterly data in the commonly used Bureau of Census Method II version X–11 program). In published statistics, single series tend to be separately adjusted. However, as documented by Wallis (1974), *separate* adjustment of series can distort the relationship between pairs of series and in particular can alter the underlying dynamic reactions. Thus, Wallis records a case of *four* period lags being incorrectly identified as *one* period after *SA* and the possibility of such a dynamic mis-specification applying to (14) merits investigation since earlier variants of (14) in the London Business School model based on unadjusted data used C_{t-4} as a regressor.

Note that the estimates in (14) again seem to imply a long-run (static equilibrium) elasticity of less than unity ($MPC = 0.74$, $APC = 0.84$) which is consistent with Fig. 3 and reasonably similar to (12) despite the very different treatment of seasonality.

(*W*) Wall *et al.* (1975) analyse total consumers' expenditure $C_t^* = Cd_t^a + C_t^a$ using *SA* data for 1955 (i)–1971 (iv) with estimation based on the transfer function methodology proposed by Box and Jenkins (1970). Their published model is

$$\hat{\dot{C}}_t^* = 0.21 + \underset{(0.08)}{0.31}\dot{Y}_t^a + \underset{(0.08)}{0.24}\dot{Y}_{t-1}^a \quad \text{where} \quad \dot{x}_t = 100(x_t - x_{t-1})/x_{t-1}, \quad (15)$$

$$R^2 = 0.56, \quad \hat{\sigma} = 0.74.$$

The relative first difference transformation was adopted to make the variables "stationary". Given their advocacy of Box–Jenkins methods, we assume that *W* estimated (15) with the residuals treated as "white noise" because they had found no evidence of residual autocorrelation.

However, (15) has *no static equilibrium* solution and is only consistent with a steady-state growth rate of about 2% pa. Indeed, the *ad hoc* mean correction of 0·21 is 35% of the mean of $\dot{C}^*$ and as a consequence, the conventionally calculated long-run elasticity is 0·55. Moreover, (15) implies that any adjustment to income changes is completed within six months and is independent of any disequilibrium between the *levels* of C_t and Y_t. Also, the use of $C_t^* = C_t^a + Cd_t^a$ may entail some aggregation bias in view of the extra variables usually included in models of durables purchases (see, for example, Williams, 1972, and Garganas, 1975). However, it is not surprising that (15) results from estimation based on data like that in Fig. 5.

IV. A STANDARDISED FRAMEWORK

The studies listed as H, B and W above satisfy the requirement that approximately the same data set (C, Y) is involved in all three cases. Nevertheless, the *results* differ in many respects and are conditioned by very different auxiliary hypotheses. Indeed, the first problem is to find enough common elements to allow direct comparisons to be made! Our approach is to re-estimate close equivalents of (13)–(15) in a standard framework which tries to isolate which factors do, and which do not, induce differences in the results. We begin by examining the roles of the data period, seasonal adjustment procedures, data transformations and functional forms since it might be anticipated that small alterations to these should not greatly change the findings of any particular study.

We chose the data series graphed in Fig. 1, with the 20 observations for 1971 (i)–1975 (iv) being used purely for forecast tests.[1] The choice of data period did not seem to be too important, and we preferred raw to SA data for the reasons noted in the discussion of H and B, namely "Wallis' effects" and our desire to "model" the seasonal behaviour of C rather than filter it out. Sims (1974) has pointed out that the use of raw data involves a potential risk of "omitted seasonals" bias if there is also seasonal noise in the error on the equation of relevance; however, the estimates recorded below do not suggest the presence of such a problem.

A major assumption which we made in order to develop analogues of the various equations is that the closest equivalent of a transformation of the form $\Delta_1 Z_t^a = Z_t^a - Z_{t-1}^a$ (in SA data) is $\Delta_4 Z_t = Z_t - Z_{t-4}$ (in raw data), since both transformed variables represent changes net of seasonal factors (we have also estimated most of the analogue equations using SA data and report below on the negligible changes this induces). The *converse* equivalence is not valid, however, since dynamics should be unaltered when a linear filter is *correctly* applied to a relationship (see Wallis, 1974; Hendry and Mizon, 1978). Also,

[1] We ignored the two observations for 1976 as being liable to considerably larger revisions than the earlier data.

we assumed that $\Delta_1 \ln x_t \simeq \Delta_1 x_t/x_{t-1}$ (where ln denotes $\log_e$) and so could approximate W's variable $\dot{C}_t^*$ in SA data by $\Delta_4 \ln (C_t + Cd_t)$ in raw data.

Finally, models in differences can be related to those in levels, by noting that there are two distinct interpretations of "differencing", a point most easily demonstrated by the following relationship:[1]

$$x_t = \beta_1 w_t + \beta_2 w_{t-4} + \beta_3 x_{t-4} + \sum_{j=1}^{4} (\theta_j + \mu_j t) Q_{jt} + v_t, \tag{16}$$

where w_t is an exogenous regressor generated by a stationary stochastic process and v_t is a stationary error.

(a) Differencing as a Filter or Operator

Applying the operation Δ_4 to equation (16) yields

$$\Delta_4 x_t = \beta_1 \Delta_4 w_t + \beta_2 \Delta_4 w_{t-4} + \beta_3 \Delta_4 x_{t-4} + \sum_{j=1}^{4} 4\mu_j Q_{jt} + \Delta_4 v_t \tag{17}$$

and hence the features which are altered comprise:

– the elimination of trends and the re-interpretation of the constant seasonal pattern in (17) as corresponding with the evolving pattern in (16);

– the autocorrelation properties of v_t (e.g. if $v_t = v_{t-4} + \nu_t$, where ν_t is white noise, then $\Delta_4 v_t$ is white noise, whereas if $v_t = \nu_t$, $\Delta_4 v_t$ is a four period simple moving-average with a coefficient of -1);

– the form of all the non-dummy variables in (16) (which reoccur as annual differences in (17)).

(b) Differencing as a Set of Coefficient Restrictions

Applying the restrictions $\beta_1 = -\beta_2$ and $\beta_3 = 1$ to (16) yields:

$$\Delta_4 x_t = \beta_1 \Delta_4 w_t + \sum_{j=1}^{4} (\theta_j + \mu_j t) Q_{jt} + v_t. \tag{18}$$

Now, the changes from (16) are the elimination of w_{t-4} and x_{t-4} as independent regressors and the occurrence of the transformed variables $\Delta_4 x_t$ and $\Delta_4 w_t$. The interpretation of the constant and the seasonal pattern is unaltered and when the restrictions are valid the autocorrelation properties of v_t are unaffected – if v_t in (16) is white noise then so is v_t in (18) (contrast the arguments presented in Granger and Newbold, 1974). One immediate and obvious application of interpretation (*b*) is the converse step of deriving (16) from (18) which allows valid comparisons of models involving differenced variables with those using level variables.

Combining all of the above approximations, we move from (15) via the equivalence of $\dot{C}_t^a$ with $\Delta_4 \ln C_t$ to:

$$\begin{aligned} \Delta_4 \ln C_t &= \delta_0 + \delta_1 \Delta_4 \ln Y_t + \delta_2 \Delta_4 \ln Y_{t-1} + e_t \\ &= \delta_0 + (\delta_1 + \delta_2) \Delta_4 \ln Y_t - \delta_2 \Delta_1 \Delta_4 \ln Y_t + e_t, \end{aligned} \tag{19}$$

[1] Equations involving variables denoted by x_t and w_t are used to illustrate simplified versions of principles which can be generalised validly to the relationship between C and Y.

which provides our version of (15). For comparison with equations in log-levels we use the unrestricted version of (19), namely

$$\ln C_t = \lambda_1 \ln Y_t + \lambda_2 \ln Y_{t-4} + \lambda_3 \ln Y_{t-1} + \lambda_4 \ln Y_{t-5} + \lambda_5 \ln C_{t-4} + \lambda_6 + e_t, \tag{20}$$

where (19) corresponds to imposing $\lambda_1 = -\lambda_2$, $\lambda_3 = -\lambda_4$ and $\lambda_5 = 1$ in (20). Since no seasonal dummy variables are introduced in (20) this procedure requires that Δ_4 in (19) removes any seasonal factors; some support for such a proposition is provided in Fig. 2 but this issue will be reconsidered below. We assumed that it was reasonable to use C_t in place of C_t^* in developing (19) for reasons presented above. Throughout, we have estimated most specifications in both linear and log-linear forms, comparing these where necessary using the likelihood criterion proposed by Sargan (1964). Thus (19) can be compared through (20) and the linear-log mapping with whatever equivalents are chosen for (13) and (14) (we chose to use C_t rather than $(C_t - G_t)$ when approximating (14) to maintain closer comparison with both (15) and (11)).

