VAR analysis in the presence of a Changing Correlation in the Structural Errors

By

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Abstract

In this paper an extension to standard VAR analysis has been proposed which allows us to investigate the possibility that the correlation structure of the shocks hitting the system is changing over time. The proposal is to estimate a VAR in the usual way and then apply Orthogonal GARCH analysis to calculate the conditional covariance matrix for each period. Calculating Impulse responses is then straightforward based on any of the standard techniques for identifying the structural VAR. this proposal is illustrated using a four variable VAR for GDP for the UK, France, Germany and Italy. This shows very clearly that standard diagnostics do not detect the changing correlation structure very well but that this change can have profound effects on the estimated impulse response and the associated policy conclusions which might be drawn from them.

JEL: C1, C2,C3 Keywords; VAR, structural change, GARCH

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Introduction

VAR analysis and the use of impulse responses has become one of the main ways to analyse reasonable size data sets following the key paper of Sims(1980). It has a number of attractions and also some disadvantages; the lack of a need for conventional theoretical restrictions is one obvious attraction although the need to impose identification to calculate the impulse responses partly offsets this. The widely used orthogonalised impulse responses has the disadvantage that the ordering of the variables changes the results. More recently the proposal of Pesaran and Shin(????) to use generalised impulse responses has overcome this disadvantage although at the cost of making the interpretation of the results a little less clear cut.

In this paper I wish to raise a question, which has not so far been addressed. What is the impact on VAR analysis if the structural errors have a changing correlation structure? This question arguably has considerable relevance to a number of areas of great policy interest. The convergence debate, which has been conducted in many contexts, may be characterised largely as one of a changing correlation structure of shocks. For example, the issue of the how similar shocks are across counries dominates the question of forming a monetary union. Clearly the convergence debate is all about this matrix of shocks changing its covariance structure.

The issue to be addressed here then is what should be the consequence of a changing correlation structure for VAR analysis and how might we approach the modelling problems in a tractable way. Section 2 of this paper sets out the basic structure of the VAR problem when the structural error covariance matrix is time varying. Section 3 discusses some problems in implementing this general framework and discusses a tractable framework. Section 4 then gives a short empirical example and section 5 concludes.

2. The Structure of VAR analysis

Consider the Structural VAR model

$$A_0 x_t = \sum_{i=1}^{p} A_i x_{t-i} + B w_t + e_{t,} \quad t = 1,2,...T \qquad 1.$$

where $x_t=(x_{1t},x_{2t},\ldots,x_{mt})'$ is an mx1 vector of jointly determined variables, w_t is a qx1 vector of exogenous or deterministic components and A_i and B are mxm and mxq coefficient matrices. I also, as usual, assume that $E(e_t)=0$, but in addition that $E(e_t e'_t)=\Psi_t$, that is that the covariance matrix of the structural shocks is time varying. For simplicity I also assume that x_t are weakly stationary, although the analysis carries over to a cointegrated VAR quite easily. The standard reduced form VAR representation of this model is

$$x_t = \sum_{i=1}^{p} \Phi_i x_{t-i} + \Psi w_t + \varepsilon_{t,} \quad t = 1,2,...T \qquad 2.$$

Where of course $\Phi_i = A_0^{-1} A_i$, $\Psi = A_0^{-1} B$ *and* $\varepsilon_t = A_0^{-1} e_t$ the covariance structure of the reduced form errors will then be given by $E(\varepsilon_t \varepsilon_t') = \Sigma_t = A_0^{-1} \Psi_t A_0^{-1\prime}$, which is of course time varying.

This model now raises two questions: How should we estimate such a system and how should we calculate impulse responses.

