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INFLATION FAN CHARTS, MONETARY POLICY AND SKEW NORMAL DISTRIBUTIONS

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ABSTRACT

Issues related to classification, interpretation and estimation of inflationary uncertainties are addressed in the context of their application for constructing probability forecasts of inflation. It is shown that confusions in defining uncertainties lead to potential misunderstandings of such forecasts. The principal source of such confusion is in ignoring the effect of feedback from the policy action undertaken on the basis of forecasts of inflation onto uncertainties. In order to resolve this problem a new class of skew normal distributions (weighted skew normal, WSN) have been proposed and its properties derived. It is shown that parameters of WSN distribution can be interpreted in relation to the monetary policy strength and symmetry. It has been fitted to empirical distributions of inflation multi-step forecast errors of inflation for 34 countries, alongside others distributions already existing in the literature. The estimation method applied is using the minimum distance criteria between the empirical and theoretical distributions. Results lead to some constructive conclusions regarding the strength and asymmetry of monetary policy and confirm the applicability of WSN to producing probabilistic forecasts of inflation.

1. INTRODUCTION

Assessing uncertainties related to future inflation is a long established element of monetary policy and indeed of most micro- and macroeconomic decisions. Without going into wide and well covered review of the problem, let us limit the exposition to stating that inflation uncertainty is one of the main elements of inflation costs. This statement is well supported in the literature, starting from the seminal Fisher (1981) paper up to more recent development by Chiu and Molico (2010). In particular, as monetary policy decisions are undertaken on the basis of evaluation of future inflation which is understood as a random variable rather than a deterministic scalar, central bankers in inflation targeting countries spend a lot of time trying to assess the probability that future inflation will be within the pre-imposed bands. The need for practical assessment of such uncertainties is obvious.

In this paper we enquire about the operational concepts of inflation uncertainties in the context of probability forecasts of inflation that is usually undertaken in central banks with an aim of providing a convenient tool for monetary policy. Such forecasts are often produced in a form of the so-called fan-charts (or ‘rivers of blood’) depicting the uncertainties related to forecasts for subsequent horizons. We argue that misunderstanding related to the nature and definitions of uncertainties results in confusions in interpretation of the fan-charts and possibly errors in its construction, which in turn might affect the monetary decision. Further on we propose a simple stochastic model describing the inflation uncertainties. This model is grounded within a new type of skew normal distribution introduced further in this paper, which parameters can be directly interpreted in the context of monetary policy.

In the empirical part we estimate our model using forecast errors of monthly inflation obtained for 38 countries for the period from January 1998 to November 2012. In order to compare the effect of forecast quality, we have used two sets of forecast errors: obtained by the simplest possible, naïve forecasting method and by a reasonably sophisticated one, seasonal autoregressive moving average method. To these data sets we fit developed in this paper weighted skew normal distribution and compared the results with that obtained by fitting already known types of skew normal distributions. Our approach enabled us to assess impact of monetary policy on the uncertainties for each country, in terms of its strengths and likely asymmetry.

2. DEFINITIONS, ASSUMPTIONS AND MISUNDERSTANDINGS

It is tempting to start by adopting one of the universal definitions of uncertainty for its use for empirical modelling of inflation. However, it seems that it isn’t any. Some attempts to generalize the notion of uncertainty can be found e.g. in Walker *et al*., 2003; (for discussion and adaptation for inferring about inflation see Kowalczyk, 2012). However, this have been criticised for incompleteness and tautology (Norton *et al*., 2006; this paper also gives a review of other concepts). Other definitions are frequently used, without much harm, in different sciences. We have decided to follow this trend referring, however, to Walker *et al.* (2003) classification, where relevant.

Even on the grounds of economics, theoretical and practical approaches used so far in this context are not much unified. The plethora of methods and techniques can, somewhat arbitrarily, be divided into assessing inflation uncertainties *ex-post* and *ex-ante*. The *ex-post* methods define uncertainty as the stochastic components of estimated time series models, usually the generalized autoregressive conditional heteroscedasticity, *GARCH*, (see e.g. Elder, 2004, Kontonikas, 2004, Daal *et al*., 2005, Fountas *et al.*, 2006, Henry *et al*., 2007, Fountas, 2010, Neandis and Savva, 2011), stochastic volatility (Berument *et al.*, 2009) and *RiskMetrix* (Hartmann and Herwartz, 2012). The *ex-post* approach, although popular among the academics, with numerous papers published worldwide, has not been widely adopted by the practitioners, who are interested in undertaking economic decisions with their consequences in the future and hence prefer forward-looking methods. Also, defining uncertainty as a stationary, past-dependent, phenomenon seems to be too narrow for practical purposes.

Among the *ex-ante*, or forward-looking, ways of defining and assessing inflation uncertainty, it is possible to identify two, not mutually exclusive, clusters. The first cluster consists of methods which aim at deriving probabilistic characteristics of the process forecasted from a dynamic model, either univariate (see e.g. Kemp, 1991, 1999) or vector autoregressive (e.g. Lütkepohl, 2006a,b), Bayesian (Cogley *et. al*, 2005) and others. In this cluster inflation uncertainty is identified by the shape (and, in particular, dispersion) of the predictive density of inflation. The approach here is to estimate, from historical data, the joint density of inflation for all periods of the forecasting horizon, usually under assumption of the perfect knowledge. Most notably, it estimates jointly the location (or the most likely outcome of inflation, which is not related directly to the uncertainty of future inflation) and other characteristics of the predicted density, which describes such uncertainty (dispersion, skewness, *etc*.). This approach of assessing uncertainties, called in this paper *uncertainty by the model*, is not however, popular among inflation modellers as well, due to its numerically complicated nature and heavy dependence on the assumption of normality.

Difficulties with methods of the first cluster lead to the development of another cluster of *ex-ante* methods, where the distributional characteristics of inflation uncertainty are derived separately from the way the point forecast is made that is, to a large extent, extraneously to the model which has been used for forecasting the mean of the expected inflation. This approach really took off with the practitioners. According to Tay and Wallis (2000), first multi-stage density forecasts of inflation derived in that way was published in USA in 1968. Bank of England published its first density forecast of inflation in 1996. From 2000 most central banks, and a lot of research institutes and professional forecasting establishments, started to produce their forecasts in this way.

All these practitioners face obvious problems of imposing an operational definition of inflation uncertainties and deciding on their distribution. There are essentially three ways to measure such uncertainties:

(*i*) By using the concept of *uncertainty by disagreement*. Uncertainty here results from differences between surveys of expectations or individually made forecasts, usually without paying much attention to the way these individual forecasts, point or probabilistic, were made. The intuition is simple here: if the forecasters don’t agree, they are uncertain. This methodology has been pioneered by Bomberger (1996) and continued, in particular, by Diebold *et al.* (1999), Giordani and Söderlind (2003), Lahiri and Liu, (2006), and Pesaran and Weale (2006). Referring to the classification of Walker *et al.* (2003), in its pure form (that is where each individual forecaster formulates point forecasts), this is *epistemic uncertainty*, that is resulted from incomplete knowledge of the system by the experts. In its more comprehensive form, where the forecasters formulate their statements about uncertainty related to non-predictability due to the randomness, it also contains an element of inherent *variability uncertainty*, that is of an unpredictable randomness.

(*ii*) By using the concept of *uncertainty by error*, that is, assuming that uncertainty is a stochastic variable. Parameters of its distribution are obtained through the analysis of past point forecast errors. In this case it is usually assumed implicitly that point forecasts are efficient, from the point of view of the optimal use of all information available at time of producing the forecast, and non-zero forecast errors appears only because of the presence of unforecastable (in mean) innovations. If this is the case, then, in Walker’s classification, this is called *variability uncertainty*. In practice, however, and also in case considered in this paper, forecasts are often made from imperfect models. It is practically impossible to include no-statistical information, experts' judgements, inside information *etc*. in an econometric model. Therefore we are assuming that uncertainties by error, understood as the differences between observed inflation and its past-dependent (econometric) forecast contain both epistemic and variability elements.

(*iii*) By imposing some arbitrary parameters on the distribution of uncertainty (*uncertainty by assumption*).

The practical problem, common to all these ways of assertion (albeit more difficult to tackle in some than in the others), is the effect of *feedback to forecast* on the distribution of the uncertainties. Existence of such feedback has been acknowledged for a long time, but not often analysed. If the forecast of inflation is taken seriously by the monetary authorities and happens to be unfavourable (that is, inflation is to be too high or too low, according to the inflation targeters), they would impose an anti-inflationary or pro-inflationary action, as the result of which inflation would miss the level originally forecasted and the forecast would prove to be inaccurate (see Clements, 2004). Such feedback creates an obvious problem of measuring point forecasts accuracy (see Granger and Pesaran, 2000) and, in further on, uncertainties. For the uncertainties by disagreement, the open question is: has the panel of forecasters imposed some guesses about the possible action of the monetary authorities or not? For the uncertainties by error, one can wonder: has the forecasting model implicitly considered some anti-inflationary action of the central bank? For the uncertainty by assumption, what are the assumptions regarding the forecasts recipients' action and, whatever they are, are they sincere?

On the basis of above classifications, we have identified two elements in the observed inflation. One element is equal to the forecast made on the grounds of all information available in the past, at the time the forecast has been produced. This is usually an econometric forecast, made on the grounds of past economic performance. As this forecast is based on information common to everybody, we are assuming that there is no disagreement in relation to this forecast. As mentioned above, even the best econometric forecast is inefficient in the sense that they are based only on the set of measurable and collectable information available in statistical data sets. Consequently, forecast errors contain some epistemic uncertainty, due to the fact that such forecasts ignore such information which is neither systematic nor directly measurable. This is why we introduce the second non-econometric forecast component, which does not reflect the past performance, but is derived on the basis of the assessment of current economic and political climate and on information of non-systematic and non-statistical nature. In practice it is often called *fine tuning* or *constant-adjustment* and its presence is often not widely advertised. Contrary to econometric forecasts, these non-econometric adds-on are usually subject to disagreement, as experts often differ in their assessments of quantitative effects of not directly measurable phenomena. Hence, differences among them constitute uncertainty by disagreement and might also contain a substantive epistemic element.

It is also assumed here, somewhat strongly, that econometric forecasts are produced without taking into account their possible feedback to inflation (they are *feedback-free*). In another words, these forecasts do not contain second guesses about the inflationary consequences of possible decisions undertaken by monetary authorities which might be based on these forecasts. In fact, this assumption is to some extent stretchy. In practice, forecasters and modellers might, often subconsciously, account for the perspective actions of monetary authorities, as their main priority is to have forecast as accurate as possible. This is ignored here. However, at least in theory, majority of professionally made point econometric forecasts published by central banks and commercial forecasting institutions claim to be feedback-free.

Let's now consider a possible feedback effect to uncertainties. On the logic, if distributions of past econometric forecast errors (that is, uncertainties by error) are used for constructing fan charts and econometric forecasts are feedback-free, uncertainties contain monetary policy feedback. In another words, if current inflation has been, to an extent, affected by past monetary decisions and forecast is policy-free, the effect of the monetary policy has to be in the errors. Uncertainties by assumption are usually claim to be feedback free, that is made under assumption that the forecasters do not assume any change in current monetary policy. However, are the forecasters sincere? After all, they are judged upon the accuracy of their forecasts and the excuse that their forecast was so good that the authorities took it seriously and changed the policy so that it becomes wrong, might not sound serious.

In the light of this 'uncertainty about uncertainty' this paper is primarily concerned with telling the monetary policy related elements of inflationary uncertainties apart from these not related. Confusing these two elements might lead to misunderstandings. Good example here could be the critique of the Bank of England forecasts by Dowd (2007) who discovered that the Bank of England overestimated the inflation uncertainty in the sense that in the period 1997-1999 the observed inflation was within an interval which has a low probability according to the Bank of England fan chart assumptions. We will refer to this observation as *Dowd Puzzle* later in the text.