The main justification for adopting the above approximations is simply that the important features of and differences between the results in (13)–(15) survive the standardisation sequence. Firstly, to illustrate the effects of changing the sample period to 1958(ii)–1970(iv) and using 1970 prices, re-estimation of (13) by least-squares yields:

$$\begin{aligned} \hat{C}_t = {} & \underset{(537)}{2556} + \underset{(0{\cdot}05)}{0{\cdot}20}Y_t + \underset{(0{\cdot}12)}{0{\cdot}35}C_{t-1} + \underset{(33)}{83}D_t^0 - \underset{(60)}{514}Q_{1t} - \underset{(57)}{74}Q_{2t} \\ & - \underset{(38)}{187}Q_{3t} + \underset{(3{\cdot}7)}{16{\cdot}3}t - \underset{(1{\cdot}3)}{6{\cdot}7}Q_{1t}t - \underset{(1{\cdot}2)}{2{\cdot}2}Q_{2t}t - \underset{(1{\cdot}2)}{0{\cdot}3}Q_{3t}t, \end{aligned} \tag{21}$$

$$R^2 = 0{\cdot}996, \quad \hat{\sigma} = 39{\cdot}4, \quad \text{dw} = 2{\cdot}2, \quad z_1(20) = 130, \quad z_2(15) = 25.$$

In (21), $D^0 = (D_{t-1} - D_t)$, which assumes that the 1968(i) announcement caused a switch in expenditure between quarters.[1] $z_1(k)$ is a test of parameter stability or one period ahead forecast accuracy using the actual future values of the regressors for the next k quarters. Letting f_t denote the forecast error, then

$$z_1(k) = \sum_{t=T+1}^{T+k} (f_t/\hat{\sigma})^2,$$

which would be distributed as χ_k^2 in large samples if the parameters in (21) remained constant.[2] $z_2(l)$ is the Pierce (1971) residual correlogram statistic, distributed as χ_l^2 in large samples when the residuals are serially independent.[3]

[1] The introduction of VAT in 1973 was treated as being similar to the 1968(i)/(ii) budget effect and hence we projected D_t^0 as $+1$, -1 in 1973(i)/(ii). This improved the forecast accuracy in these quarters and demonstrated the value of investigating "special effects".

[2] A significant value for z_1 indicates *both* an incorrect model *and* a change in the stochastic properties of the variables in the "true" data generation process of C_t, whereas an insignificant value for z_1 only shows that the latter has not occurred and is fully consistent with an incorrect model for C_t (see Hendry, 1977*a*). Note that a large value of z_1 occurs when the variance of the forecast errors is large relative to the variance of the sample residuals.

[3] The results of Hendry (1977*b*) and Davis *et al.* (1977) suggest that "small" values of z_2 should be treated with caution, and do not necessarily indicate the absence of residual autocorrelation.

The only noticeable differences between (13) and (21) are the change in $\hat{\sigma}$ due to the change in the base of the implicit deflator for C, and the increase in the coefficient of C_{t-1}. However, z_2 indicates the presence of significant autocorrelation (actually, of fourth order) and z_1 strongly rejects parameter stability (when comparing equations, it should be noted that z_1 is *not* a measure of *absolute* forecast accuracy).

Next, we estimated two analogues of (14) from the same sample, namely

$$\underset{}{\hat{C}_t} = \underset{(142)}{509} + \underset{(0\cdot08)}{0\cdot18}Y_t + \underset{(0\cdot11)}{0\cdot75}C_{t-1} + \underset{(49)}{68}D_t^0 - \underset{(63)}{812}Q_{1t} + \underset{(46)}{32}Q_{2t} - \underset{(26)}{169}Q_{3t}, \tag{22}$$

$$R^2 = 0\cdot990, \quad \hat{\sigma} = 62\cdot0, \quad \text{dw} = 2\cdot1, \quad z_1(20) = 190, \quad z_2(15) = 86,$$

which is, not surprisingly, also similar to (12); and

$$\hat{C}_t = \underset{(112)}{734} + \underset{(0\cdot06)}{0\cdot28}Y_t - \underset{(0\cdot06)}{0\cdot09}\Delta_1 Y_t + \underset{(0\cdot08)}{0\cdot59}C_{t-4} + \underset{(44)}{71}D_t^0 - \underset{(58)}{251}Q_{1t} - \underset{(27)}{92}Q_{2t} - \underset{(27)}{81}Q_{3t} \tag{23}$$

$$R^2 = 0\cdot993, \quad \hat{\sigma} = 53\cdot2, \quad \text{dw} = 1\cdot3, \quad z_1(20) = 129, \quad z_2(12) = 21,$$

which is reasonably similar to (14). ($\Delta_1 Y_t$ was included in (23) to allow for a one-lag income effect but dw still indicates considerable first-order residual autocorrelation.) Lastly, for (15):

$$\Delta_4 \hat{C}_t = \underset{(10\cdot1)}{78\cdot5} + \underset{(0\cdot04)}{0\cdot34}\Delta_4 Y_t - \underset{(0\cdot04)}{0\cdot14}\Delta_1 \Delta_4 Y_t + \underset{(22)}{61}\Delta_4 D_t^0, \tag{24}$$

$$R^2 = 0\cdot70, \quad \hat{\sigma} = 40\cdot5, \quad \text{dw} = 1\cdot7, \quad z_1(20) = 94\cdot0, \quad z_2(16) = 20.$$

We have quoted (24) in linear (rather than log) form for immediate comparison with (21)–(23) but despite this change of functional form, (24) reproduces the main features of (15) (a long-run elasticity of about 0·5, a large and significant intercept and "white noise" errors). All of the re-estimated analogues are rejected by the forecast test, although this does not affect our ability to choose between them; an explanation for the overall poor forecasts is provided in Section IX. Equation (22) also exhibits very marked four period autocorrelation and re-estimation assuming an error of the form $u_t = \rho_4 u_{t-4} + \epsilon_t$, yielded $\hat{\rho}_4 = 0\cdot98(0\cdot03)$, $z_2(12) = 22$ and $\hat{\sigma} = 42\cdot9$. Consequently, all of the models estimated from the raw data require some allowance for four-period effects.

A similar story emerges if *SA* data are used and seasonal dummies are omitted from all of the models. Specifically for (22), (23) and (24) we obtained (for the rather different data period 1963 (i)–1973 (i), forecasting 1973 (ii)–1975 (ii)):

$$\hat{C}_t^a = \underset{(154)}{250} + \underset{(0\cdot04)}{0\cdot18}Y_t^a + \underset{(0\cdot06)}{0\cdot75}C_{t-1}^a + \underset{(29)}{101}D_t^0, \tag{22a}$$

$$R^2 = 0\cdot990, \quad \hat{\sigma} = 39\cdot3, \quad \text{dw} = 2\cdot5, \quad z_1(8) = 35\cdot5, \quad z_2(9) = 8\cdot4;$$

$$\hat{C}_t^a = \underset{(145)}{567} + \underset{(0\cdot03)}{0\cdot41}Y_t^a - \underset{(0\cdot05)}{0\cdot20}\Delta_1 Y_t^a + \underset{(0\cdot05)}{0\cdot44}C_{t-4}^a + \underset{(30)}{81}D_t^0, \tag{23a}$$

$$R^2 = 0\cdot990, \quad \hat{\sigma} = 41\cdot2, \quad \text{dw} = 1\cdot5, \quad z_1(8) = 138, \quad z_2(6) = 6\cdot3;$$

$$\Delta_4 \hat{C}_t^a = \underset{(11)}{66} + \underset{(0\cdot03)}{0\cdot37}\Delta_4 Y_t^a - \underset{(0\cdot04)}{0\cdot16}\Delta_1 \Delta_4 Y_t^a + \underset{(23)}{86}\Delta_4 D_t^0, \qquad (24a)$$

$$R^2 = 0\cdot80, \quad \hat{\sigma} = 43\cdot9, \quad \mathrm{dw} = 1\cdot8, \quad z_1(8) = 26\cdot2, \quad z_2(10) = 20;$$

$$\Delta_1 \hat{C}_t^a = \underset{(8\cdot6)}{21} + \underset{(0\cdot08)}{0\cdot31}\Delta_1 Y_t^a - \underset{(0\cdot05)}{0\cdot11}\Delta_1^2 Y_t^a + \underset{(19)}{69}\Delta_1 D_t^0, \qquad (15a)$$

$$R^2 = 0\cdot52, \quad \hat{\sigma} = 44\cdot5, \quad \mathrm{dw} = 2\cdot9, \quad z_1(8) = 7\cdot3, \quad z_2(10) = 24.$$

These results support our contention that the choices of the exact data period and of the seasonal adjustment procedures do not markedly affect the *estimates* obtained, although it should be noted that the goodness of fit ranking of the models on *SA* data is the *opposite* of that prevailing with raw data. Only (24*a*) has an error variance close to its raw data counterpart, and hence the *selection* of equations is greatly altered by *SA*.