3. Estimation

In principal the specification of the likelihood function for (2) is quite straightforward once we have specified a parametric form for the evolution of Σ_t. A natural choice would be to specify a multivariate GARCH process and a number of alternative specifications exist in the literature, Kraft and Engle(1982), Bollerslev, Engle and Wooldridge(1988), Hall Miles and Taylor(1990), Hall and Miles(1992), Engle and Kroner(1995). If we define the VECH operator in the usual way as a stacked vector of the lower triangle of a symmetric matrix then we can represent the standard generalization of the univariate GARCH model as

$$VECH(\Sigma_t) = C + A(L)VECH(\varepsilon_t\varepsilon_t') + B(L)VECH(\Sigma_{t-1}) \qquad 3.$$

where C is an (N(N+1)/2) vector and A_i and B_i are (N(N+1)/2)x(N(N+1)/2) matrices. Estimation of such a model is, in principle, quite straightforward as the log likelihood is proportional to the following expression.

$$l = \sum_{t=1}^{T} \ln|\Sigma_t| + \varepsilon'_t \Sigma_t^{-1} \varepsilon_t \qquad 4.$$

However this general formulation rapidly produces huge numbers of parameters as N rises (for just 1 lag in A and B and a 5 variable system we generate 465 parameters to be estimated) so for anything beyond the simplest system this will almost certainly be intractable. A second problem with this system is that without fairly complex restrictions on the system the conditional covariance matrix cannot be guaranteed to be positive semi definite. So much of the literature in this area has focused on trying to find a parameterization which is both flexible enough to be useful and yet is also reasonably tractable.

One of the most popular formulations was first proposed by Baba, Engle, Kraft and Kroner, sometimes referred to as the BEKK(see Engle and Kroner(1993)) representation, this takes the following form

$$\Omega_t = C'C + \sum_{i=1}^{q} A'_i \varepsilon_{t-i}\varepsilon'_{t-i} A_i + \sum_{j=1}^{p} B'_j \Omega_{t-j} B_{t-j} \qquad 5.$$

This formulation guarantees positive semi definiteness of the covariance matrix almost surely and reduces the number of parameters considerably. However even this model can give rise to a very large number of parameters and further simplifications are often applied in terms of making A and B symmetric or diagonal.

Orthogonal GARCH

Any of the multivariate GARCH models listed above are severely limited in the size of model, which is tractable. Even a restricted BEKK model becomes largely unmanageable for a system above 4 or 5 variables. An alternative approach, however which can be applied, potentially to a system of any size rests on the use of principal components and is sometimes referred to as orthogonal GARCH (see Ding(1994). Consider a set of n stochastic variables X, which have a covariance structure V. Principal components then produces a set of n variables (P), which contain all the variation of X but are also orthogonal to each other. The standard principal component representation can be written as follows.

$$X_i = \mu_i + \sum_{j=1}^{n} \omega_{ij} p_j \quad \text{i=1...n} \qquad 6.$$

so if all n principal components are used each x_i can be exactly reproduced by weighting the principal components together with the correct loading weights. Now by simply taking the variance of both sides of this equation we can see that

$$VAR(X) = V = W(VAR(P))W' = W\Psi W' \qquad 7.$$

The advantage of this is of course that as the principal components are orthogonal Ψ will be a diagonal matrix with zeros on all non diagonal elements. From applying principal components we know W, we then simply have to derive a set of univariate GARCH models to each principal component to derive estimates of the conditional variance at each point in time and apply the above formulae to derive an estimate of the complete covariance matrix V. The conditional variance may be obtained from any chosen procedure (GARCH, EGARCH or even an EWMA model of the squared errors)

There are however two further issues here;

i) as the principal components are ordered by their explanatory power we often find that a subset of them produces a very high degree of explanatory power. It may then only be deemed necessary to use the first k principal components. It is even suggested that this helps to remove noise from the system as the minor principal components may be reflecting pure random movements. This can easily be done but it introduces an error term into the principal components representation above and the resulting covariance matrix may no longer be positive definite.

ii) Equation (7) above is true exactly for the average of the whole period the principal components are calculated for but it does not necessarily hold at each point in the sample. So this is really only delivering an approximation. It may then be useful to apply the procedure to a moving window of observations so that the W matrix also effectively becomes time varying, Yhap(2003) has carried out an extensive Monte Carlo analysis of this technique and it seems to work well up to sample sizes of 500.