It is almost universally agreed that distributions of inflation uncertainties might be skewed. Type of skew distribution usually applied in central banks forecasts is two-piece skew normal distribution (see e.g. Wallis, 2004). This distribution is mostly applied for modelling uncertainties by error used in most central banks and uncertainties by assumption used by the Bank of England. In the literature the subject of explanation or interpretation of such skewness is usually not tackled in details. Skewness is sometimes positive, sometimes negative and the fact of its existence is not much commented on. It seems that there is a consensus regarding the statement by Wallis that: 'the degree of skewness shows their collective assessment of the balance of risks on the upside and downside of the forecast' (Wallis, 2004).

While not disputing the fact that risk assessment is an essential factor in explaining skewness in the distribution of inflation uncertainties, this paper concentrates on another possible reason of skewness, namely in explaining how the feedback free uncertainties differ, in terms of distribution, from feedback related uncertainties in the light of a strengths and asymmetry of monetary policy. We argue that, for countries with sound monetary policy, the differences between these distributions result from the effectiveness of the monetary policy, sensitivity of the monetary policy boards to forecast signals and the accuracy of such forecasts. We have shown that an uncomplicated and natural policy rule gives, under the assumption of normally distributed feedback free uncertainties, such feedback related uncertainties that can be explained by a fairly general skew normal distribution that is introduced in this paper. This distribution is called herein the weighted skew-normal distribution, WSN, and the well-known Azzalini skew normal (1985, 1986) is the special case. Parameters of WSN directly reflect the accuracy of fine tuning, efficiency of monetary policy and effective inflation bands, which are the limits of inflation outside which the authorities might be willing to undertake anti-inflationary action. It can be shown that dispersion of the feedback related uncertainties is usually greater than the dispersion of the feedback free uncertainties that might explain the Dowd Puzzle.

3. MONETARY POLICY FEEDBACK INTO UNCERTAINITES

On the basis of general reflections given in Section 2, we analyse the following simple model. Inflation observed in time $t+h$, π_{t+h}, is split into two parts: predictable from the past and nonpredictable from the past. However, inflation nonpredictable from the past can still be forecastable through non-econometric means (fine tuning, or experts' corrections), especially if potential policy decisions can be foreseen. Let's decompose π_{t+h} as:

$$\pi_{t+h} = \hat{\pi}_{t+h} + Z_{t+h} \quad ,$$

where Z_{t+h} contains both pure variability uncertainty and these elements of epistemic uncertainty which are not predictable econometrically. As discussed in Section 2, it is assumed (perhaps too strongly) that the predictable inflation from the past, $\hat{\pi}_{t+h}$, cannot be affected by the monetary policy. The epistemic elements of Z_{t+h} are in fact predictable non-econometrically, e.g. by experts who based their adjustment to the econometric forecast on the basis of non-quantified data, inside information, etc. Due to stochastic nature we call it *imperfect knowledge* in time t regarding inflation in time $t+h$ and denote by Y_{t+h}. We also allow this imperfect knowledge to create a feedback into uncertainties. Let us also denote the feedback-free uncertainties, possibly containing both variability and epistemic elements, by X_{t+h}. If imperfect knowledge contains some substantive information regarding X_{t+h}, this knowledge should be used in a non-econometric way for improving forecast of inflation. In this case, Y_{t+h} should be positively correlated with X_{t+h} . Strictly speaking, such correlation is equal to zero only if X_{t+h} contain only variability (unpredictable in any sense) elements and no epistemic elements. It is equal to one, if X_{t+h} contains only epistemic elements. In an intermediate case, the correlation between two variables tells about the share of the epistemic element in feedback-free uncertainty.

Further in the text we will concentrate on the distributions of Z_{t+h} , X_{t+h} and Y_{t+h} separately for each period of time, so that we can drop the subscripts h and t for the sake of clarity of notation. We will also assume that X_{t+h} and Y_{t+h} have identical variances and zero means, that is Y_{t+h} explains inflation relatively to the econometric forecast, which is regarded as a benchmark and does not contain any stochastic elements. With these assumptions we have arrived at the following model:

$$Z = X + \alpha \cdot Y \cdot I_{Y>\bar{m}} + \beta \cdot Y \cdot I_{Y<\bar{k}} \quad , \tag{1}$$

$$(X,Y) \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 \end{bmatrix} \right) \quad ;$$

where $I_{Y>\bar{m}} = \begin{cases} 1 & \text{if } Y > \bar{m} \\ 0 & \text{otherwise} \end{cases}$, $I_{Y<\bar{k}} = \begin{cases} 1 & \text{if } Y < \bar{k} \\ 0 & \text{otherwise} \end{cases}$.

In this model inflation uncertainties by error represented by Z are affected by the fact that monetary authorities act upon information derived from imperfect knowledge of inflation. We assume that, on the basis of this knowledge, they undertake an anti-inflationary action, which would eventually lead to a reduction in inflation around its forecasted value (and then to an increase in uncertainty by error), if the imperfect knowledge signal is outside certain thresholds.

The thresholds $\bar{m}$ and $\bar{k}$ denote the levels of imperfect knowledge regarding respectively 'high' and 'low' inflation deviations which, if breached, signal to the monetary authorities the necessity of undertaking an anti-inflationary decision (if $\bar{m}$ is breached from below) of pro-inflationary (if $\bar{k}$ is breached from above). Effectiveness of such decisions depends on the strength of the monetary policy towards inflation expressed by α for anti-inflationary policy and by β for pro-inflationary one.

Although formally $\bar{m}\in\mathbb{R}$, $\bar{k}\in\mathbb{R}$, $\alpha\in\mathbb{R}$, $\beta\in\mathbb{R}$ and $-1<\rho<1$, rational behaviour of the policy makers and forecasters implies that $\bar{m}\geq 0$, $\alpha\leq 0$, $\bar{k}\leq 0$ $\beta\leq 0$ and $0<\rho<1$. The greater absolute values of α and β become, the greater would be the effect of the monetary policy on inflation.

Random variable Z defines a family of distributions which, for reasons described further in the text, is named the *weighted skew-normal* variables and abbreviated by $\mathrm{WSN}_{\sigma}(\alpha,\beta,\bar{m},\bar{k},\rho)$. For notational simplicity it is convenient to normalize WSN in such way that σ=1 and:

$$\frac{Z}{\sigma}=Z^{*}\sim\mathrm{WSN}_{1}\left(\alpha\,,\beta,m,k,\rho\right)\quad ,$$

where $m=\bar{m}/\sigma$ and $k=\bar{k}/\sigma$. The probability density function (*pdf*) of Z^{*} is given by:

$$\begin{aligned}f_{WSN_1}(t)=&\frac{1}{\sqrt{A_\alpha}}\varphi\left(\frac{t}{\sqrt{A_\alpha}}\right)\Phi\left(\frac{B_\alpha t-mA_\alpha}{\sqrt{A_\alpha(1-\rho^2)}}\right)+\frac{1}{\sqrt{A_\beta}}\varphi\left(\frac{t}{\sqrt{A_\beta}}\right)\Phi\left(\frac{-B_\beta t+kA_\beta}{\sqrt{A_\beta(1-\rho^2)}}\right)\\&+\varphi(t)\cdot\left[\Phi\left(\frac{m-\rho t}{\sqrt{1-\rho^2}}\right)-\Phi\left(\frac{k-\rho t}{\sqrt{1-\rho^2}}\right)\right]\end{aligned}\quad , (2)$$

where φ and Φ denote respectively the density and cumulative distribution functions of the standard normal distribution, and:

$$A_\alpha=1+2\alpha\rho+\alpha^2\,,\qquad B_\alpha=\alpha+\rho\quad ,$$
$$A_\beta=1+2\beta\rho+\beta^2\,,\qquad B_\beta=\beta+\rho\quad .$$

Moment generating function of $\mathrm{WSN}_1(\alpha,\beta,m,k,\rho)$ is given by:

$$R_{WSN_1}(u)=e^{\frac{u^2}{2}A_\alpha}\Phi(B_\alpha u-m)+e^{\frac{u^2}{2}A_\beta}\Phi(k-B_\beta u)+e^{\frac{u^2}{2}}\cdot\left[\Phi(m-\rho u)-\Phi(k-\rho u)\right]\quad . \quad (3)$$

If in (1) $\alpha=-2\rho$ and $\beta=m=0$, the distribution of Z coincides with the Azzalini (1985, 1986), skew-normal $\mathrm{SN}(\xi)$ distribution with *pdf* $f_{\mathrm{SN}}(t;\xi)=2\varphi(t)\Phi(\xi t)$, where $\xi=\dfrac{-\rho}{\sqrt{1-\rho^2}}$.

It can be shown that *pdf* for weighted skew-normal variable $\mathrm{WSN}_1(\alpha,\beta,m,k,\rho)$ given in (2) can be interpreted as a weighted sum (hence – the name for the distribution) of *pdf*'s for two Azzalini-type skew normal densities with different ξ's and a *pdf* of conditional distribution of $\left(X\,|k\leq Y\leq m\right)$.

The expected value of Z^{*} is: $E(Z^{*})=\alpha\cdot\varphi(m)-\beta\cdot\varphi(k)$. Variance of Z^{*} can be computed from the usual moments decomposition $Var(Z^{*})=E(Z^{*2})-[E(Z^{*})]^2$ where:

$$E(Z^{*2})=A_\alpha+\left[1-A_\alpha\right]\Phi(m)+\left[B_\alpha^2-\rho^2\right]m\varphi(m)+\left[A_\beta-1\right]\Phi(k)-\left[B_\beta^2-\rho^2\right]k\varphi(k)\quad .$$

Proofs of the above formulated properties of WSN_1, generalisations and technique for derivation of moments are given in Appendix A.

As the moments can be interpreted in the context of uncertainties (especially variance), it is interesting to evaluate their dependence on the characteristics of the monetary policy: decision thresholds (k and m), strength of anti-inflationary (α) and output-stimulative (β) monetary policies, and the degree of predictability of X from imperfect knowledge, measured by ρ.

Variability uncertainty, that is non-predictable from Y, element in X, is given by:

$$U = X - E(X \mid Y) = X - \rho Y \quad ,$$

$$(U,Y) \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} (1-\rho^2)\sigma^2 & 0 \\ 0 & \sigma^2 \end{bmatrix} \right) \quad .$$

In another words, U is the feedback-free uncertainty net of epistemic element. Further on we can retrieve the epistemic element of X from Z as:

$$V = Z - E(X \mid Y) = Z - \rho Y \quad .$$

Although V does not contain the epistemic element of X, it is contaminated by it through feedback, as

$$V = Z - E(X \mid Y) = Z - \rho Y = U + \alpha \cdot Y \cdot I_{Y > \bar{m}} + \beta \cdot Y \cdot I_{Y < \bar{k}} \quad ,$$

where the feedback element is equal to $\alpha \cdot Y \cdot I_{Y > \bar{m}} + \beta \cdot Y \cdot I_{Y < \bar{k}}$. Further in the text we refer to V, not very precisely, as *net uncertainties*. Distribution of V is also related to WSN, as

$$\frac{1}{\sigma\sqrt{1-\rho^2}} V \sim \mathrm{WSN}_1\left(\frac{\alpha}{\sqrt{1-\rho^2}}, \frac{\beta}{\sqrt{1-\rho^2}}, \frac{\bar{m}}{\sigma}, \frac{\bar{k}}{\sigma}, 0 \right) \quad .$$

In order to evaluate the relationship between the parameters of WSN and main quantitative characteristics of uncertainty, let us at first consider a fully symmetric case, where the thresholds are fixed at, $\bar{k} = -\bar{m}$ and the anti-and pro-inflationary policy are of identical strength ($\alpha = \beta$). In this case the skewness of the distribution of uncertainties is zero. Figure 1 shows variances of uncertainties in cases where the policy reaction is reasonably infrequent (is undertaken if forecasted inflation exceed its one standard deviation, that is when $\bar{k} = -\bar{m} = 1$, and when it happens every time, that is when $\bar{k} = -\bar{m} = 0$. The parameters α and β representing the monetary policy inflationary effect change from 0 to -0.99, and ρ, representing the non-econometric predictability of feedback-free inflation, changes from 0 to 0.90.