Thus (i)–(iv) can be eliminated as the *main* factors accounting for the differences in (13)–(15) and we can proceed to consider (v)–(vii) which represent more important differences in methodology. At this stage, our standardised analogues of (13)–(15) can be nested as special cases of the model

$$\begin{aligned} C_t = {} & \xi_0 + \xi_1 Y_t + \xi_2 Y_{t-4} + \xi_3 \Delta_1 Y_t + \xi_4 \Delta_1 Y_{t-4} + \xi_5 C_{t-4} \\ & + \sum_{j=1}^{3} \xi_{5+j} Q_{jt} + \xi_9 D_t^0 + \xi_{10} D_{t-4}^0 + \xi_{11} C_{t-1} + \xi_{12} t \\ & + \sum_{j=1}^{3} \xi_{12+j} Q_{jt} t + \epsilon_t, \end{aligned} \qquad (25)$$

and hence (21)–(24) can all be tested directly against the estimated version of (25).

V. ON MULTICOLLINEARITY

Can sensible estimates of (25) be obtained given the general misapprehension that "severe collinearity problems are bound to be present"? To resolve this, consider the well-known formula (see, for example, Theil, 1971, p. 174):

$$\hat{\xi}_i^2/\widehat{\mathrm{var}(\hat{\xi}_i)} = (T-m)\, r_i^2/(1-r_i^2) \quad (i = 1, \ldots, m), \qquad (26)$$

where r_i is the partial correlation between the regressand and the ith regressor allowing for the influence of the other $(m-1)$ regressors in the equation. The left-hand side of (26) is the square of the conventionally calculated t statistic to test $H_0: \xi_i = 0$.

A crucial point is that a partial correlation like r_i can *increase* as m increases to $m+n$ even if the n added variables are highly (but not perfectly) collinear *provided they are relevant to explaining the regressand.* Thus t values can increase even though the moment matrix requiring inversion in least squares becomes "more singular" in the sense of having a smaller determinant or a smaller ratio of the least to the greatest eigenvalue (compare, for example, the analysis assuming that the true model is known in Johnston, 1972, ch. 5.7). In effect, the issue is that "collinearity problems" are likely to occur in conjunction with omitted variables problems. If the n initially excluded regressors are important in determining the regressand, then adding them may well help resolve what appears to be a collinearity problem between the m originally included variables, since "small" t values can arise from downward biases in $\hat{\xi}_i$ as well as from "large" values of $\widehat{\text{var}}(\hat{\xi}_i)$. Consequently, it is *not* universally valid to assume that a group of badly determined estimates indicates the presence of collinearity (to be solved by *reducing* the dimensionality of the parameter space) rather than omitted variables bias (solved by *increasing* the dimensionality of the parameter space). To illustrate these points consider the following estimates of a special case of (25) (which incidentally immediately demonstrates some mis-specification of (23)):

$$\underset{}{\hat{C}_t} = \underset{(627)}{2516} + \underset{(0{\cdot}06)}{0{\cdot}24 Y_t} - \underset{(0{\cdot}06)}{0{\cdot}07\Delta_1 Y_t} + \underset{(0{\cdot}12)}{0{\cdot}38 C_{t-4}} + \underset{(4)}{13t} + \underset{(40)}{65 D_t^0} + \hat{S}_t, \tag{27}$$

$$R^2 = 0{\cdot}994, \quad \hat{\sigma} = 48{\cdot}8, \quad \text{dw} = 1{\cdot}6, \quad z_1(20) = 119, \quad z_2(12) = 37.$$

Conventionally, $\Delta_1 Y_t$ is "insignificant" (but see Bock *et al.*, 1974, for an analysis of some of the consequences of using a "preliminary test" estimator in which "insignificant" regressors are excluded prior to re-estimation), and the trend coefficient is significant. Now compare (27) with the equation in which every regressor also re-occurs with a four-period lag:[1]

$$\begin{aligned}\hat{C}_t = {} & \underset{(695)}{921} + \underset{(0{\cdot}05)}{0{\cdot}31 Y_t} - \underset{(0{\cdot}05)}{0{\cdot}09\Delta_1 Y_t} + \underset{(0{\cdot}13)}{0{\cdot}65 C_{t-4}} + \underset{(5)}{4t} - \underset{(0{\cdot}06)}{0{\cdot}25 Y_{t-4}} \\ & + \underset{(0{\cdot}05)}{0{\cdot}16\Delta_1 Y_{t-4}} + \underset{(0{\cdot}11)}{0{\cdot}16 C_{t-8}} + \underset{(33)}{67 D_t^0} - \underset{(34)}{46 D_{t-4}^0} + \hat{S}_t,\end{aligned} \tag{28}$$

$$R^2 = 0{\cdot}997, \quad \hat{\sigma} = 38{\cdot}7, \quad \text{dw} = 2{\cdot}1, \quad z_1(20) = 110, \quad z_2(8) = 12.$$

Patently, despite including three more regressors, the t values for Y_t, $\Delta_1 Y_t$ and C_{t-4} are *all* considerably larger in (28) than in (27), whereas the trend coefficient has become negligible in (28), and reveals the possibility of explaining the behaviour of C_t by economic variables alone (the seasonal dummies are also insignificant in (28)).

VI. SELECTION OF THE "BEST" EQUATION

We now return to choosing between the various equations on statistical criteria. Even before estimating (25) it can be seen that (28) encompasses (23) and allows immediate rejection of the latter. Moreover, adding C_{t-1} to (28) cannot worsen the goodness of fit and so (22) can be rejected also. Testing (21) proves more of a problem since in Hendry (1974), (21) was chosen in preference to an equation similar to (28) (but excluding $\Delta_1 Y_t$, $\Delta_1 Y_{t-4}$ and C_{t-8}) whereas for the present data, (28) fits marginally better. Strictly, (28) is not nested within the (initially)

[1] Equation (28) can be derived from (27) by assuming that the residuals in (27) follow a simple fourth order autoregressive process, then carrying out the usual "Cochrane–Orcutt" transformation, but ignoring the parameter restrictions implied by the autoregressive transform.

general equation (25), although this is only due to the presence of the insignificant regressor C_{t-8} and so can be ignored. Direct estimation of (25) yields:

$$\begin{aligned}\hat{C}_t = &\underset{(692)}{1618} + \underset{(0\cdot06)}{0\cdot28}Y_t + \underset{(0\cdot15)}{0\cdot14}C_{t-1} + \underset{(0\cdot15)}{0\cdot40}C_{t-4} - \underset{(0\cdot06)}{0\cdot15}Y_{t-4} - \underset{(0\cdot06)}{0\cdot09}\Delta_1 Y_t \\ &+ \underset{(0\cdot06)}{0\cdot11}\Delta_1 Y_{t-4} + \underset{(34)}{91}D_t^0 - \underset{(34)}{11}D_{t-4}^0 + \underset{(4\cdot4)}{7\cdot6}t + \underset{(129)}{212}Q_{1t} + \underset{(77)}{180}Q_{2t} \\ &+ \underset{(104)}{278}Q_{3t} + \underset{(1\cdot87)}{2\cdot95}Q_{1t}t + \underset{(1\cdot60)}{3\cdot88}Q_{2t}t + \underset{(1\cdot52)}{4\cdot63}Q_{3t}t,\end{aligned} \tag{29}$$

$$R^2 = 0\cdot997, \quad \hat{\sigma} = 36\cdot2, \quad \text{dw} = 2\cdot0, \quad z_1(20) = 114.$$

The fit of (29) is little better than either of (21) or (28) even though many of the four lagged variables and evolving seasonals appear to be significant on t tests. Thus, given either set of variables, the additional explanatory power of the other set is small and so to a considerable extent we re-confirm their substitute roles. Relative to Hendry (1974), the four period lags are more important in the larger sample.

The most interesting outcome is that (24) cannot be rejected against (29) by testing the joint significance of all the restrictions using an F-test based on the residual sums of squares ($F(12, 31) = 1\cdot9$). Thus at the chosen significance level (using, for example, the S-method discussed by Savin, 1977) no other subset of the restrictions can be judged significant either; alternatively, individual t tests on restrictions would need to be significant at (at least) the 0·4 % level to preserve the overall size of the test at 5 % when considering 12 restrictions. On this basis, (24) seems to provide an adequate parsimonious description of the data (although other equations are also not significantly worse than (29) at the 5 % level), and it seems that the Δ_4 transform satisfactorily removes seasonality.