The suggestion being proposed here is then to estimate a standard VAR as in (2), which gives consistent parameter estimates even if the covariance structure of the errors is time varying, and then to use the orthogonal GARCH model to generate the time varying covariance matrix of the estimated residuals. This process is only limited by degrees of freedom constraints on the VAR in the usual way so that any conventional VAR, which is tractable, may also have the error covariance structure decomposed in this way.

The Impulse Response Function

In general the impulse response function may be simply described following Koop et al(1996) as,

$$I(n, \delta, \Omega_{t-1}) = E(x_{t+n} \mid \varepsilon_t = \delta, \Omega_{t-1}) - E(x_{t+n} \mid \Omega_{t-1}) \qquad 8.$$

where Ω_t is the information set at time t and δ is a vector of shocks applied at time t. Different choices of the structure of δ characterise different schemes of identification. The dominant procedure is the orthogonalised residuals originally proposed by Sims, here we simply define

$$P_t P_t^{'} = \Sigma_t \qquad 9.$$

Using a Cholesky decomposition where P is a lower triangular matrix then, defining p_{it} as the ith row of P_t we may define the orthoganalised impulse response for the ith variable by setting.

$$\delta = p_{it}^{'}\sigma \qquad 10.$$

where σ is a suitable scaling factor, normally one standard deviation. (10) Will of course be dependent on the ordering of the variables in the decomposition, (9), as is well known.

An alternative method is the generalised impulse response of Koop et al(1996) or Pesaran et al(1998), here rather than using an identifying assumption such as orthogonality they simply take the estimated structure of the covariance matrix so that if we take s_{it} to be the ith row of Σ_t then the generalised impulse response is generated by setting

$$\delta = s_{it}^{'}\sigma$$

The issue being considered here is not which alternative is preferable but that whichever method is used the results will vary over time if Σ is time varying. Also it is obvious that once an estimate of Σ_t is available the application of any of these identification schemes is entirely straightforward, even on a time varying basis.

4. An Example

In this section a standard VAR model of GDP for the UK, France, Italy and Germany is first estimated in the usual way and then the conditional correlation structure of the errors is assessed using the orthogonal GARCH technique above. I then illustrate the potential change, which can occur in the impulse responses if they are calculated based on the conditional covariance matrix at different points in time. This is an interesting application given the importance of the question of the correlation of economic shocks to the formation of a currency union within the standard optimal currency area literature and hence the issue of the UK's membership of the European Monetary Union. Clearly we should be making the judgement over entry based on an estimate of the current conditional correlation of errors not some average over the past. And if convergence has been taking place we might expect this to show up in the conditional correlation structure of the residuals.

The starting place is therefore to estimate a standard VAR for the log of the four GDP series, quarterly data is used for the period 1973Q3-2002Q2. A VAR(2) is chosen based on the Schwarts criteria and a series of joint F tests of excluding each lag (the probability level of these tests were lag 1=0.0000, lag 2=0.0069, lag 3=0.78 and lag 4=0.15). The following diagnostics confirm that this VAR seems to be reasonably well specified.

	Germany	France	UK	Italy
Serial Corr	0.12	0.47	0.07	0.1
ARCH	0.59	0.86	0.008	0.34
Hetero	0.74	0.41	0.02	0.14
Hetero-X	0.69	0.59	0.25	0.14

Figures in each cell denote probability levels, hence a figure less than 0.05 denotes rejection at the 5% critical value. Serial corr is a test of up to 5th order serial correlation, ARCH is a test of up to a fourth order ARCH process, Hetero tests for heteroskedasticity related to any of the lagged variables, Hetero-X allows for cross products of all the variables.