Figure 1: Variance of uncertainties by error, symmetric case

restrained reaction: $\bar{k} = -\bar{m} = 1$ immediate reaction: $\bar{k} = -\bar{m} = 0$

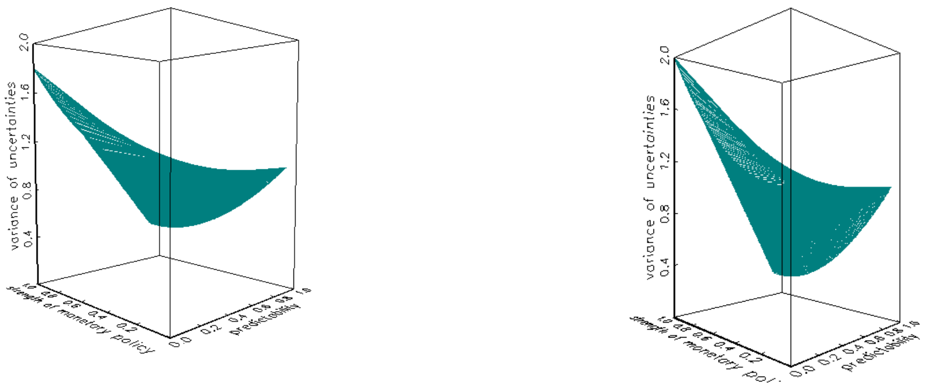


Figure 1 reveals the nonlinear nature of the influence of the parameters of the uncertainties distribution on its variance. In both cases variance reaches minimum when the predictability is about 0.25. The maximum is for the strongest policy and no predictability. For the restrained reaction $\bar{k} = -\bar{m} = 1$ the speed of increase in variance of uncertainty with the increase of strength of monetary policy is visibly smaller than in the case of immediate reaction $\bar{k} = -\bar{m} = 0$.

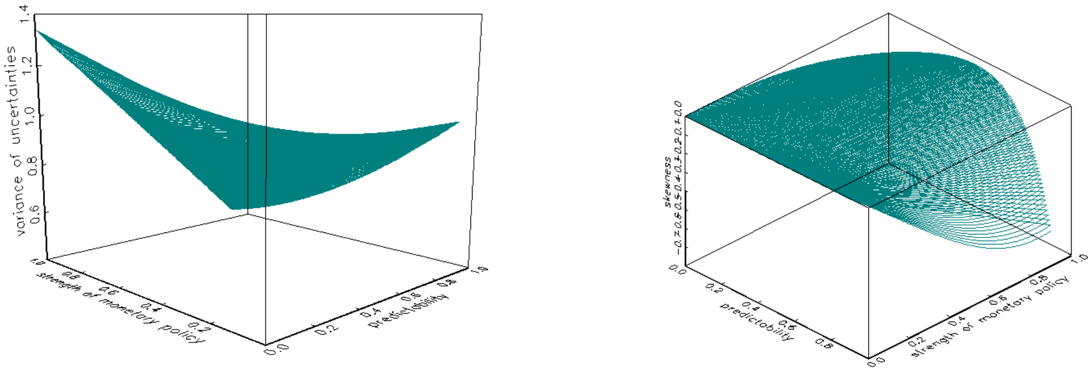
On the first sight, this result is counterintuitive; why should uncertainty regarding inflation increase in case when predictability is high and monetary policy strong? In fact this is a direct consequence of our assumption that point forecast of inflation are feedback-free and the uncertainties are measured by error, that by past econometric forecast errors. Point forecasts are therefore, by implication, less accurate in relation to observed inflation than they are in relation to forecast-free inflation, as monetary policy actions result in the increase in forecast errors. This in turn results in an increased average uncertainty, measured by variance.

The result above sheds a new light on the Dowd Puzzle, according to which variation of the Bank of England inflation uncertainties was too large (see Dowd, 2007). The published Bank of England forecasts claim to be feedback-free and are made under the assumption of the monetary policy being unchanged. Although the Bank of England uncertainties are by assumption rather than by error, it is likely that they reflect, to an extent, variation of the past forecast errors. If this is the case and (*a*) accuracy of fine-tuning, that is non-econometric adjustments to the forecast is good, that is, ρ is high, and (*b*) monetary policy is reasonably efficient, that is α and β are markedly negative, there is no surprise that the Bank of England uncertainties are high and, at the same time, inflation is close to its target.

Figure 2 shows the case of an extreme asymmetric policy where only anti-inflationary policy is undertaken (or is effective) and $\bar{m} = 1$, $\beta = 0$. In this case, evidently, the distribution of the uncertainties becomes skewed.

Figure 2: Variance and skewness of uncertainties by error, asymmetric case, $\bar{m} = 1$, $\beta = 0$

Variance   Skewness



As for the symmetric case, Figure 2 shows that dispersion of the uncertainties falls with the increase of the policy and, separately, predictability. For skewness there is also a nonlinearity. The general pattern is similar to that of dispersion; increase in negative skewness with the increase in non-econometric predictability and strength of the monetary policy (please note the change of axis, here, for a better view). With the increase in monetary policy strength, skewness remains negligible until the non-econometric predictability reaches 0.45. After this point, it raises sharply. We might conjecture therefore that the smallest degree of skewness in the distribution of the uncertainties is for the moderately effective fine tuning forecast.

4. FOUR SIMULATED FAN CHARTS

Formula (1) suggests a convenient way of generating random numbers from $\mathrm{WSN}_1(\alpha,\beta,m,k,\rho)$ distribution. A straightforward algorithm is:

Step 1: generate a pair of random numbers (x, y) from a bivariate normal distribution with zero means, unitary variance and covariance equal to ρ.

Step 2: (a) if $y \le m$ and $y \ge k$: return $z = x$,

(b) if $y > m$: return $z = x + \alpha y$,

(c) if $y < k$: return $z = x + \beta y$.

Simple simulation illustrates the differences between the econometric and feedback-free uncertainties by constructing fan charts. Suppose that inflation is a pseudo-martingale process, that is the best 'econometric' (based on the history of inflation) predictor of inflation in time t + 1 is inflation observed in time t. The process of generating inflation data is:

1. Feedback-free uncertainties and imperfect knowledge:

   $$(\varepsilon_{xt},\varepsilon_{yt}) \sim N\left(0,\begin{bmatrix}1 & \rho\\ \rho & 1\end{bmatrix}\right) \quad , t = -H+1,\ldots,H \quad ,$$

   where H (H>0) denotes the maximum forecast horizon.

2. Uncertainties by error:

   $$\varepsilon_{zt} = \begin{cases}\varepsilon_{xt} + \alpha\cdot\varepsilon_{yt}\cdot I_{\varepsilon_{yt}>m} + \beta\cdot\varepsilon_{yt}\cdot I_{\varepsilon_{yt}<k}, & \text{for } t>0\\ \varepsilon_{xt}, & \text{for } t\le 0\end{cases} \quad ,$$

   where

   $$I_{\varepsilon_{yt}>m} = \begin{cases}1, & \text{if } \varepsilon_{yt} > m\\ 0, & \text{otherwise}\end{cases} \quad , \quad I_{\varepsilon_{yt}<k} = \begin{cases}1, & \text{if } \varepsilon_{yt} < k\\ 0, & \text{otherwise}\end{cases} \quad .$$

3. Net uncertainties:

   $$\varepsilon_{vt} = \varepsilon_{zt} - \rho\varepsilon_{yt} \quad .$$

4. Inflation:

   $$\pi_t = \pi_{t-1} + \varepsilon_{zt} \quad , \quad t = -H+1,\ldots, H \quad . \tag{4}$$

We have simulated fan charts with two types of uncertainties: uncertainties by error, that is directly from (4), and with net uncertainties, that is from:

$$\pi_t^* = \pi_{t-1}^* + \varepsilon_{vt} \quad , \quad t = -H+1,\ldots, H \quad .$$

In order to highlight the difference (in our set-up) in the roles of non-positive t's (t = -H+1,…,0) which correspond to 'observed' time periods and positive t's (t = 1,…,H) which correspond to forecasts periods (up to the period H) we will use notation h for time moments between 1 and H. In all simulations H = 12, that is, forecasts are made for 1,2,..,12 steps ahead. Let's also note that we normalise the series such that $\pi_0 = \pi_0^* = 0$.

We simulate 25,000 of 'realisations' of inflation $\pi_h^{(i)}$ and $\pi_h^{*(i)}$, i = 1, 2,…, 25000, h=1, 2, ..., 12. Figure 3 presents fan-charts for uncertainties by error and net uncertainties for a moderately asymmetric case, where the asymmetry is induced by the different strengths of the anti-inflationary and output-stimulating policies the former being stronger. The parameters of the WSN distribution are: $\alpha = -0.75$, $\beta = -1.00$ $m = -k = 1.00$, $\rho = 0.25$. The coefficient of correlation is set at the level corresponding to the minimum variance of the uncertainties by error, as shown in Figure 2. The dotted lines represent respectively 0.14, 0.29, 0.43, 0.57, 0.71 and 0.86 quantile of the simulated distribution. Solid line which goes from the beginning to the end of the scale represents a single simulated series of z_t which mimics realized inflation. The horizontal line at the level of zero represents the 'martingale' prediction, that is the expected level of inflation in case of full symmetricity and when no information regarding expected inflation can be retrieved from its distribution. The fan charts are centred around mean, with median marked alongside. Table 1 gives the basic descriptive characteristics: means, medians, skewness coefficients excess kurtosis and the *p*-values for the Jarque-Bera normality statistics.

Figure 3: Simulated fan charts, different policy strengths, $\alpha \neq \beta$

Fan chart based on uncertainties by error $\pi_t = \pi_{t-1} + \varepsilon_{zt}$

Fan chart based on net uncertainties $\pi_t^* = \pi_{t-1}^* + \varepsilon_{vt}$

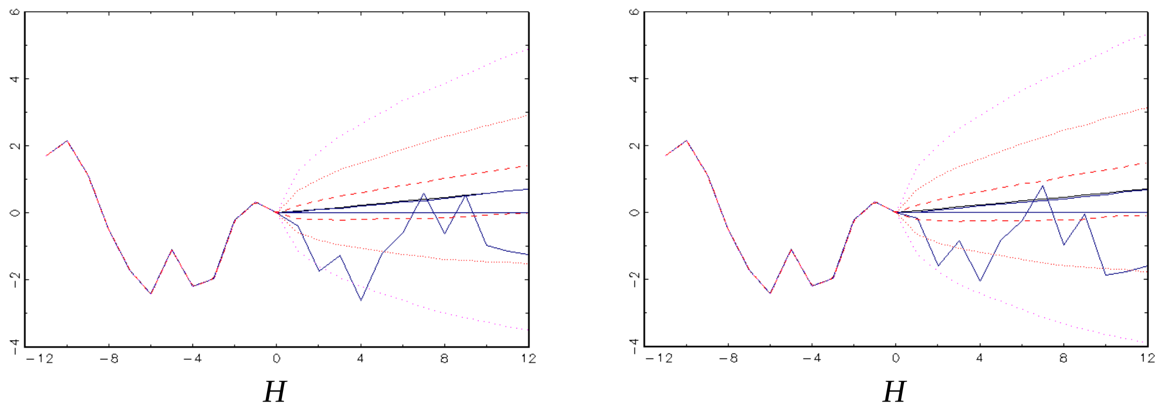


Table 1: Basic descriptive characteristics of uncertainties, $\alpha \neq \beta$

h	uncertainties by error						net uncertainties					
	mean	med	st.dev	skew	Ex.krt	J-Bpv	mean	med	st.dev	Skew	Ex.krt	J-Bpv
1	0.05	0.02	1.13	0.12	0.10	0.00	0.05	0.02	1.25	0.16	0.25	0.00
2	0.11	0.10	1.60	0.07	0.01	0.00	0.11	0.08	1.77	0.11	0.06	0.00
3	0.17	0.14	1.96	0.06	-0.03	0.00	0.17	0.14	2.17	0.08	0.02	0.00
4	0.22	0.20	2.26	0.04	-0.01	0.05	0.23	0.21	2.50	0.06	0.04	0.00
5	0.29	0.27	2.52	0.03	-0.01	0.08	0.29	0.25	2.78	0.06	0.03	0.00
6	0.35	0.32	2.77	0.04	0.00	0.03	0.35	0.31	3.06	0.06	0.04	0.00
7	0.40	0.36	2.99	0.05	0.00	0.00	0.40	0.36	3.31	0.08	0.04	0.00
8	0.46	0.44	3.20	0.04	-0.03	0.02	0.46	0.42	3.53	0.07	0.00	0.00
9	0.52	0.51	3.39	0.04	-0.01	0.04	0.53	0.49	3.75	0.06	0.02	0.00
10	0.59	0.59	3.59	0.04	0.02	0.02	0.60	0.55	3.97	0.06	0.05	0.00
11	0.66	0.66	3.77	0.04	0.03	0.05	0.67	0.63	4.17	0.05	0.05	0.00
12	0.72	0.70	3.92	0.04	0.01	0.07	0.72	0.68	4.34	0.05	0.04	0.00