Moreover, if (24) were close to the correct data generation process then we would expect just the sort of result shown in (28) (the fits are similar, the lag polynomial in C_t has a root near unity, the seasonal dummies are insignificant and four period lags of income variables have roughly equal magnitudes, opposite signs to current dated equivalents). Tentatively accepting such a hypothesis, (22) and (23) would constitute poor approximations to (24) and hence are easy to reject whereas (21) is a reasonable approximation and is not easily discarded (see Figs. 1 and 4). Also, the relationship between (24) and (28) corresponds closely with the interpretation of differences as arising from coefficient restrictions but does not cohere with the "filtering" interpretation. The large change in the constant term is probably due to collinearity, since the exact unrestricted equivalent of (24) is:

$$\begin{aligned}\hat{C}_t = &\underset{(76)}{150} + \underset{(0\cdot04)}{0\cdot32}Y_t - \underset{(0\cdot04)}{0\cdot33}Y_{t-4} - \underset{(0\cdot04)}{0\cdot12}\Delta_1 Y_t + \underset{(0\cdot04)}{0\cdot16}\Delta_1 Y_{t-4} \\ &+ \underset{(0\cdot04)}{0\cdot995}C_{t-4} + \hat{\phi}_1 D_t^0 + \hat{\phi}_2 D_{t-4}^0,\end{aligned} \tag{30}$$

$$R^2 = 0\cdot996, \quad \hat{\sigma} = 40\cdot7, \quad \text{dw} = 1\cdot8, \quad z_1(20) = 82\cdot9, \quad z_2(12) = 19.$$

The coefficients of (30) correspond very closely with those of the unrestricted equation which would be anticipated if (24) validly described the expenditure relationship, although the large standard error of the intercept in (30) compared to (24) is a distinct anomaly requiring explanation in due course. In summary, the evidence points strongly to accepting (24) as the best simple description of the data despite the loss of long-run information and the theoretical drawbacks discussed in Section III.

VII. MEASUREMENT ERRORS

Zellner and Palm (1974) note that difference transformations can substantially alter the ratio of the "systematic" variance to the measurement error variance of time series. Since large measurement error variances in regressors can cause large downward biases in estimated coefficients (see for example, Johnston, 1972, ch. 9.3) it is possible that the low-income elasticities in (24) could be caused by the effects of the Δ_4 transform enhancing relative measurement errors.

A formal mis-specification analysis of a simple model where observations are generated by a first-order autoregressive process with coefficient ψ_1 and first-order autoregressive measurement error with coefficient ψ_2, reveals that $\psi_1 > \psi_2$ is a necessary and sufficient condition for differencing to induce a relative increase in measurement error variance. The amount by which the measurement error bias in the coefficient of a differenced regression exceeds the corresponding bias in the regression in levels depends directly and proportionately on $(\psi_1 - \psi_2)$. Davidson (1975) found that data revisions were highly autoregressive, and although by itself this does not imply that the unknown errors also will be autoregressive, two other factors argue for the magnitude of $(\psi_1 - \psi_2)$ being small for Y_t. First, if there were large measurement errors in $\Delta_4 Y_t$ these would occur one period later in $\Delta_4 Y_{t-1}$ which would create a negative first order moving average error on (24) and we could find no evidence of this – nor did Wall *et al.* (1975) in their similar equation (15). Secondly, we re-estimated (24) by weighted least squares assuming a measurement error variance of 50 % of the variance of $\Delta_4 Y_t$ yet there was no noticeable increase in the coefficients. All of these points together, though individually rather weak, suggest that errors-in-variables biases do *not* explain the low long-run elasticities. Conversely, any simultaneity bias which might arise from least squares estimation would tend to cause upward biased coefficients and hence can be discarded as an explanation also. Thus we return to Figs. 2 and 5 which originally indicated the source of the problem: the variance of $\Delta_4 Y_t$ is much larger than that of $\Delta_4 C_t$ and so any model like (24) must end up having "small" coefficients.

VIII. A SIMPLE DYNAMIC MODEL

In one sense, the above results simply reproduce the familiar problem of reconciling short-run and long-run consumption behaviour. However, there is a more serious difficulty since the original set of models included several distributed lag variants of permanent income and/or life-cycle theories (see Section III above) yet in a direct comparison, the statistical evidence favoured the model which accounted for only short-run behaviour. Clearly, therefore, either some new implementation of the *PIH* or *LCH* is required or (assuming that we do not wish to canvass a new theory) an account must be provided of why the evidence takes the form which it does. Naturally, we prefer the latter course.

Fisher (1962) advocated using equations involving only differenced variables to facilitate the study of short-run behaviour without having to specify trend dominated long-run components. The main defects in this strategy are that one loses almost all *a priori* information from economic theory (as most theories rely on steady-state arguments) and all long-run information in the data (yet Granger's "typical spectral shape" suggests that economic data are highly informative about the long-run: see Granger, 1966). Moreover, as noted when discussing (15), it seems inappropriate to assume that short-run behaviour is independent of disequilibria in the levels of the variables.

A simple modification of equations in differences can resolve these three problems. Consider a situation in which an investigator accepts a non-stochastic steady-state theory that $X_t = KW_t$, where K is constant on any given growth path, but may vary with the growth rate. In logs, letting $x_t = \ln X_t$, etc., the theory becomes:

$$x_t = k + w_t. \tag{31}$$

The differenced variable equivalent is:

$$\Delta_1 x_t = \Delta_1 w_t. \tag{32}$$

However, to assume that (32) had a white-noise error would deny the existence of any "long-run" relationship like (31), and to assume that (31) had a stationary error process would cause a negatively autocorrelated error to occur on (32). Furthermore, the Δ_1 operator in (32) could just as validly have been Δ_4 – on all of these points the theory is unspecific.

On the basis of (31), the investigator wishes to postulate a stochastic disequilibrium relationship between x_t and w_t, which will simplify to (31) in steady state. In the absence of a well-articulated theory of the dynamic adjustment of x_t to w_t, it seems reasonable to assume a general rational lag model of the form:

$$\alpha(L)\, x_t = k^* + \beta(L)\, w_t + v_t, \tag{33}$$

where $\alpha(L)$ and $\beta(L)$ are polynomials in the lag operator L of high enough order that v_t is white noise. For simplicity of exposition we consider the situation where both polynomials are first order:

$$x_t = k^* + \beta_1 w_t + \beta_2 w_{t-1} + \alpha_1 x_{t-1} + v_t. \tag{34}$$

Clearly, (31) and (32) are the special cases of (34) when $\beta_2 = \alpha_1 = 0$, $\beta_1 = 1$, and $\beta_1 = -\beta_2 = 1$, $k^* = 0$, $\alpha_1 = 1$ respectively; these coefficient restrictions force behaviour to be in steady state at all points in time. However, to ensure that for *all* values of the estimated parameters, *the steady-state solution of* (34) *reproduces* (31) one need only impose the coefficient restriction $\beta_1 + \beta_2 + \alpha_1 = 1$ or

$$\beta_1 = -\beta_2 + \gamma \quad \text{and} \quad \alpha_1 = 1 - \gamma$$

yielding the equation:

$$\Delta_1 x_t = k^* + \beta_1 \Delta_1 w_t + \gamma(w_{t-1} - x_{t-1}) + v_t \tag{35}$$

(which is more general than (31) or (32) but less general than (34)). The specification of (35) is, therefore, guided by the long-run theory; there is no loss of long-run information in the data since (35) is a reformulated "levels equation"; and compared with the "short-run" model:

$$\Delta_1 x_t = k^* + \beta_1 \Delta_1 w_t \tag{36}$$

the vital "initial disequilibrium" effect is provided by $\gamma(w_{t-1} - x_{t-1})$. Consequently, (35) does indeed resolve the three problems noted above (it is straightforward to generalise the analysis to equations of the form of (33)). An important example of this class of model is the real-wage variable formulation used by Sargan (1964).