Given the analysis later it is particularly interesting to note that this VAR has very little signs of ARCH or Heteroskedasticity. In addition it appears to be quite stable as recursive estimation shows very little sign of instability (a recursive one period ahead chow test finds only 1 period out of 320 tests which exceed the 1%critical value, the results are detailed in appendix A).

We then apply the Orthogonal GARCH model to the residuals of this VAR. The standard correlation matrix of the residuals is presented below

	Germany	France	UK	Italy
Germany	1.0	0.37	0.25	0.28
France	0.37	1.0	0.27	0.44
UK	0.25	0.27	1.0	0.26
Italy	0.28	0.44	0.26	1.0

Here we see a picture of a relatively high correlation between France and Italy and France and Germany but uniformly low correlations between the UK and the other three countries. But of course if there is any systematic movement over time in these correlations then this table will in effect simply be recording the average.

The details of the orthogonal GARCH estimation will not be given in detail as there is no particular intuition to attach to the GARCH models for the principal components. Given the lack of serial correlation in the residuals the mean equation for each principal component is specified as a simple constant and a GARCH(1,1) model was specified for the conditional variance. This seemed to be adequate in all four estimated equations.

The results of the Orthogonal GARCH process are given in Figures 1 and 2. Figure 1 shows the conditional correlations for the UK with Germany France and Italy while figure 2 shows the correlation BETWEEN Germany France and Italy. The Broad picture is fairly simple, the correlation between Germany France and Italy seems to be quite stable with little sign of any trend. The correlation with the UK however show quite strong trends especially with France and Italy, the 1970’s and early 1980s exhibits quite strong variations in the correlation with average values below 0.2 and some values actually being negative, by the last half of the 1990s the correlation was much more stable and averages over 0.4. There has however been little trend change in the correlation with Germany.

Clearly the change which has taken place in the correlation structure could profoundly affect the impulse responses of this system no matter what form of identification structure we use to identify them. To illustrate this point we present the Generalised impulse response to a shock to Italy calculated first on the basis of the covariance matrix calculated for 1979Q3 (which exhibits the most extreme negative correlation for the UK) and for 2002Q1 which is fairly representative of the final years of the period. The conclusions of these two pictures are very clear; in the first chart the UK is an obvious outlier and responds very differently to the other European countries. In figure 4 the UK is responding in a way, which is almost identical to the other countries.

5. Conclusion

In this paper an extension to standard VAR analysis has been proposed which allows us to investigate the possibility that the correlation structure of the shocks hitting the system is changing over time. The proposal is to estimate a VAR in the usual way and then apply Orthogonal GARCH analysis to calculate the conditional covariance matrix for each period. Calculating Impulse responses is then straightforward based on any of the standard techniques for identifying the structural VAR. this proposal is illustrated using a four variable VAR for GDP for the UK, France, Germany and Italy. This shows very clearly that standard diagnostics do not detect the changing correlation structure very well but that this change can have profound effects on the estimated impulse response and the associated policy conclusions which might be drawn from them.