Results given in Table 1 illustrate the difference between two types of the uncertainties. The uncertainties by error are usually marginally non-normal, with *p*-values of Jarque-Bera statistics being on the verge of significance. The net uncertainties are clearly non-normal, with all Jarque-Bera statistics being virtually zeros. As this is the case for martingale-type processes, variance of the simulated series increases. However, variance of the net uncertainties is bigger than the variance of uncertainties by error.

Figure 4 and Table 2 show analogous results for another asymmetric case, where the strengths of the anti-inflationary and output-stimulating policies are identical ($\alpha = \beta$), but the thresholds ($m \neq -k$). We set $\alpha = \beta = -1.5$, $m = 1$, $k = -0.5$.

Figure 4: simulated fan charts, different thresholds, $m \neq -k$

Fan chart based on uncertainties by error

$$\pi_t = \pi_{t-1} + \varepsilon_{zt}$$

Fan chart based on net uncertainties

$$\pi_t^* = \pi_{t-1}^* + \varepsilon_{vt}$$

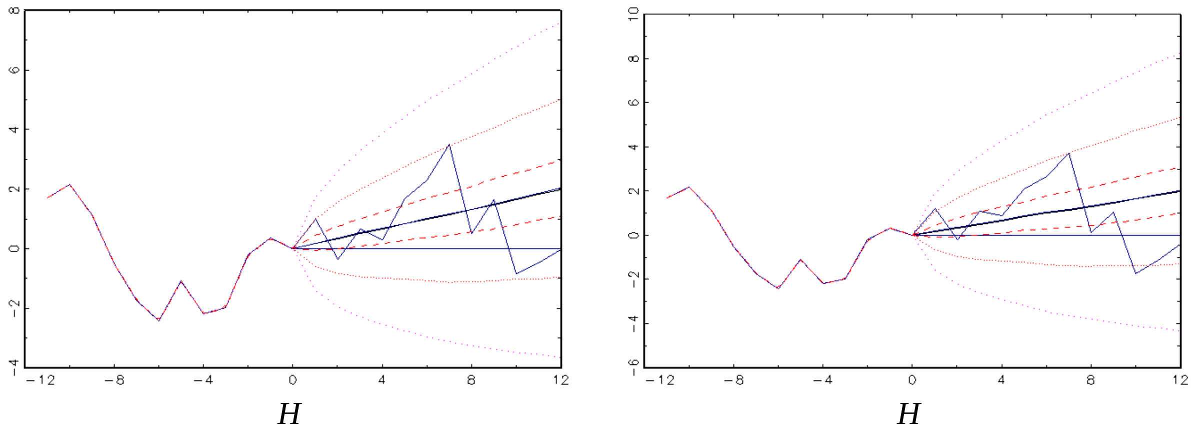


Table 2: Basic descriptive characteristics of uncertainties, $m \neq -k$

For. hor	uncertainties by error						net uncertainties					
	mean	med	st.dev	skew	Ex.krt	J-Bpv	mean	med	st.dev	skew	Ex.krt	J-Bpv
1	0.16	0.19	1.51	-0.15	0.31	0.00	0.17	0.19	1.70	-0.15	0.40	0.00
2	0.33	0.35	2.15	-0.10	0.08	0.00	0.33	0.36	2.41	-0.09	0.12	0.00
3	0.50	0.53	2.62	-0.10	0.09	0.00	0.50	0.54	2.95	-0.09	0.13	0.00
4	0.65	0.69	3.03	-0.10	0.09	0.00	0.66	0.69	3.40	-0.09	0.13	0.00
5	0.82	0.85	3.38	-0.08	0.06	0.00	0.83	0.87	3.79	-0.08	0.09	0.00
6	0.99	1.02	3.72	-0.07	0.05	0.00	0.99	1.05	4.17	-0.07	0.07	0.00
7	1.14	1.17	4.02	-0.04	0.05	0.01	1.14	1.17	4.51	-0.04	0.07	0.00
8	1.30	1.33	4.28	-0.03	0.01	0.12	1.31	1.32	4.80	-0.03	0.03	0.11
9	1.47	1.48	4.55	-0.04	0.01	0.05	1.47	1.48	5.10	-0.04	0.02	0.05
10	1.65	1.66	4.82	-0.04	0.05	0.02	1.65	1.65	5.41	-0.03	0.06	0.01
11	1.82	1.86	5.07	-0.04	0.06	0.00	1.83	1.86	5.69	-0.04	0.07	0.00
12	1.98	2.03	5.27	-0.05	0.03	0.01	1.99	2.03	5.92	-0.05	0.03	0.01

In this case means are also close to each other, non-normality, indicated by Jarque-Bera statistics, is evident, and standard deviations of the net uncertainties are visibly greater than that of the uncertainties by error. This once again illustrates the source of misunderstandings in the Dowd Puzzle. As the Bank of England uncertainties are net (or very close to, if the assertion that they are feedback free is to be taken seriously), there is no wonder that their dispersion is bigger than that given by the measurement of the accuracy of point forecasts.

5. EMPIRICAL RESULTS: ESTIMATION OF UNCERTAINTIES

Parameters of distributions of inflationary uncertainties by error have been estimated for 38 countries: all OECD countries and Brazil, China, India, Indonesia, South Africa and the

Russian Federation. We have alternatively used not-deseasonalised monthly data on monthly inflation (that is on changes in CPI in relation to previous month, called further in the paper *monthly inflation*), and monthly data on annual inflation (changes in CPI in relation to the corresponding month of the previous year, called *annual inflation*). The data series are of various lengths and all ends at February 2013. He longest series starting at January 1949 is for Canada (770 observations) and the shortest are for Estonia (182 observations) and China (242 observations). The raw CPI data can be downloaded from: http://stats.oecd.org/. Clearly in most of these countries inflation targeting in its revealed form has never been implemented. Some of them (e.g. the EU countries) do not have independent monetary policy since the creation of the Euro. Nevertheless, from the type of the best fitted distribution to forecast errors of a particular country, and possibly interpretation of the estimated parameters on this distribution, one might conclude about the predominant patterns of monetary policy effects on inflation.

At the first step, econometric forecast errors have been computed separately for each series in the following way.

1. Orders of seasonal and non-seasonal integration have been identified using the Taylor (2003) test which takes into account the possibility of the presence of unit roots at frequencies other than tested.
2. Initial (starting) period for estimation has been defined as a maximum of the first 80 observations and the 20% of the series length.
3. Forecasts have been made recursively, by using the estimated seasonal autoregressive moving average model (*SARMA*) for the initial period and then by updating the estimation period by one observation at a time and re-estimating the model. The *SARMA* model of a series y_t is defined as:

   $$\phi(B)\Phi(B^s)\Delta^{\kappa}\Delta^{D}y_t = \theta(B)\Theta(B^s)u_t \quad ,$$

   where u_t denotes not autocorrelated, possibly skew normal, residuals, B is the lag operator, $\Delta^{\kappa} = (1-B)^{\kappa}$ is the regular difference operator, κ is the order of integration of the regular part of y_t, $\Delta^{D} = (1-B^s)^{D}$ is the seasonal difference operator for a seasonal I(D) process, $\phi(B) = (1-\phi_1 B - \cdots - \phi_p B^p)$ is the polynomial of order p in the lag operator B and similarly, the seasonal *AR* operator is defined as $\Phi(B^s) = (1-\Phi_1 B^s - \cdots - \Phi_P B^{sP})$. Moving average polynomials, θ and Θ, are defined by their orders denoted by q and Q respectively. The orders p, P, q and Q have been obtained by the Gómez and Maravall (1998) procedure which is based on an automatic lag selection criterion that minimises the Bayesian Information Criteria (BIC). The algorithm applied here is equivalent to the well-known TRAMO-SEATS and X-11 adjustment methods. Forecasts have been made for up to 12 periods ahead. In this paper we present the results for the forecast horizons equal to 1 and 4.

Next, on the basis of the observed and forecasted inflation, forecast errors have been computed and regarded further as the realisations of uncertainties by error. For each country three parameters of the WSN distribution: α, β and σ have been estimated. In order to reduce the computational burden we have assumed that the thresholds are identical for all countries as $m = -k = 1$ and $\rho = 0.25$.

We have compared fit of WSN with that of another three-parameters skew normal distribution, which is popular among fan-chart modellers, namely the two-piece skew normal, TPN (see John, 1982 and Kimber, 1985; for its discussion and use in the context of fan-chart modelling see e.g. Tay and Wallis, 2000). The popular representation of its *pdf* is:

$$f_{TPN}(t)=\begin{cases}A\exp\left\{-(t-\mu)^2/2\sigma_1^2\right\} & \text{if } t\le\mu\\ A\exp\left\{-(t-\mu)^2/2\sigma_2^2\right\} & \text{if } t>\mu\end{cases} ,$$

where $A=\left(\sqrt{2\pi}(\sigma_1+\sigma_2)/2\right)^{-1}$. Three parameters to be estimated are σ_1^2 , σ_2^2 and μ .

Maximum likelihood estimation of parameters of skew normal distributions, albeit formally straightforward, as the density functions are known, is usually numerically awkward, with possible bias and convergence problems (see e.g. Pewsey, 2000, Monti, 2003). For this reason we have decided to apply the minimum distance estimators (*MDE's*) rather than maximum likelihood. Appropriately defined *MDE*'s are asymptotically efficient and asymptotically equivalent to the maximum likelihood estimators (see Basu *et. al* 2011). Additional advantage is ease of their interpretation, as the measures of fit of the theoretical to the empirical distribution, and the possibility of comparison across the model, in order to search for the one which gives the best fit.