In (35) consider any steady-state growth path along which

$$\Delta_1 \ln X_t = \Delta_1 x_t = g = \Delta_1 w_t,$$

then the solution of (35) with $v_t = 0$ is:

$$g = k^* + \beta_1 g + \gamma(w_{t-1} - x_{t-1}) \tag{37}$$

or assuming $\gamma \neq 0$,

$$X_t = KW_t \quad \text{where} \quad K = \exp\{[k^* - g(1 - \beta_1)]/\gamma\} \tag{38}$$

(implicitly, in (31) $k = [k^* - g(1 - \beta_1)]/\gamma$). Thus for any constant growth rate, if $\gamma \neq 0$, (35) automatically generates a long-run elasticity of unity for all values of the parameters, whereas if $\gamma = 0$, the elasticity is β_1. (Interpreting X_t as C_t and W_t as Y_t, then the derived APC ($= MPC$ in steady-state growth) is a decreasing function of the growth rate g, consonant with inter-country evidence (see Modigliani, 1975). Note that the above analysis remains valid even if $k^* = 0$ (in which case $K < 1$ for $\gamma, g > 0$ and $1 > \beta_1 > 0$), so that the theory entails no restrictions on the presence or absence of an intercept in (35).[1]

If, from the steady-state solution (38), the growth rate of W_t changes from g to g_1 the ratio of X to W will gradually change from K to

$$K_1 = \exp\{[k^* - g_1(1 - \beta_1)]/\gamma\}$$

and hence even prolonged movements in one direction of the observed X/W ratio do not rule out a long-run unit elasticity hypothesis for a *given* growth rate. If g is a variable, then $(X/W)_t$ will not be constant either, although the data will be consistent with a model like (35). The important implication of this is that a variable, or even trending, observed APC does not by itself refute a unit-elasticity model (the unit elasticity restriction is easily tested in (35) by including w_{t-1} as a separate regressor and testing its coefficient for significance from zero). Estimation of (31) requires that the *data* satisfy a unit elasticity restriction (this will be false out of steady state) whereas estimation of (35) only requires that the *model* satisfy this restriction and that the data are consonant with the model.

[1] If $k^* = 0$, then $K = 1$ when $g = 0$. Consequently, care must be exercised when simulating to *equilibrium* a model containing equations of the form of (35) for a subcategory of expenditure.

The estimation of restricted dynamic models like (35) from finite samples does not seem to have been the subject of any investigations to date. Consequently, we undertook a pilot simulation study of least-squares estimation of δ_1 and δ_2 in

$$\Delta_1 x_t = \delta_1 \Delta_1 w_t + \delta_2(w_{t-1} - x_{t-1}) + v_t \quad (t = 1, \ldots, T) \tag{35*}$$

for

$$(\delta_1, \delta_2) = (0.5, 0.1), \quad v_t \sim NI(0, 1)$$

and

$$w_t = 0.8 w_{t-1} + u_t \quad \text{with} \quad u_t \sim NI(0, 9), \quad \text{independent of } v_t.$$

The results are shown in Table 2 for 100 random replications. For $T \geqslant 34$, the biases are very small, S.E. provides an accurate estimate of S.D. and $H_0: \delta_i = 0$

Table 2

*Simulation Findings for (35)**

	$(\hat{\delta}_1 - \delta_1)$				$(\hat{\delta}_2 - \delta_2)$			
T	14	34	54	74	14	34	54	74
Bias	0.010	−0.013	−0.003	0.003	−0.026	−0.009	−0.004	−0.007
S.D.	0.106	0.062	0.041	0.044	0.108	0.060	0.043	0.038
S.E.	0.090	0.058	0.044	0.038	0.105	0.060	0.045	0.039
H_0 rejected	99	100	100	100	15	41	60	82

S.D. denotes the sampling standard deviation, S.E. the average estimated standard error and "H_0 rejected" shows the frequency with which the null hypothesis $H_0: \delta_i = 0$ was rejected when the nominal test size was 0.05. The values for δ_1, δ_2, and σ_v^2/σ_u^2 were based on empirical estimates of analogous consumption functions.

is rejected with considerable frequency. This contrasts favourably both with the bias which would arise from estimating μ in a simple dynamic model of the form:

$$x_t = \mu x_{t-1} + u_t$$

(where the bias is approximately equal to $-2\mu/T$ and so has the same sign but is about 5 times as large as the corresponding bias in $\hat{\delta}_2$ in Table 2) as well as with the biases and the variances which would be obtained from unrestricted estimation of:

$$x_t = \beta_1 w_t + \beta_2 w_{t-1} + \alpha_1 x_{t-1} + v_t. \tag{34*}$$

Thus there may be an "estimation" advantage from formulating dynamic equations as in (35), although for small δ_2, it may not be easy to establish $\hat{\delta}_2$ as significant at the 0.05 level unless T is relatively large.

When the appropriate lag length in (35) is four periods, the resulting model can be written as:

$$(X/W)_t = K^*(X/W)_{t-4}^{1-\gamma}(W_t/W_{t-4})^{\beta_1 - 1}. \tag{39}$$

For small γ the historical seasonal pattern of the *APC* will persist with modifications from any "seasonality" in $\Delta_4 \ln W_t$. Note that (35) and (39) are stable dynamic processes for $2 > \gamma > 0$, and that K is relatively robust to changes in the values of β_1 and $\gamma > 0$ (contrast the properties of the solved long-run *MPC* from (12)). However, K is not a continuous function of γ at $\gamma = 0$ (switching from zero to infinity) which reflects dynamic instability in (39) at $\gamma = 0$.

The solved distributed lag representation of (35) is:

$$x_t = k^*/\gamma + \sum_{j=0}^{\infty} \mu_j w_{t-j} + u_t, \tag{40}$$

where

$$u_t = (1-\gamma)\, u_{t-1} + v_t \quad \text{and} \quad \mu_0 = \beta_1, \quad \mu_j = (1-\gamma)^{j-1}\gamma(1-\beta_1) \quad (j \geq 1).$$

The mean lag is $(1-\beta_1)/\gamma$ which could be very large for γ close to zero, but, depending on the magnitude of β_1, much of the adjustment could occur instantaneously (for example, the median lag could be less than one period). If v_t is white noise, then (40) will manifest considerable autocorrelation for small γ, no matter how long a distributed lag is used for w_t.

The final feature of (35) is of crucial importance; *if* the growth rate g is relatively constant, then X_t will be approximately equal to KW_t and hence from (31), $(x_{t-1} - w_{t-1}) \simeq k$. In such a state of the world, the intercept and $(w_{t-1} - x_{t-1})$ would be almost perfectly collinear in (35). A similar collinearity also must affect any attempt to estimate (34) unrestrictedly. Although either regressor could be dropped without much loss to the goodness of fit, setting $k^* = 0$ does not affect the long-run behaviour (see (38) above) but setting $\gamma = 0$ *does*. This phenomenon at last provides a potential explanation both for the discrepant behaviour of the standard error of the intercept between (24) and (30) *and* for the low elasticity of the former equation since *the initial disequilibrium effect has been excluded from* (24), *but is still indirectly present in* (30).

However, before considering empirical variants of (35) it seems worth commenting on the relationship between equations like (35) and the four main theories of consumers' behaviour discussed in Section III. First, it is clear that both (34) and (35) resemble Brown's (1952) model; also, the term $\gamma(w_{t-1} - x_{t-1})$ could be interpreted as a "ratchet" to the "short-run" relationship (36) (compare Duesenberry, 1949) although it is a "ratchet" which operates in either direction for any sustained change in the growth rate of w_t. The distributed lag form (40) could be interpreted as an empirical approximation to "permanent income" in a model which always satisfies a long-run steady-state unit elasticity postulate (see Friedman, 1957). Moreover, using $C_t = x_t$ and $Y_t = w_t$, (35) corresponds to a transformed "life-cycle" model. For example, the wealth model of Ball and Drake (1964) is the special case of (35) in which $\beta_1 = \gamma$ and Deaton (1972) presents a modified life-cycle model of the same form but with revaluations of wealth as an additional variable. More recently, Deaton (1977) presents a savings equation closely similar to (35) but with the rate of inflation as an additional regressor (this study is discussed in Section IX below). Similar reasoning applies to models using changes in liquid assets in consumption equations (see Townend, 1976).

Nevertheless, as stressed above, the transformations involved in deriving the *PIH* and *LCH* (or eliminating any stock variable) significantly affect the properties of the error process, and it is possible (at least in principle) to distinguish between the contending hypotheses on this basis, subject to requiring that the error on the "true" model is white noise. Even so, it is exceedingly hard in practice to decide in a time-series context *alone* which relationships are "autonomous" and which are merely "good approximations". In terms of modelling any relationship between C and Y, the only really definite conclusion is that it seems vital to include some factor to account for the effect represented by $(w_{t-1} - x_{t-1})$.