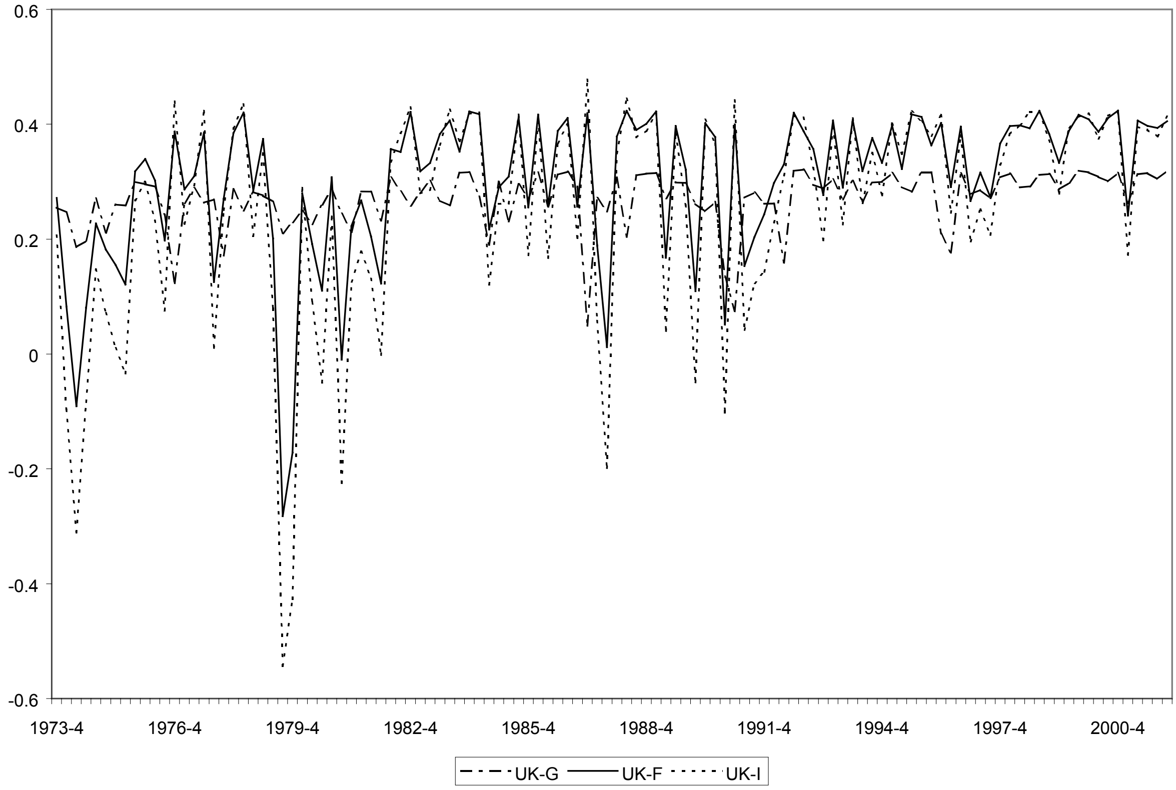


Figure 1: The Conditional Correlation between the shocks to the UK and Germany, France and Italy