The minimum distance criteria can be defined in different ways. In this paper we have concentrated on the Cressie and Read (1984) family of power divergence disparities, defined as:

$$PD_\lambda(d_n\,,\,f_\theta)=\frac{1}{\lambda_{CR}(\lambda_{CR}+1)}\sum_{i=1}^{m+1}d_n(i)\left[\left(\frac{d_n(i)}{f_\theta(i)}\right)^{\lambda_{CR}}-1\right] \quad , \tag{5}$$

where d_n is the empirical (frequency) distribution of data in *m* disjoint intervals and f_θ is the corresponding density function, where the density depends on the vector of parameters θ. For $\lambda_{CR}=1$ formula (5) gives the Pearson χ^2 (*PCS*) measure, for $\lambda_{CR}=-1/2$ the Hellinger twice squared distance (*HD*) and for $\lambda_{CR}=-2$ the Neyman χ^2 measure (*NCS*). For $\lambda_{CR}\to 0$ and $\lambda_{CR}\to -1$ the continuous limits of the right-hand side expression in (5) are respectively the likelihood disparity (*LD*) and the Kullback-Leibler divergence (*KLD*) statistics. Cressie and Read (1984) advocate setting $\lambda_{CR}=3/2$. Although we have computed the minimum distance criteria for all λ_{CR}'s listed above, for further inference we have decided to concentrate on the *HD* distance estimator. Its properties have been well researched in the context of skew normal distributions (see Greco, 2011), and it is known that it is reasonably robust to the presence of outliers, which might appear in a large sample of inflation forecast errors, especially for longer forecast horizons.

Due to numerical problems related to computing the theoretical probabilities of intervals we made a deviation from the established tradition of computing the f_θ densities and obtained the estimates of the densities by simulation. Random number generators of the distributions considered here are straightforward (for WSN see Section 4 above and for TPN see Nakatsuma, 2003). We have simulated the densities over a wide parameters space and, by a grid search, we have located the minimum. Details of the method, called the simulated minimum distance estimator, *SMDE*, are given in Charemza *et. al* (2012); similar approach have been used by Dominicy and Veredas (2013). The version of *SMDE* applied here can be defined as:

$$\hat{\omega}_n^{SMDE}=\arg\min_{\omega\in\Omega}\left\{\mu_w\left(d(g_n\,,\,f_{r,\omega})\right)_{r=1}^{R}\right\} \quad ,$$

where $f_{r,\omega}$ is the approximation of the *pdf* , f_{ω} , of a random variable obtained by generating r = 1,2,…,R replications (drawings) from a distribution with parameters ω ($\omega \in \Omega \subset \mathbb{R}^k$), g_n denotes the density of empirical sample of size n, μ_w is an aggregation operator based on R replications, which deals with the problem of the 'noisy' criterion function (median, in this case), and $d(\bullet,\bullet)$ is the distance measure.

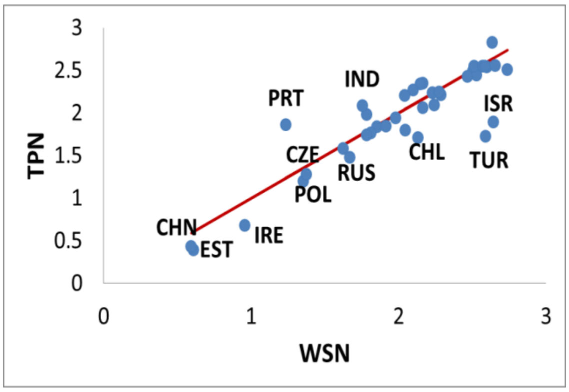
The Hellinger minimum distance measures obtained for all countries for WSN and TPN are given in Appendix B, Table B1. Tables B2 and B3 contain respectively the *SMDE* parameters' estimates for monthly and annual inflation forecasts errors.

Figure 5 illustrates the differences between the logarithms of Hellinger distances obtained for the estimated WSN and TPN distributions. Explanation of labels is given at the bottom of Appendix B.

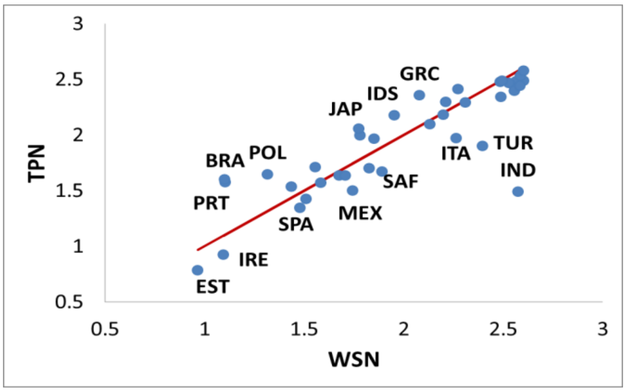
Figure 5: WSN and TPN logarithms of Hellinger distances

monthly inflation

5.a: *fhor*= 1

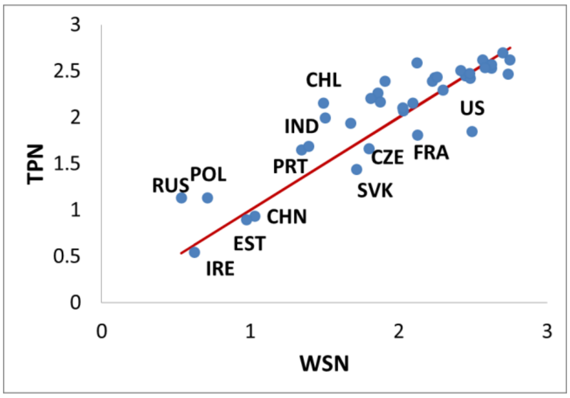


5.b: *fhor* = 4

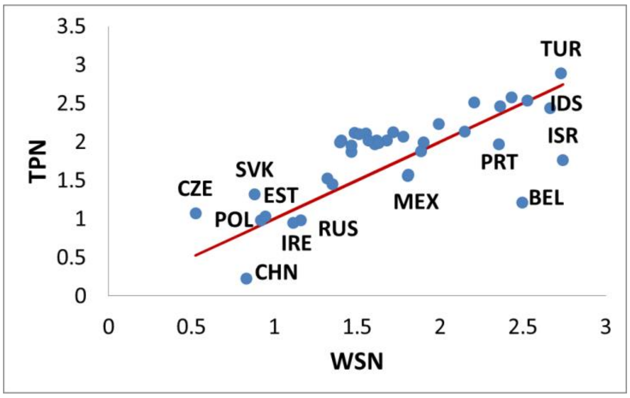


annual inflation

5.c: *fhor* = 1



5.d: *fhor* = 4



In each panel straight 45 degree line represents the points for which the Hellinger distances for the WSN and TPN distribution would be identical. If the dot representing particular a country is below this line, TPN distribution has a better fit than WSN distribution and *vice versa*.

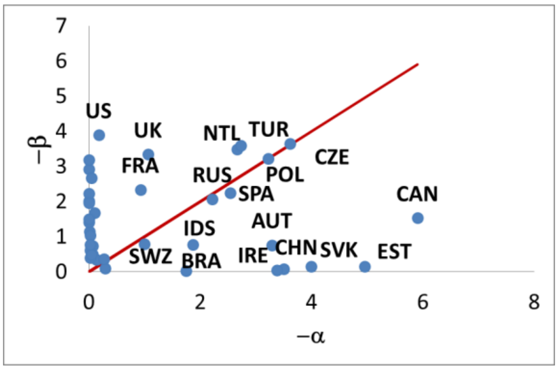
The choice of distribution on the grounds of its fit differs for the monthly and annual inflations. For annual inflation, WSN fits better than TPN in 22 cases out of 38 for the one-step ahead forecast horizon (58%), and for 27 cases for four-steps ahead forecast horizon (71%). For monthly inflation, TPN usually fits better: in 28 cases for one-step ahead forecast (74%) and in 27 cases for four-step ahead forecast (71%). Anyway, substantial differences between the distance measures for these two distributions are rare.

Figure 6 depicts the comparison between estimated α and β parameters (multiplied by -1, for the clarity of graphs). On Figure 6 deviations from the 45 degree line downwards denotes the dominance of anti-inflationary policy and deviations upwards, of the pro-inflationary (output-stimulative) policy.

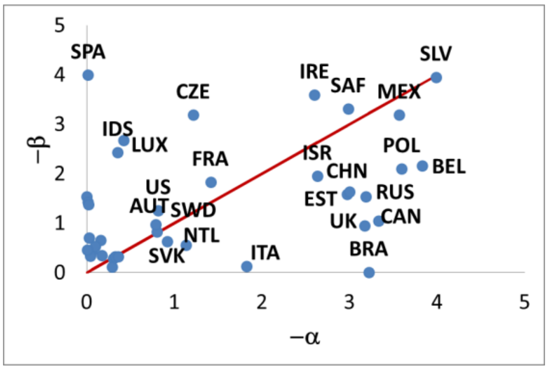
Figure 6: Estimated α and β parameters

monthly inflation

6a: *fhor*= 1

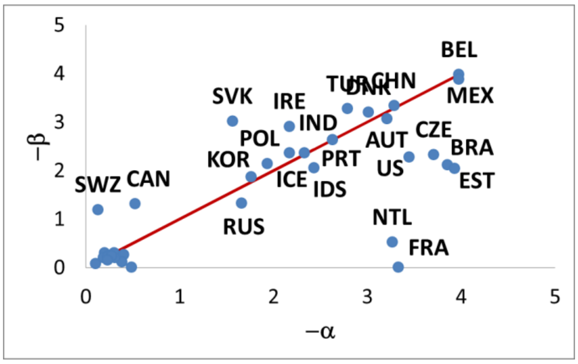


6b: *fhor* = 4

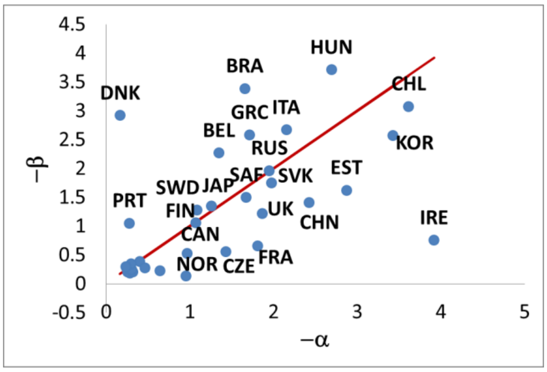


annual inflation

6c: *fhor* = 1



6d: *fhor* = 4



Results differ markedly for different forecast horizons and for the types of inflation forecasted. Not surprisingly, for monthly inflation and short forecast horizon of one month, for a number of countries the estimated α's are very close to zero, with β's being of a non-zero value. This is in line with the widespread assertion that anti-inflationary monetary policy is impotent in a very short run, while output-stimulating policy might have a quicker, albeit short-lived, effect, on inflation uncertainty due to asymmetric information nature of the dynamic budget constraint (see Greenwald and Stiglitz, 1990). Generally, the results seem to be smoother for the annual inflation, where there are fewer estimates close to the boundary of zero. Most interesting are the results for four-steps ahead forecast, which might reveal possible footprints of medium-term effects of monetary policy. In panel 6b, in the area of stronger anti-inflationary policy, are countries pursuing for a long time the 'dirty float' policy aiming at stabilizing of domestic currency under prolonged appreciation pressure such as China and Russia, Canada, which official monetary policy is anti-inflationary rather than output-stimulating, Brazil, with its periodic drastic anti-inflationary measures and UK, where Bank of England seems to be more preoccupied with keeping inflation at bay rather than stimulation of output. This is, evidently, just a snapshot and data given at Figure 6 deserves more detailed analysis.

6. CONCLUSIONS AND DIRECTIONS FOR FURTHER RESEARCH

It appears that forecast errors of inflation might tell us more than just by how much inflation experts err. But, in order to squeeze out more information out of them, the statistical distribution which describes these errors has to be identified and estimated. In this respect our results are promising, but by no means complete. More has to be done on the evaluation of forecast error uncertainties and relaxing assumptions regarding thresholds and imperfect knowledge effects on uncertainties. Nevertheless, it is already possible to learn more about the different types and nature of inflation uncertainty, and also on the effects of monetary policy actions onto it.