Returning to the problem of reconciling the estimates in (24) and (30), consider the alternative restriction of dropping the intercept and retaining $(C_{t-4} - Y_{t-4})$, which in log terms yields:

$$\Delta_4 \ln \hat{C}_t = \underset{(0{\cdot}04)}{0{\cdot}49}\Delta_4 \ln Y_t - \underset{(0{\cdot}05)}{0{\cdot}17}\Delta_1 \Delta_4 \ln Y_t - \underset{(0{\cdot}01)}{0{\cdot}06}\ln (C/Y)_{t-4} + \underset{(0{\cdot}004)}{0{\cdot}01}\Delta_4 D_t^0, \quad (41)$$

$$R^2 = 0{\cdot}71, \quad \hat{\sigma} = 0{\cdot}0067, \quad \text{dw} = 1{\cdot}6, \quad z_1(20) = 80{\cdot}7, \quad z_2(12) = 23.$$

A relationship like (41) can be derived from a simple "feedback" theory in which consumers plan to spend in each quarter of a year the same as they spent in that quarter of the previous year ($\ln C_t = \ln C_{t-4}$) modified by a proportion of their annual change in income ($+0{\cdot}49\Delta_4 \ln Y_t$), and by whether that change is itself increasing or decreasing ($-0{\cdot}17\Delta_1 \Delta_4 Y_t$) (compare Houthakker and Taylor, 1970); these together determine a "short-run" consumption decision which is altered by $-0{\cdot}06 \ln (C_{t-4}/Y_{t-4})$, the feedback from the previous C/Y ratio ensuring coherence with the long-run "target" outcome $C_t = KY_t$. The parameterisation of (41) is determined by the choice of a set of plausible decision variables which incorporate relatively independent items of information, allowing agents to assess their reactions separately to changes in each variable. This seems a "natural" parameterisation to adopt, and as the small standard errors in (41) show, the resulting parameters are precisely estimated. Moreover, if any omitted decision variables can be re-formulated as orthogonal to the already included regressors, then the potentially serious problem of "omitted variables bias" is transformed to a problem of estimation efficiency. In practical terms, previously estimated coefficients will not change radically as new explanatory variables are added (see equation (45) below). The use of transformed variables like $\Delta_4 \ln C_t$, etc., is *not* because we want to "seasonally adjust" and/or achieve "stationarity" (with the attendant loss of spectral power at low frequencies noted by Sims, 1974) but because $\Delta_4 \ln C_t$ represents a sensible decision variable when different commodities are being purchased in different quarters of the year.

The significant value of z_1 in (41) reveals that other factors need to be included to provide a full account of the behaviour of C_t and this aspect is considered in Section IX. Nevertheless, (41) seems consistent with the salient features of the data in Figs. 1–6 and straightforwardly explains the large difference between the short-run and long-run *MPC*. The impact elasticity is 0·32, rising to 0·49 after one quarter, the remaining 51 % of the adjustment taking a considerable time to occur, which matches the relatively small value of the variance of consumption relative to that of income noted earlier. With only three "economic" variables, the model seems a reasonably parsimonious explanation of trend, cycle and seasonal components. Also it provides a suitable basis for discussing why the studies by *H*, *B* and *W* reached their published results.

First, a model like (41) could never be detected by any methodology in which the first step was to difference data and then only investigate the properties of the differenced series (as Wall *et al.*, 1975, do). Subject to that restriction, (24) (or its log equivalent) provides an excellent approximation in terms of goodness of fit despite its apparent lack of coherence with steady-state theory and long-run evidence.

Next, the lag structure of (41) could not be detected by researchers who only investigated lags of one or two periods and never used diagnostic tests for higher order residual autocorrelation (see Ball *et al.*, 1975). *The use of SA data does not justify neglecting higher-order lags.* If a model like (39) constitutes the true data generation process then this should not be greatly altered by filtering out seasonal frequencies from the data. Indeed, re-estimating (41) on the *SA* data used earlier yields:

$$\Delta_4 \ln \hat{C}_t^a = \underset{(0\cdot04)}{0\cdot44}\Delta_4 \ln Y_t^a - \underset{(0\cdot05)}{0\cdot19}\Delta_1 \Delta_4 \ln Y_t^a - \underset{(0\cdot01)}{0\cdot06} \ln (C^a/Y^a)_{t-4} + \underset{(0\cdot003)}{0\cdot01}\Delta_4 D_t^0, \quad (41a)$$

$$R^2 = 0\cdot79, \quad \hat{\sigma} = 0\cdot0063, \quad \text{dw} = 1\cdot7, \quad z_1(8) = 29\cdot0, \quad z_2(6) = 18.$$

The coefficients are very similar to (41), but the use of *SA* data has created considerable negative fourth-order residual autocorrelation (e.g. a coefficient at four lags of $-0\cdot7$ in a tenth-order residual autoregression) which would induce any investigator who did not *previously* believe in a model like (41) to select an equation with considerably less emphasis on four period effects.

Lastly, despite estimating equations with four period lags similar to unrestricted variants of (41), Hendry (1974) selected (11) as his preferred equation. The seasonal pattern for C_t seems to evolve whereas that for $\ln C_t$ does not (see Figs. 1 and 6 above) and hence the use of the untransformed data appears to have been one factor determining Hendry's choice. Further since C/Y was relatively constant over the period to 1967, the inclusion of an intercept in all the models considered by Hendry would greatly reduce the partial significance of four-period lagged variables. Both of these effects favour the incorrect selection of the evolving seasonals model as the best description of the data. Moreover, it is interesting that if a model like (39) is assumed as a data generation process, and w_t is highly correlated with w_{t-1} then regressing x_t on w_t and x_{t-1} will yield estimates like those in (12) when the data are *not* prior seasonally adjusted, and the true partial coefficient of x_{t-1} is zero. In summary, therefore, (41) seems to have the requisite properties to explain why previous researchers' methodologies led to their published conclusions.

Finally, in terms of the levels of the variables, equation (39) becomes:

$$x_t = k^* + \beta_1 w_t + (\gamma - \beta_1) w_{t-4} + (1-\gamma) x_{t-4} + v_t. \quad (42)$$

Such an equation can be approximated closely by:

$$x_t = k^{**} + \beta_1 w_t + u_t \quad \text{where} \quad u_t = (1-\gamma) u_{t-4} + e_t. \quad (43)$$

The mis-specification of (42) as (43) entails restricting the coefficient of w_{t-4} to be $(\gamma\beta_1 - \beta_1)$ instead of $(\gamma - \beta_1)$. This mis-specification will be negligible for small γ and $\beta_1 > 0$. Consequently, it is easy to approximate incorrectly the four period dynamics by fourth-order autocorrelation. Since γ is small, imposing the further restriction that the autocorrelation coefficient is unity will not noticeably worsen the fit and provides an alternative sequence whereby an incorrect differenced model might be selected (for a more general discussion of this last issue see Hendry and Mizon, 1978).

IX. INFLATION EFFECTS

Deaton (1977) has presented evidence for a disequilibrium effect of inflation on Consumers' Expenditure, which he interprets as consumers mistaking unanticipated changes in inflation for relative price changes when sequentially purchasing commodities. Since the forecast period contains inflation rates which are considerably greater than any observed during the sample used for estimation (the graph of $\Delta_4 \ln P_t$, where P_t is the implicit deflator of C_t, is shown in Fig. 7), Deaton's analysis offers a potential explanation for the poor forecast performance of all the estimated models.

In view of the functional form of the models (24) and (41), the regressors $\Delta_4 \ln P_t$ and $\Delta_1 \Delta_4 \ln P_t$ were included to represent the level and rate of change of inflation. Retaining the same sample and forecast periods yielded the results shown in equations (44) and (45) respectively (for comparability, we have chosen the log equivalent of (24)):

$$\Delta_4 \ln \hat{C}_t = \underset{(0{\cdot}04)}{0{\cdot}022} + \underset{(0{\cdot}05)}{0{\cdot}34}\Delta_4 \ln Y_t - \underset{(0{\cdot}05)}{0{\cdot}16}\Delta_1 \Delta_4 \ln Y_t + \underset{(0{\cdot}003)}{0{\cdot}01}\Delta_4 D_t^0 - \underset{(0{\cdot}07)}{0{\cdot}21}\Delta_4 \ln P_t - \underset{(0{\cdot}14)}{0{\cdot}15}\Delta_1 \Delta_4 \ln P_t, \tag{44}$$

$$R^2 = 0{\cdot}81, \quad \hat{\sigma} = 0{\cdot}0055, \quad \text{dw} = 1{\cdot}8, \quad z_1(20) = 146, \quad z_2(16) = 15;$$

$$\Delta_4 \ln \hat{C}_t = \underset{(0{\cdot}04)}{0{\cdot}47}\Delta_4 \ln Y_t - \underset{(0{\cdot}05)}{0{\cdot}21}\Delta_1 \Delta_4 \ln Y_t - \underset{(0{\cdot}02)}{0{\cdot}10}\ln (C/Y)_{t-4} + \underset{(0{\cdot}003)}{0{\cdot}01}\Delta_4 D_t^0 - \underset{(0{\cdot}07)}{0{\cdot}13}\Delta_4 \ln P_t - \underset{(0{\cdot}15)}{0{\cdot}28}\Delta_1 \Delta_4 \ln P_t, \tag{45}$$

$$R^2 = 0{\cdot}77, \quad \hat{\sigma} = 0{\cdot}0061, \quad \text{dw} = 1{\cdot}8, \quad z_1(20) = 21{\cdot}8, \quad z_2(12) = 19.$$