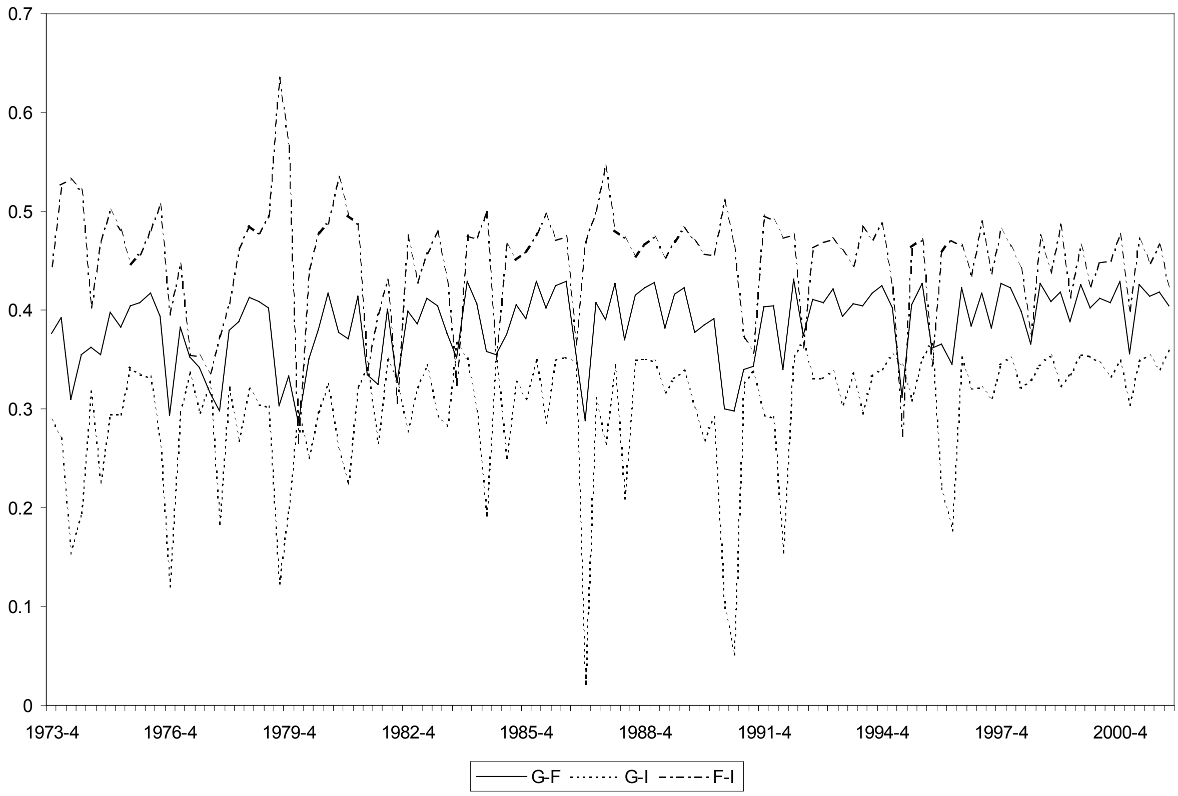


Figure 2: The conditional Correlation between Germany France and Italy.