In the light of current results, perspective for further work looks promising. It is possible to construct two complementary types of fan charts: one derived directly from the observed forecast errors interpreted as monetary policy restricted uncertainties, and the other one, derived indirectly (using computed parameters of the weighted skew normal distribution), from the net uncertainties which are free of the epistemic element, if only forecasters are not trying to second guess the policy makers. Both types of fan charts could be used for different practical purposes, and possibly by different users; the former one by central bankers and other policy decision makers and the latter one by ‘end users’, who do not have direct influence on monetary policy. It might also be worthwhile to compute feedback correction to forecasts and to find out to what extent forecasters might not be sincere in their claim that they have not tried to second guess possible policy outcomes which might be undertaken on the basis of information they provide.

APPENDIX A

Properties of weighted skew-normal distribution

Definition 1. Let

$$(X_0,Y_0)\sim N\left(\begin{bmatrix}0\\0\end{bmatrix},\begin{bmatrix}1&\rho\\\rho&1\end{bmatrix}\right)\text{ with }|\rho|<1, \tag{A.1}$$

and

$$Z^{(1)}=X+\alpha\cdot Y\cdot I_{Y_0>m}+\beta\cdot Y\cdot I_{Y_0<k}\quad , \tag{A.2}$$

where $I_{Y>\bar{m}}$ is an indicator of $\{Y_0>m\}$ and is equal to 1 if $Y_0>m$ and 0 otherwise. Similarly, $I_{Y_0<k}$ is an indicator of $\{Y_0<k\}$. We will call distribution of $Z^{(1)}$ defined by(A.1)-(A.2) *standard weighted skew normal* and the family of standard weighted skew normal distributions will be denoted as WSN_1, and we will write $Z^{(1)}\sim\mathrm{WSN}_1(\alpha,\beta,m,k,\rho)$.

The probability density function (*pdf*) of $Z^{(1)}$ is given by:

$$\begin{aligned}f_{\mathrm{WSN}_1}(t)=&\frac{1}{\sqrt{A_\alpha}}\varphi\left(\frac{t}{\sqrt{A_\alpha}}\right)\Phi\left(\frac{B_\alpha t-mA_\alpha}{\sqrt{A_\alpha(1-\rho^2)}}\right)+\frac{1}{\sqrt{A_\beta}}\varphi\left(\frac{t}{\sqrt{A_\beta}}\right)\Phi\left(\frac{-B_\beta t+kA_\beta}{\sqrt{A_\beta(1-\rho^2)}}\right)\\&+\varphi(t)\cdot\left[\Phi\left(\frac{m-\rho t}{\sqrt{1-\rho^2}}\right)-\Phi\left(\frac{k-\rho t}{\sqrt{1-\rho^2}}\right)\right]\end{aligned} \tag{A.3}$$

where φ and Φ denote respectively the density and cumulative distribution functions of the standard normal distribution, and functions $A_\bullet$ and $B_\bullet$ are :

$$A_\tau=1+2\tau\rho+\tau^2,\qquad\qquad B_\tau=\tau+\rho\quad , \tag{A.4}$$

Definition 1 can be straightforward generalised in the following way:

Definition 2. Let

$$(X,Y)\sim N\left(\begin{bmatrix}\mu_X\\\mu_Y\end{bmatrix},\begin{bmatrix}\sigma_X^2&\rho\sigma_X\sigma_Y\\\rho\sigma_X\sigma_Y&\sigma_Y^2\end{bmatrix}\right)\text{ with }|\rho|<1. \tag{A.5}$$

The weighted skew-normal distribution $\mathrm{WSN}_\sigma^{(\mu_X,\mu_Y)}(\alpha,\beta,m,k,\rho)$ is defined as the distribution of random variable Z given by

$$Z=X+\alpha\cdot Y\cdot I_{Y>m}+\beta\cdot Y\cdot I_{Y<k}\quad , \tag{A.6}$$

where $I_{Y>m}$ and $I_{Y<k}$ are corresponding indicators.

When $\mu_X=\mu_Y=0$ in (A.5), for Z defined by (A.6) we will omit superscript and write: $\mathrm{Z}\sim\mathrm{WSN}_\sigma(\alpha,\beta,m,k,\rho)$ in order to simplify notation.

NOTE 1. A standard re-parameterisation of (A.5)-(A.6) shows that if $Z\sim\mathrm{WSN}_\sigma^{(\mu_X,\mu_Y)}(\alpha,\beta,m,k,\rho)$ then it can be represented as:

$$Z = \sigma_X Z^{(1)} + \mu_X + \mu_Y \cdot \left(\alpha I_{Y_0 > \frac{m-\mu_y}{\sigma_Y}} + \beta I_{Y_0 < \frac{k-\mu_y}{\sigma_Y}} \right) \quad , \tag{A.7}$$

where Y_0 and $Z^{(1)}$ are jointly defined by (A.1)-(A.2) and, hence, $Z^{(1)} \sim \mathrm{WSN}_1\left(\alpha \frac{\sigma_Y}{\sigma_X}, \beta \frac{\sigma_Y}{\sigma_X}, \frac{m-\mu_Y}{\sigma_Y}, \frac{k-\mu_Y}{\sigma_Y}, \rho \right)$.

PROPOSITION 1. The *pdf* $f_{\mathrm{WSN}_1}(\bullet)$ for weighted skew-normal variable $\mathrm{WSN}_1(\alpha, \beta, m, k, \rho)$ can be represented as a weighted sum (hence – the name for the distribution) of *pdf*'s for two Azzalini-type skew normal densities with different λ's and a *pdf* of conditional variable $\left(X \,|\, k \le Y \le m \right)$.

PROOF.

Let $f_{\mathrm{WSN}_1}(t) = \alpha_1 f_1(t) + \alpha_2 f_2(t) + \alpha_3 f_3(t)$,

where:

$$f_1(t) = \frac{1}{\Phi(-m)} \frac{1}{\sqrt{A_\alpha}} \varphi\left(\frac{t}{\sqrt{A_\alpha}} \right) \Phi\left(\frac{B_\alpha t - m A_\alpha}{\sqrt{A_\alpha (1-\rho^2)}} \right) \quad , \qquad \alpha_1 = \Phi(-m)$$

$$f_2(t) = \frac{1}{\Phi(k)} \frac{1}{\sqrt{A_\beta}} \varphi\left(\frac{t}{\sqrt{A_\beta}} \right) \Phi\left(\frac{-B_\beta t + k A_\beta}{\sqrt{A_\beta (1-\rho^2)}} \right) \quad , \qquad \alpha_2 = \Phi(k)$$

$$f_3(t) = \frac{1}{\Phi(m) - \Phi(k)} \varphi(t) \cdot \left[\Phi\left(\frac{m - \rho t}{\sqrt{1-\rho^2}} \right) - \Phi\left(\frac{k - \rho t}{\sqrt{1-\rho^2}} \right) \right], \quad \alpha_3 = \Phi(m) - \Phi(k)$$

Note that

1) $\Phi(-m) + \Phi(k) + \left[\Phi(m) - \Phi(-k) \right] = 1$;

2) $\int_{-\infty}^{+\infty} f_i dt = 1$, $i = 1, 2, 3$;

3) functions f_1 and f_2 for $m = k = 0$ and $\alpha = -2\rho$ are *pdf*'s for Azzalini (1985, 1986), skew-normal $\mathrm{SN}(\lambda_1)$ and $\mathrm{SN}(\lambda_2)$ with $\lambda_1 = -\rho / \sqrt{1-\rho^2}$ and $\lambda_2 = \rho / \sqrt{1-\rho^2}$ correspondingly;

4) function f_3 is a *pdf* of $\left(X \,|\, k \le Y \le m \right)$ where $\left(X, Y \right)$ is defined by (A.1).

Q.E.D.

PROPOSITION 2. Moment generating function for $\mathrm{WSN}_1(\alpha, \beta, m, k, \rho)$ is given by :

$$R_{\mathrm{WSN}_1}(u) = e^{\frac{u^2}{2} A_\alpha} \Phi(B_\alpha u - m) + e^{\frac{u^2}{2} A_\beta} \Phi(k - B_\beta u) + e^{\frac{u^2}{2}} \cdot \left[\Phi(m - \rho u) - \Phi(k - \rho u) \right], \tag{A.8}$$

where Φ denotes cumulative distribution functions of the standard normal distribution, and $A_\bullet$ and $B_\bullet$ are given by (A.4).

The PROOF consists of applying standard tools for integrating:

$$R_{WSN_1}(u) = E\left(e^{u\cdot Z_1}\right)$$

$$= \frac{1}{2\pi\sqrt{1-\rho^2}} \int_{-\infty}^{+\infty} dx \left[\int_{-\infty}^{k} e^{u[x+\beta\cdot y]} + \int_{k}^{m} e^{ux} + \int_{m}^{+\infty} e^{u[x+\alpha\cdot y]} \right] \cdot e^{-\frac{x^2-2\rho xy+y^2}{2(1-\rho^2)}} dy$$

Q.E.D.

PROPOSITION 3.

Let R_{WSN_1} be a moment generating function (MGF) given by(A.8), then

$$R'_{\mathrm{WSN}_1}(0) = \alpha\cdot\varphi(m) - \beta\cdot\varphi(k);$$

$$R''_{\mathrm{WSN}_1}(0) = A_\alpha + [1-A_\alpha]\Phi(m) + \left[B_\alpha^2-\rho^2\right]m\varphi(m) + \left[A_\beta-1\right]\Phi(k) - \left[B_\beta^2-\rho^2\right]k\varphi(k);$$

$$\begin{aligned} R^{(3)}_{\mathrm{WSN}_1}(0) = {} & \varphi(m)\cdot\left\{B_\alpha\cdot\left[3A_\alpha + B_\alpha^2\left(m^2-1\right)\right] - \rho\cdot\left[3+\rho^2\left(m^2-1\right)\right]\right\} + \\ & \varphi(k)\cdot\left\{-B_\beta\cdot\left[3A_\beta + B_\beta^2\left(k^2-1\right)\right] + \rho\cdot\left[3+\rho^2\left(k^2-1\right)\right]\right\} \end{aligned};$$

$$\begin{aligned} R^{(4)}_{\mathrm{WSN}_1} = {} & 3\cdot\left\{A_\alpha^2 + \Phi(m)\cdot\left[1-A_\alpha^2\right]\right\} + m\cdot\varphi(m)\cdot\left\{\left(3-m^2\right)\cdot\left(\rho^4-B_\alpha^4\right) + 6\cdot\left(A_\alpha B_\alpha^2-\rho^2\right)\right\} \\ & + 3\cdot\Phi(k)\cdot\left[A_\beta^2-1\right] - k\cdot\varphi(k)\left\{\left(3-k^2\right)\cdot\left(\rho^4-B_\beta^4\right) + 6\cdot\left(A_\beta B_\beta^2-\rho^2\right)\right\} \end{aligned}.$$

PROOF.