Both equations confirm Deaton's result that inflation was significantly reducing Consumers' Expenditure prior to 1971. Also, the inclusion of inflation effects in (45) has resolved the forecast problem: the considerable fall in the *APC* after 1971 (see Fig. 3) can be explained by the sharp increase in inflation and the five year ahead ex-post predictions from (45) satisfy the parameter stability test (Fig. 8 shows the plots of $\Delta_4 \ln C_t$ and $\widehat{\Delta_4 \ln C_t}$ over the period to 1975(iv)). Nevertheless, simply including the two additional regressors does *not of itself* guarantee an improved forecasting performance as z_1 in (44) shows. This outcome is easy to understand on the hypothesis that (45) constitutes the "true" model, since the behaviour of C/Y is negatively influenced by changes in P_t and so the approximation of C/Y by a constant is very poor over the forecast period.

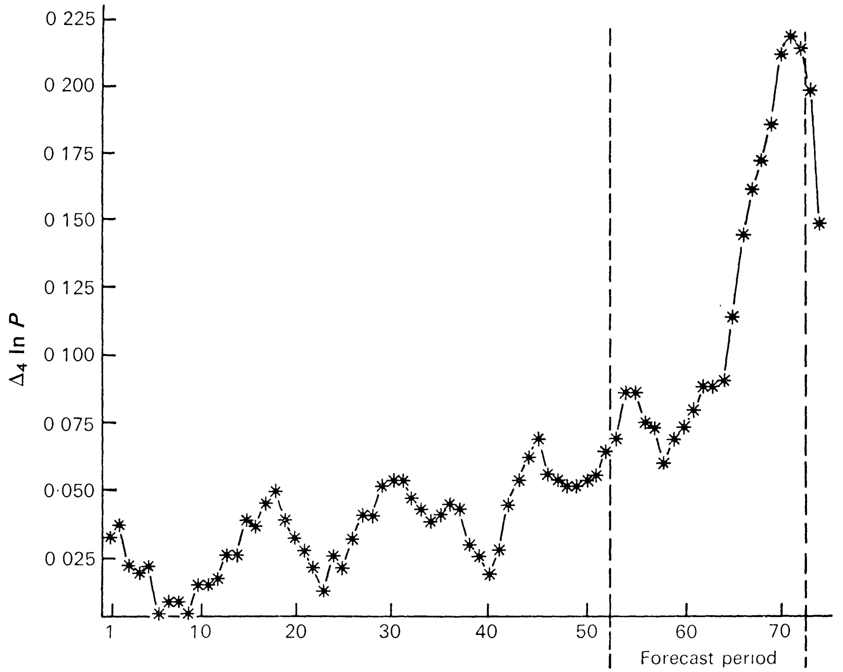

Fig. 7. Annual rate of change of prices.

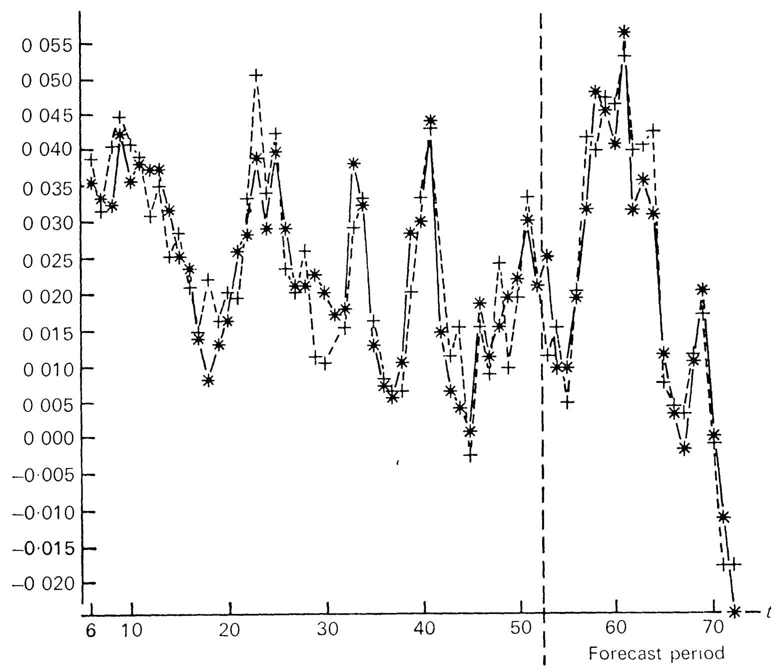

Fig. 8. Actual and predicted values of annual change in consumption.
——, $\Delta_4 \ln \hat{C}$. +——+, $\Delta_4 \ln C$.

Consonant with this argument, and illustrating the robustness of the parameter choice in (41), the only parameter estimate to be substantially altered by the inclusion of $\Delta_4 \ln P_t$ and $\Delta_1 \Delta_4 \ln P_t$ is the coefficient of $\ln (C/Y)_{t-4}$. The fact that (44) has a lower value of $\hat{\sigma}$ than (45) is evidence against suppressing the intercept, and indeed an intercept is significant if added to (45). However, $\ln (C/Y)_{t-4}$ loses significance if this is done and $z_1(20) = 137$. Thus, (44) and (45) exhibit an interesting conflict between goodness of fit and parameter stability as criteria for model selection. Bearing in mind that the forecast period is very different in several respects from the estimation period, the predictive accuracy of (45) is rather striking. Adding this to the earlier theoretical arguments, we have no hesitation in dropping the constant term instead of $\ln (C/Y)_{t-4}$.

On a steady-state growth path with constant annual real income growth rate g and inflation rate μ, (45) yields the solution:

$$C = KY \quad \text{where} \quad K = \exp(-5{\cdot}3g - 1{\cdot}3\mu). \tag{46}$$

When $g = 0{\cdot}02$ and $\mu = 0{\cdot}05$ (as roughly characterised the 1960s) $K = 0{\cdot}84$, whereas if μ increases to $0{\cdot}15$, K falls to $0{\cdot}74$ (which is similar to the 1970s). Variations in the rate of inflation induce substantial changes in the ratio of C to Y.

There are a number of theories in addition to Deaton's which would lead one to anticipate significant inflation effects in (45). For example, during periods of rapid inflation, the conventional measure of Y_t ceases to provide a good proxy for "real income" (note that equation (9) above holds when capital gains and losses are accounted for in Y_t) and $\Delta_4 \ln P_t$, etc., "pick up" this effect. Models like (10) based on the *LCH* but transformed to eliminate wealth, should manifest negative inflation effects of the form $\Delta_4 \ln P_t$ through the erosion of the real value of the liquid assets component of A_t. Although one might expect agents to alter the composition of their wealth portfolio by shifting into real assets such as housing when inflation is rapid, it is not clear how this would affect expenditure decisions. In terms of empirical evidence, Townend (1976) found a real Net Liquid Assets variable (N) to be significant in his specification of the consumption function together with negative inflation effects (based on Almon lags). Using Townend's data for N_t (1963 (iii)–1975 (i), retaining the last two years' data for a forecast test) the only form which yielded significant results when added to (45) was $\Delta_1 \ln N_t$ (which could be due in considerable measure to the joint endogeneity of C_t and N_t):

$$\begin{aligned}\Delta_4 \ln \hat{C}_t = {} & \underset{(0{\cdot}04)}{0{\cdot}47}\Delta_4 \ln Y_t - \underset{(0{\cdot}05)}{0{\cdot}28}\Delta_1 \Delta_4 \ln Y_t - \underset{(0{\cdot}02)}{0{\cdot}05} \ln (C/Y)_{t-4} \\ & - \underset{(0{\cdot}17)}{0{\cdot}27}\Delta_1 \Delta_4 \ln P_t + \underset{(0{\cdot}003)}{0{\cdot}008}\Delta_1 D_t^0 + \underset{(0{\cdot}05)}{0{\cdot}11}\Delta_1 \ln N_t + \underset{(0{\cdot}06)}{0{\cdot}01}\Delta_4 \ln P_t,\end{aligned} \tag{47}$$

$$R^2 = 0{\cdot}86, \quad \hat{\sigma} = 0{\cdot}0059, \quad \text{dw} = 1{\cdot}8, \quad z_1(8) = 41, \quad z_2(12) = 26.$$