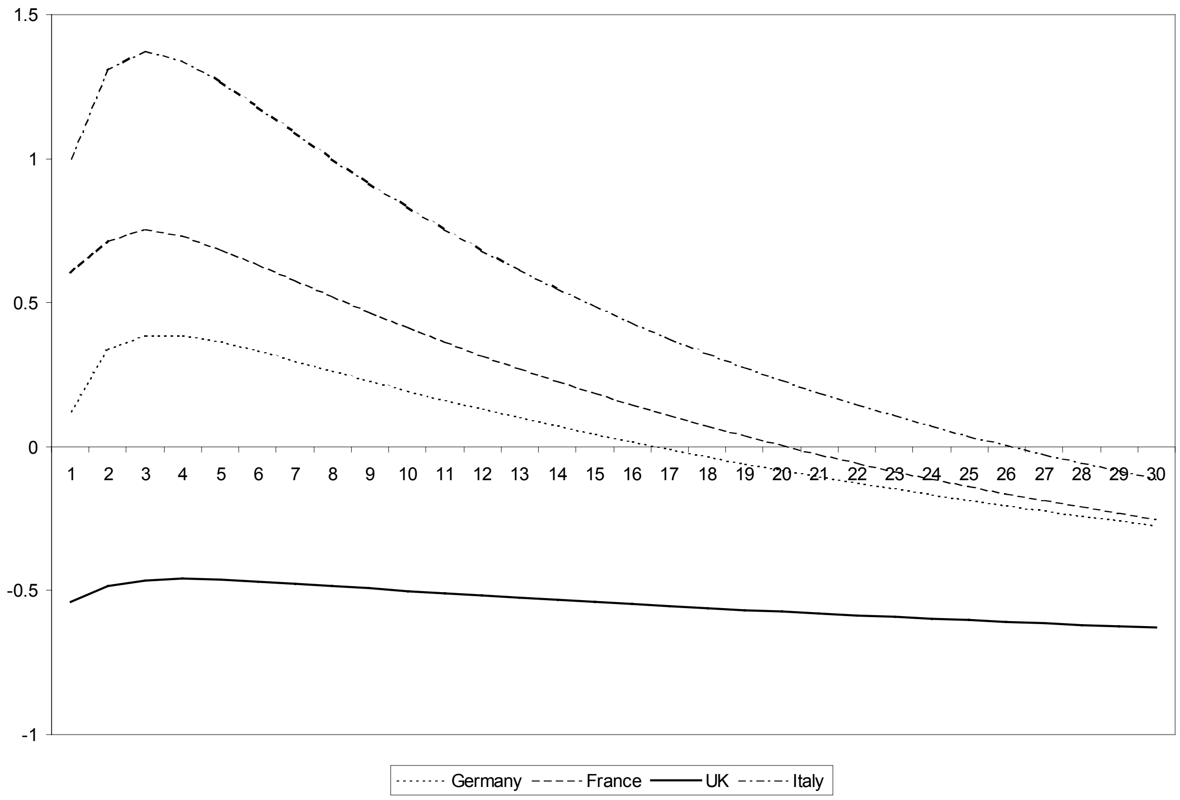


Figure 3: Generalised Impulse Response based on the covariance matrix in 1979Q3

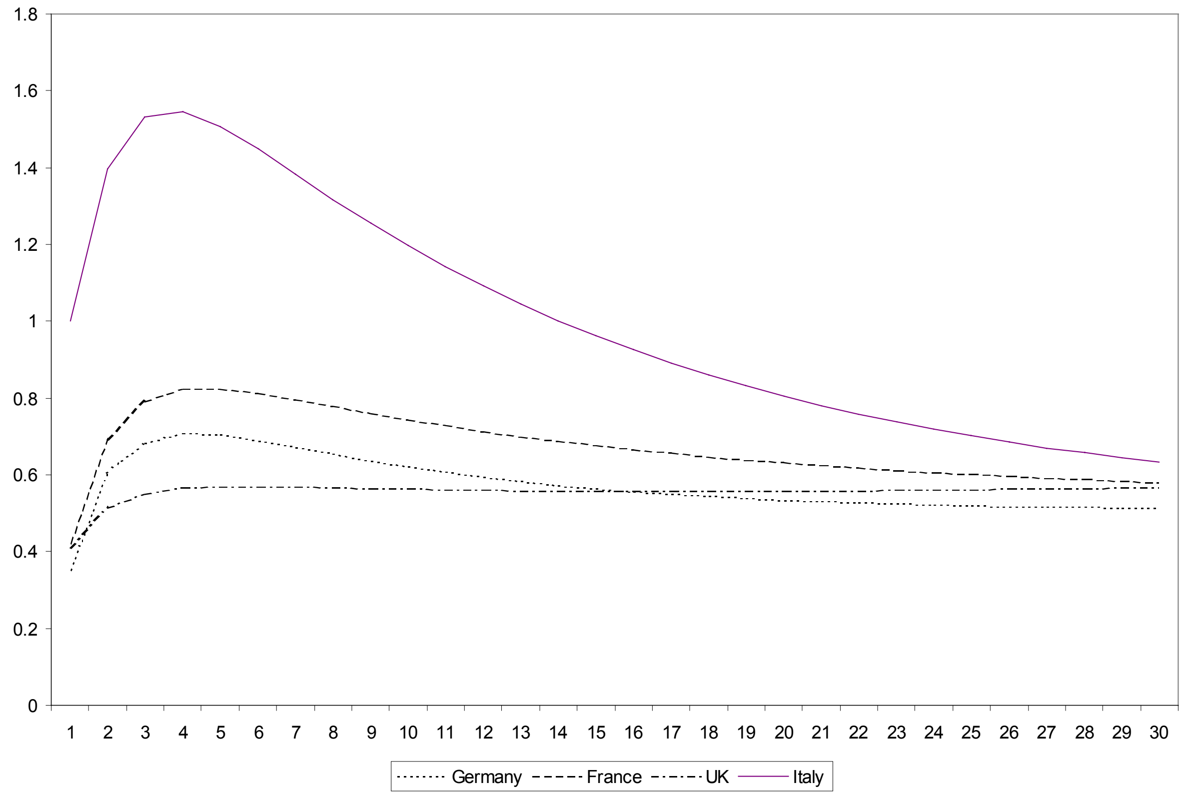
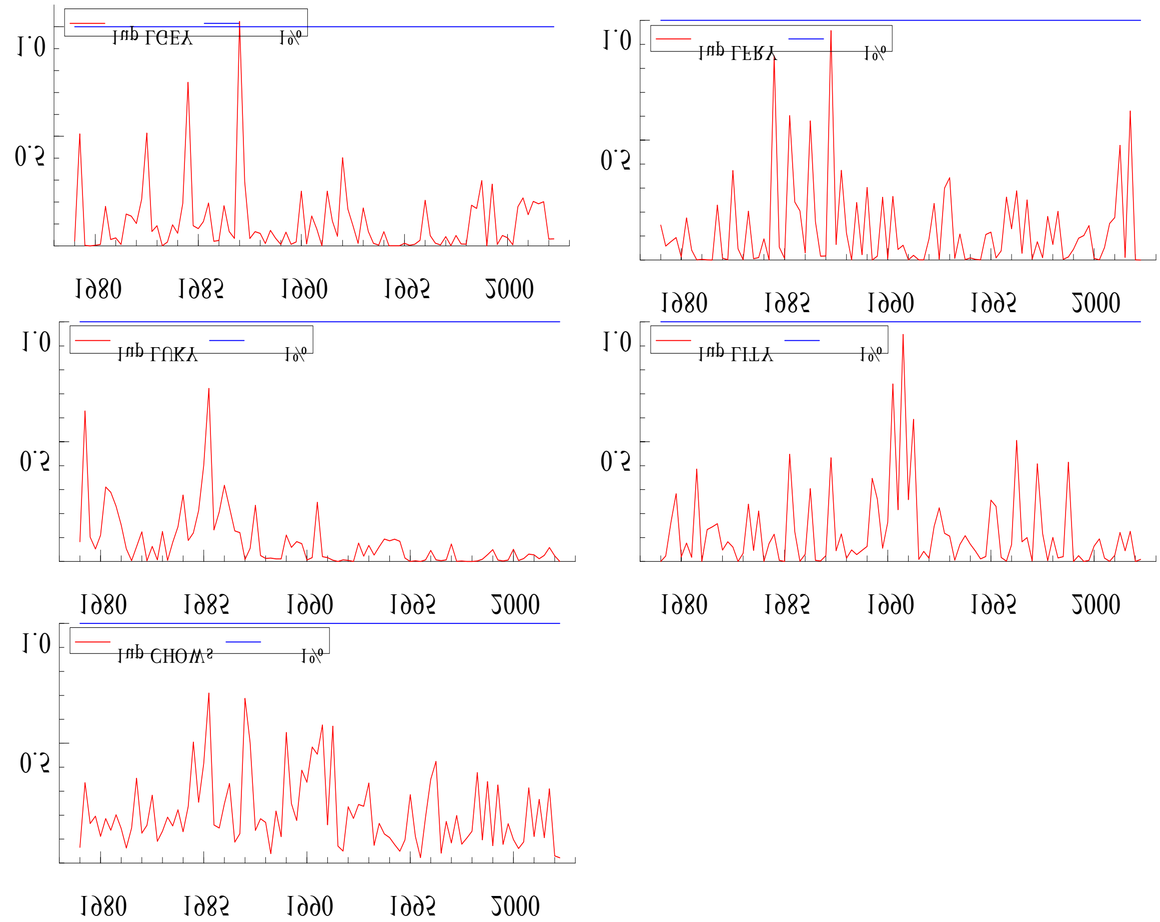


Figure 4: Generalised Impulse Response based on the covariance matrix in 2002Q1

Appendix A: Recursive chow tests for the VAR(2)



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