Let $g_{a,b,c}(u) = e^{\frac{au^2}{2}}\Phi(bu+c)$ (A.9)

Taylor expanding: $e^{\frac{au^2}{2}} = 1 + \frac{au^2}{2} + \frac{a^2u^4}{8} + \ldots$ (A.10)

and $\Phi(bu+c) = \Phi(c) + b\varphi(c)\cdot u - \frac{b^2c}{2}\varphi(c)\cdot u^2 + \frac{b^3(c^2-1)}{3!}\varphi(c)\cdot u^3 + \frac{b^4c(3-c^2)}{4!}\varphi(c)\cdot u^4 + \ldots$
(A.11)

Substituting (A.10) and (A.11) to (A.9) yields:

$$g_{a,b,c}(u) =$$

$$= \left(1 + \frac{au^2}{2} + \frac{a^2u^4}{8} + \ldots\right)\cdot\left(\Phi(c) + b\varphi(c)\cdot u - \frac{b^2c}{2}\varphi(c)\cdot u^2 + \frac{b^3(c^2-1)}{3!}\varphi(c)\cdot u^3 + \frac{b^4c(3-c^2)}{4!}\varphi(c)\cdot u^4 + \ldots\right)$$

$$= \Phi(c) + b\varphi(c)\cdot u + \frac{1}{2}\left[a\cdot\Phi(c) - b^2c\cdot\varphi(c)\right]\cdot u^2 + \frac{1}{3!}\left[3a + b^2(c^2-1)\right]b\varphi(c)\cdot u^3$$

$$+ \frac{1}{4!}\left[3a^2\Phi(c) - 6ab^2c\varphi(c) + b^4c(3-c^2)\varphi(c)\right]\cdot u^4 + \ldots$$

Therefore:

$g'_{a,b,c}(0) = b\varphi(c)$ (A.12)

$$g^{"}_{a,b,c}(0) = a\cdot\Phi(c) - b^2 c\cdot\varphi(c) \tag{A.13}$$

$$g^{(3)}_{a,b,c}(0) = \left[3a + b^2(c^2-1)\right]b\varphi(c) \tag{A.14}$$

$$g^{(4)}_{a,b,c}(0) = 3a^2\Phi(c) - 6ab^2c\varphi(c) + b^4c(3-c^2)\varphi(c) \tag{A.15}$$

Bearing in mind that

$$R_{\mathrm{WSN}_1}(u) = g_{A_\alpha, B_\alpha, (-m)}(u) + g_{A_\beta,(-B_\beta),k}(u) + g_{1,(-\rho),m}(u) - g_{1,(-\rho),k}(u) \quad . \tag{A.16}$$

Taking derivative of the both sides of (A.16) and substituting (A.12)-(A.15) complete the proof.

Q.E.D.

COLORRARY. Calculation of moments for WSN_1 is based on the basic property of MGF, that is $R^{(j)}_{\mathrm{WSN}_1}(0) = E\left(Z^j\right)$ ($Z \sim \mathrm{WSN}_1(\alpha,\beta,m,k,\rho),\ j\in\mathbb{N}$), Proposition 3 and the usual formulae for central moments give:

$$Var\left(Z_0\right) = E\left(Z_0^2\right) - \left[E\left(Z_0\right)\right]^2;$$

$$Sk\left(Z_0\right) = \frac{E(Z_0^3) - 3\cdot E(Z_0^2)\cdot E(Z_0) + 2\cdot\left[E(Z_0)\right]^3}{\left[Var(Z_0)\right]^{3/2}};$$

$$ExKu(Z_0) = \frac{E(Z_0^4) - 4\cdot E(Z_0^3)\cdot E(Z_0) + 6\cdot E(Z_0^2)\cdot\left[E(Z_0)\right]^2 - 3\left[E(Z_0)\right]^4}{\left[Var(Z_0)\right]^2} - 3.$$

PROPOSITION 4. Moment generating function R_{WSN} for $Z \sim \mathrm{WSN}_\sigma^{(\mu_X,\mu_Y)}(\alpha,\beta,m,k,\rho)$ defined by (A.5)-(A.6) can be is given by

$$\begin{aligned} R_{\mathrm{WSN}}(u) = {} & e^{\frac{(u\sigma_X)^2}{2}A_{\alpha\frac{\sigma_Y}{\sigma_X}}}\Phi\left(B_{\alpha\frac{\sigma_Y}{\sigma_X}}u\sigma_X - \frac{m-\mu_Y}{\sigma_Y}\right) + e^{\frac{(u\sigma_X)^2}{2}A_{\beta\frac{\sigma_Y}{\sigma_X}}}\Phi\left(\frac{k-\mu_Y}{\sigma_Y} - B_{\beta\frac{\sigma_Y}{\sigma_X}}u\sigma_X\right) + \\ & + e^{\frac{(u\sigma_X)^2}{2}}\cdot\left[\Phi\left(\frac{m-\mu_Y}{\sigma_Y} - \rho u\sigma_X\right) - \Phi\left(\frac{k-\mu_Y}{\sigma_Y} - \rho u\sigma_X\right)\right] \end{aligned},$$

The PROOF consists of applying standard tools for integrating and (A.8) to

$$R_{\mathrm{WSN}}(u) = E\left(e^{u\cdot Z}\right) =$$

$$= \frac{1}{2\pi\sqrt{1-\rho^2}}\int_{-\infty}^{+\infty}dx\left[\int_{-\infty}^{\frac{k-\mu_Y}{\sigma_Y}} e^{u\left[\sigma_X\left\{x+\beta\frac{\sigma_Y}{\sigma_X}\cdot y\right\}+\mu_X+\beta\mu_Y\right]} + \int_{\frac{k-\mu_Y}{\sigma_Y}}^{\frac{m-\mu_Y}{\sigma_Y}} e^{u\{\sigma_X x+\mu_X\}} + \int_{\frac{m-\mu_Y}{\sigma_Y}}^{+\infty} e^{u\left[\sigma_X\left\{x+\alpha\frac{\sigma_Y}{\sigma_X}\cdot y\right\}+\mu_X+\alpha\mu_Y\right]}\right]\cdot e^{-\frac{x^2-2\rho xy+y^2}{2(1-\rho^2)}}dy$$

Q.E.D.

Appendix B: Estimates of the distributions of forecasts errors

Table B1- Hellinger distances for SARMA forecasts errors

	Monthly inflation				Annual inflation			
	WSN		TPN		WSN		TPN	
	fhor=1	fhor =4	fhor =1	fhor =4	fhor =1	fhor =4	fhor =1	fhor =4
AUT	319.3	306.3	312	301.1	282.8	79.91	279.7	99.1
BEL	145.9	162.9	226	199.6	108	313.8	117.8	16.36
BRA	42.26	12.67	38.29	37.56	502	271.3	502	378.4
CAN	193.8	135	164.9	124.9	175.9	25.33	267.3	103.7
CHL	134.8	47.48	51.77	43.55	31.09	24.9	142.6	98.38
CHN	4.057	27.35	2.476	3.453	10.78	6.802	8.538	1.69
CZE	22.53	50.82	15.79	43.34	63.45	3.355	46.25	11.85
DNK	71.38	71.01	69.68	92.2	424.2	98.47	339.5	170.1
EST	3.929	9.235	2.718	6.12	9.501	8.334	7.817	9.662
FIN	174.8	186.9	125.6	260.2	262.9	30.48	322.9	130.3
FRA	65.15	38.43	58.67	37.27	134.4	64.13	64.09	35.91
GER	336.4	398.8	282	308.6	198.6	9.921	196.9	129.1
GRC	141.2	120.1	218.9	226.7	80.96	47.92	244.9	103.9
HUN	60.85	60.52	95.86	99.31	72.34	32.48	183.8	126.2
ICE	95.09	368	88.05	304	64.99	20.92	161.8	33.71
IND	56.76	375.7	121.8	30.9	22.26	77.16	45	75.89
IDS	125.7	89.77	187.9	150.2	133.2	463.7	386.4	272.6
IRE	9.104	12.43	4.837	8.414	4.209	14.43	3.494	9.573
ISR	438.8	35.97	79.21	51.53	563.4	552.8	417.9	57.81
ITA	431.8	182.9	678.5	94.26	383.5	160.8	380.5	322.1
JAP	110	59.31	160.2	114.7	125.1	41.43	143.5	105.2
KOR	110.9	360.1	63.73	249.8	32.19	40.51	98.92	92.52
LUX	325.1	337.2	358.6	292.8	304.9	42.69	265.9	97.58
MEX	169.1	55.35	173.1	31.61	168.7	64.57	245.9	37.28
NTL	370.9	313.1	354.9	306.9	300.8	141.8	297.6	135.2
NOR	397.5	386.1	352.1	339.4	422.5	52.18	374.4	132.2
POL	23.71	20.76	19.09	44.34	5.185	8.841	13.66	10.72
PRT	17.19	12.56	72.83	39.88	24.74	227.9	48.59	92.44
RUS	46.81	32.29	30.52	26.56	3.45	13.03	13.64	8.851
SVK	82.42	67.05	71.16	50.51	52.22	7.652	27.6	20.9
SLV	546.1	204	323.8	196.6	548.1	232	294.9	292.7
SAF	60.93	77.6	55.72	46.74	106.6	22.47	127.4	28.17
SPA	145.7	30.22	116.1	22.27	47.65	337	86.05	342
SWD	292.5	382.8	271.2	277.8	368.8	29.27	421.3	73.9
SWZ	450.7	308.8	360.2	219.6	381.2	37.14	346.8	105.1
TUR	387	249.3	53.98	79.65	75.2	539.2	148.3	771.4
UK	188.3	158.4	175.7	151.8	181.3	60.01	271.7	116.2
US	383.3	401.5	356.1	379.7	313.2	29.23	318.5	90.31

Table B2: Parameters' estimates, Hellinger measures, monthly inflation

country	fhor	α	β	σ	se(α)	se(β)	se(σ)
AUT	1	-3.29100	-0.75630	2.32200	0.21870	0.36770	0.25900
	4	-0.80330	-0.82160	3.46600	1.25400	0.43760	0.83410
BEL	1	-0.06816	-0.72740	0.01002	0.21550	0.73560	0.00007
	4	-3.83200	-2.15400	0.01002	0.53100	0.27250	0.00007
BRA	1	-1.74900	-0.02128	0.24260	0.97730	0.06730	0.27390
	4	-3.22300	-0.00355	0.27630	0.32480	0.01123	0.16730
CAN	1	-5.90500	-1.52200	3.61100	0.45820	0.50000	0.78830
	4	-3.33100	-1.04200	3.57500	0.85010	1.01800	0.33340
CHL	1	-0.00781	-2.01700	0.01024	0.02469	0.45160	0.00075
	4	-0.00063	-1.53600	0.01005	0.00201	0.30350	0.00016
CHN	1	-3.50200	-0.07345	3.52700	0.31060	0.23230	0.01894
	4	-2.97700	-1.57700	3.98000	0.30650	0.43250	0.06354
CZE	1	-3.61300	-3.64700	3.14100	0.04222	0.14900	0.81540
	4	-1.22100	-3.18200	3.39700	0.69190	0.56270	0.39360
DNK	1	-0.03154	-0.60660	0.01002	0.09975	0.40030	0.00006
	4	-0.03072	-0.70390	0.01001	0.09713	0.70790	0.00002
EST	1	-4.95700	-0.14300	3.09800	1.78100	0.45240	0.32830
	4	-3.00400	-1.62600	3.97900	0.39100	0.42420	0.06620
FIN	1	-0.01553	-1.13100	0.01001	0.04912	0.54000	0.00003
	4	-0.06448	-0.44790	0.01002	0.19340	0.09635	0.00006
FRA	1	-0.93090	-2.33100	2.84400	1.61000	0.54020	0.88530
	4	-1.41600	-1.82400	3.74800	2.35300	1.97300	0.21260
GER	1	-0.11000	-1.67100	3.55800	0.15810	1.23500	0.11580
	4	-0.16300	-0.64970	3.70800	0.48900	0.93100	0.87720
GRC	1	-0.29880	-0.08624	0.01001	0.17630	0.46130	0.00002
	4	-0.30760	-0.28130	0.01001	0.20270	0.12400	0.00004
HUN	1	-0.00887	-1.41400	0.01004	0.02803	0.08158	0.00012
	4	-0.01677	-1.42700	0.01024	0.05304	0.04078	0.00076
ICE	1	-0.00784	-2.91700	0.01050	0.02478	0.11710	0.00158
	4	-0.17610	-0.34410	0.07698	0.20200	0.32930	0.79760
IND	1	-0.00563	-1.48600	0.01047	0.01779	0.14490	0.00149
	4	-0.33060	-0.32330	0.07300	0.03362	0.49560	0.81020
IDS	1	-1.87400	-0.76600	0.01001	0.14560	1.12000	0.00004
	4	-0.35570	-2.42700	0.01015	0.39310	1.60500	0.00047
IRE	1	-3.37600	-0.03222	3.95600	0.04782	0.09665	0.13150
	4	-2.60300	-3.58900	3.70900	0.13700	0.21860	0.41480
ISR	1	-0.06614	-0.48700	0.04443	0.29680	0.52810	0.90050
	4	-2.63500	-1.94600	0.01008	0.77620	0.67800	0.00027
ITA	1	-0.02989	-1.03300	0.01000	0.09452	0.23160	0.00001
	4	-1.82800	-0.12980	3.45100	0.28950	0.41040	0.72530
JAP	1	-0.13670	-0.34300	0.01001	0.06984	0.41090	0.00002
	4	-0.04683	-0.33460	0.01002	0.14050	0.28380	0.00006