The main impacts of adding $\Delta_1 \ln N_t$ to (45) are the halved coefficient of $\ln (C/Y)_{t-4}$ (in an *LCH* framework, these are proxies) and the dramatic change to almost zero in the coefficient of $\Delta_4 \ln P_t$, consistent with the hypothesis that

$\Delta_4 \ln P_t$ is a proxy for the erosion of the value of liquid assets from inflation. Nevertheless, the effect of accelerating inflation retains a large negative coefficient. The marked deterioration in the forecast performance of (47) suggests an incorrect specification and hence we decided to omit N_t from further consideration, attributing its significance in (47) to simultaneity.[1]

To test the validity of the various restrictions imposed on (45) (price level homogeneity, exclusion restrictions and the unit income elasticity) we estimated the general unrestricted model:

$$\ln C_t = \sum_{j=0}^{5} (\alpha_j \ln Y_{t-j} + \beta_j \ln P_{t-j}) + \sum_{j=1}^{5} \lambda_j \ln C_{t-j} + \epsilon_t. \tag{48}$$

The results are shown in Table 3, and Table 4 records the equivalent values derived from the restricted model (45) (it seemed spurious to include five lagged values of D_t^0 in (48), although doing so does not greatly alter the results, $\hat{\sigma}$ falling to 0·0059). The restrictions are not rejected on a likelihood ratio test, and indeed the two sets of estimates are rather similar. Moreover, to two decimal digits, $\Sigma\hat{\beta}_j = 0$ and $\Sigma\hat{\alpha}_j \simeq 1 - \Sigma\hat{\lambda}_j$ favouring the hypotheses of price homogeneity and a unit elasticity for income.

Finally, re-estimation of (45) assuming $\Delta_4 \ln Y_t$, $\Delta_1 \Delta_4 \ln Y_t$, $\Delta_4 \ln P_t$ and $\Delta_1 \Delta_4 \ln P_t$ to be endogenous and using instrumental variables[2] yielded the outcome shown in (45)*:

$$\begin{aligned} \Delta_4 \ln \hat{C}_t = {} & \underset{(0\cdot04)}{0\cdot48}\Delta_4 \ln Y_t - \underset{(0\cdot06)}{0\cdot20}\Delta_1 \Delta_4 \ln Y_t - \underset{(0\cdot07)}{0\cdot12}\Delta_4 \ln P_t \\ & - \underset{(0\cdot18)}{0\cdot28}\Delta_1\Delta_4 \ln P_t - \underset{(0\cdot02)}{0\cdot09} \ln (C/Y)_{t-4} + \underset{(0\cdot004)}{0\cdot007}\, \Delta_4 D_t^0, \end{aligned} \tag{45*}$$

$$\hat{\sigma} = 0\cdot0061, \quad \text{dw} = 1\cdot7, \quad z_1(20) = 22, \quad z_2(12) = 19, \quad z_3(10) = 16,$$

where $z_3(l)$ is the test for validity of the choice of instrumental variables discussed by Sargan (1964) and is distributed as χ_l^2 in large samples when the instruments are independent of the equation error. It is clear that the coefficient estimates and the goodness of fit are hardly altered, providing no evidence of simultaneity biases.

An interesting result emerges from estimating (45) over the entire sample period (to 1975 (iv)):

$$\begin{aligned} \Delta_4 \ln \hat{C}_t = {} & \underset{(0\cdot03)}{0\cdot48}\Delta_4 \ln Y_t - \underset{(0\cdot04)}{0\cdot23}\Delta_1 \Delta_4 \ln Y_t - \underset{(0\cdot01)}{0\cdot09} \ln (C/Y)_{t-4} \\ & + \underset{(0\cdot002)}{0\cdot006}\Delta_4 D_t^* - \underset{(0\cdot02)}{0\cdot12}\Delta_4 \ln P_t - \underset{(0\cdot10)}{0\cdot31}\Delta_1 \Delta_4 \ln P_t, \end{aligned} \tag{45**}$$

$$R^2 = 0\cdot85, \quad \hat{\sigma} = 0\cdot0062, \quad \text{dw} = 2\cdot0, \quad z_2(12) = 23.$$

[1] Other regressors which were added to (47) without yielding significant results were unemployment, the relative price of durables to non-durables and short-term interest rates. The largest t value was for $\Delta_1 \Delta_4$ ln (unemployment) and Bean (1977) reports a significant value for this variable in a variant of (45). Note that, if the significance of $\Delta_1 \ln N_t$ *is* due to simultaneity, then the vanishing of the *direct* effect of $\Delta_4 \ln P_t$ on $\Delta_4 \ln C_t$ provides *no* evidence on the "erosion of the value of real liquid assets" hypothesis.

[2] The instruments used were $\ln Y_{t-j}$ ($j = 1, \ldots, 5$), $\ln P_{t-j}$ ($j = 1, 4, 5$), $\ln F_{t-j}$, $\ln E_{t-j}$, $\ln I_{t-j}$ ($j = 0, 4$) (where F_t, E_t, I_t respectively denote the real value of current government expenditure, exports and gross domestic fixed capital formation) and the predetermined variables in the regression.

D_t^* is D_t^0 extended to allow for the introduction of VAT – see footnote 1, p. 674. Manifestly, the coefficient estimates and $\hat{\sigma}$ are hardly changed from (45), as would be expected given the value for $z_1(20)$ on equation (45). R^2 has, therefore, increased, and the coefficient standard errors are smaller, especially for $\Delta_4 \ln P_t$. However, the equivalent long period estimates of (44) alter considerably, with $\hat{\sigma}$ increasing to 0·0063 and $z_2(16)$ to 28. Thus, the overall data set does not offer much evidence against deleting the intercept, and strongly favours retaining $\ln (C/Y)_{t-4}$. From the longer sample period, a significant coefficient for $\Delta_4 \ln Y_{t-2}$ also can be established, creating a "smoother" distributed lag of C_t on Y_t.

Table 3

Unrestricted Estimates of (47)

j	0	1	2	3	4	5
$\ln C_{t-j}$	—	0·12 (0·17)	0·02 (0·04)	−0·06 (0·04)	0·98 (0·05)	−0·11 (0·17)
$\ln Y_{t-j}$	0·25 (0·06)	0·10 (0·08)	−0·06 (0·07)	0·11 (0·07)	−0·18 (0·07)	−0·16 (0·08)
$\ln P_{t-j}$	−0·59 (0·21)	0·50 (0·29)	−0·23 (0·24)	0·12 (0·24)	0·44 (0·28)	−0·24 (0·21)

$R^2 = 0·997$, $\hat{\delta} = 0·0062$, dw $= 2·2$.

Table 4

Solved Estimates from (45)

j	0	1	2	3	4	5
$\ln C_{t-j}$	—	0	0	0	0·90	0
$\ln Y_{t-j}$	0·26	0·21	0	0	−0·16	−0·21
$\ln P_{t-j}$	−0·41	0·28	0	0	0·41	−0·28

X. SUMMARY AND CONCLUSIONS

A simple dynamic model which conforms with a range of theoretical requirements and matches all of the salient features of the data was used to explain various recently published relationships between Consumers' Expenditure on Non-durables and Disposable Income. Extended to allow for the effects of inflation noted by Deaton (1977), the model produces an acceptable set of post-sample predictions over twenty quarters using the actual data for incomes and prices. While noting the implications of the analyses of Leamer (1974, 1975) for an exercise like that described above, we feel that our "prejudiced search for an acceptable model" has not been fruitless. We conclude that it is worth while trying to explain the *complete* set of existing findings; that restrictions derived from economic theories can be valuable in econometric modelling if correctly implemented to restrict the *model* but not the *data*; that seasonal adjustment of data can confuse the *selection* of an appropriate dynamic specification; that "multicollinearity" is not necessarily resolved by *restricting* the parameter space rather than by *enlarging* it, and that econometric relationships can predict accurately over periods in which the behaviour of the regressors is sufficiently different that mechanistic time-series methods will fail. However, we do not conclude that our model represents the "true" structural relationship since there are several important issues which have not been considered (including changes in income distribution and direct wealth effects). Hopefully, our methods, models and results will facilitate future work on these problems.

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