country	fhor	α	β	σ	se(α)	se(β)	se(σ)
KOR	1	-0.00514	-2.21400	0.01196	0.01624	0.08186	0.00621
	4	-0.29150	-0.11300	0.07792	0.16280	0.40160	0.79460
LUX	1	-0.27750	-0.35050	0.01000	0.35260	0.38860	0.00000
	4	-0.42110	-2.66800	2.98300	0.69220	0.34180	0.69270
MEX	1	-0.00405	-3.17600	0.01242	0.01280	0.58330	0.00766
	4	-3.57100	-3.18300	0.01322	0.66790	0.55900	0.01018
NTL	1	-2.66700	-3.47500	2.37900	0.84440	0.36260	0.93130
	4	-1.13800	-0.55770	2.50100	0.44750	1.27200	1.20600
NOR	1	-0.04477	-2.67400	3.55600	0.14160	0.14410	0.39440
	4	-0.36040	-0.31680	3.95800	0.36110	0.95040	0.83170
POL	1	-3.21900	-3.22400	0.01486	0.44580	0.42960	0.01536
	4	-3.59400	-2.09500	0.88250	0.74100	0.20540	0.26910
PRT	1	-0.00384	-1.95200	0.01004	0.01213	0.60650	0.00014
	4	-0.02383	-1.37100	0.01014	0.07534	0.21850	0.00043
RUS	1	-2.21900	-2.06000	3.97900	0.57090	0.44170	0.06716
	4	-3.19100	-1.53500	3.29700	0.02902	0.30110	0.30030
SVK	1	-3.99100	-0.15170	3.14100	0.02717	0.47970	0.19340
	4	-0.92130	-0.62760	3.97200	0.62820	0.03926	0.92240
SLV	1	-0.02488	-0.39930	0.07361	0.42730	0.25080	0.80820
	4	-3.99100	-3.94900	0.01617	0.02636	0.14450	0.01745
SAF	1	-0.04627	-0.49130	0.01001	0.14630	0.47010	0.00002
	4	-2.98500	-3.30800	3.35400	0.17480	1.35500	0.02267
SPA	1	-2.54600	-2.23200	0.01007	0.96810	0.02470	0.00021
	4	-0.01906	-3.99000	0.01286	0.05390	0.02931	0.00809
SWD	1	-0.02819	-0.76830	0.01002	0.08913	0.60610	0.00005
	4	-0.78850	-0.96560	3.74200	0.46960	1.03000	0.19290
SWZ	1	-0.99560	-0.78080	2.39300	1.12500	0.06057	1.04300
	4	-0.10080	-0.52280	3.54300	0.18730	0.64140	0.06814
TUR	1	-2.72700	-3.58600	0.01449	0.02263	0.80180	0.01420
	4	-0.00948	-0.45890	0.05800	0.47600	0.06673	0.85760
UK	1	-1.06300	-3.33400	3.56300	0.32420	0.58910	0.37330
	4	-3.17600	-0.94740	2.38300	0.43030	0.54570	0.43940
US	1	-0.18170	-3.88400	2.01300	0.06849	0.36740	1.23600
	4	-0.81940	-1.25100	2.68200	0.06138	0.92070	0.37420

Table B3: Parameters' estimates, Hellinger measures, annual inflation

country	fhor	α	β	σ	se(α)	se(β)	se(σ)
AUT	1	-3.20700	-3.07400	3.17600	0.52960	0.90410	0.58750
	4	-0.25780	-0.20640	0.01001	0.18670	0.13910	0.00002
BEL	1	-3.97600	-3.98400	0.01000	0.06821	0.04437	0.00000
	4	-1.34700	-2.26900	3.92800	0.29560	0.91920	0.22680
BRA	1	-3.85100	-2.12200	0.19090	0.46980	1.65100	0.43740
	4	-1.66000	-3.38900	1.55100	0.69710	0.59620	0.67960
CAN	1	-0.52750	-1.31900	0.01001	0.35570	0.62880	0.00004
	4	-0.96800	-0.53040	0.01000	0.02522	0.66540	0.00001
CHL	1	-0.37440	-0.18530	0.01000	0.31670	0.07595	0.00001
	4	-3.61500	-3.07900	0.01127	0.20580	0.89000	0.00403
CHN	1	-3.28600	-3.34200	3.44200	0.23320	0.44920	0.25410
	4	-2.42400	-1.40500	0.01108	1.44300	0.11100	0.00341
CZE	1	-3.70700	-2.33400	3.82200	0.59280	0.71580	0.56340
	4	-1.43400	-0.55940	0.01001	0.02023	0.25500	0.00002
DNK	1	-3.01300	-3.20900	3.78800	1.09600	1.04100	0.16610
	4	-0.16790	-2.91900	0.01007	0.48100	0.63060	0.00022
EST	1	-3.93100	-2.05400	3.36500	0.21720	1.43700	0.51630
	4	-2.87200	-1.62400	0.01008	0.53220	0.43000	0.00024
FIN	1	-0.26570	-0.25830	0.01000	0.31720	0.29500	0.00001
	4	-1.07500	-1.06200	0.01000	0.36300	0.32130	0.00001
FRA	1	-3.33300	-0.00603	0.01004	0.07977	0.01808	0.00013
	4	-1.81300	-0.65340	3.62200	0.16850	0.04232	1.19700
GER	1	-0.19730	-0.31170	0.01002	0.36800	0.02482	0.00006
	4	-2.28000	-1.75800	0.01002	0.12680	0.00729	0.00006
GRC	1	-0.36450	-0.18400	0.01000	0.34660	0.07189	0.00001
	4	-1.71800	-2.58200	0.01013	0.13300	0.06914	0.00042
HUN	1	-0.48750	-0.01581	0.01001	0.48240	0.45600	0.00004
	4	-2.69300	-3.71200	0.01398	1.09800	0.10090	0.01258
ICE	1	-2.17100	-2.36300	0.01001	0.21680	0.62230	0.00002
	4	-2.84400	-2.30000	0.01165	0.39180	0.18980	0.00523
IND	1	-2.33200	-2.37100	0.01147	0.28950	0.59780	0.00464
	4	-0.28740	-0.18330	0.06580	0.40270	0.43240	0.83290
IDS	1	-2.43100	-2.05500	0.01004	0.40790	0.42670	0.00013
	4	-0.29820	-0.23990	0.07408	0.06895	0.25260	0.80680
IRE	1	-2.17200	-2.91800	3.93300	0.72160	0.62610	0.21260
	4	-3.92000	-0.75550	0.01004	0.75990	0.36520	0.00012
ISR	1	-0.40480	-0.27310	0.13420	0.26820	0.65430	0.61660
	4	-3.01100	-0.30580	0.44380	0.41470	0.55070	0.64700
ITA	1	-0.24040	-0.25090	0.01000	0.23890	0.20730	0.00001
	4	-2.15500	-2.66900	0.01000	0.26770	0.66830	0.00000
JAP	1	-0.18690	-0.20580	0.01002	0.08075	0.13730	0.00006
	4	-1.25800	-1.35400	0.01002	0.57610	0.73910	0.00005

country	fhor	α	β	σ	se(α)	se(β)	se(σ)
KOR	1	-1.76200	-1.87300	0.01002	0.00757	0.35590	0.00007
	4	-3.42600	-2.57600	0.01365	0.20910	0.45400	0.01154
LUX	1	-0.30510	-0.30920	0.01000	0.04471	0.03254	0.00001
	4	-0.25220	-0.26950	0.01002	0.27670	0.32860	0.00005
MEX	1	-3.97500	-3.88700	0.01000	0.07006	0.31960	0.00000
	4	-2.42700	-3.89200	0.01319	1.43100	0.34090	0.01009
NTL	1	-3.26600	-0.53010	3.15700	0.80230	0.66440	0.36100
	4	-0.32330	-0.21020	0.01001	0.00986	0.15050	0.00002
NOR	1	-0.23260	-0.22620	0.01000	0.26210	0.28140	0.00000
	4	-0.95590	-0.13660	0.01008	0.01293	0.57980	0.00027
POL	1	-1.93100	-2.15300	0.01073	0.03571	0.27500	0.00232
	4	-0.40660	-0.39230	0.12520	0.27400	0.22860	0.64510
PRT	1	-2.63100	-2.63800	0.01115	0.22330	0.24800	0.00364
	4	-0.28080	-1.05200	0.30100	0.38190	0.79660	0.08920
RUS	1	-1.66000	-1.33500	0.01001	0.31760	0.68030	0.00003
	4	-1.95200	-1.96400	0.01182	0.40570	0.36770	0.00576
SVK	1	-1.56300	-3.02500	3.98000	1.63500	1.47200	0.44230
	4	-1.97900	-1.75400	0.01000	0.18580	0.02054	0.00002
SLV	1	-0.38790	-0.12440	0.05135	0.29120	0.11240	0.87860
	4	-0.64600	-0.22750	0.22640	0.48690	0.21360	0.32500
SAF	1	-0.30570	-0.21160	0.01002	0.04292	0.15490	0.00007
	4	-1.67400	-1.49700	0.01034	0.27340	0.18160	0.00106
SPA	1	-0.10670	-0.08300	0.01002	0.32000	0.23100	0.00007
	4	-0.30420	-0.24920	0.03749	0.04981	0.28200	0.92250
SWD	1	-0.27470	-0.24600	0.01000	0.13590	0.22190	0.00001
	4	-1.08600	-1.27600	0.01000	0.39750	0.51710	0.00000
SWZ	1	-0.12930	-1.20100	3.60400	0.60310	0.75600	0.26170
	4	-0.46210	-0.27500	0.01000	0.42640	0.34500	0.00001
TUR	1	-2.79100	-3.27700	0.01295	0.78850	0.76760	0.00933
	4	-0.29660	-0.34900	0.28330	0.43210	0.09166	0.14500
UK	1	-0.23170	-0.16170	0.01000	0.21520	0.00523	0.00001
	4	-1.86800	-1.22200	0.01000	0.34010	0.68890	0.00001
US	1	-3.44500	-2.28300	3.65900	0.23590	0.37090	0.93970
	4	-0.23430	-0.29630	0.01003	0.25710	0.55120	0.00010

Country symbols

AUT	Austria	FRA	France	JAP	Japan	SLV	Slovenia
BEL	Belgium	GER	Germany	KOR	Korea	SAF	South Africa
BRA	Brazil	GRC	Greece	LUX	Luxembourg	SPA	Spain
CAN	Canada	HUN	Hungary	MEX	Mexico	SWD	Sweden
CHL	Chile	ICE	Iceland	NTL	Netherlands	SWZ	Switzerland
CHN	China	IND	India	NOR	Norway	TUR	Turkey
CZE	Czech Rep	IDS	Indonesia	POL	Poland	UK	United Kingd
DNK	Denmark	IRE	Ireland	PRT	Portugal	US	United States
EST	Estonia	ISR	Israel	RUS	Russia		
FIN	Finland	ITA	Italy	SVK	Slovak Rep		

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